L'Ansatz cellulaire 2

séminaire algo

INRIA

2 Mars 2009

xgv

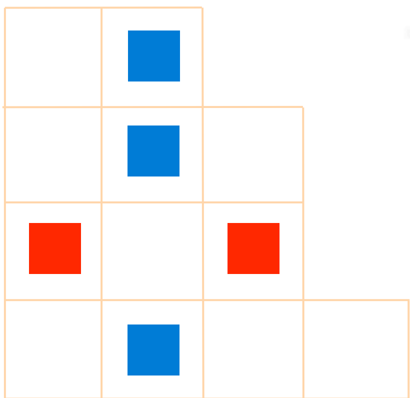
LaBRI, CNRS

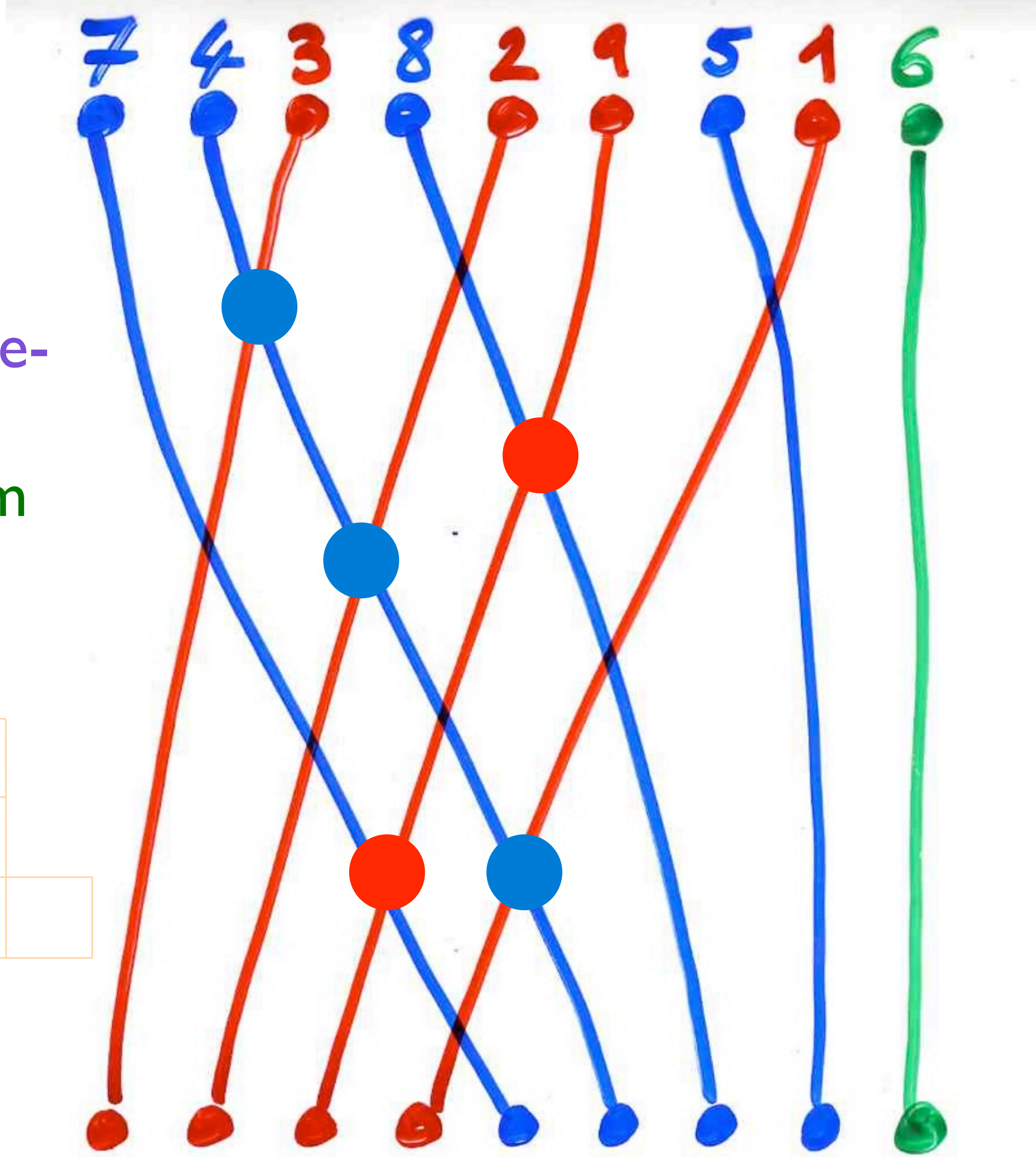
Bordeaux

§ 1

Up-down and Genocchi sequences

"exchange-fusion" algorithm





(789) (4321) 5 6

Def. Genocchi sequence of a permutation

$$\sigma = \sigma(1) \quad \sigma(n)$$

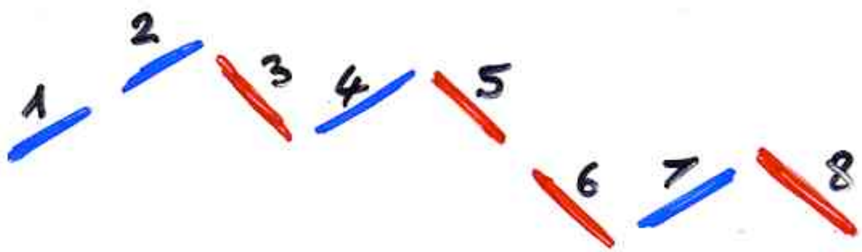
$$G(\sigma) = z_1 \ldots z_{n-1}$$

$$z_x = \begin{cases} a & \text{(ascent)} \\ d & \text{(descent)} \end{cases} \qquad x = \sigma(i) \begin{matrix} < \sigma(i+1) \\ > \sigma(i+1) \end{matrix}$$

$1 \leqslant x \leqslant n-1$ "value" "index"

<u>convention</u> : $\sigma(n+1) = 0$ ($\sigma(n)$ is a descent)

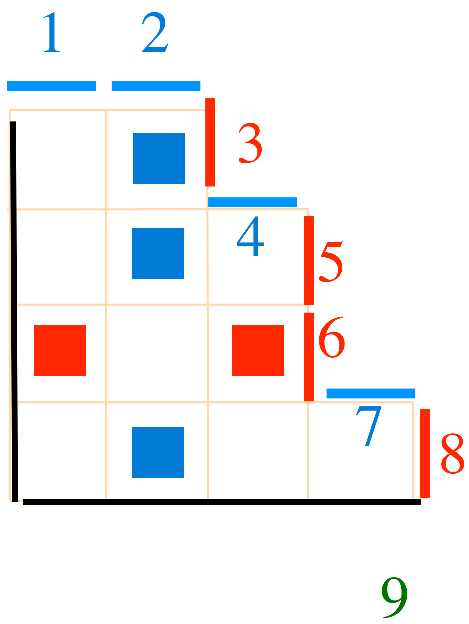
<u>ex</u> : $\sigma = (8\ 5\ 3\ 2\ 7\ 9\ 1\ 4\ 6)$



σ permutation

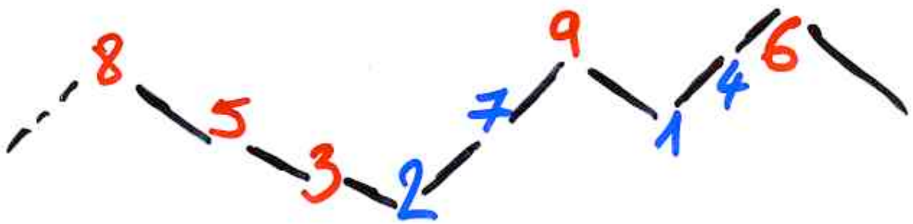
(valeur) x $\begin{cases} \text{avance} \\ \text{recul} \end{cases}$ ssi (indice) x $\begin{cases} \text{montée} & \sigma^{-1}(x) < \sigma^{-1}(x+1) \\ \text{descente} & \sigma^{-1}(x) > \sigma^{-1}(x+1) \end{cases}$

convention : $\sigma(n)$ descente



$$\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 7 & 4 & 3 & 8 & 2 & 9 & 5 & 1 & 6 \end{pmatrix}$$

$$\sigma^{-1} = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 8 & 5 & 3 & 2 & 7 & 9 & 1 & 4 & 6 \end{pmatrix}$$



“Genocchi shape” of a permutation

alternating sequence d a d a d ... a d a

Prop. (Dumont, 1974)
The nb of permutations on $\{1, 2, .., 2n\}$ having an alternating Genocchi sequence is the Genocchi numbers G_{2n+2}

nombres de Genocchi

$$G_{2n} = 2(2^{2n} - 1) B_{2n}$$

Bernoulli

$$2^{2n} G_{2n+2} = (n+1) T_{2n+1}$$

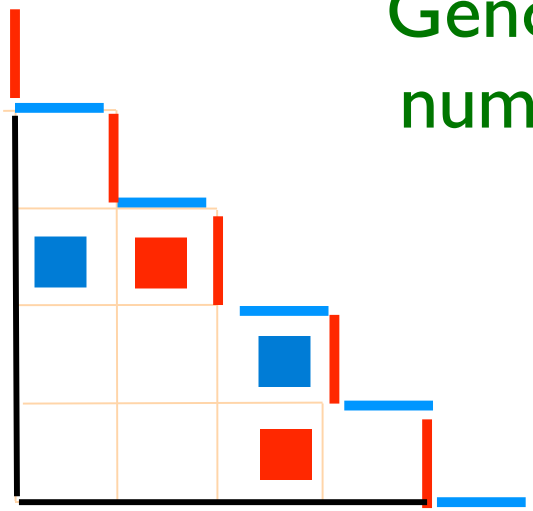
nombres de Genocchi

$$G_{2n} = 2(2^{2n} - 1)\, B_{2n}$$

Bernoulli

$$2^{2n} G_{2n+2} = (n+1)\, T_{2n+1}$$

Genocchi numbers



alternating shape

Angelo Genocchi
1817 - 1889

$$\mathrm{tg}(t) = \sum_{n\geq 0} T_{2n+1} \frac{t^{2n+1}}{(2n+1)!}$$

$$\frac{1}{\cos(t)} = \sum_{n\geq 0} E_{2n} \frac{t^{2n}}{(2n)!}$$

E_{2n} $\{1, 5, 61, 1385, \ldots\}$

nombres sécant (d'Euler)

T_{2n+1} $\{1, 2, 16, 272, 7936, \ldots\}$

nombres tangents

Permutations alternantes

D. André (1880)

$$\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 6 & 2 & 9 & 7 & 8 & 4 & 5 & 1 & 3 \end{pmatrix}$$

Hinc igitur calculo inſtituto reperietur:

$$A = 1$$
$$B = 1$$
$$C = 3$$
$$D = 17$$
$$E = 155 = 5.31$$
$$F = 2073 = 691.3$$
$$G = 38227 = 7.5461 = 7.\frac{127.129}{3}.$$
$$H = 929569 = 3617.257$$
$$I = 28820619 = 43867.9.73 \quad \&c.$$


EULER

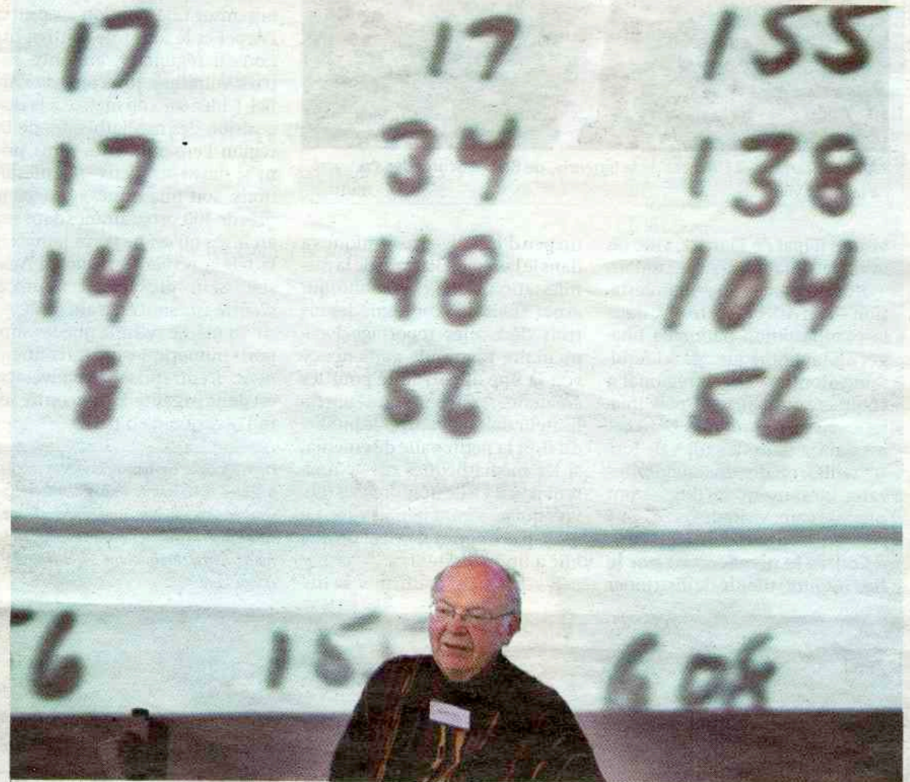
BORDEAUX 1. Le professeur Donald Knuth consacre sa vie à la programmation informatique, considérée comme un art. Il vient d'être sacré docteur honoris causa à Bordeaux

L'ermite de l'informatique

Bernard Broustet

Une sommité de l'informatique mondiale a séjourné en Gironde ces derniers jours. Donald Knuth, 69 ans, a été sacré mardi docteur honoris causa de l'université Bordeaux 1, après avoir été lundi au centre d'une journée d'échanges qui réunissait une bonne partie du gratin français et européen de la recherche en informatique (1).

Depuis son premier contact, il y a un demi-siècle, avec un monumental et dinosaurien IBM 650, Donald Knuth n'a cessé d'être habité par la passion de l'informatique. Physicien, puis mathématicien de formation, ce géant affable et modeste a voué sa vie à ce qu'il appelle « l'art de la programmation informatique ». Car, à ses yeux, plus qu'une technique, c'est une forme d'activité qui requiert à la fois rigueur, intuition et sens esthétique. Les programmes informatiques réussis ont une sorte de beauté à laquelle même les non-spécialistes peuvent être sensibles.



Donald Knuth, à Bordeaux, le 29 octobre. À 69 ans, il animait une journée d'échanges avec le gratin européen de la recherche en informatique PHOTO LAURENT THEILLET

Une encyclopédie. Au long de sa carrière académique (pour l'essentiel à l'université californienne de Stanford), Donald Knuth a fait preuve d'une grande fécondité, en jouant notamment un rôle essentiel dans le développement de langages toujours utilisés par la communauté des mathématiciens. Mais, à 55 ans, le professeur Knuth a décidé de prendre sa retraite de Stanford. Il trouve que les fonctions administratives sont trop absorbantes pour lui permettre de mener à bien l'œuvre entamée à la fin des années 60 sous le titre de « Art of computer programming », sorte d'encyclopédie de l'algorithmique et de la programmation informatique. Donald Knuth a publié, il y a quelque temps déjà, les trois premiers volumes de cette gigantesque somme, traduite en russe, en japonais, en polonais, etc. mais pas en français. Le quatrième tome est pour bientôt. Et Donald Knuth se dit décidé à poursuivre sa tâche tant qu'il en aura la force. Ses ouvrages, dont les ventes cumulées au fil des ans approchent le million d'exemplaires, visent essentiellement les informaticiens et créateurs de programmes. Une communauté certes minoritaire à travers le monde, mais qui se trouve investie d'une mission considérable. En quelques décennies, l'écriture informatique a aidé à résoudre d'innombrables problèmes. « Mais il y en a tant d'autres qui attendent des solutions, notamment dans le domaine médical », affirme le professeur émérite de Stanford.

Un chèque de 2,56 dollars. Pour mener à bien sa tâche, Donald Knuth s'est imposé une vie d'ermite. D'ordinaire, sa journée débute par la bibliothèque ou la piscine. Après quoi, il passe tout le reste de son temps à sa table de travail, dimanche compris. Il n'a plus d'e-mail depuis le début des années 90, considérant que le courrier électronique représente une perte de temps, dès lors qu'on veut aller au fond des choses et non pas rester à leur surface. Une secrétaire lui fait passer les messages considérés comme les plus urgents. Pour le reste, Donald Knuth demande qu'on lui écrive par courrier ordinaire ou par fax, dont il prend parfois connaissance avec des mois de retard. Il s'oblige, en revanche, à tenir aussi scrupuleusement que possible sa promesse d'envoyer un chèque de 2,56 dollars à tout lecteur ayant détecté une erreur dans un de ses livres. Par ailleurs, pour se détendre, il pratique l'orgue, appris dans sa prime jeunesse auprès de son père qui partagea sa vie entre la musique et l'enseignement.

L'orgue de Sainte-Croix. Donald Knuth n'est pas fermé aux choses de ce monde. Sur son site Internet, à la rubrique « Questions qui ne me sont pas fréquemment posées », il demande entre autres : « Pourquoi mon pays a-t-il le droit d'occuper l'Irak ? », « Pourquoi mon pays ne soutient-il pas une Cour internationale de justice ? » Mais cet homme de conscience ne se veut pas militant, pas plus qu'il n'aspire au vedettariat et à la richesse. « Beaucoup de gens, dit-il, ont tendance à considérer que l'informatique, c'est surtout des histoires de business, d'entreprise. Ce n'est pas mon cas. » Sortant de sa semi-réclusion, Donald Knuth s'est donc laissé convaincre d'accepter les hommages de l'université de Bordeaux, après celles de Harvard, d'Oxford, de Tübingen. Il a eu le coup de foudre pour la beauté et l'agrément de la ville. Et il n'oubliera sans doute pas de sitôt l'orgue illustre de l'église Sainte-Croix (2), sur lequel il a eu le bonheur d'exercer son talent.

(1) Ces journées étaient organisées par le Laboratoire bordelais de recherche en informatique (Labri).

(2) Thierry Semenoux, professeur d'orgue au conservatoire de Bordeaux, a joué dans ce domaine un rôle de cicérone auprès de Donald Knuth.

§2 Some Parameters

The maximum letter of the blocks of letters reaching the ground level are:

- for the columns of T (red threads), the left-to-right maximum elements of the values of the permutation s less than the last letter s(n+1),
- for the rows of T (blue threads), the right-to-left maximum elements of the values of the permutation s bigger than the last letter

(3 proofs comming 3 different methodologies: by P. Nadeau , O.Bernardi and xgv)

This gives an interpretation of the two parameters on alternative tableaux:

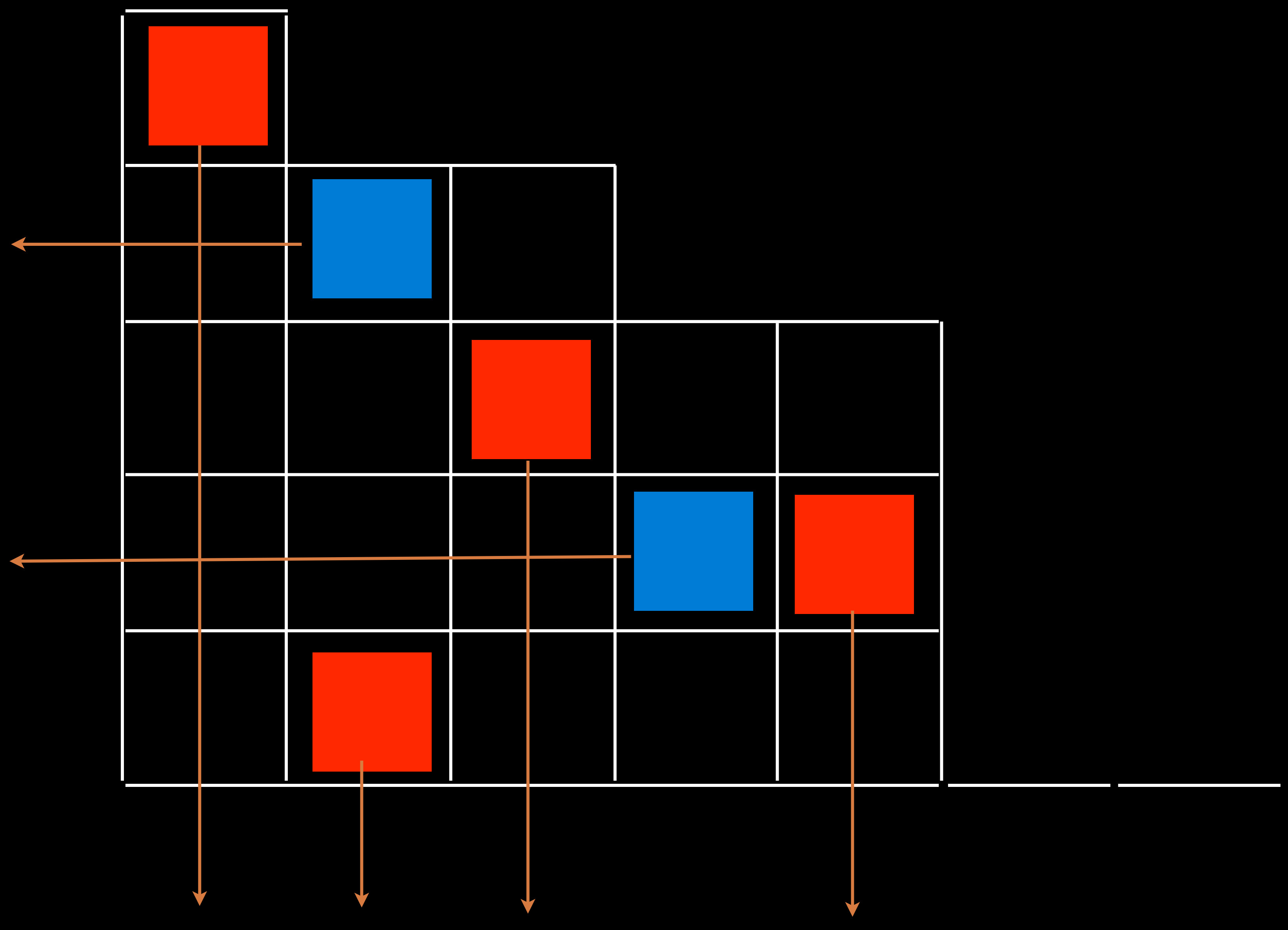
- number of “open” columns (i.e. columns without a red cell)
- number of “open” rows (i.e. rows without a blue cell)

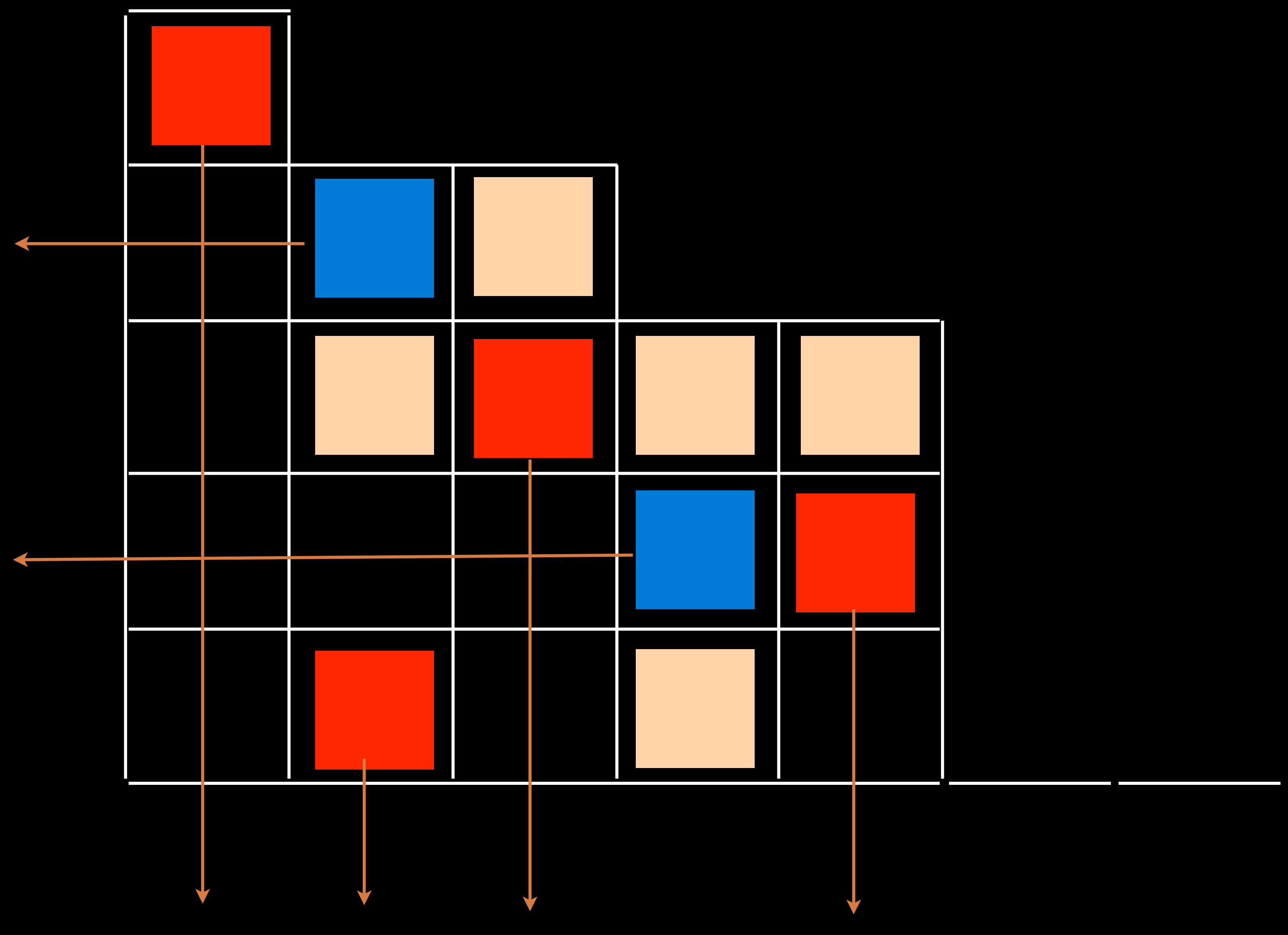
Def- Crossing of an alternative tableau:
a non-colored cell which is
neither at the left of a blue cell
neither below a red cell

crossing of alternating tableau

↕

exchange in the "exchange-fusion"
algorithm





From work of Corteel, Nadeau, Steingrimsson, Williams

we know that parameter

"number of crossing" in alternating tableaux :

same distribution as

"q-analog of Laguerre histories"

The number of crossings of the alternative tableau has been be characterized by O.Bernardi on the corresponding permutation s.

It is the number of pairs (x,y), $x=s(i)$, $y=s(j)$, $1\leq i<j\leq n+1$, such that there exist two integers k, $l\geq 0$ such that: the set of the values $x+1, x+2, ..., x+k$, $y+1, .., y+l$ are located between x and y (in the word s), and $x+k+1$ is located (in s) at the right of y and $y+l+1$ is located (in s) at the left of x (with the convention of $n+2$ at the left of all the values).

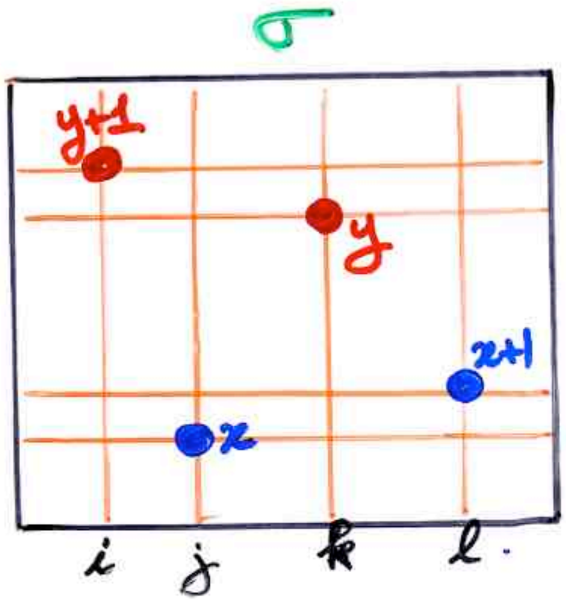
Permutations

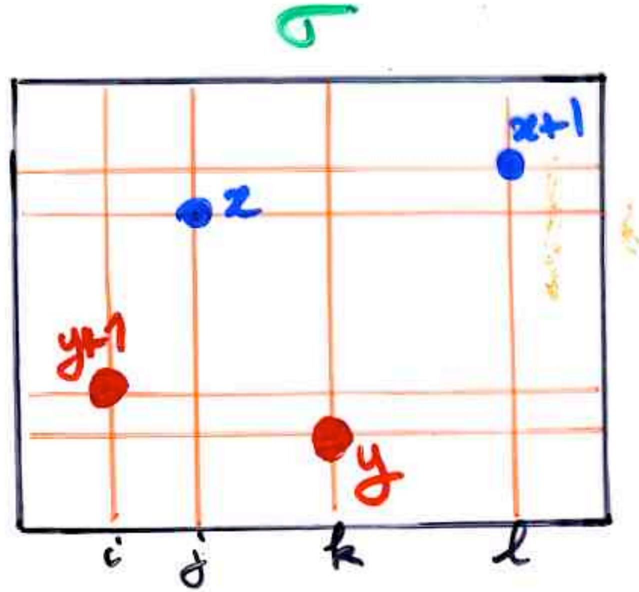
with no subsequence of the type

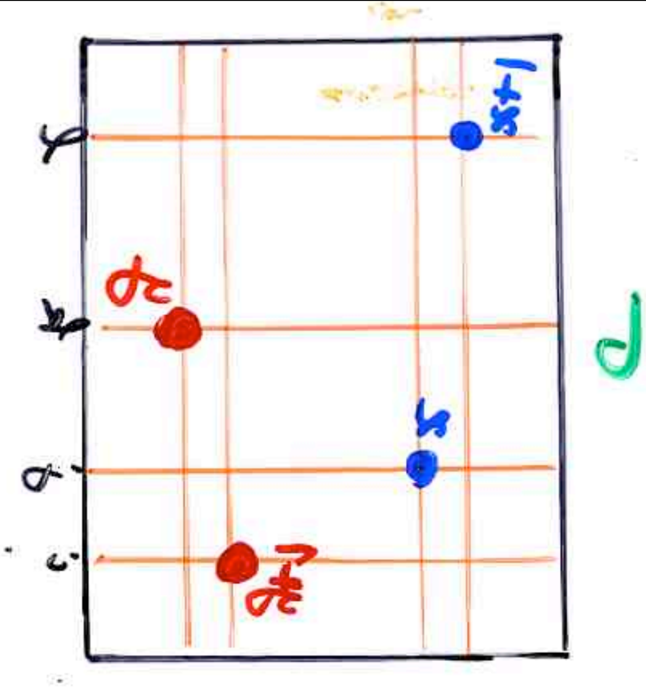
$$\cdots (y+1) \cdots x \cdots y \cdots (x+1) \cdots$$

ex: $\sigma = 6\ 4\ 5\ 3\ 9\ 7\ 8\ (10)\ 1\ 2$

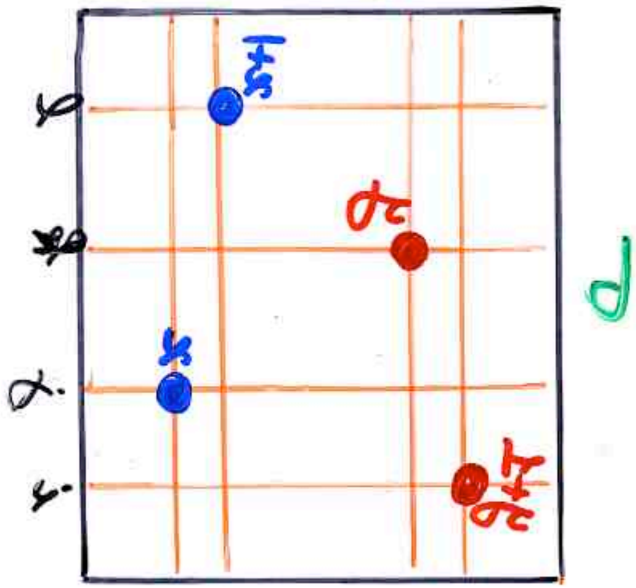
Prop. (O. Bernardi, 2008)
The number of such permutations on n elements is C_n Catalan number


σ
y+1
y
x+1
x
i
j
k
l


σ
x+1
x
y+1
y
i
j
k
l

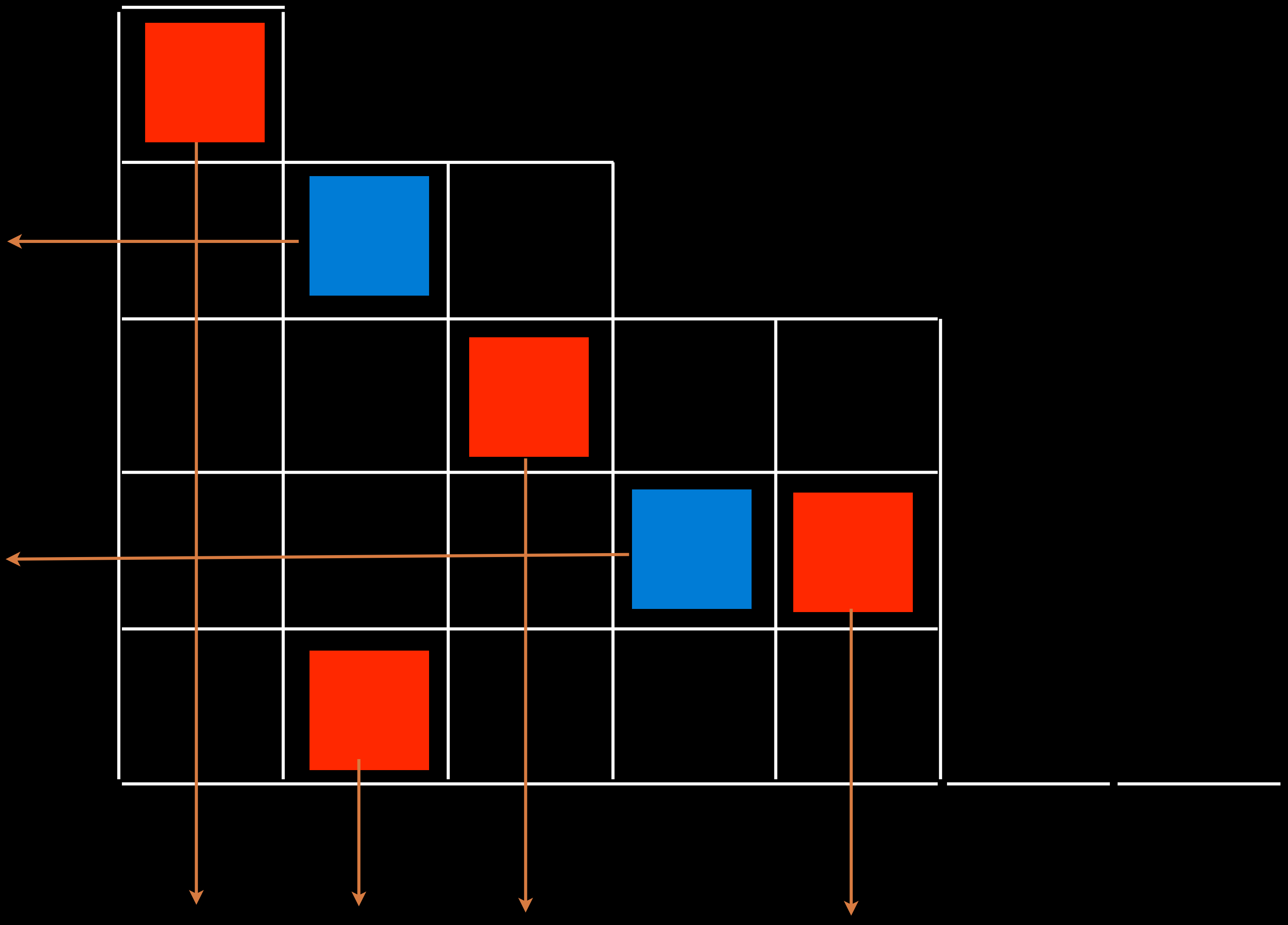


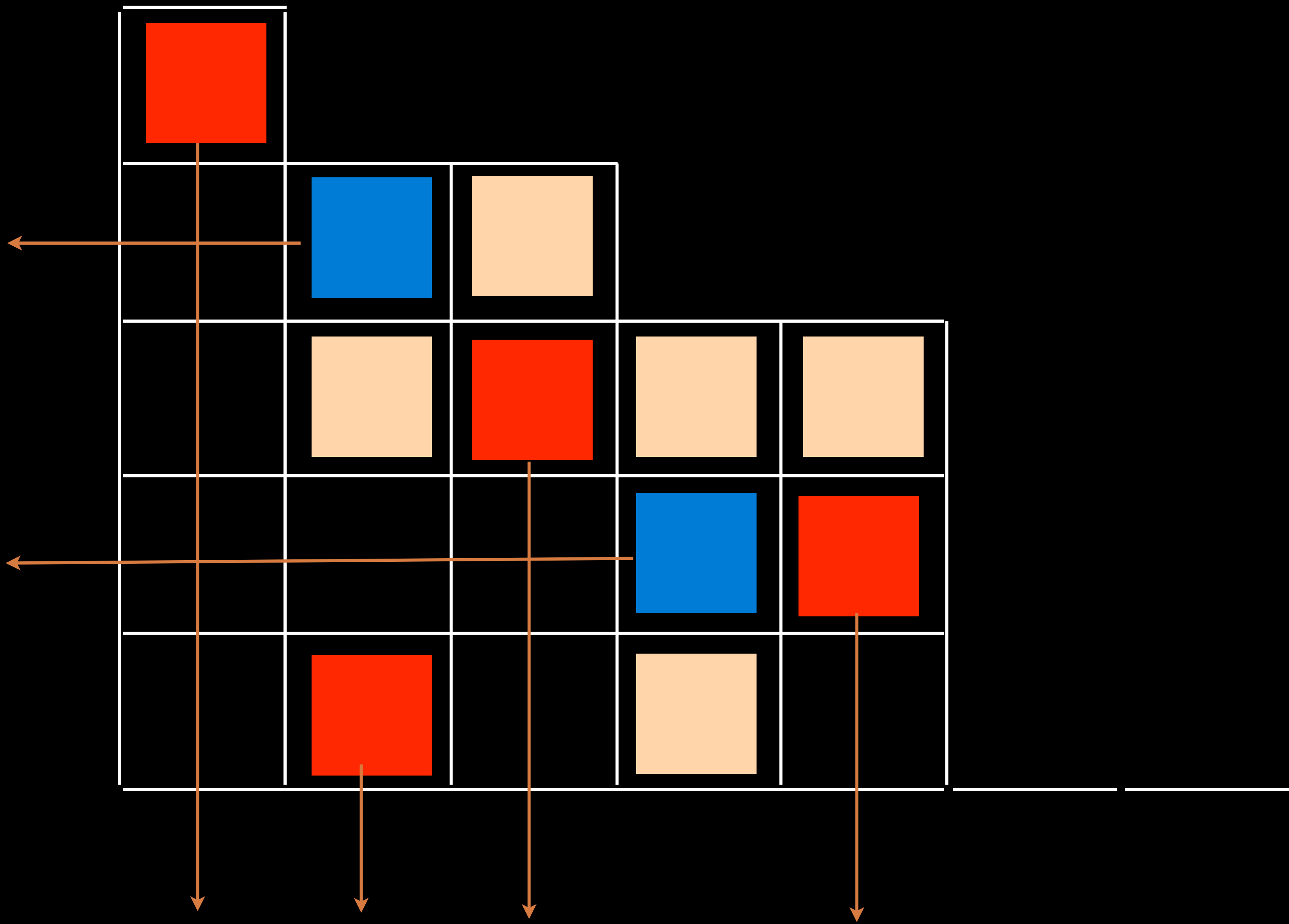
31- 24



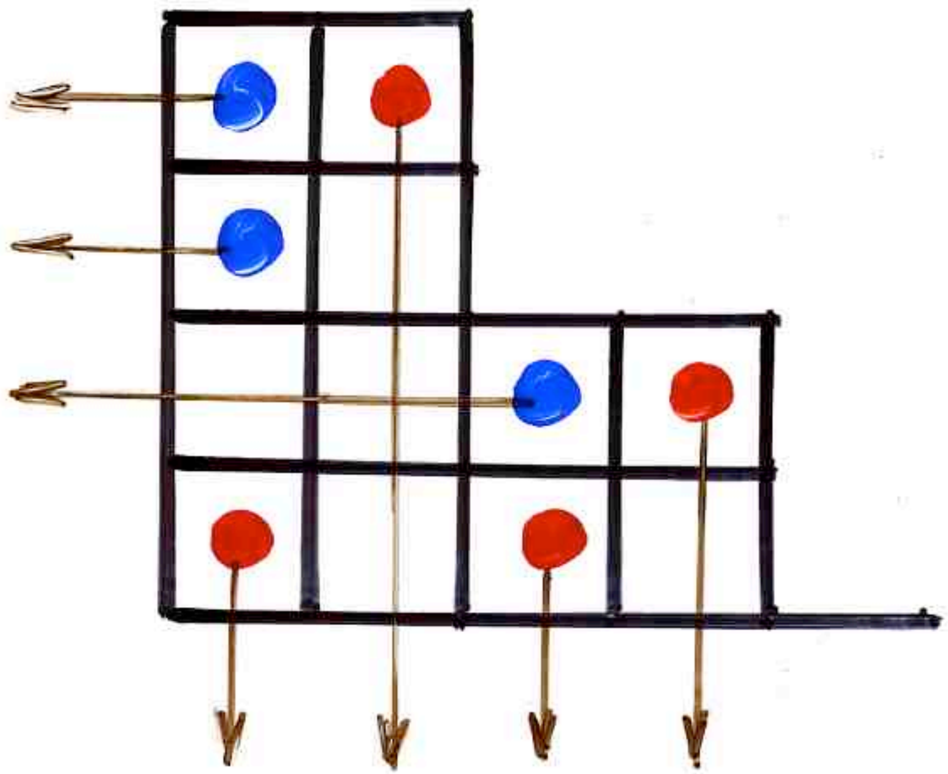
24 - 31

§3 tableaux alternatifs de Catalan

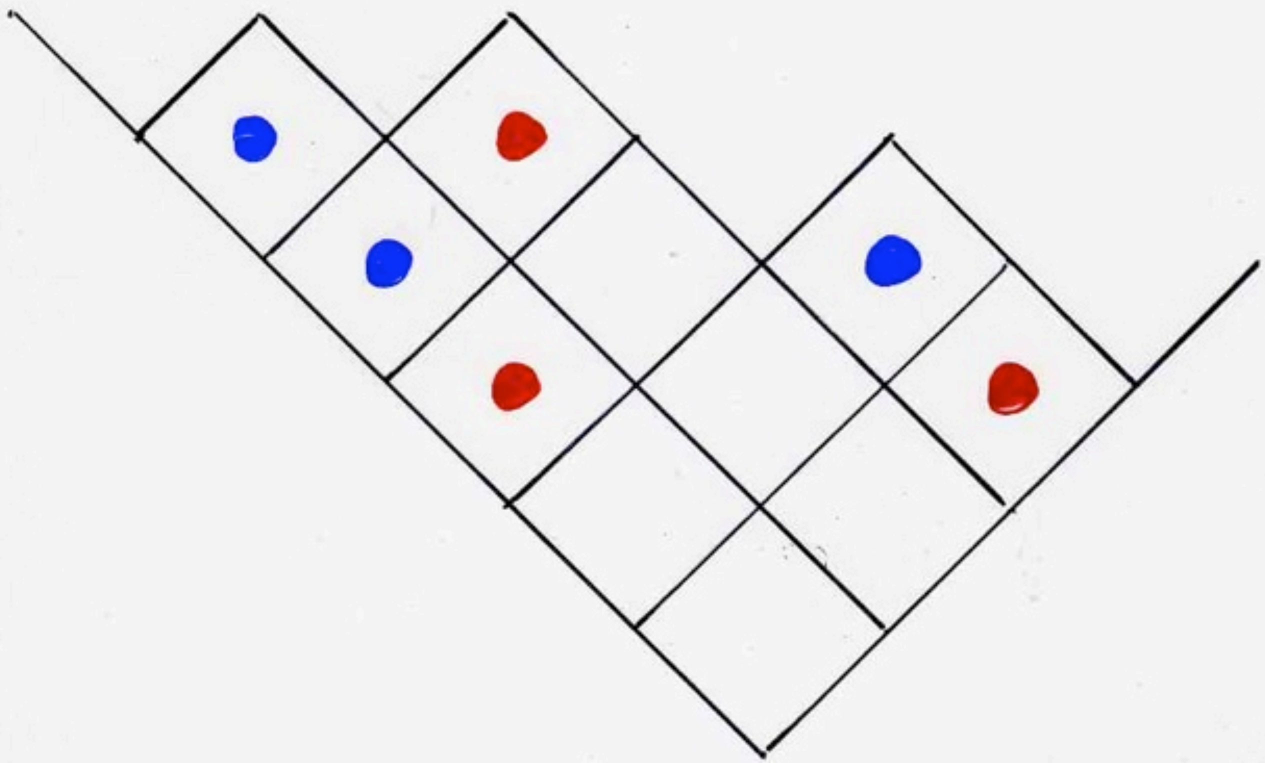


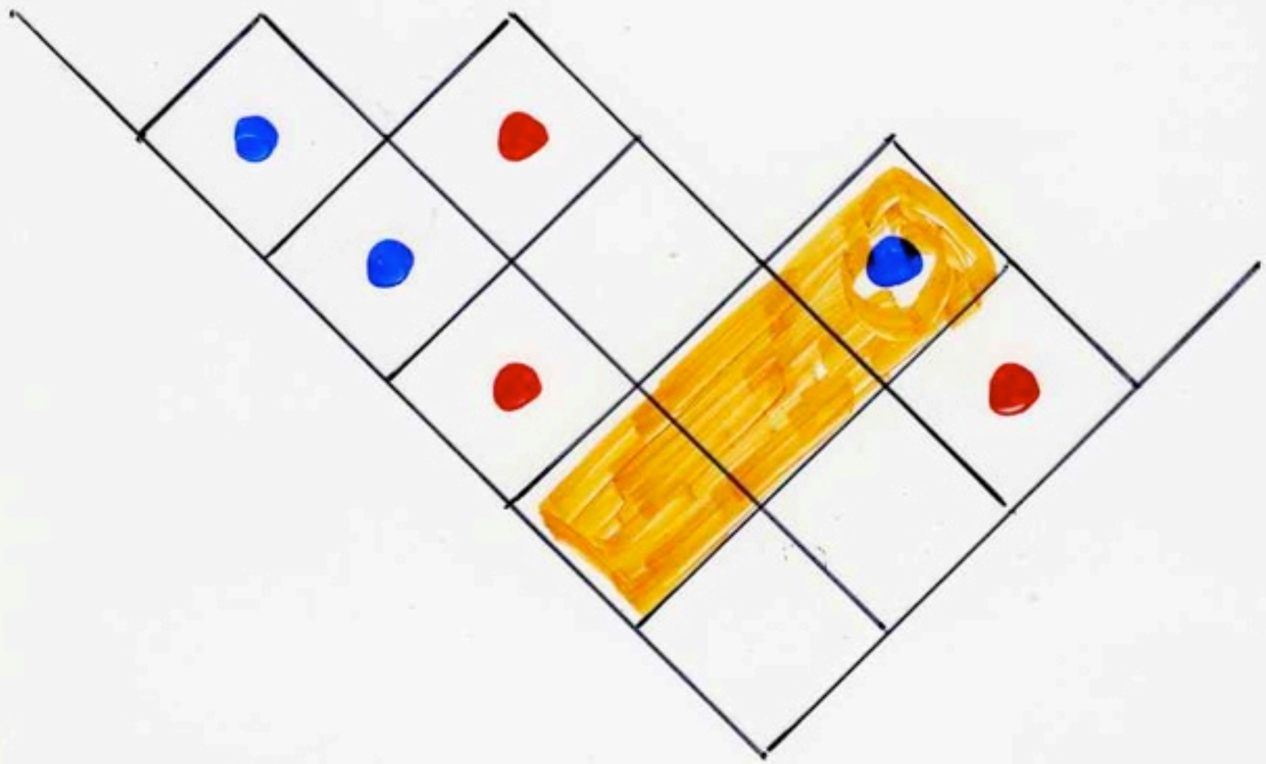


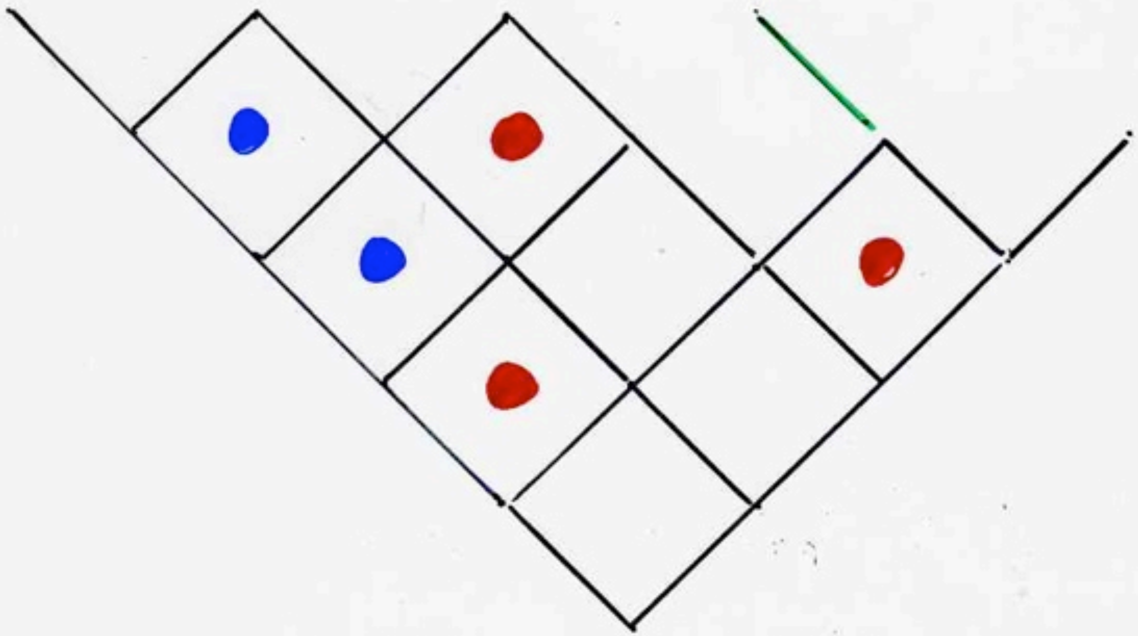
Def Catalan alternative tableau T
alt. tab. without cells ☒
i.e: every empty cell is below a red cell or on the left of a blue cell

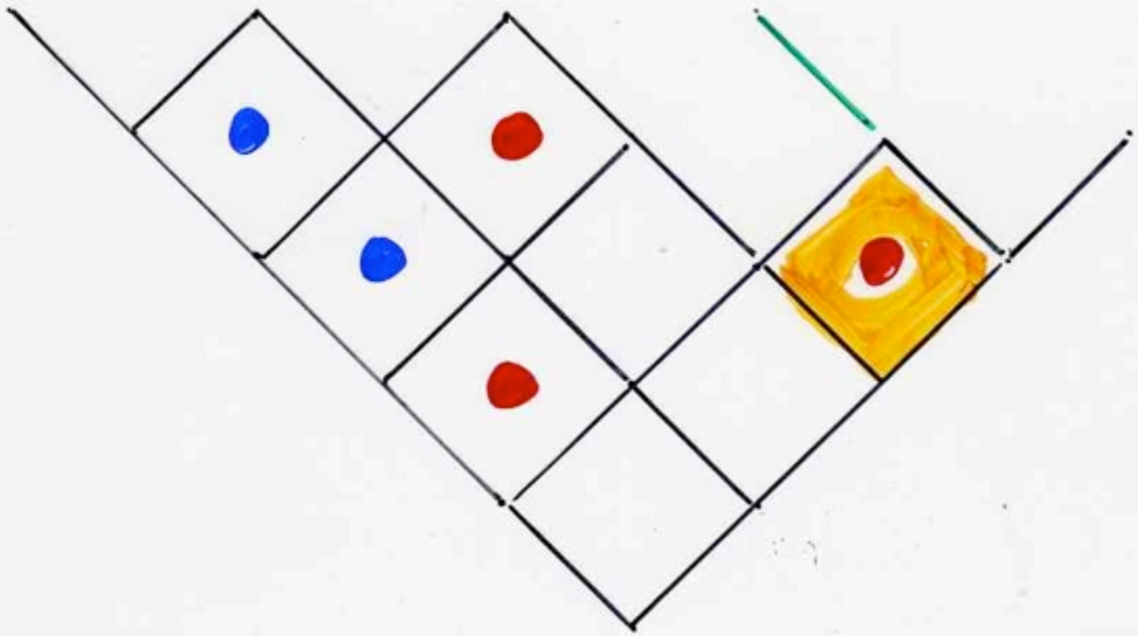


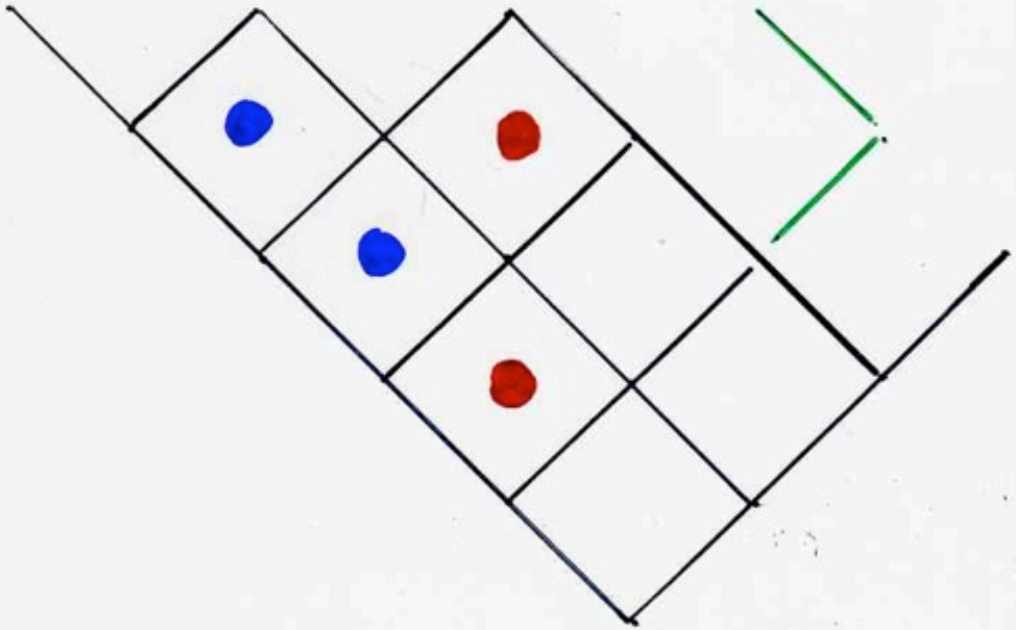
Bijection
tableaux alternatifs de Catalan
arbres binaires

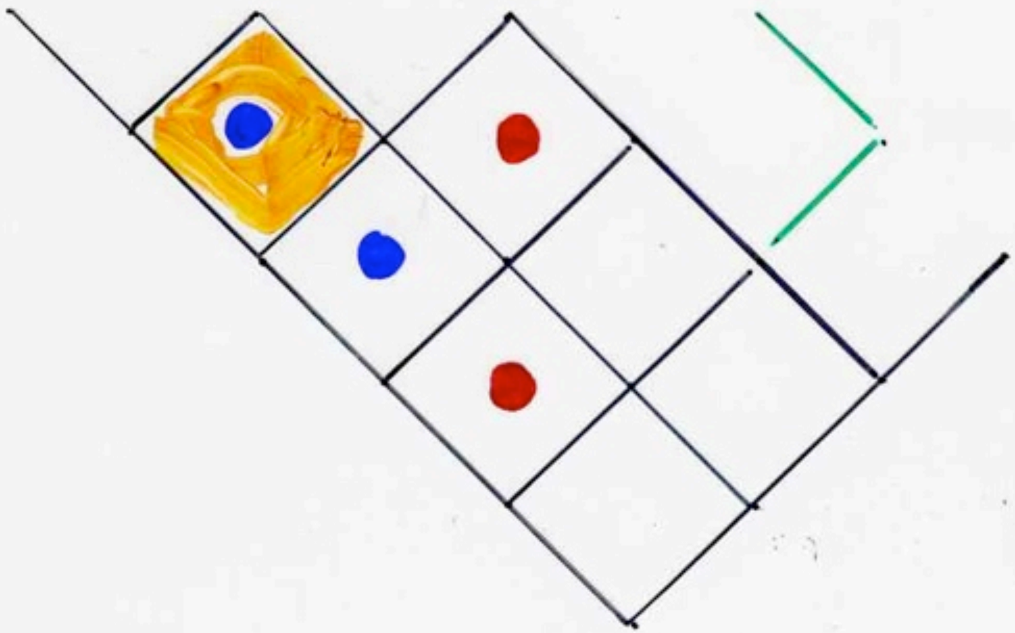


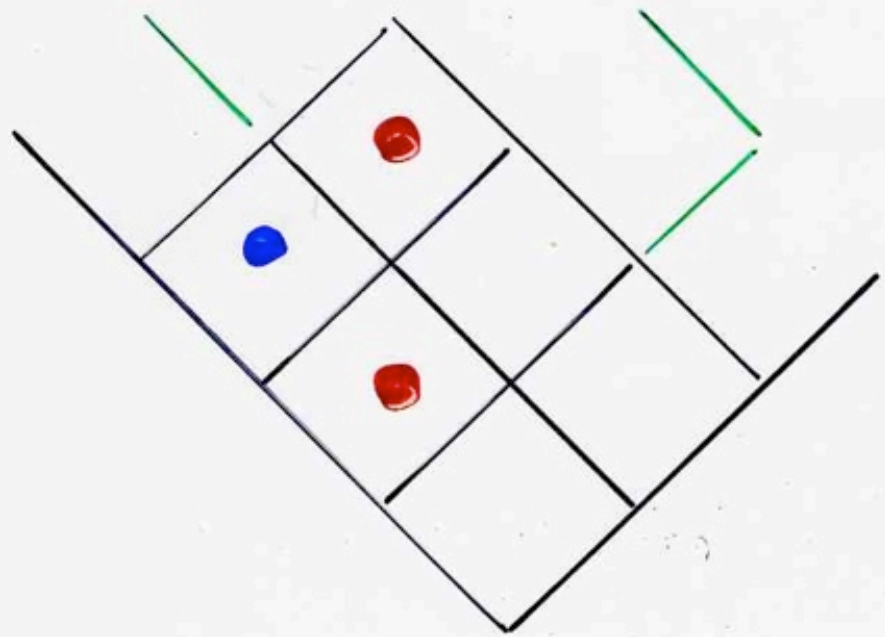


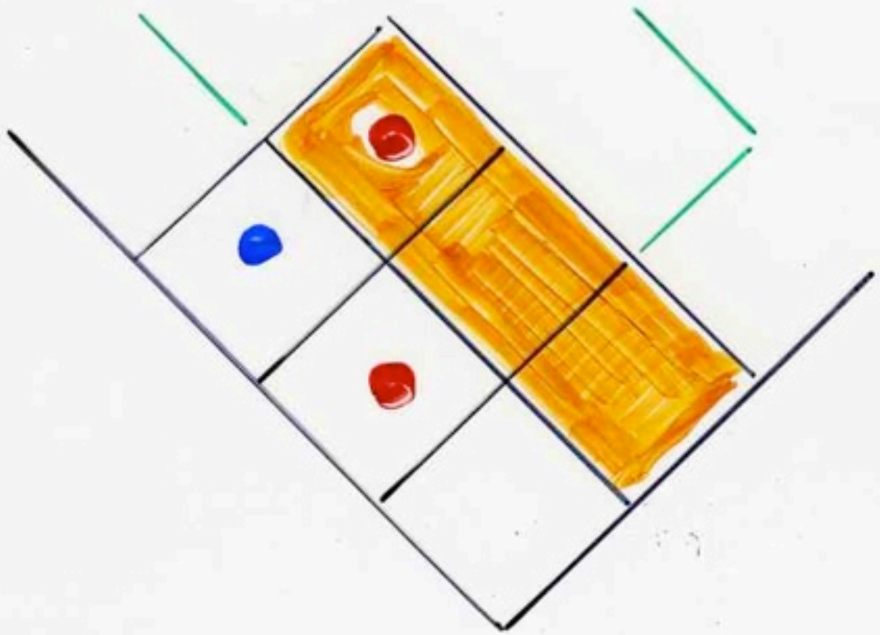


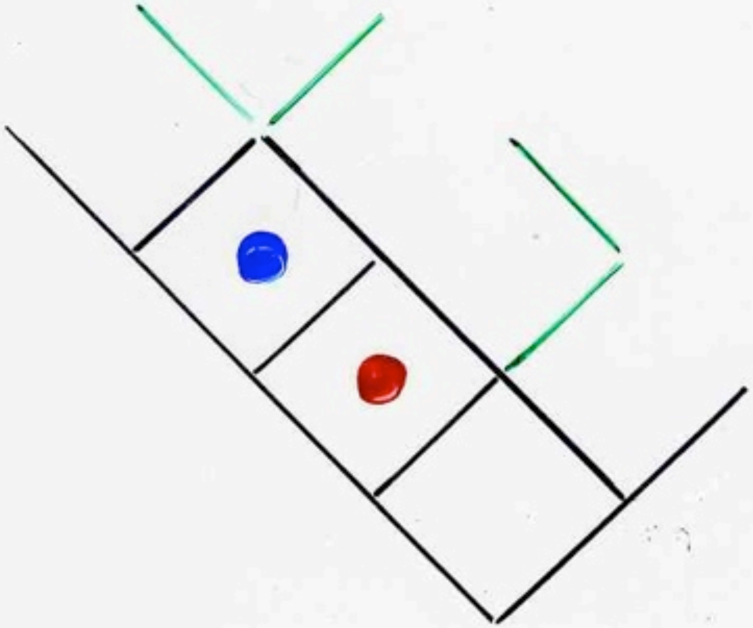


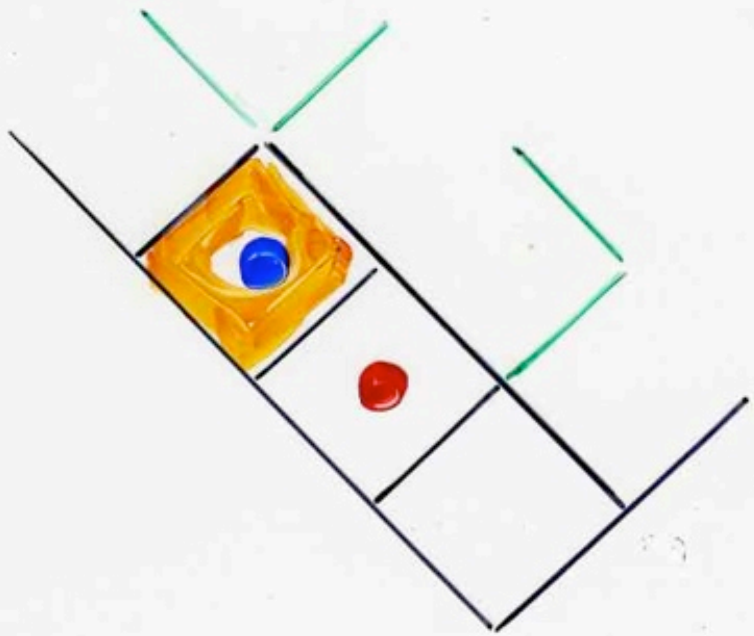


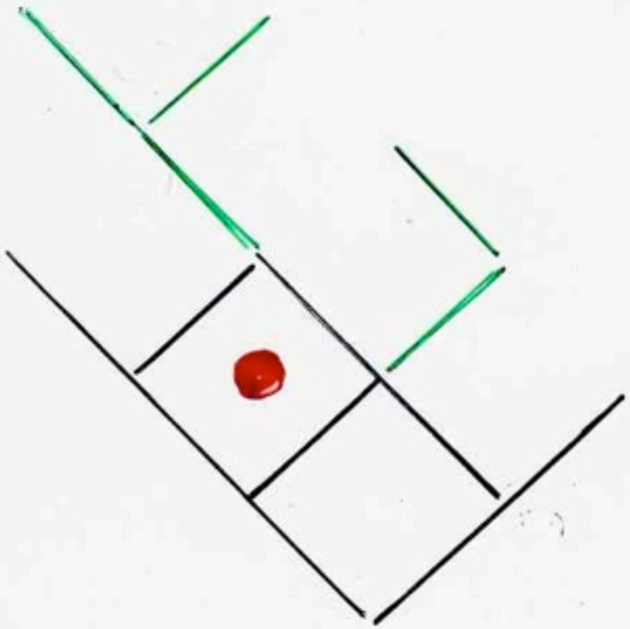


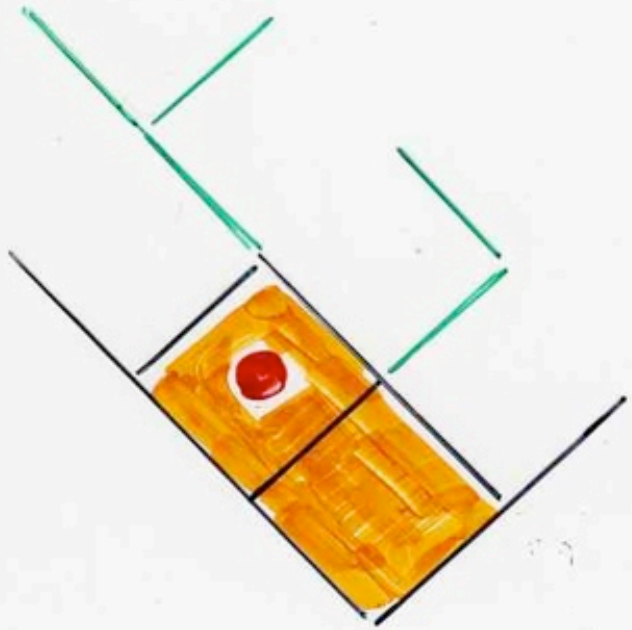


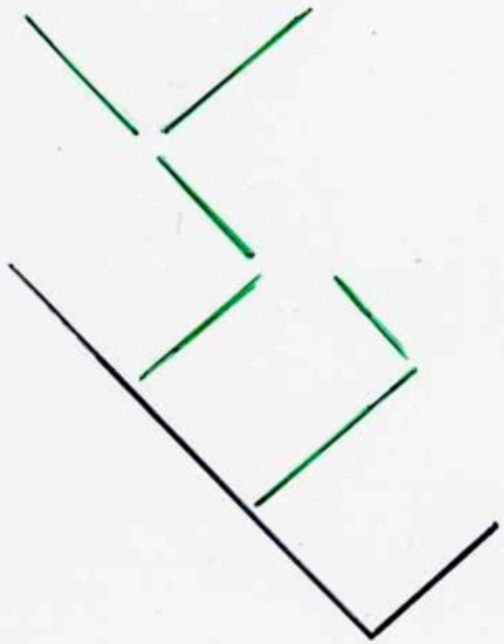






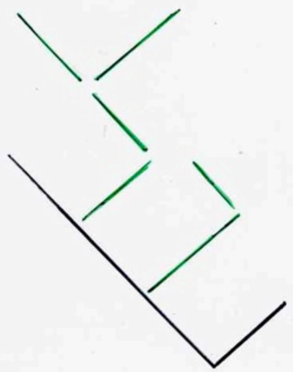


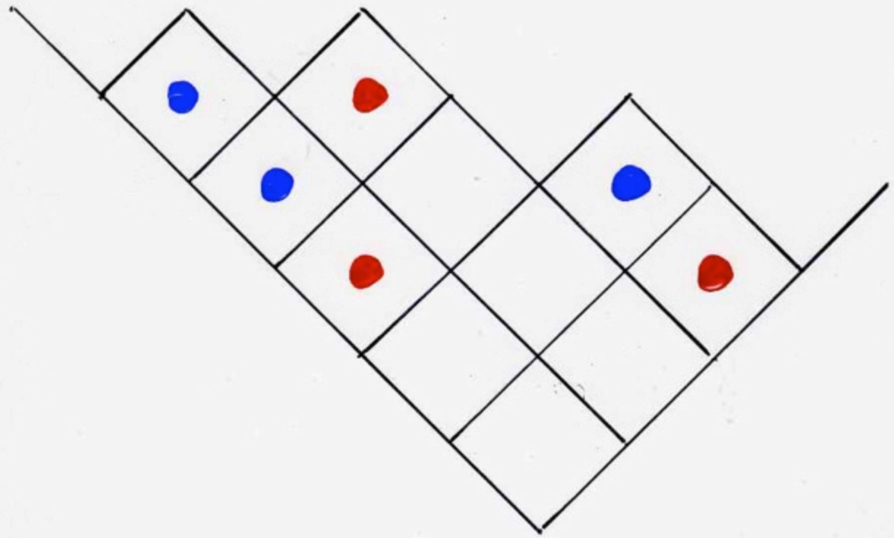




Bijection

tableaux alternatifs de Catalan taille n ⟷ arbres binaires n arêtes

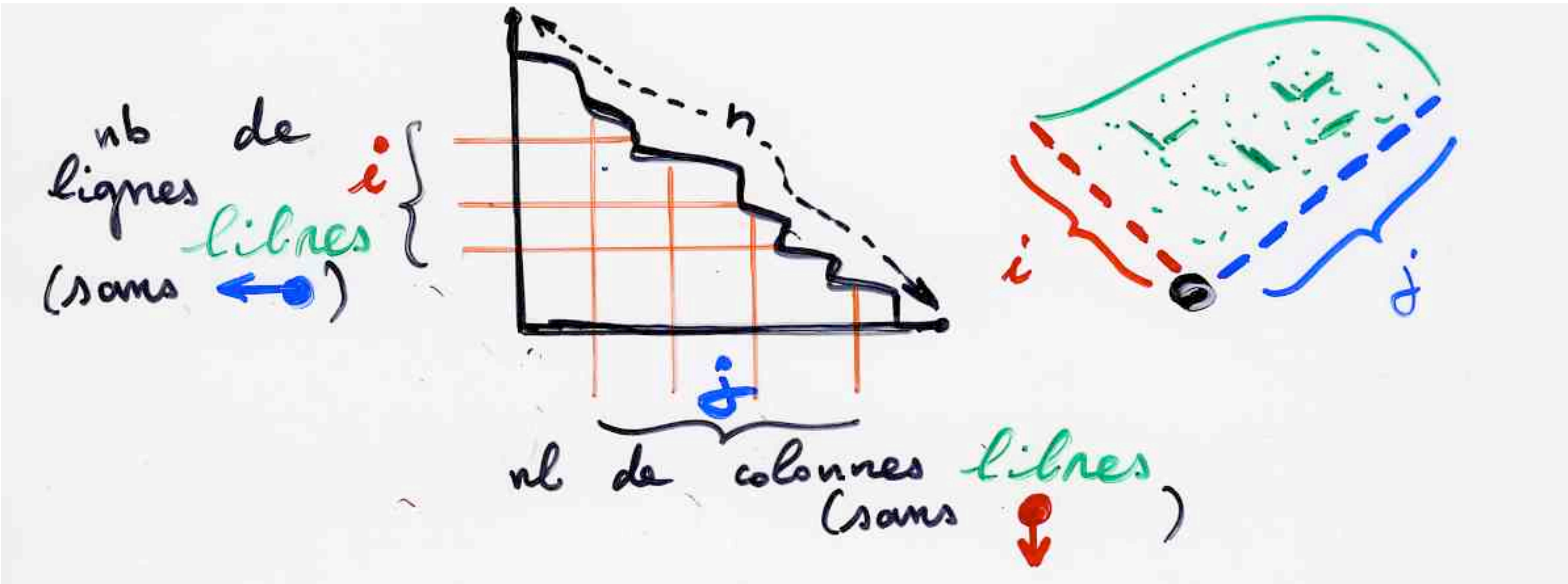




profil (bord) du diagramme de Ferrers ⟵ - - ⟶ canopée

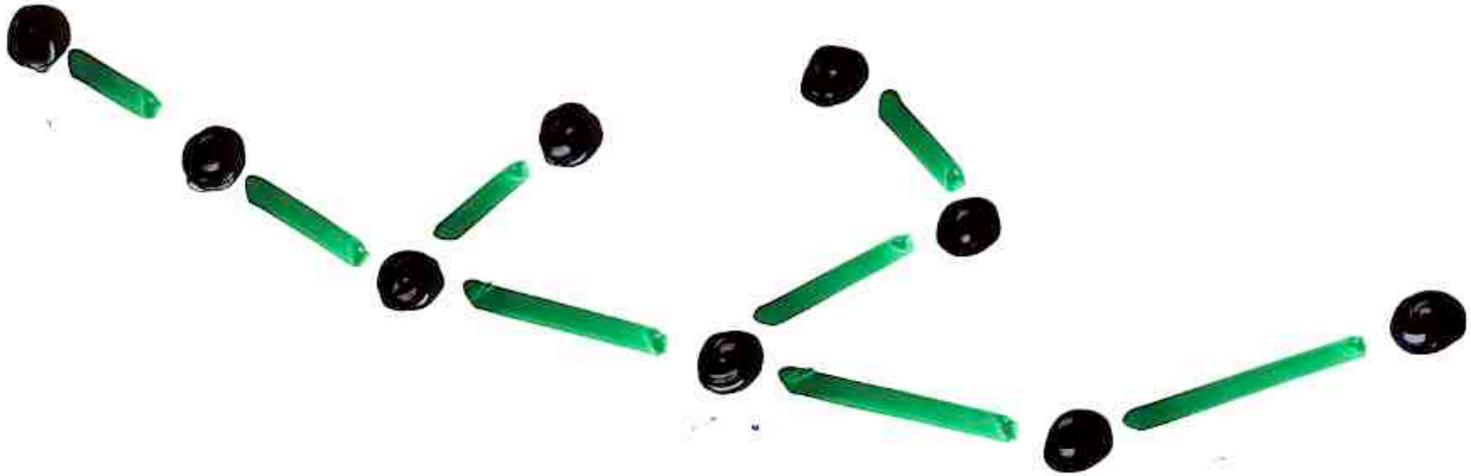
Bijection tableaux alternatifs de Catalan taille n ⟷ arbres binaires n arêtes

profil (bord) du diagramme de Ferrers ⟵---⟶ canopée

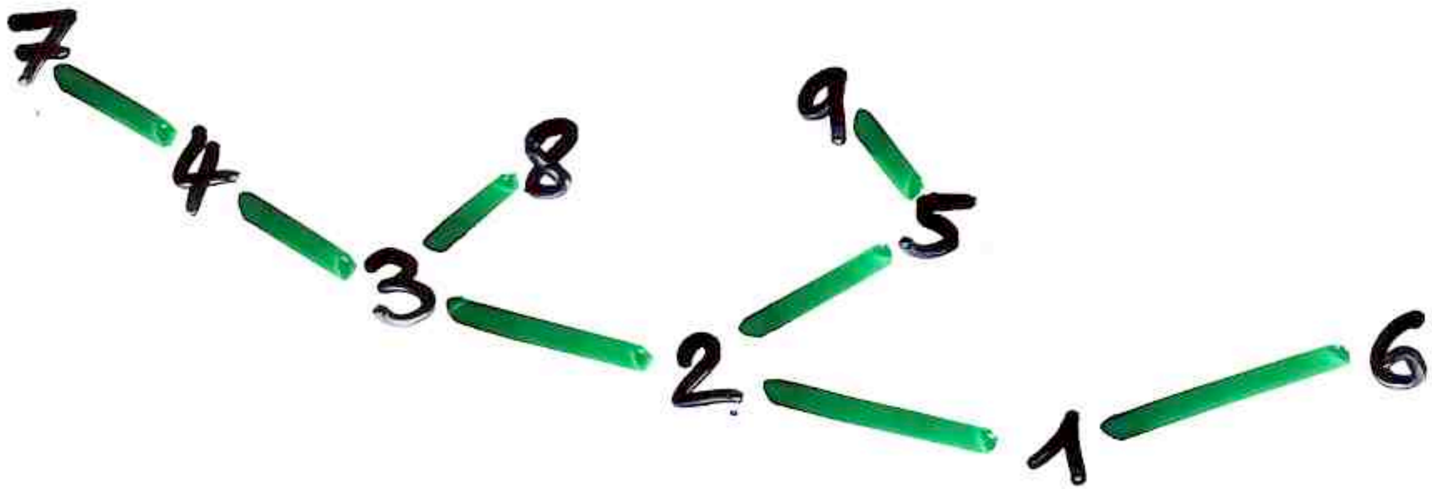


§4 increasing binary trees

Def- Increasing binary tree

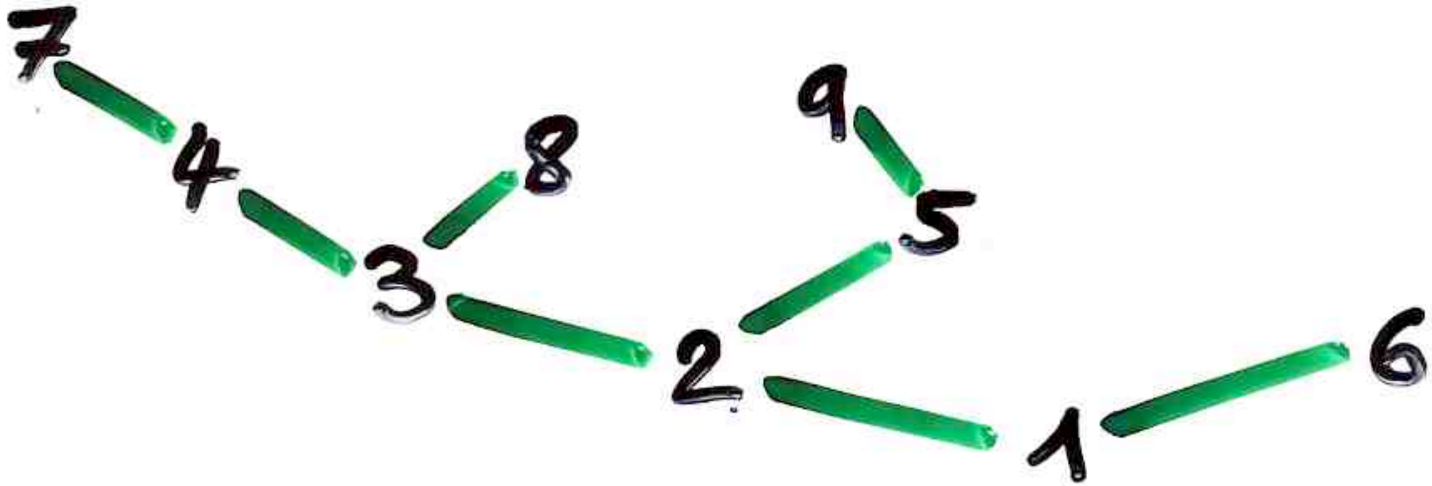


Def. Increasing binary tree



Bijection

increasing binary tree T ⟷ permutation σ



$\sigma = 7\ 4\ 3\ 8\ 2\ 9\ 5\ 1\ 6$

Bijection

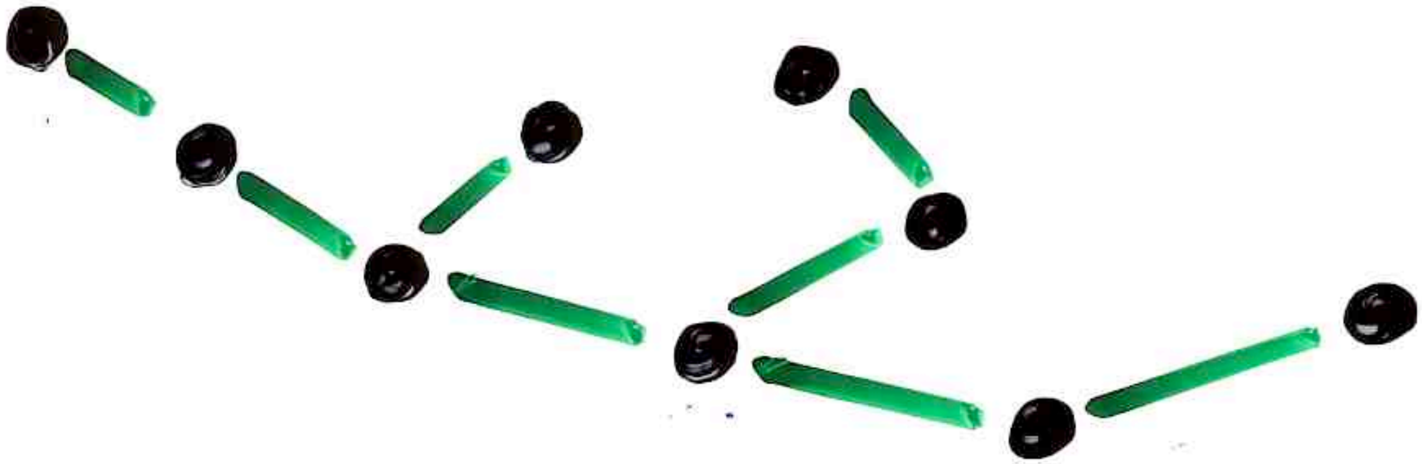
increasing binary tree T ⟷ permutation σ

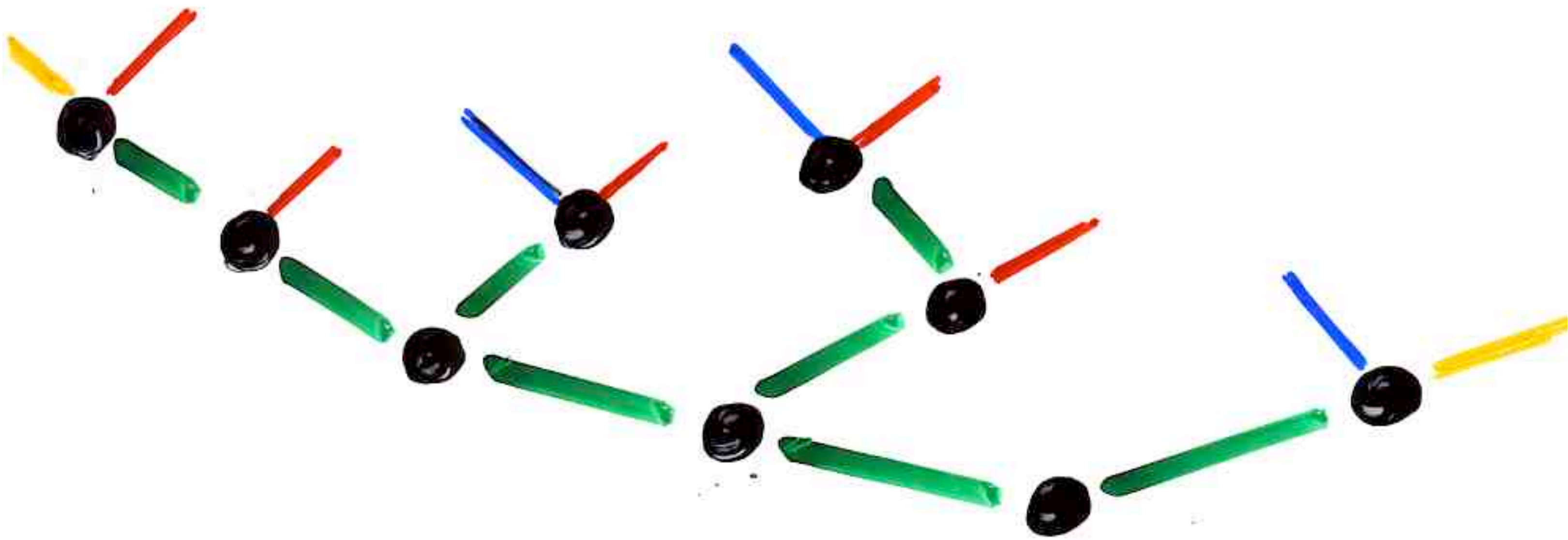
$T \xrightarrow{\pi} \sigma$ symmetric order of vertices (or "projection")

$\sigma \xrightarrow{\delta} T$ "déployé"

word $w = u\,m\,v$ m (unique) minimum letter

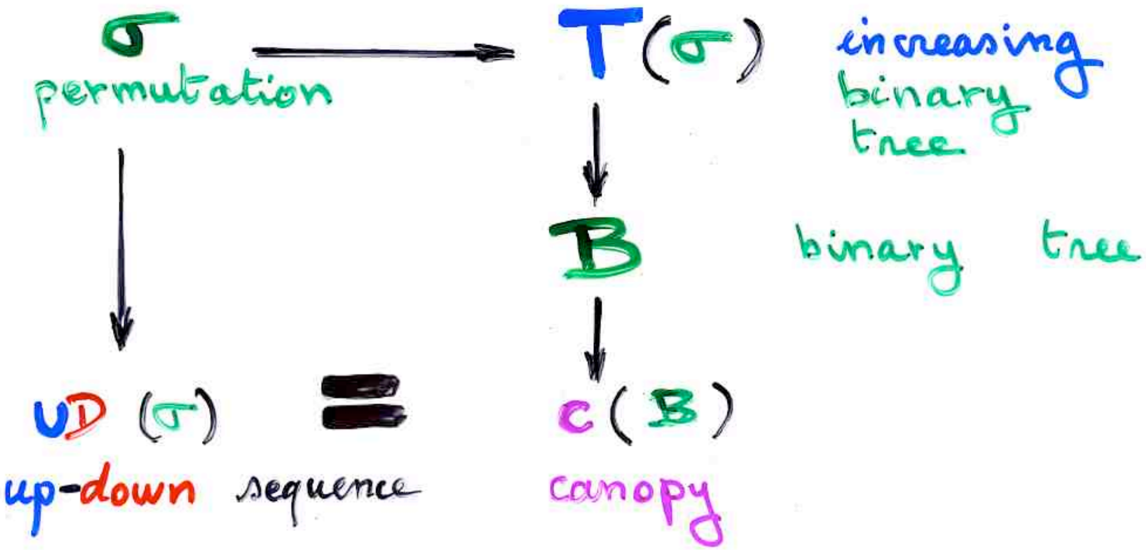
$\delta(w) = \delta(u) \; m \; \delta(v)$

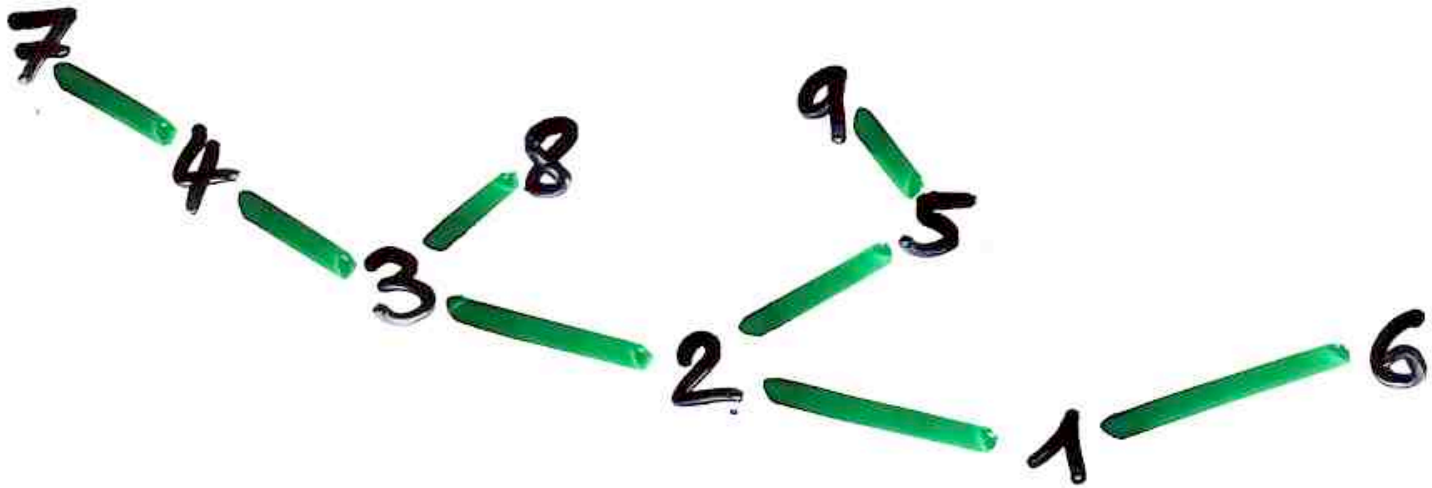




canopy of a binary tree

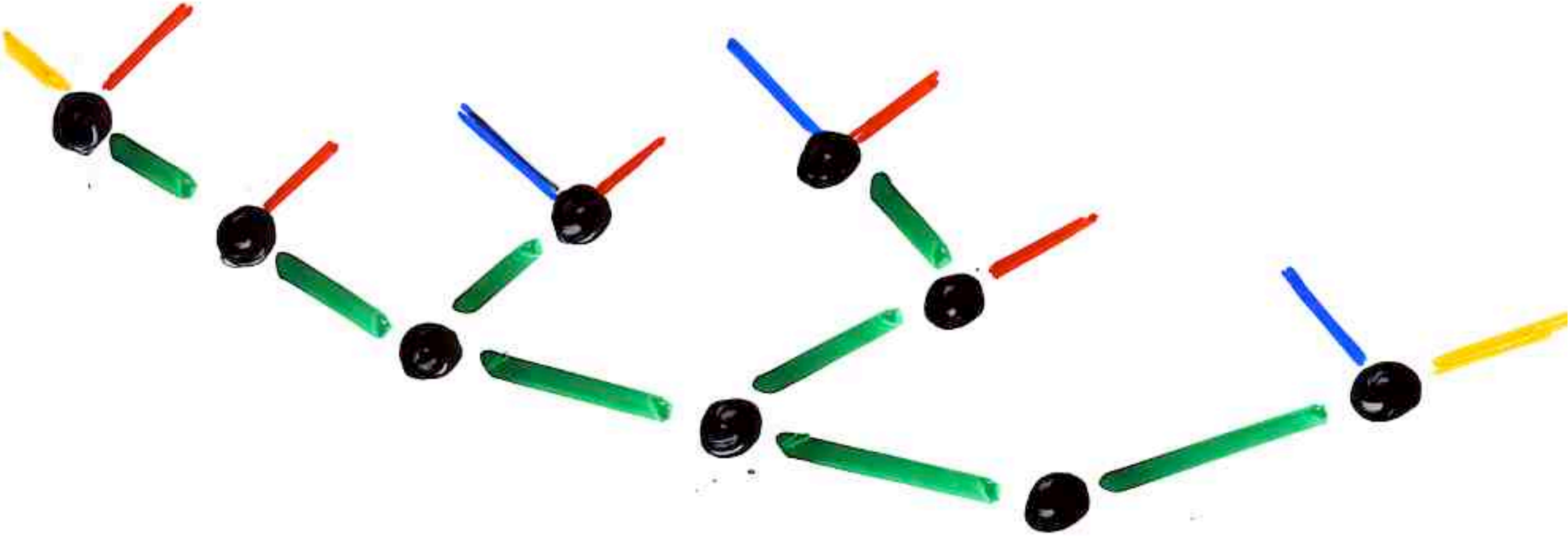
$C(B) = - - + - + - - +$


σ
permutation
T(σ)
increasing binary tree
B
binary tree
UD(σ)
up-down sequence
=
c(B)
canopy



σ = 7 4 3 8 2 9 5 1 6 ...

up-down sequence − − + − + − − +



$\sigma = 7\ 4\ 3\ 8\ 2\ 9\ 5\ 1\ 6 \ldots$

up-down sequence $\quad - \quad - \quad + \quad - \quad + \quad - \quad - \quad +$

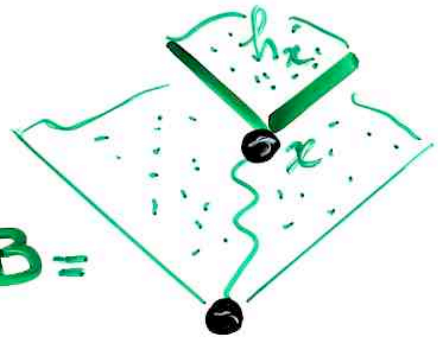
"hook-length formula"

$$\frac{n!}{\prod_x h_x}$$

n nb of vertices

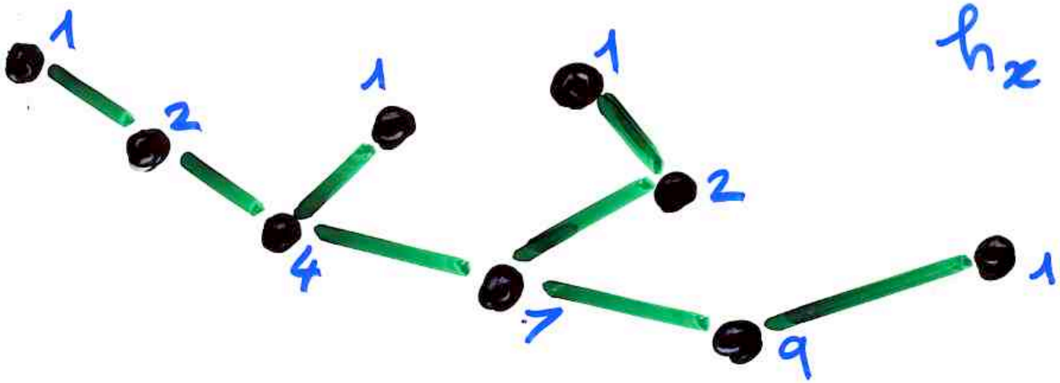
product of size of sub-trees

nb of increasing binary tree for a binary tree B

$B =$ 

ex:

"hook-length" h_x



$$\frac{9!}{2^2 . 4 . 7 . 9} = 360$$

§5 Pagodes

⑤ les pagodes et la formule $2n-g-d$

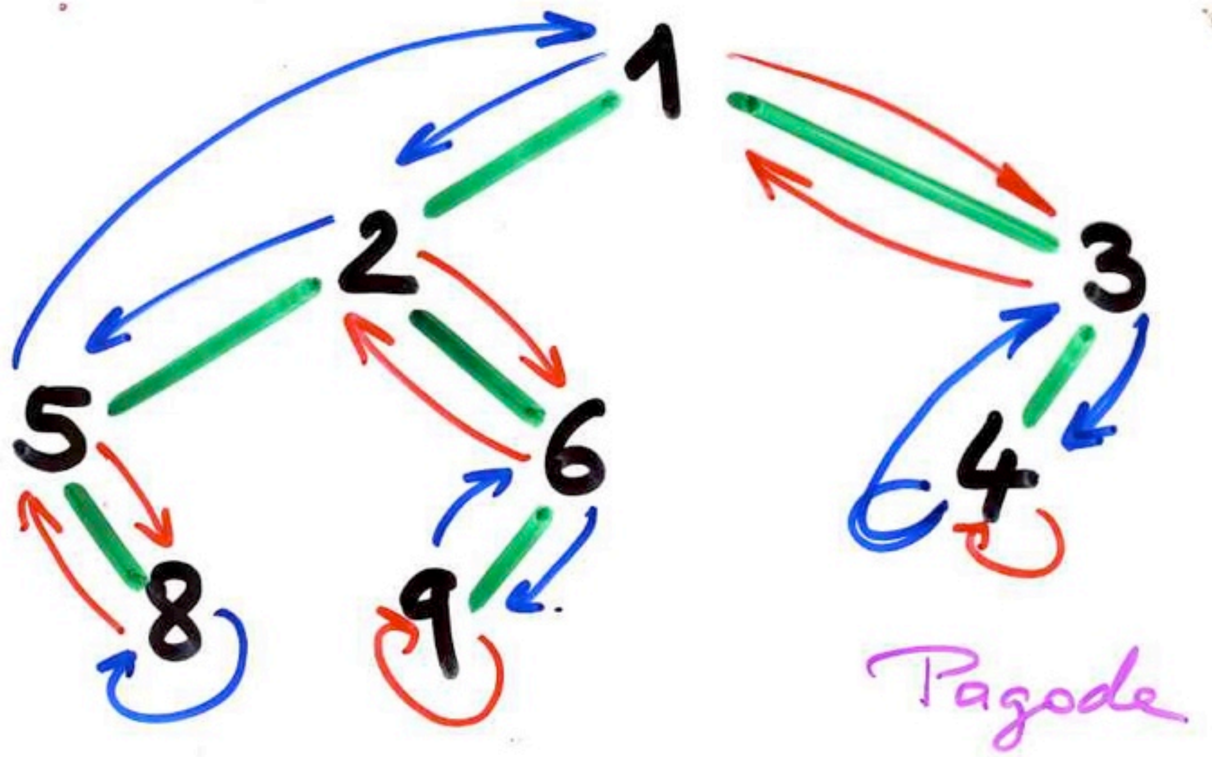
Françon, Viennot, Vuillemin (1978)

représentation des **files de priorité**

avec des arbres linéaires croissants,

mis en machine avec la structure

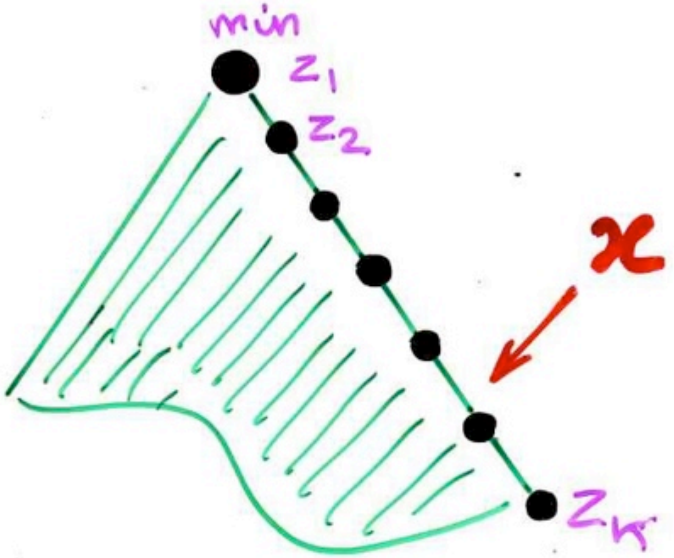
"Pagode"



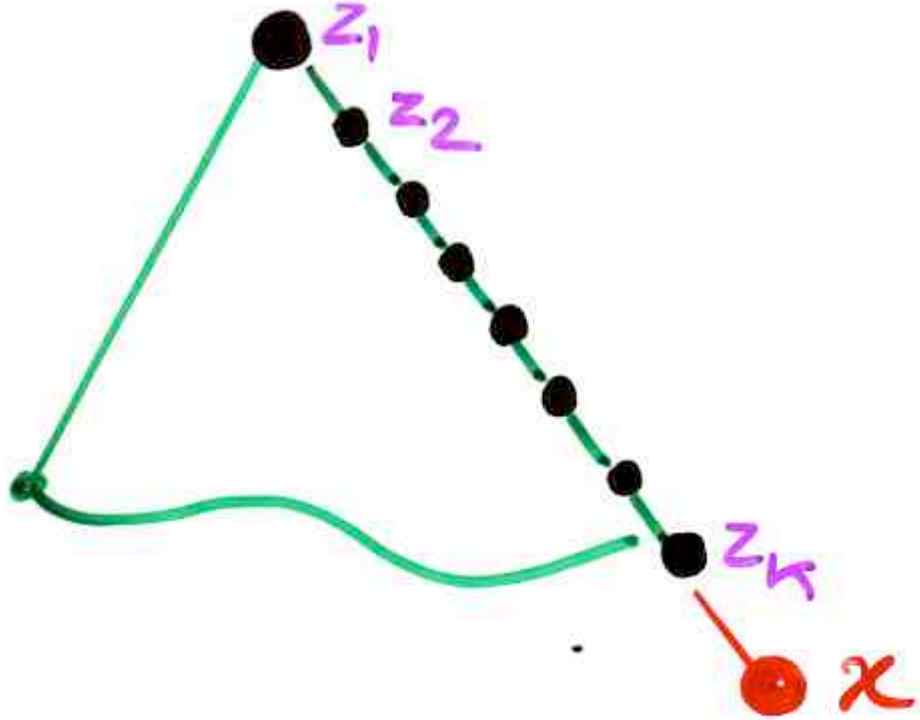
$w = 5, 8, 2, 9, 6, 1, 4, 3$

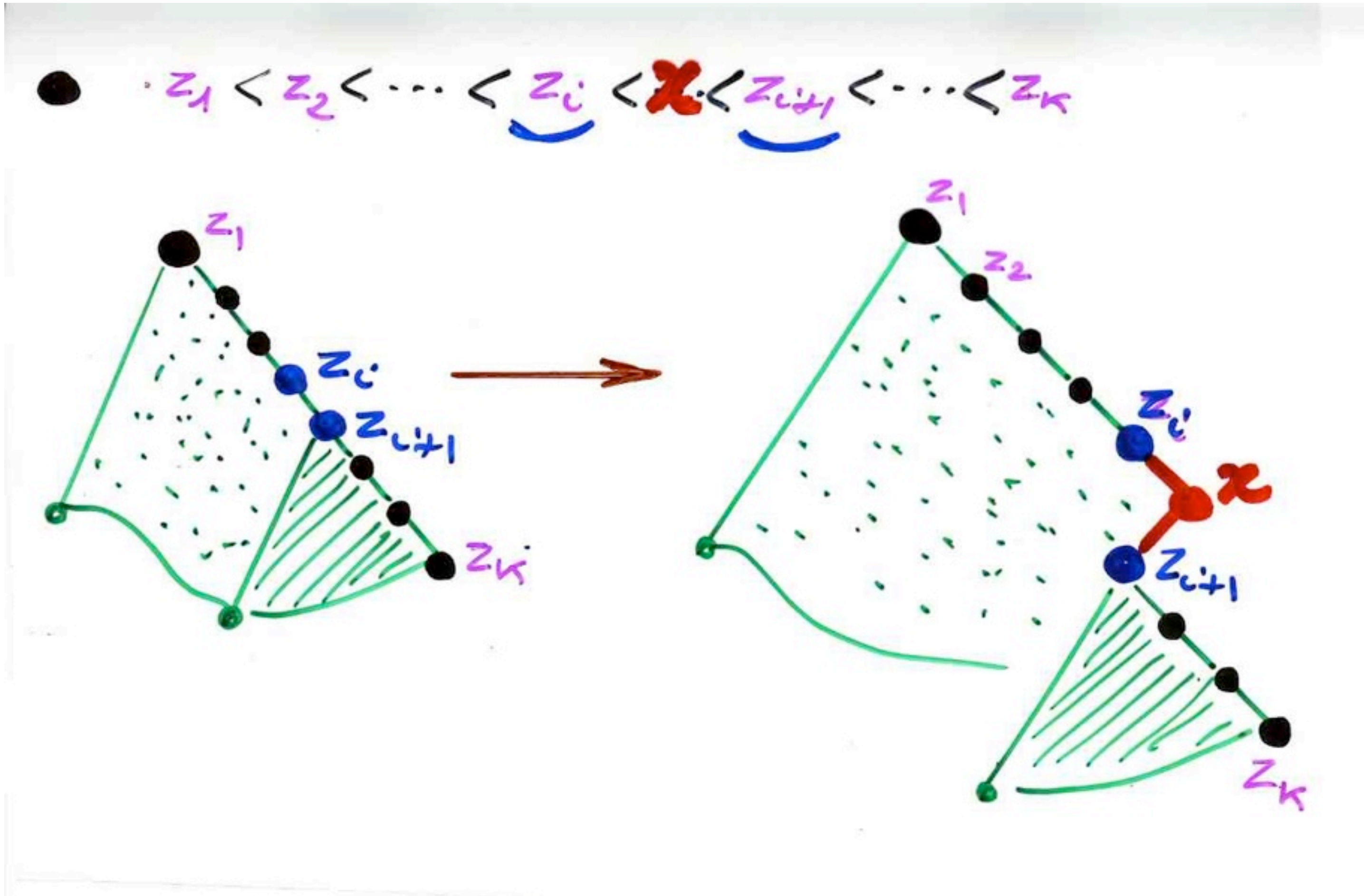
Insertion d'un élément dans une file de priorité

$\delta(w\ )$ $\delta(wx)$

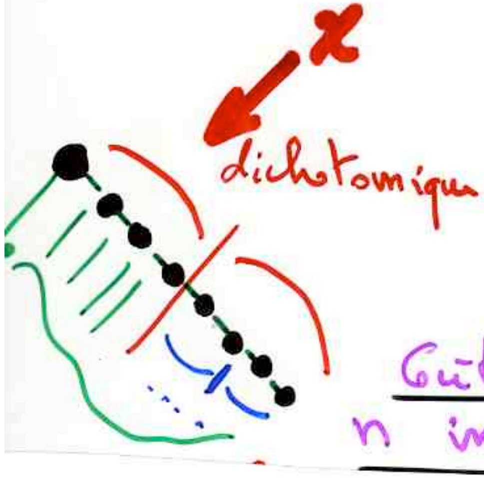


- $z_1 < z_2 < \dots < z_k < x$

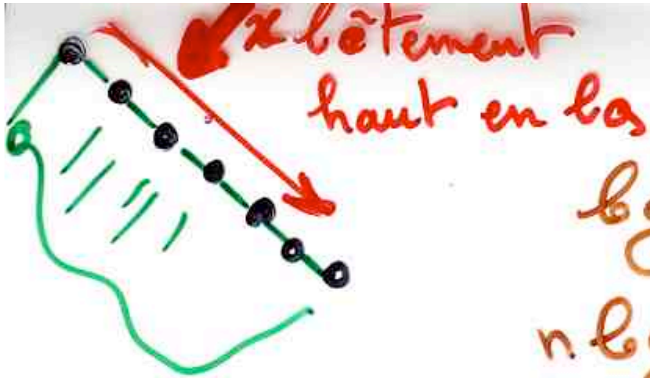



z1 < z2 < ··· < zi < x < zi+1 < ··· < zk
z1
zi
zi+1
zk
z1
z2
zi
x
zi+1
zk

Coût d'une insertion

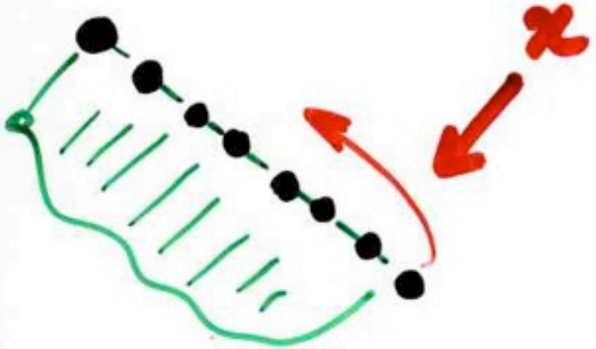


	Coût moyen	Cas le pire
	$\log n$	$\log n$
Coût n insertions	$n \log n$	$n \log n$



	Coût moyen	Cas le pire
une insertion	$\log n$	n
n insertions	$n \log n$	n^2

de bas en haut

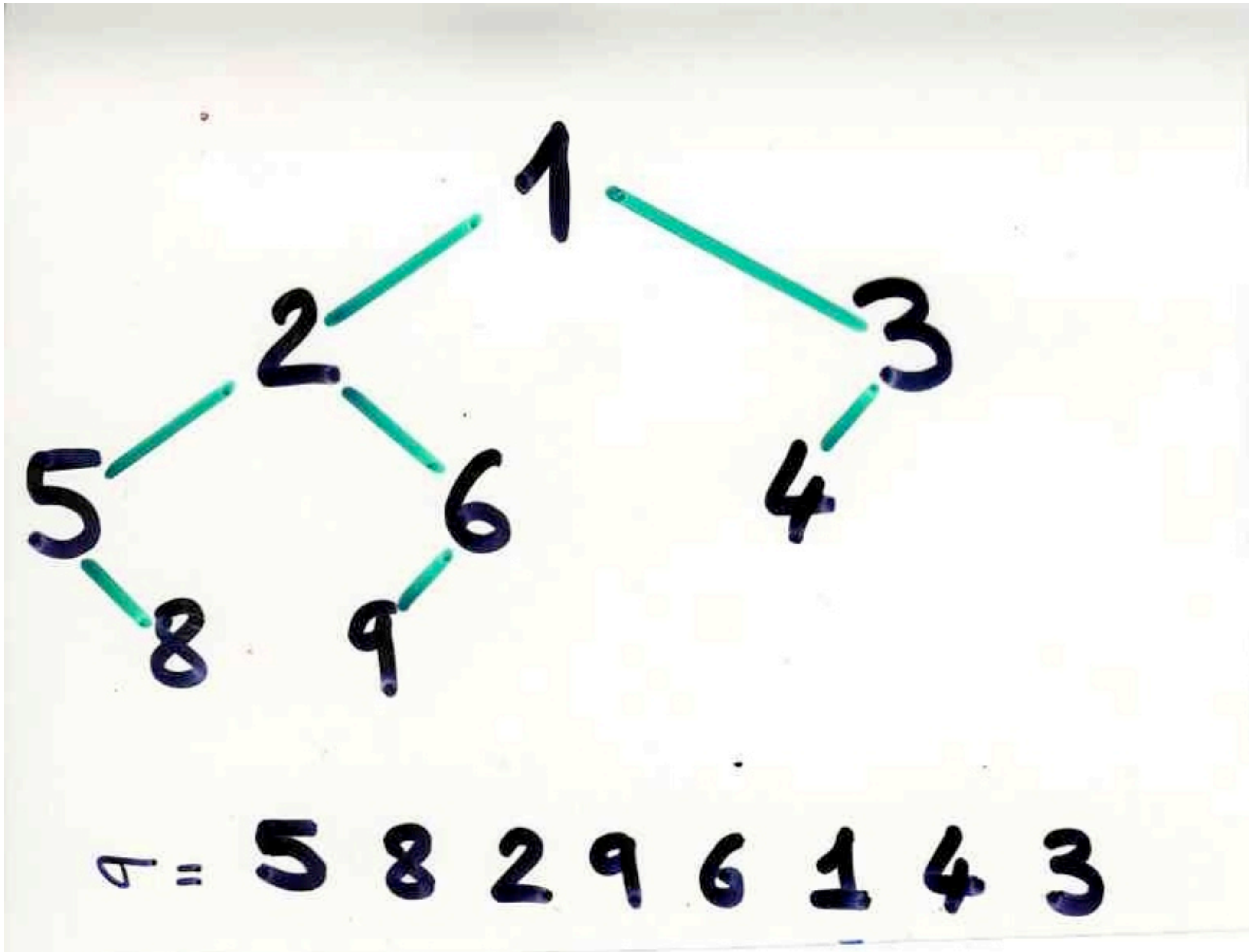


Coût moyen	Cas le pire
$\log n$	une insertion n
$2n - 2\log n$	n insertions $2n - 3$!!

optimum en lecture "off-line"

Prop - le nombre de comparaisons effectuées pour construire $\delta(w)$ par insertions successives de $w_1, w_2, \dots, w_n$ (pour $w = w_1 w_2 \dots w_n$) est égal à :

$$2n - g - d$$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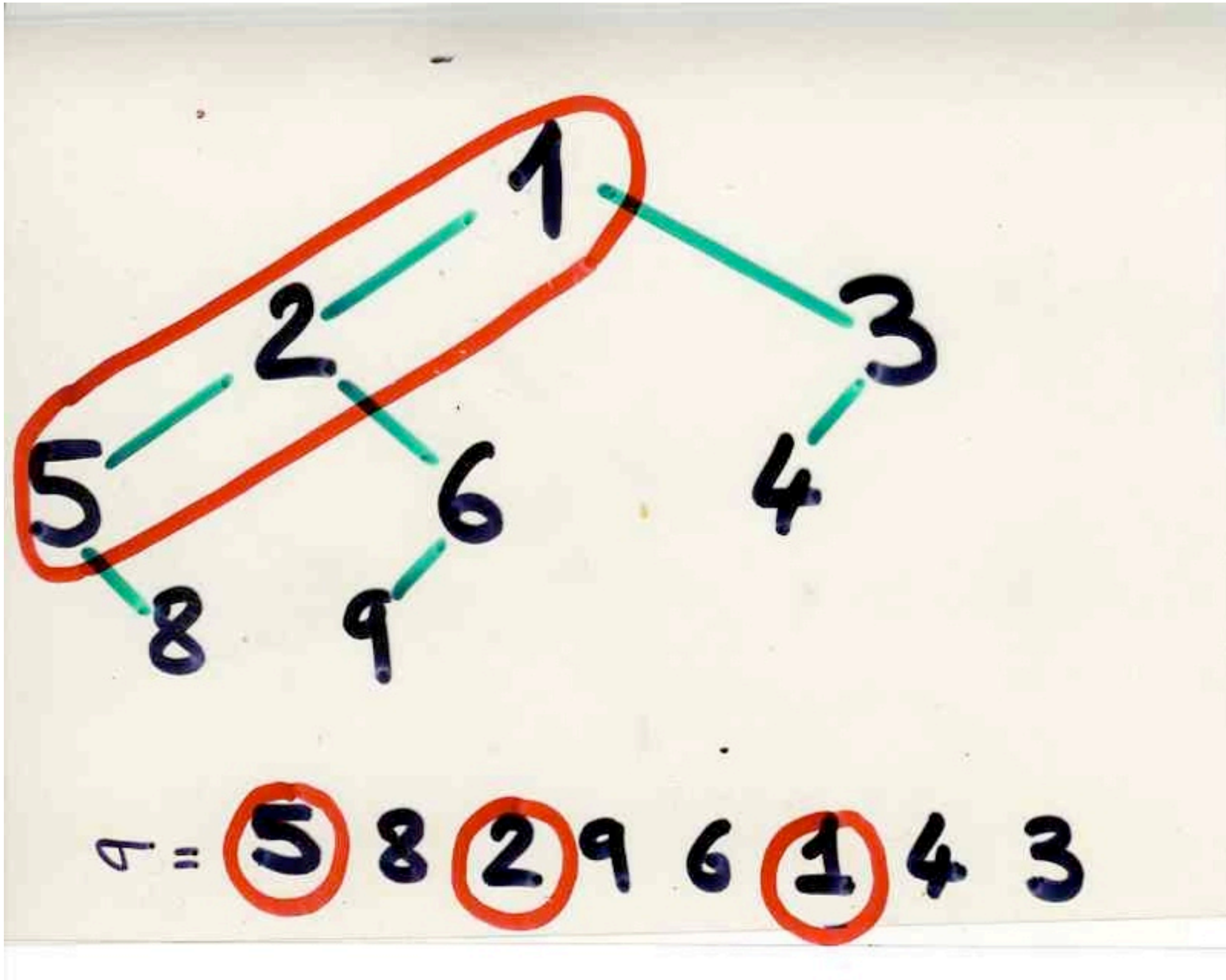
σ = 5 8 2 9 6 1 4 3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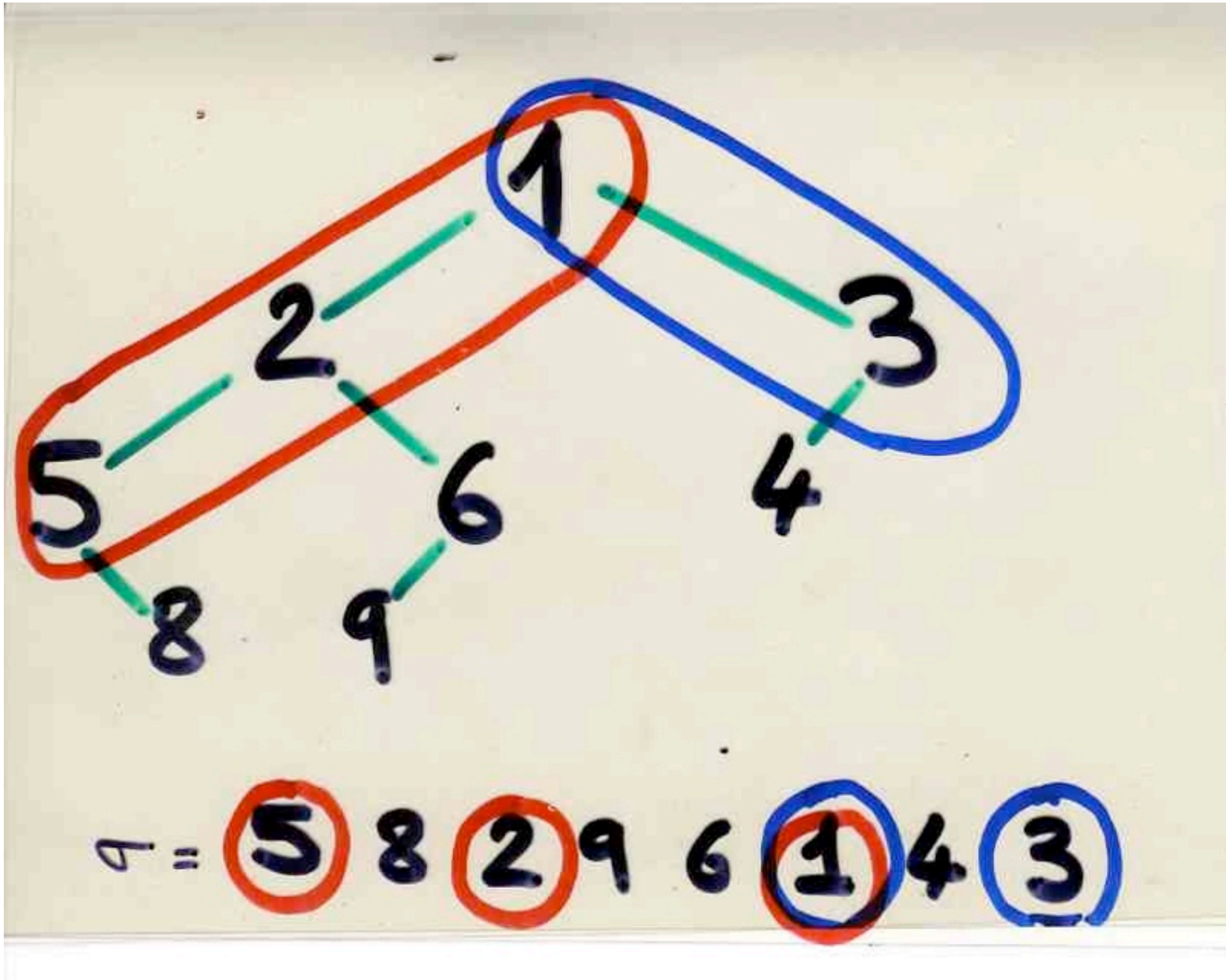
σ = 5 8 2 9 6 1 4 3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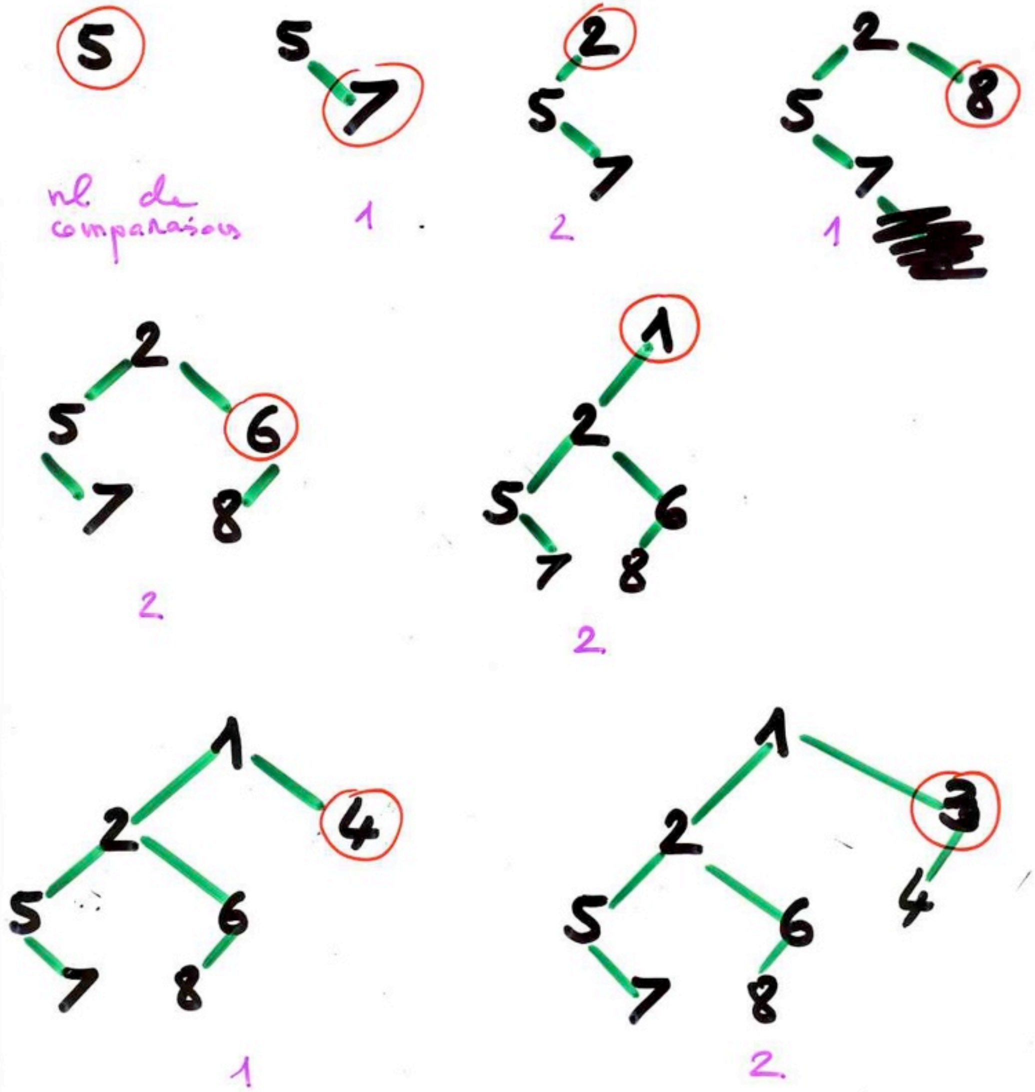
σ = 5 8 2 9 6 1 4 3

ex - $\sigma = 5\ 7\ 2\ 8\ 6\ 1\ 4\ 3$

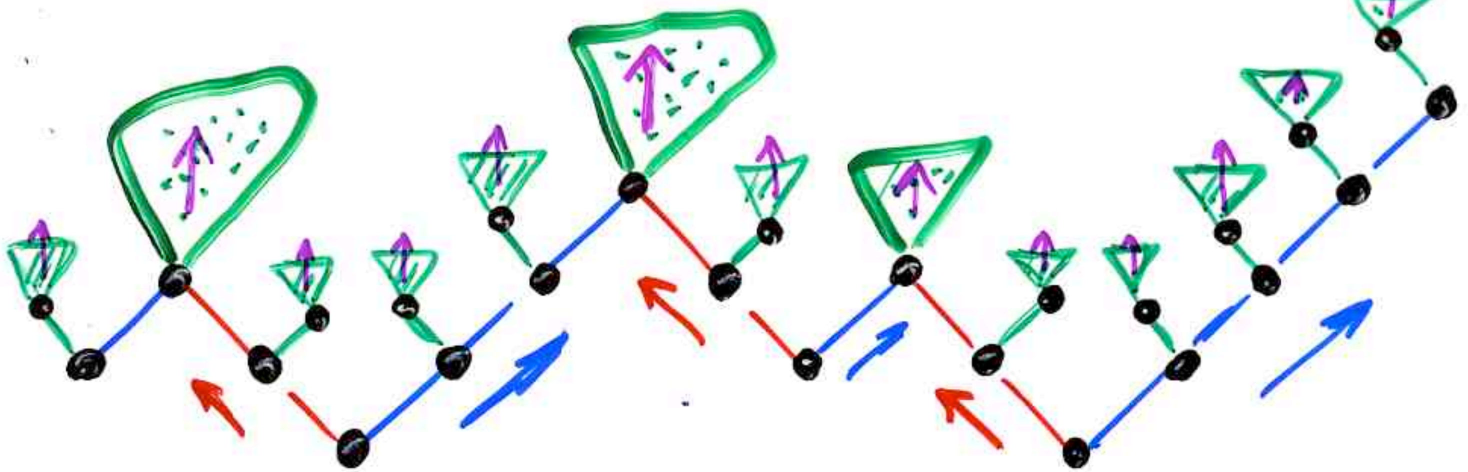


Total - 1+2+1+2+2+1+2 (11)

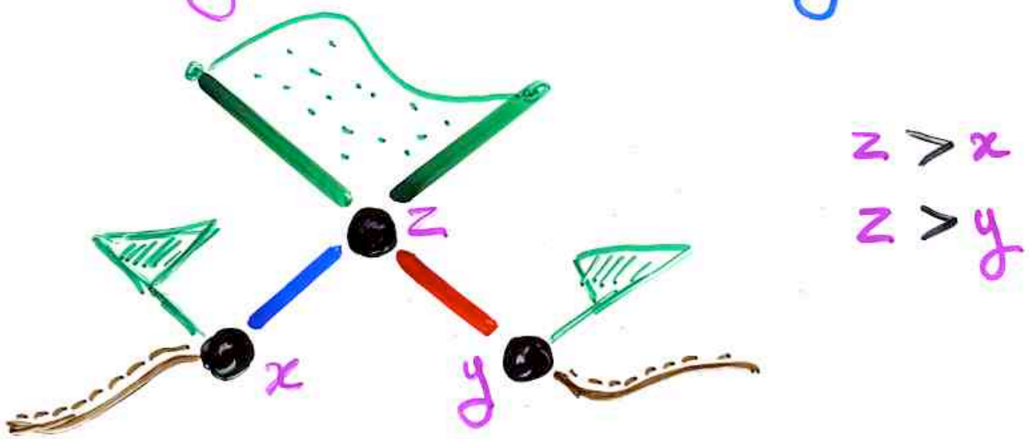
$2n - g - d = 16 - 3 - 2 = 11$

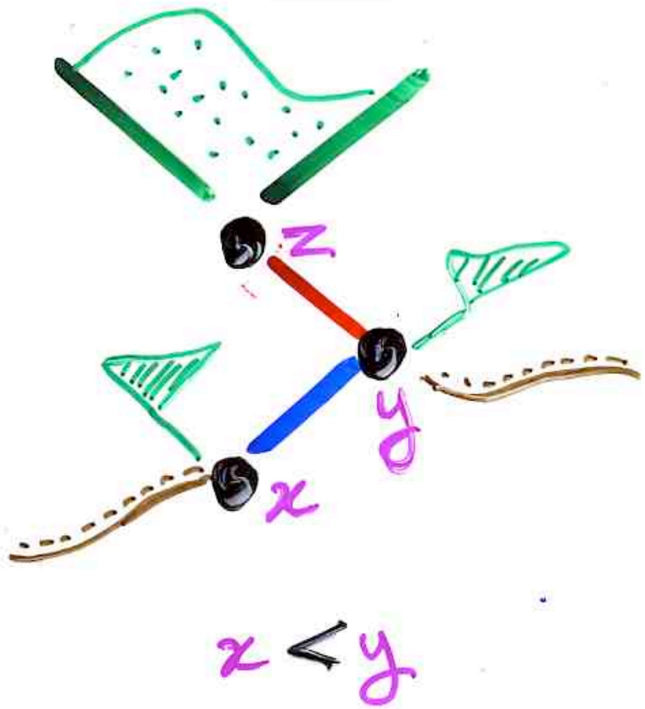
§ 6 “jeu de taquin” for increasing binary trees

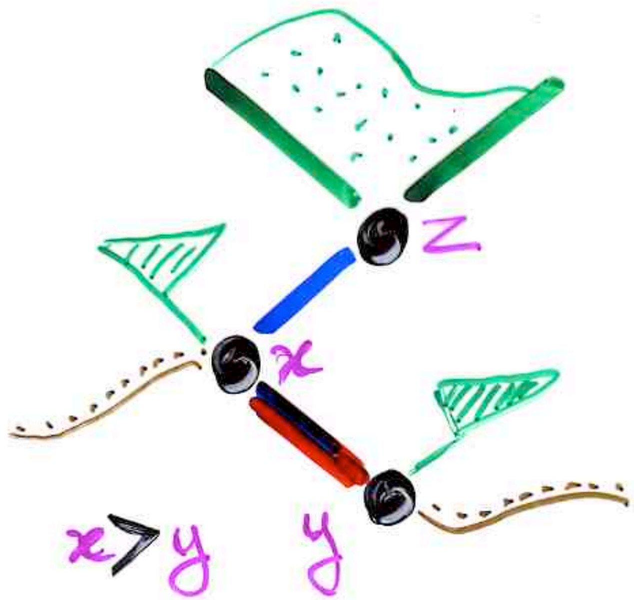
Def- Increasing woods ("buissons" croissants)



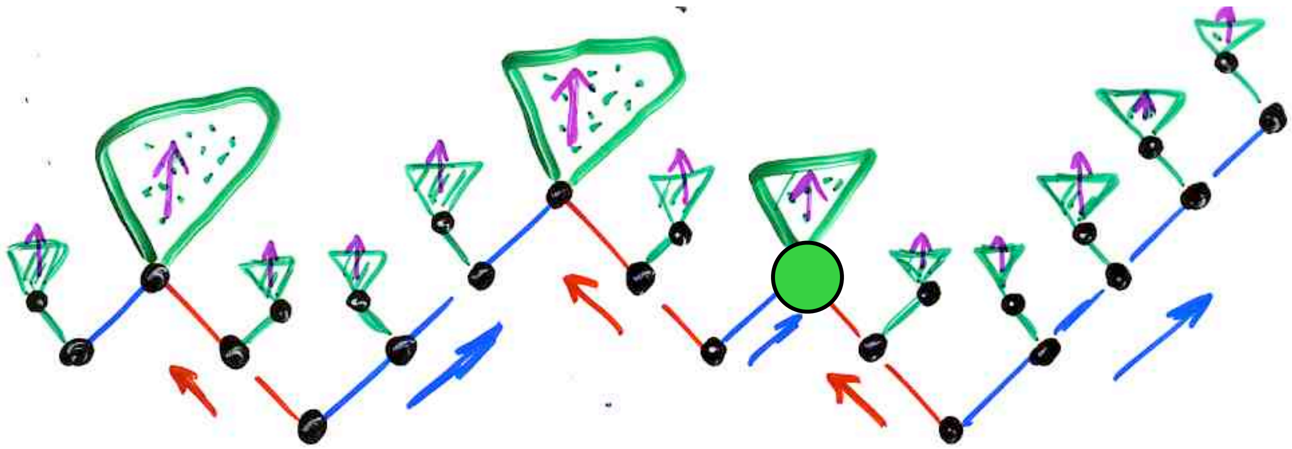
Def - sliding in an increasing woods

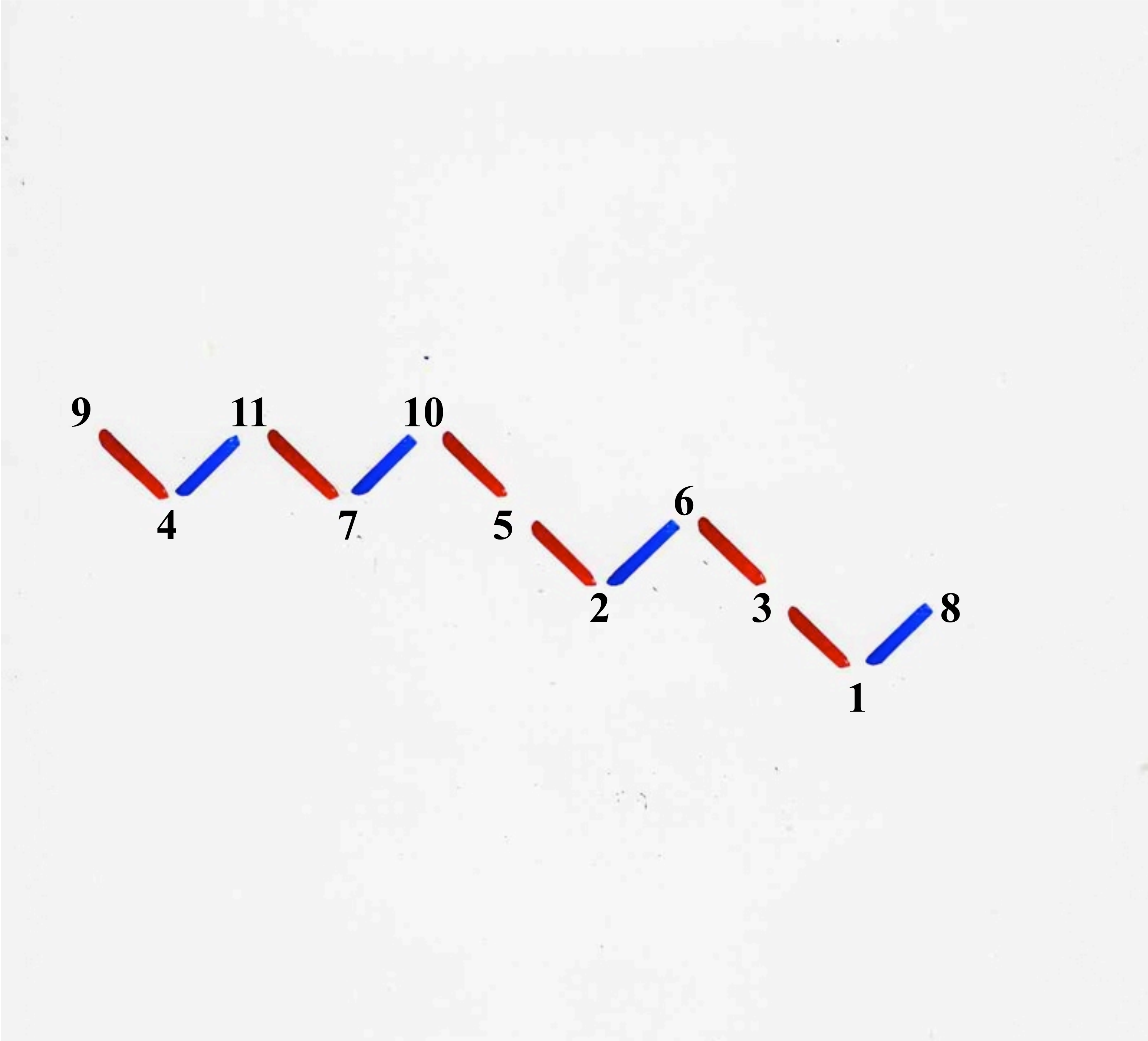






"jeu de taquin" for increasing woods

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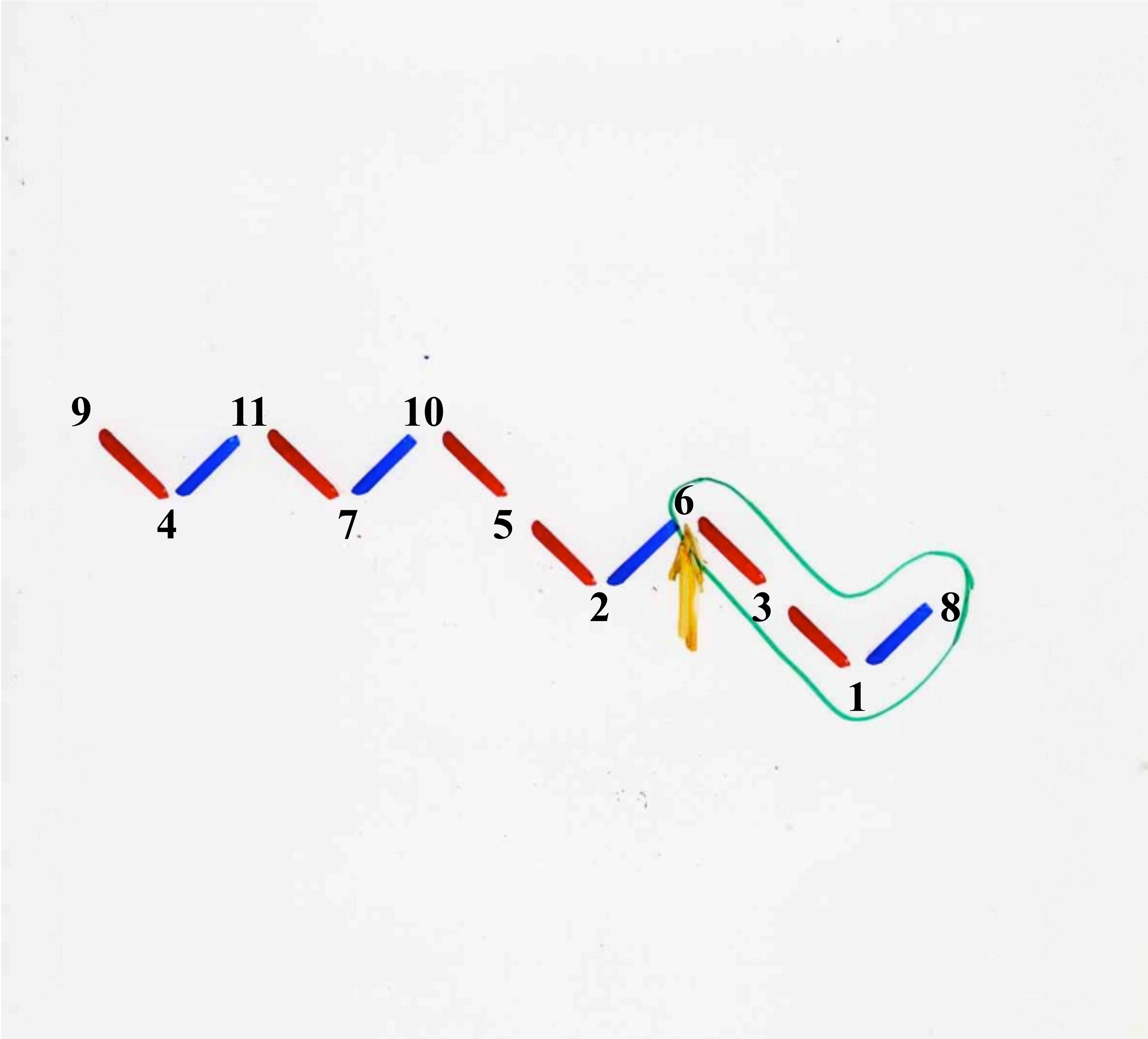
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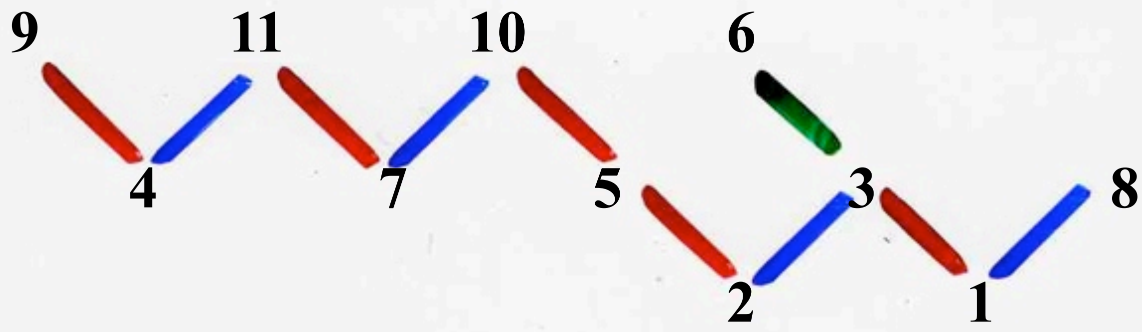
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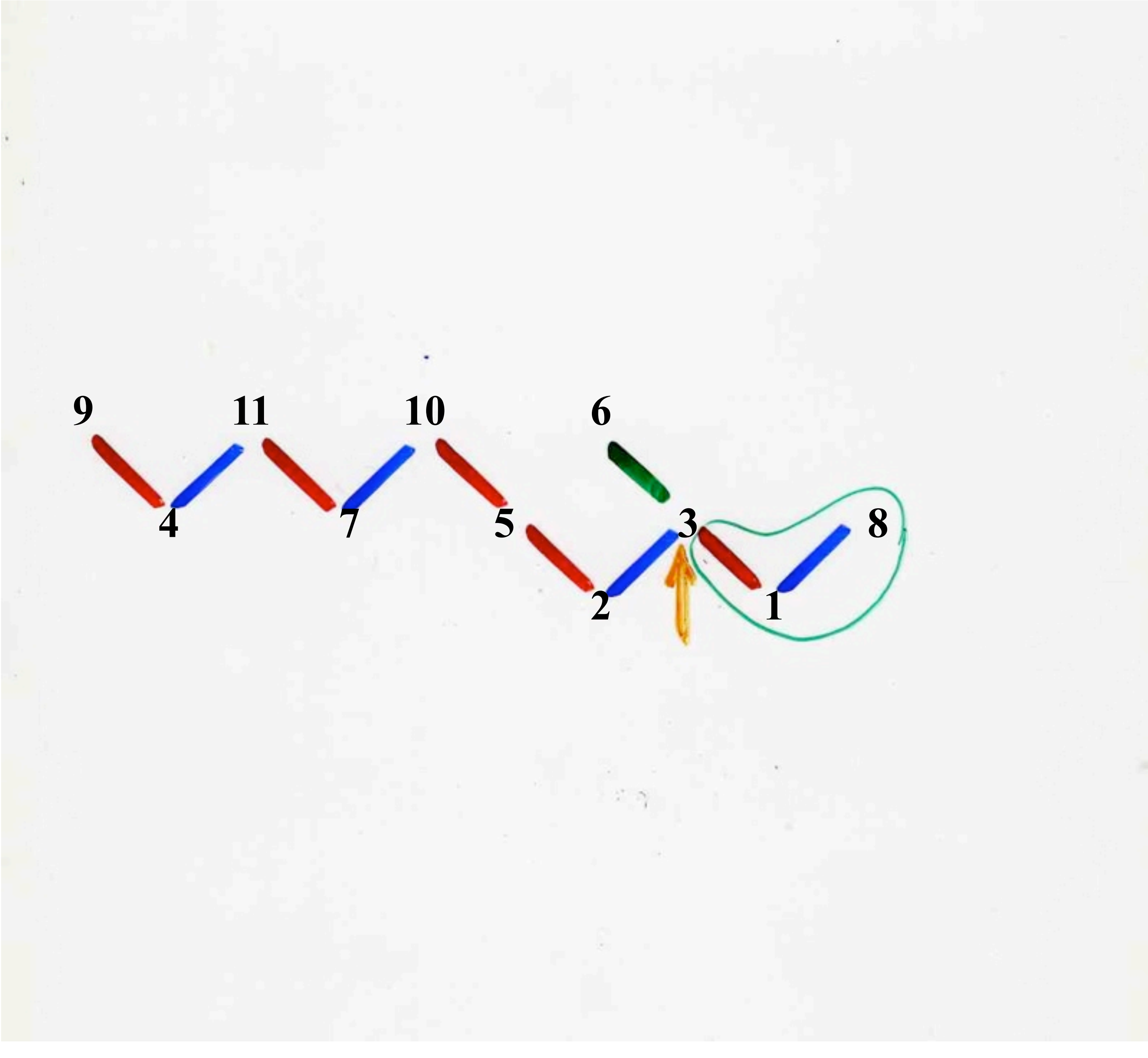
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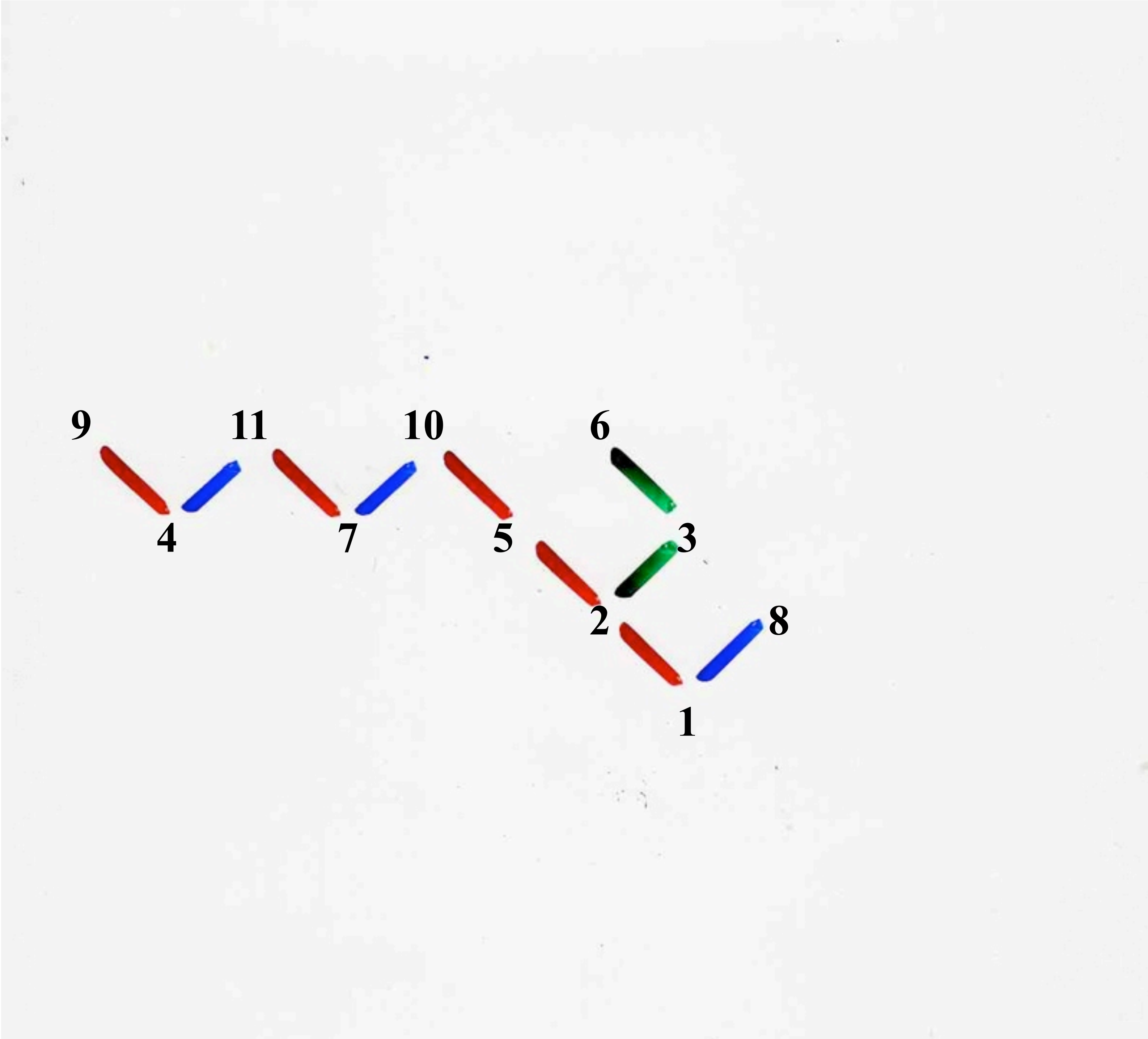

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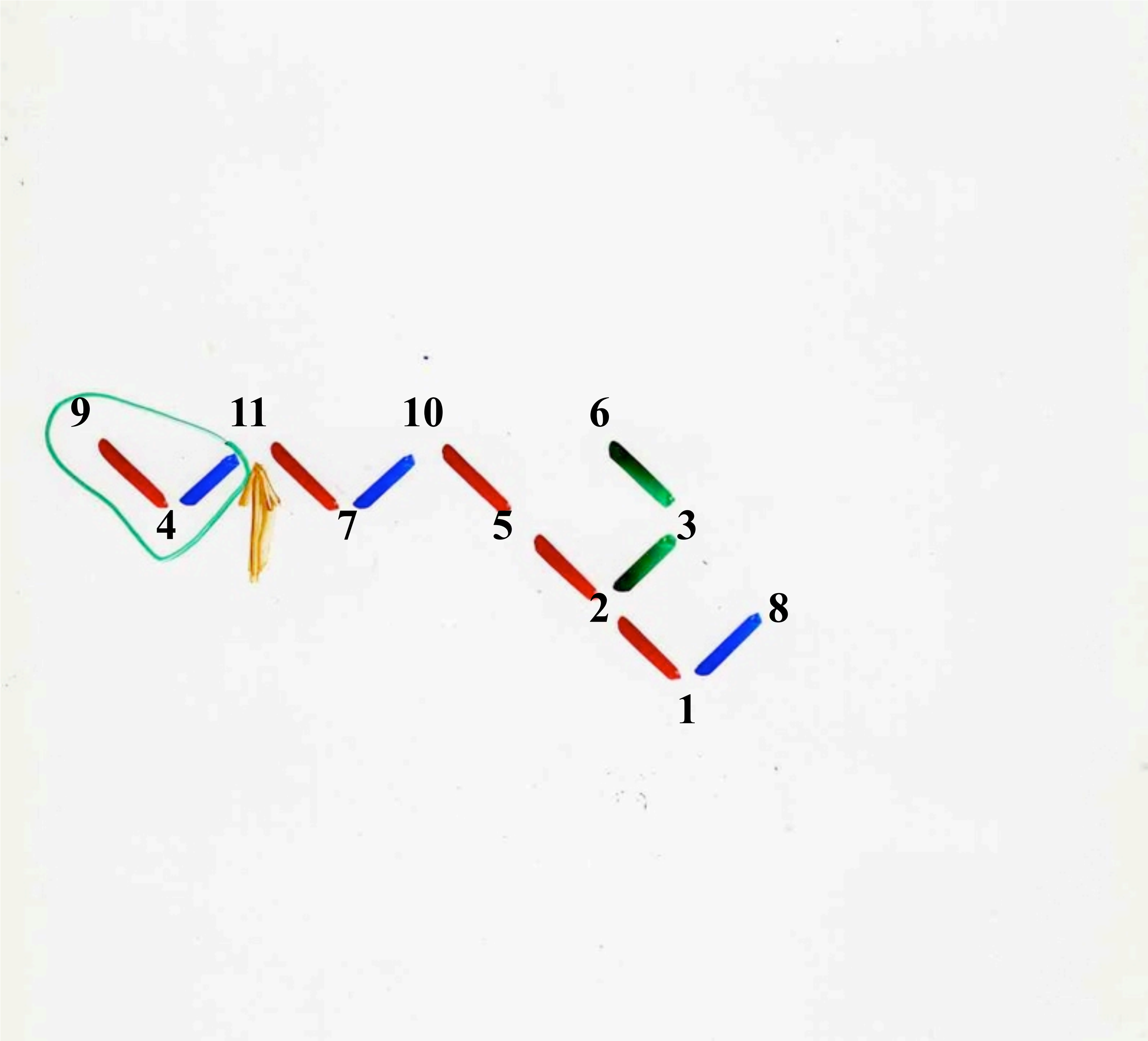
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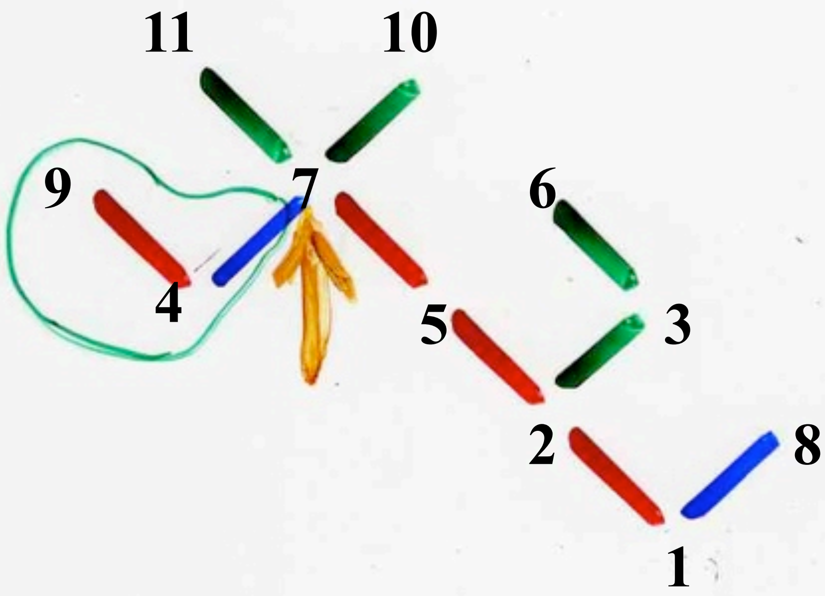
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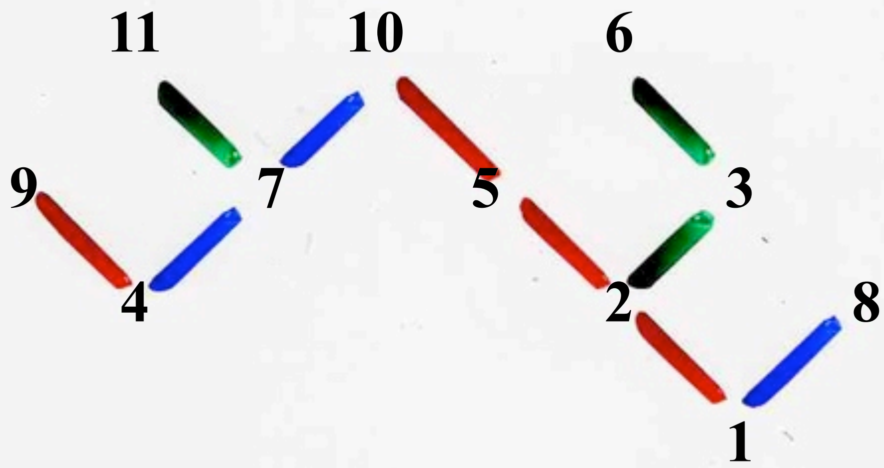

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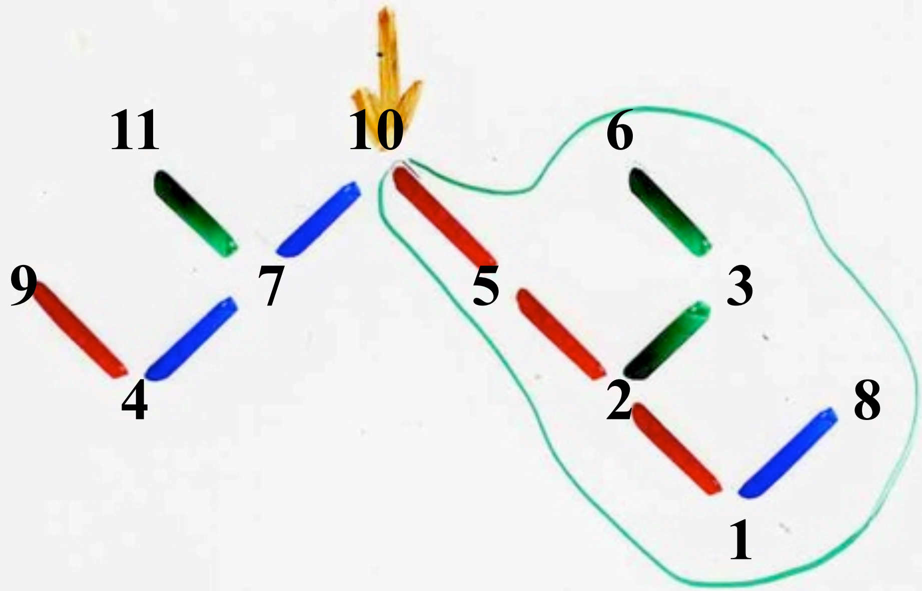
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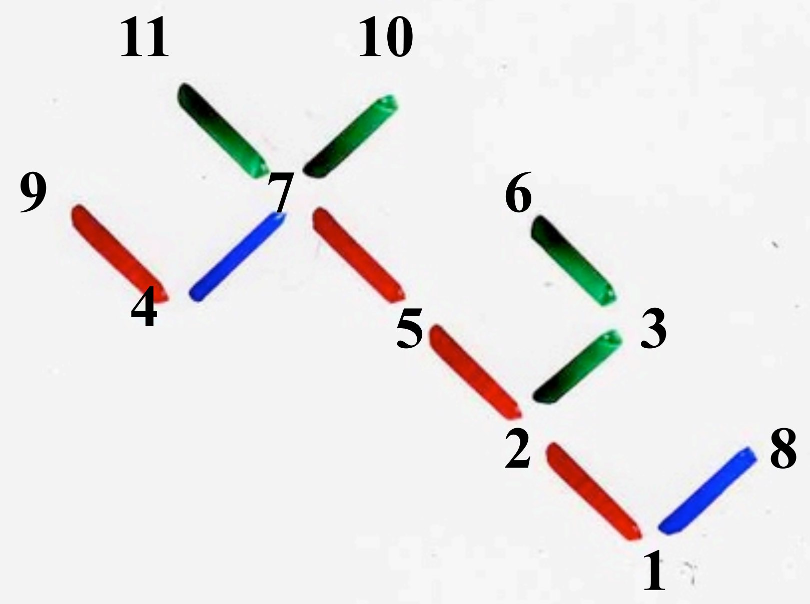

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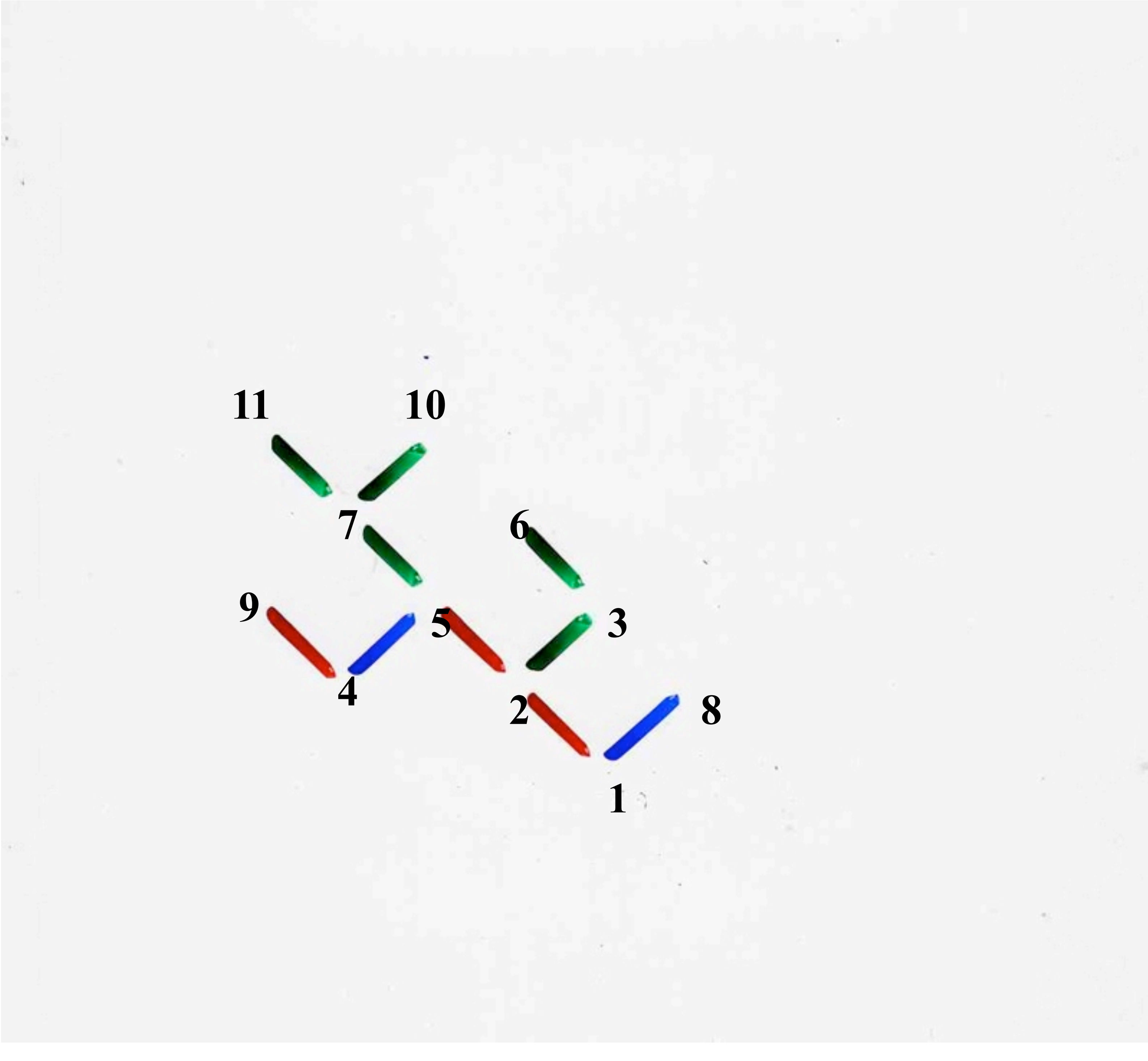
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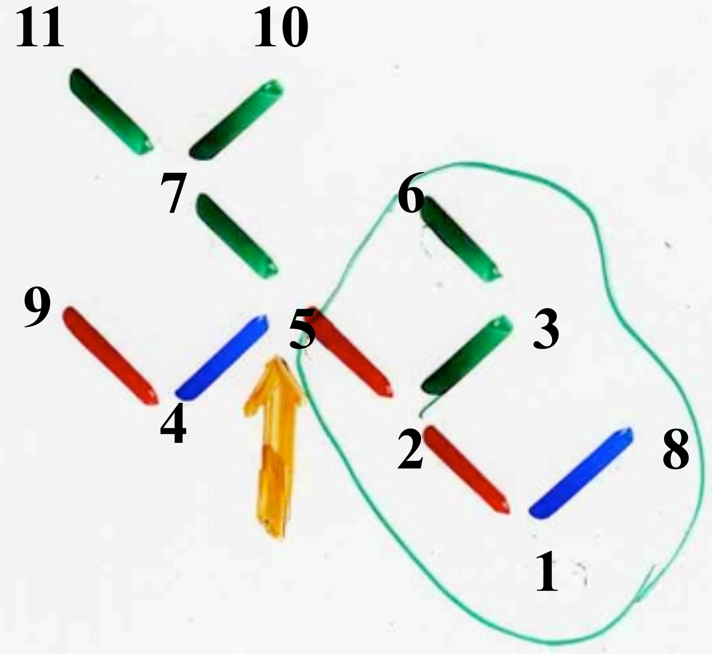
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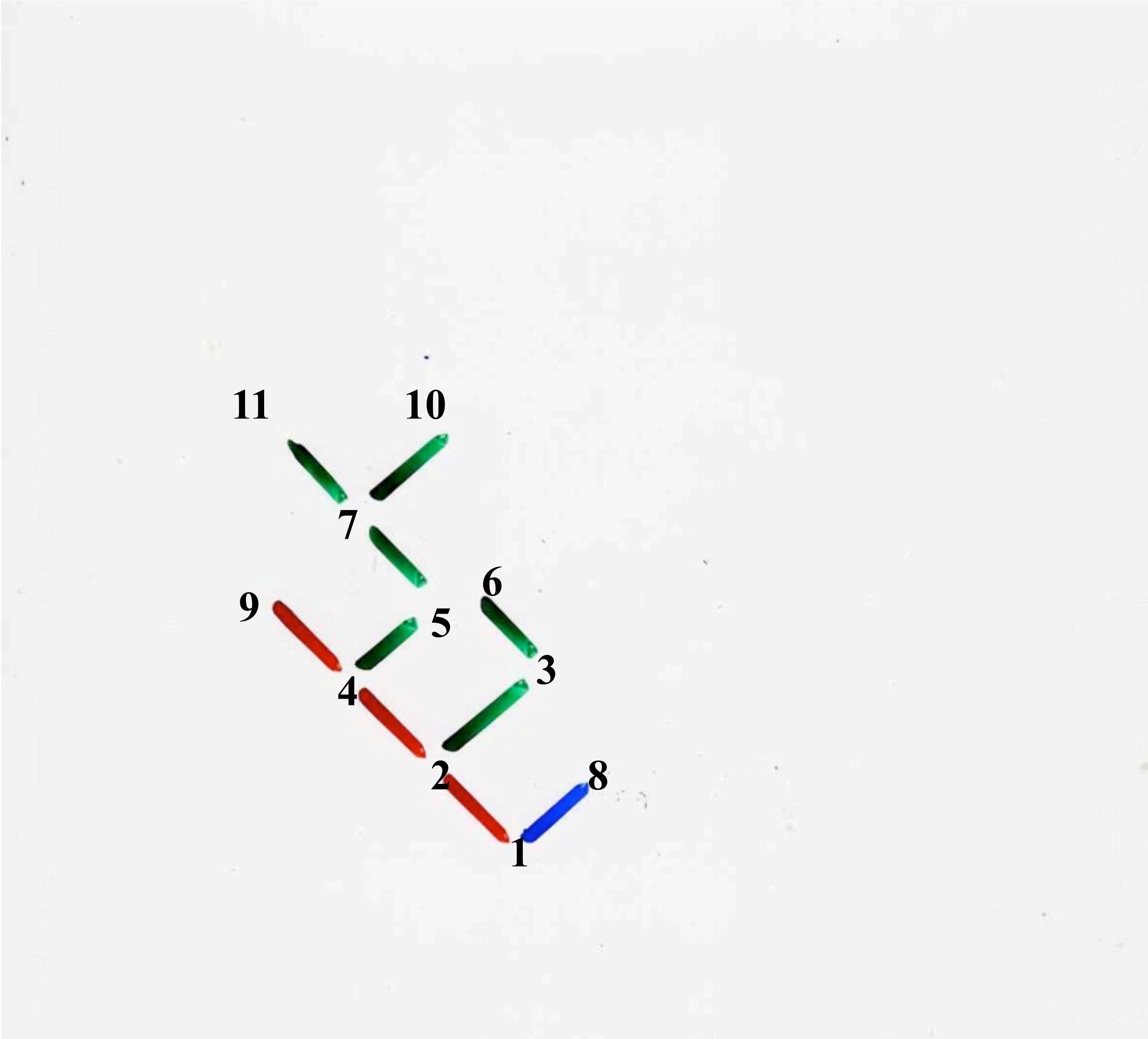

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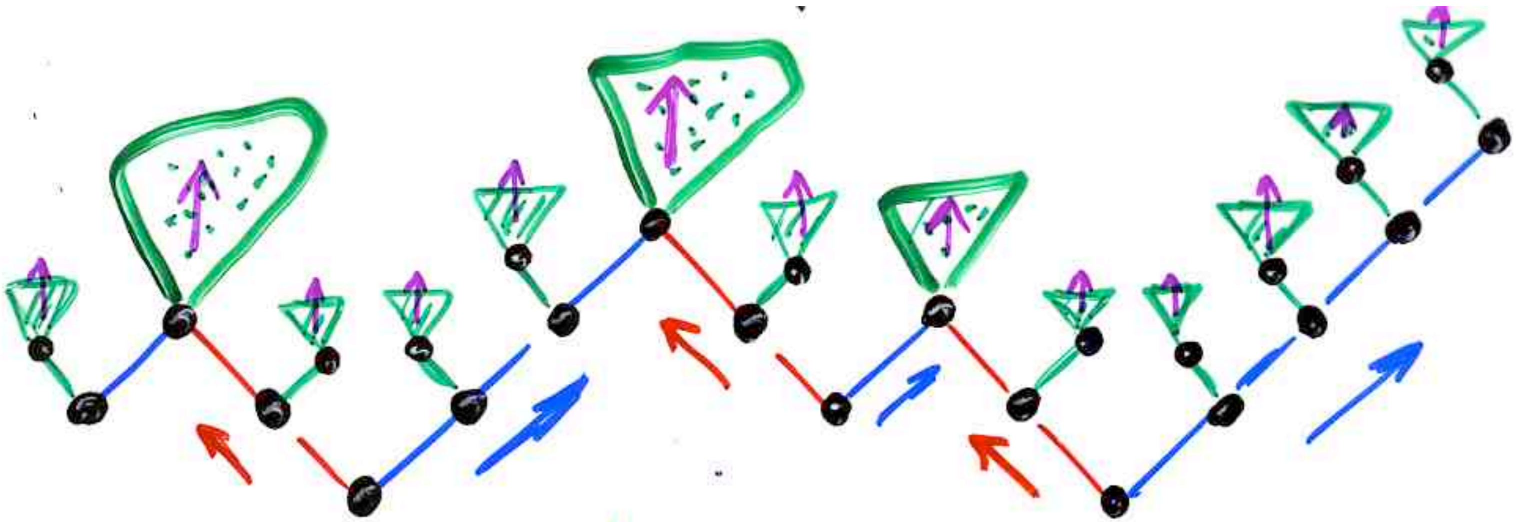


symmetric order for woods

Lemma. invariance of the symmetric order through slidings of an increasing woods

Cor. $\sigma \rightarrow$ UD-wood 

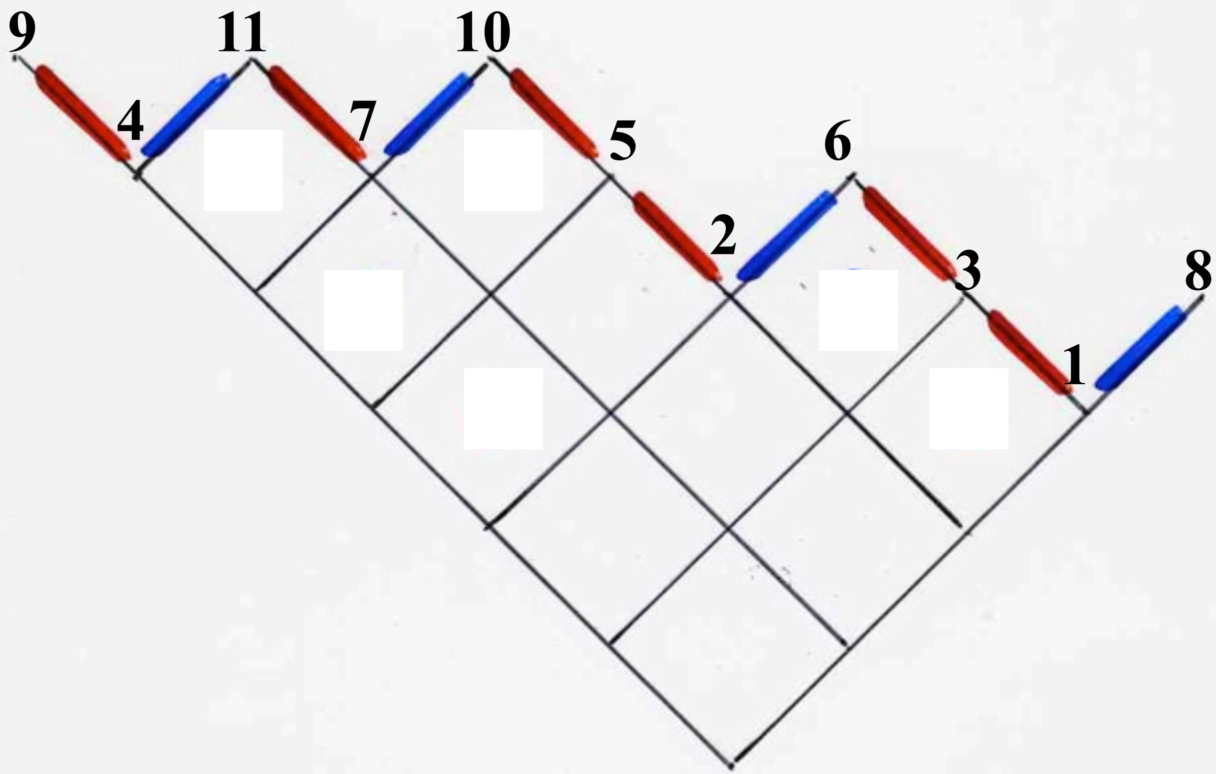
$\downarrow$ "jeu de taquin"

$T(\sigma) = \delta(\sigma)$ déployé

increasing binary tree

“jeu de taquin”

pour un arbre binaire croissant $\longrightarrow$ tableau alternatif de Catalan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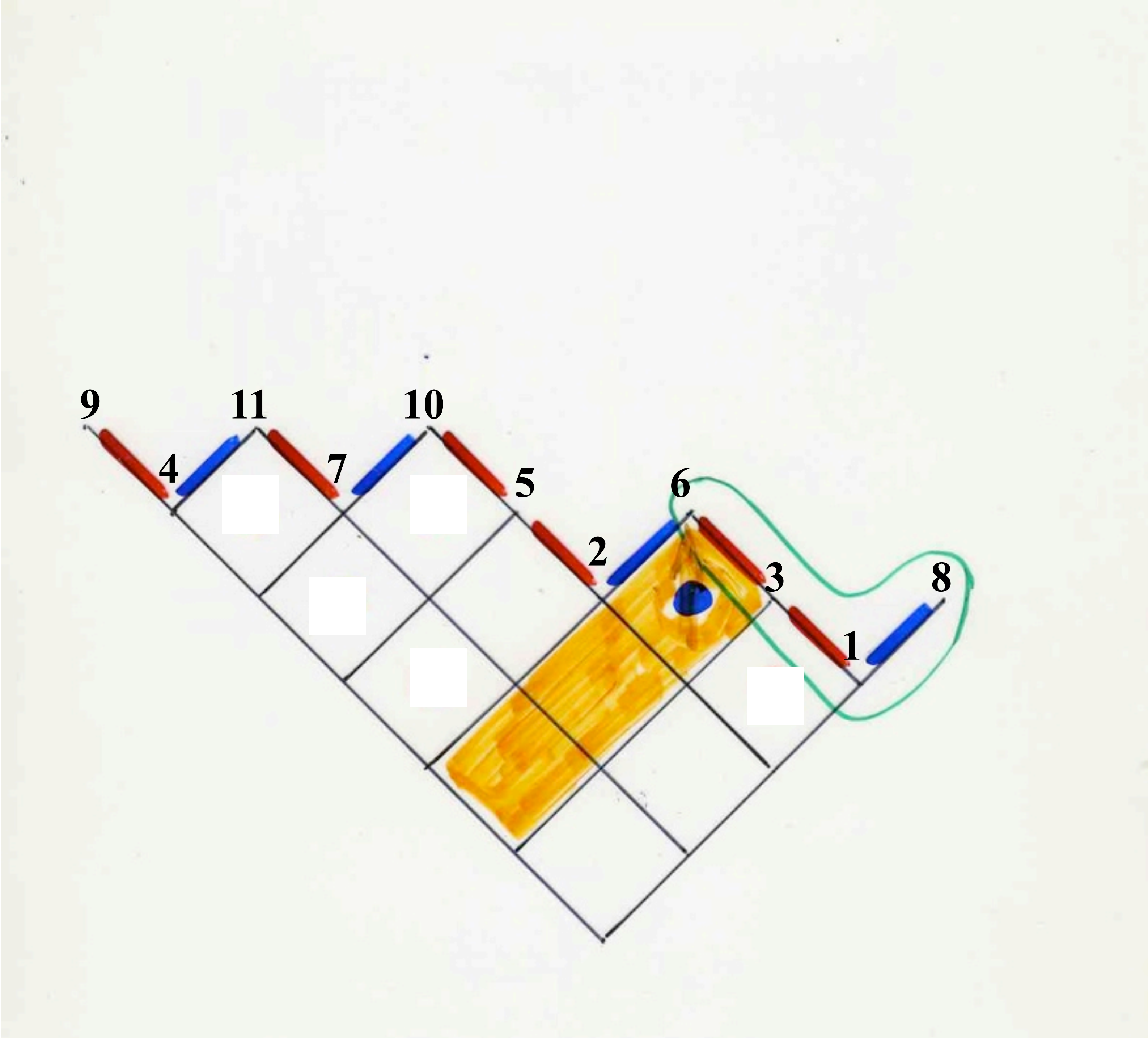
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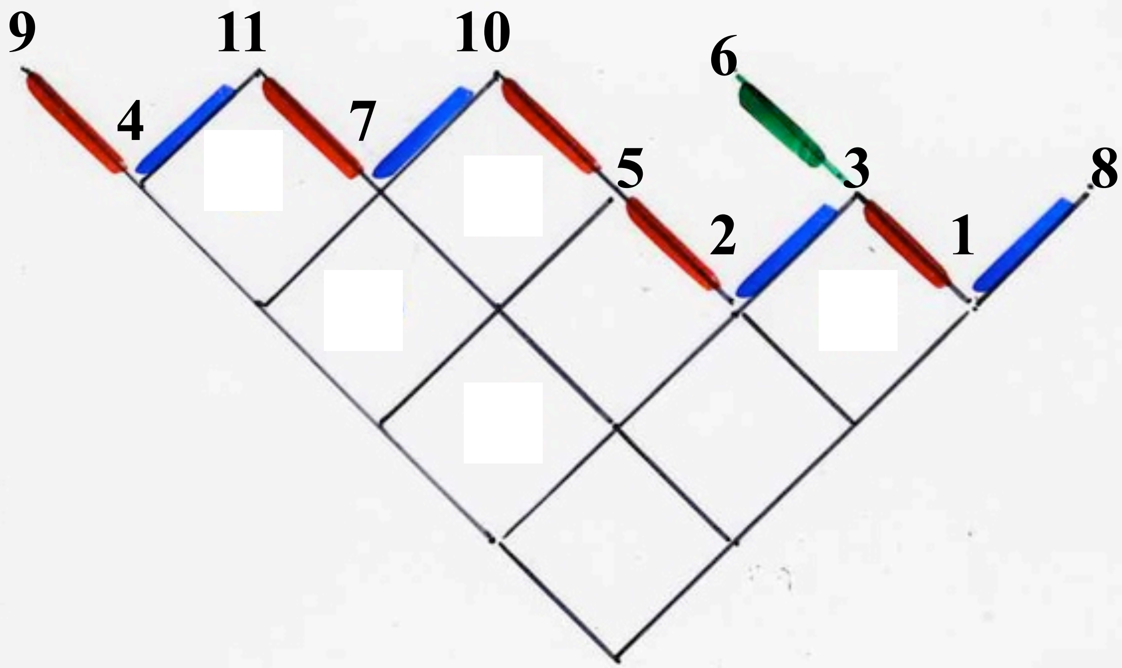
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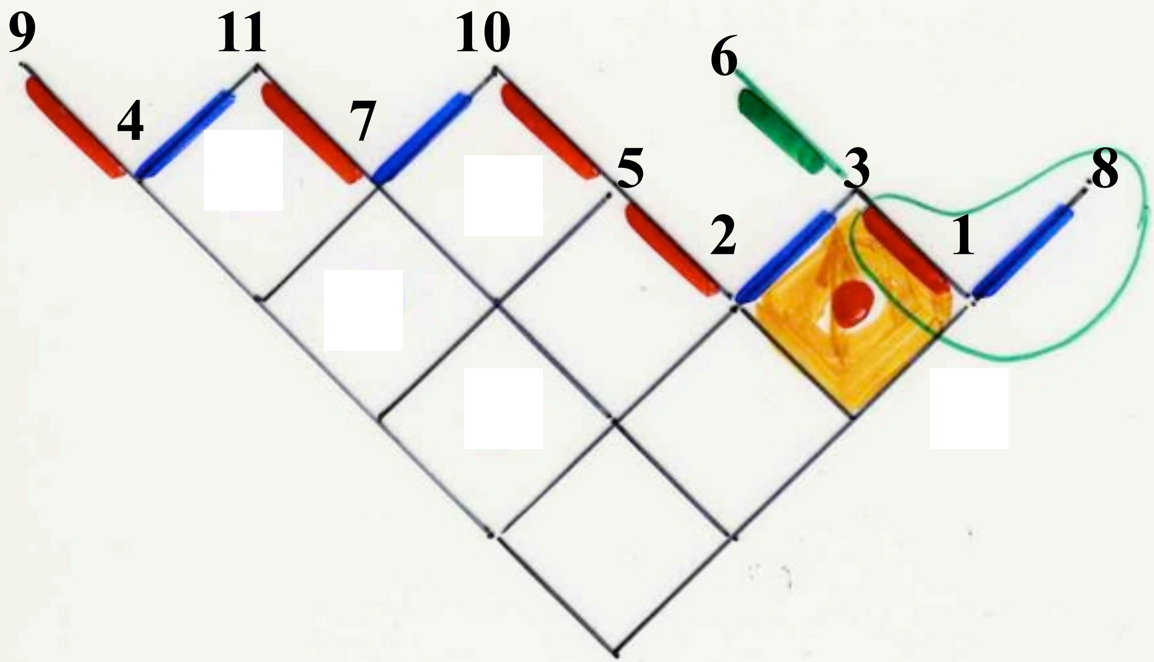
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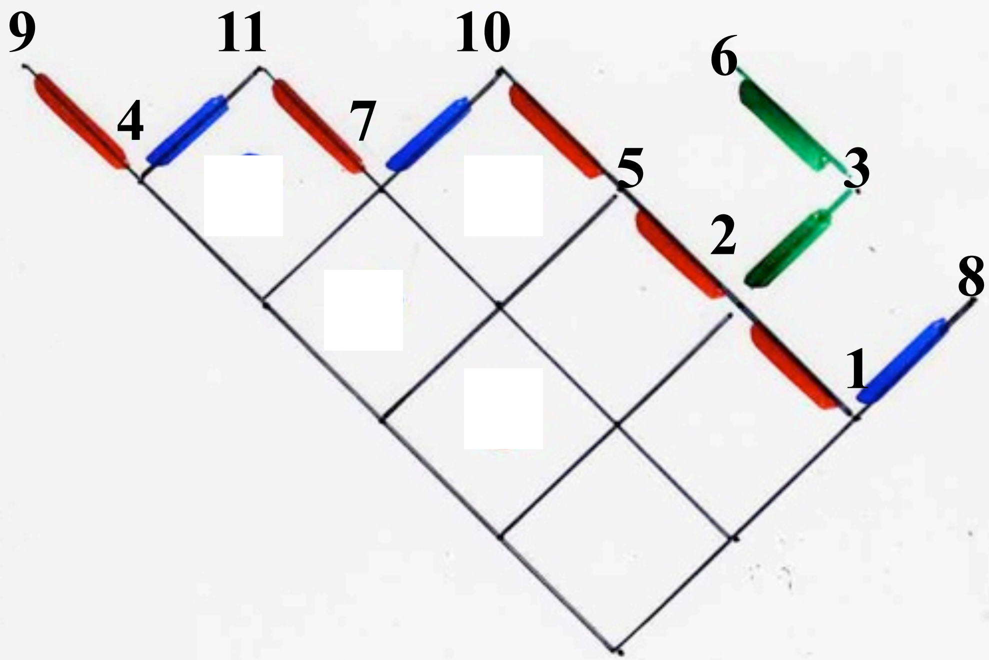

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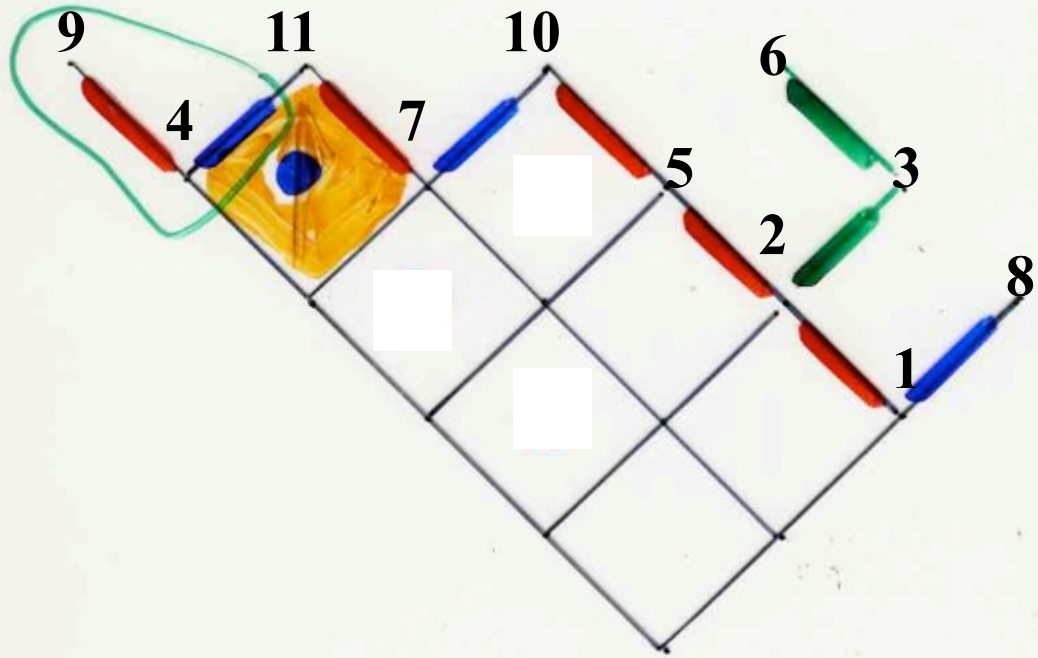

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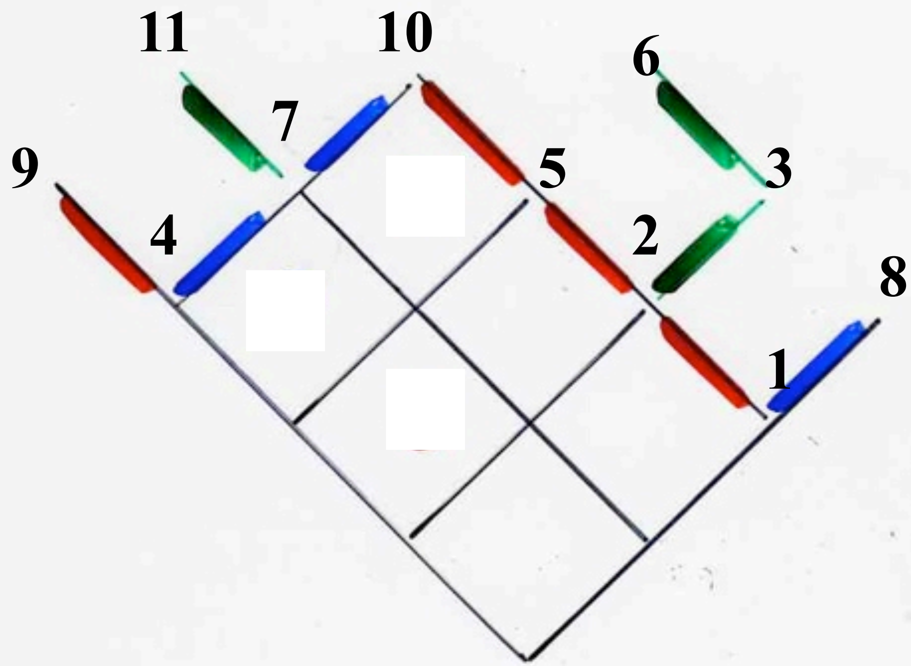

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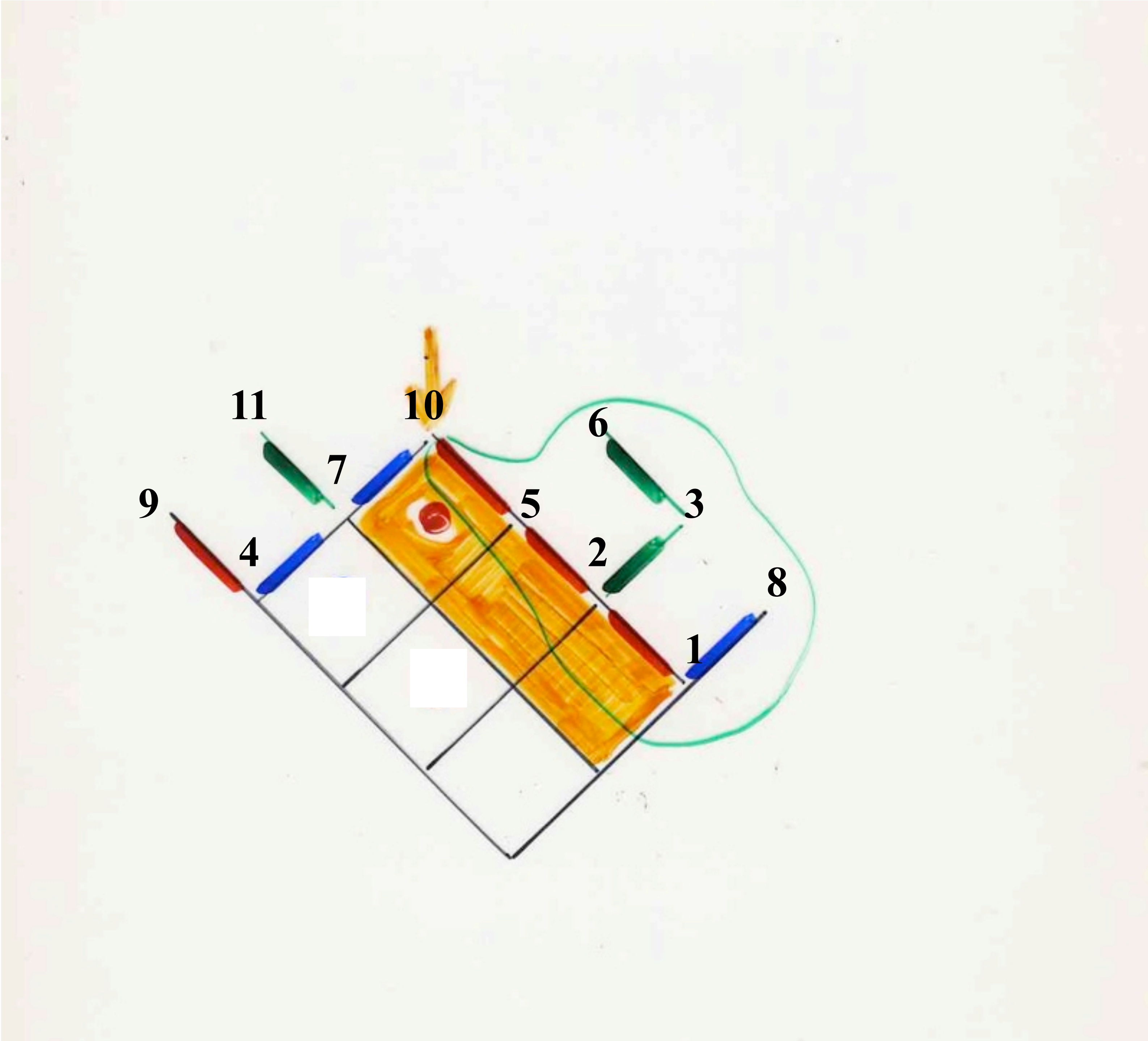

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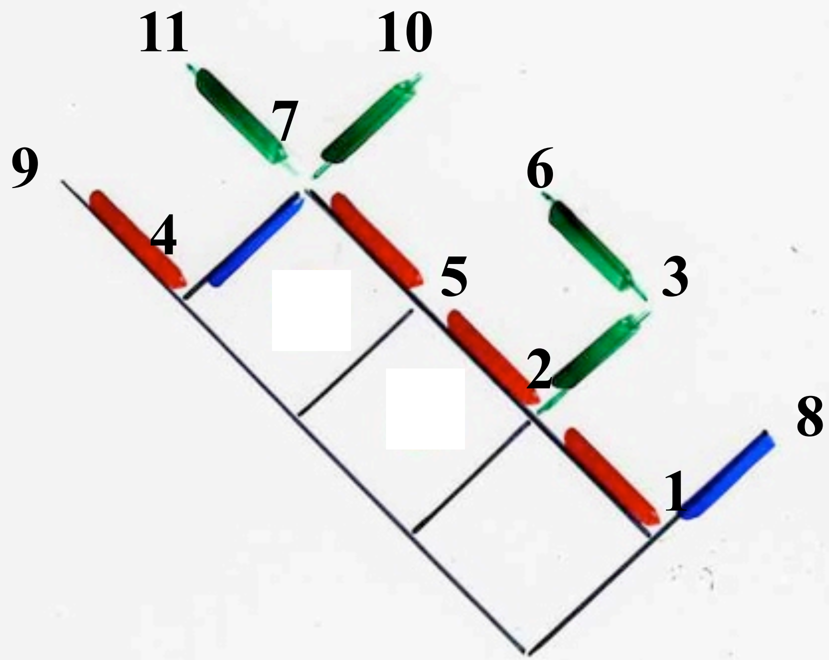

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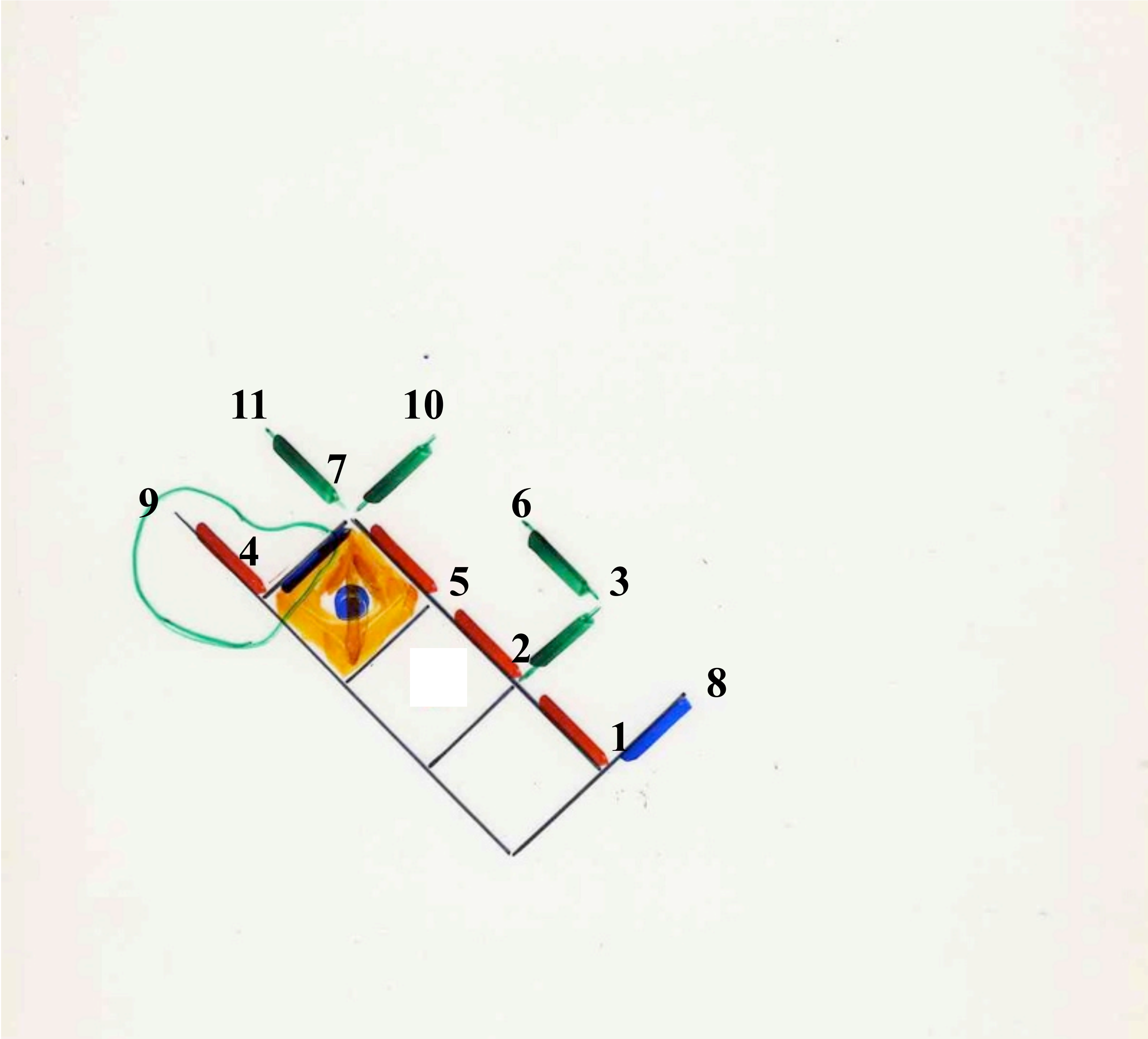

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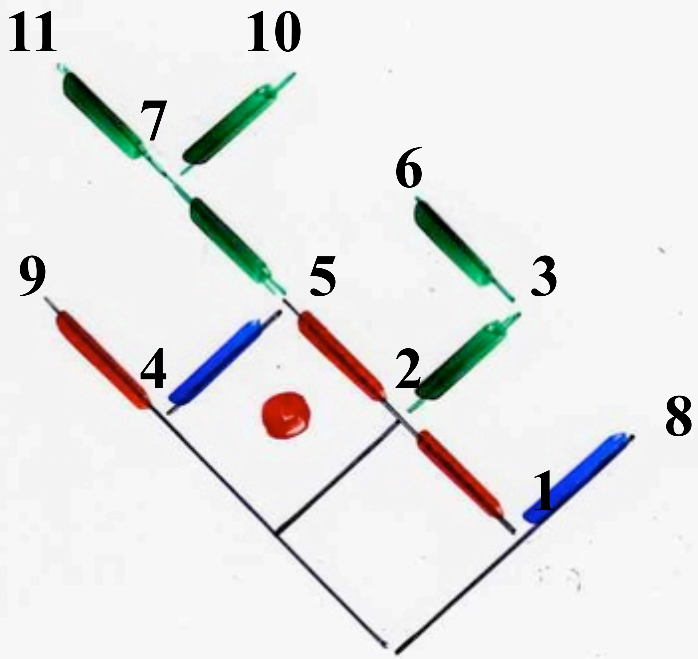
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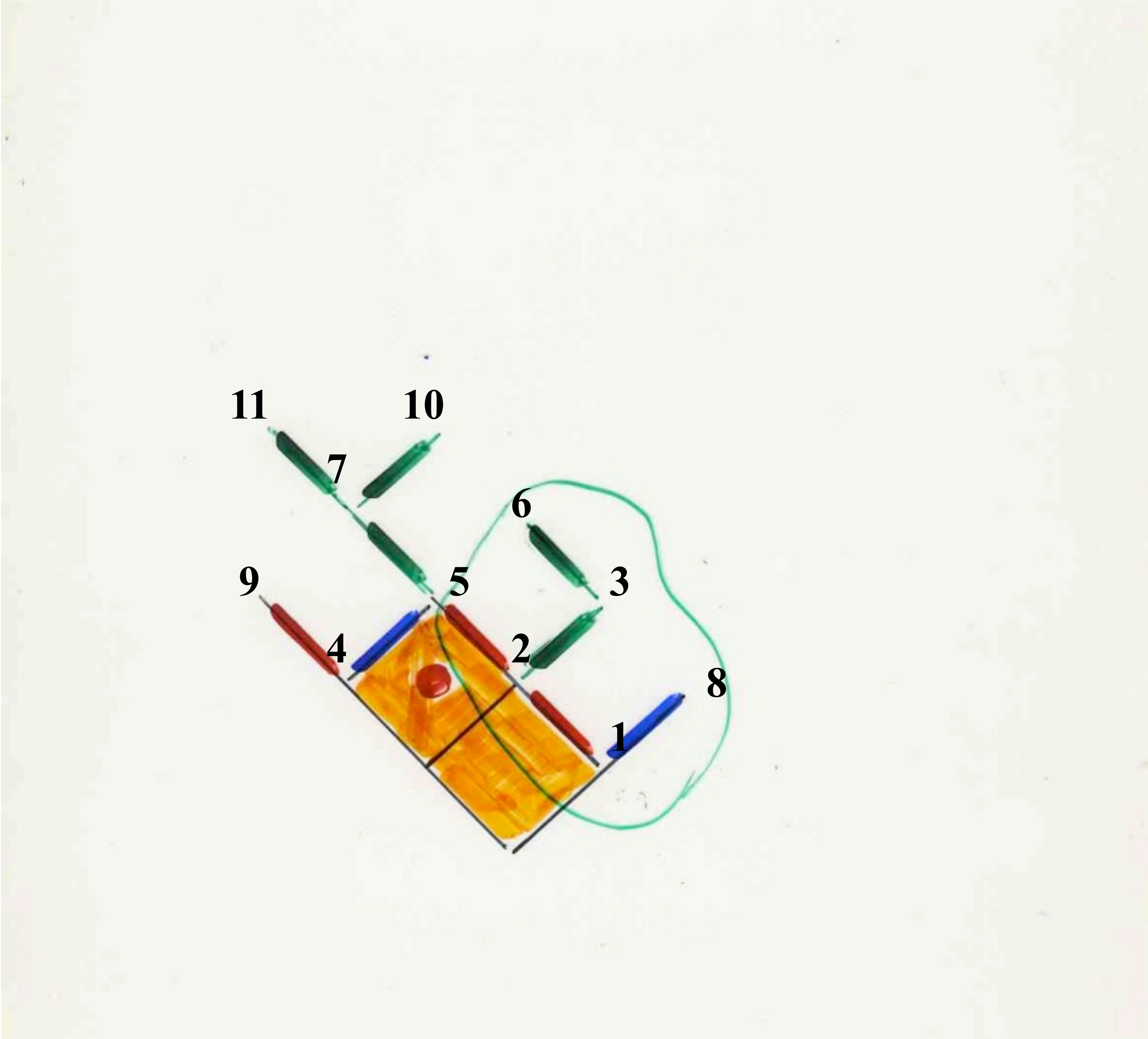

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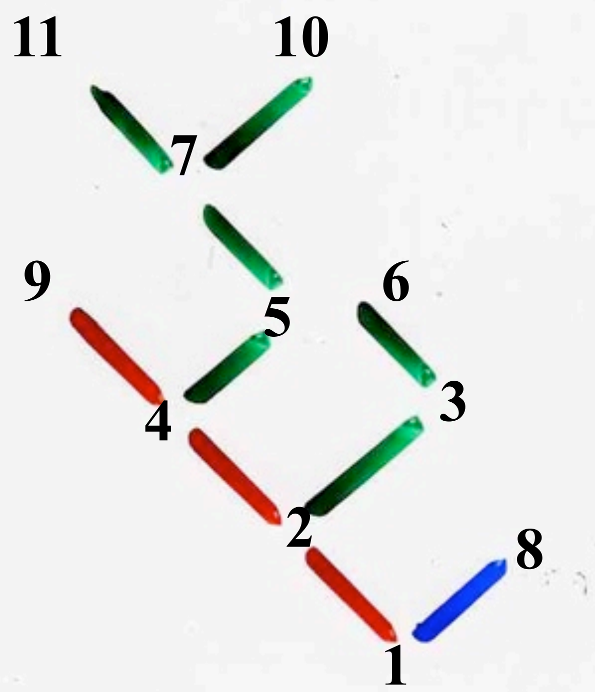
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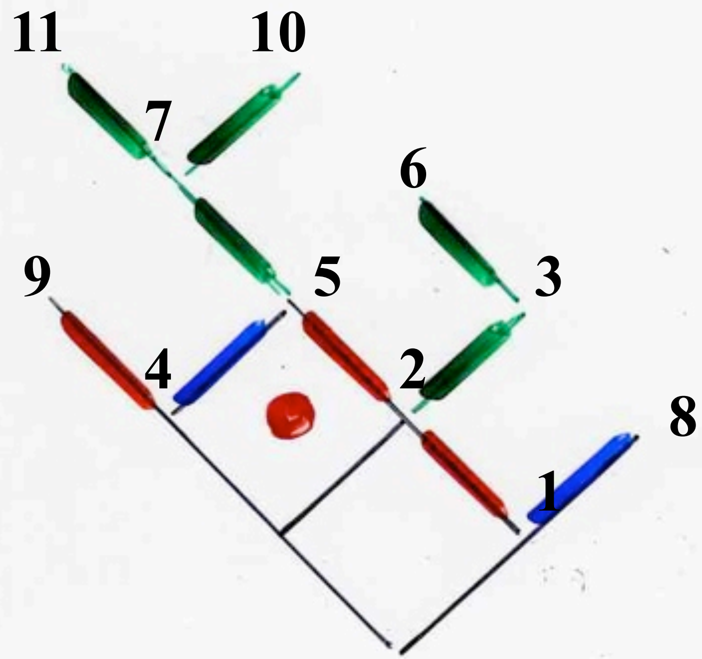

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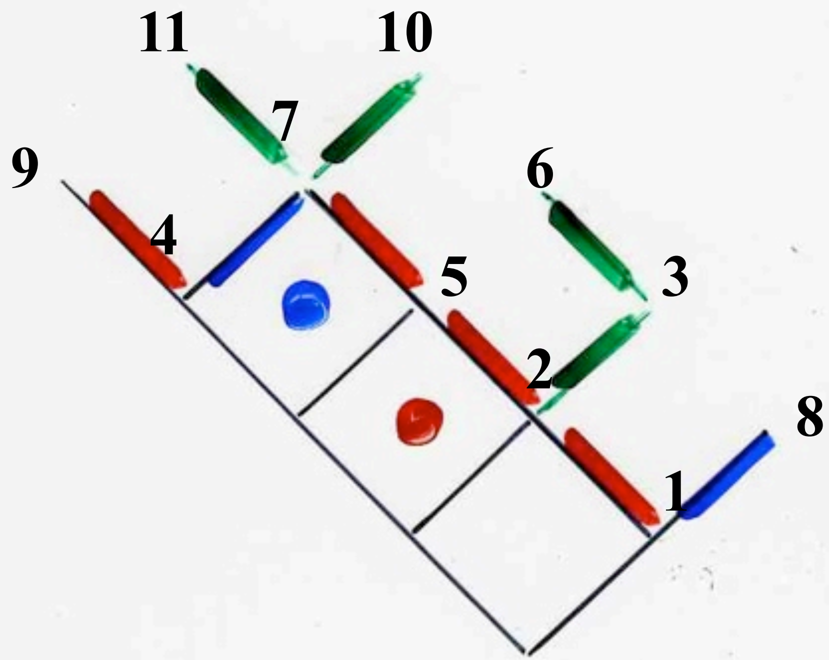
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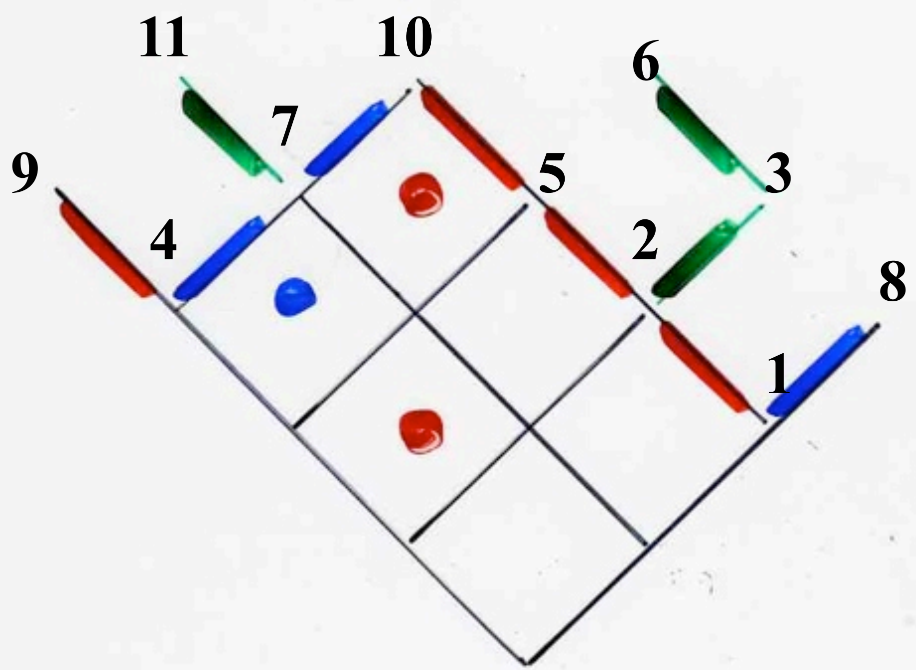

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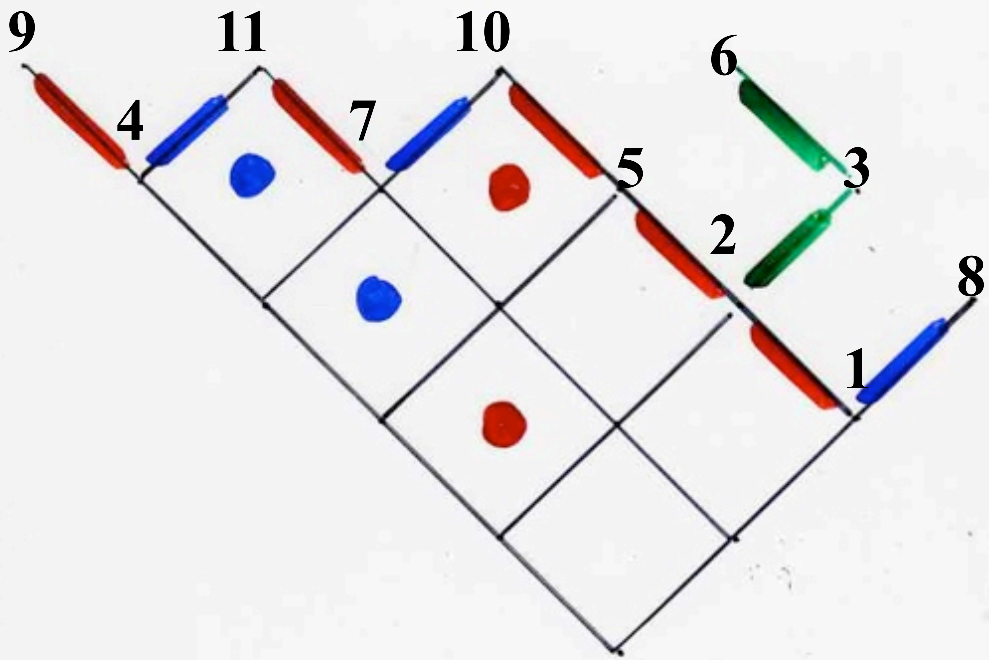
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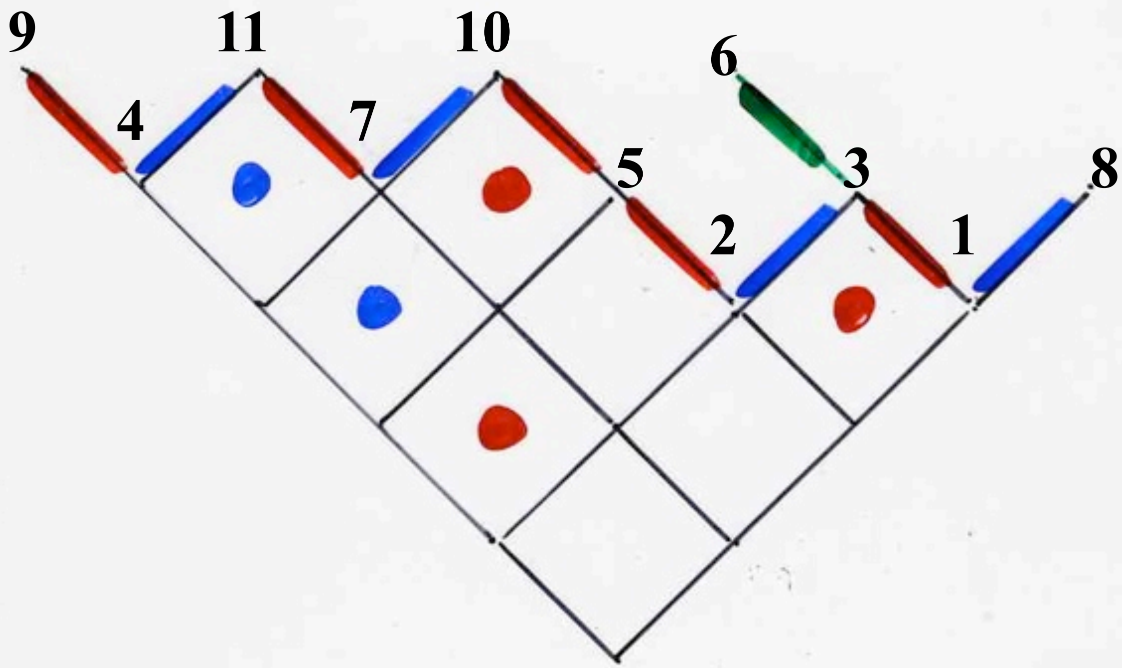
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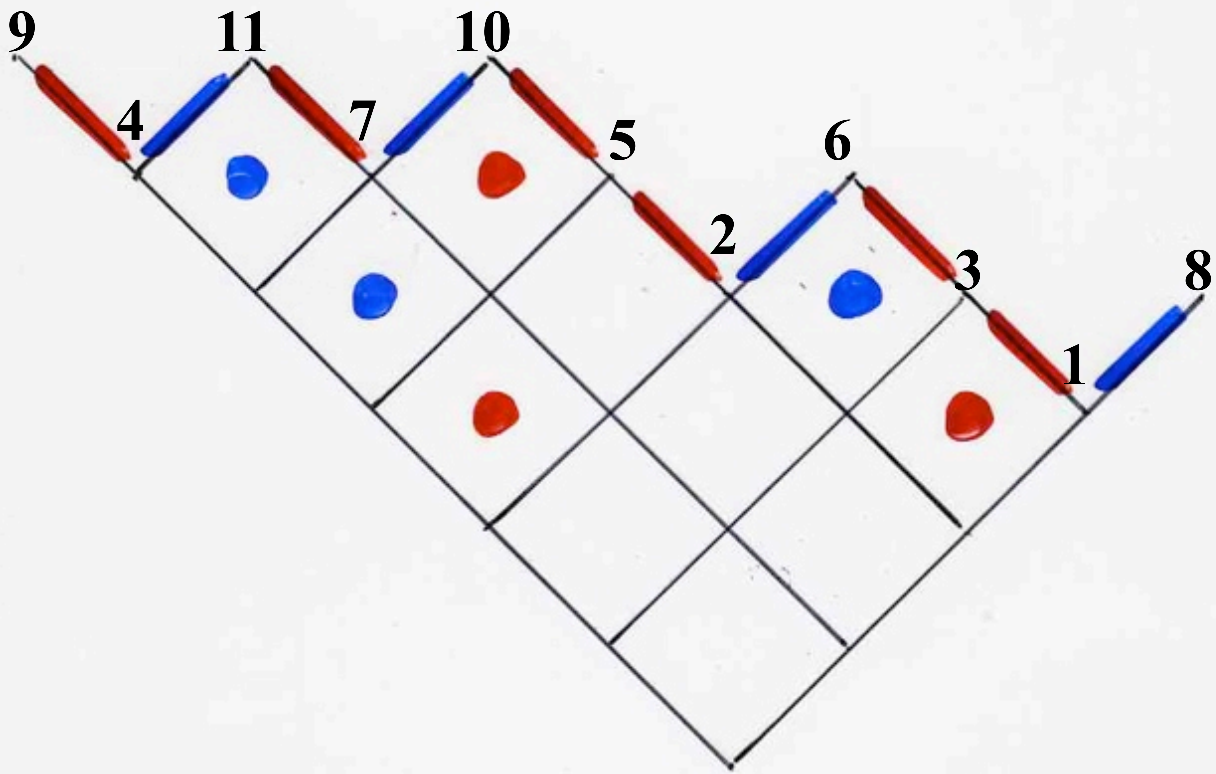

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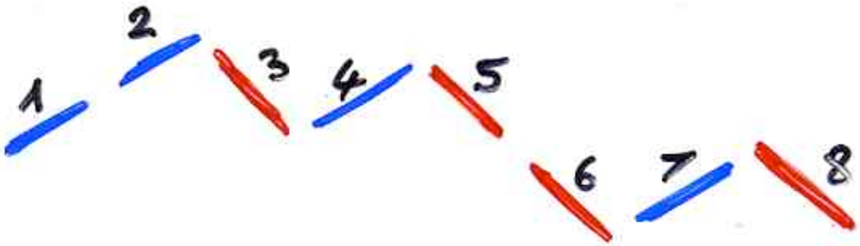
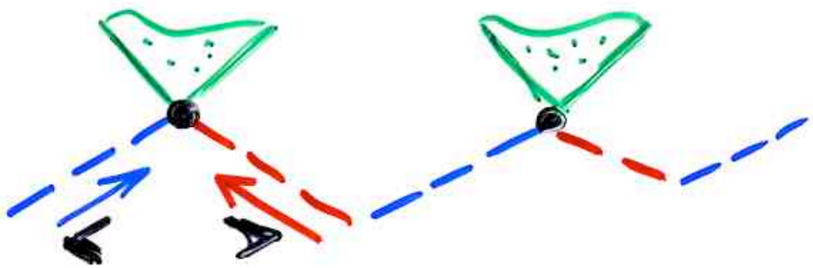
§ 7 alternative binary trees

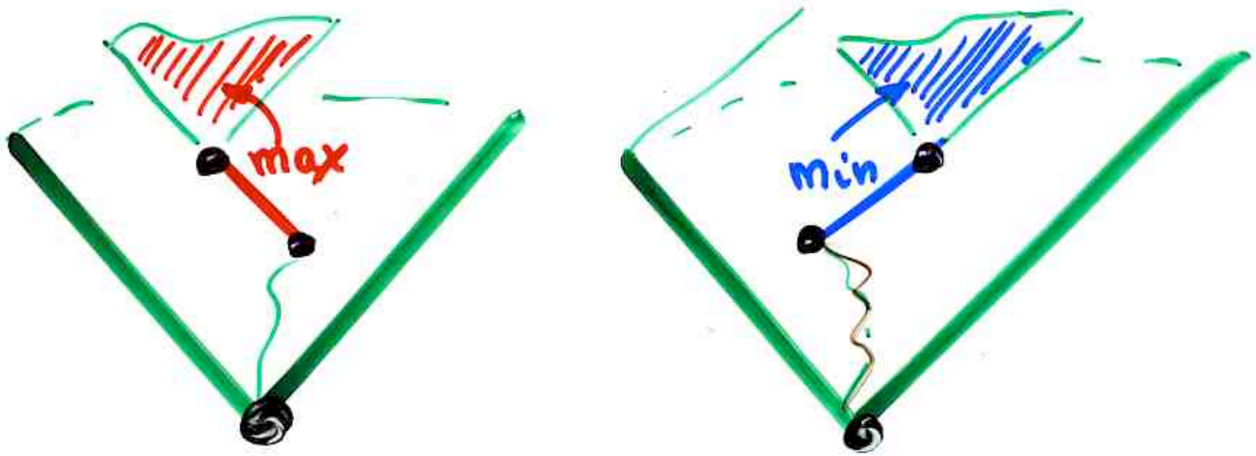


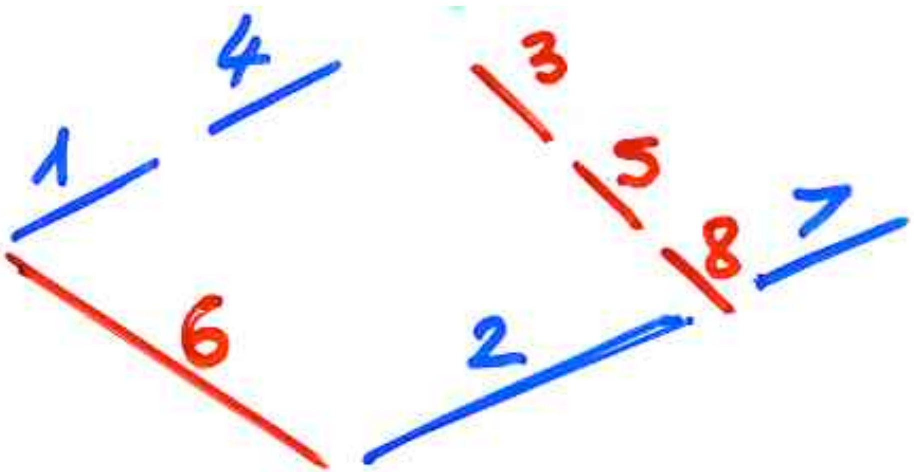
9

Def. edge-alternative woods

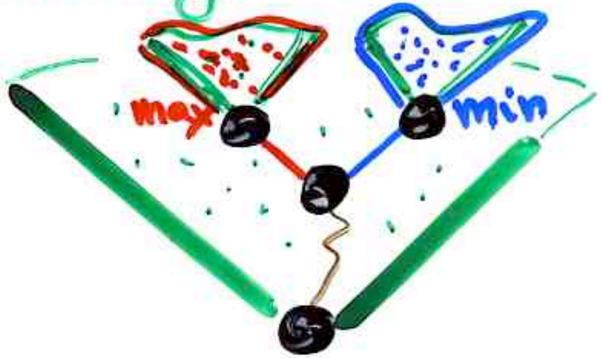


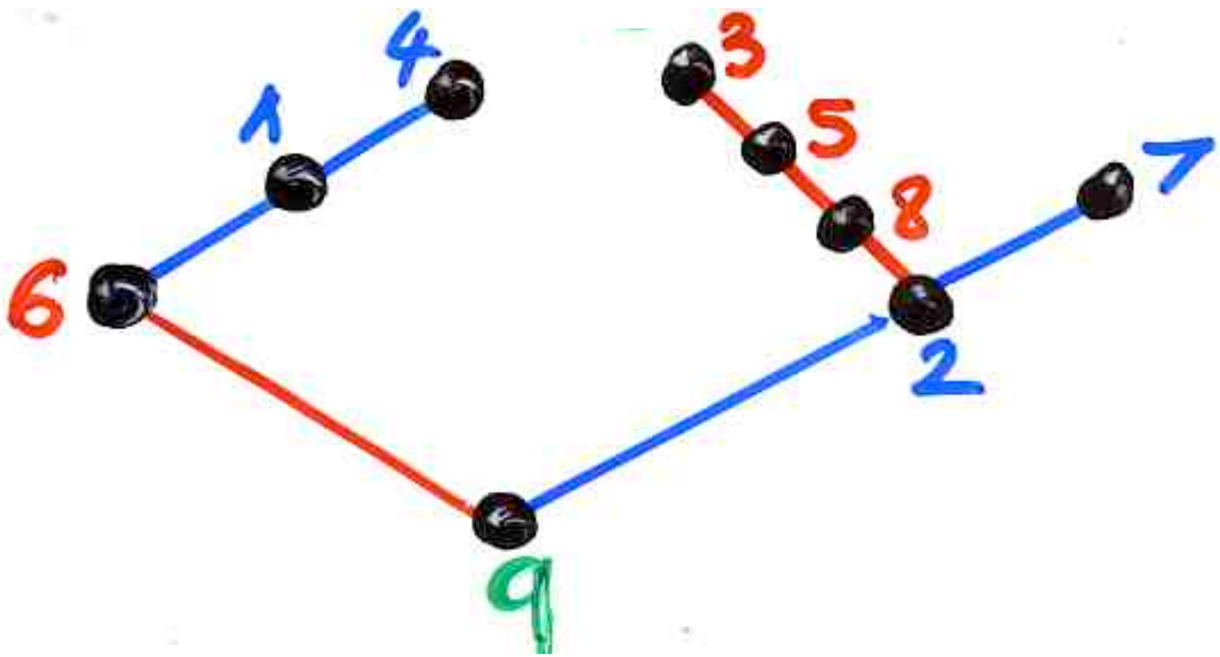
Def edge-alternative binary trees





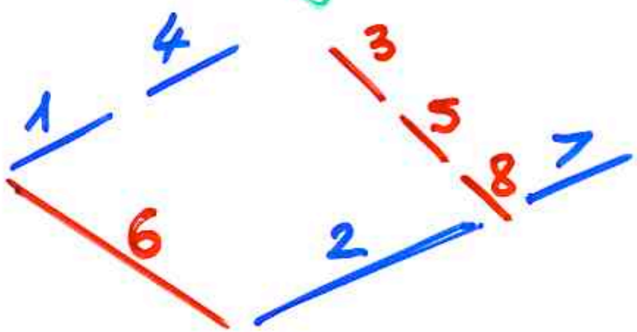
Def alternative binary tree



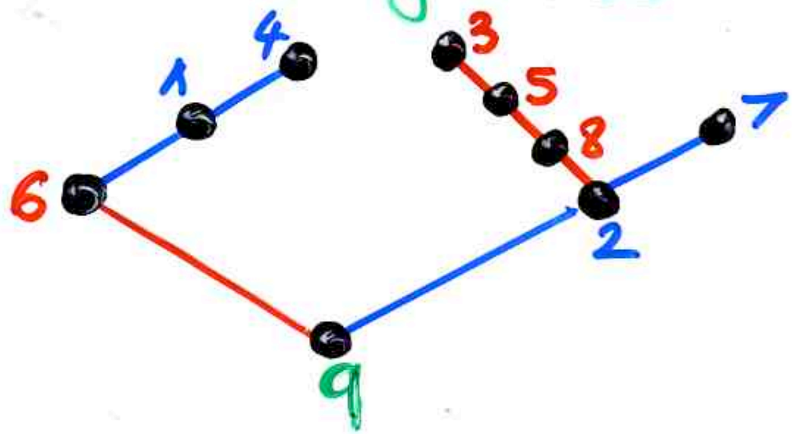


bijection

edge-alternative binary tree

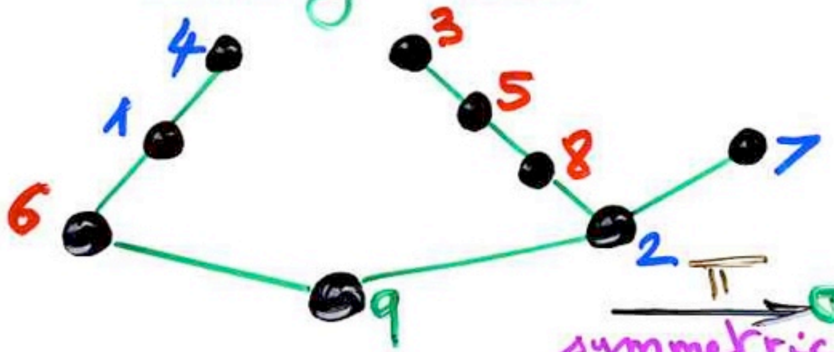


alternative binary tree



bijection

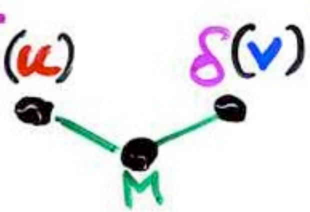
alternative binary trees — permutations



$\xrightarrow[\text{symmetric order (projection)}]{\pi} \sigma = (6\ 1\ 4\ 9\ 3\ 5\ 8\ 2\ 7)$

$M = \max(\sigma)$

$\bar{\delta}(\sigma) =$



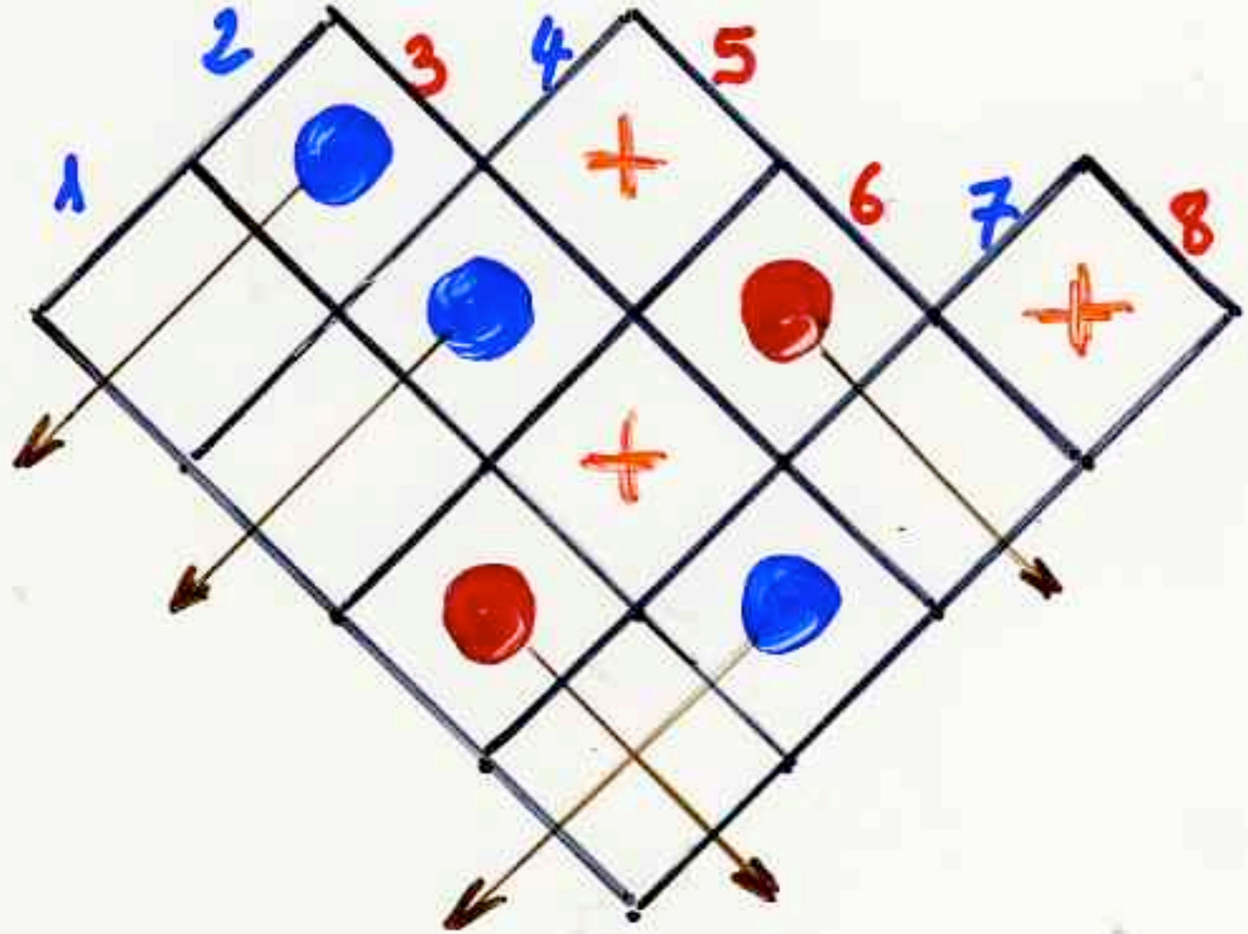
$\xleftarrow{\bar{\delta}}$ $\sigma = u M v$

$\bar{\delta}$ alternative "déployé"

"hook length"
formula
same as for
increasing
binary tree

$$\frac{n!}{\prod_{x} h_x}$$

§ 8
jeu de taquin
for
alternative
binary
trees



1
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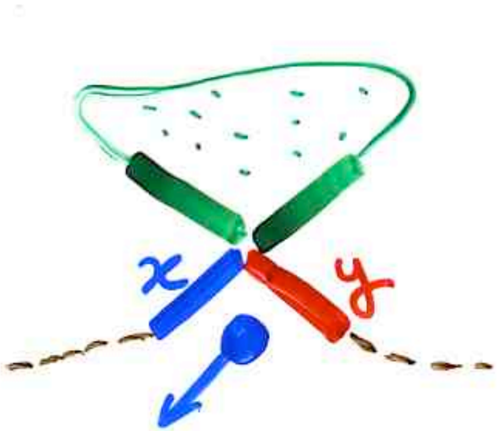
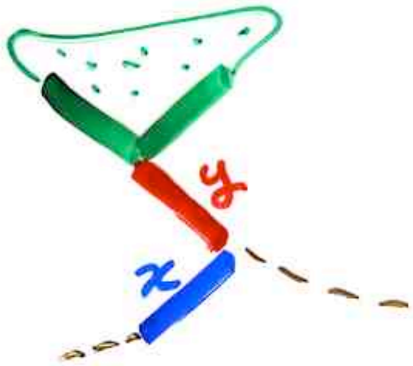
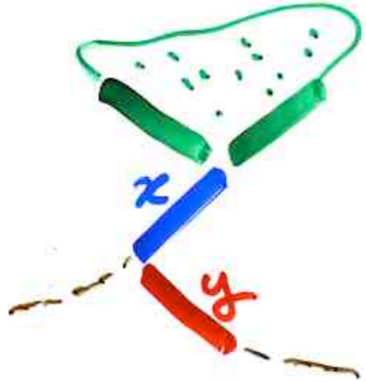
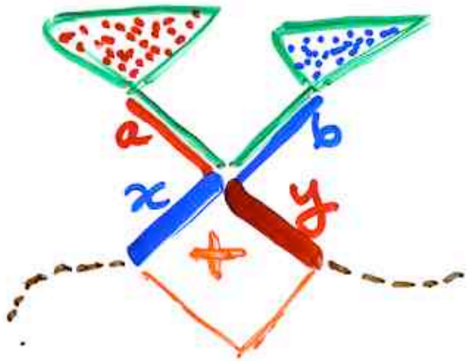

x
y

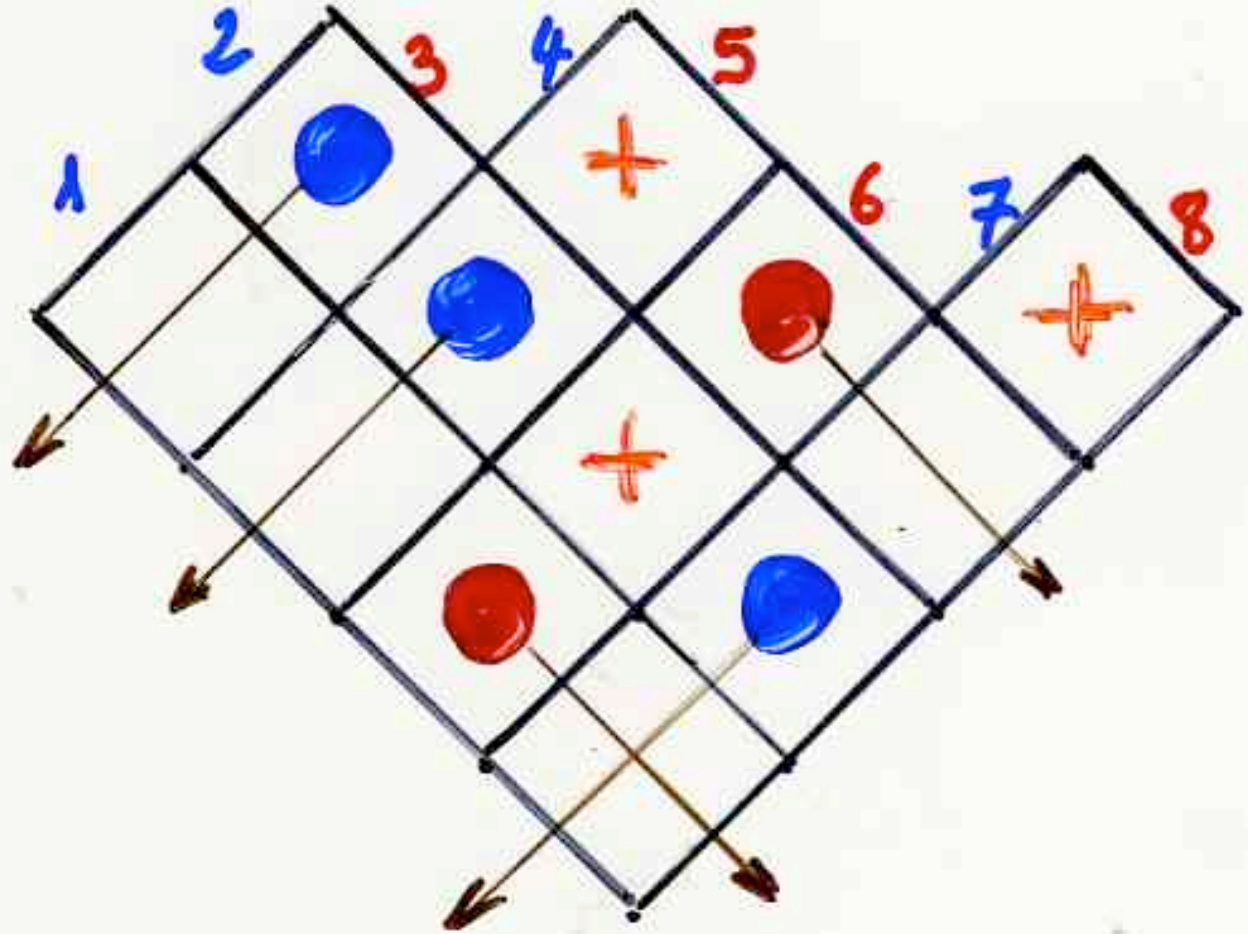

y
x


x
y


x
y


a
b
x
y


b
a
y
x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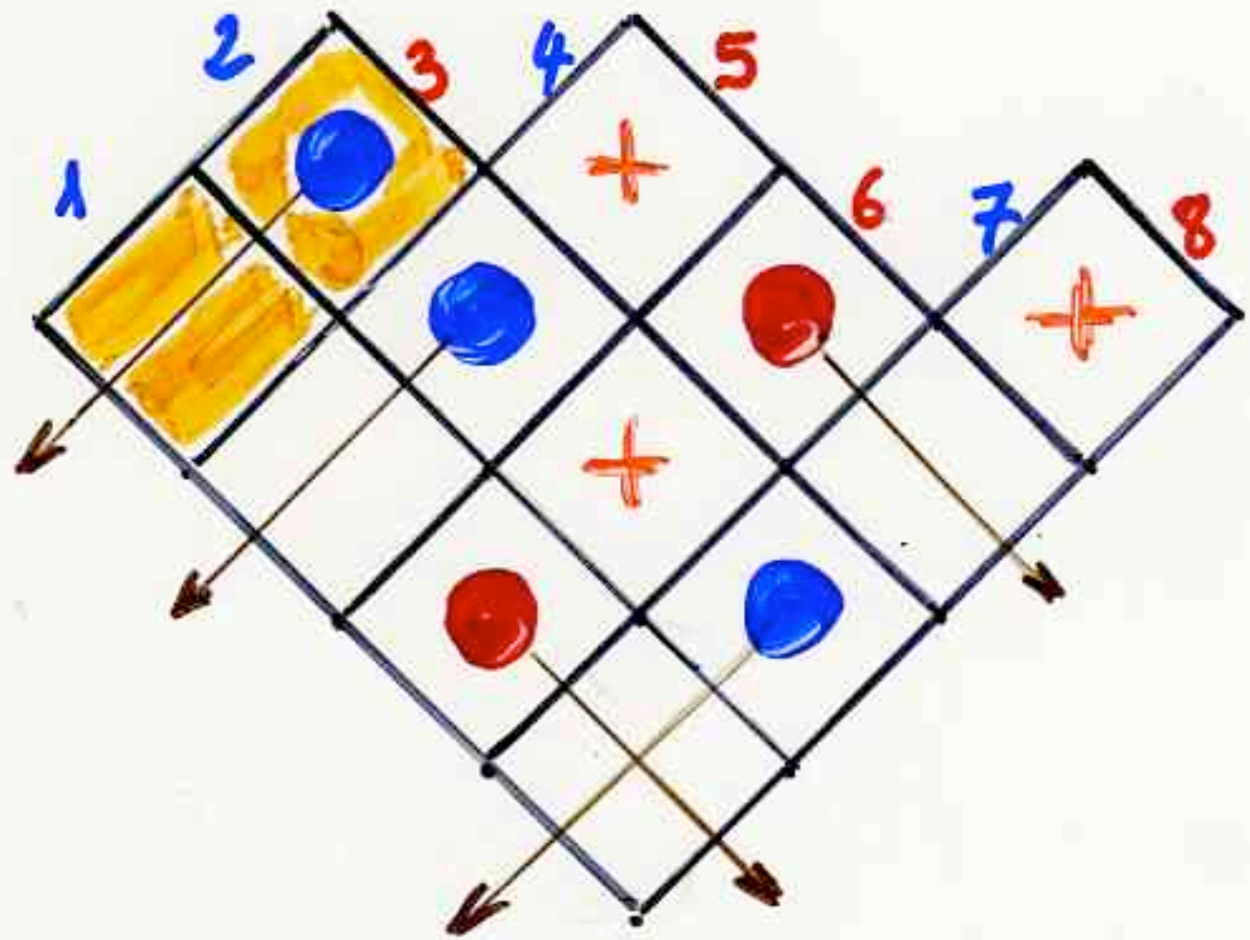
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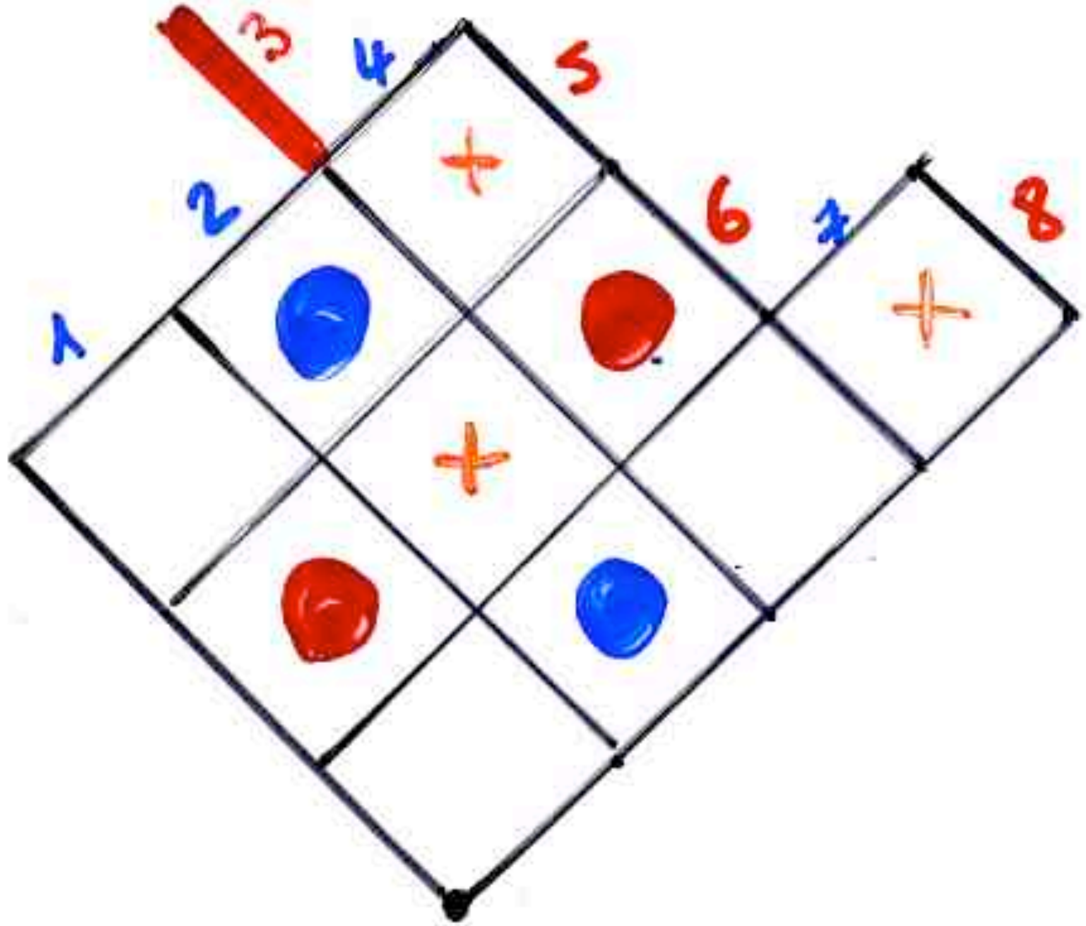
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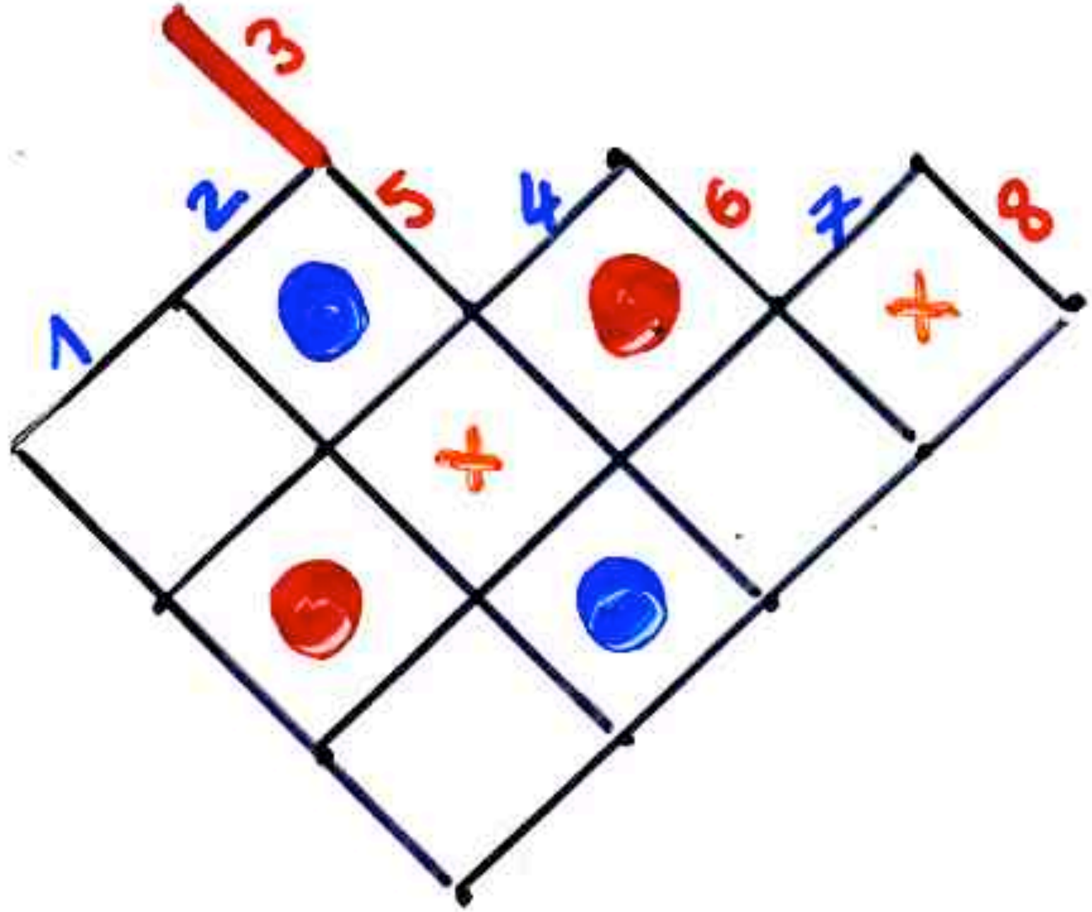
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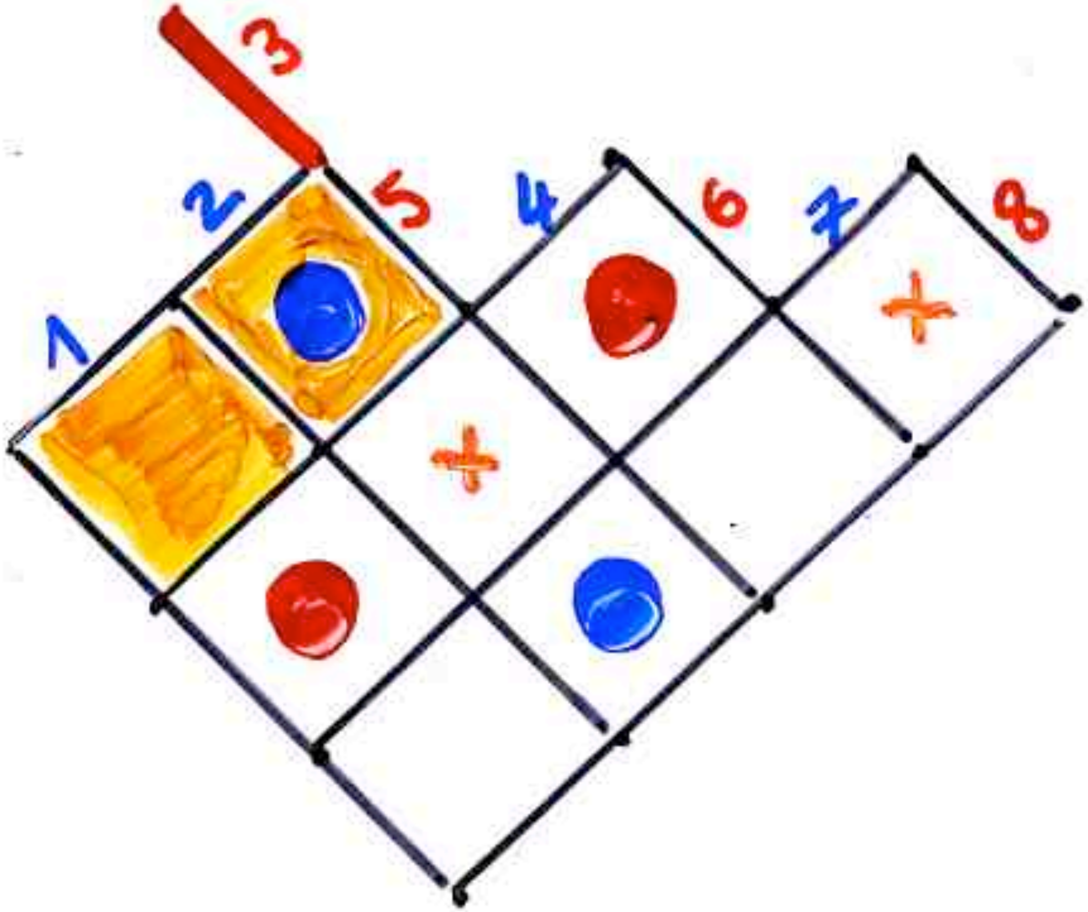
1



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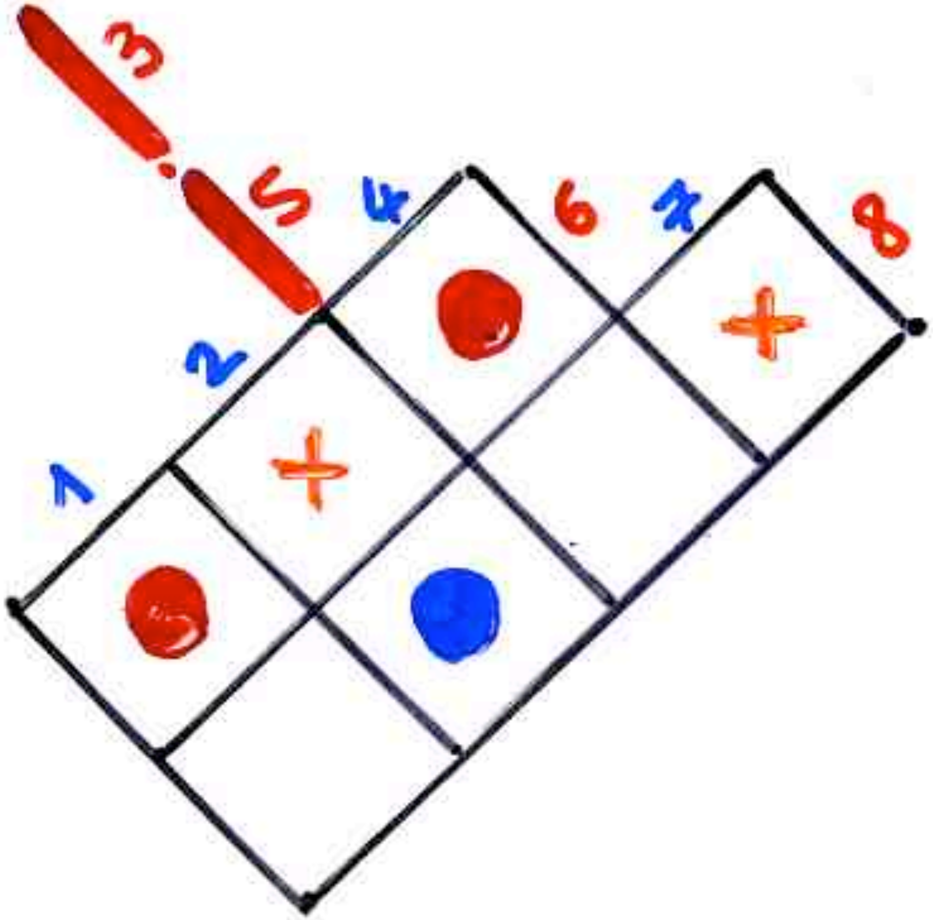
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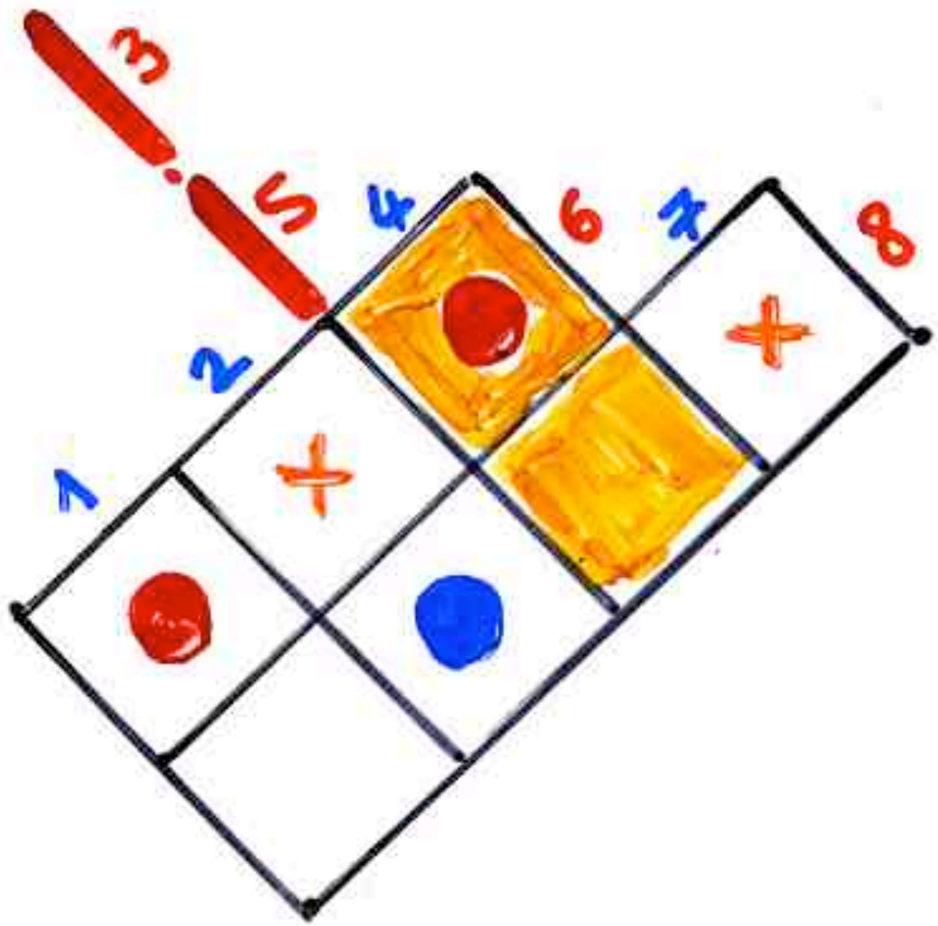

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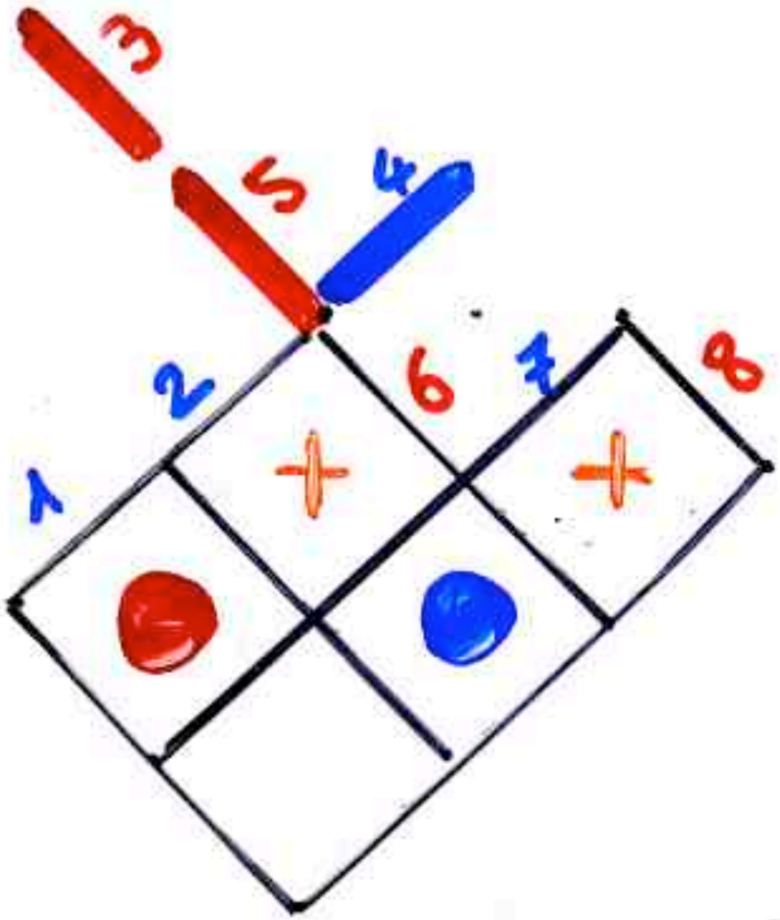

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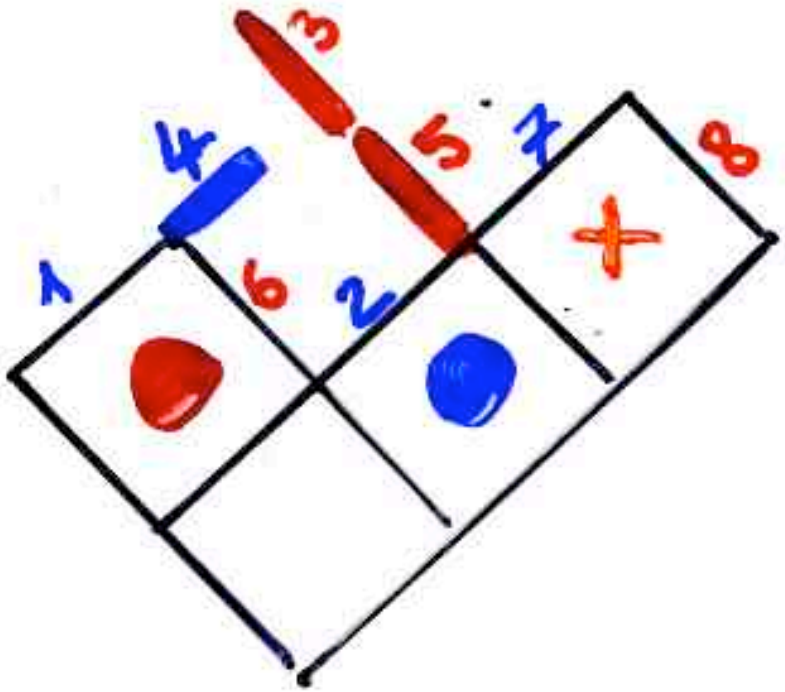

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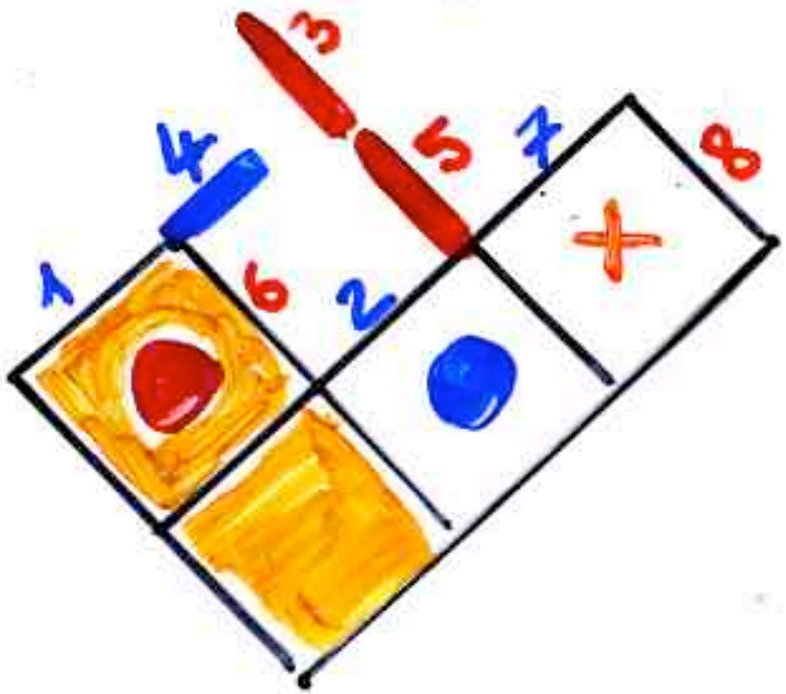

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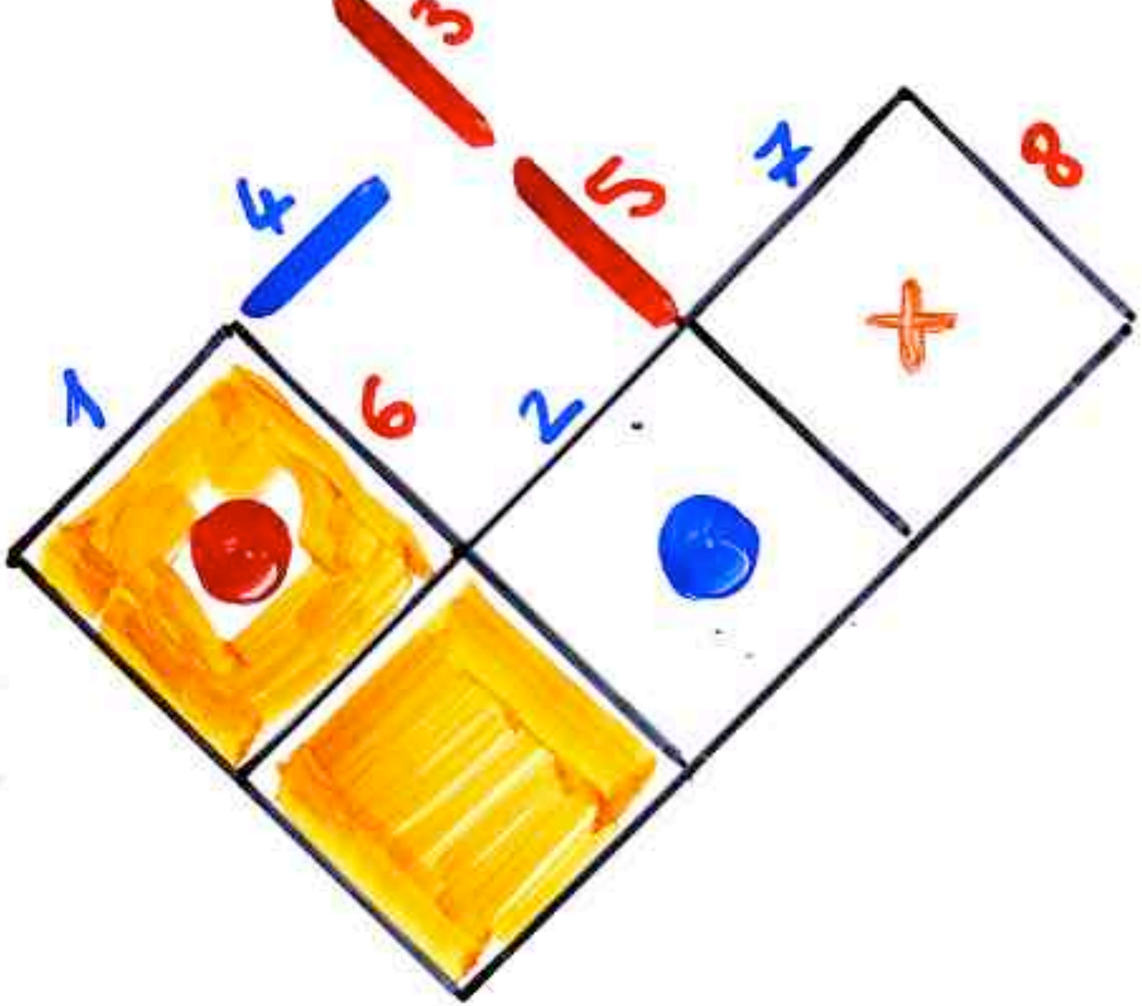
3
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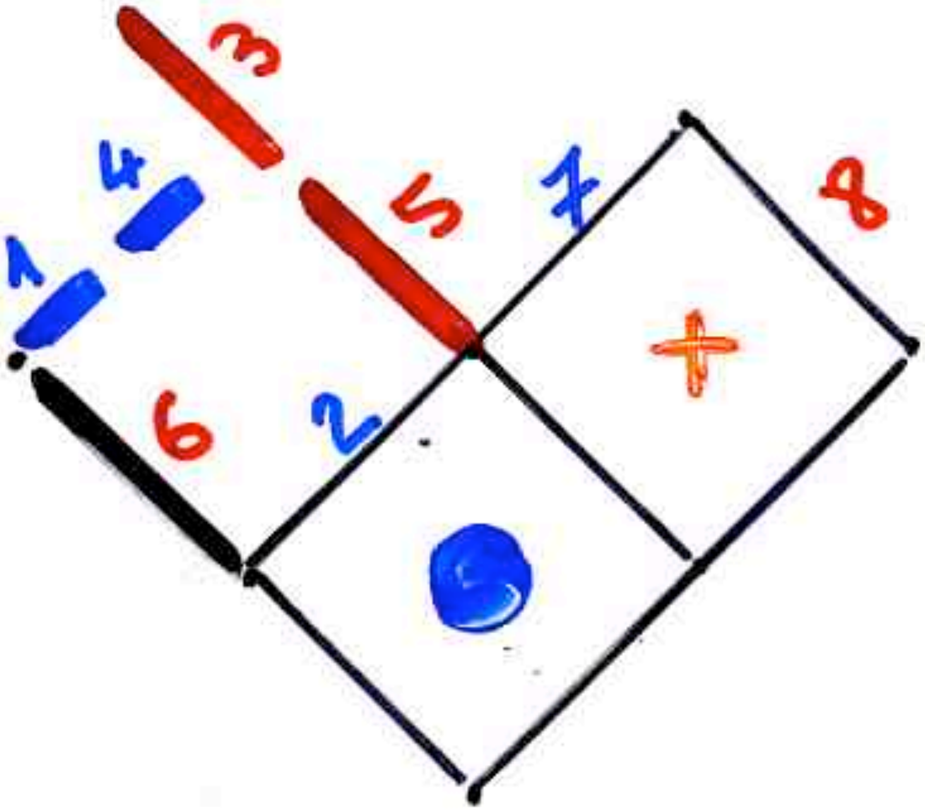
8
1
6
2



3
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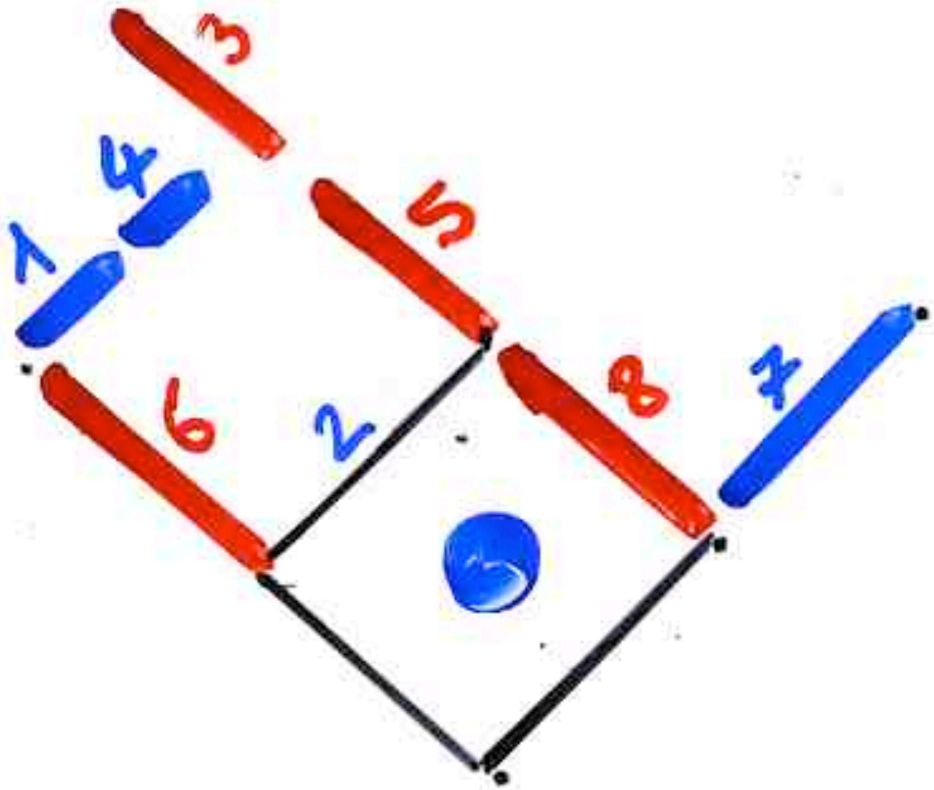
2



3
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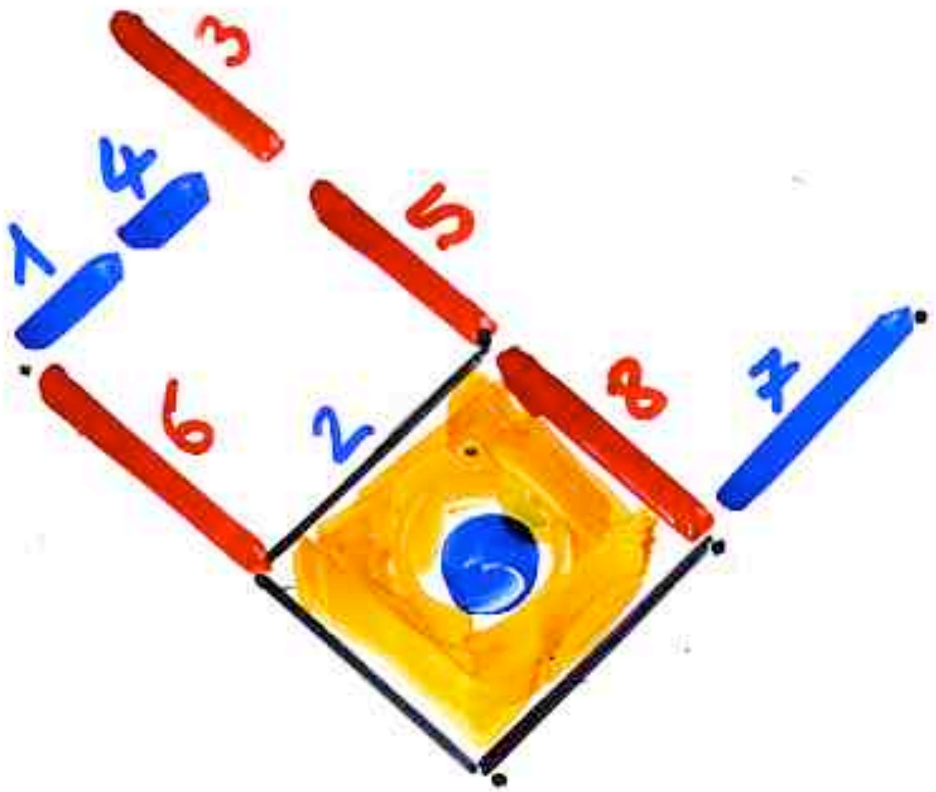

3
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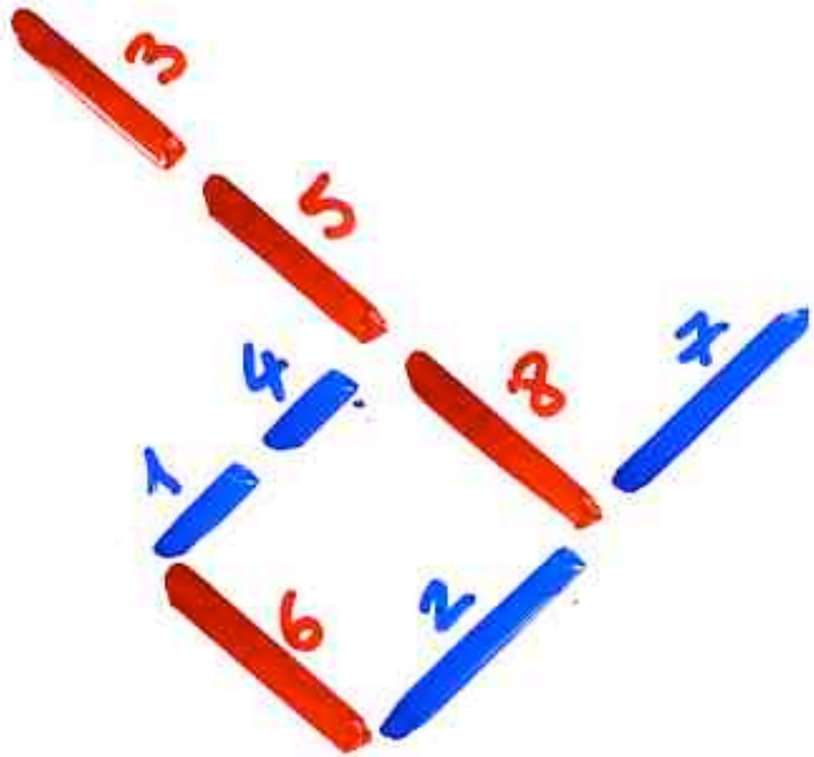
2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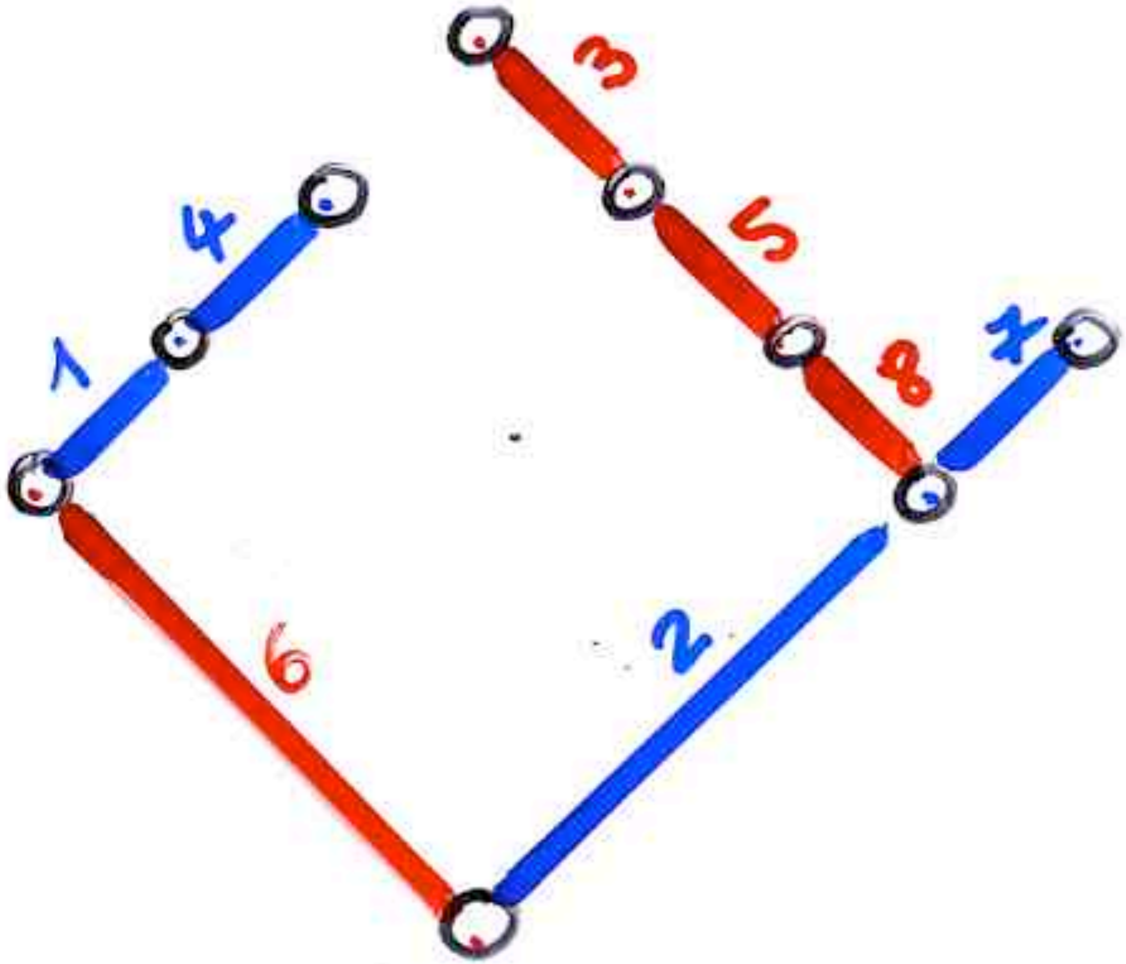
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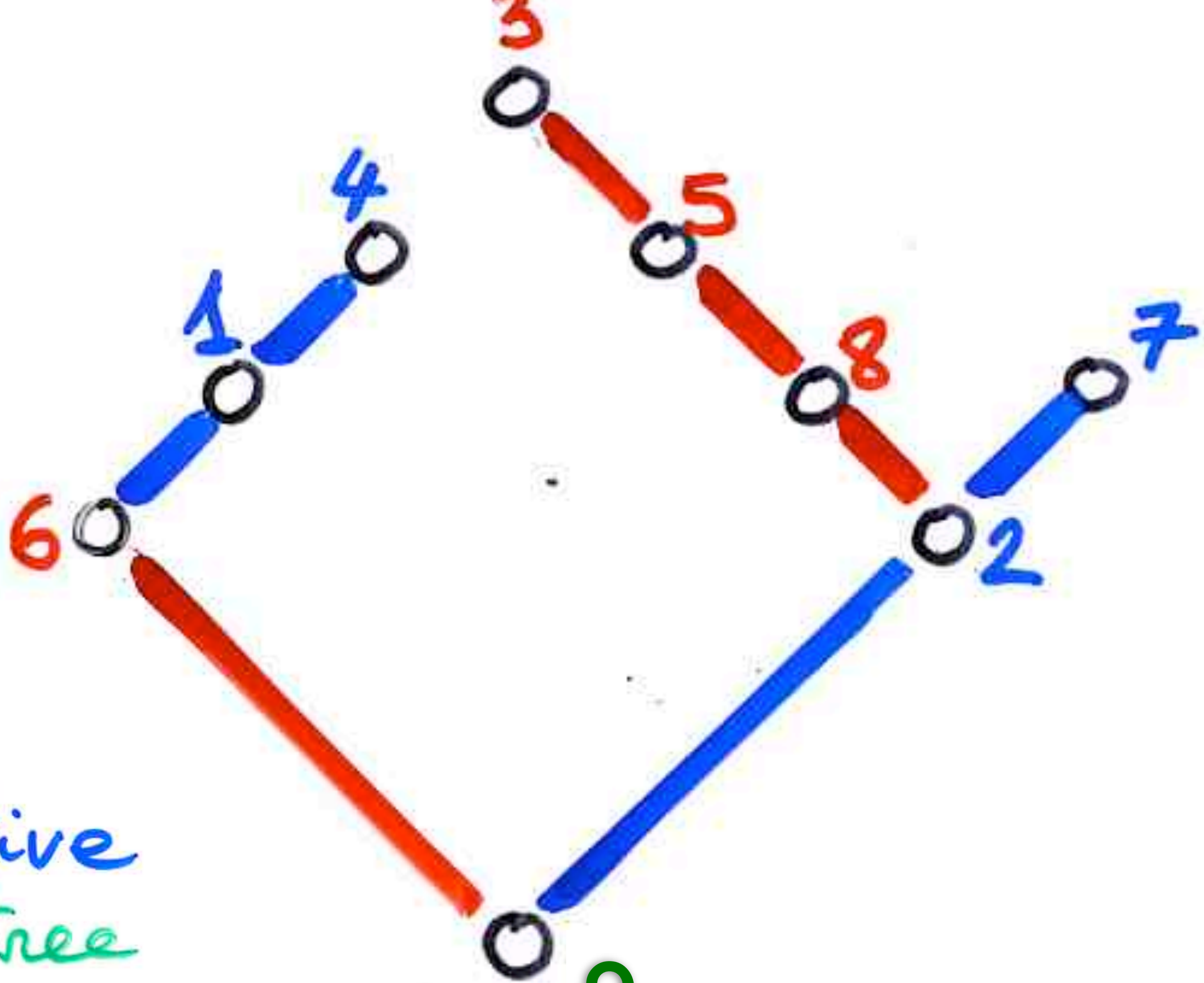
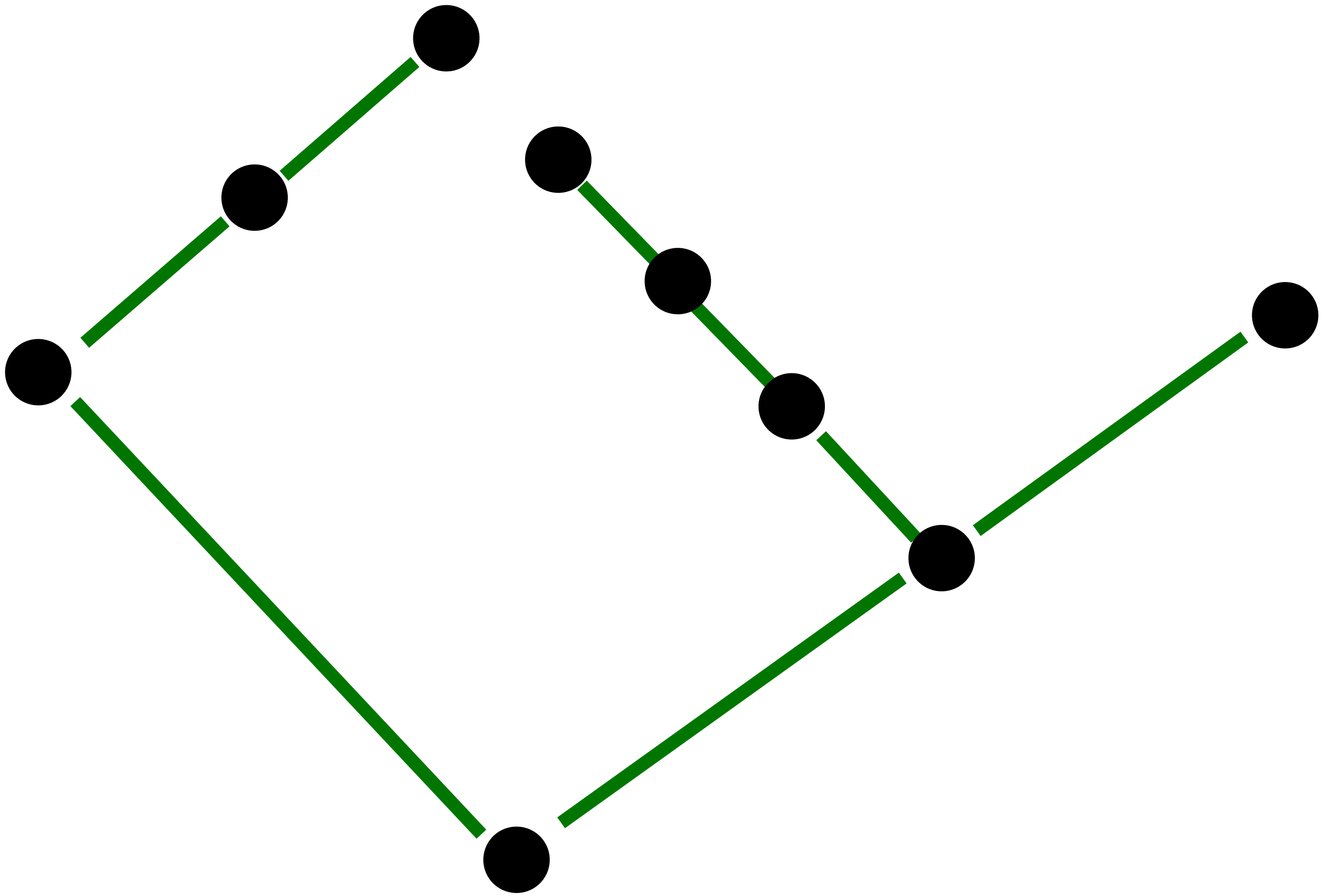
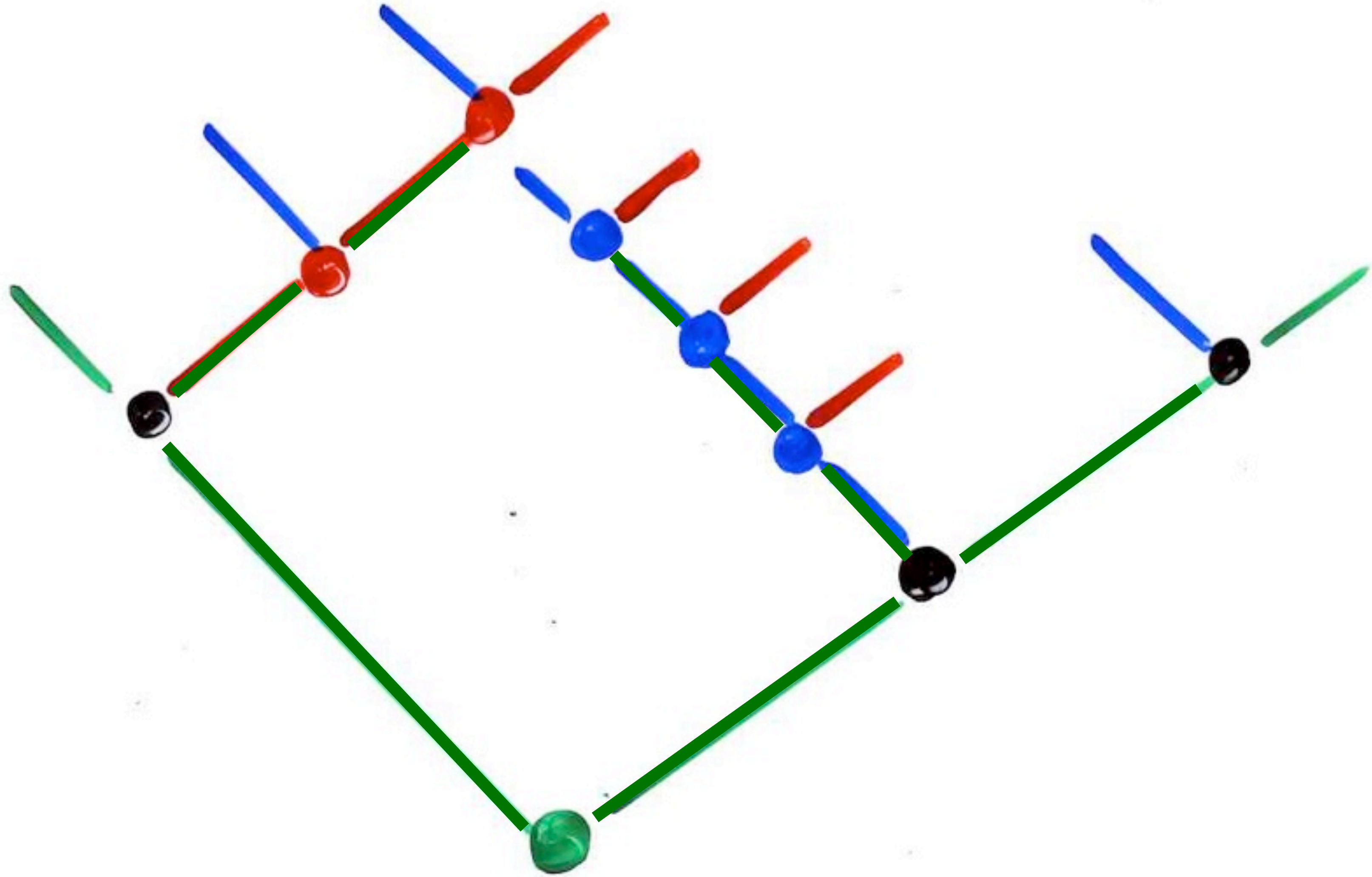
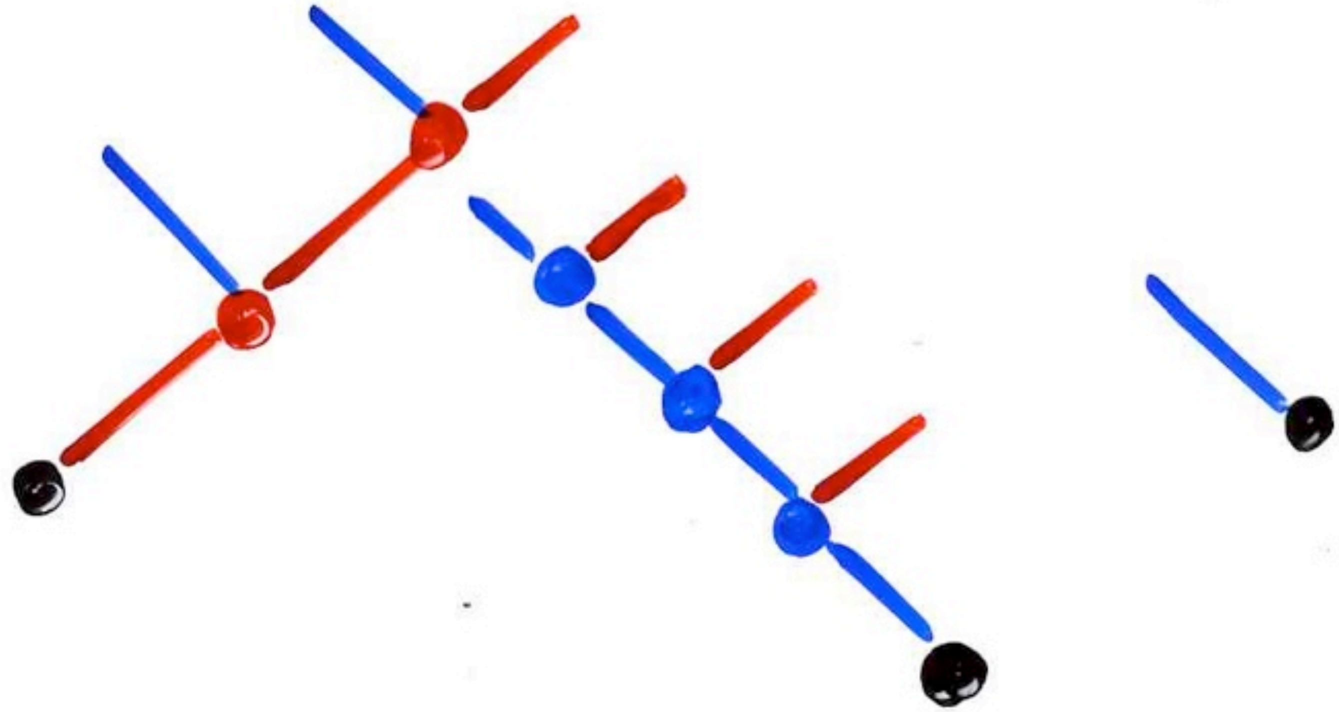


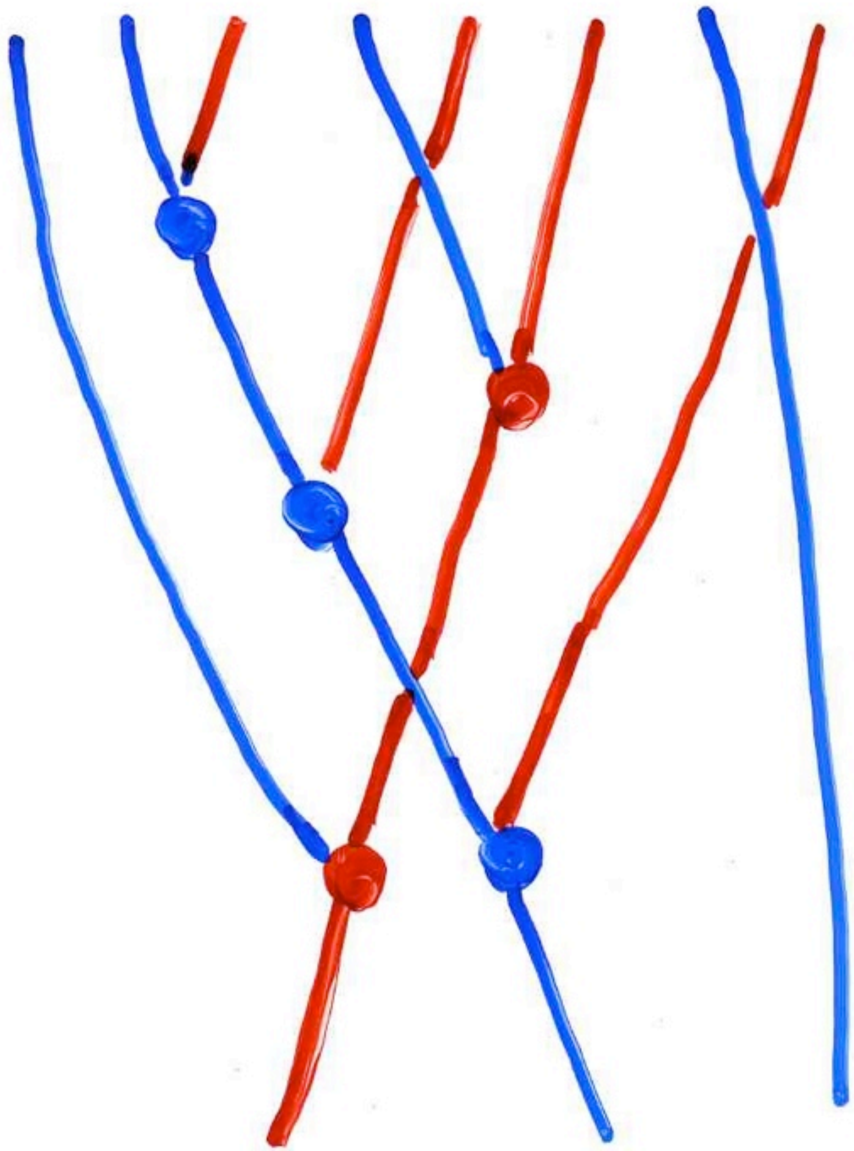
alternative binary tree
(P. Nadeau)

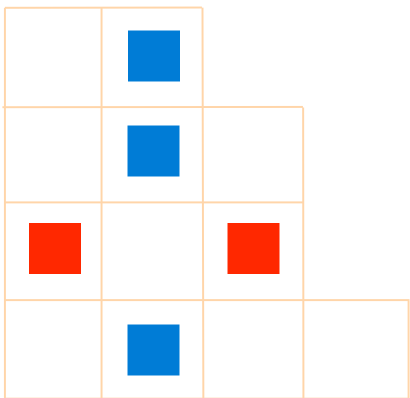
9

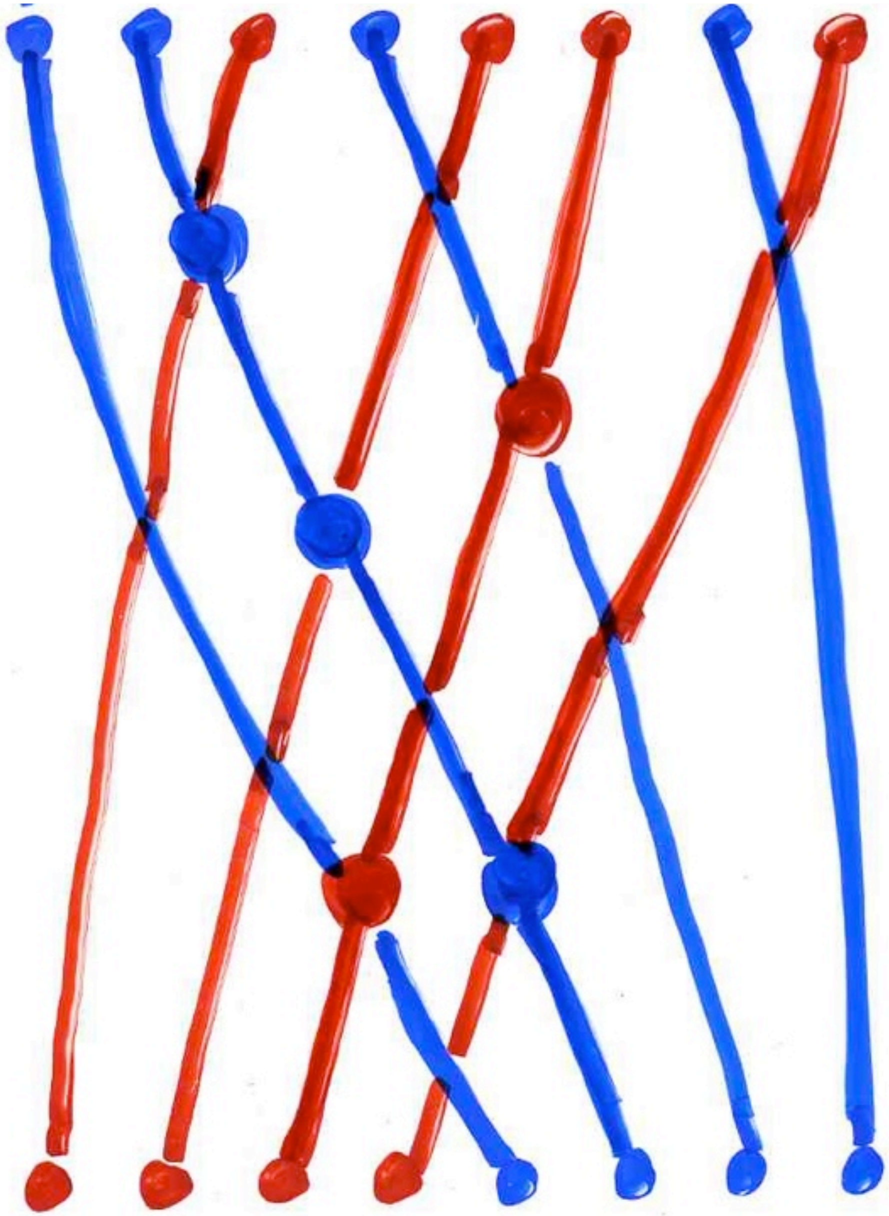






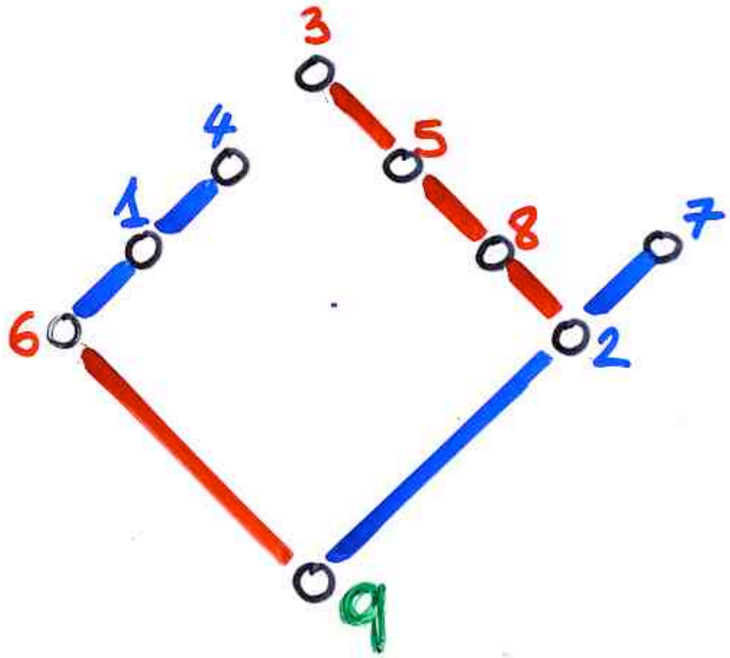


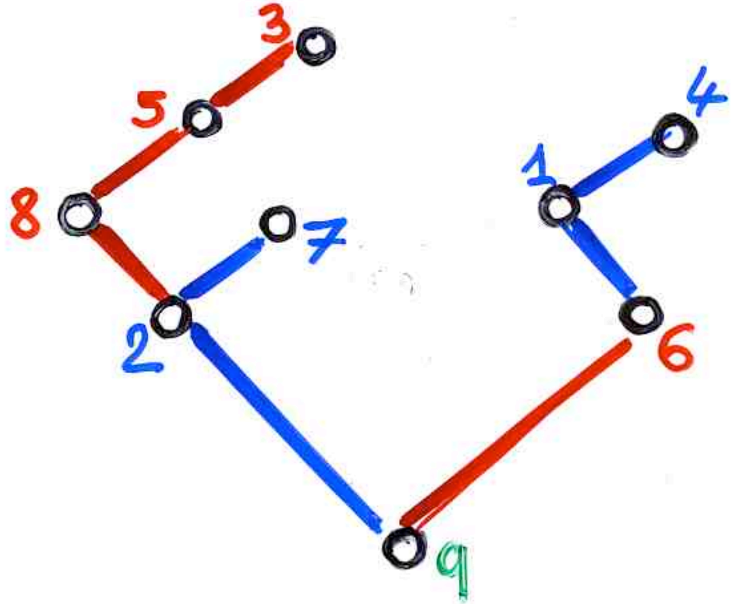




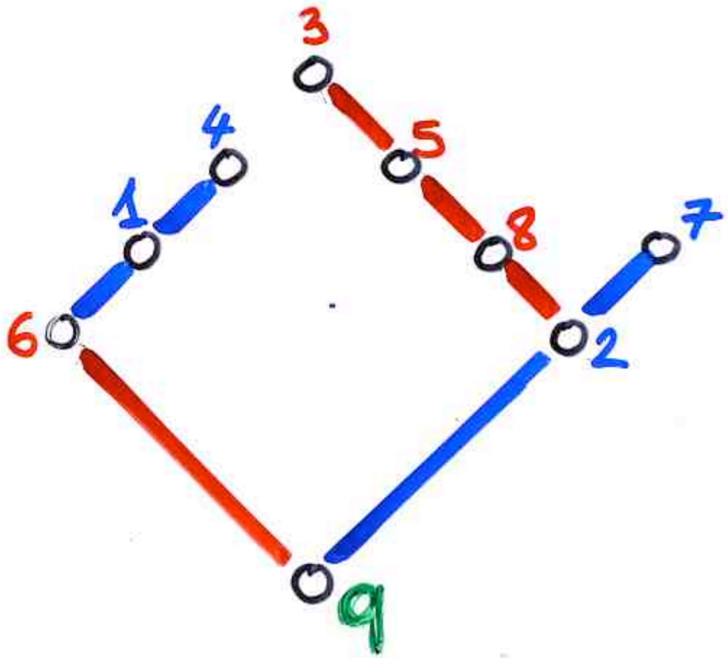
§ 9

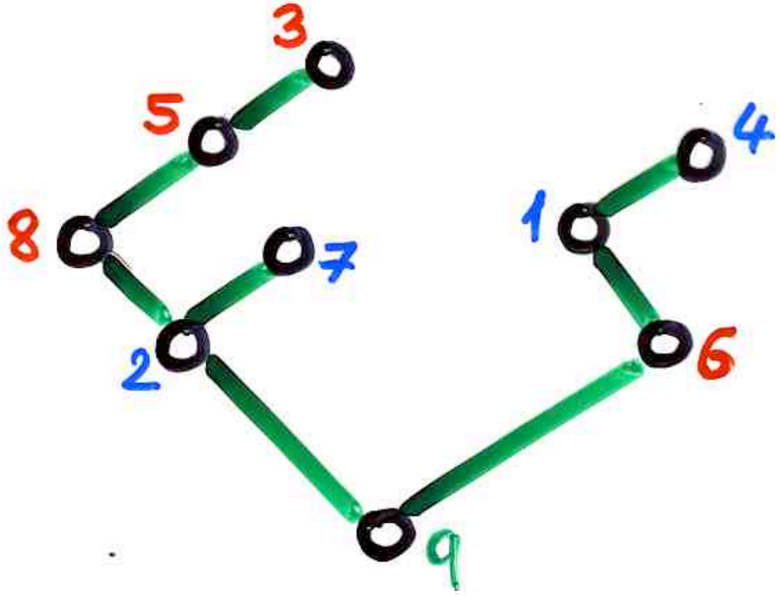
The twisted symmetric order





"twisted"
symmetric
order

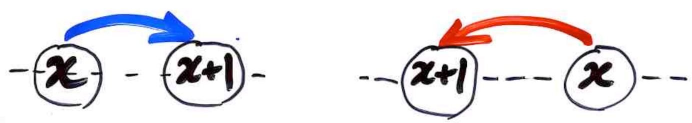


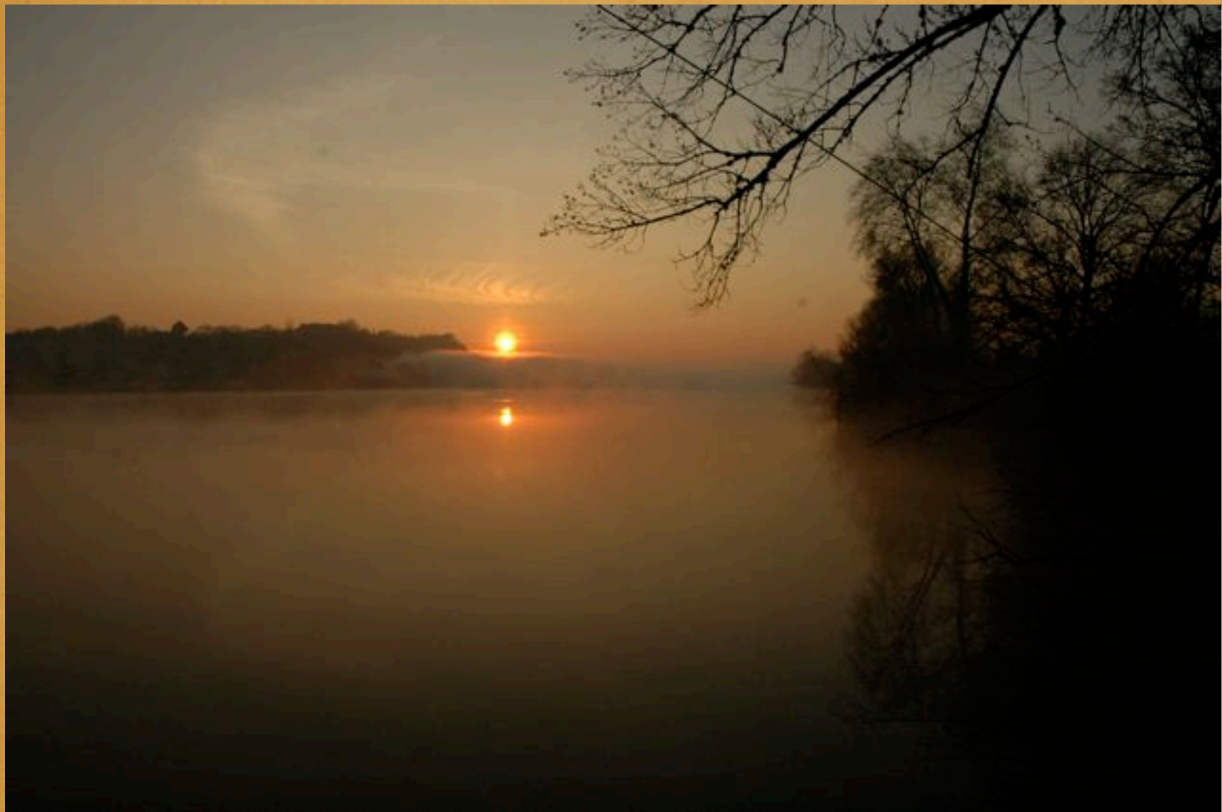


$$\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 8 & 5 & 3 & 2 & 7 & 9 & 1 & 4 & 6 \end{pmatrix}$$

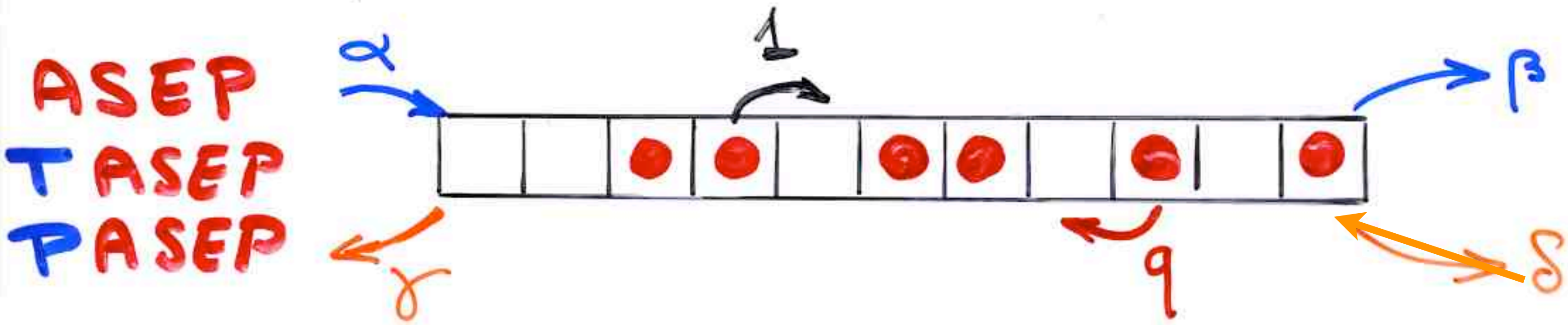
"twisted"
symmetric
order

$$\sigma^{-1} = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 7 & 4 & 3 & 8 & 2 & 9 & 5 & 1 & 6 \end{pmatrix}$$





§ 10 The PASEP


ASEP
TASEP
PASEP
α
1
β
γ
q
δ

boundary induced phase transitions

molecular diffusion
linear array of enzymes
biopolymers
traffic flow
------
formation of shocks

$$P_n(\tau_1, \ldots, \tau_n) = f_n(\tau_1, \ldots, \tau_n) / Z_n$$

$$Z_n = \sum_{\tau} f_n(\tau_1, \ldots, \tau_n)$$

partition function

Derrida, Evans, Hakim, Pasquier (1993)

"matrix ansatz"

D E matrices,

V column vector, W row vector

$$\begin{cases} DE = qED + D + E \\ (\beta D - \delta E)|V\rangle = |V\rangle \\ \langle W|(\alpha E - \gamma D) = \langle W| \end{cases}$$

Then

$$f_n(\tau_1, \dots, \tau_n)$$

Derrida, Evans, Hakim, Pasquier (1993)

"matrix ansatz"

D E matrices,

V column vector, W row vector

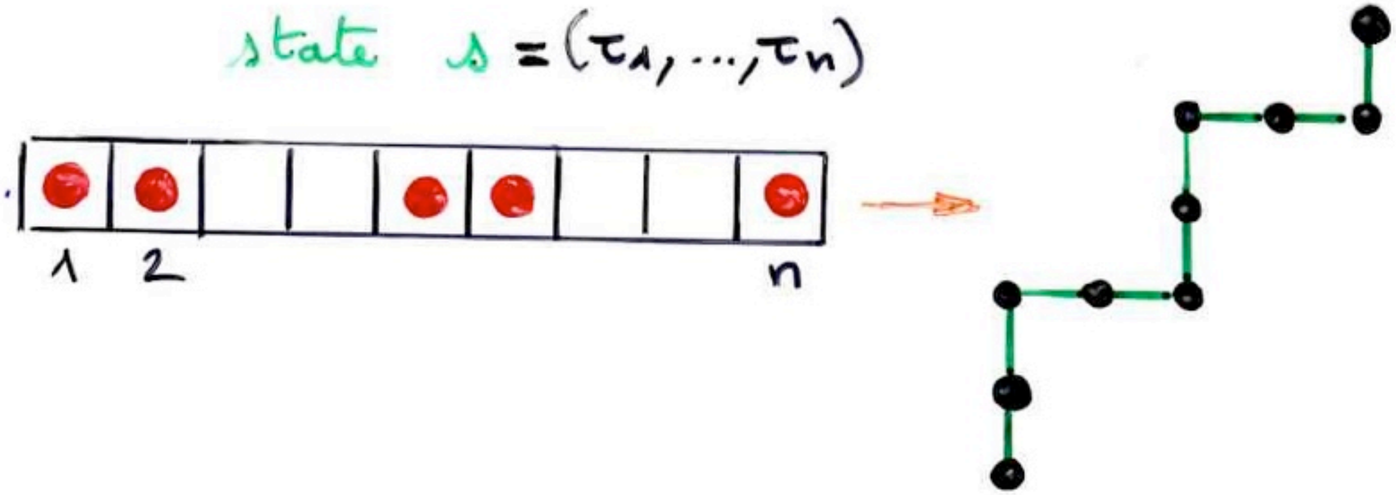
TASEP

q=0

$$\begin{cases} DE = \square + D + E \\ (\beta D - \square)|V\rangle = |V\rangle \\ \langle W|(\alpha E - \square) = \langle W| \end{cases}$$

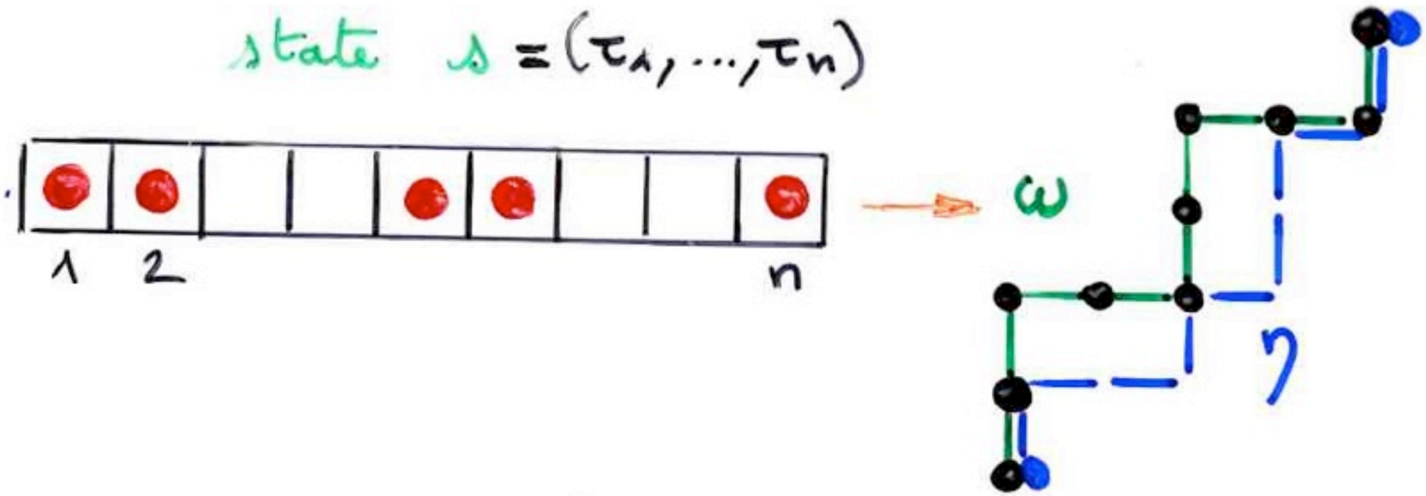
Then

$$f_n(\tau_1, \dots, \tau_n) = \langle W | \prod_{i=1}^{n} (\tau_i D + (1-\tau_i) E) | V \rangle$$



$\mathcal{P}_n(s) =$

Shapiro, Zeilberger, 1982



$$P_n(s) = \frac{1}{C_{n+1}} \left(\begin{array}{c} \text{number of paths } \eta \\ \text{below the path } \omega \\ \text{associated to } s \end{array} \right)$$

Shapiro, Zeilberger, 1982

TASEP

Brak, Essam (2003), Duchi, Schaeffer, (2004),
Angel (2005), xgv, (2007)

(P) ASEP

Brak, Corteel, Essam, Parviainen, Rechnitzer (2006)
Corteel, Williams (2006)
Josuat-Vergès (2008)

Derrida, ...
Mallick, ... Golinelli, Mallick (2006)

Stationary probability with alternative tableaux

q-analog

$$DE = qED + E + D$$

$$w(E,D) = \sum_{T \text{ alternative tableau with profile } w} q^{k(T)} E^{i(T)} D^{j(T)}$$

$k(T)$ = nb of $\boxtimes$

$i(T)$ = nb of columns without red cell

$j(T)$ = nb of rows without blue cell

$$\begin{cases} DE = qED + D + E \\ DV = \bar{\beta} V & \bar{\beta} = 1/\beta \\ WE = \bar{\alpha} W & \bar{\alpha} = 1/\alpha \end{cases}$$

$$W E^i D^j V = \bar{\alpha}^i \bar{\beta}^j \underbrace{WV}_{1}$$

<u>Cor.</u> The stationary probability associated to the state $\tau = (\tau_1, \dots, \tau_n)$ (PASEP) is

$$\text{proba}_\tau(q;\alpha,\beta) = \frac{1}{Z_n} \sum_{T} q^{L(T)} \alpha^{-f(T)} \beta^{-u(T)}$$

(T: alternative tableaux, profile τ)

$f(T)$ nb of rows without ● cell

$u(T)$ nb of columns without ● cell

$L(T)$ nb of cells ⊠

Corteel, Williams (2006)
permutation tableaux

- Orthogonal polynomials

→ Sasamoto (1999)

→ Blythe, Evans, Colaiori, Essler (2000)

q- Hermite polynomial

$\alpha, \beta, q \qquad \gamma = \delta = 1$

$$D = \frac{1}{1-q} + \frac{1}{\sqrt{1-q}}\,\hat{a}$$

$$E = \frac{1}{1-q} + \frac{1}{\sqrt{1-q}}\,\hat{a}^+$$

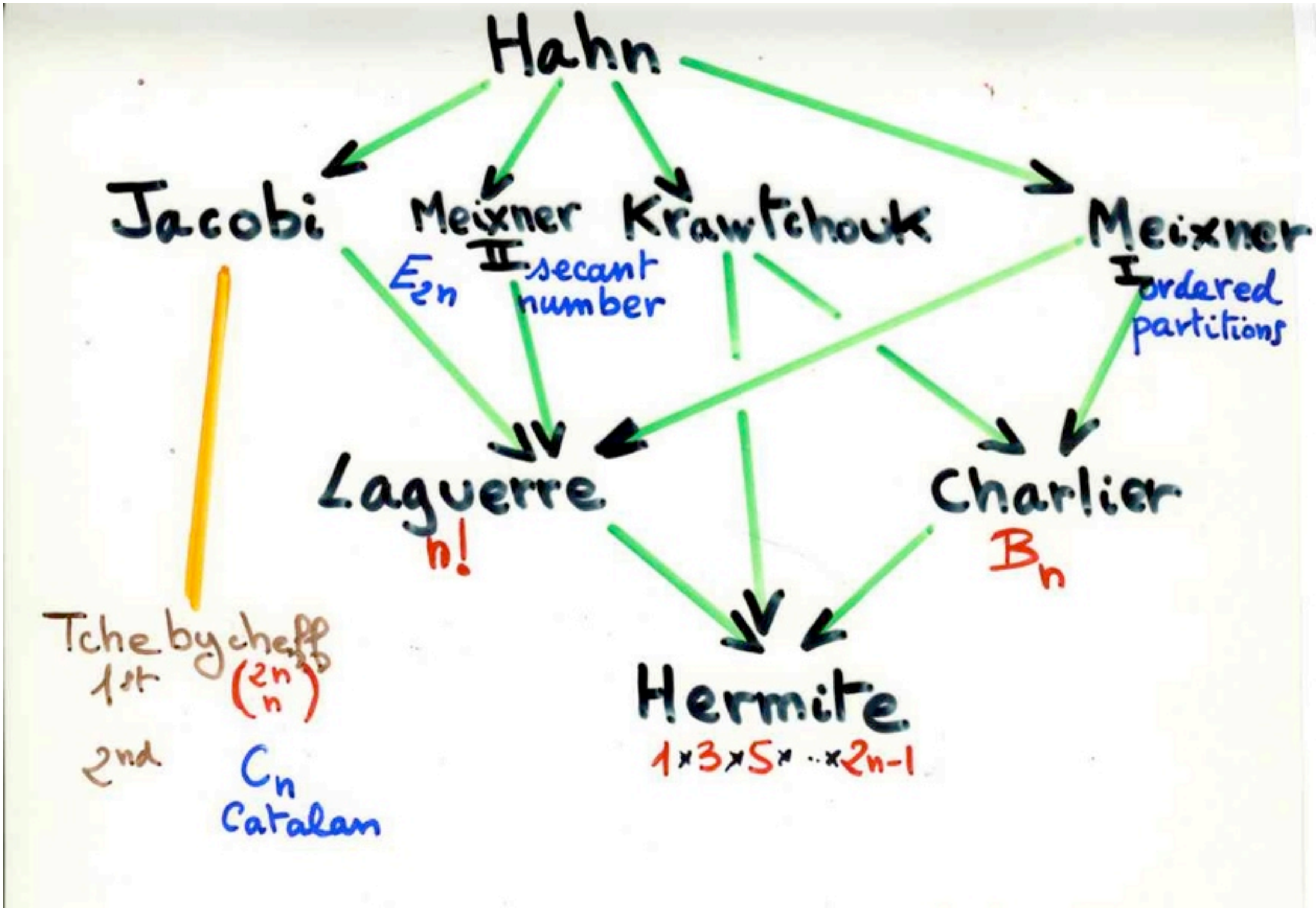
$$\hat{a}\,\hat{a}^+ - q\,\hat{a}^+\,\hat{a} = 1$$

→ Uchiyama, Sasamoto, Wadati (2003)

$\alpha, \beta, \gamma, \delta, \qquad q$

Askey-Wilson polynomials

Askey-Wilson

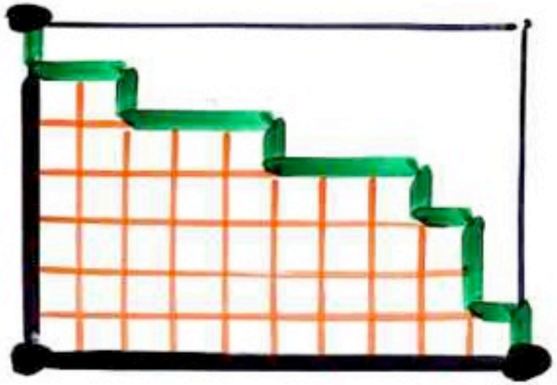


§ 11

Permutation tableaux

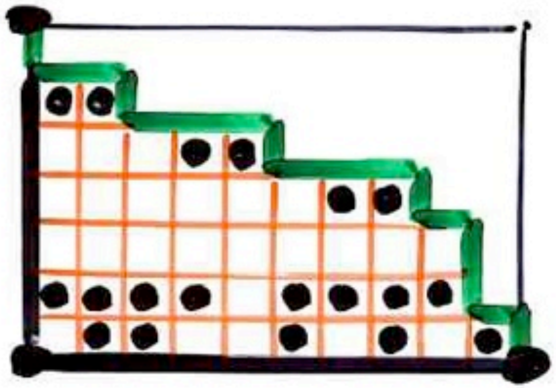
Permutation Tableau

Ferrers diagram $F \subseteq k \times (n-k)$ rectangle



Permutation Tableau

Ferrers diagram $F \subseteq k \times (n-k)$ rectangle



filling of the cells with 0 and 1

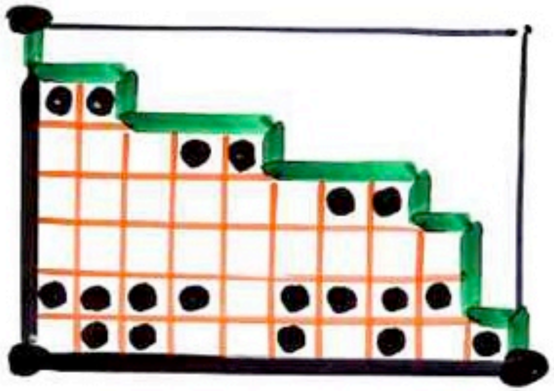
(i)

(ii)

□ = 0 ⊡ = 1

Permutation Tableau

Ferrers diagram $F \subseteq k \times (n-k)$
rectangle



filling of the cells with 0 and 1

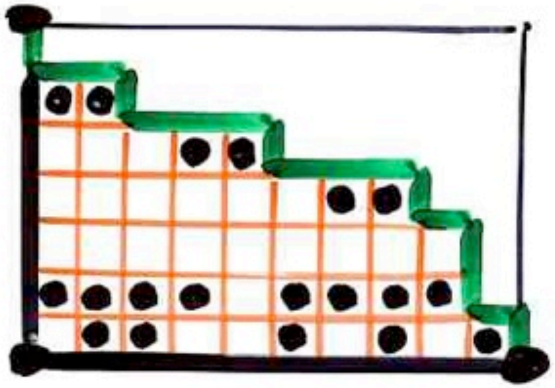
(i) in each column : at least one 1

(ii)

□ = 0   ▣ = 1

Permutation Tableau

Ferrers diagram $F \subseteq k \times (n-k)$
rectangle



filling of the cells with 0 and 1

(i) in each column : at least one 1

□ = 0 ⊡ = 1

(ii)
$$\begin{matrix} 1 & \cdots & 0 \\ & & \vdots \\ & & 1 \end{matrix}$$
forbidden

permutation tableau

A. Postnikov (2001, ...)

totally nonnegative part of the Grassmannian

E. Steingrímsson, L. Williams (2005)

Corteel, Williams (2006) PASEP

Partially Asymmetric Exclusion Process

M. Josuat-Vergès (2007)

The total number of permutation tableaux (n fixed, $1 \leqslant k \leqslant n$) is

$$n!$$

bijection

permutations $\longleftrightarrow$ permutation tableaux

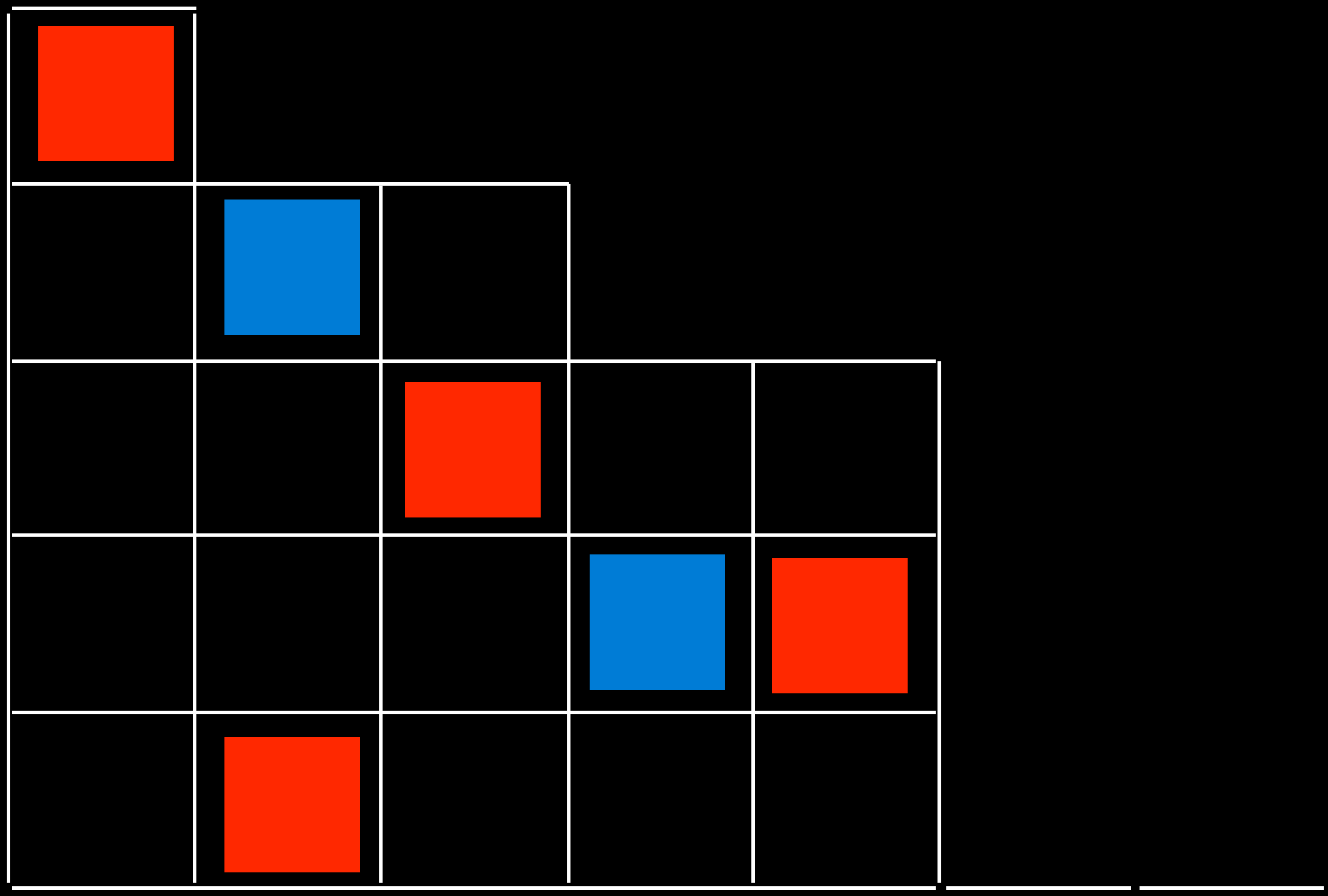
- Postnikov, Steingrímsson, Williams (2005)
- Corteel (2006)
- Corteel, Nadeau (2007)

bijection

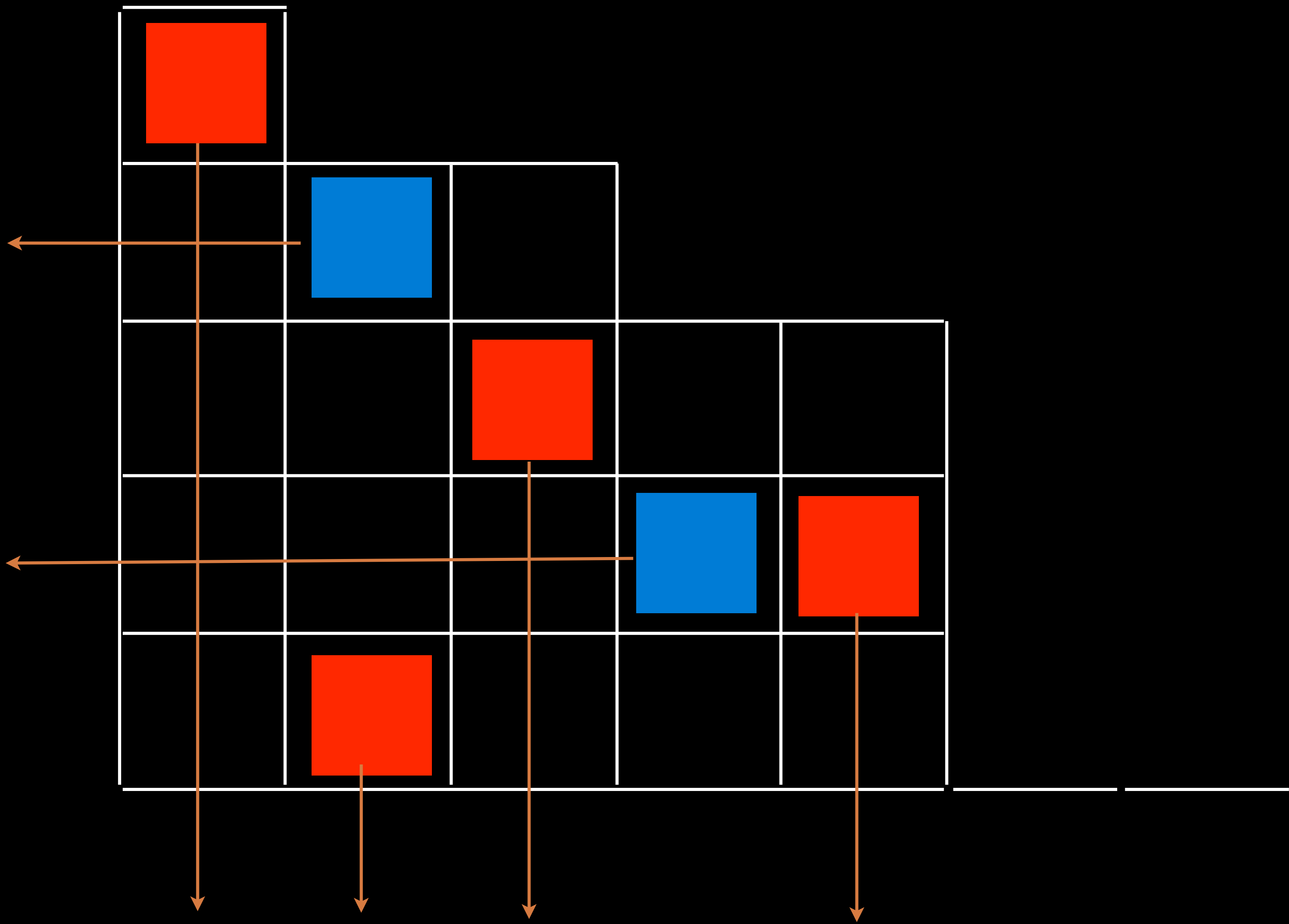
alternative tableaux size n

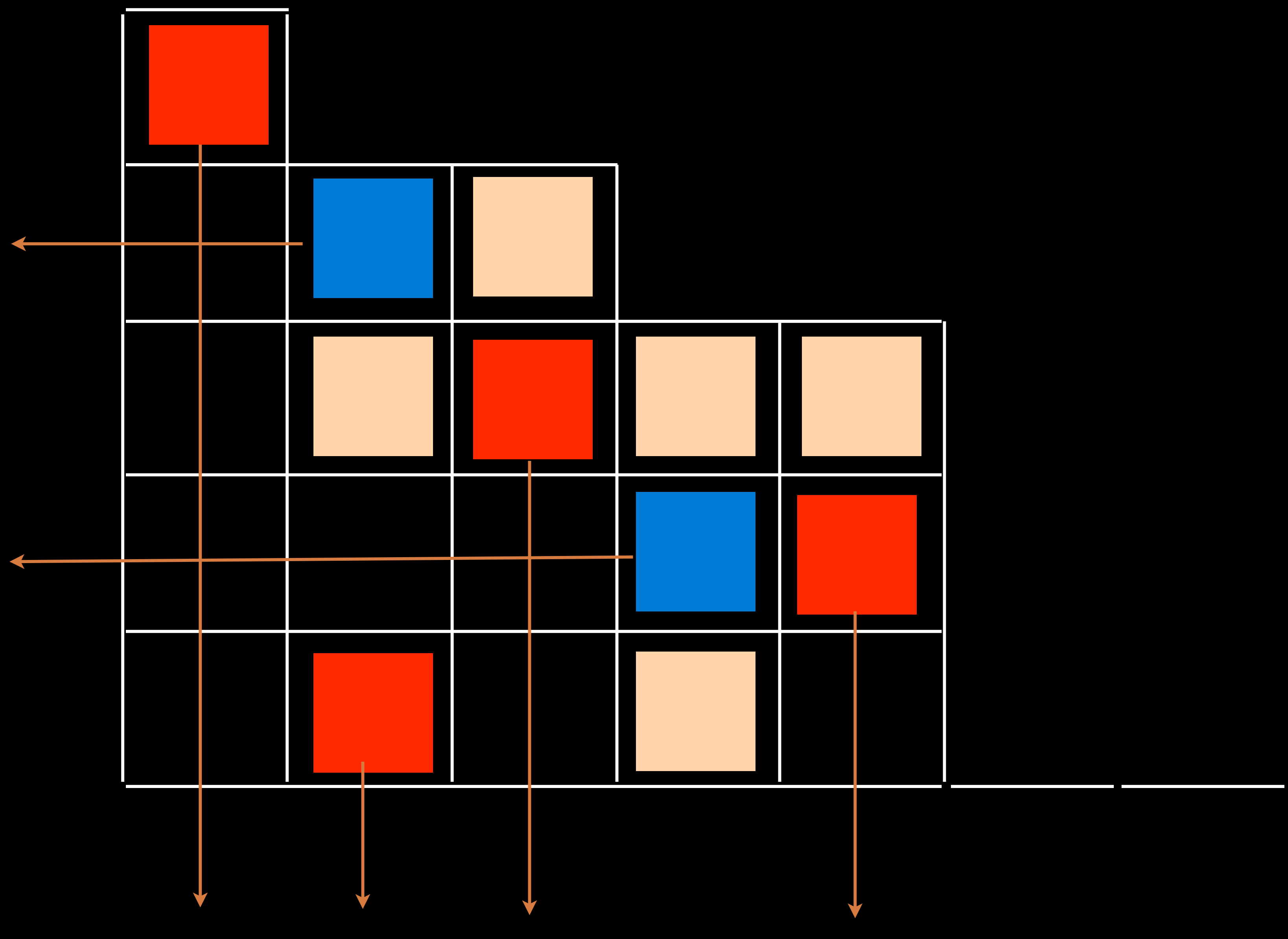
$\updownarrow$

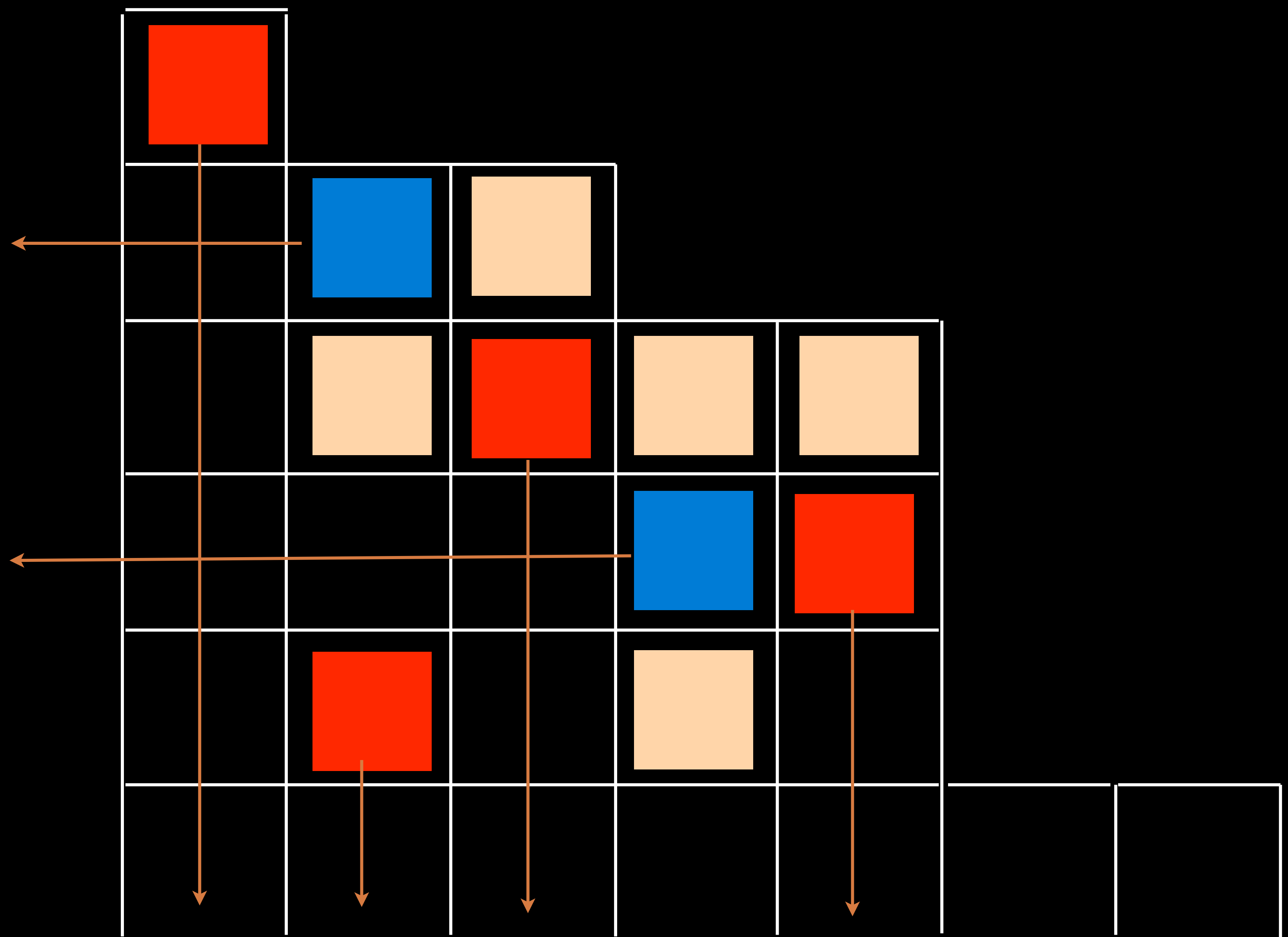
permutation tableaux size $(n+1)$

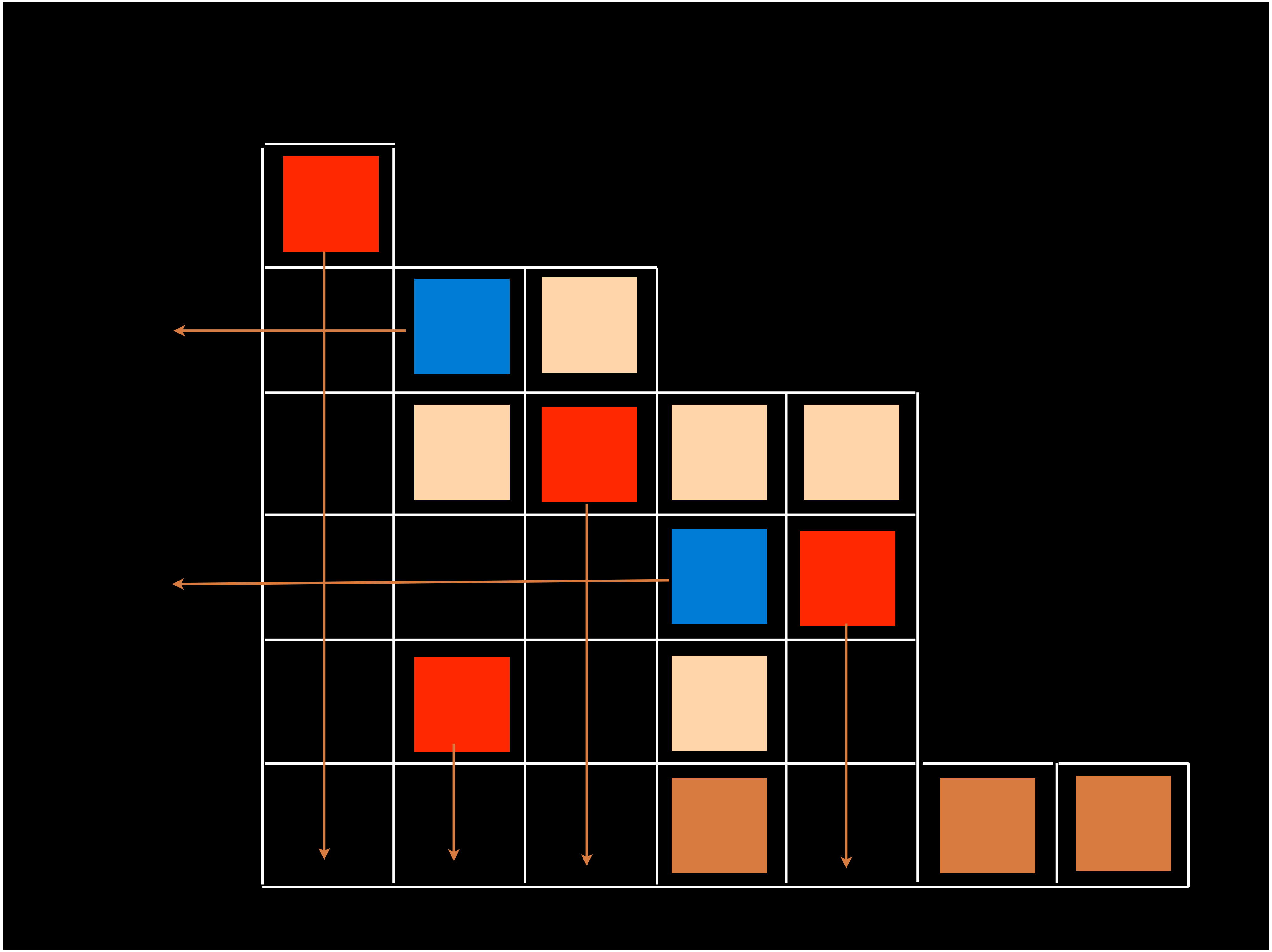


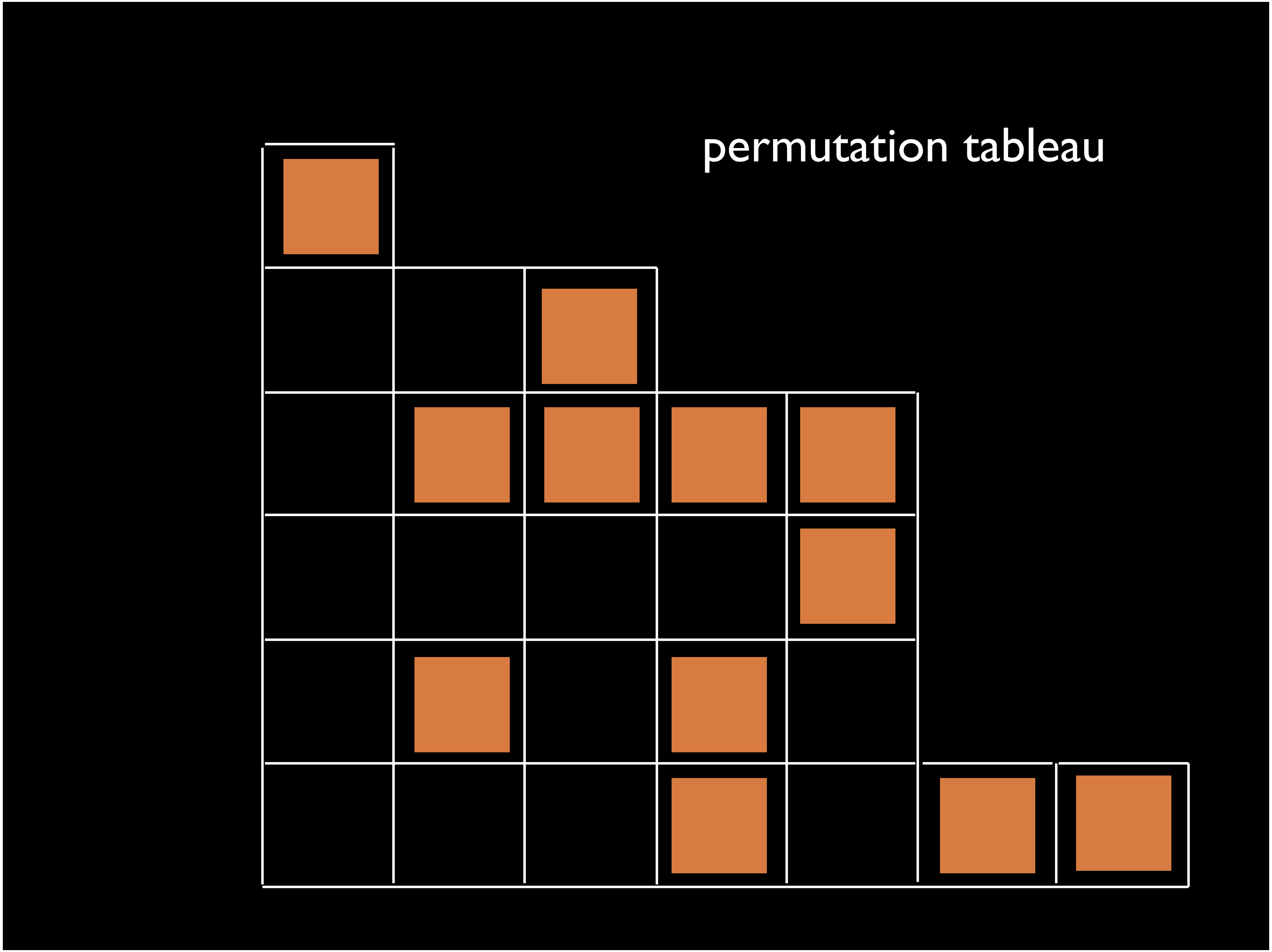
alternative tableau



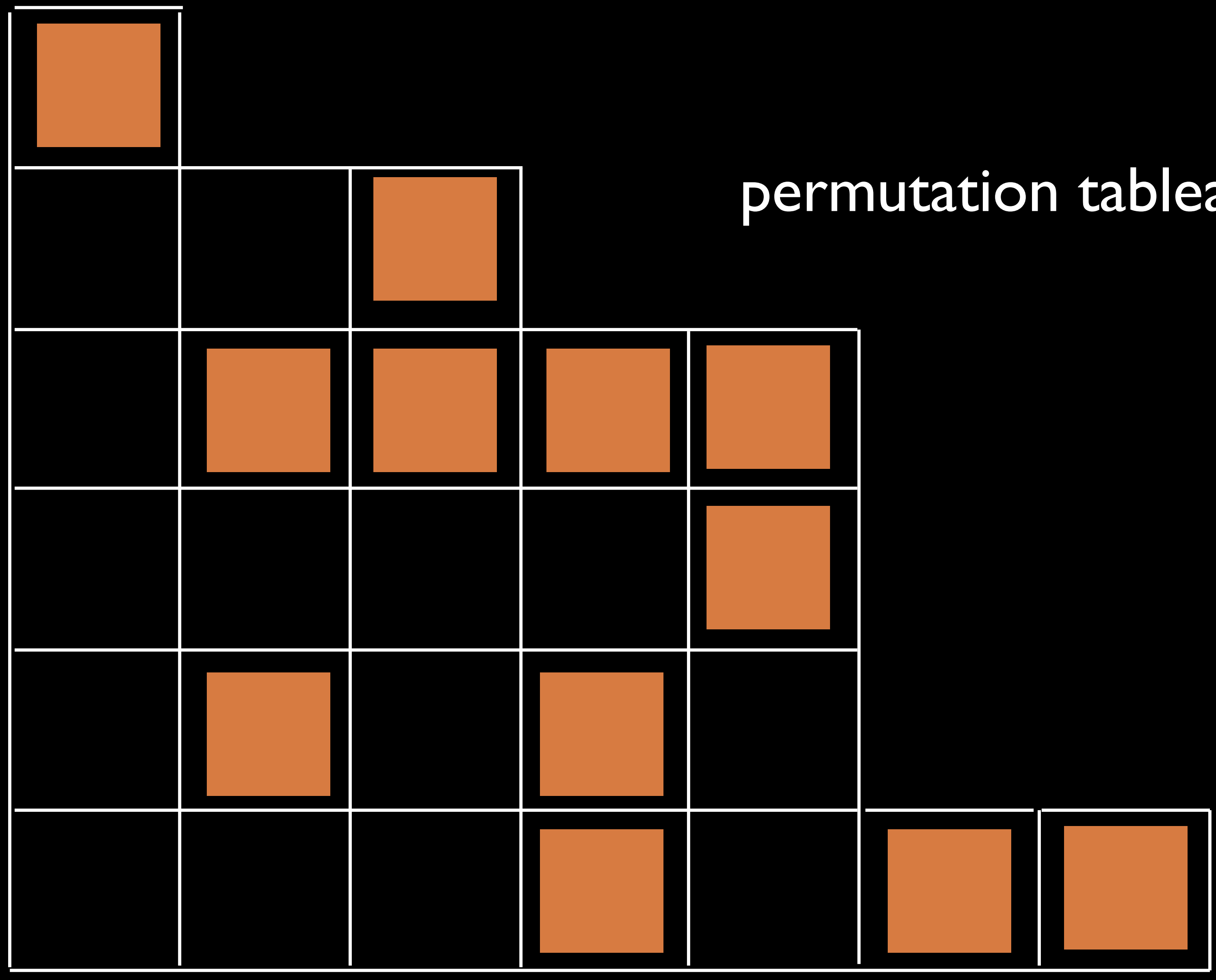




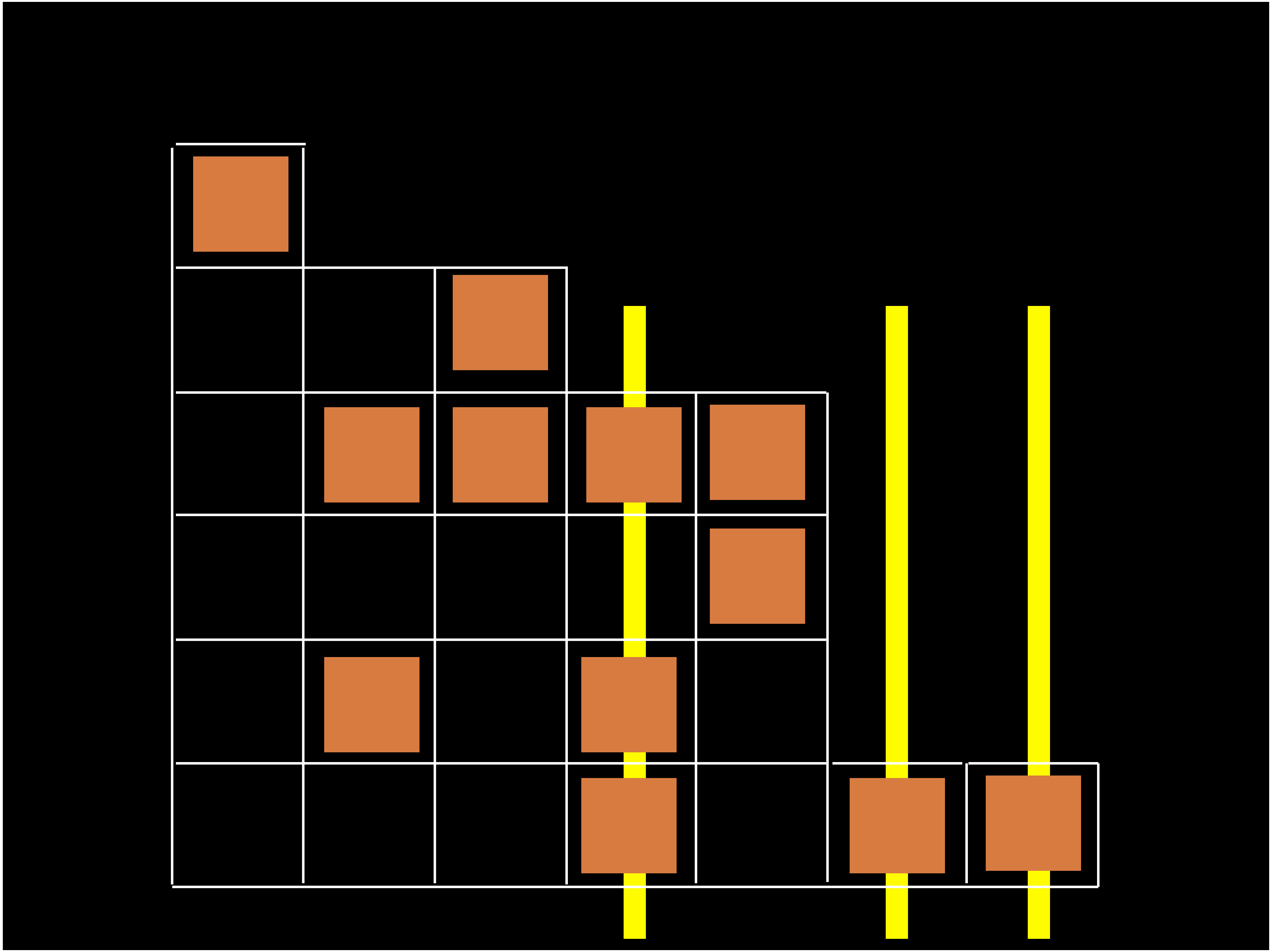


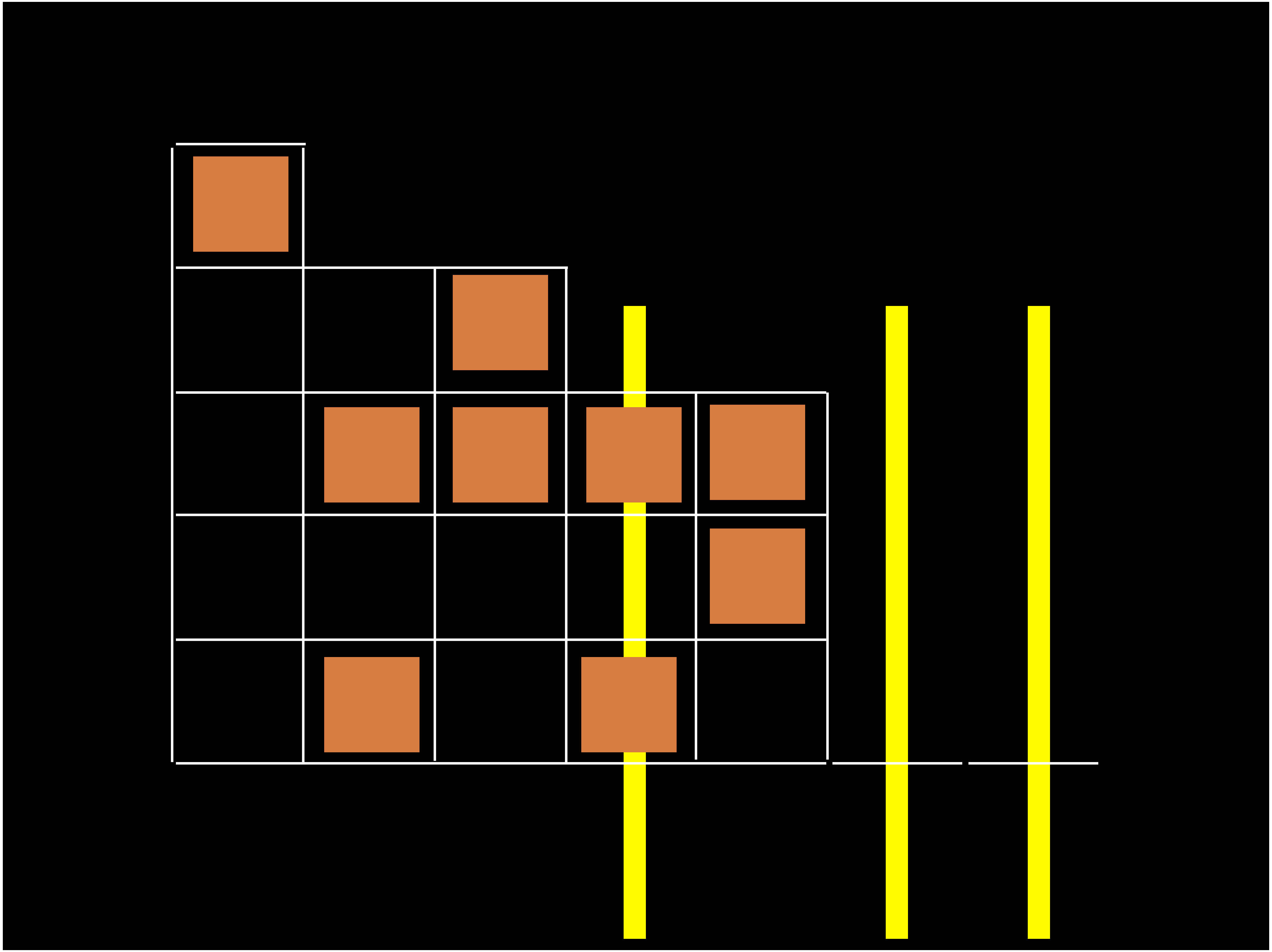

permutation tableau

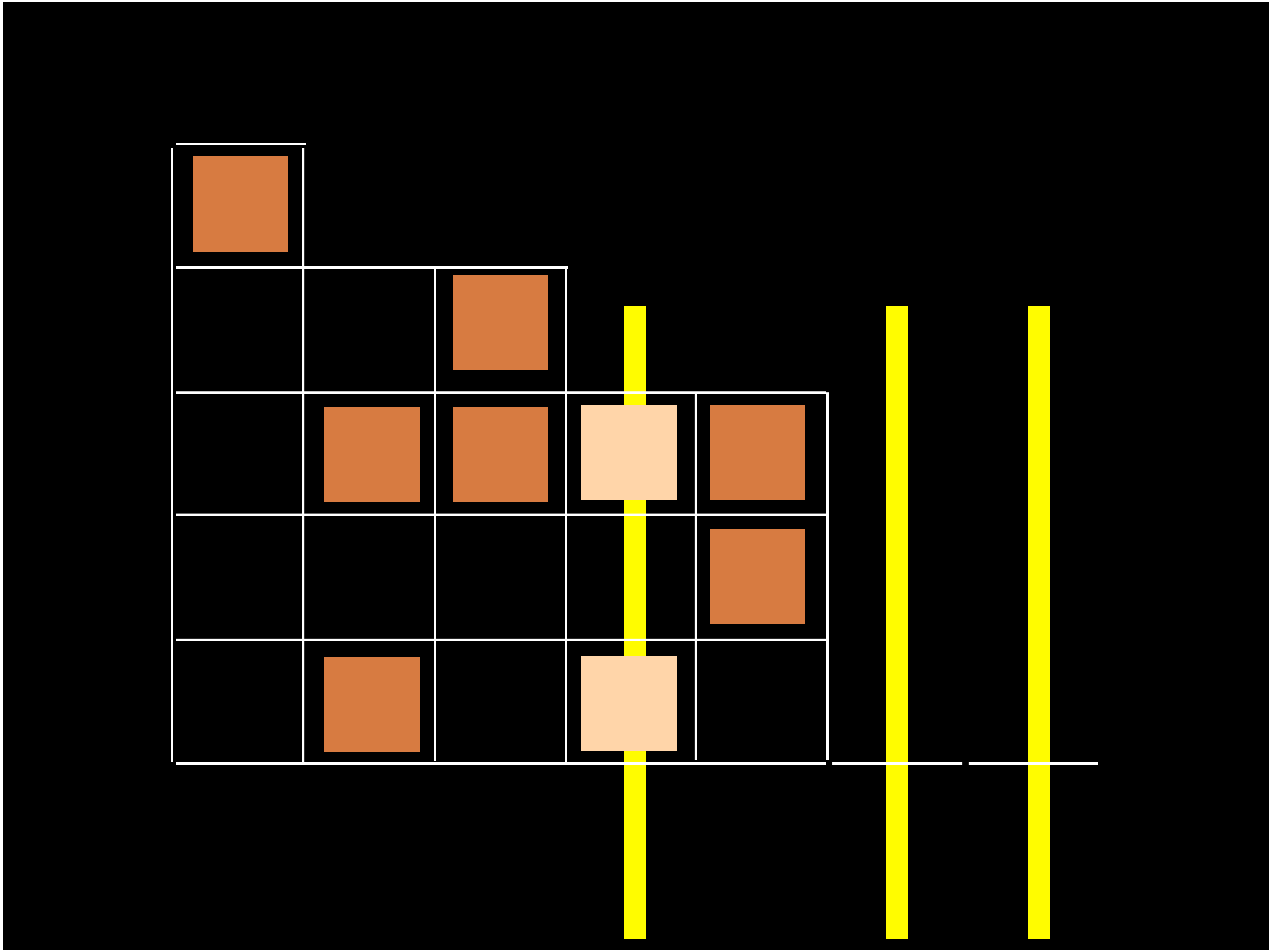
inverse bijection $\psi = \varphi^{-1}$

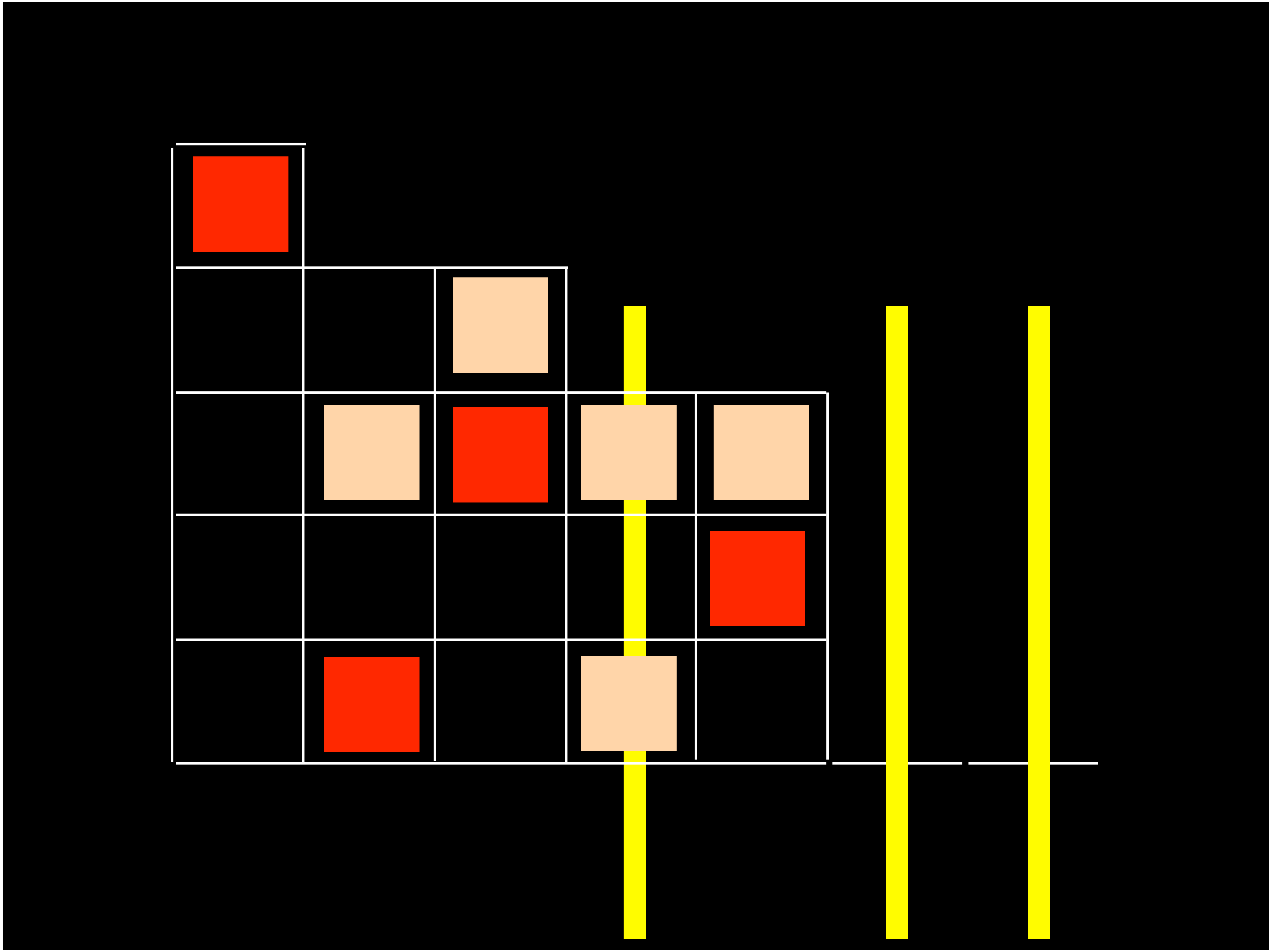


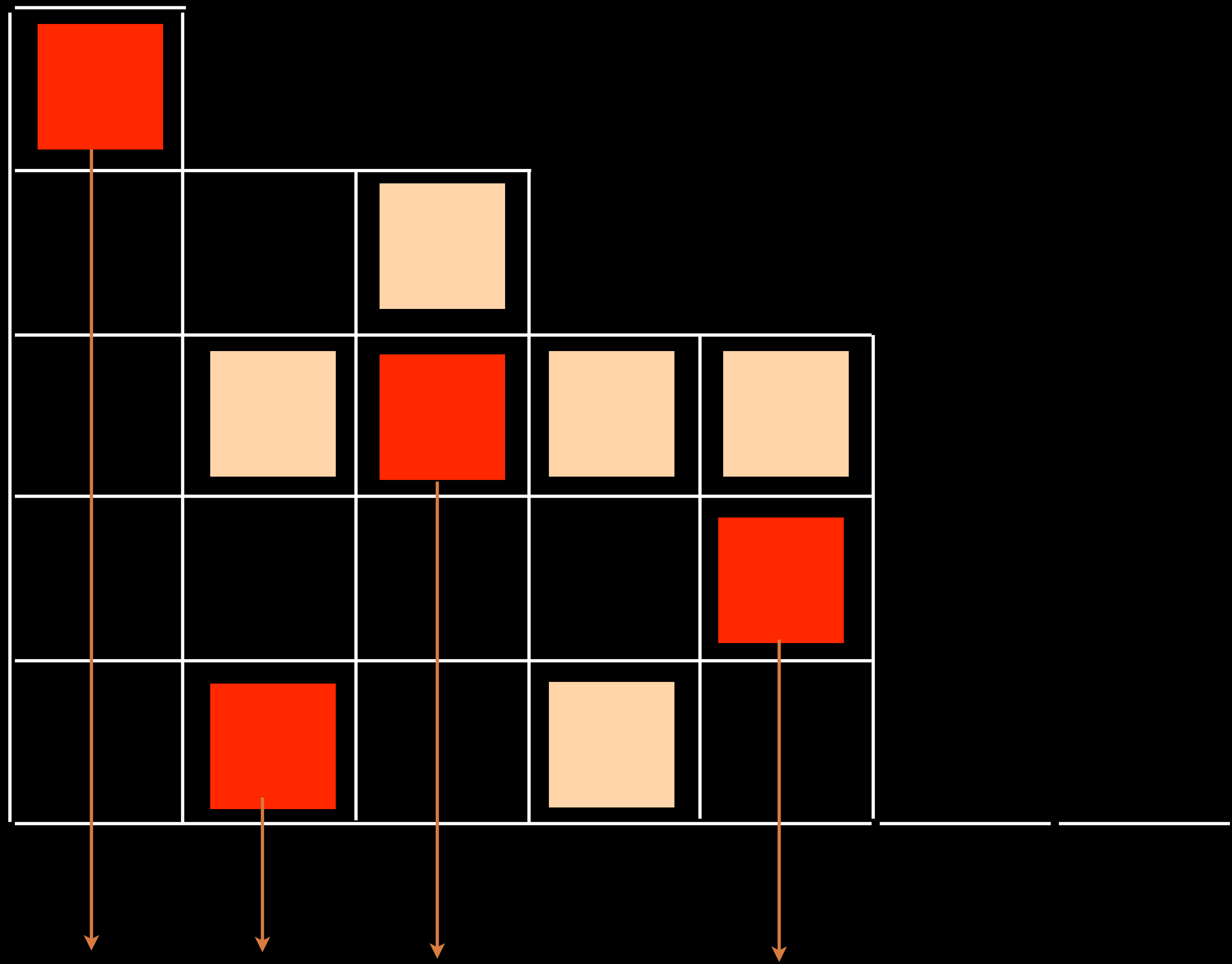
permutation tableau

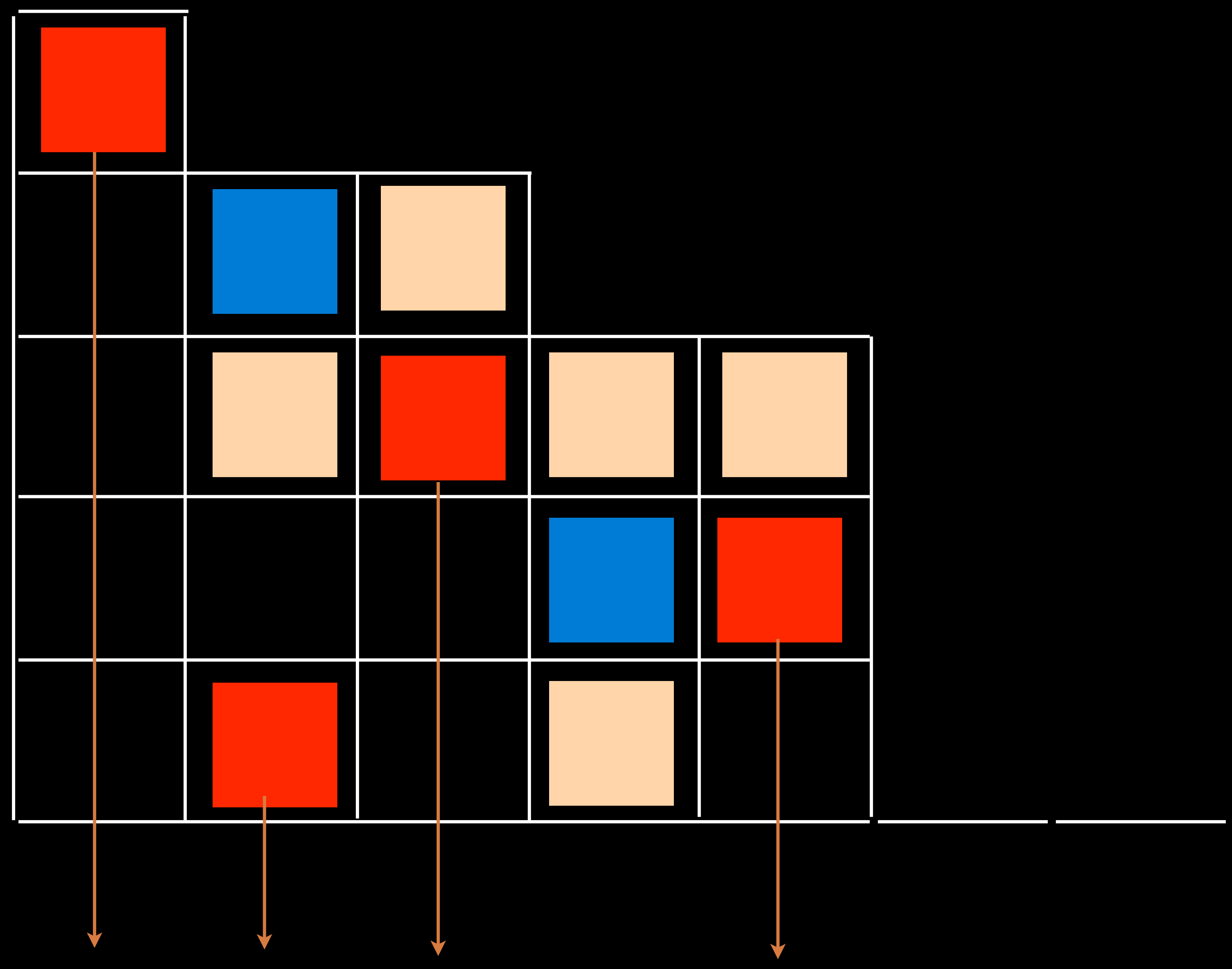




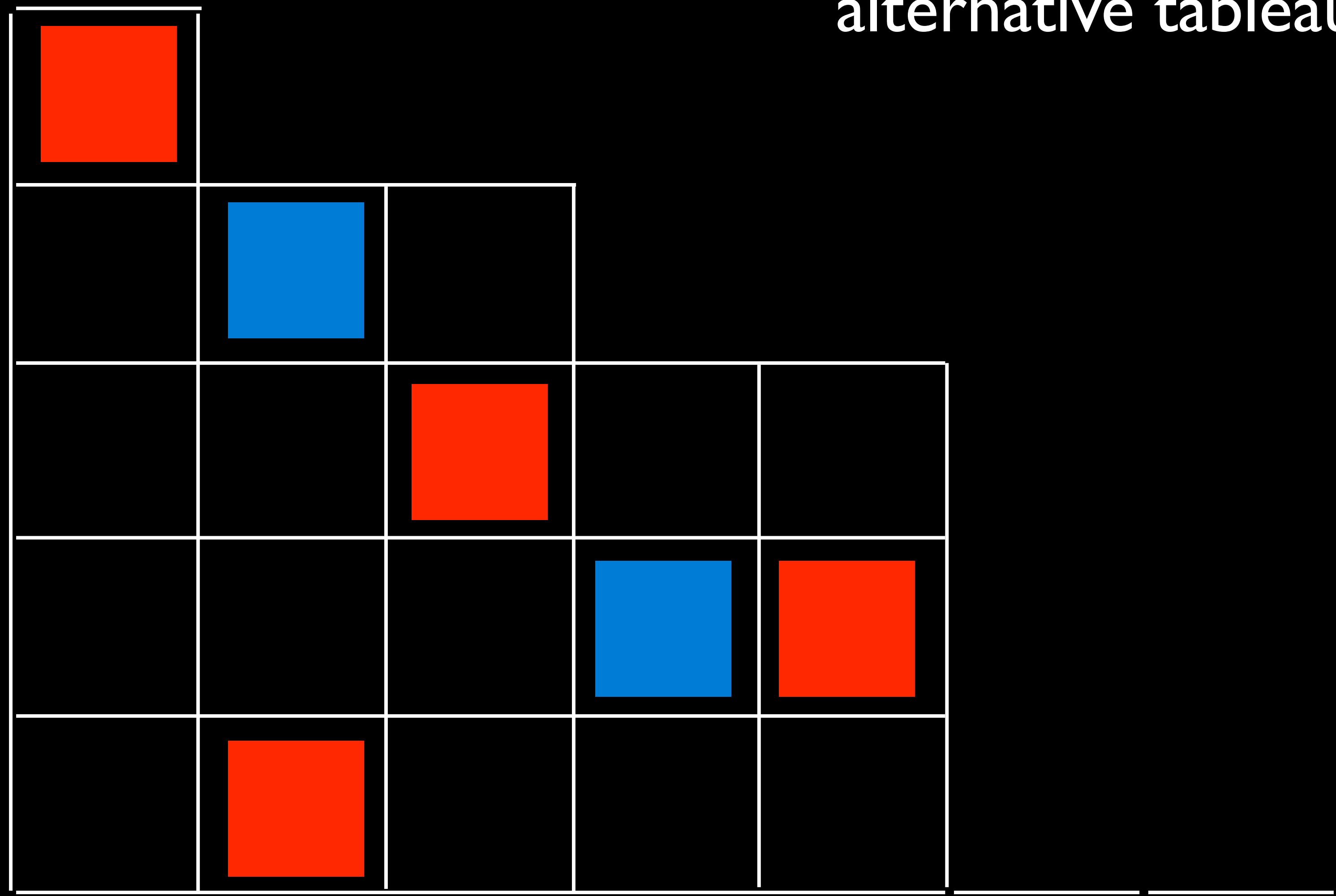




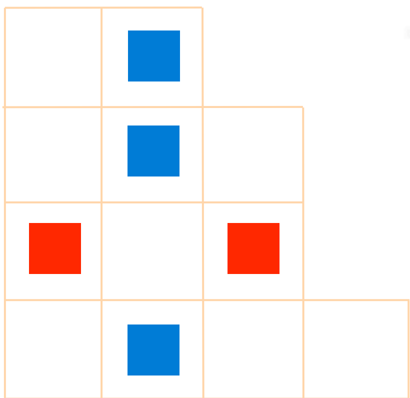


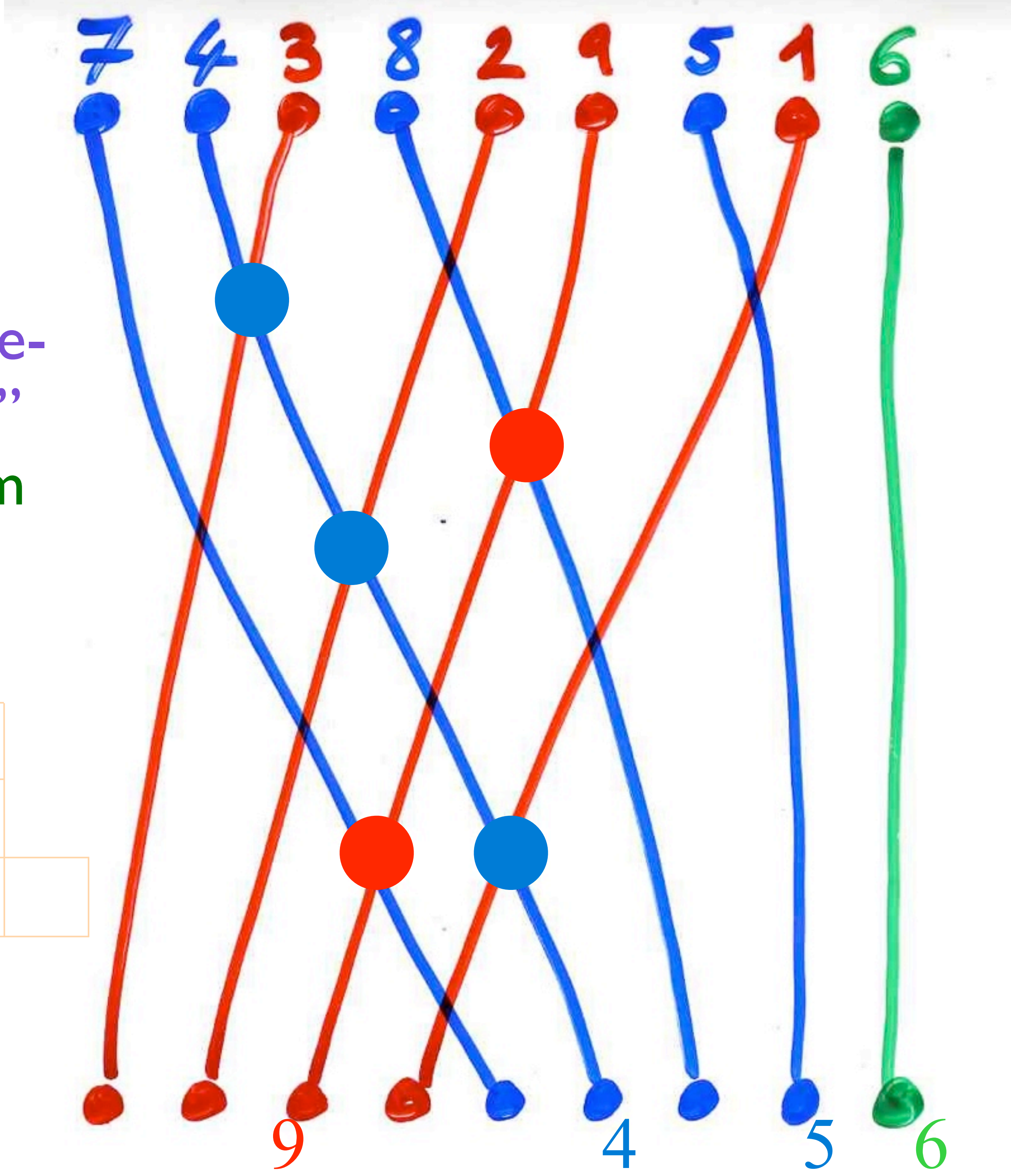


alternative tableau

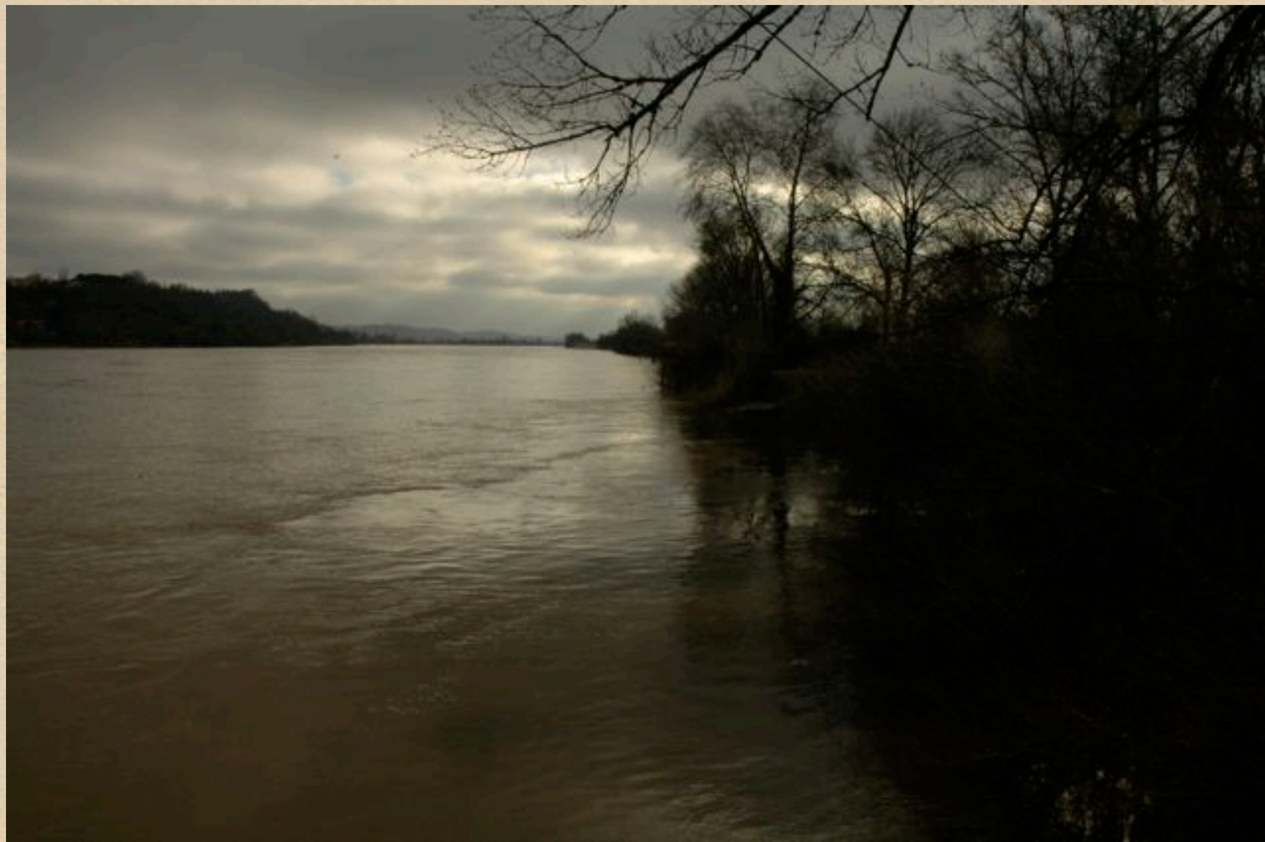


“exchange-deletion” algorithm





§ 12 quantum mechanics: spin chain model



Spin chains and combinatorics

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Institute for High Energy Physics
142284 Protvino, Moscow region, Russia

The XXZ quantum spin chain model with periodic boundary conditions is one of the most popular integrable models which has been investigating by the Bethe Ansatz method during the last 35 years [3]. It is described by the Hamiltonian

$$H_{XXZ} = -\sum_{j=1}^{N} \left\{ \sigma_j^x \sigma_{j+1}^x + \sigma_j^y \sigma_{j+1}^y + \Delta \sigma_j^z \sigma_{j+1}^z \right\}, \qquad \vec{\sigma}_{N+1} = \vec{\sigma}_1. \tag{1}$$

The nonzero wave function components are

$$N=3: \quad \psi_{001}=1;$$
$$N=5: \quad \psi_{00011}=1, \quad \psi_{00101}=2;$$
$$N=7: \quad \psi_{0000111}=1, \quad \psi_{0001101}=\psi_{0001011}=3, \quad \psi_{0010011}=4 \quad \psi_{0010101}=7.$$

All components not included in the list can be obtained by shifting. Notice that the components of the ground state are positive in accordance with the Perron–Frobenius theorem.

Let us continue the list. For $N=9$ the components of the eigenvector with the energy $-27/2$ and $S_z=-1/2$ are

$$\psi_{000001111}=1, \quad \psi_{000010111}=4, \quad \psi_{000011011}=6, \quad \psi_{000100111}=7,$$
$$\psi_{000101011}=17, \quad \psi_{000101101}=14, \quad \psi_{000110011}=12, \quad \psi_{001001011}=21,$$
$$\psi_{001010011}=25, \quad \psi_{001010101}=42.$$

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$$\psi_{000001111}=1, \quad \psi_{000010111}=4, \quad \psi_{000011011}=6, \quad \psi_{000100111}=7,$$
$$\psi_{000101011}=17, \quad \psi_{000101101}=14, \quad \psi_{000110011}=12, \quad \psi_{001001011}=21,$$
$$\psi_{001010011}=25, \quad \psi_{001010101}=42.$$

We omit nonzero components which can be obtained by the reflection of the order of sites since this transformation is a symmetry of our state, as it is for the ground state. For example, we have

$$\psi_{000011101}=\psi_{000010111}=4.$$

1 , 2 , 7 , 42 , 429 , ...

?

M1791 0, 1, 2, 7, 32, 181, 1214, 9403, 82508, 808393, 8743994, 103459471, 1328953592, 18414450877, 273749755382, 4345634192131, 73362643649444, 1312349454922513
$a(n) = n.a(n-1) + (n-2)a(n-2)$. Ref R1 188. [0,3; A0153, N0706]

$$\text{E.g.f.: } (1-x)^{-3} e^{-x}.$$

M1792 1, 1, 2, 7, 32, 181, 1232, 9787, 88832, 907081, 10291712, 128445967, 1748805632, 25794366781, 409725396992, 6973071372547, 126585529106432
Expansion of $1/(1-\sinh x)$. Ref ARS 10 138 80. [0,3; A6154]

M1793 0, 1, 1, 2, 7, 32, 184, 1268, 10186, 93356, 960646, 10959452, 137221954, 1870087808, 27548231008, 436081302248, 7380628161076, 132975267434552
Stochastic matrices of integers. Ref DUMJ 35 659 68. [0,4; A0987, N0707]

M1794 1, 2, 7, 33, 192
Permutations of length n with n in second orbit. Ref C1 258. [2,2; A6595]

M1795 1, 2, 7, 34, 209, 1546, 13327, 130922, 1441729, 17572114, 234662231, 3405357682, 53334454417, 896324308634, 16083557845279, 306827170866106
$a(n) = 2n.a(n-1) - (n-1)^2 a(n-2)$. Ref SE33 78. [0,2; A2720, N0708]

M1796 1, 2, 7, 34, 257, 2606, 32300, 440564, 6384634
Polyhedra with n nodes. Ref GR67 424. UPG B15. Dil92. [4,2; A0944, N0709]

M1797 2, 7, 35, 219, 1594, 12935, 113945, 1070324, 10586856, 109259633, 1168384157, 12877168147, 145656436074, 1685157199175, 19886174611045
Two-rowed truncated monotone triangles. Ref JCT A42 277 86. Zei93. [1,1; A6947]

M1798 1, 1, 2, 7, 35, 228, 1834, 17382, 195866, 2487832, 35499576, 562356672, 9794156448, 186025364016, 3826961710272, 84775065603888, 2011929826983504
Coefficients of iterated exponentials. Ref SMA 11 353 45. [0,3; A0154, N0710]

M1799 1, 2, 7, 35, 228, 1834, 17582, 195866, 2487832, 35499576, 562356672, 9794156448, 186025364016, 3826961710272, 84775065603888, 2011929826983504
Expansion of $\ln(1 + \ln(1+x))$. [0,2; A3713]

M1800 1, 0, 1, 2, 7, 36, 300, 3218, 42335, 644808
Circular diagrams with n chords. Ref BarN94. [0,4; A7474]

M1801 1, 2, 7, 36, 317, 5624, 251610, 33642660, 14685630688
$n \times n$ binary matrices. Ref CPM 89 217 64. SLC 19 79 88. [0,2; A2724, N0711]

M1802 2, 7, 37, 216, 1780, 32652
Semigroups of order n with 2 idempotents. Ref MAL 2 2 67. SGF 14 71 77. [2,1; A2787, N0712]

M1803 1, 2, 7, 37, 266, 2431, 27007, 353522, 5329837, 90960751, 1733584106, 36496226977, 841146804577, 21065166341402, 569600638022431
$a(n) = (2n-1)a(n-1) + a(n-2)$. Ref RCI 77. [0,2; A1515, N0713]

M1804 1, 1, 2, 7, 38, 291, 2932, 36961, 561948, 10026505, 205608536, 4767440679, 123373203208, 3525630110107, 110284283006640, 3748357699560961
Forests of labeled trees with n nodes. Ref JCT 5 96 68. SIAD 3 574 90. [0,3; A1858, N0714]

M1805 1, 1, 2, 7, 40, 357, 4824, 96428, 2800472, 116473461
n-element partial orders contained in linear order. Ref nbh. [0,3; A6455]

M1806 1, 2, 7, 41, 346, 3797, 51157, 816356, 15050581, 314726117, 7359554632, 190283748371, 5389914888541, 165983936096162, 5521346346543307
Planted binary phylogenetic trees with n labels. Ref LNM 884 196 81. [1,2; A6677]

M1807 1, 1, 2, 7, 41, 376, 5033, 92821, 2257166, 69981919, 2694447797, 126128146156, 7054258103921, 464584757637001, 35586641825705882, 3136942184333040727
Hammersley's polynomial $p_n(1)$. Ref MASC 14 4 89. [0,3; A6846]

M1808 1, 2, 7, 42, 429, 7436, 218348, 10850216, 911835460, 129534272700, 31095744852375, 12611311859677500, 8639383518297652500
Robbins numbers: $\Pi(3k+1)!/(n+k)!$, $k = 0 \ldots n-1$. Ref MINT 13(2) 13 91. JCT A66 17 94. [1,2; A5130]

M1809 1, 2, 7, 42, 582, 21480, 2142288, 575016219, 415939243032, 816007449011040, 4374406209970747314, 64539836938720749739356
Antisymmetric relations on n nodes. Ref PAMS 4 494 53. MIT 17 23 55. [1,2; A1174, N0715]

M1810 0, 1, 2, 7, 44, 361, 3654, 44207, 622552, 10005041, 180713290, 3624270839, 79914671748, 1921576392793, 50040900884366, 1403066801155039
Modified Bessel function $K_n(1)$. Ref AS1 429. [0,3; A0155, N0716]

M1811 0, 1, 2, 7, 44, 447, 6749, 142176, 3987677, 143698548, 6470422337, 356016927083, 23503587609815, 1833635850492653, 166884365982441238
$a(n)=n(n-1)a(n-1)/2+a(n-2)$. [0,3; A1046, N0717]

M1812 1, 2, 7, 44, 529, 12278, 565723, 51409856, 9371059621, 3387887032202, 2463333456292207, 3557380311703796564, 10339081666350180289849
Sum of Gaussian binomial coefficients $[n,k]$ for $q=4$. Ref TU69 76. GJ83 99. ARS A17 328 84. [0,2; A6118]

M1813 2, 7, 52, 2133, 2590407, 3374951541062, 5695183504479116640376509, 16217557574922386301420514191523784895639577710480
Free binary trees of height n. Ref JCIS 17 180 92. [1,1; A5588]

M1814 1, 1, 2, 7, 56, 2212, 2595782, 3374959180831, 5695183504489239067484387, 16217557574922386301420531277071365103168734284282
Planted 3-trees of height n. Ref RSE 59(2) 159 39. CMB 11 87 68. JCIS 17 180 92. [0,3; A2658, N0718]

M1807 1, 1, 2, 7, 41, 376, 5033, 92821, 2257166, 69981919, 2694447797, 126128146156, 7054258103921, 464584757637001, 35586641825705882, 3136942184333040727
Hammersley's polynomial $p_n(1)$. Ref MASC 14 4 89. [0,3; A6846]

M1808 1, 2, 7, 42, 429, 7436, 218348, 10850216, 911835460, 129534272700, 31095744852375, 12611311859677500, 8639383518297652500
Robbins numbers: $\Pi(3k+1)!/(n+k)!$, $k = 0 \, . \, . \, n-1$. Ref MINT 13(2) 13 91. JCT A66 17 94. [1,2; A5130]

M1809 1, 2, 7, 42, 582, 21480, 2142288, 575016219, 415939243032, 816007449011040, 4374406209970747314, 64539836938720749739356
Antisymmetric relations on n nodes. Ref PAMS 4 494 53. MIT 17 23 55. [1,2; A1174, N0715]

M1807 1, 1, 2, 7, 41, 376, 5033, 92821, 2257166, 69981919, 2694447797, 126128146156, 7054258103921, 464584757637001, 35586641825705882, 3136942184333040727
Hammersley's polynomial $p_n(1)$. Ref MASC 14 4 89. [0,3; A6846]

M1808 1, 2, 7, 42, 429, 7436, 218348, 10850216, 911835460, 129534272700, 31095744852375, 12611311859677500, 8639383518297652500
Robbins numbers: $\Pi(3k+1)!/(n+k)!$, $k = 0 \ldots n-1$. Ref MINT 13(2) 13 91. JCT A66 17 94. [1,2; A5130]

M1809 1, 2, 7, 42, 582, 21480, 2142288, 575016219, 415939243032, 816007449011040, 4374406209970747314, 6453983693872074 9739356
Antisymmetric relations on n nodes. Ref PAMS 4 494 53. MIT 17 23 55. [1,2; A1174, N0715]

§ 13 ASM

Alternating sign matrices

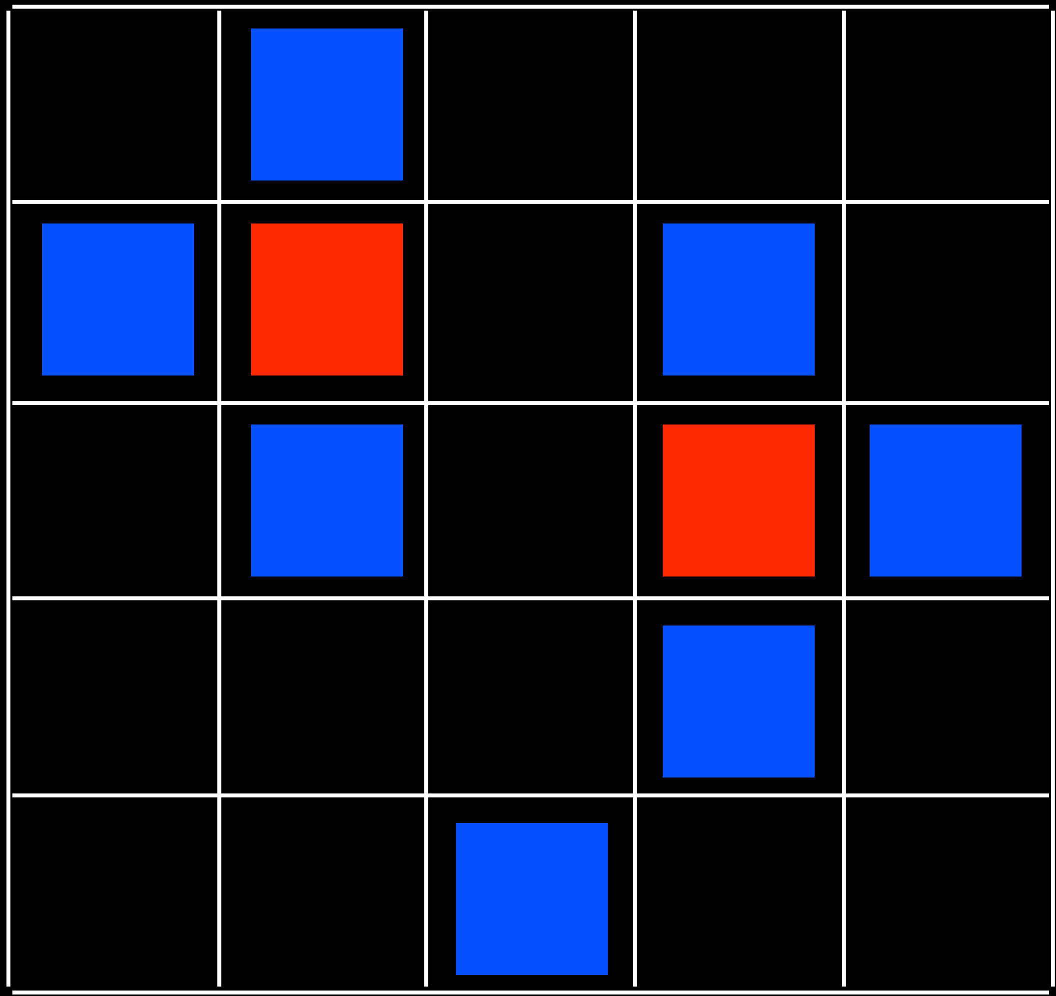
Def- **ASM** alternating sign matrix

$$\begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 & 1 \\ 0 & 1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

(i) entries: 0, 1, −1

(ii) sum of entries in each $\begin{cases}\text{row} \\ \text{column}\end{cases}$ = 1

(iii) non-zero entries alternate in each $\begin{cases}\text{row} \\ \text{column}\end{cases}$



Permutation σ

$$\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 3 & 5 & 2 & 4 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 1 & 0 \\ 1 & -1 & 1 \\ 0 & 1 & 0 \end{pmatrix} \quad + \; 6$$

permutations

1, 2, 7, 42, 429, ...

“What else have you got in your pocket?” he
went on, turning to A
“Only a thimble,”
“Hand it over her
Then they all crow
while the Dodo solem

Lewis Carroll

"Alice aux pays des merveilles"

C. I. Dodgson (1866)

Condensation of determinants

$$det(M) = \frac{M_{NO} M_{SE} - M_{NE} M_{SO}}{M_C}$$

§12

an alternative approach to alternating sign matrices

$A, A', B, B',$

commutations

$$\begin{cases} BA = AB + A'B' \\ B'A' = A'B' + AB \end{cases}$$

$$\begin{cases} B'A = AB' \\ BA' = A'B \end{cases}$$

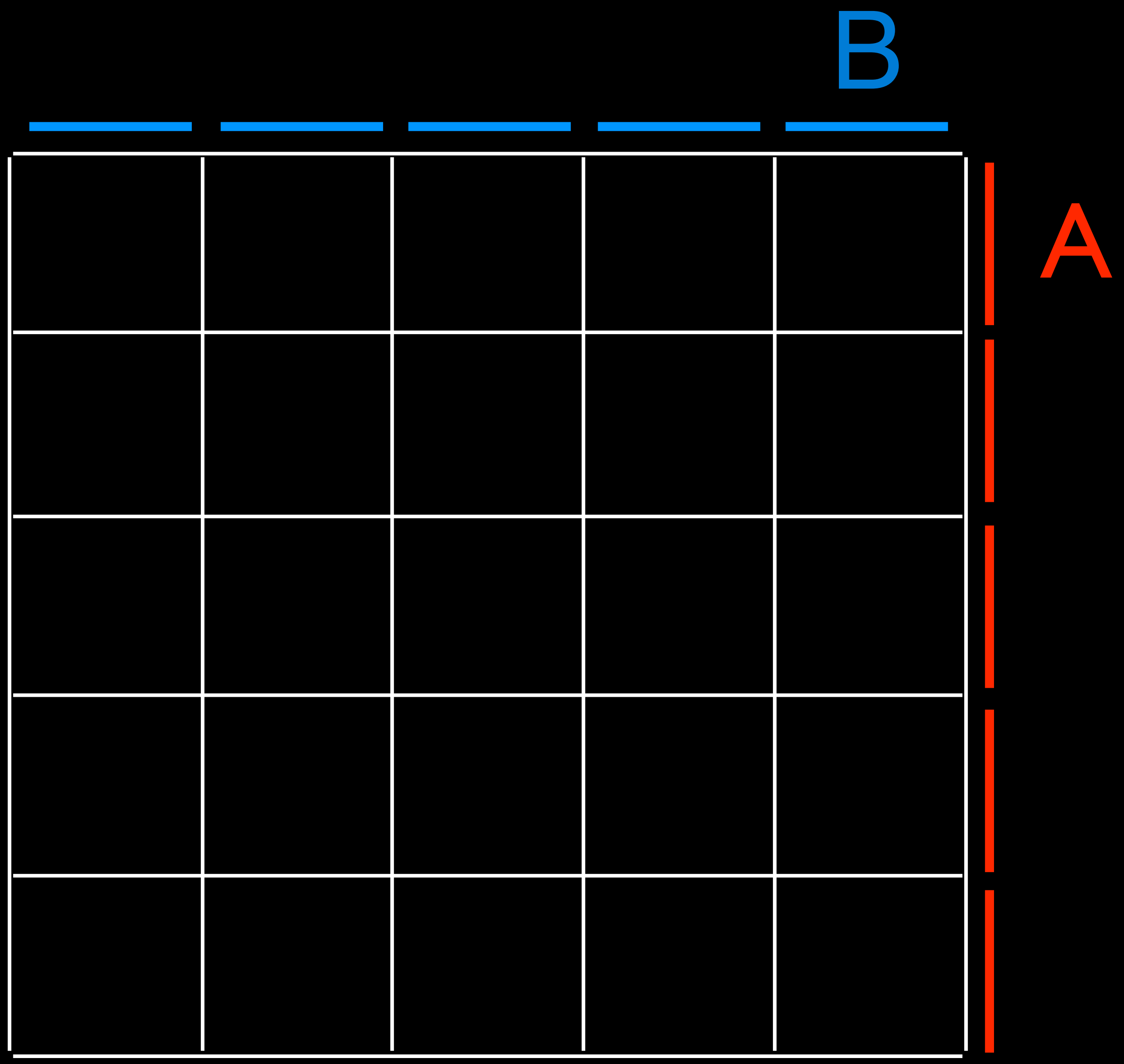
Lemma. Any word $w(A, A', B, B')$ in letters A, A', B, B', can be uniquely written

$$\sum c(u,v;w)\ \underbrace{u(A,A')}_{\substack{\text{word}\\ \text{in } A,\, A'}}\ \underbrace{v(B,B')}_{\substack{\text{word}\\ \text{in } B,\, B'}}$$

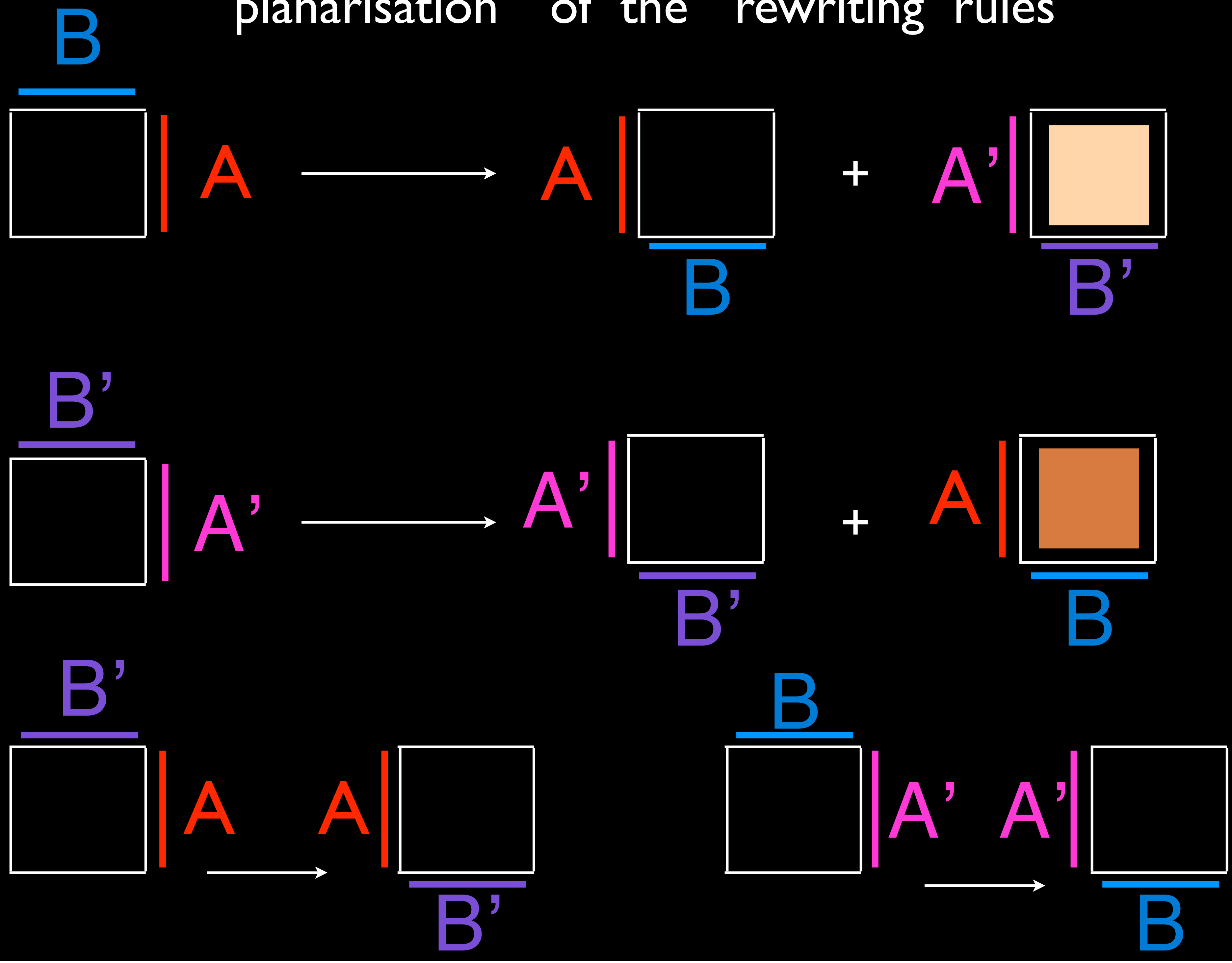
Prop. For $w = B^n A^n$

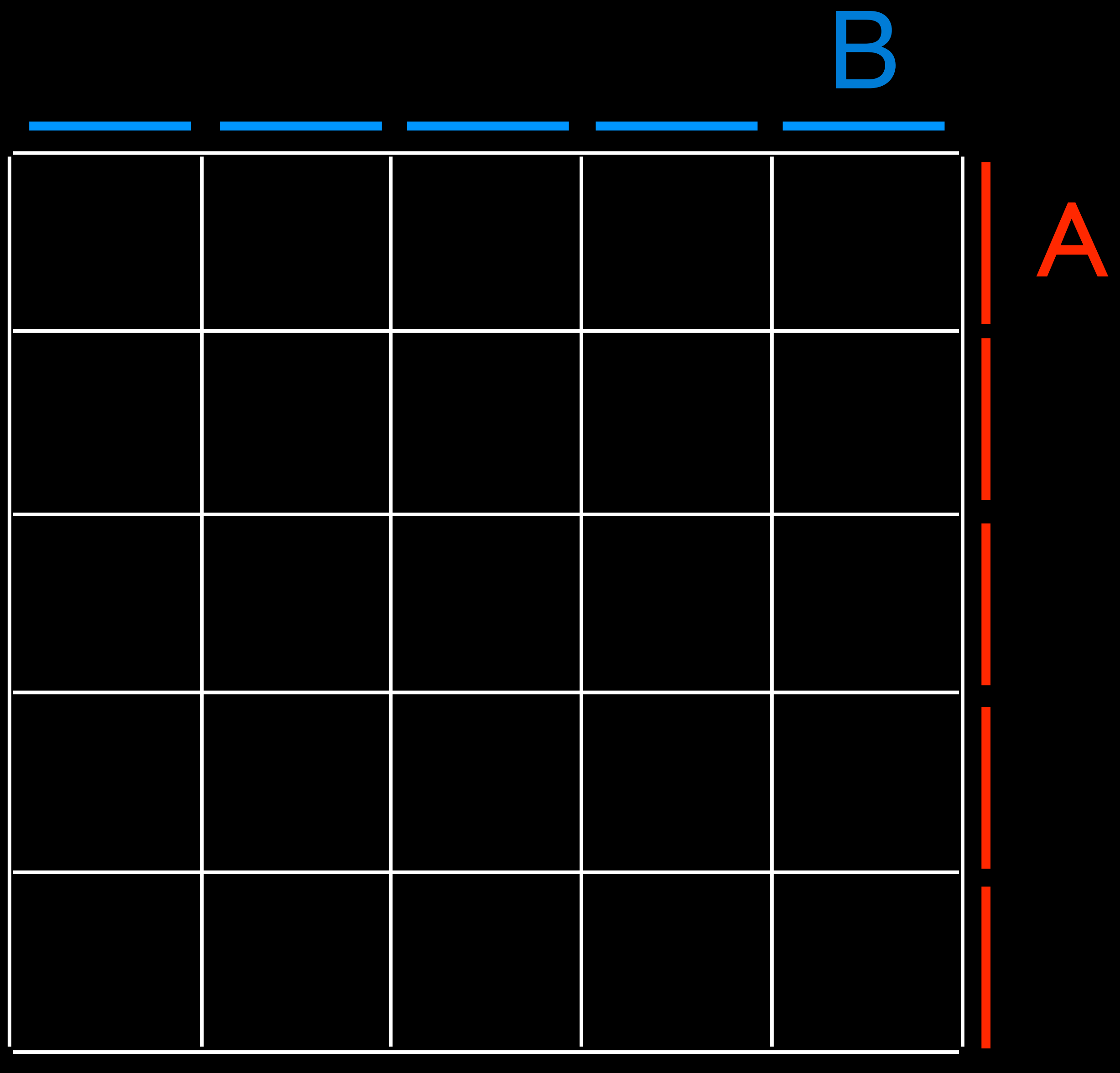
$u = A'^n$, $v = B'^n$

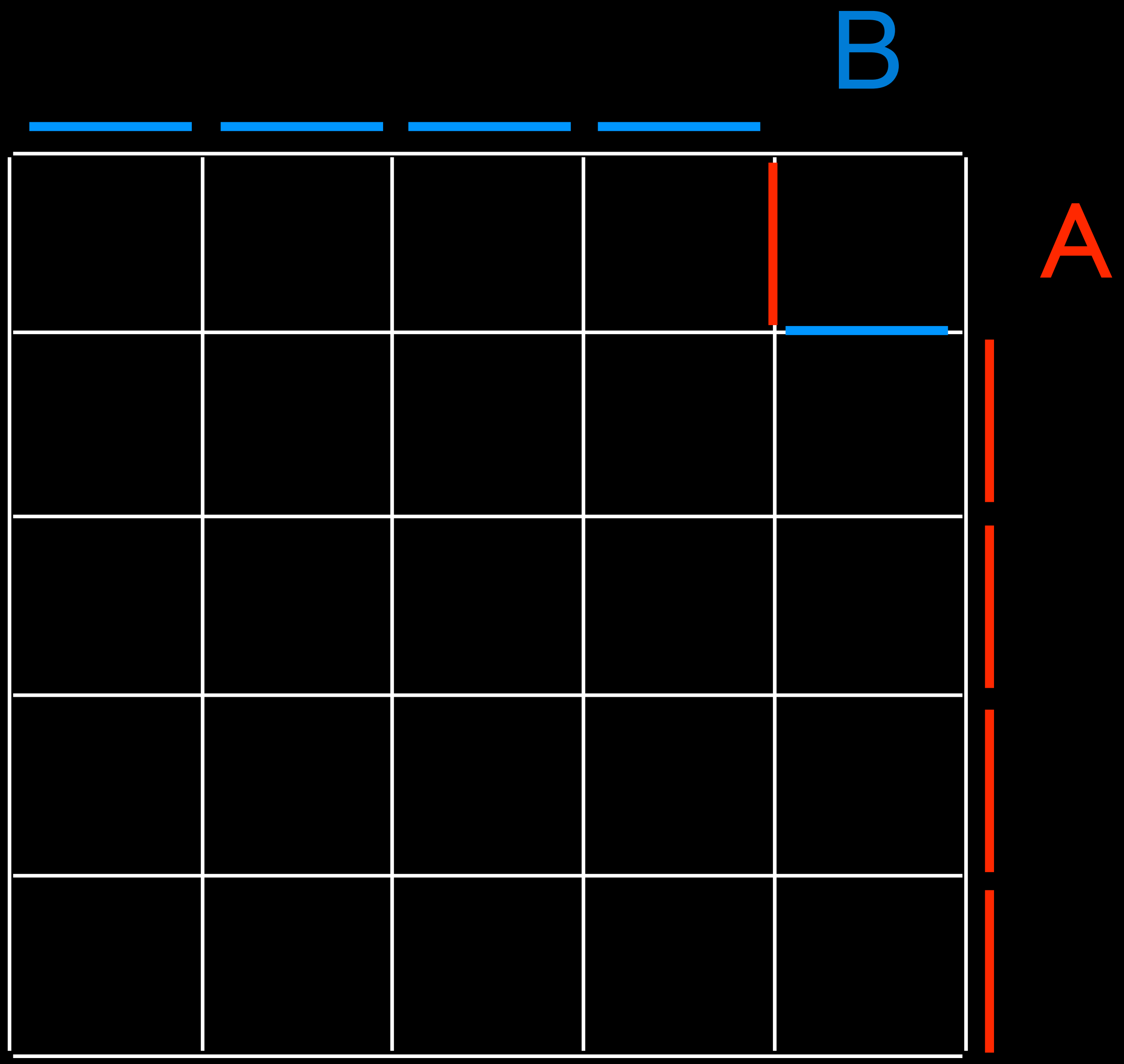
$c(u,v;w)$ = the number of $n \times n$ ASM (alternating sign matrices) "planar" proof:

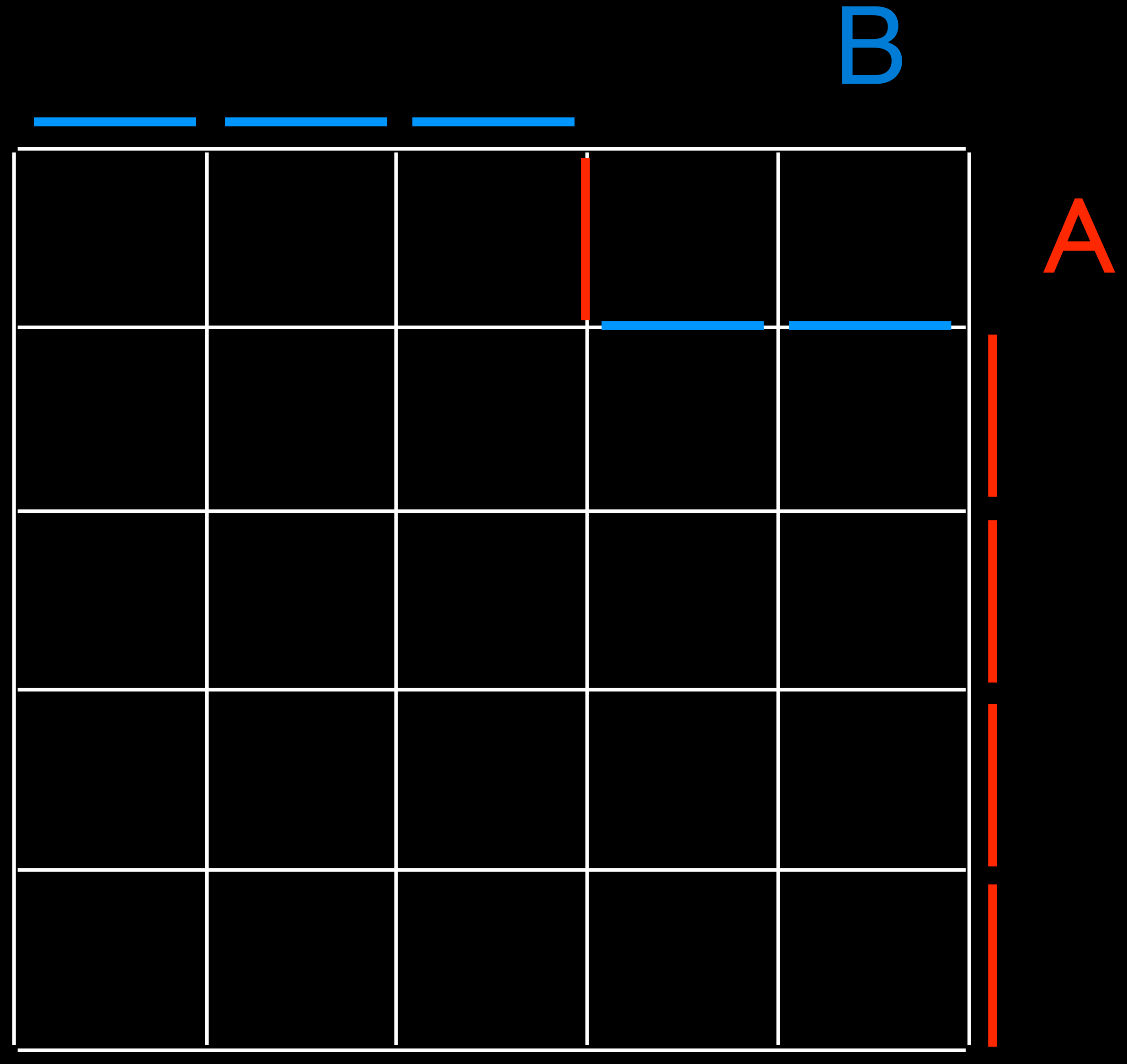


“planarisation” of the “rewriting rules”




B
A


B
A


B
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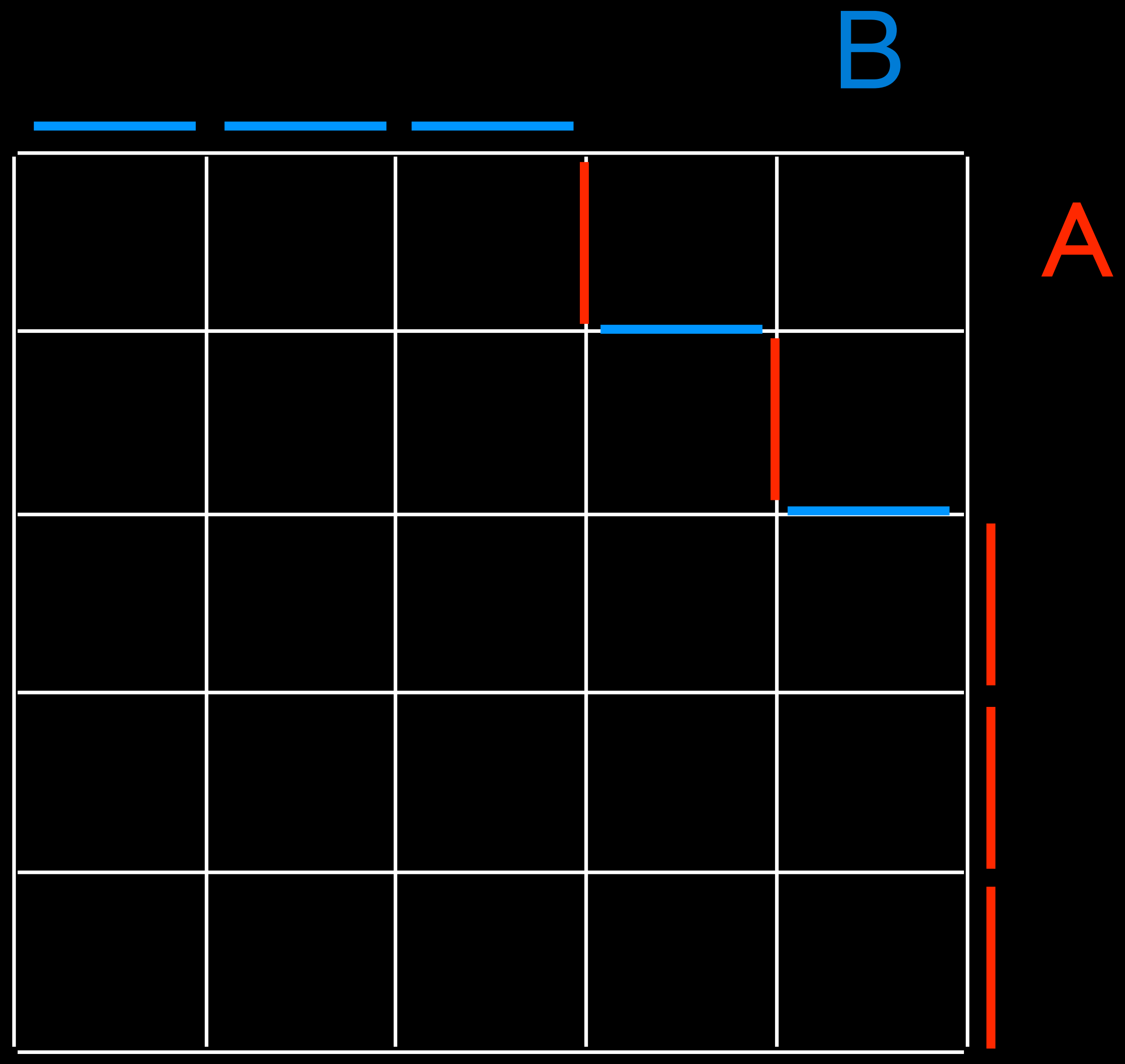
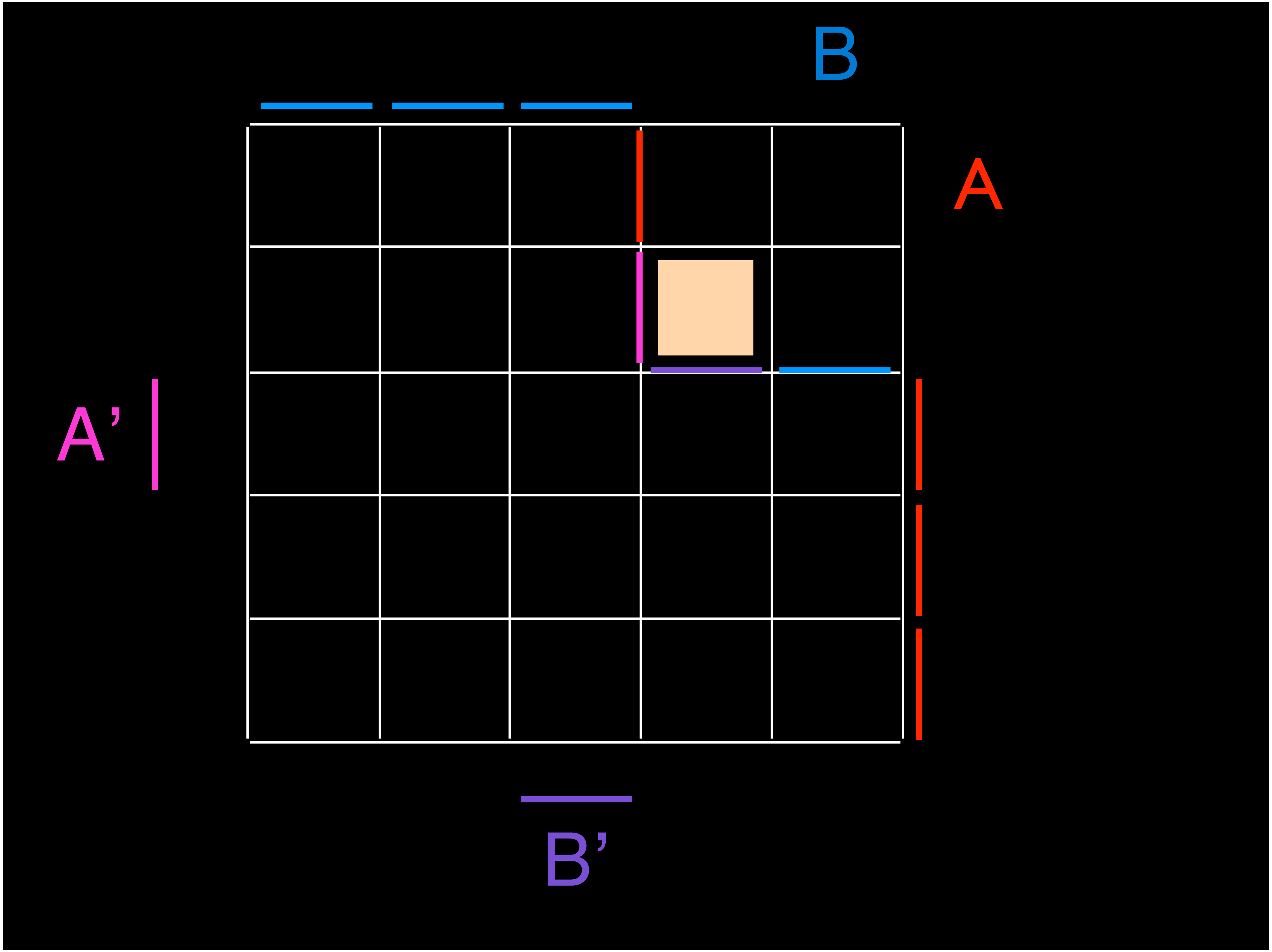
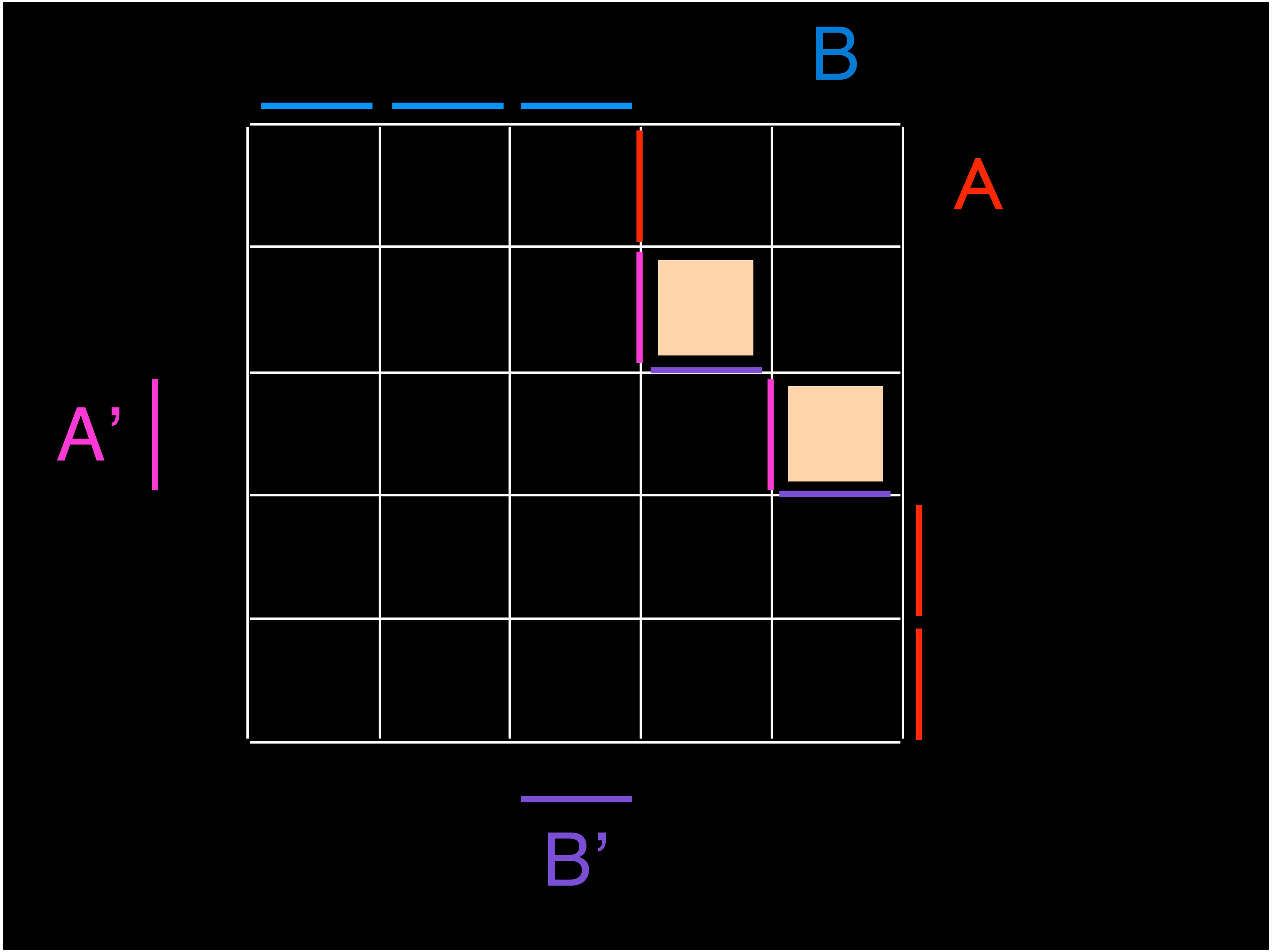

B
A


B
A
A'
B'



B
A
A'
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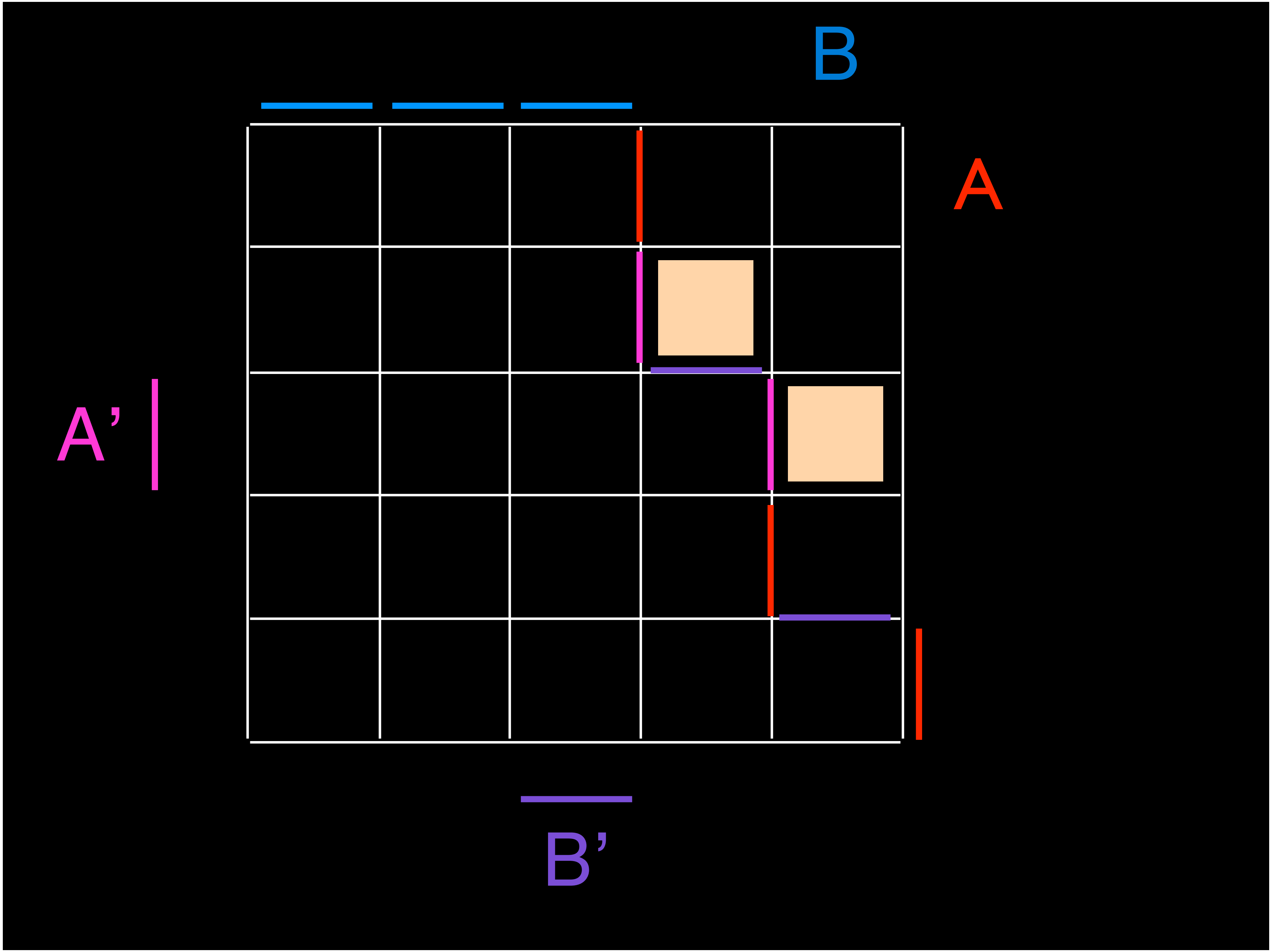

B
A
A’
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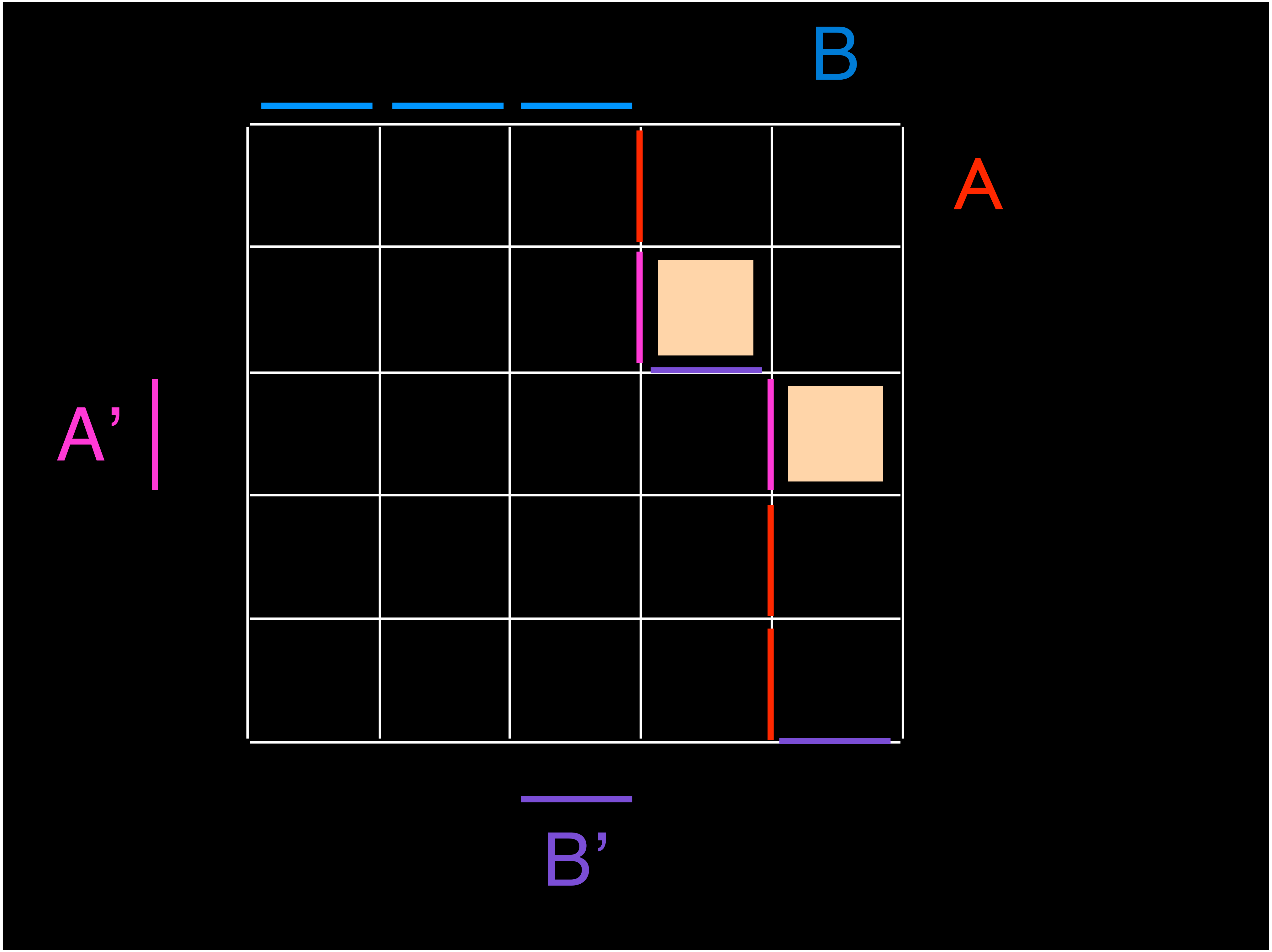

B
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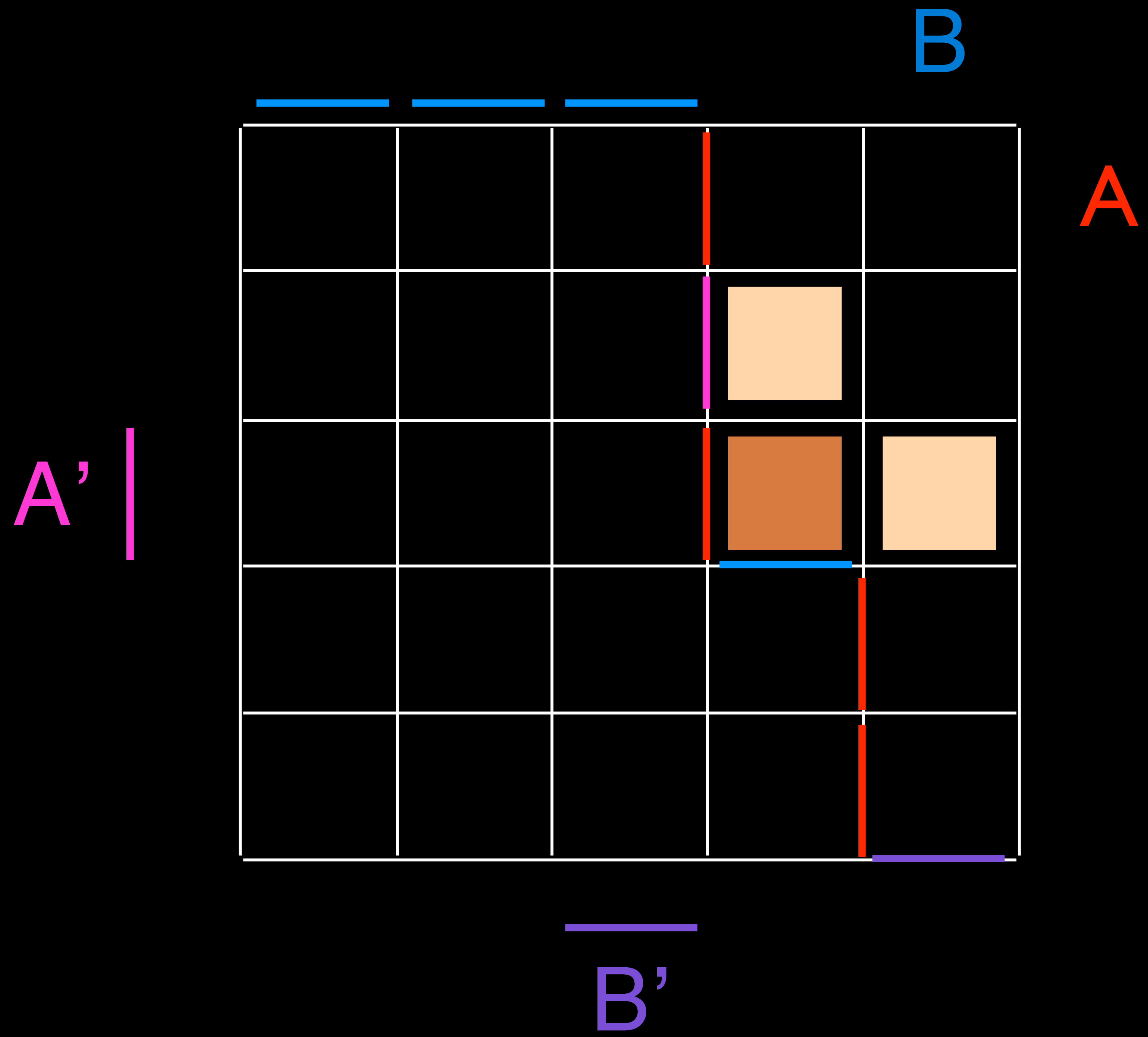
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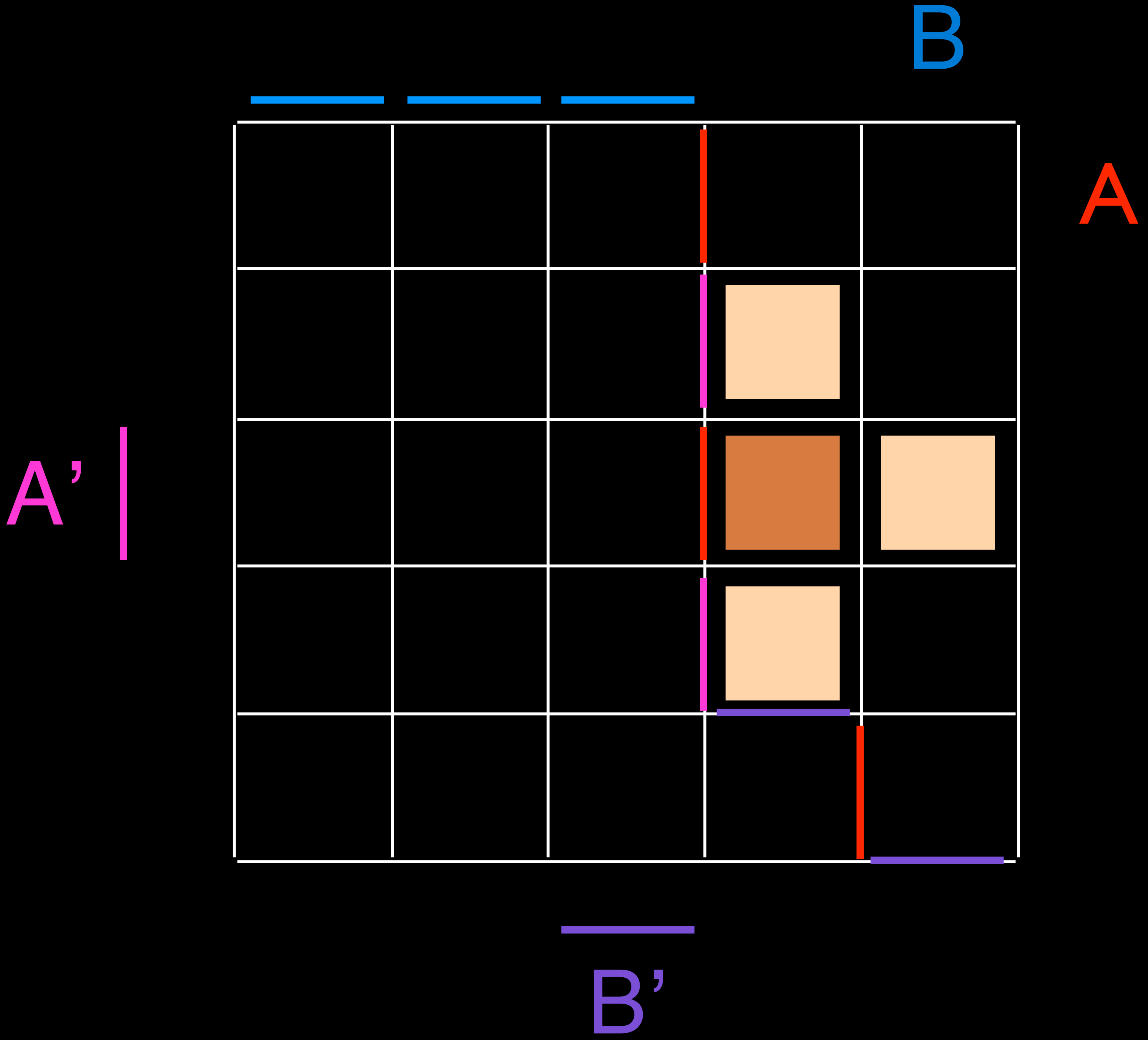
B
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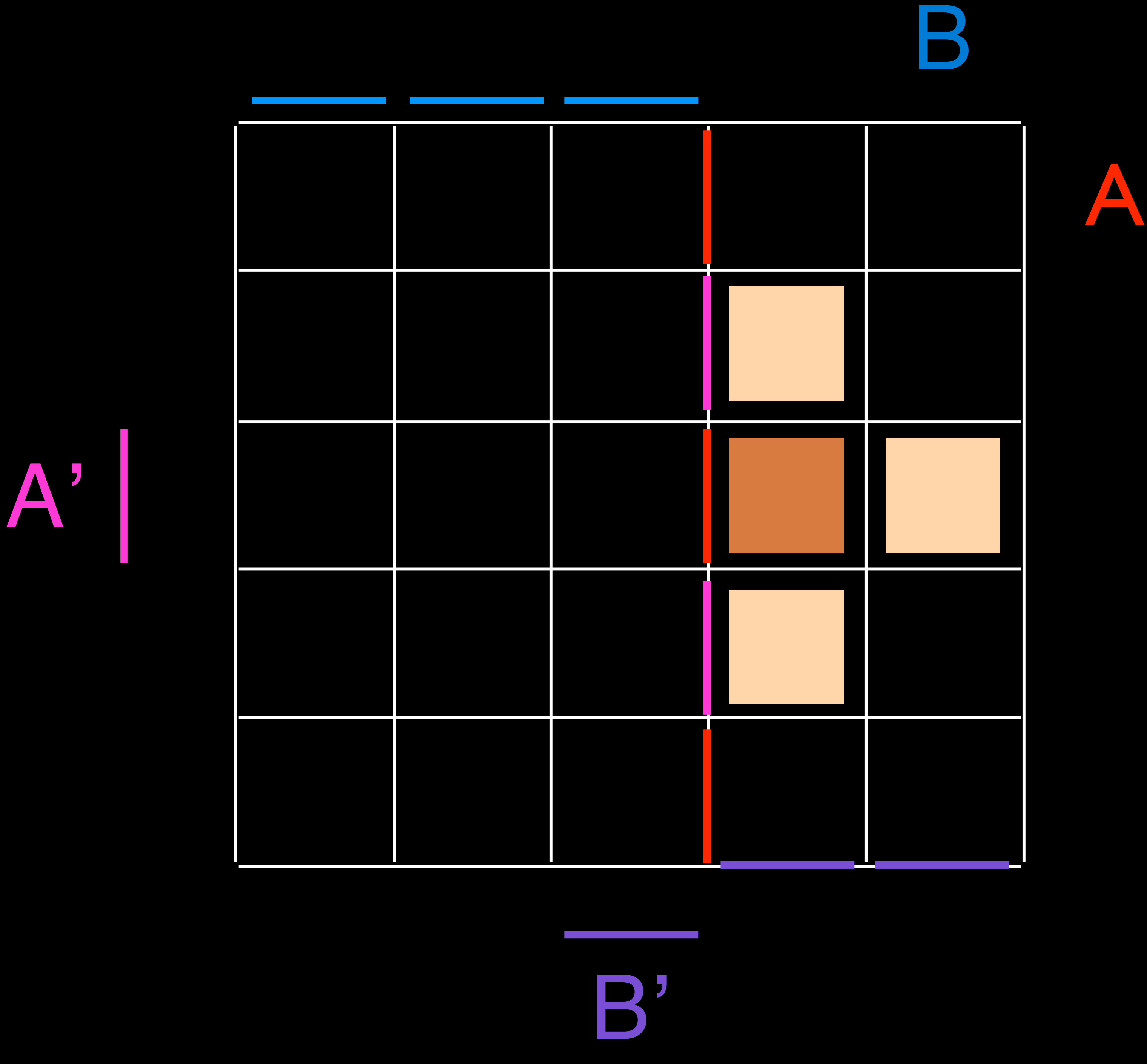
B
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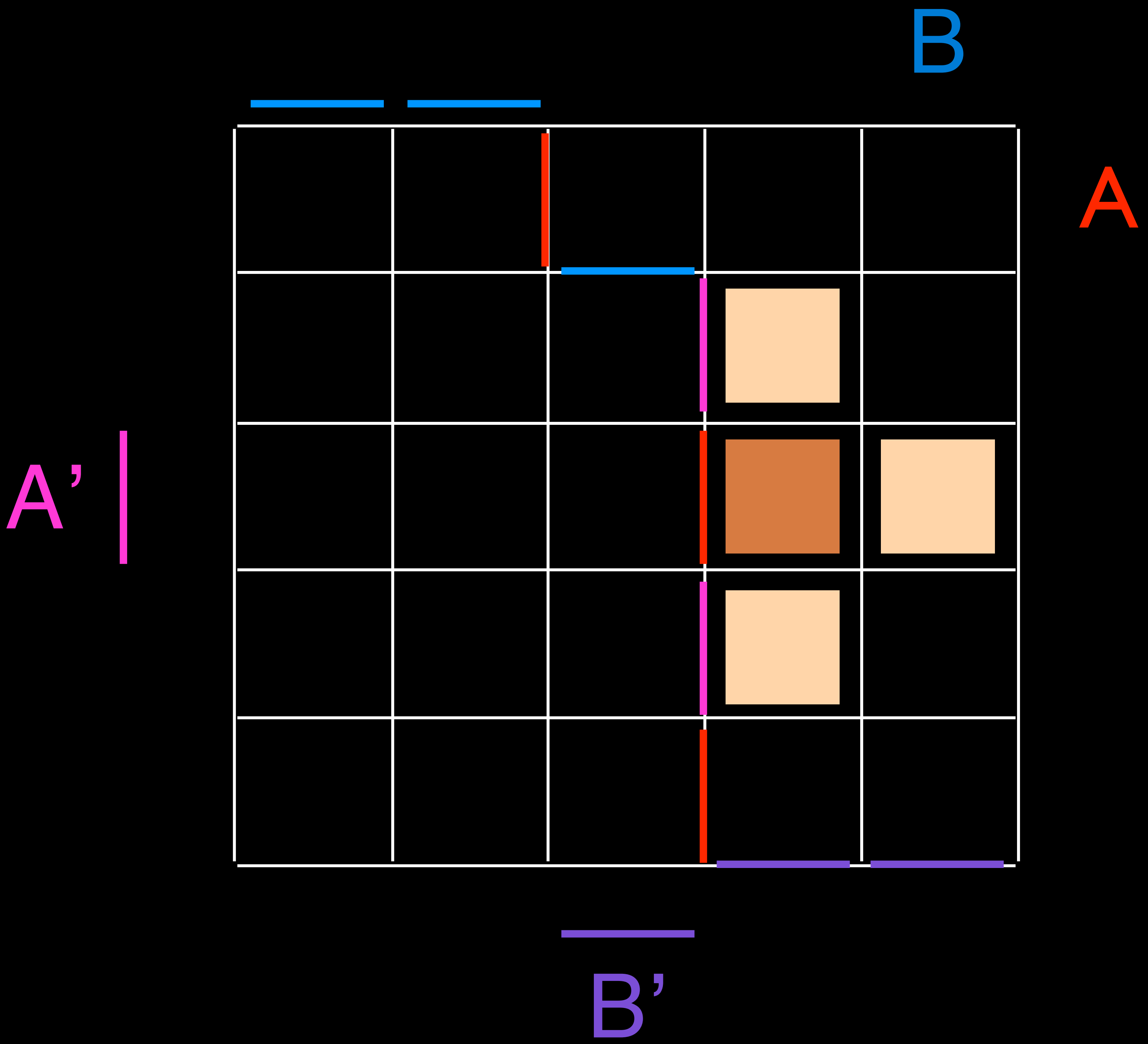

B
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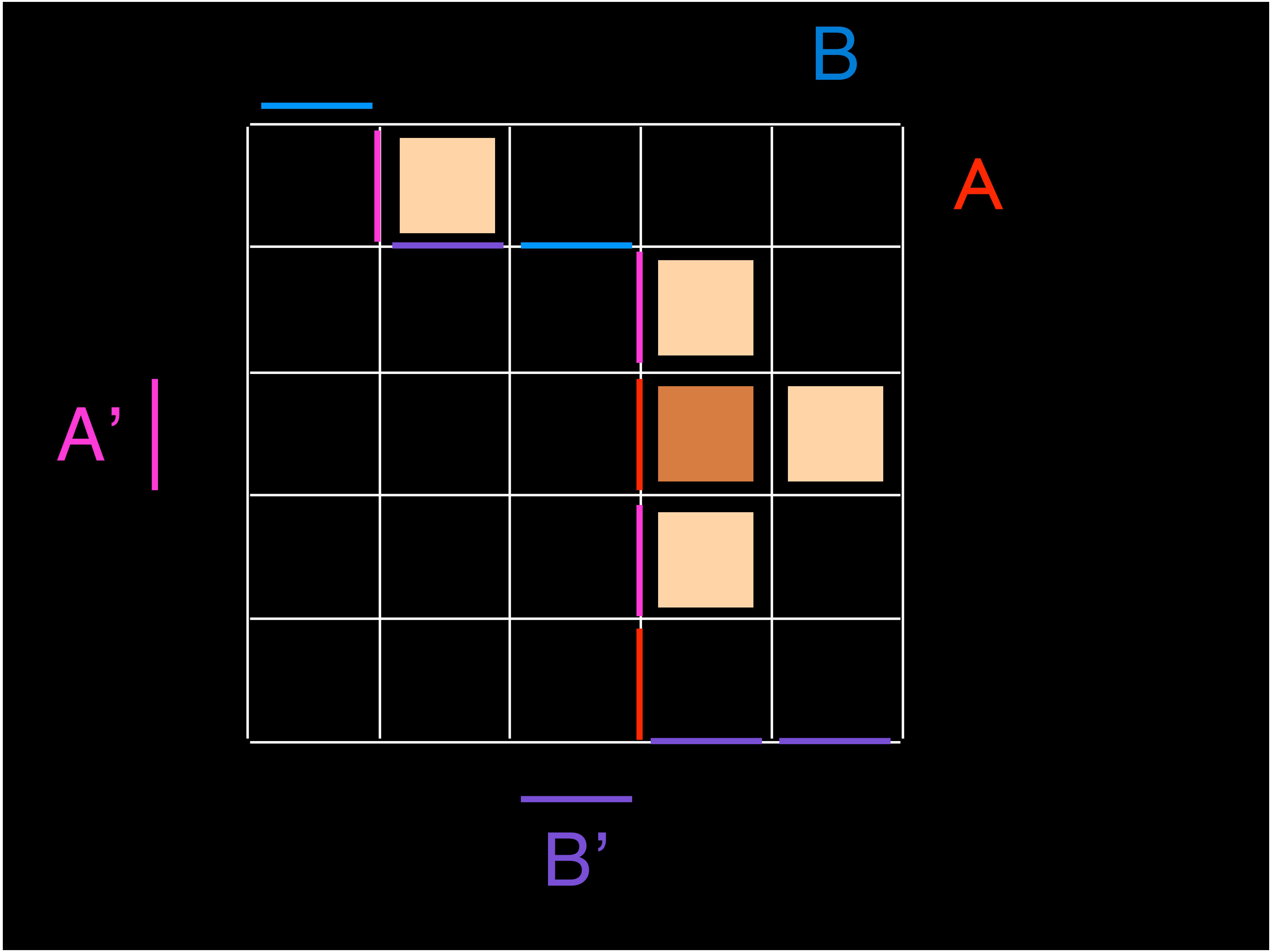
B
A
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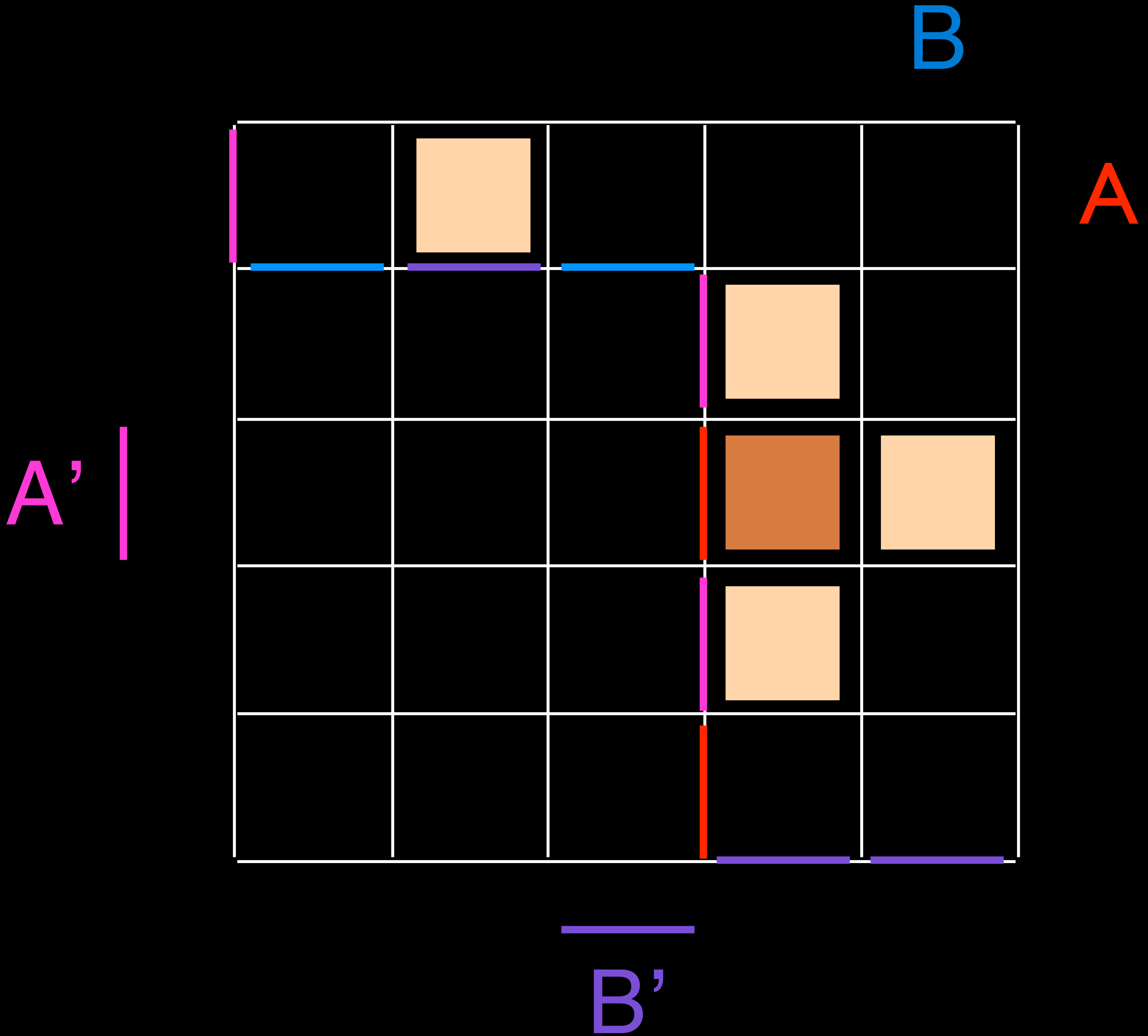
B
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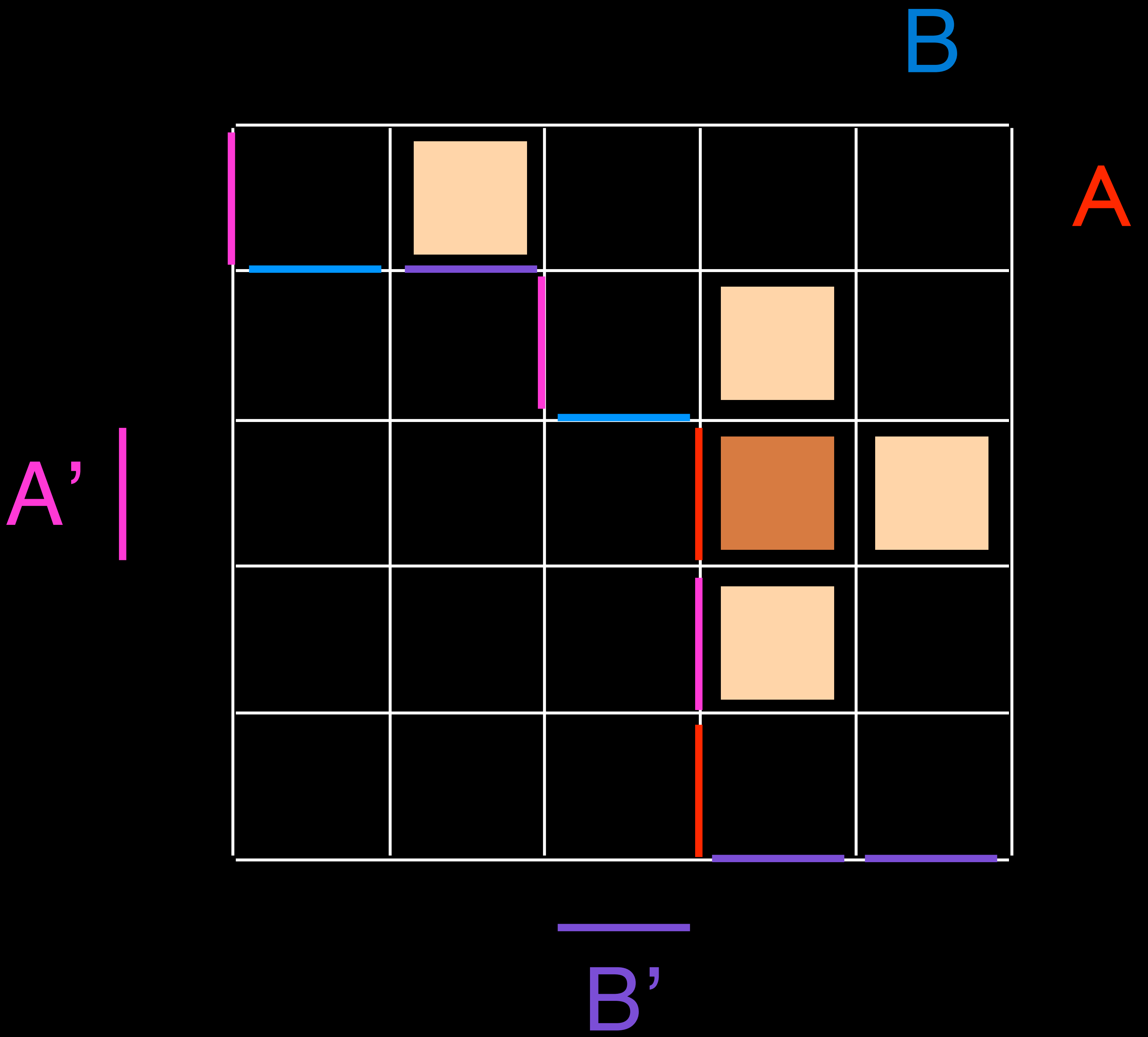
B
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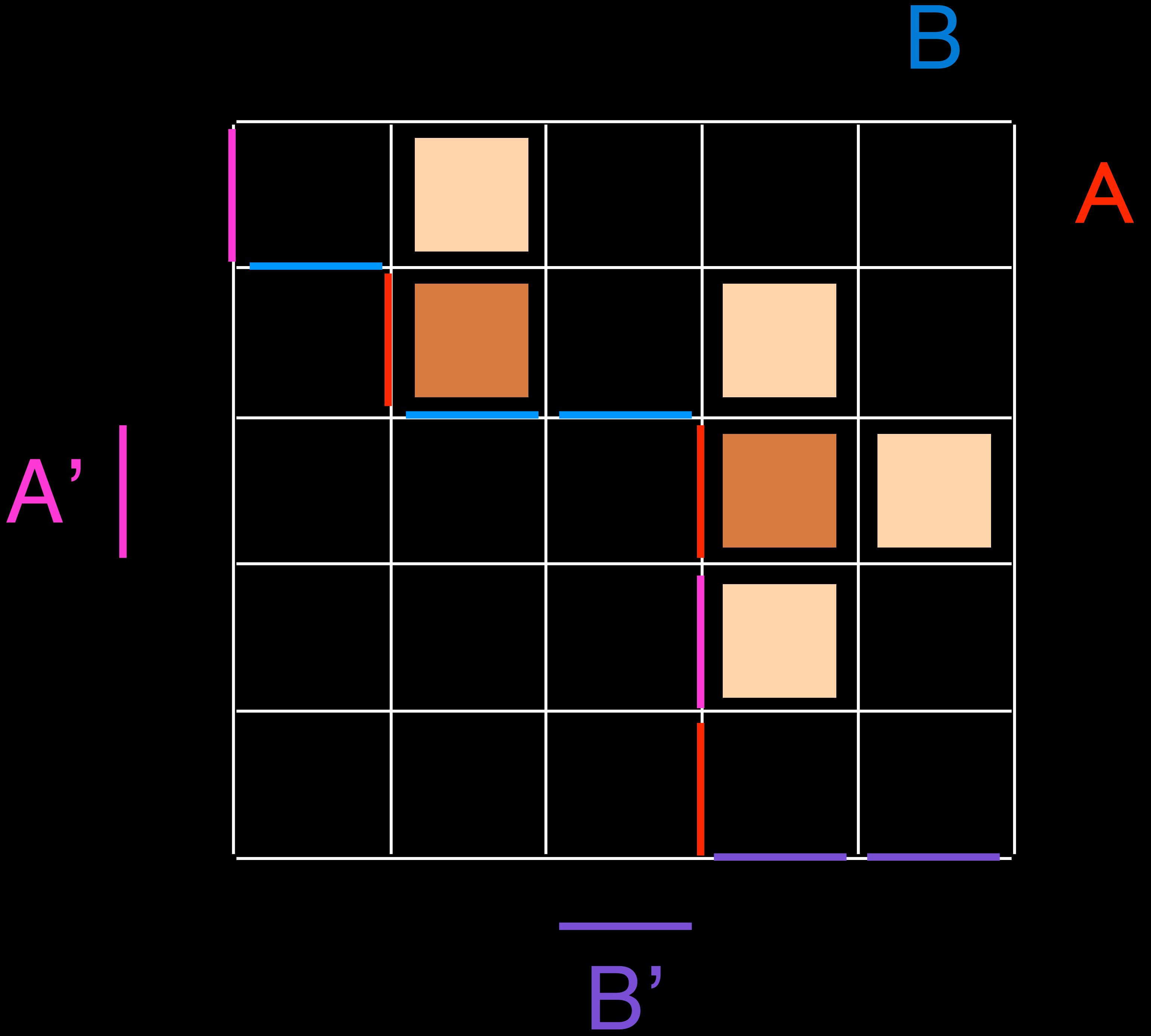
B
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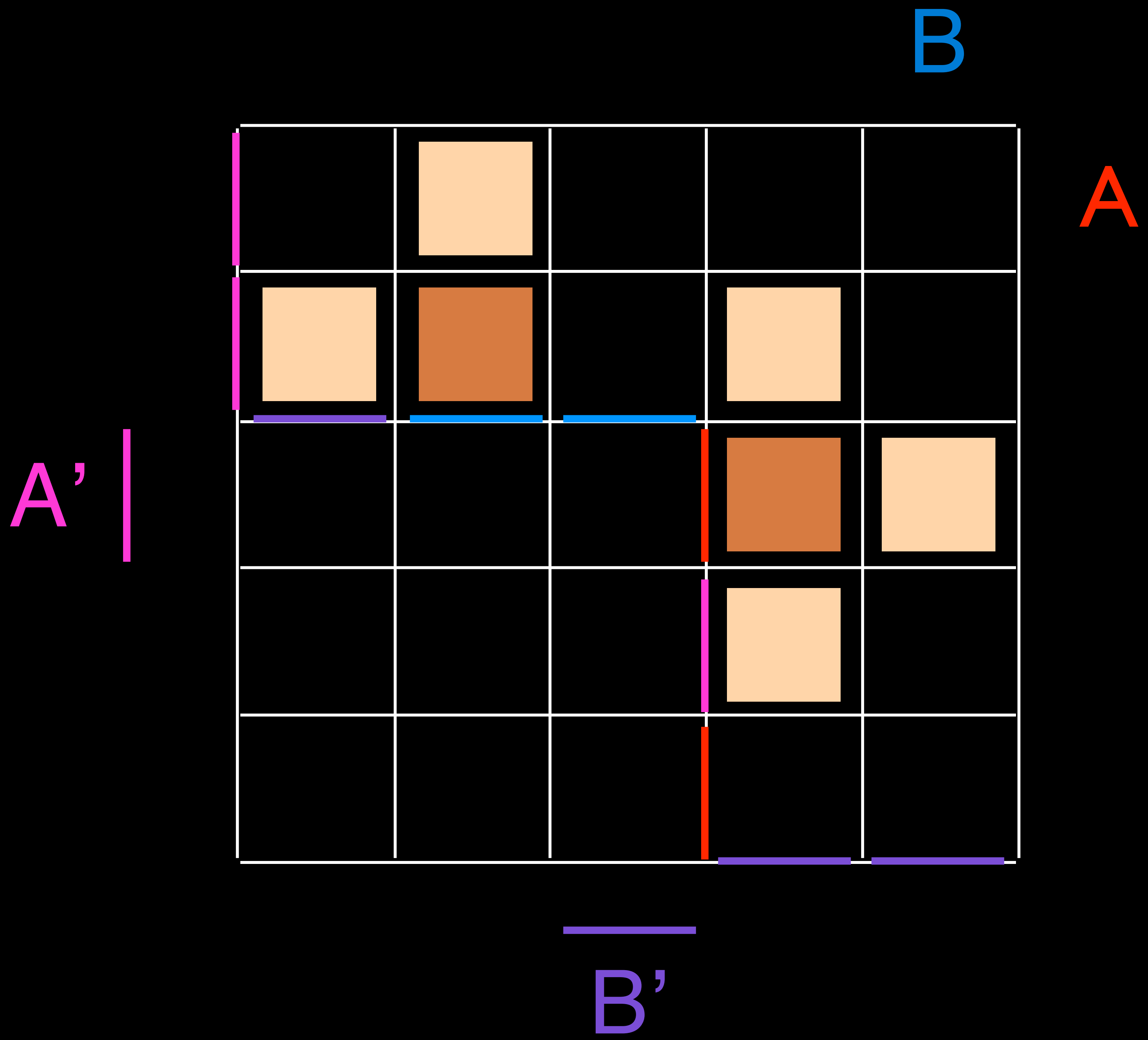
B
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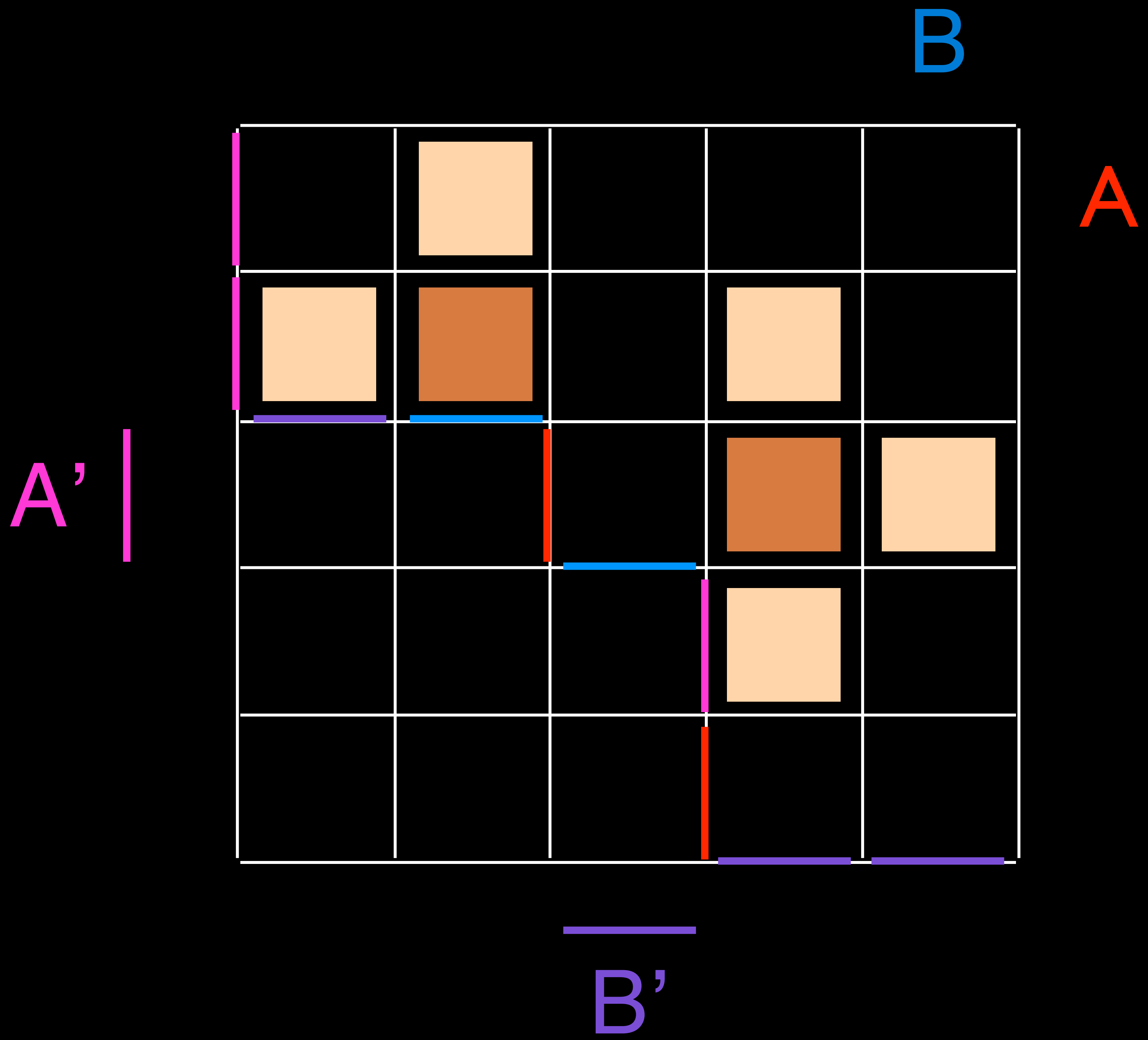
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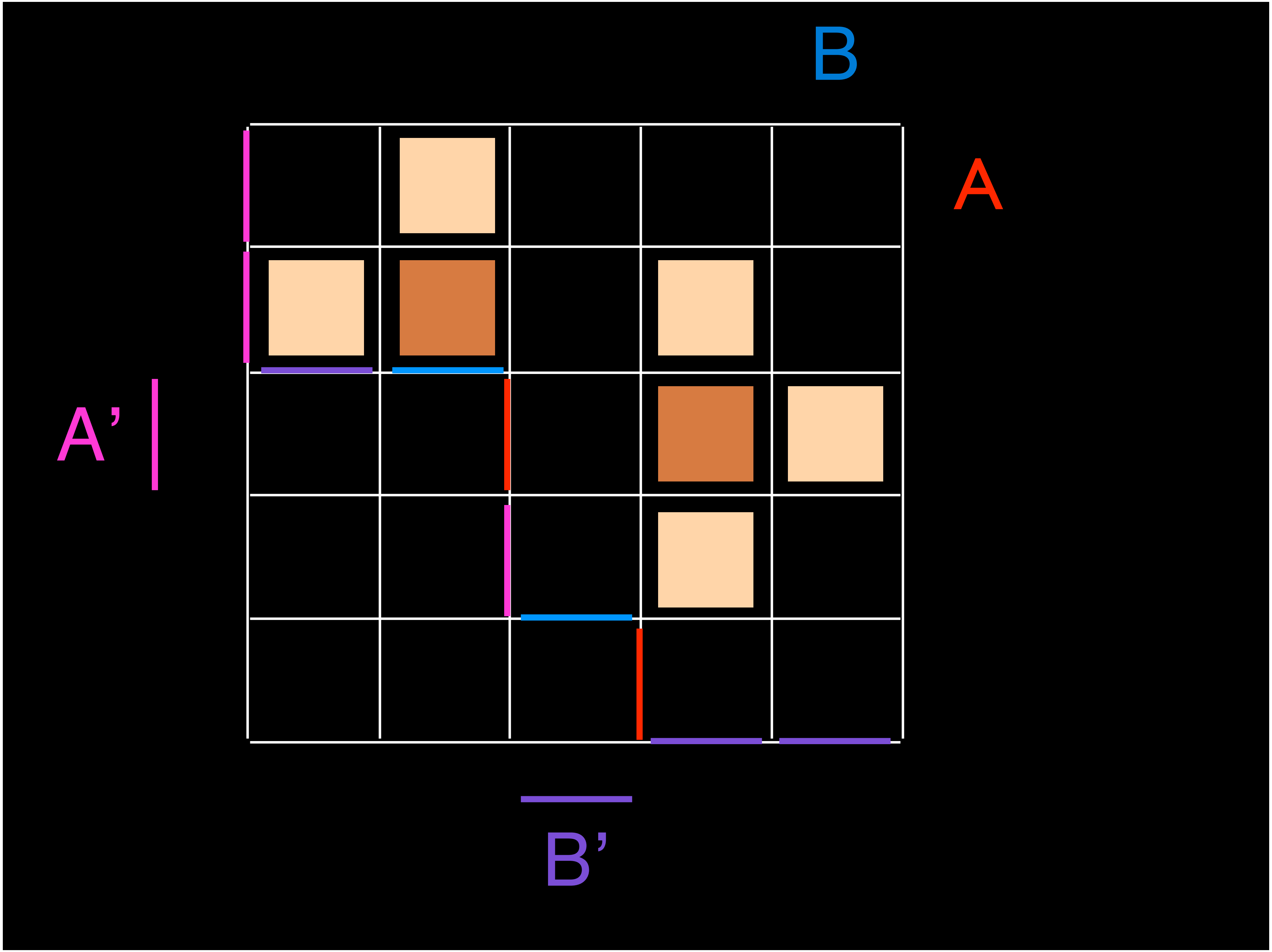

B
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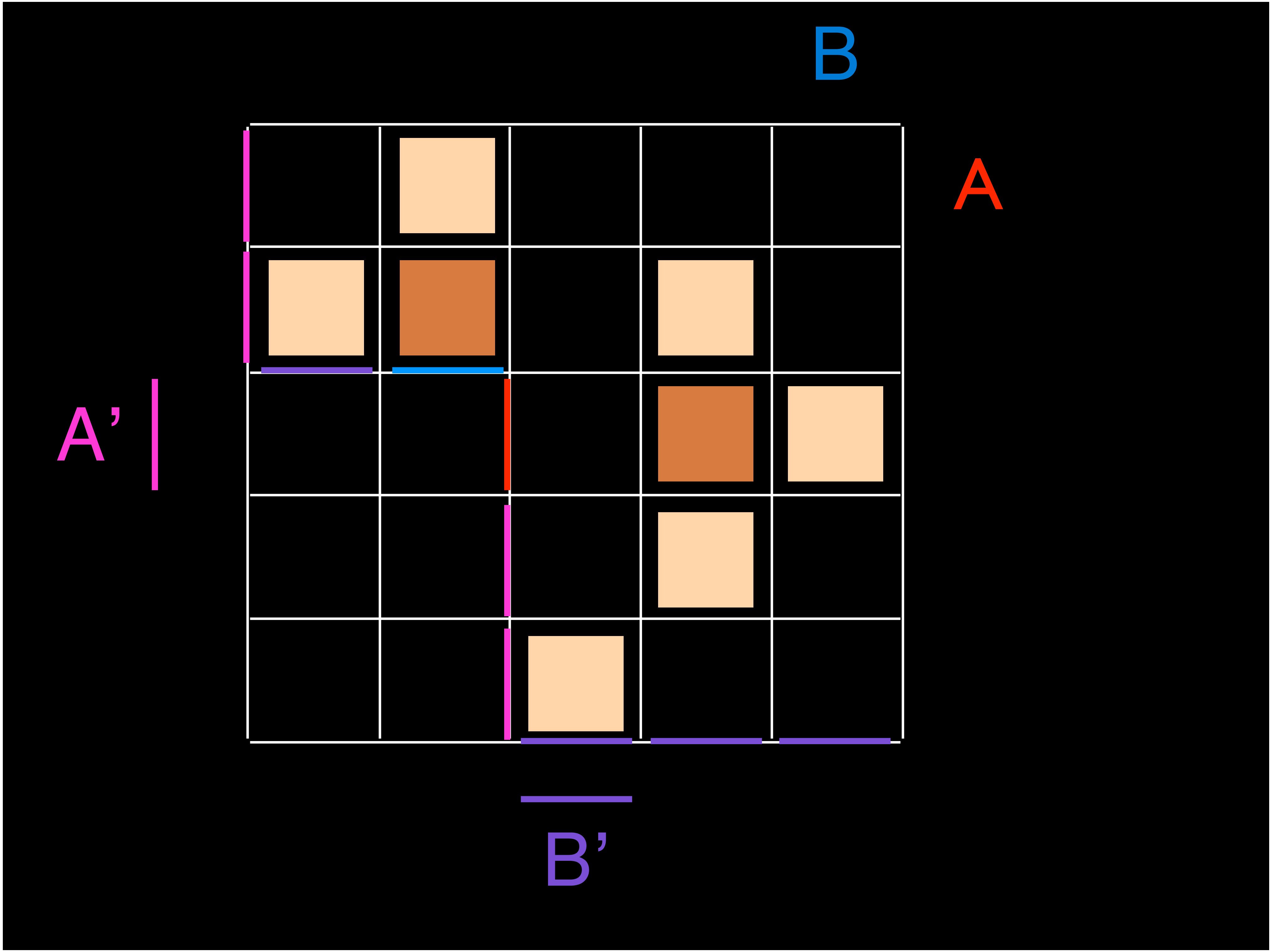
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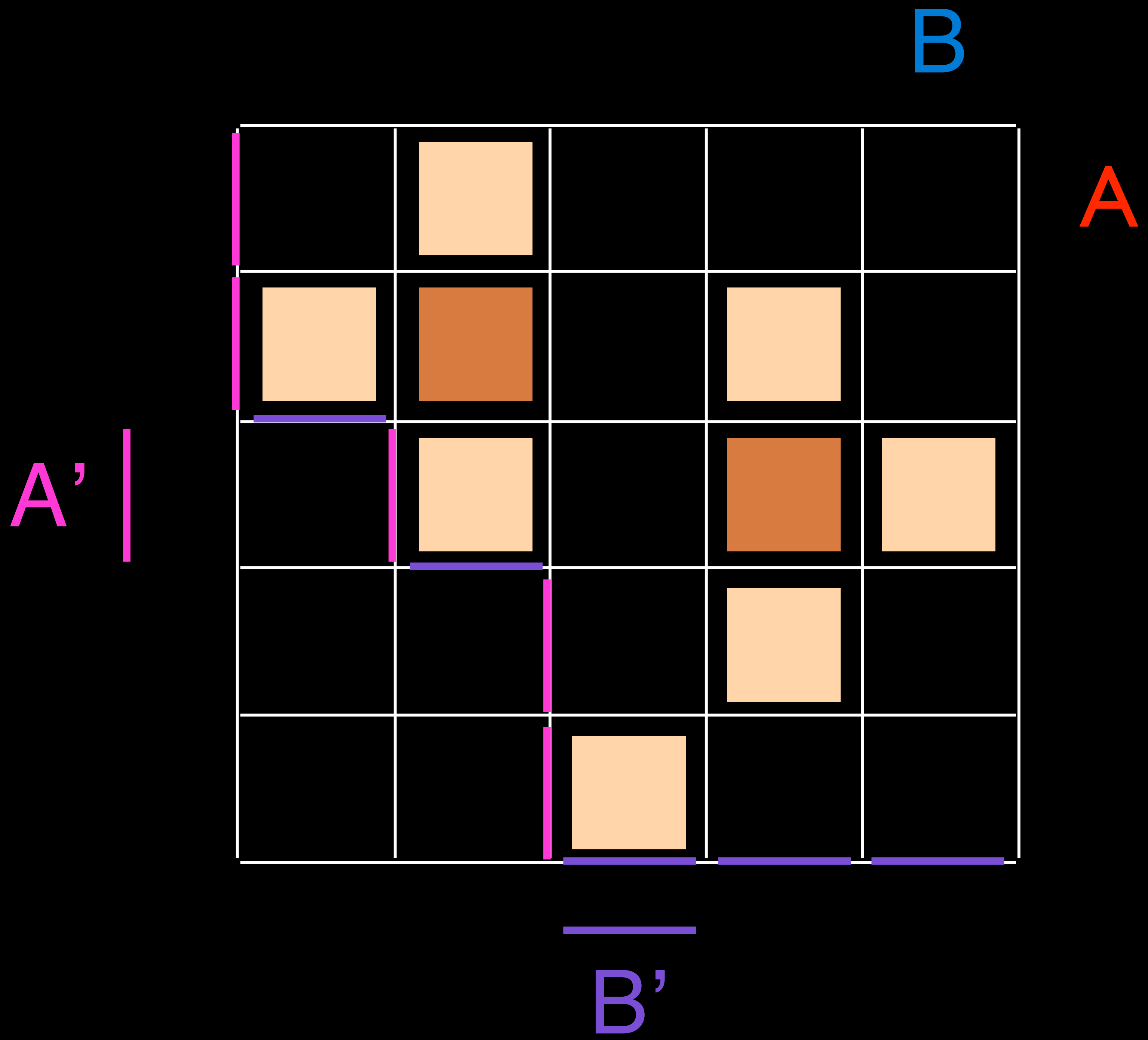
B
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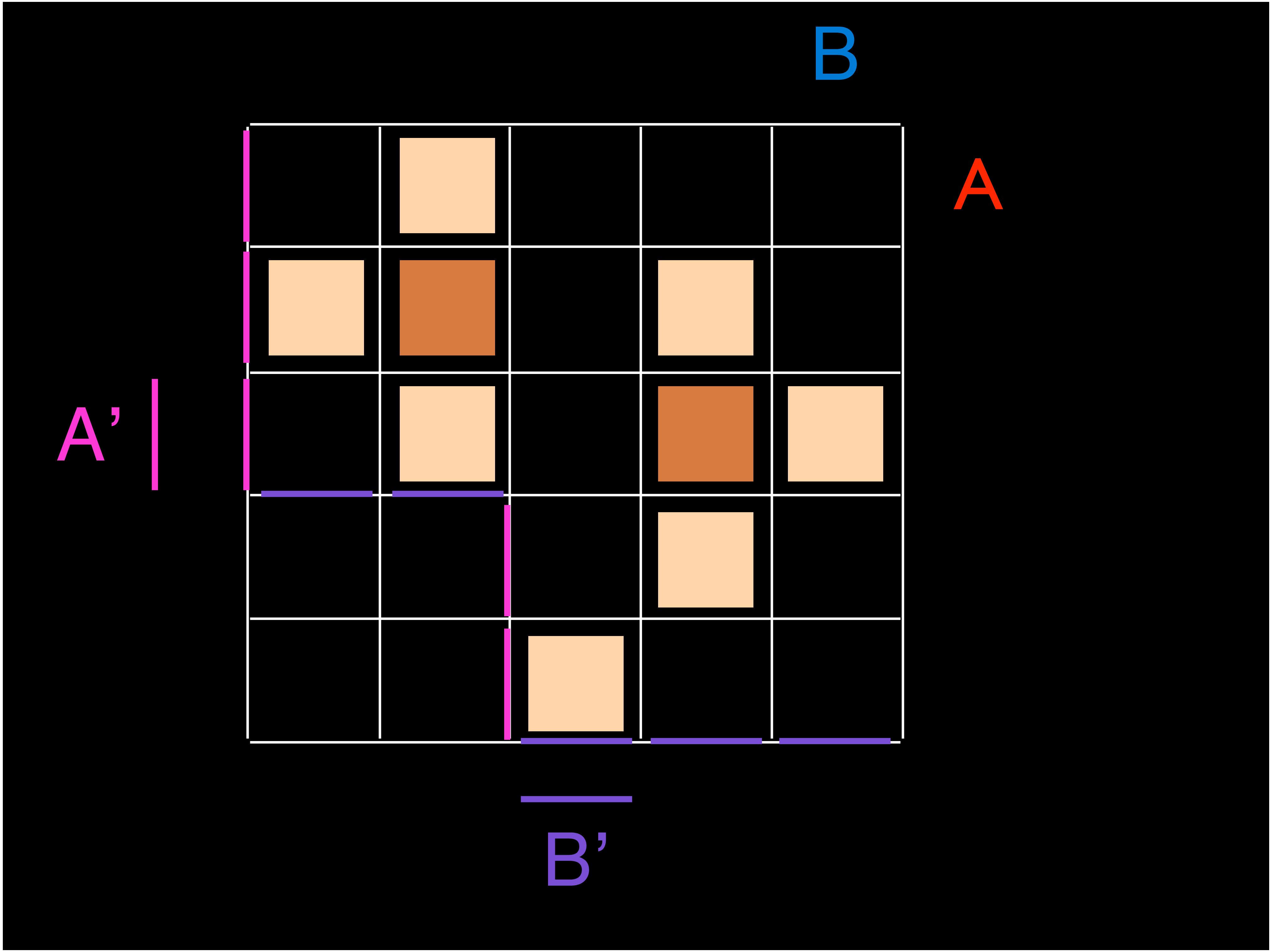

B
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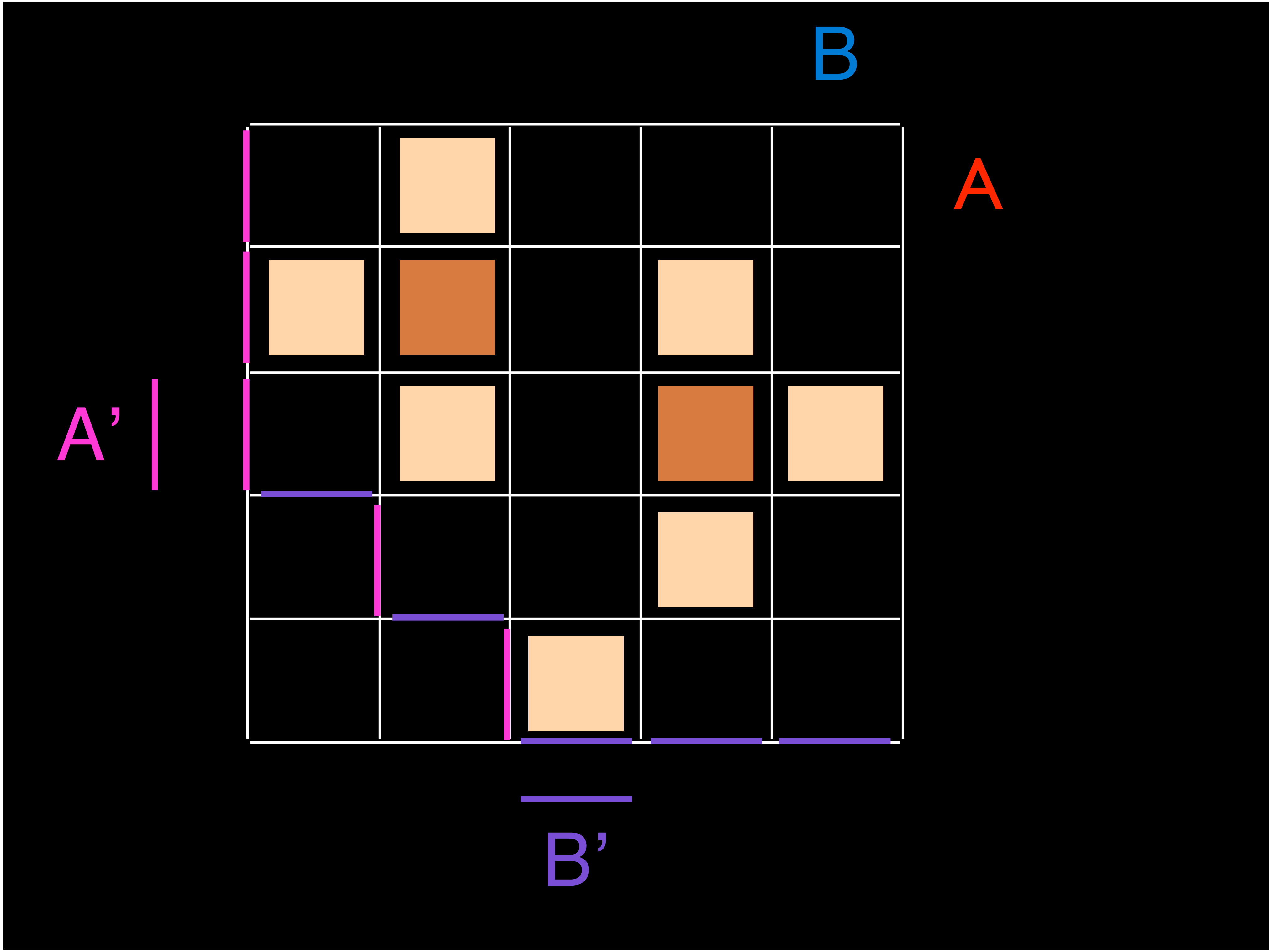

B
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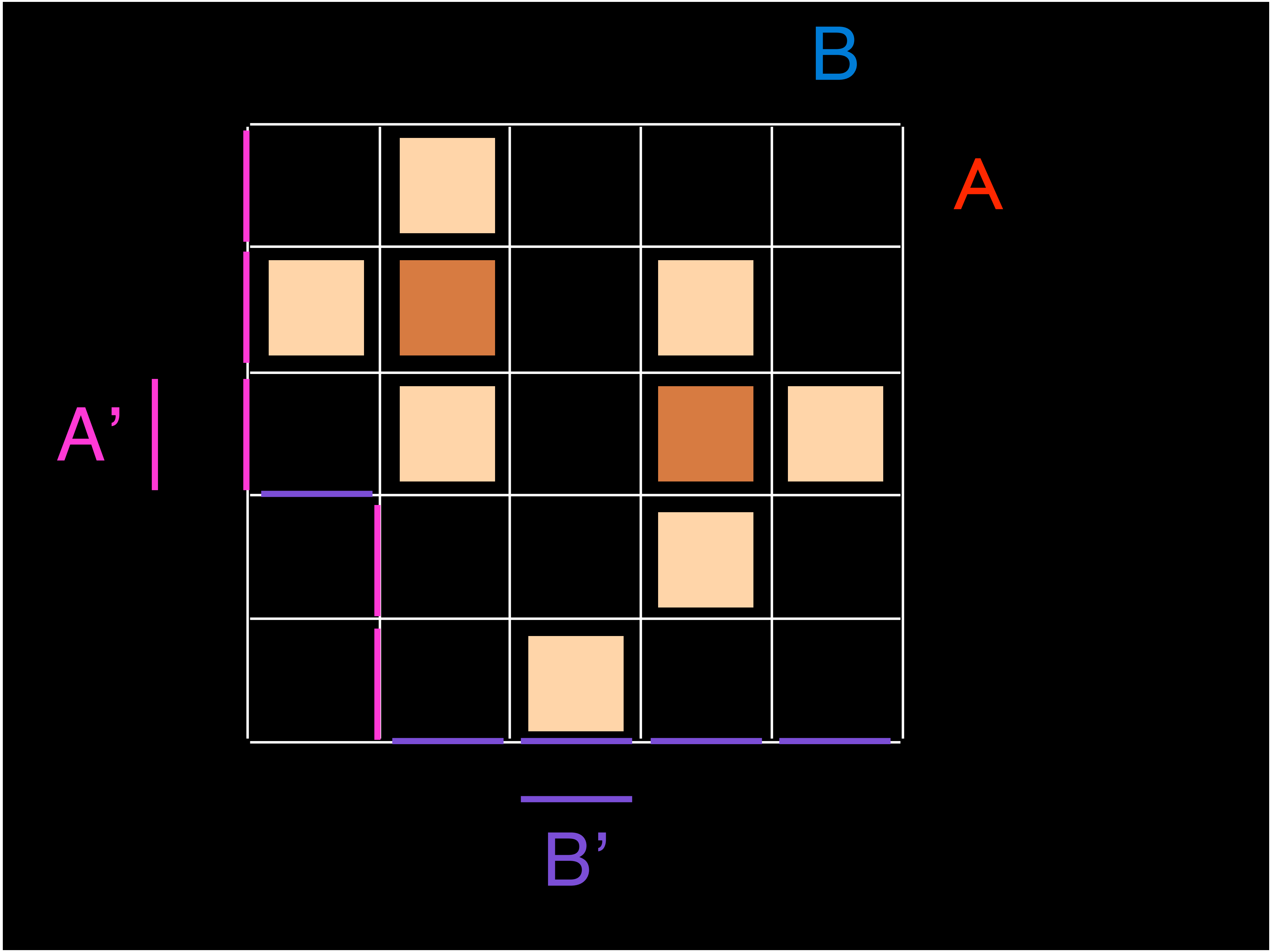

B
A
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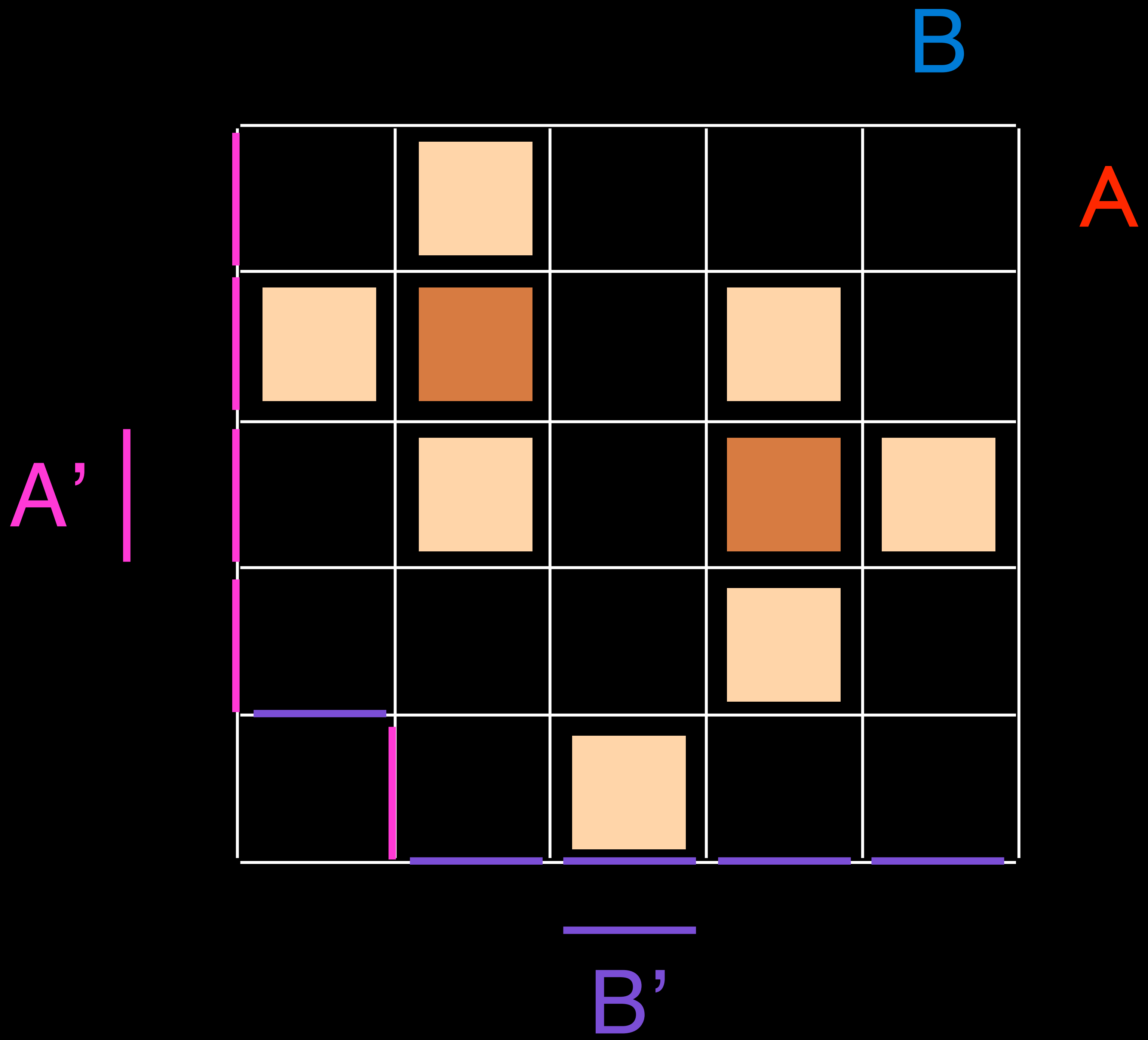
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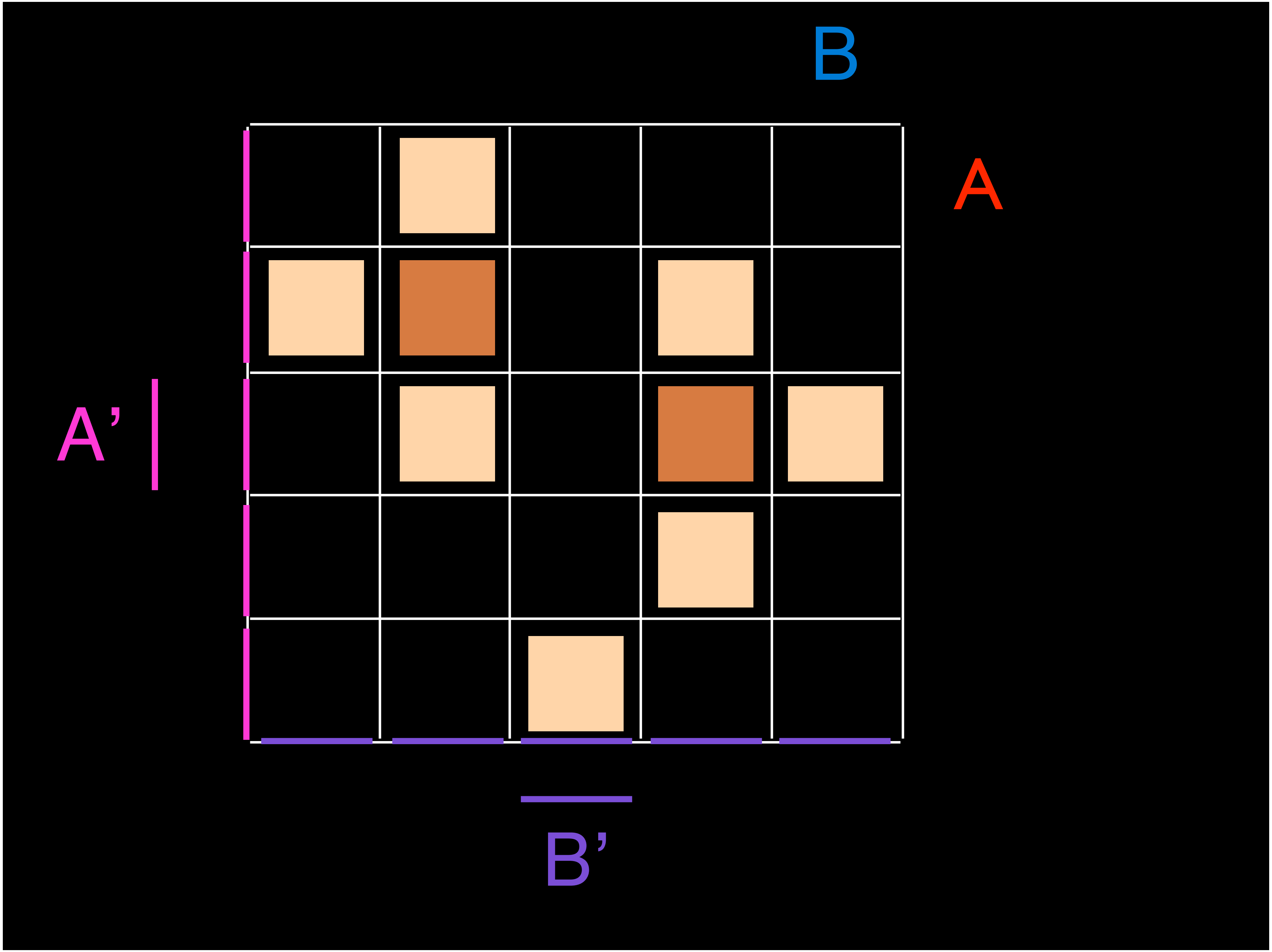
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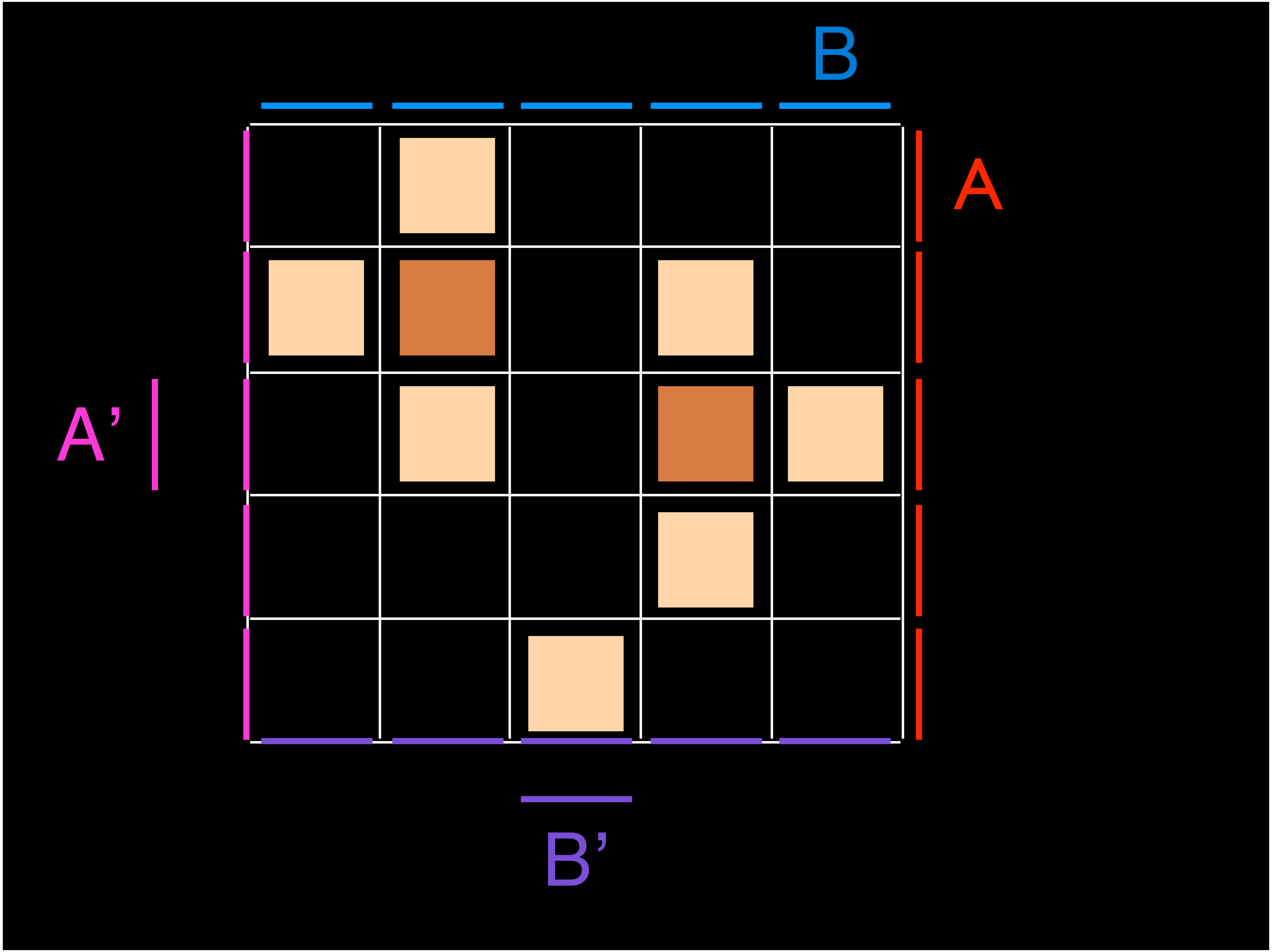
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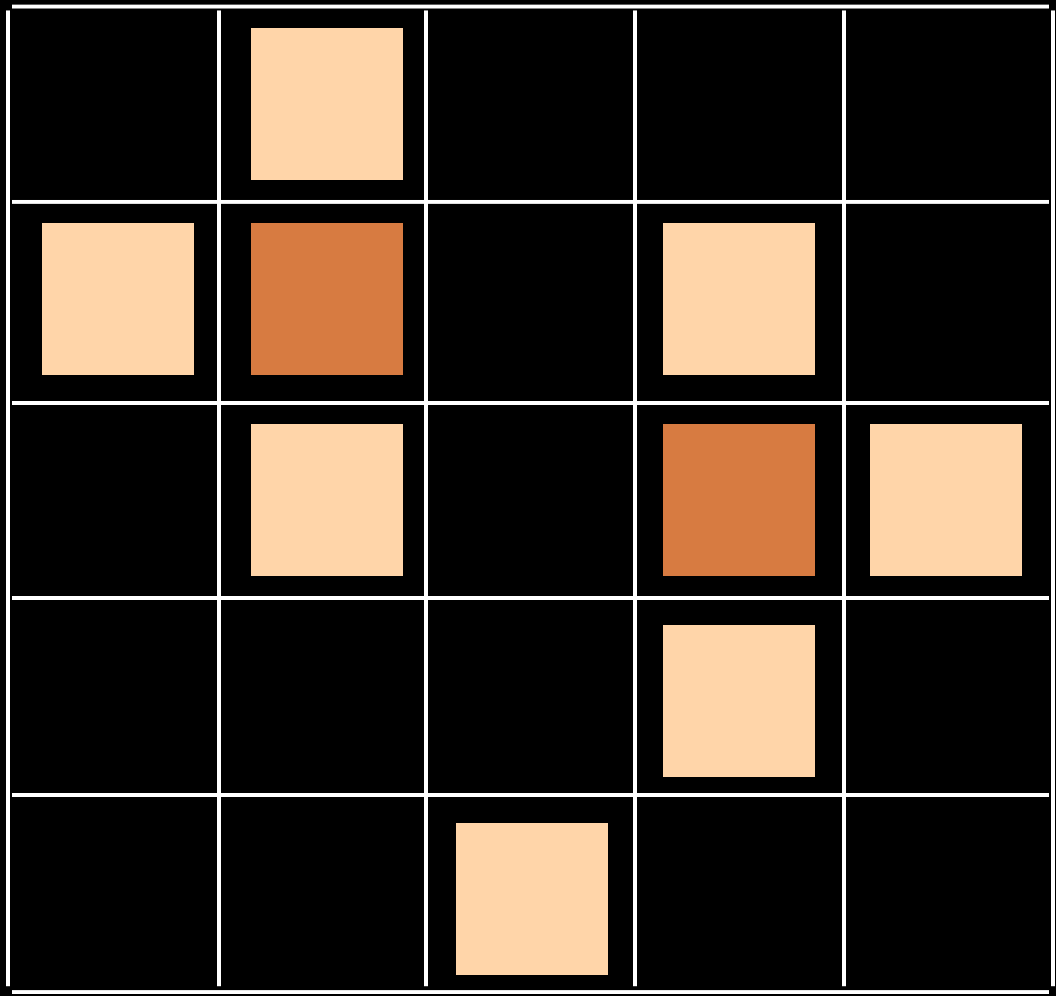

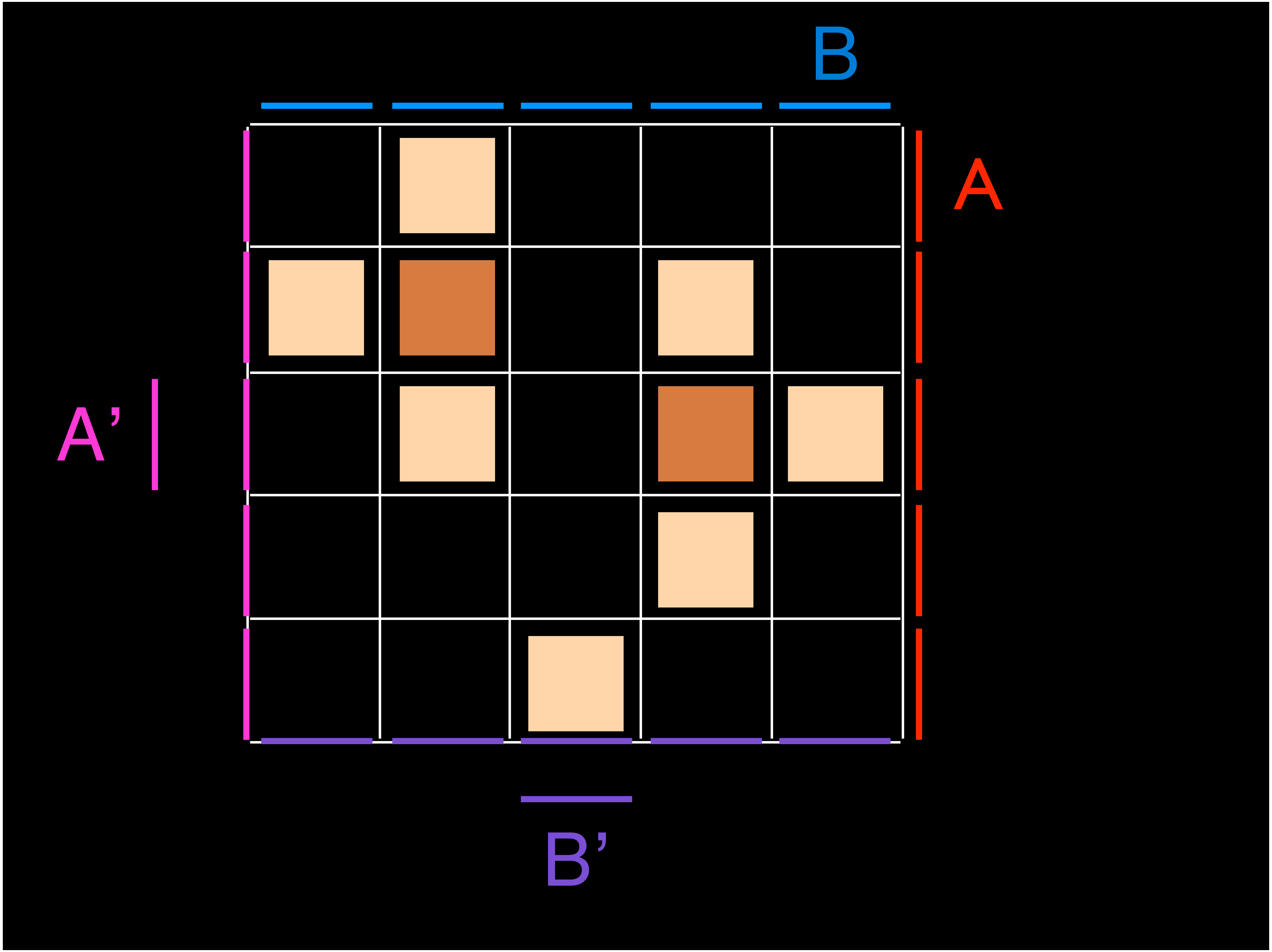

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8- parameters quadratic algebra

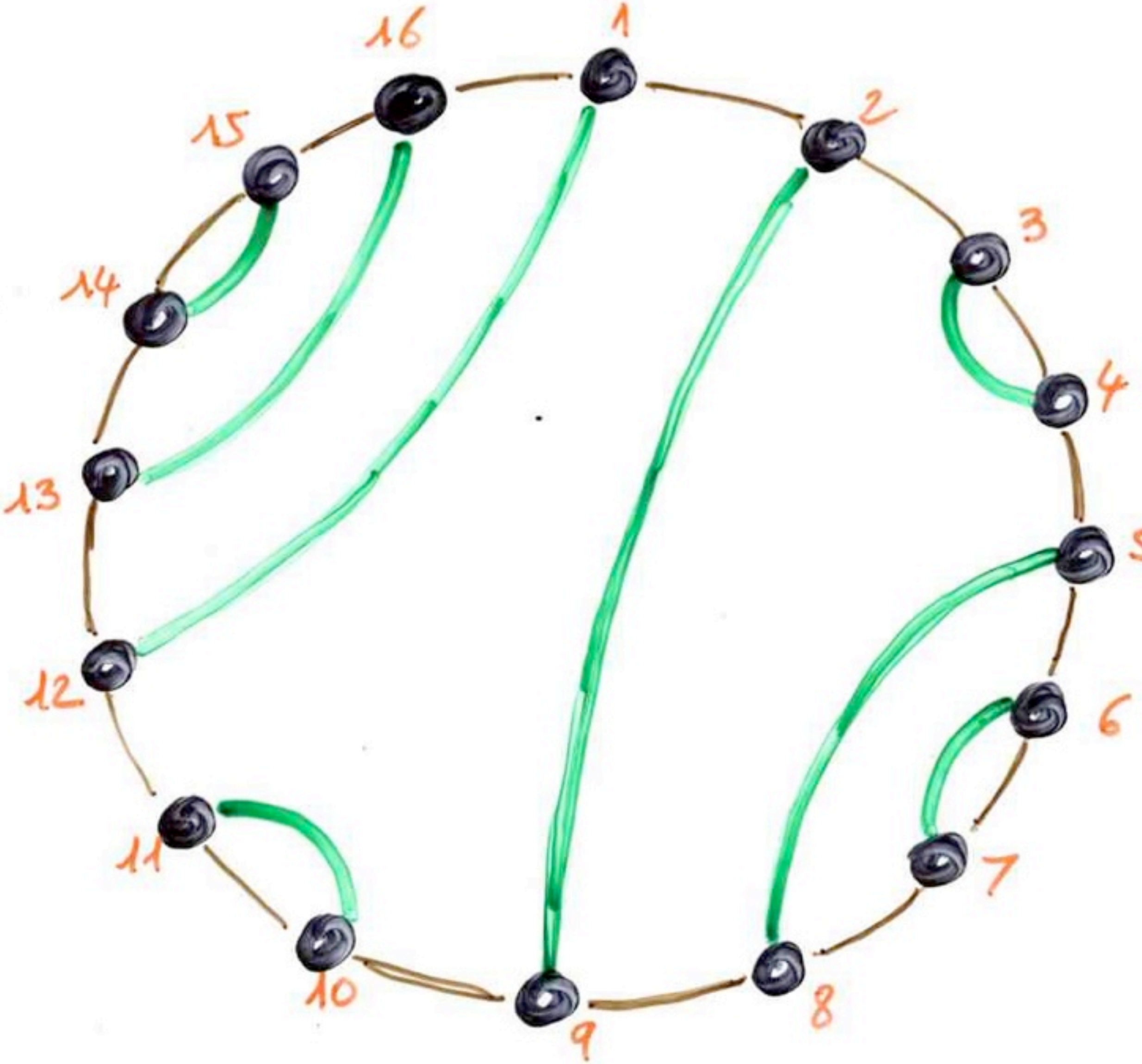
commutations

$$\begin{cases} BA = q_1 AB + q_2 A'B' \\ B'A' = q_3 A'B' + q_4 AB \end{cases}$$

$$\begin{cases} B'A = q_5 AB' + q_6 A'B \\ BA' = q_7 A'B + q_8 AB' \end{cases}$$

§ 14 Razumov - Stroganov conjecture
(2000, ...)

Markov chain
on
chord diagrams



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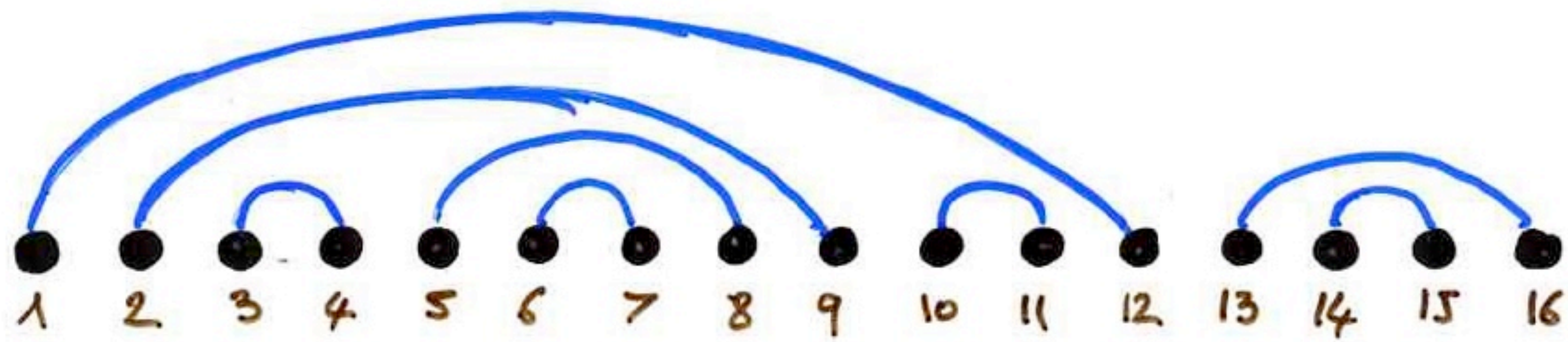
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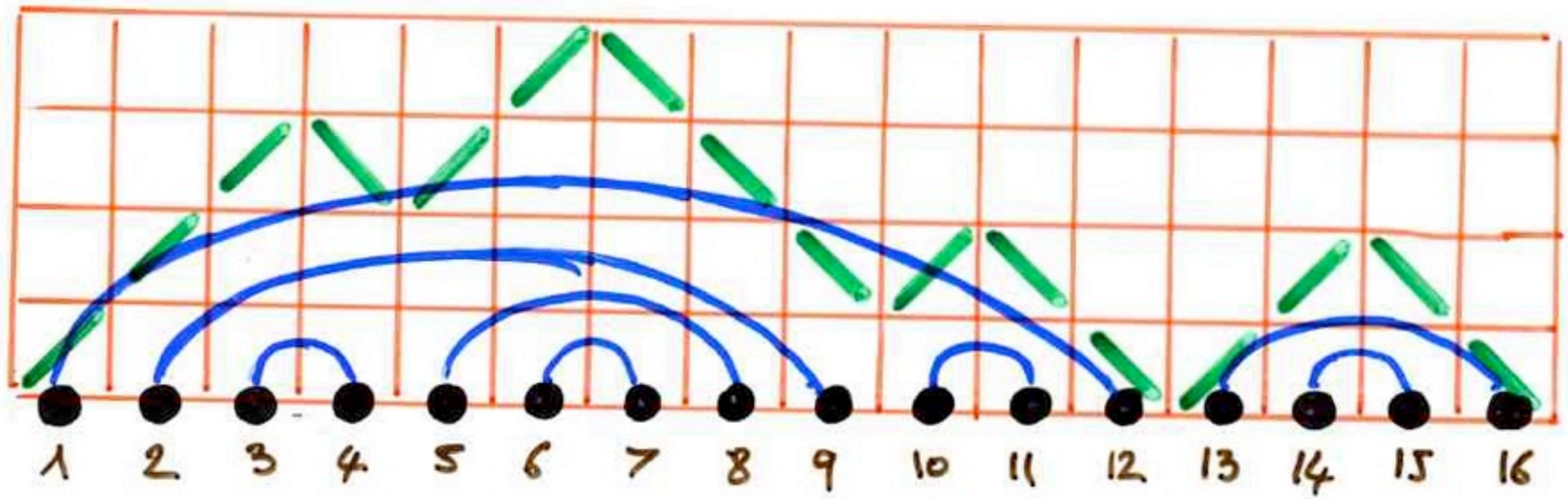

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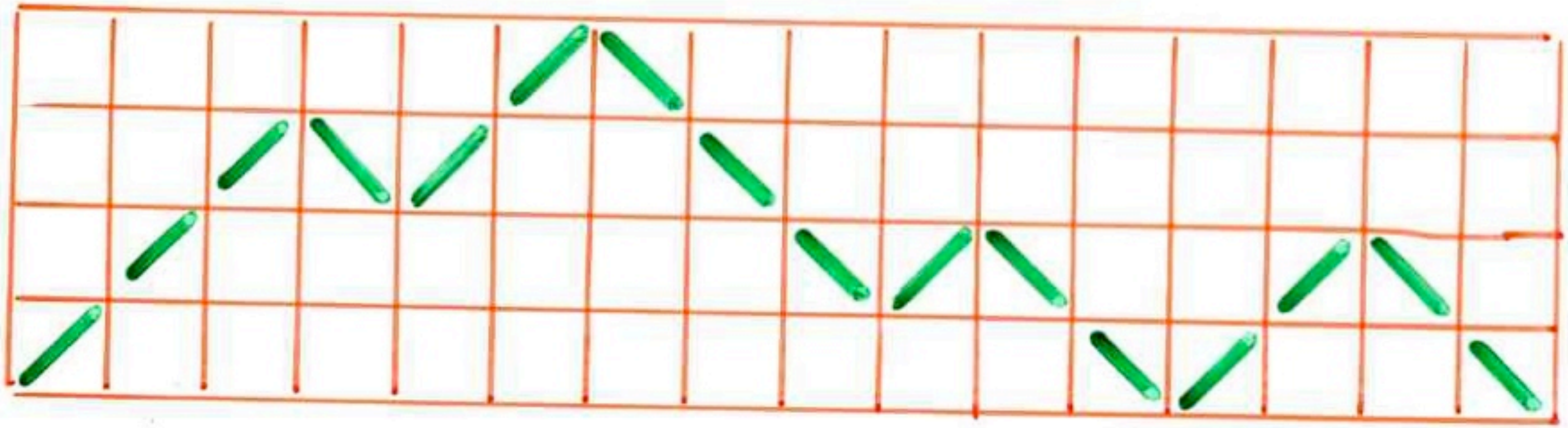

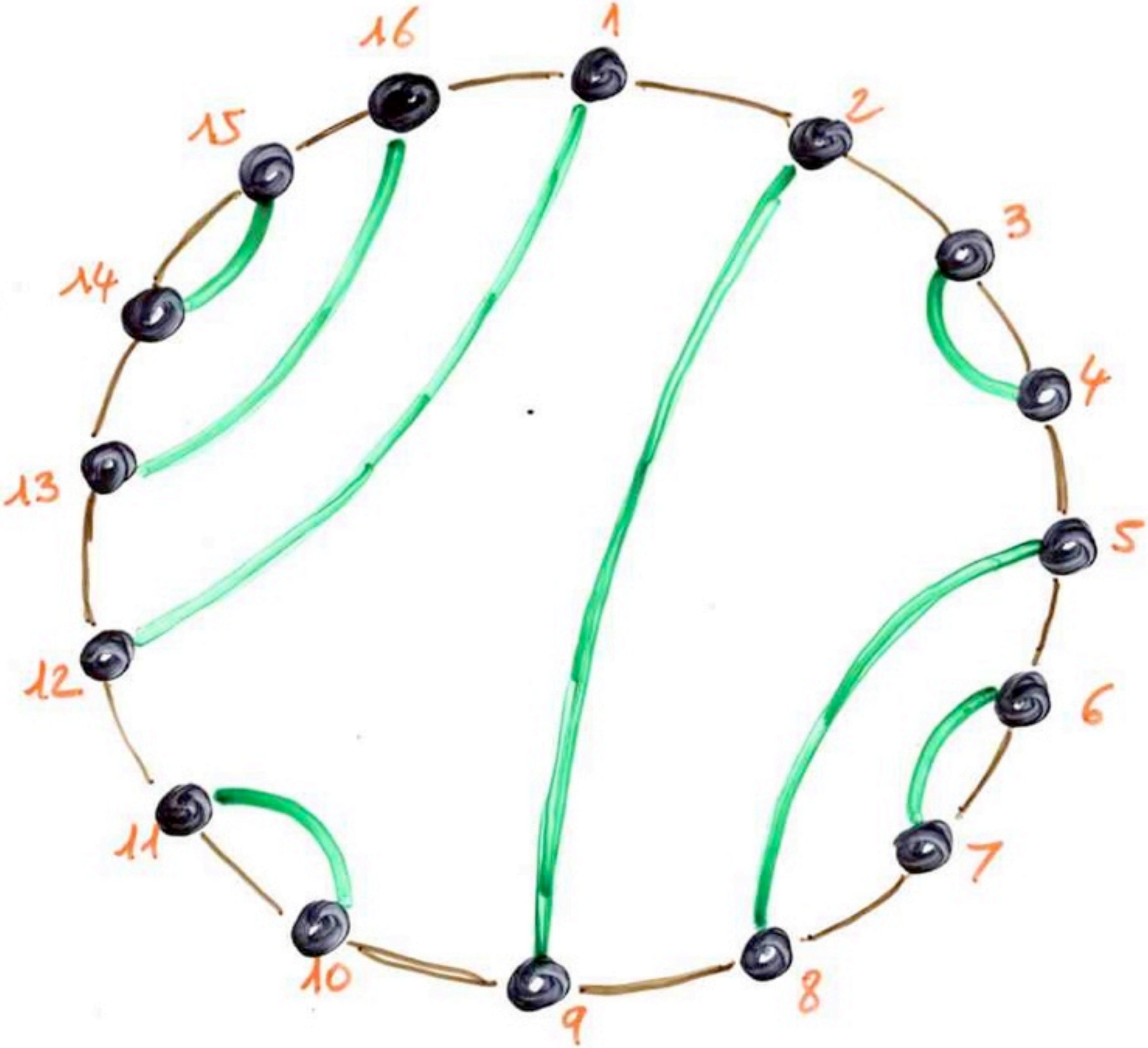
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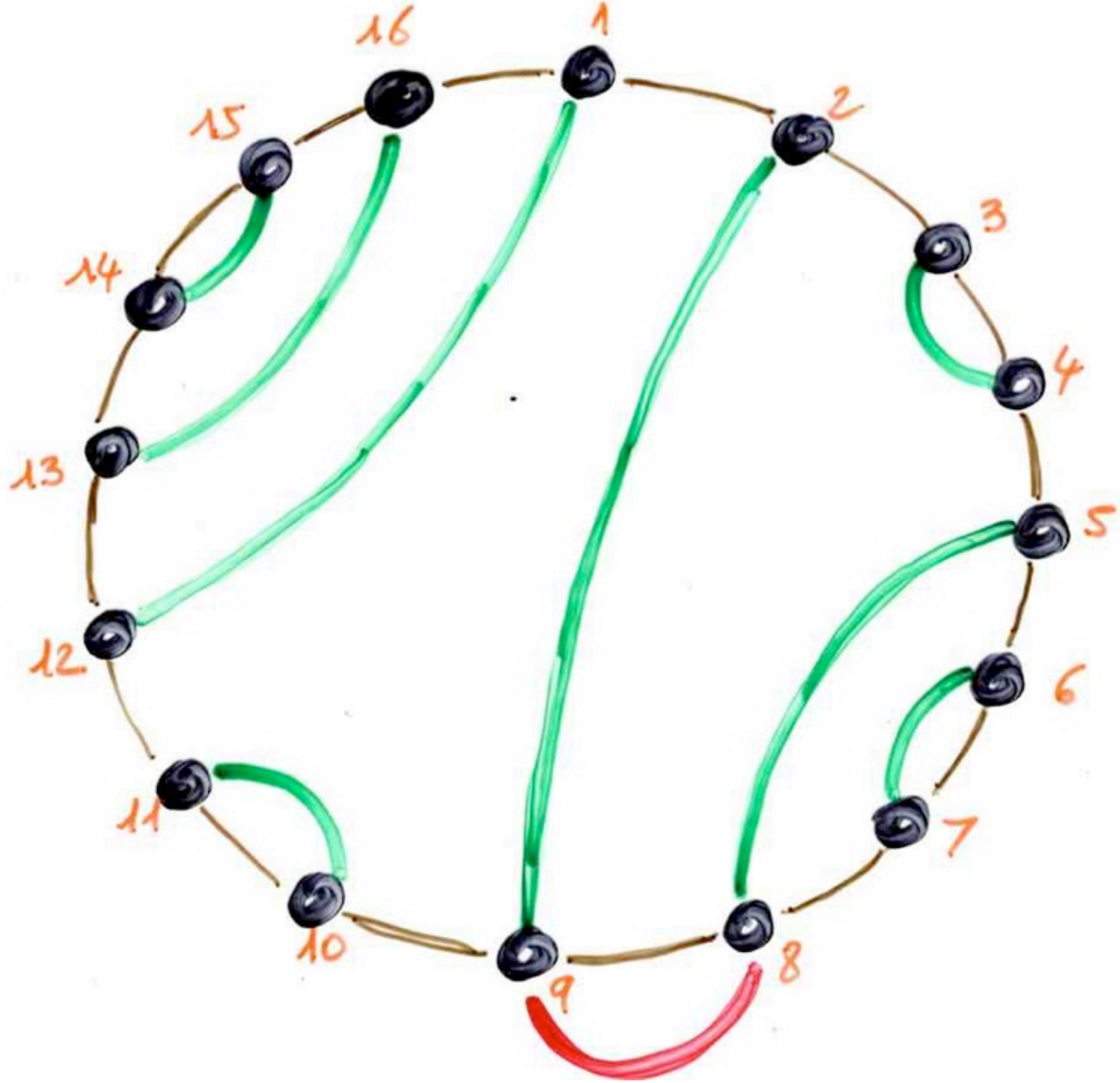

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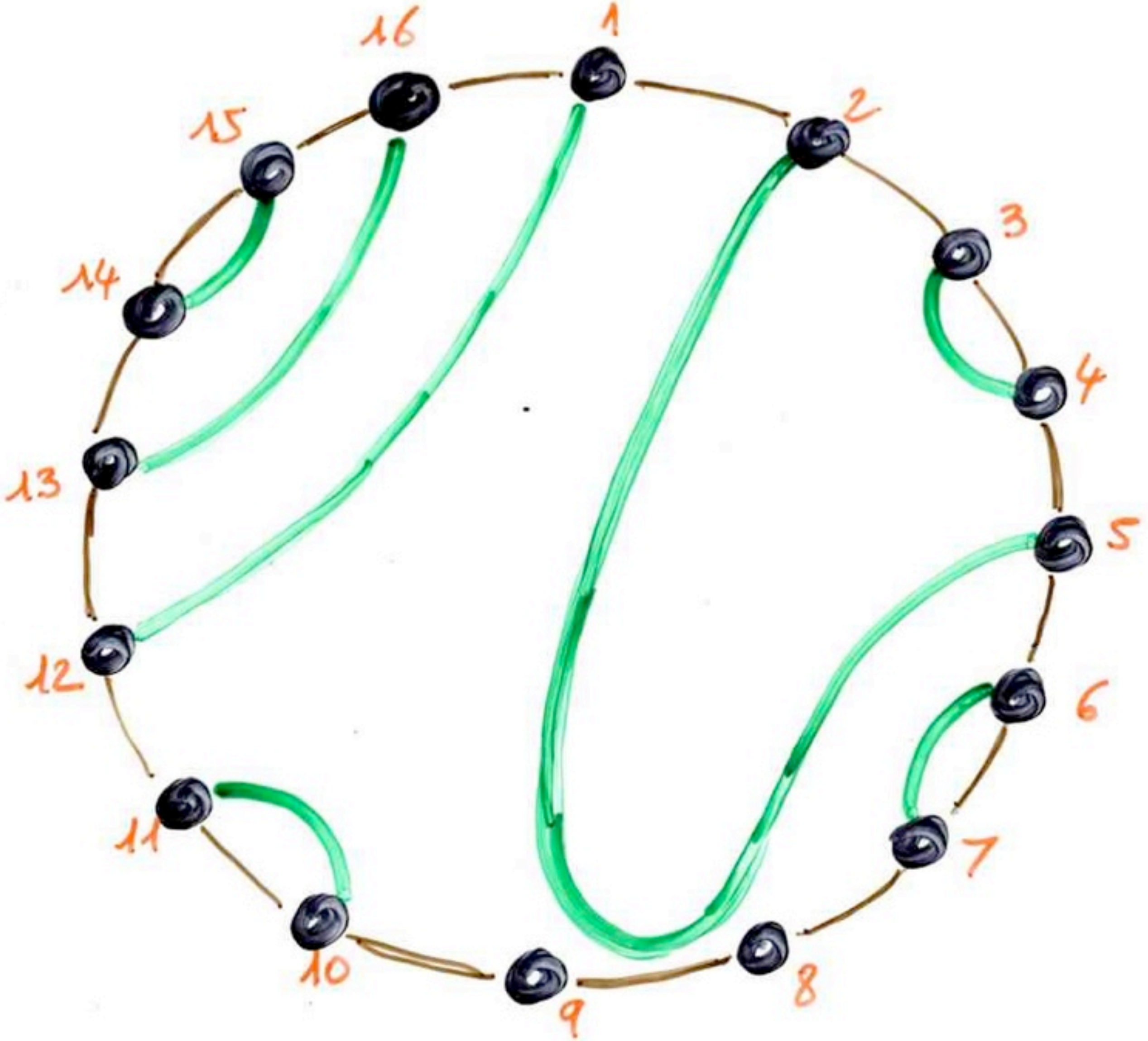
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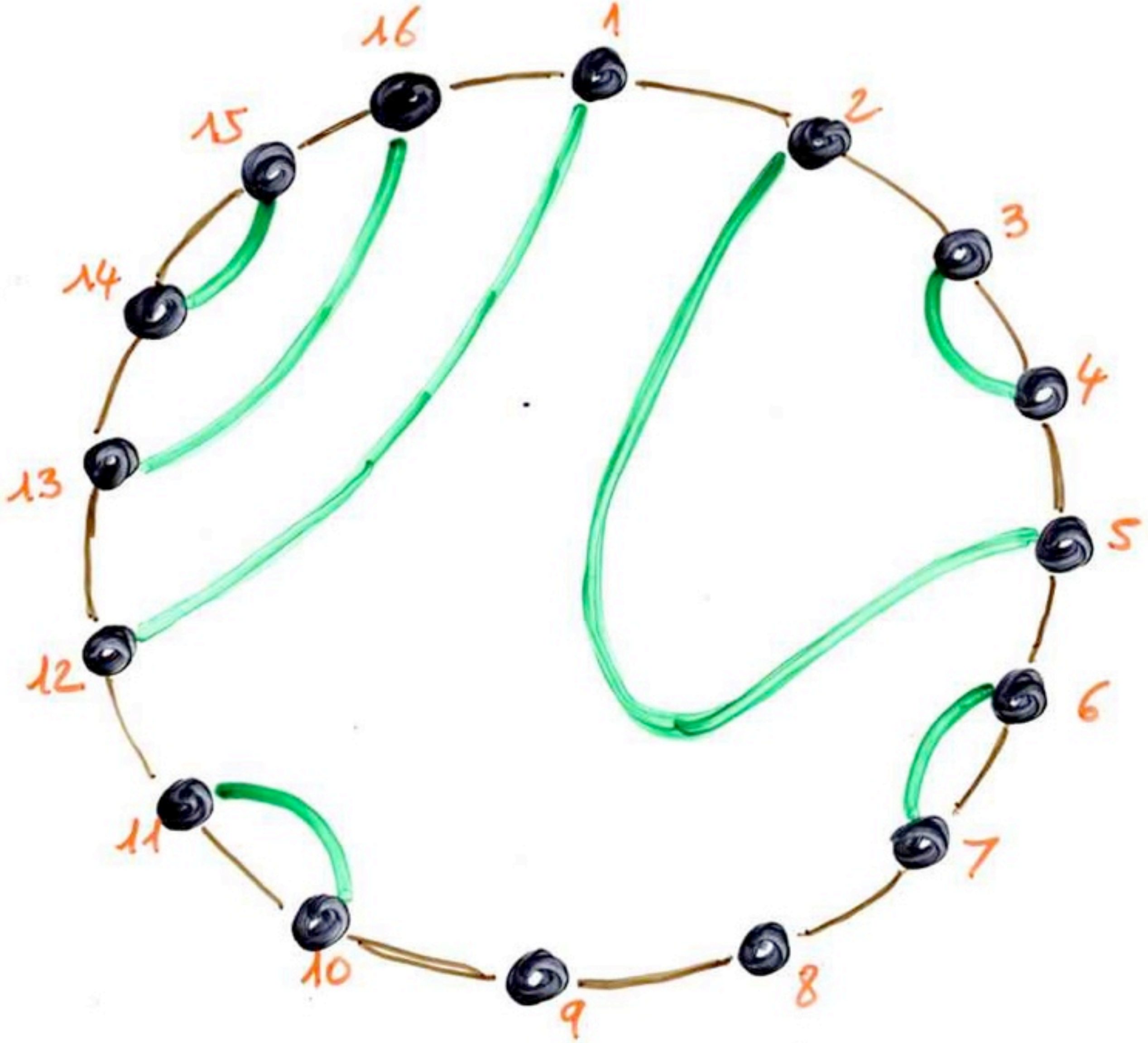
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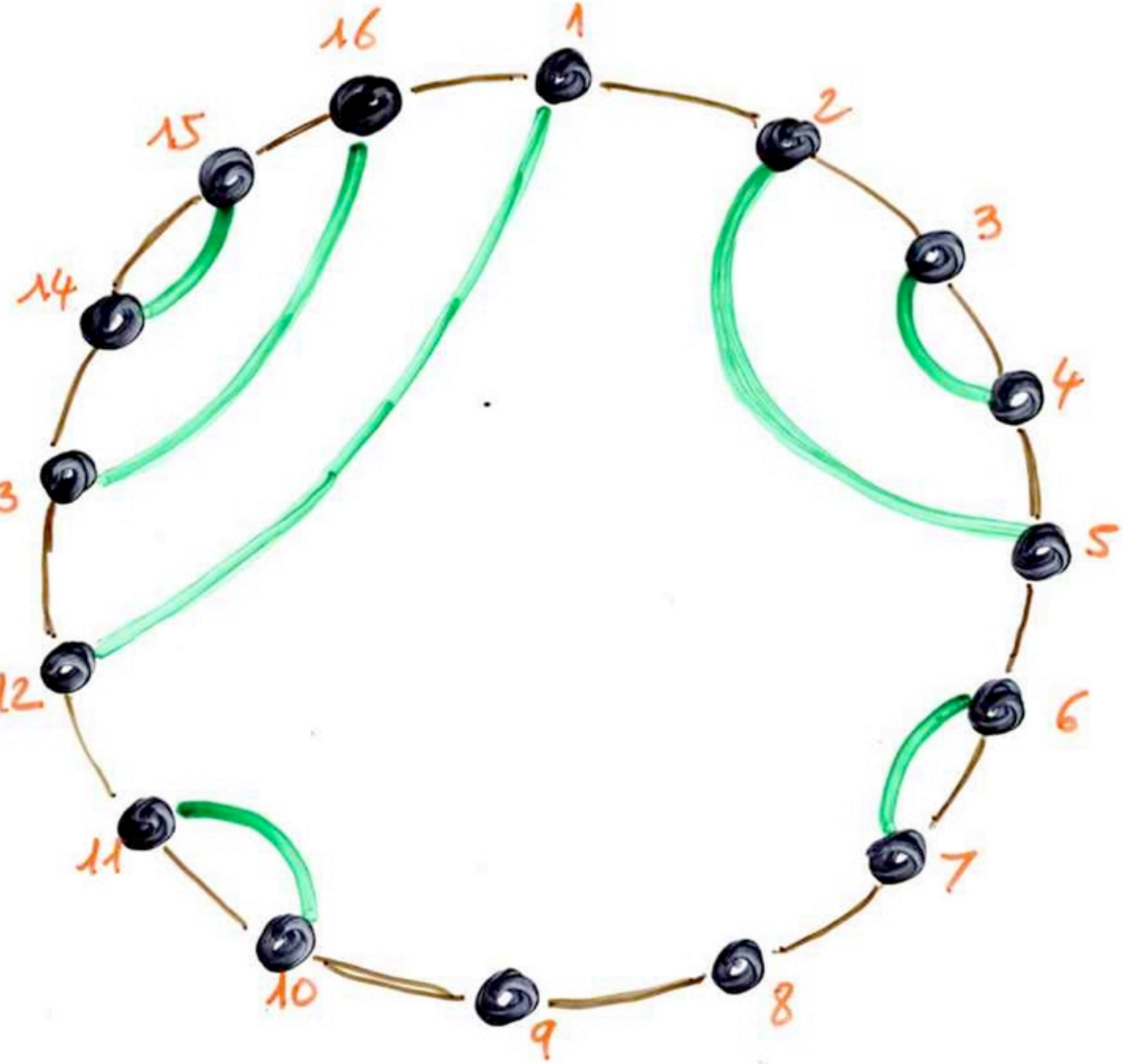

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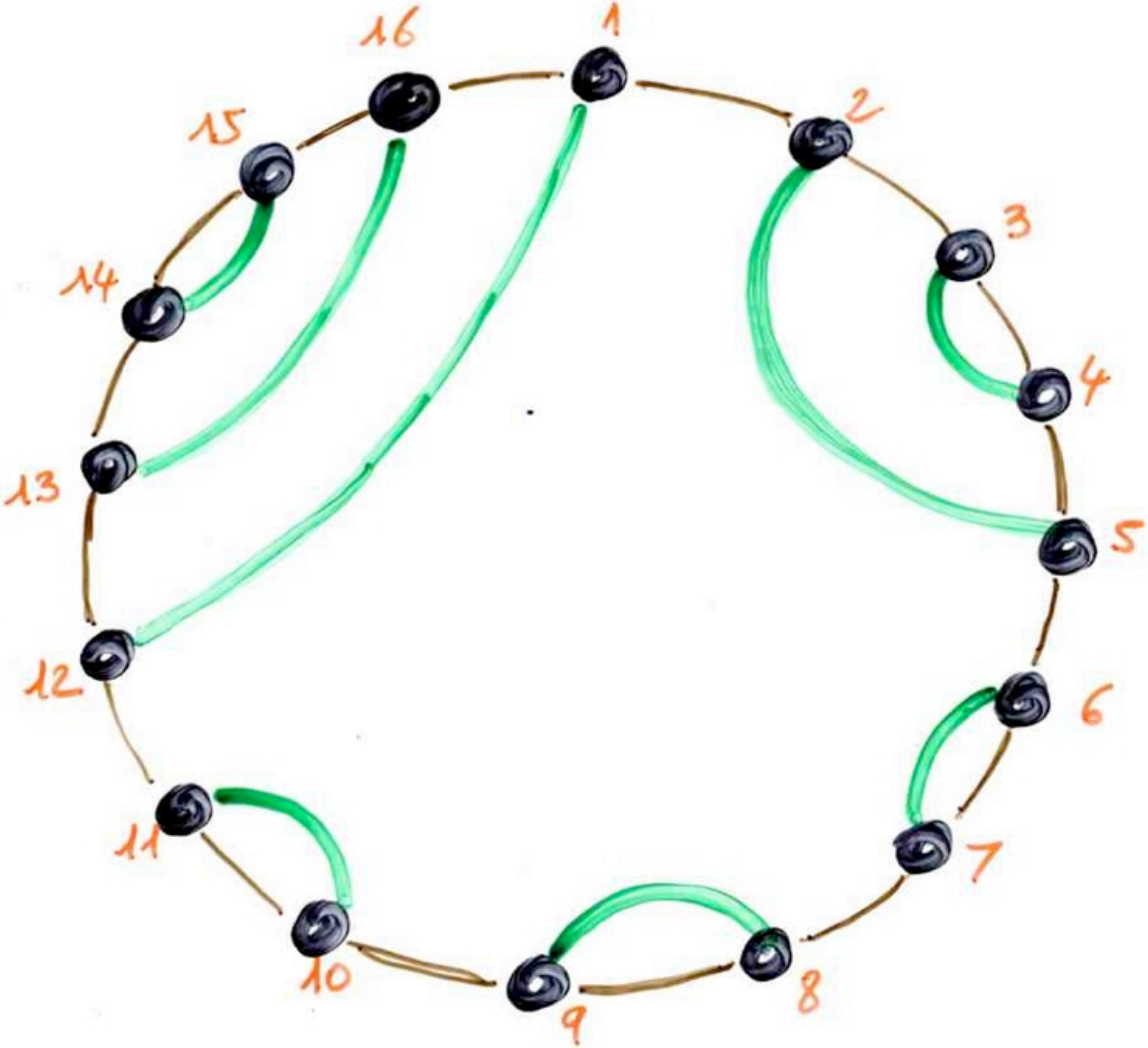
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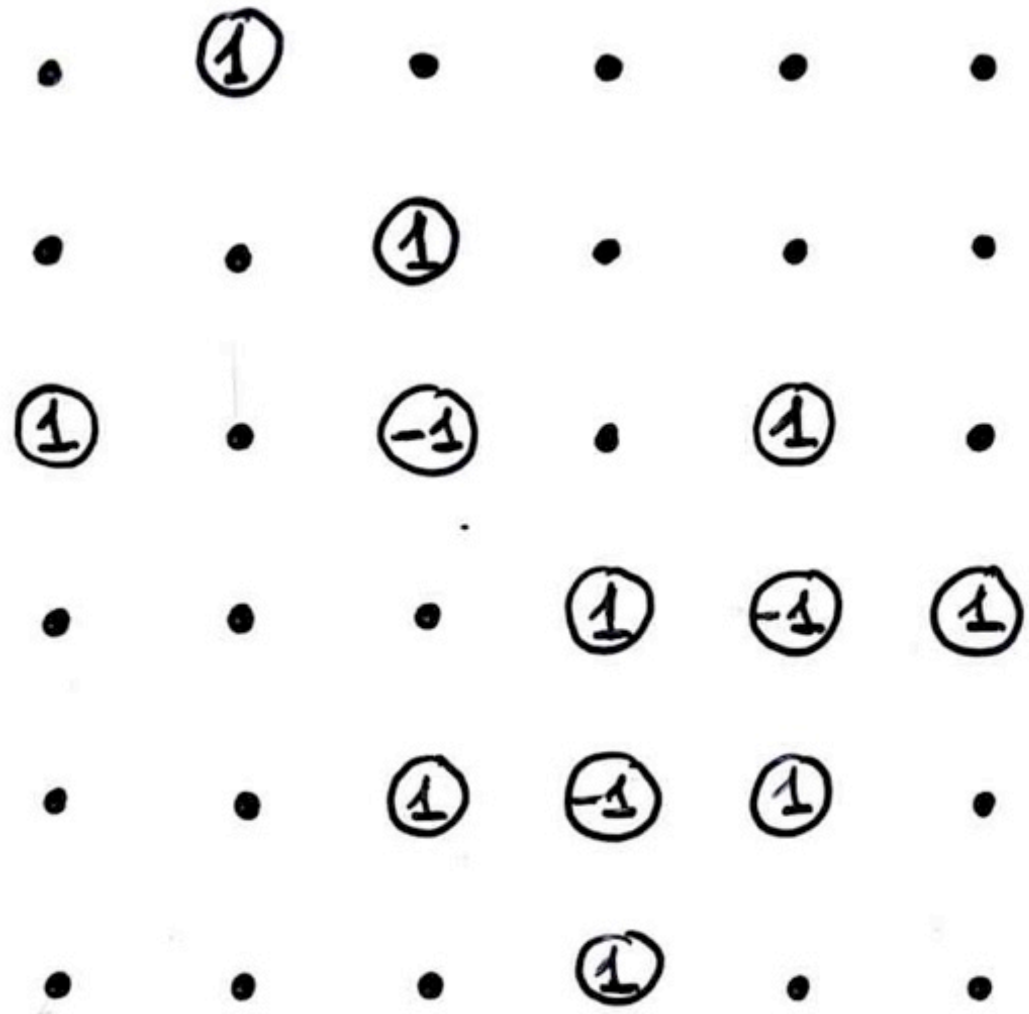
stationary probabilities

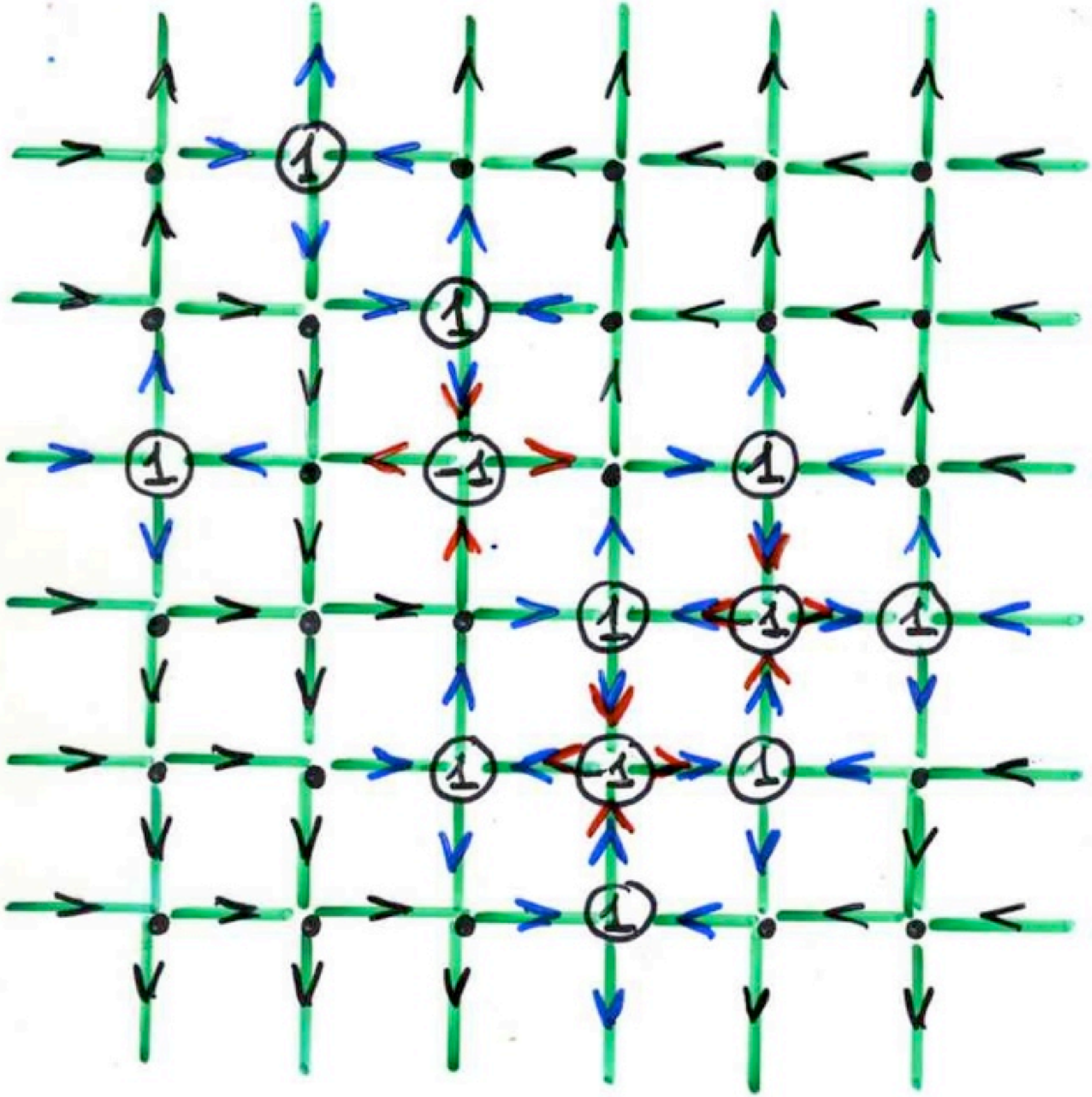
FPL

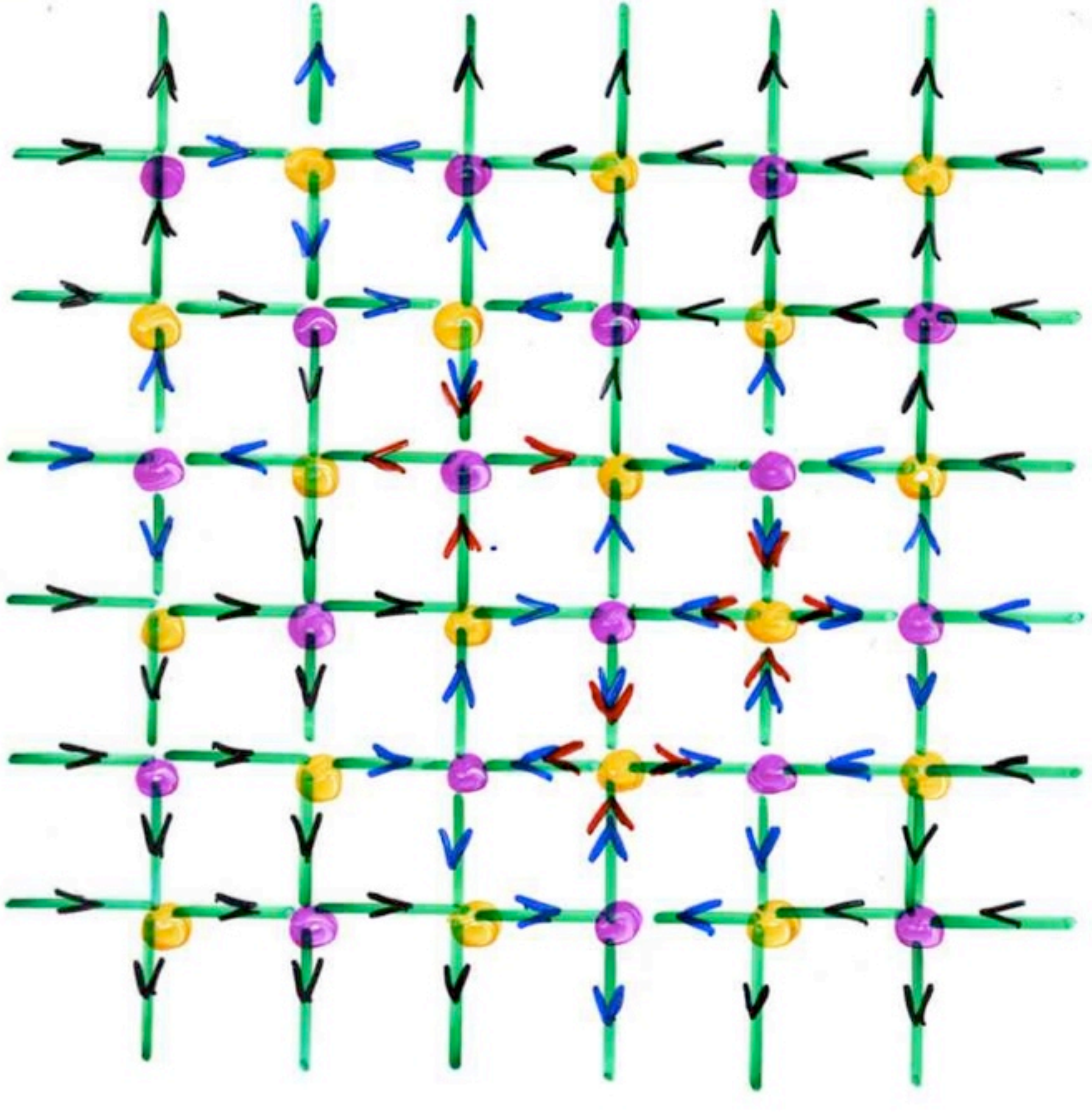
"Fully packed loop configurations"

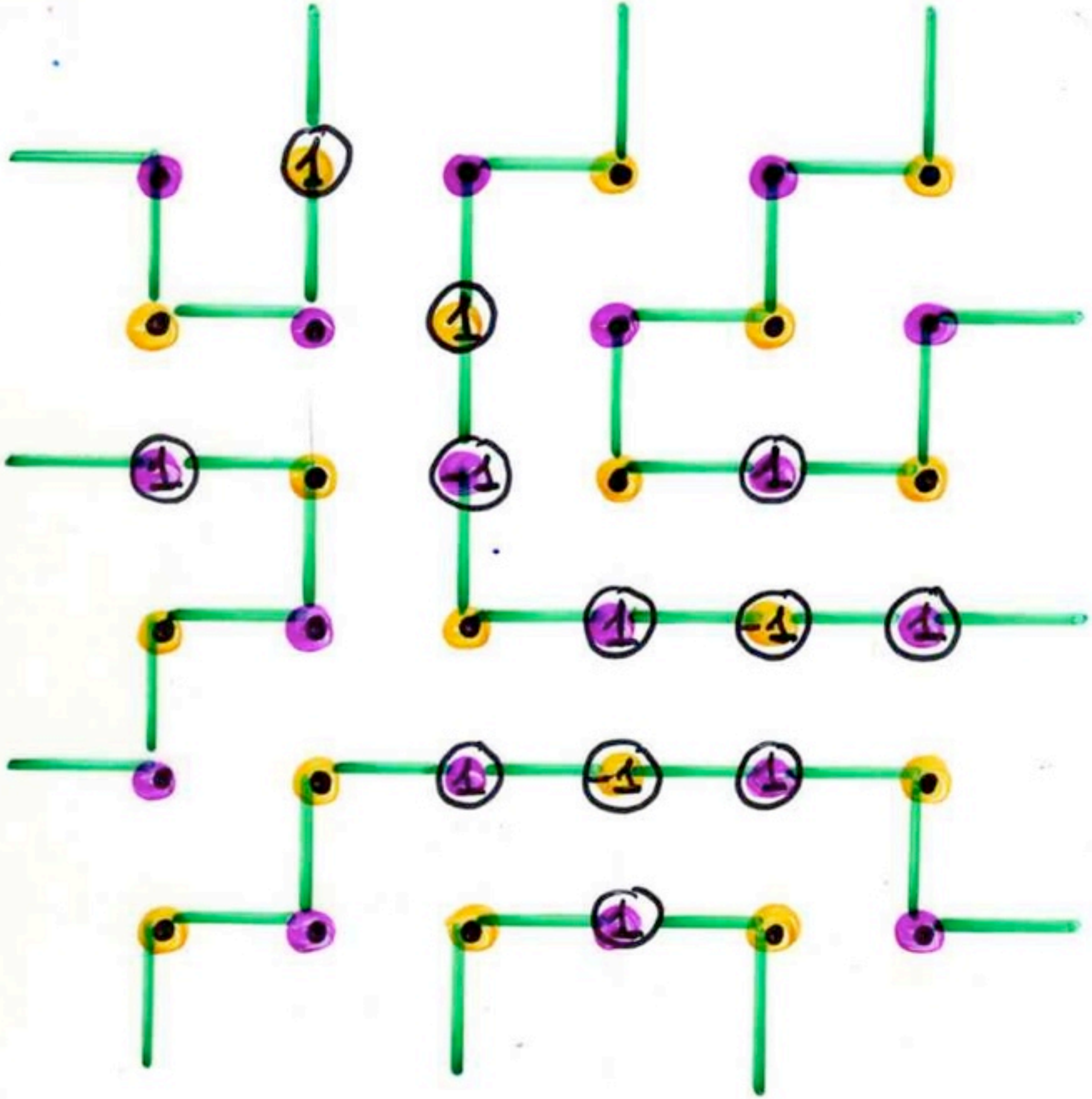
The
bijection
AMS
FPL



The 6-vertex model







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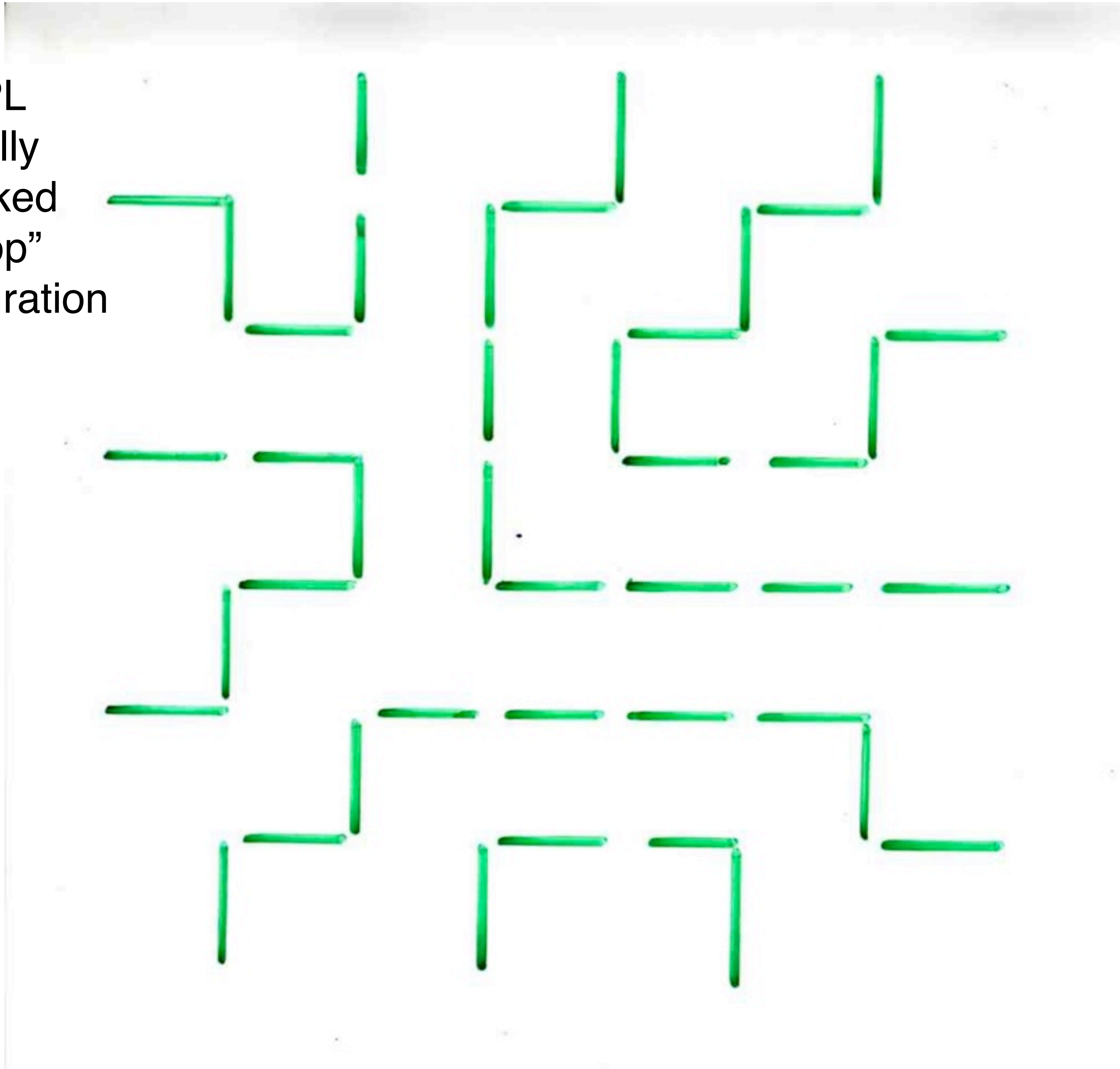
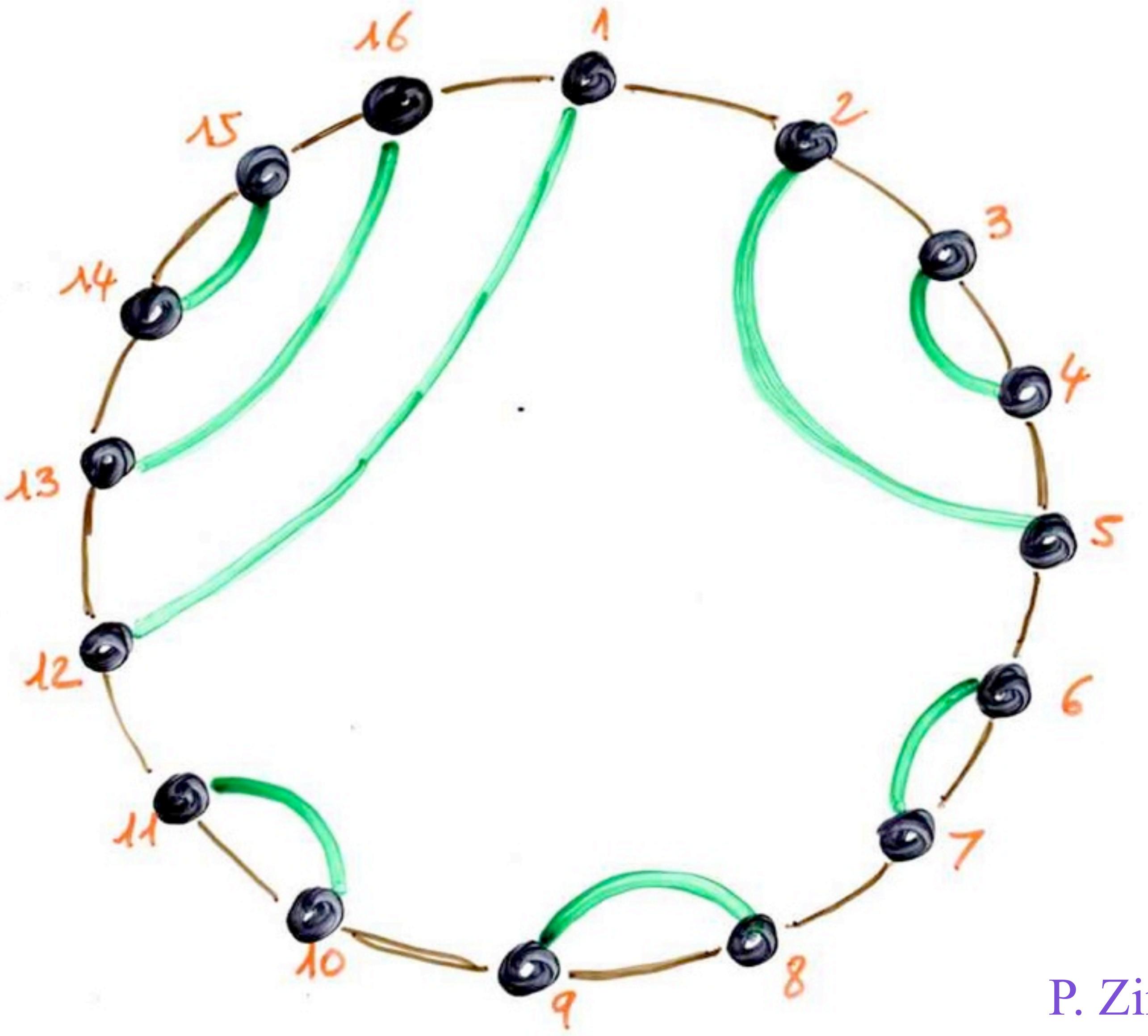
FPL
"Fully
Packed
Loop"
configuration

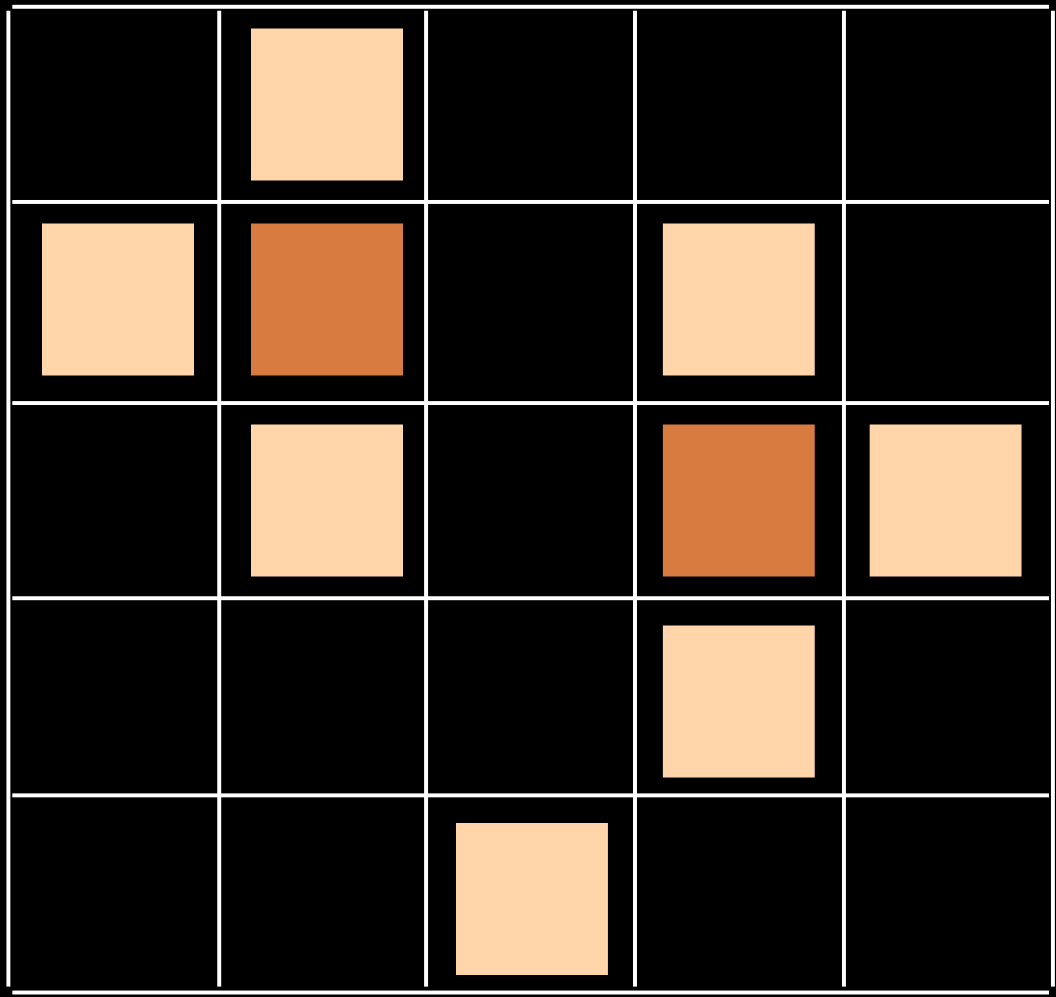


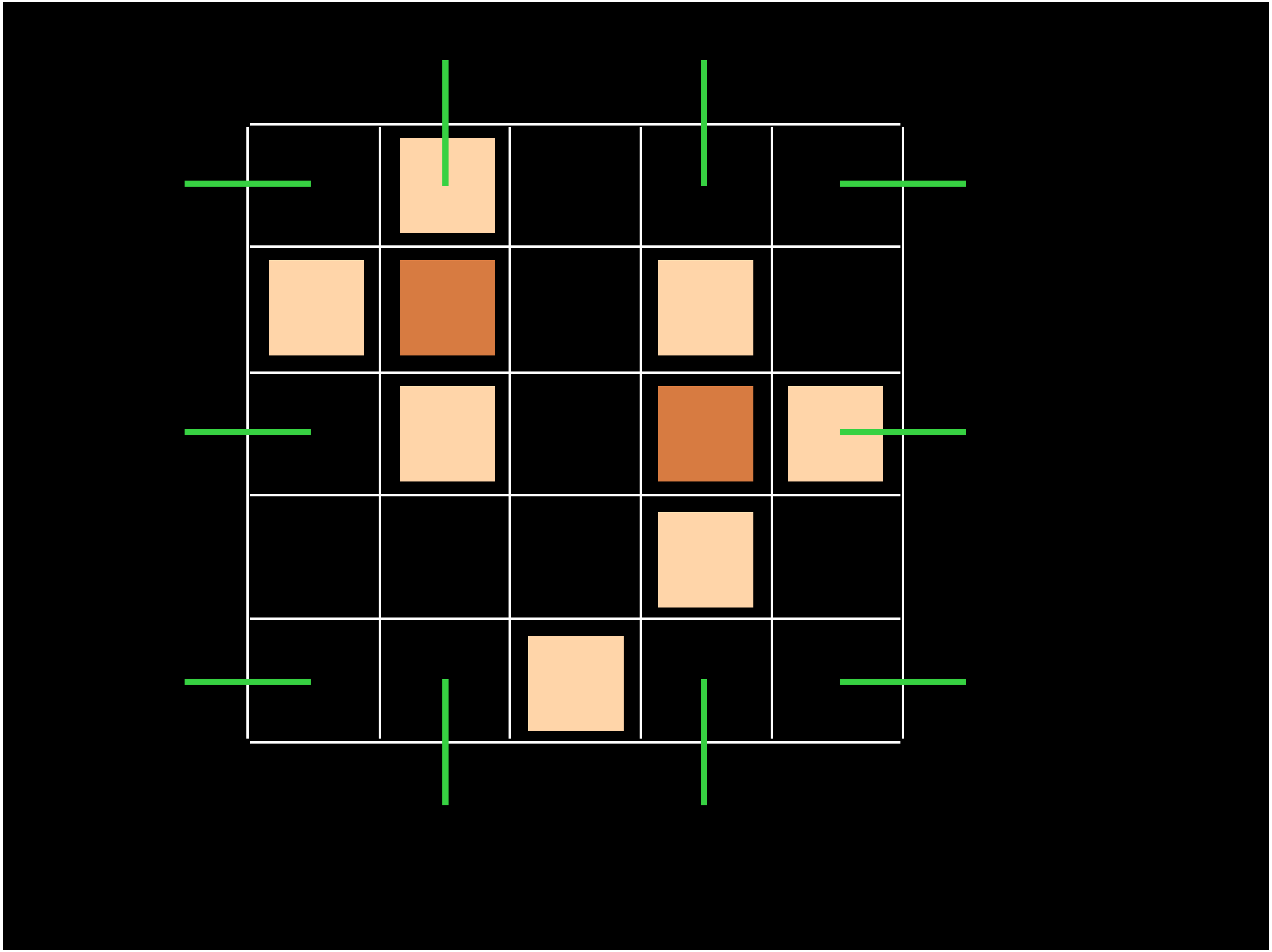
Razumov-Stroganov conjecture

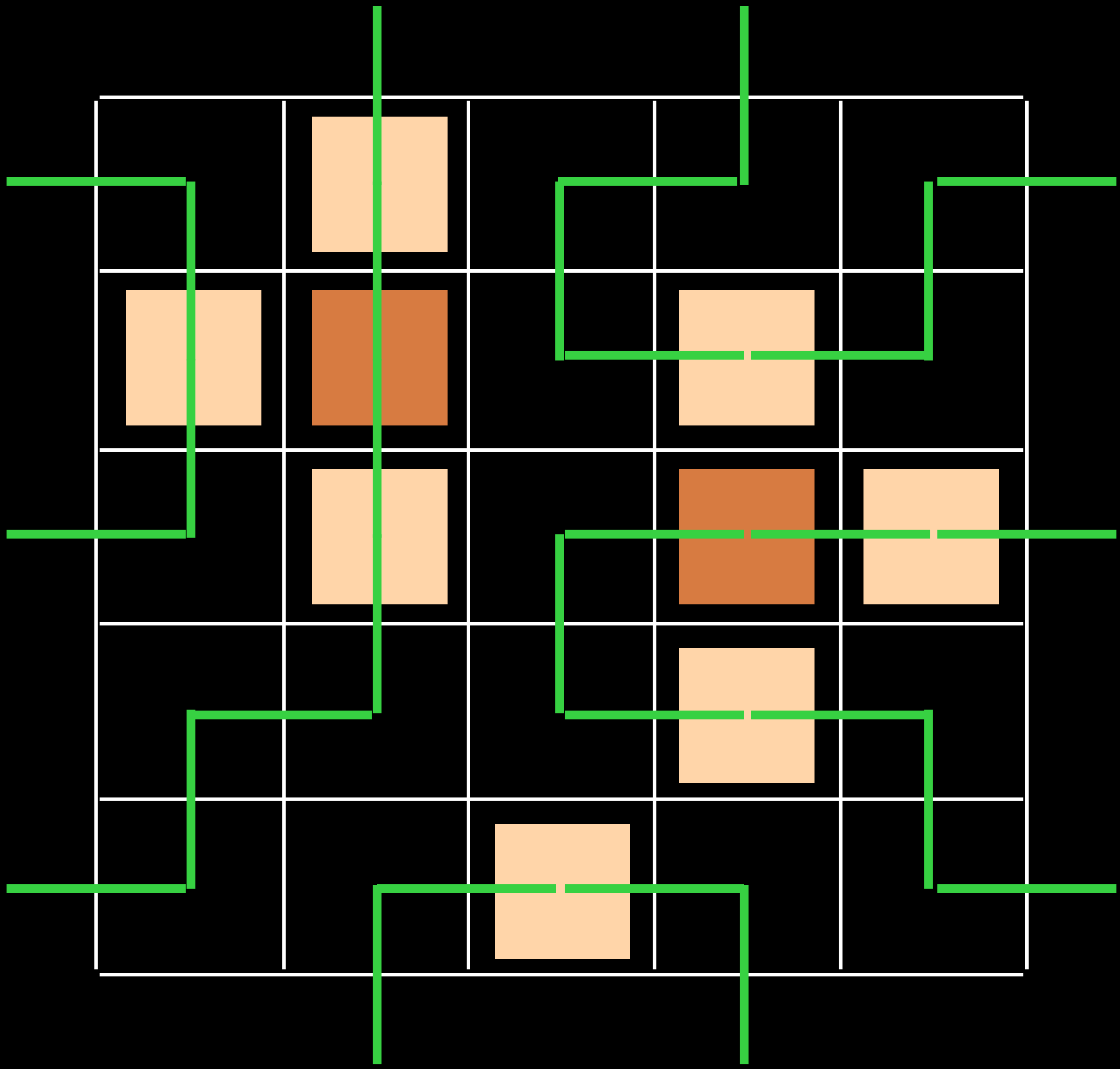


stationary
probabilities

Di Francesco,
P. Zinn-Justin (2005)







some perspectives

Questions.

- find a "combinatorial representation" for operators A, A′, B, B′.
- analogue of RSK (Robinson-Schensted-Knuth) for ASM ?
- analogue of "local rules" (Fomin)
- direct proof of the formula

$$A_n = \prod_{j=1}^{n} \frac{(3j-2)!}{(n+j-1)!}$$

(nb of ASM of size n)

= 1, 2, 42, 429, ...

?

(with P. Nadeau)
another representation of operators D and E
with triangulations of regular polygons

hypercube -- associahedron -- permutohedron -- alternohedron
(Loday-Ronco) (Lascoux-Schützenberger)

Razumov-Stroganov conjecture

spin chain Heisenberg XXZ model

Novelli, Thibon, Williams (April 2008)

Hall-Littlewood functions, Tevlin' bases (2007)

conjectures

- Orthogonal polynomials

→ Sasamoto (1999)

→ Blythe, Evans, Colaiori, Essler (2000)

q-Hermite polynomial

$\alpha, \beta, q \qquad \gamma = \delta = 1$

$$D = \frac{1}{1-q} + \frac{1}{\sqrt{1-q}}\,\hat{a}$$

$$E = \frac{1}{1-q} + \frac{1}{\sqrt{1-q}}\,\hat{a}^{+}$$

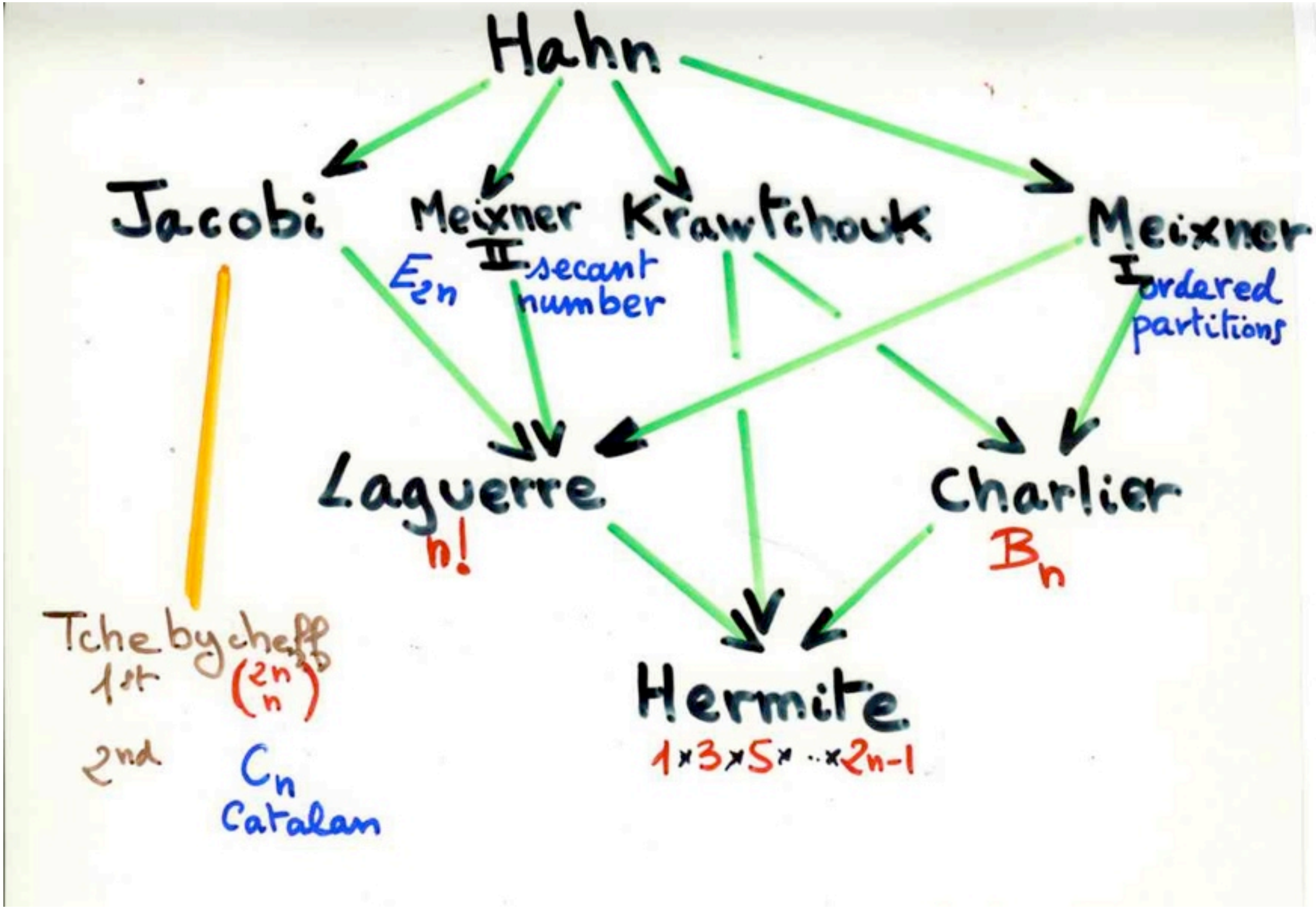
$$\hat{a}\,\hat{a}^{+} - q\,\hat{a}^{+}\hat{a} = 1$$

→ Uchiyama, Sasamoto, Wadati (2003)

$\alpha, \beta, \gamma, \delta, \qquad q$

Askey-Wilson polynomials

Askey-Wilson



references:

xgv website :

http://www.labri.fr/perso/viennot/

Recherche, cv, publications, exposés, diaporamas, livres, petite école, photos: voir ma page personnelle ici

Vulgarisation scientifique voir la page de l'association Cont'Science

downloadable papers, slides and lecture notes, etc ... here

(the summary on page "recherches" and most slides are in english)

→ **page "video"**

"Alternative tableaux, permutations and asymmetric exclusion process"

conference 23 April 2008,

Isaac Newton Institute for Mathematical science

or http://www.newton.cam.ac.uk/ (page "web seminar")

page “exposés”

An alternative approach to alternating sign matrices, (pdf 9,3 Mo) Workshop on **“Combinatorics and Statistical Physics”,** The Erwin Schrödinger International Institute for Mathematicals Physics (**ESI**), Vienna, 20 May 2008.

Growth diagrams for Young tableaux, Robinson-Schensted correspondance and some quadratic algebra coming from physics, exposé au CMUP (Centro de Matematica da Universidade do Porto), Portugal, 17 Sept 2008 **slides** (13,1 Mo)

Alernating sign matrices: at the crossroads of algebra, combinatorics and physics", exposé au CMUC (Centro de Matematica da Universidade do Coimbra), Portugal, 26 Sept 2008

TASEP:

→ **page "exposés"**

Catalan numbers, permutation tableaux and asymmetric exclusion process (pdf, 4,8 Mo)
GASCOM'06, Dijon, Septembre 2006, aussi: Journées Pierre Leroux, Montréal, Septembre 2006

Robinson-Schensted-Knuth: RSK1 (pdf, 9,1 Mo)
groupe de travail de combinatoire, Bordeaux, LaBRI, Février 2005
Robinson-Schensted-Knuth: RSK2 (pdf,10,8Mo)
groupe de travail de combinatoire, Bordeaux, LaBRI, Février 2005

survey paper on Robinson-Schensted correspondence:
[30] *Chain and antichain families, grids and Young tableaux*,
Annals of Discrete Maths., 23 (1984) 409-464.

from xgv website :

→ **A Combinatorial theory of orthogonal polynomials**

[4] *Une théorie combinatoire des polynômes orthogonaux*, Lecture Notes UQAM, 219p., Publication du LACIM, Université du Québec à Montréal, 1984, réed. 1991.

→ page **"petite école"**

Petite école de combinatoire LaBRI, année 2006/07
"Une théorie combinatoire des polynômes orthogonaux, ses extensions, interactions et applications"

Chapitre 2, Histoires et moments, (17, 23 Nov , 1, 8, 15 Dec 2006)

Chapitre X Histoires et opératerus (10 and 12 January 2007)

→ page **"cours"**

Cours au Service de Physique Théorique du CEA, Saclay Sept-Oct 2007
"Eléments de combinatoire algébrique"

Ch 4 - (9,4 Mo) théorie combinatoire des polynômes orthogonaux et fractions continues *from xgv website :*

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Paper: FV bijection

[21] (avec J. Françon) *Permutations selon les pics, creux, doubles montées et doubles descentes, nombres d'Euler et nombres de Genocchi*, Discrete Maths., 28 (1979) 21-35

survey paper: Genocchi, Euler (tangent and secant numbers), Jacobi elliptic functions

[6] *Interprétations combinatoires des nombres d'Euler et de Genocchi* (pdf, 9,2 Mo)
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http://www.fpsac.cn/PDF-Proceedings/Talks/87.pdf

MERCI BEAUCOUP !

§1 Genocchi sequences §2 Some Parameters

§3 tableaux alternatifs de Catalan Bijection TAC --- AB

§4 ABC §5 Pagodes

§6 Taquin pour ABC Taquin ABC -- TAC

§7 AB alternatifs

§8 Taquin pour ABA §9 ordre symétrique twisté

§10 PASEP

§11 Permutation Tableau

§12 spin chain §13 ASM §14 RS conjecture

perspectives

Ansatz cellulaire 1 RSK Le gâteau