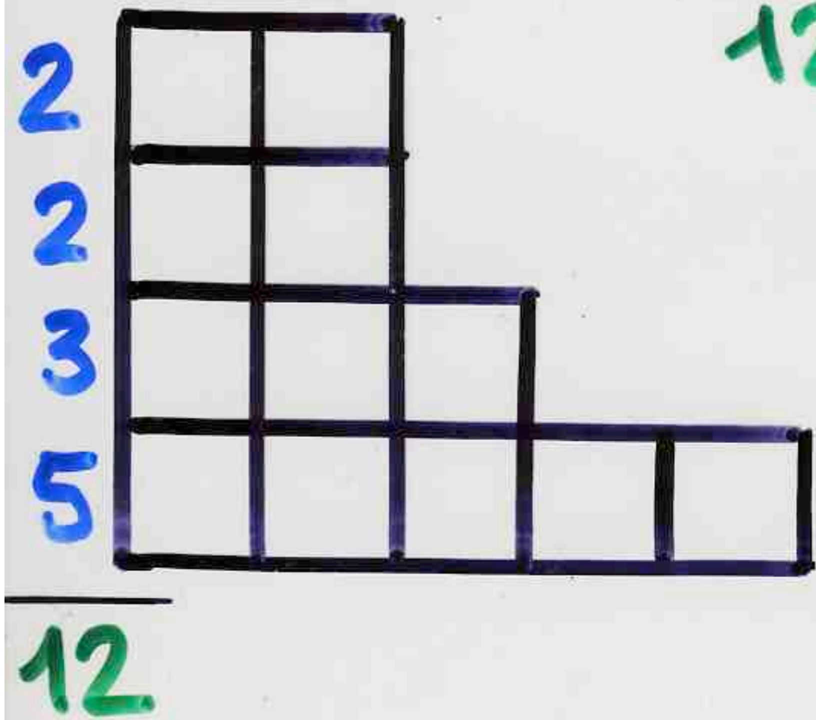
Young tableaux

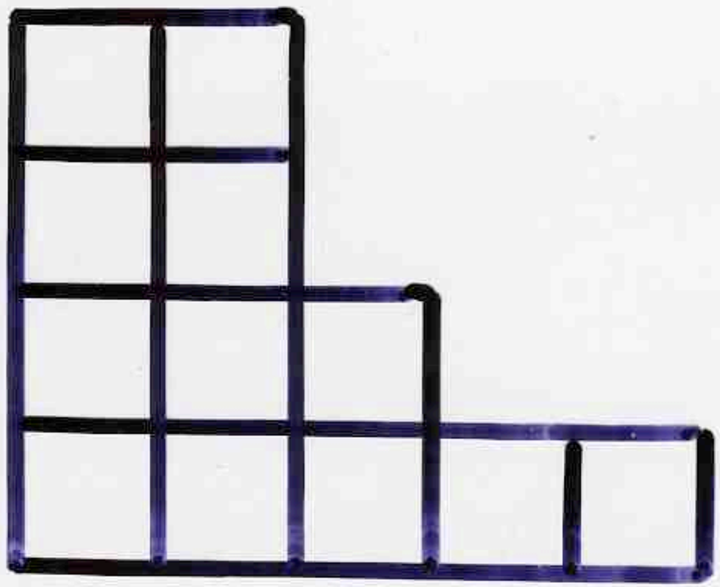
$12 = n = 5+3+2+2$

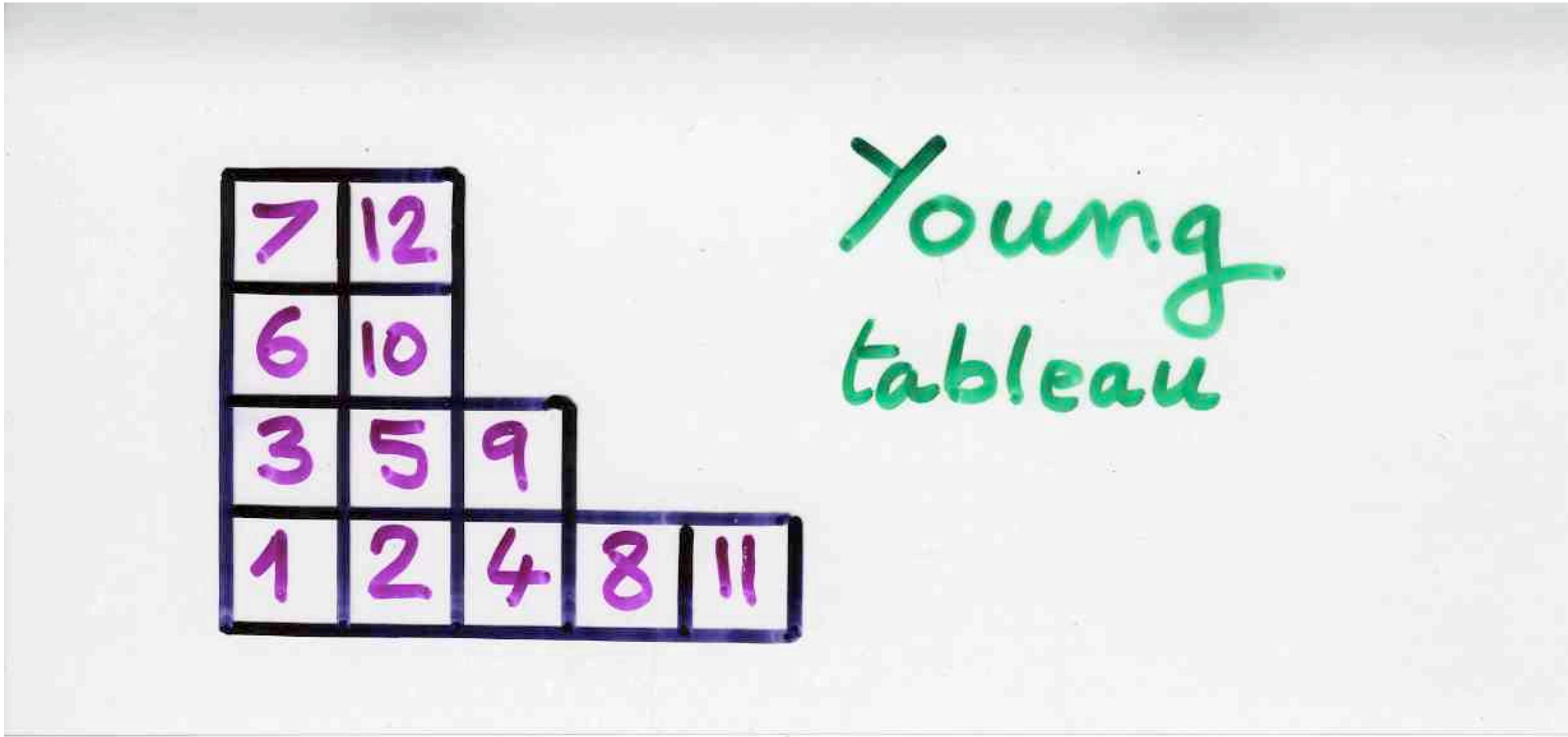


Ferrers

diagram

Partition of n




7 12
6 10
3 5 9
1 2 4 8 11
Young
tableau

An introduction to RSK

G. de B. Robinson, 1938

C. Schensted, 1961

G fini

$$|G| = \sum_{\varphi} \deg^2(\varphi)$$

φ représentation irréductible

$$n! = \sum_{\lambda} f_\lambda^2$$

ordre

groupe fini $\mathfrak{S}_n$

λ représentations irréductibles

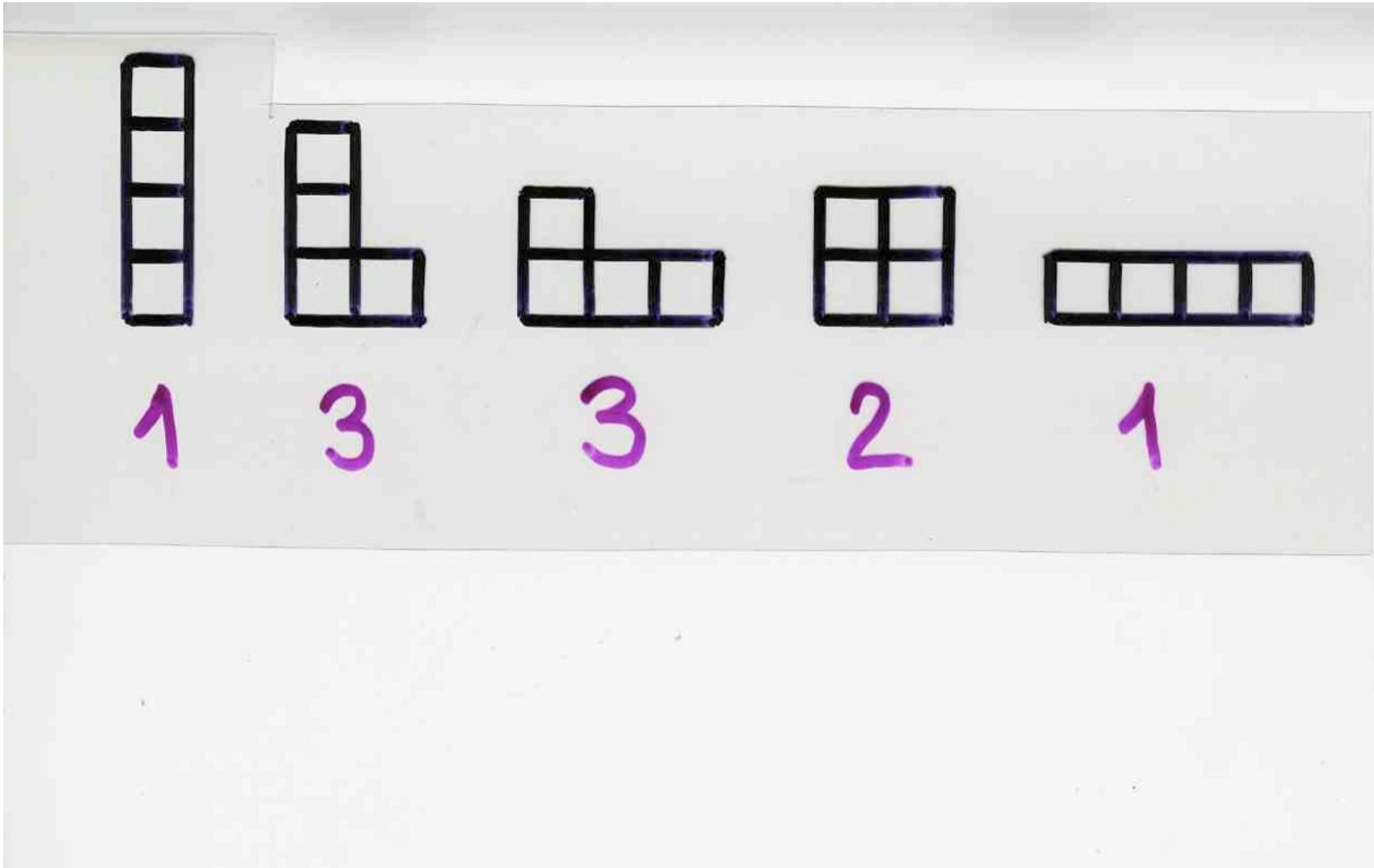
degré

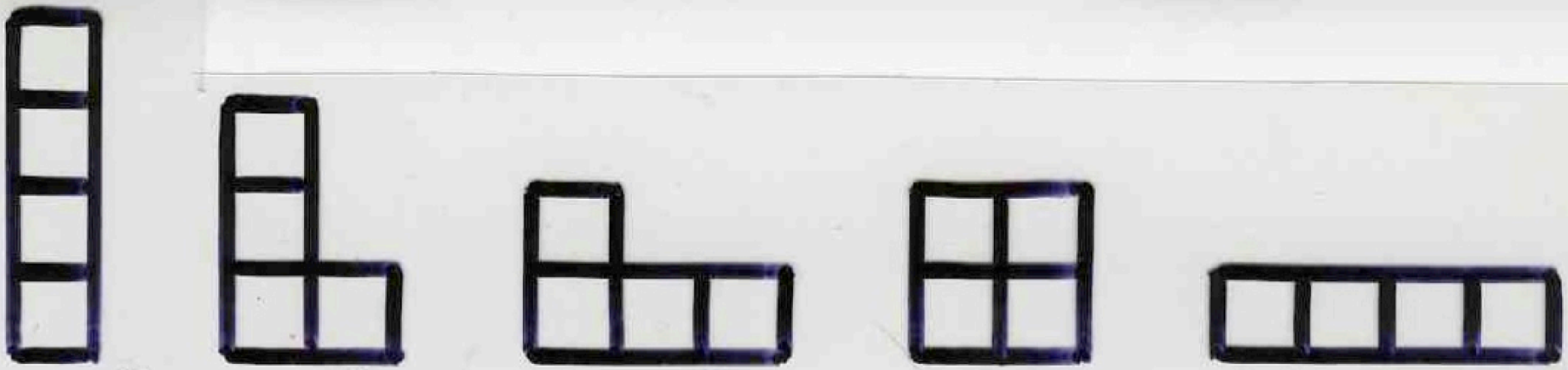
nombre de permutations

$$n! = \sum_{\lambda} \left(f_\lambda \right)^2$$

forme
n cases

nombre de tableaux de Young de forme λ

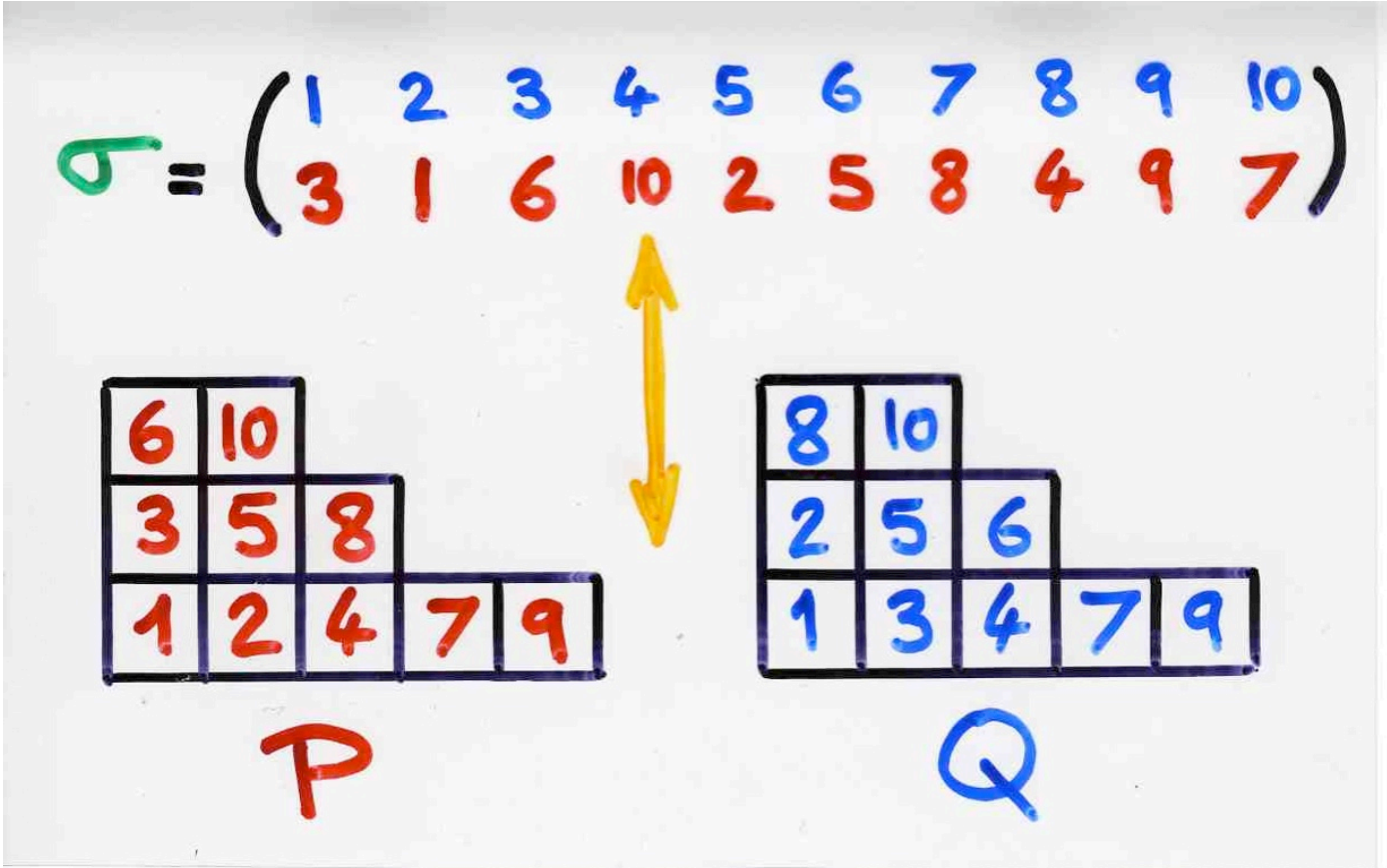

1
3
3
2
1



$$1^2 + 3^2 + 3^2 + 2^2 + 1^2$$

$$= 1 + 9 + 9 + 4 + 1$$

$$= 24 = 4!$$



The Robinson-Schensted correspondence between permutations and pair of (standard) Young tableaux with the same shape

RSK with Schensted's insertions

1	2	3	4	5	6	7	8	9	10
3	1	6	10	2	5	8	4	9	7

1	2	3	4	5	6	7	8	9	10
3	1	6	10	2	5	8	4	9	7

1					

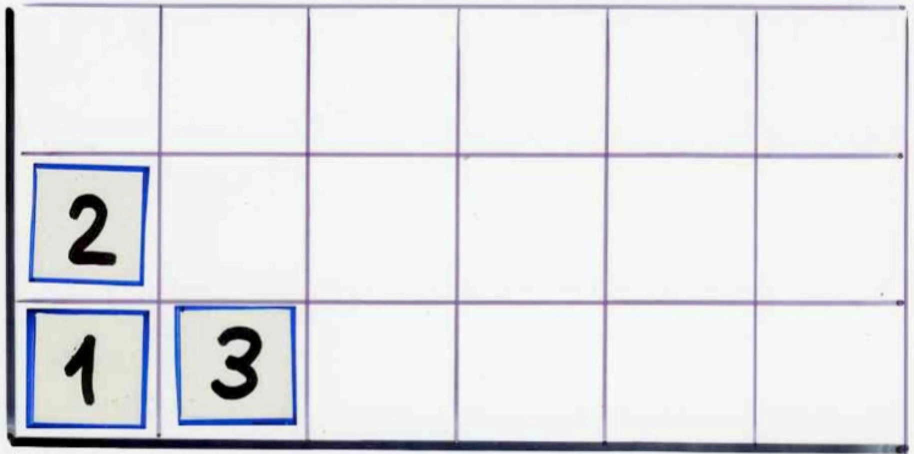
3					

1	2	3	4	5	6	7	8	9	10
3	1	6	10	2	5	8	4	9	7

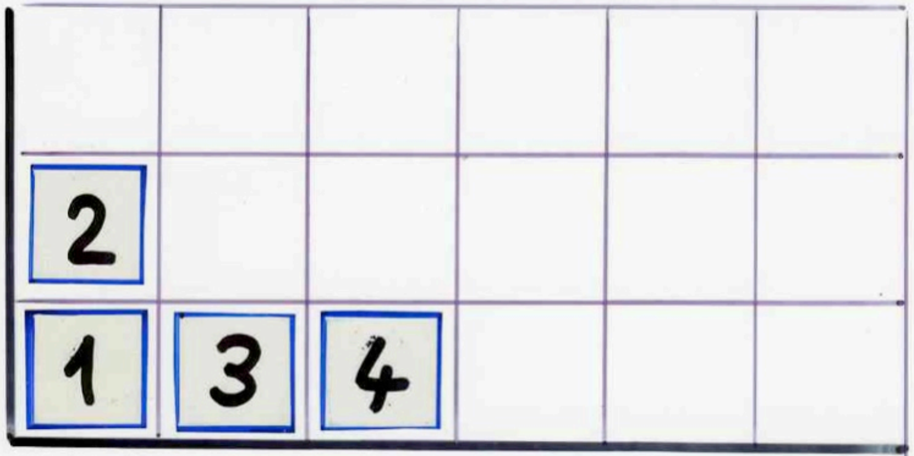
2					
1					

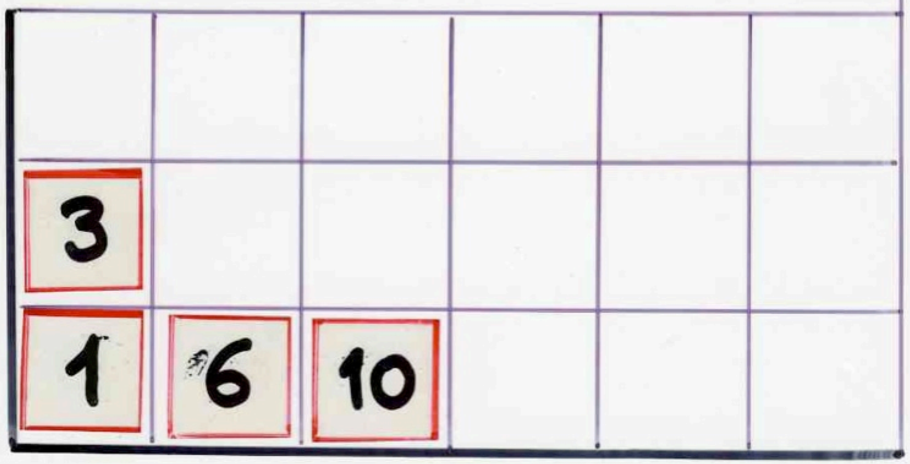
3					
1					

1	2	3	4	5	6	7	8	9	10
3	1	6	10	2	5	8	4	9	7

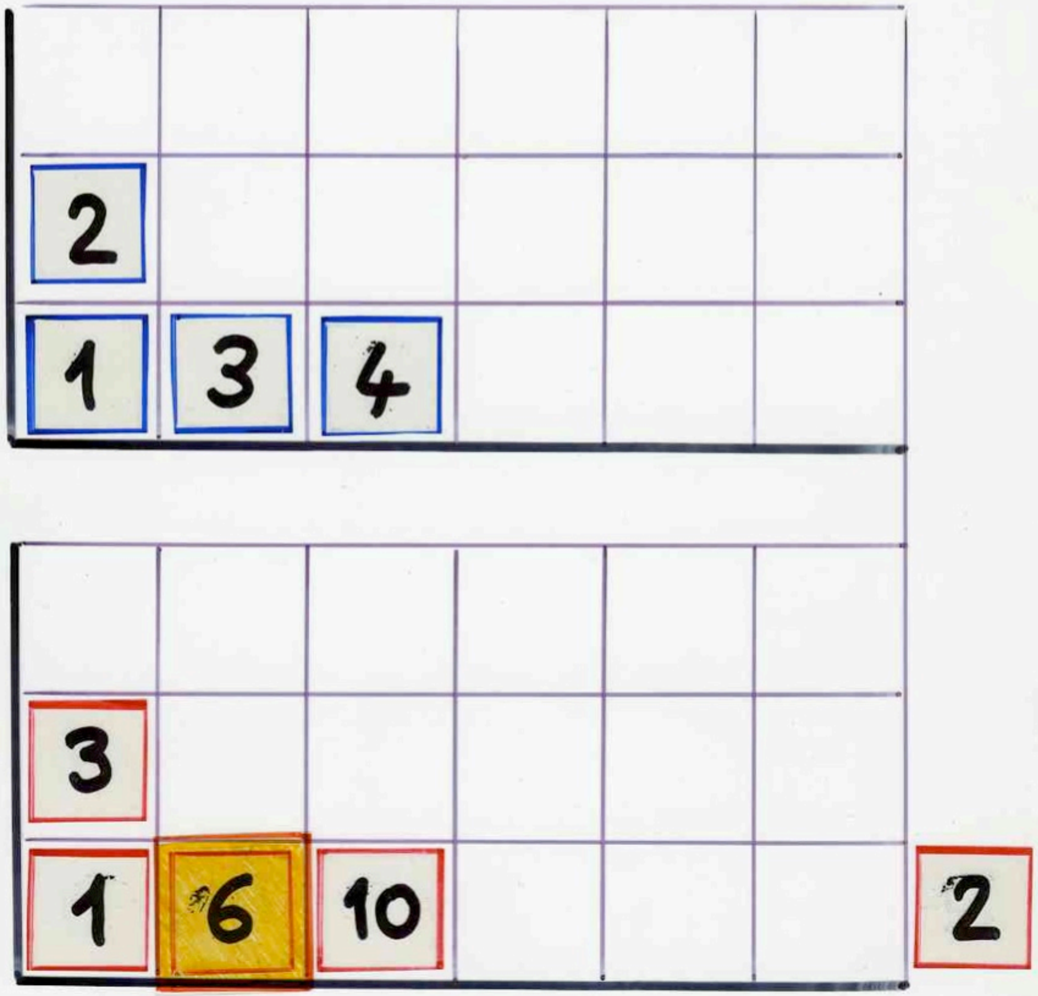


1	2	3	4	5	6	7	8	9	10
3	1	6	10	2	5	8	4	9	7

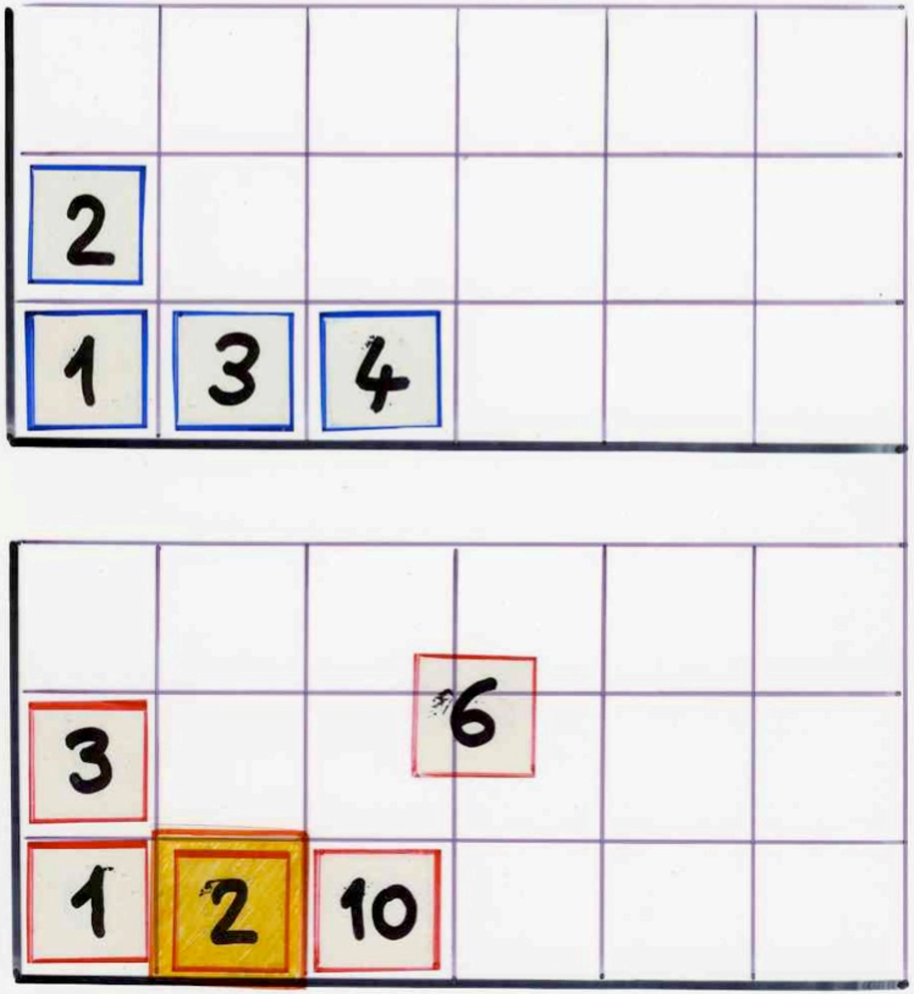




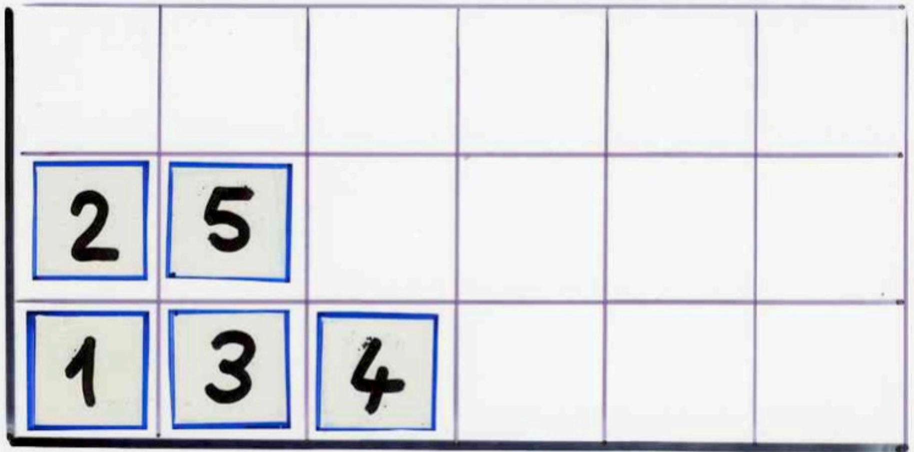
1	2	3	4	5	6	7	8	9	10
3	1	6	10	2	5	8	4	9	7



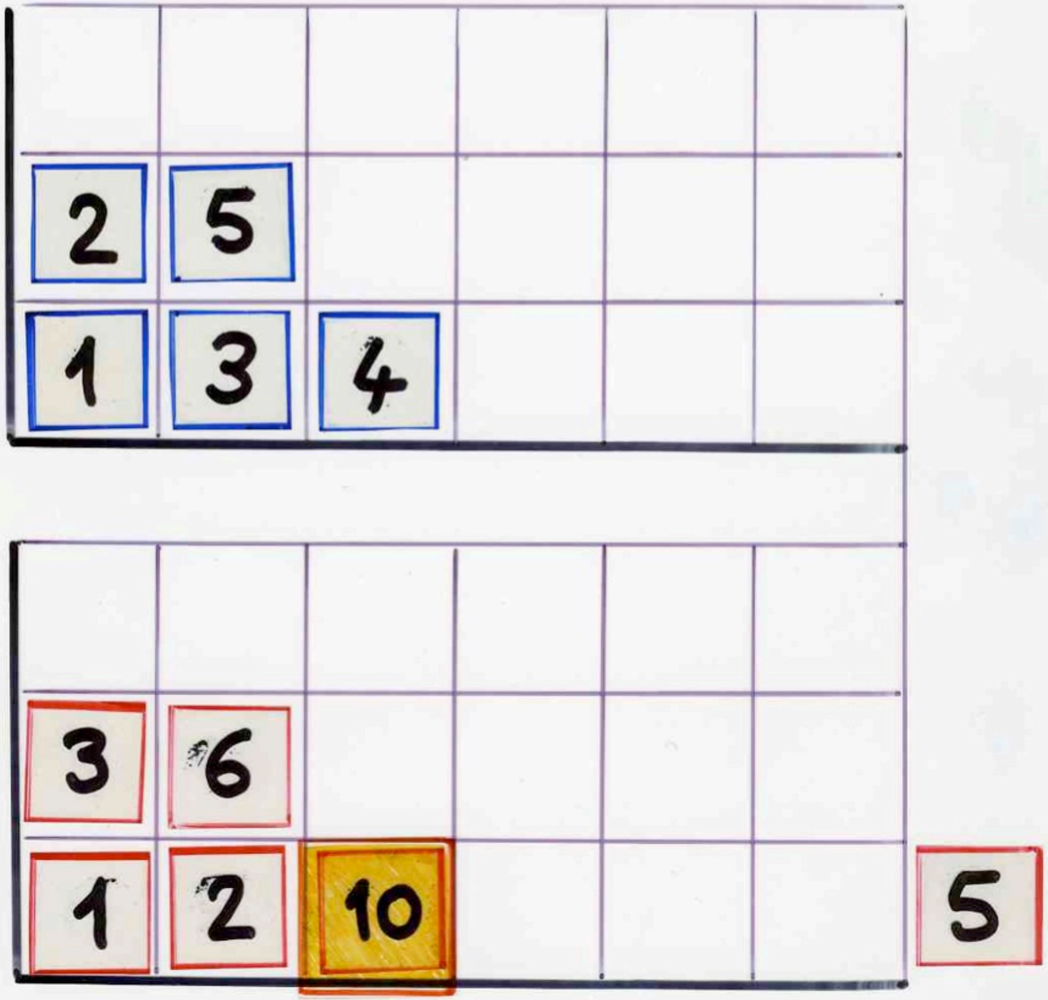
1	2	3	4	5	6	7	8	9	10
3	1	6	10	2	5	8	4	9	7



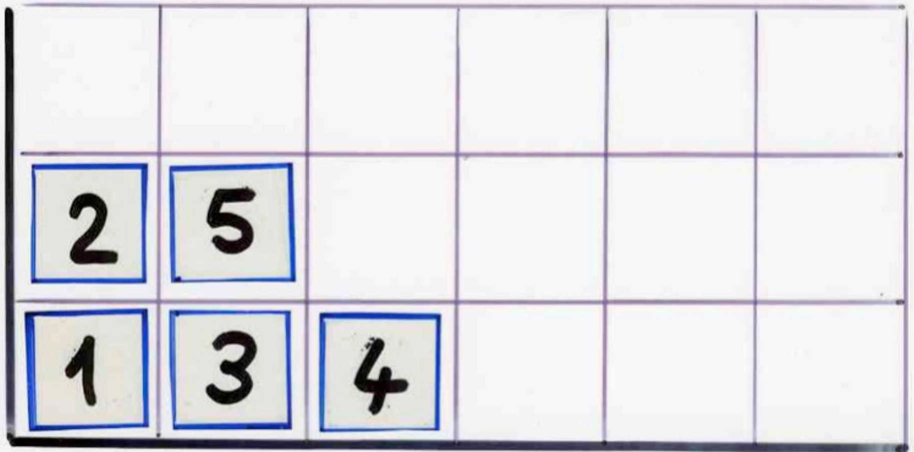
1	2	3	4	5	6	7	8	9	10
3	1	6	10	2	5	8	4	9	7

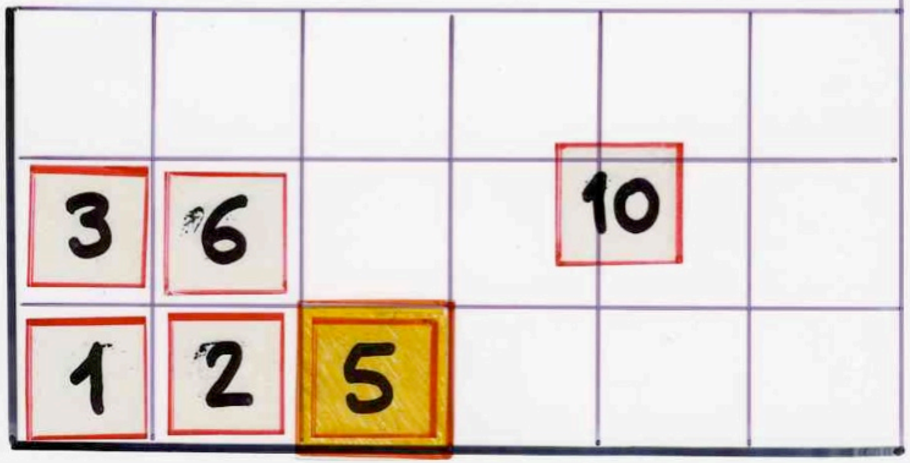


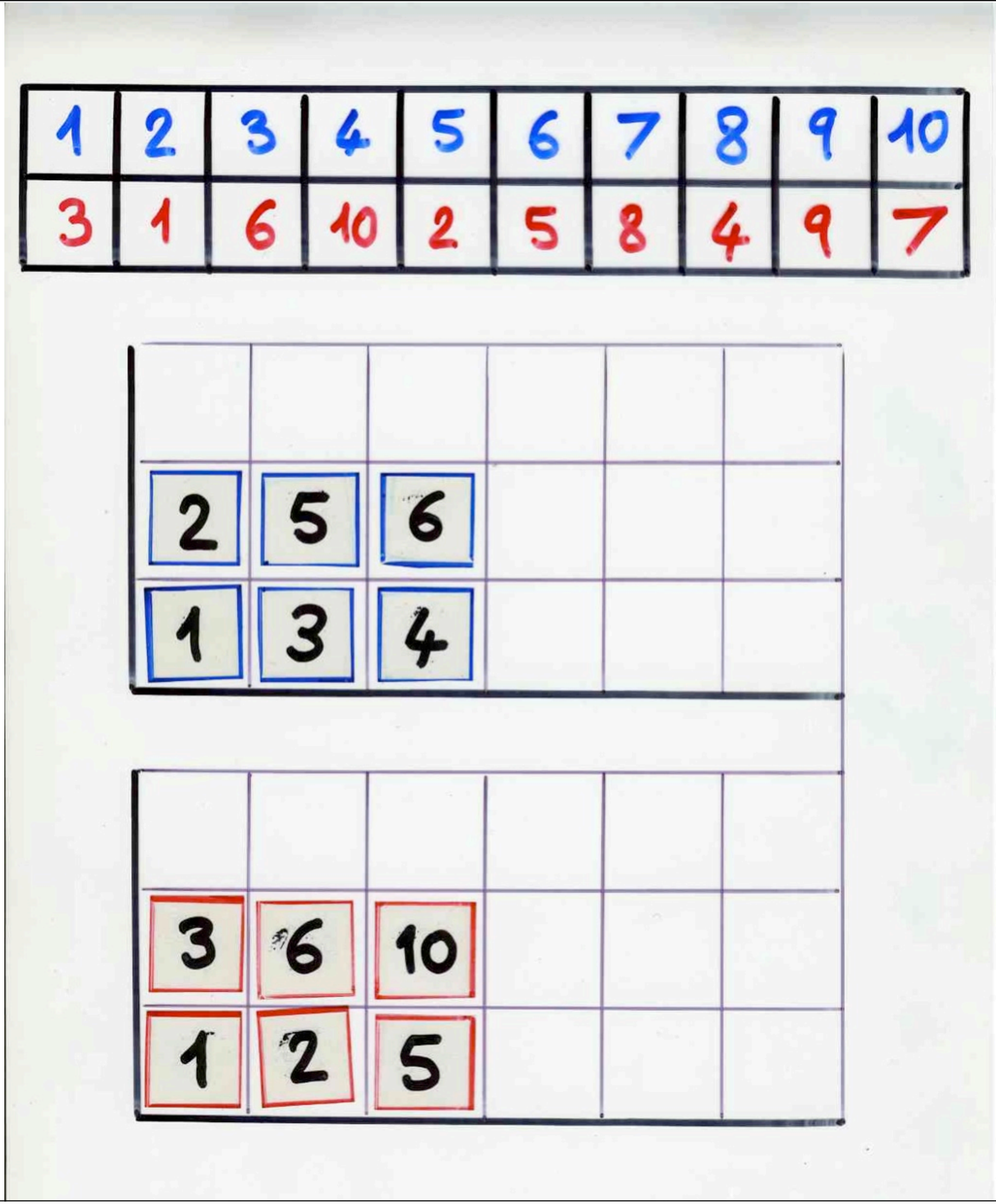
1	2	3	4	5	6	7	8	9	10
3	1	6	10	2	5	8	4	9	7



1	2	3	4	5	6	7	8	9	10
3	1	6	10	2	5	8	4	9	7





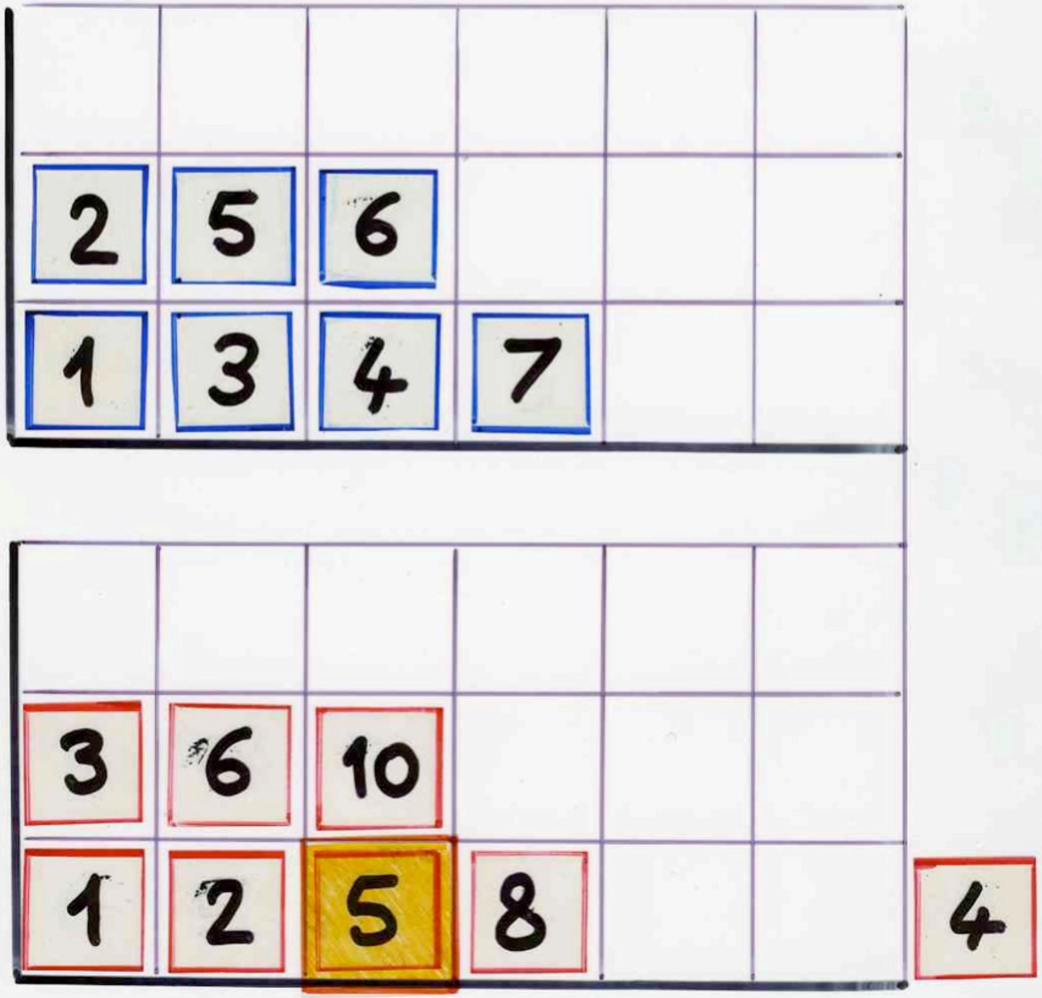

1 2 3 4 5 6 7 8 9 10
3 1 6 10 2 5 8 4 9 7
2 5 6
1 3 4
3 6 10
1 2 5

1	2	3	4	5	6	7	8	9	10
3	1	6	10	2	5	8	4	9	7

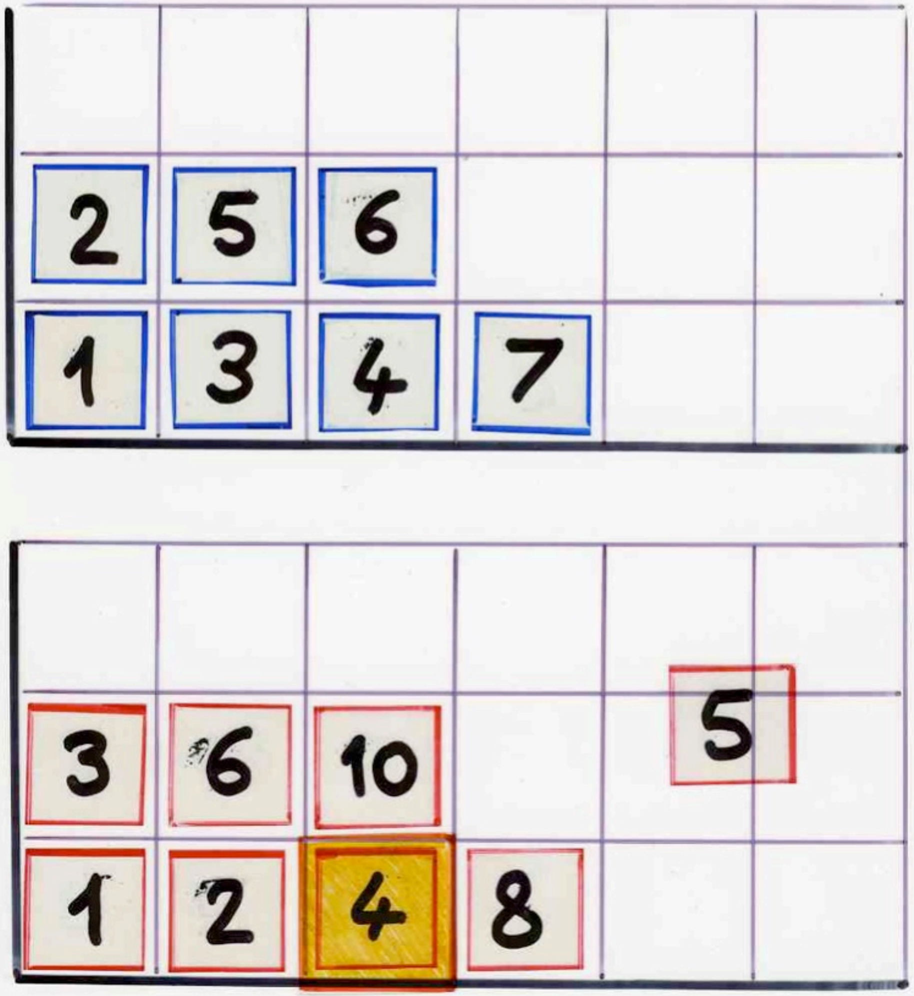
2	5	6			
1	3	4	7		

3	6	10			
1	2	5	8		

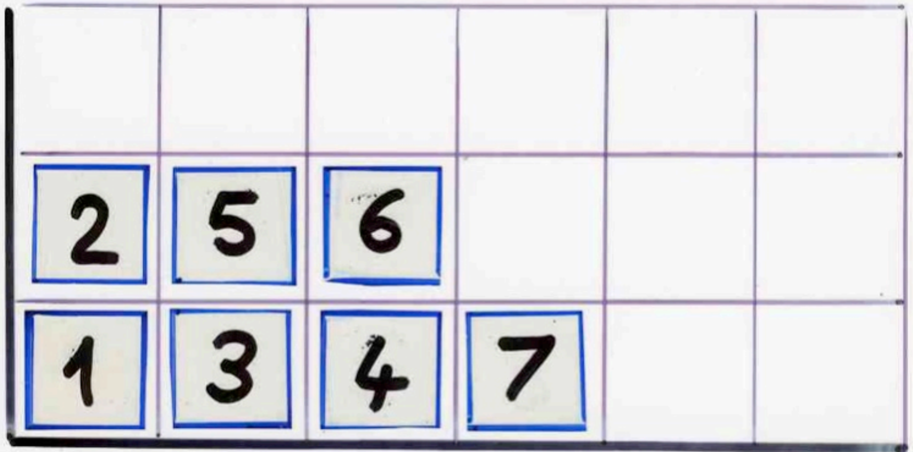
1	2	3	4	5	6	7	8	9	10
3	1	6	10	2	5	8	4	9	7

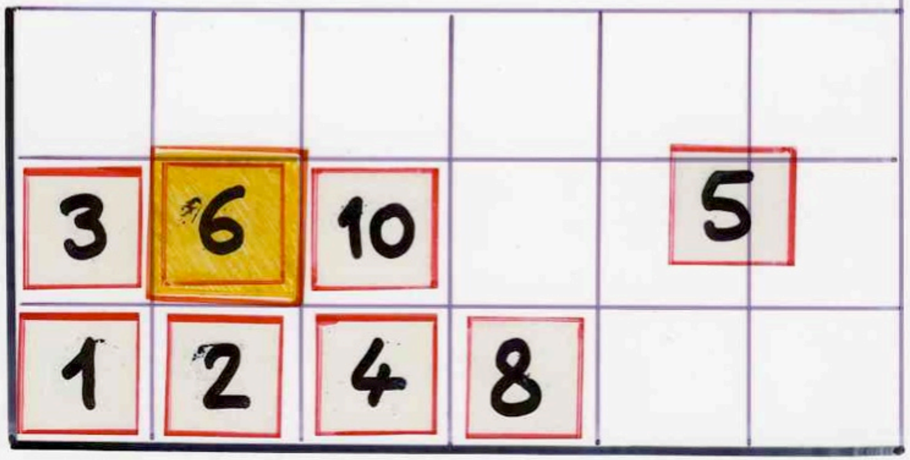


1	2	3	4	5	6	7	8	9	10
3	1	6	10	2	5	8	4	9	7

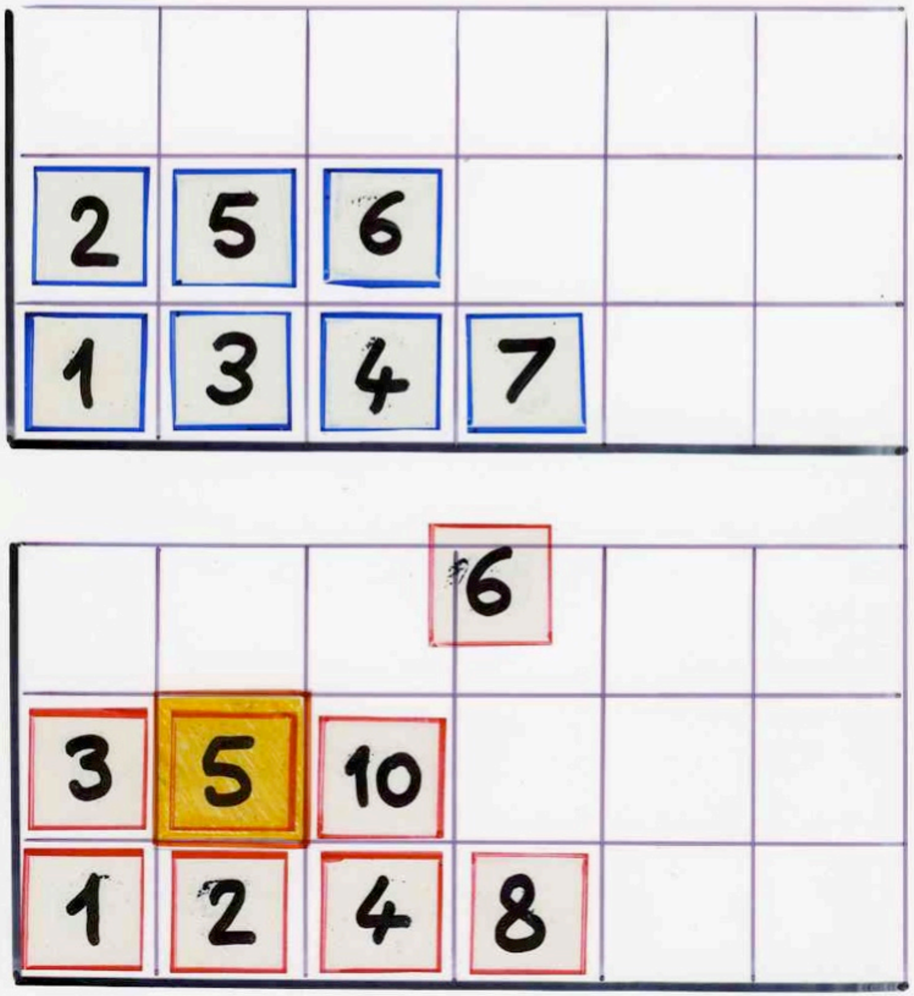


1	2	3	4	5	6	7	8	9	10
3	1	6	10	2	5	8	4	9	7





1	2	3	4	5	6	7	8	9	10
3	1	6	10	2	5	8	4	9	7



1	2	3	4	5	6	7	8	9	10
3	1	6	10	2	5	8	4	9	7

8					
2	5	6			
1	3	4	7		

6					
3	5	10			
1	2	4	8		

1	2	3	4	5	6	7	8	9	10
3	1	6	10	2	5	8	4	9	7

8					
2	5	6			
1	3	4	7	9	

6					
3	5	10			
1	2	4	8	9	

1	2	3	4	5	6	7	8	9	10
3	1	6	10	2	5	8	4	9	7

8					
2	5	6			
1	3	4	7	9	

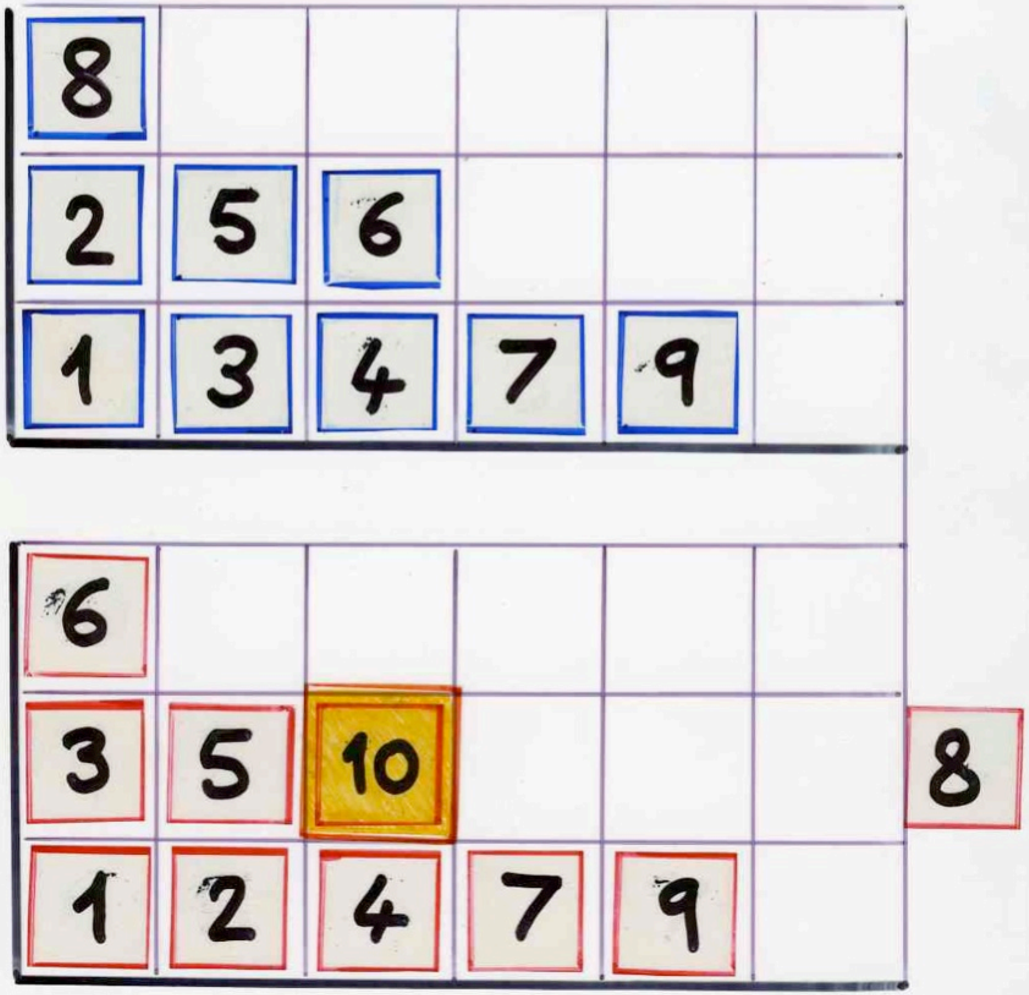
6						
3	5	10				
1	2	4	8	9		7

1	2	3	4	5	6	7	8	9	10
3	1	6	10	2	5	8	4	9	7

8					
2	5	6			
1	3	4	7	9	

6						
3	5	10				8
1	2	4	7	9		

1	2	3	4	5	6	7	8	9	10
3	1	6	10	2	5	8	4	9	7



1	2	3	4	5	6	7	8	9	10
3	1	6	10	2	5	8	4	9	7

8					
2	5	6			
1	3	4	7	9	

6						10
3	5	8				
1	2	4	7	9		

1	2	3	4	5	6	7	8	9	10
3	1	6	10	2	5	8	4	9	7

8	10				
2	5	6			
1	3	4	7	9	

6	10				
3	5	8			
1	2	4	7	9	

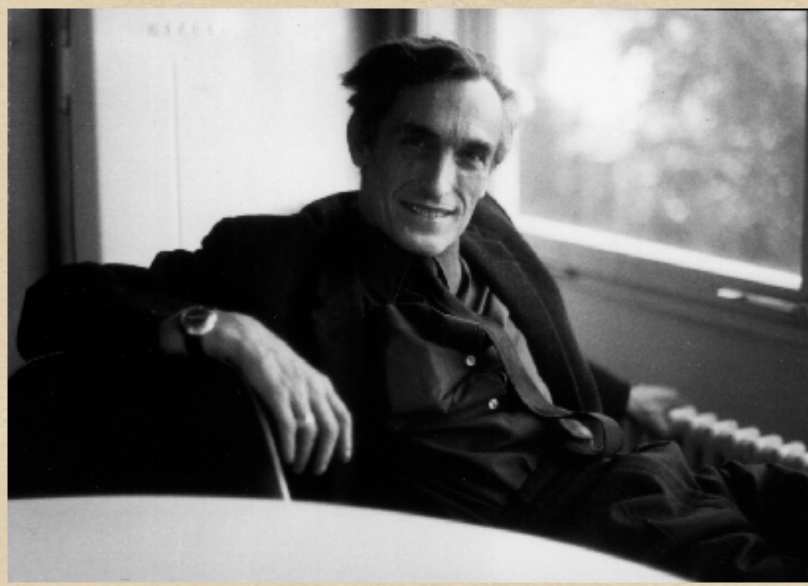
$$\sigma \longleftrightarrow (P, Q)$$

$$\sigma^{-1} \longleftrightarrow (Q, P)$$

Donald Knuth (1972)

" The unusual nature of these coincidences might lead us to suspect that some sort of withcraft is operating behind the scenes "

Jeu de taquin

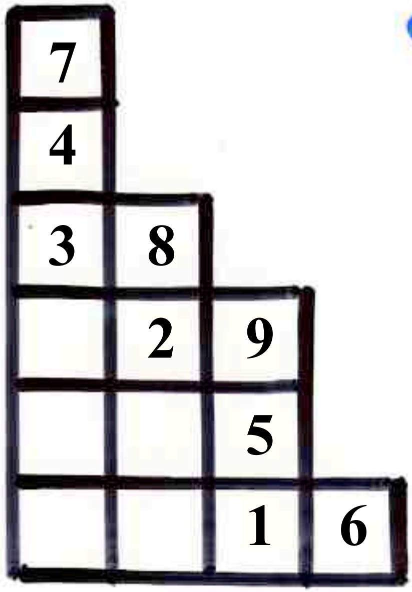


M.P. Schützenberger

$$\sigma = 7\ 4\ 3\ 8\ 2\ 9\ 5\ 1\ 6$$

"jeu de taquin"

M.P. Schützenberger

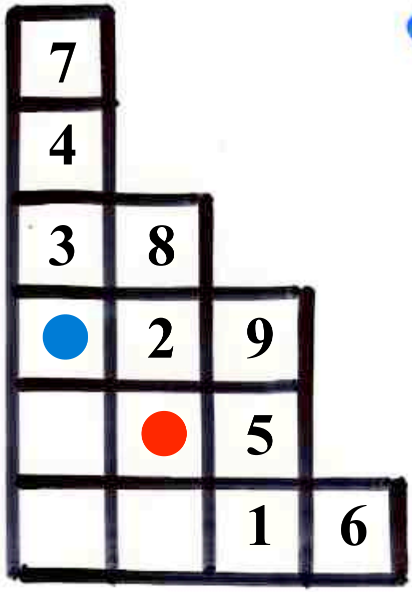


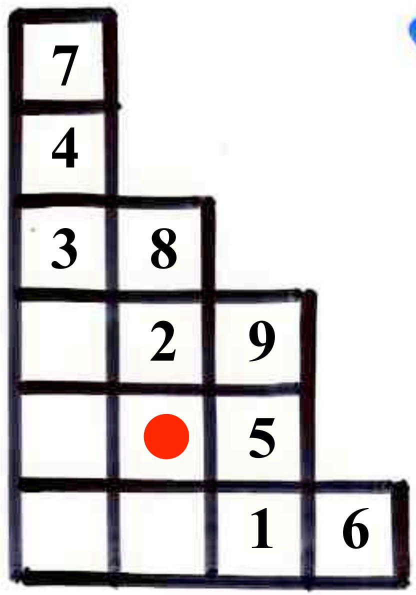
$\sigma = 7\ 4\ 3\ 8\ 2\ 9\ 5\ 1\ 6 \ldots$

up-down sequence $-\ -\ +\ -\ +\ -\ -\ +$

"jeu de taquin"

M.P. Schützenberger



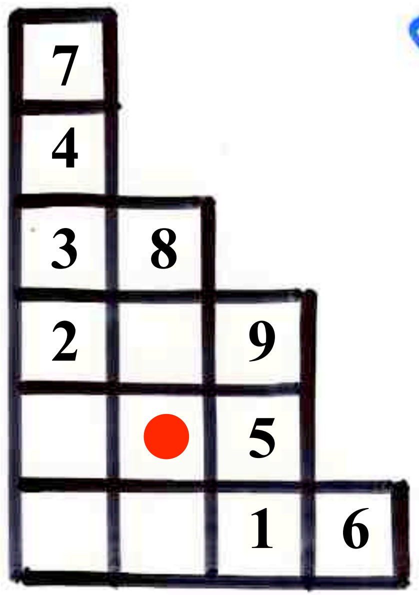


"jeu de taquin"

M.P. Schützenberger

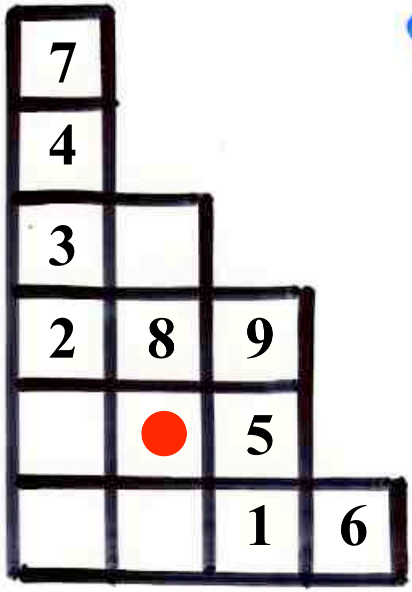
"jeu de taquin"

M.P. Schützenberger



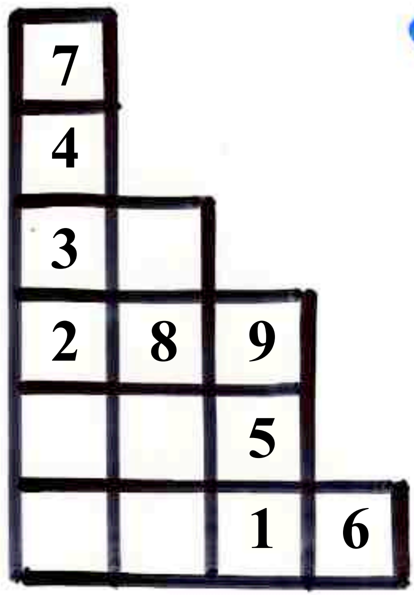
"jeu de taquin"

M.P. Schützenberger



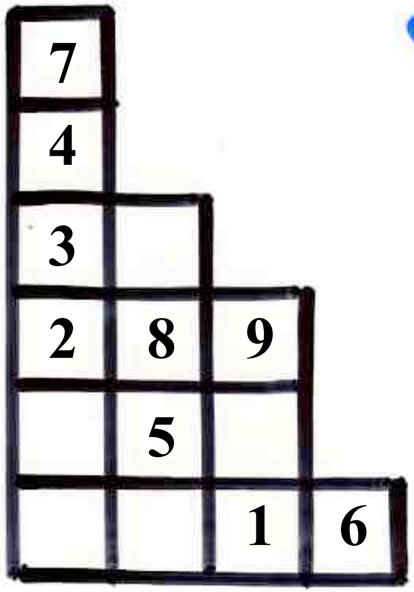
"jeu de taquin"

M.P. Schützenberger



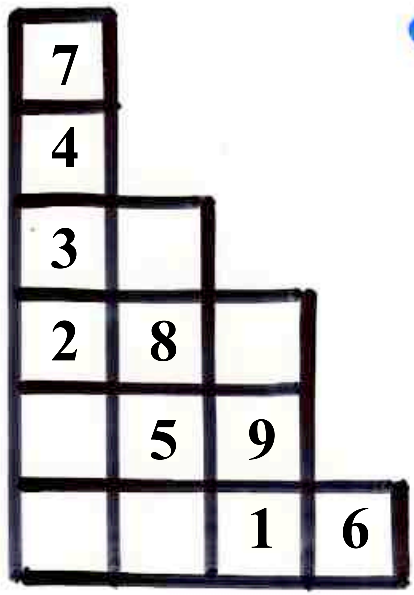
"jeu de taquin"

M.P. Schützenberger



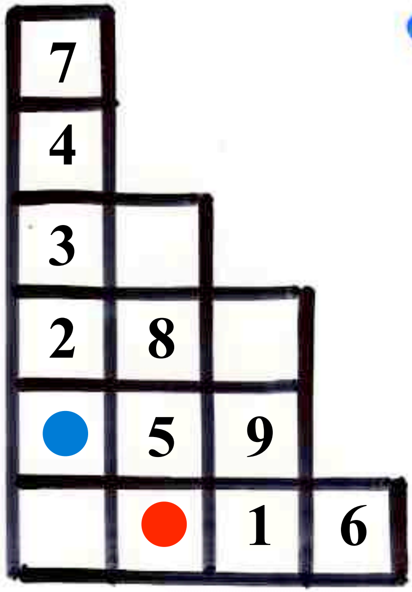
"jeu de taquin"

M.P. Schützenberger



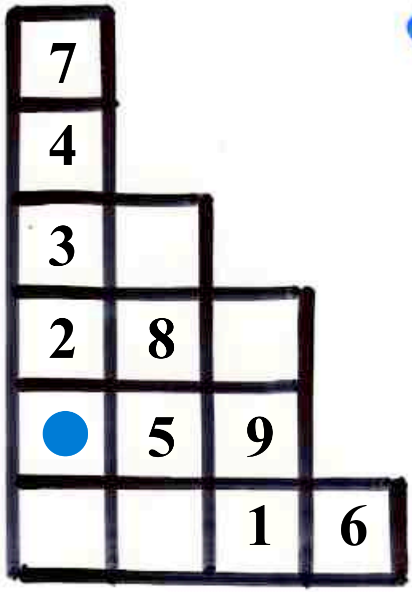
"jeu de taquin"

M.P. Schützenberger



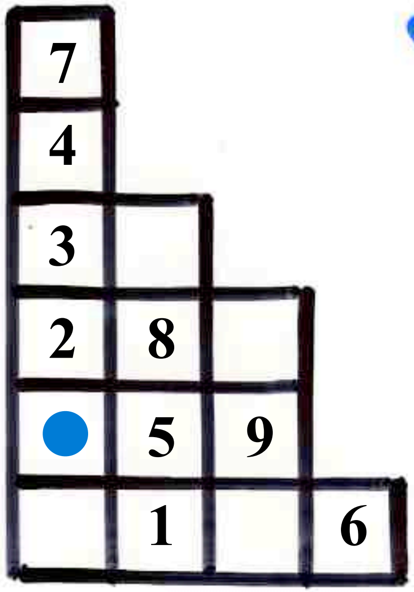
"jeu de taquin"

M.P. Schützenberger



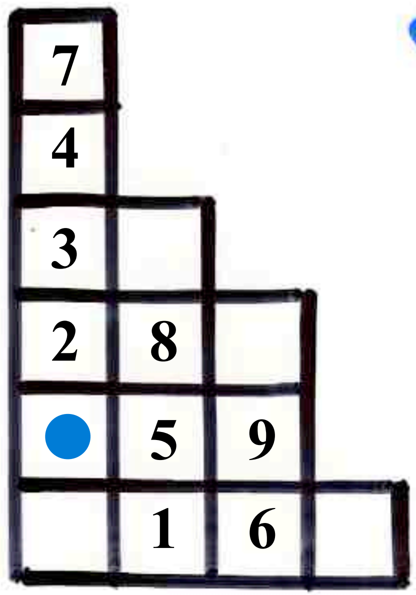
"jeu de taquin"

M.P. Schützenberger



"jeu de taquin"

M.P. Schützenberger



"jeu de taquin"

M.P. Schützenberger

7			
4			
3			
2	8		
	5	9	
	1	6	

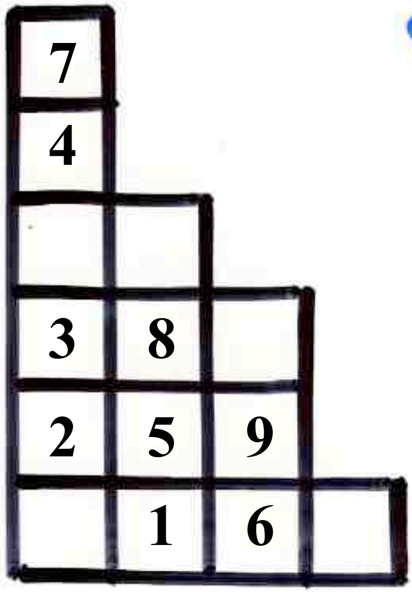
"jeu de taquin"

M.P. Schützenberger

7			
4			
3			
	8		
2	5	9	
	1	6	

"jeu de taquin"

M.P. Schützenberger



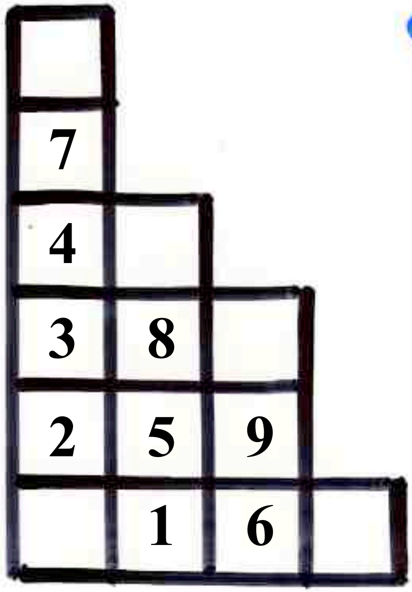
"jeu de taquin"

M.P. Schützenberger

7			
4			
3	8		
2	5	9	
	1	6	

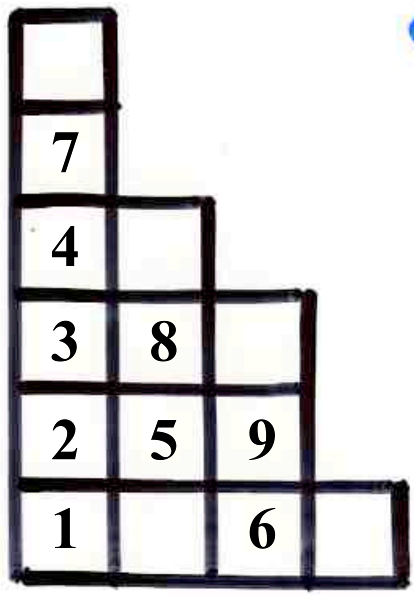
"jeu de taquin"

M.P. Schützenberger



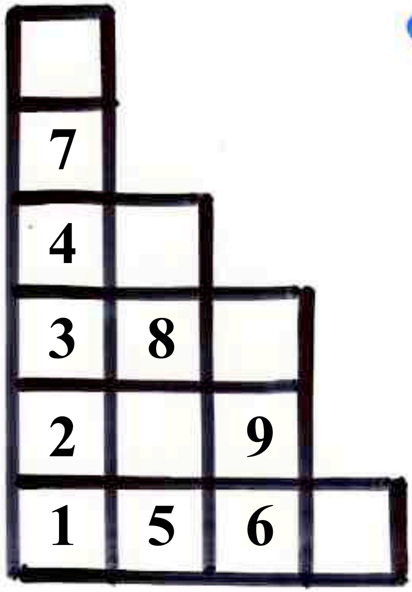
"jeu de taquin"

M.P. Schützenberger



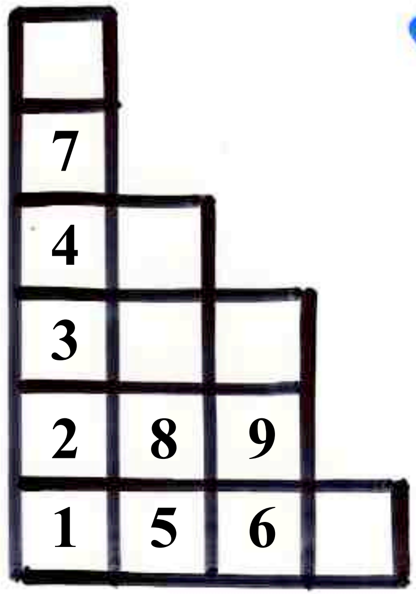
"jeu de taquin"

M.P. Schützenberger



"jeu de taquin"

M.P. Schützenberger



"jeu de taquin"

M.P. Schützenberger

$$P(\sigma) \quad \begin{array}{|c|c|c|c|} \hline \\ \cline{1-1} 7 \\ \cline{1-2} 4 & \\ \cline{1-3} 3 & & \\ \cline{1-3} 2 & 8 & 9 \\ \hline 1 & 5 & 6 & \\ \hline \end{array}$$

Young tableau

$$\sigma = 7\ 4\ 3\ 8\ 2\ 9\ 5\ 1\ 6$$

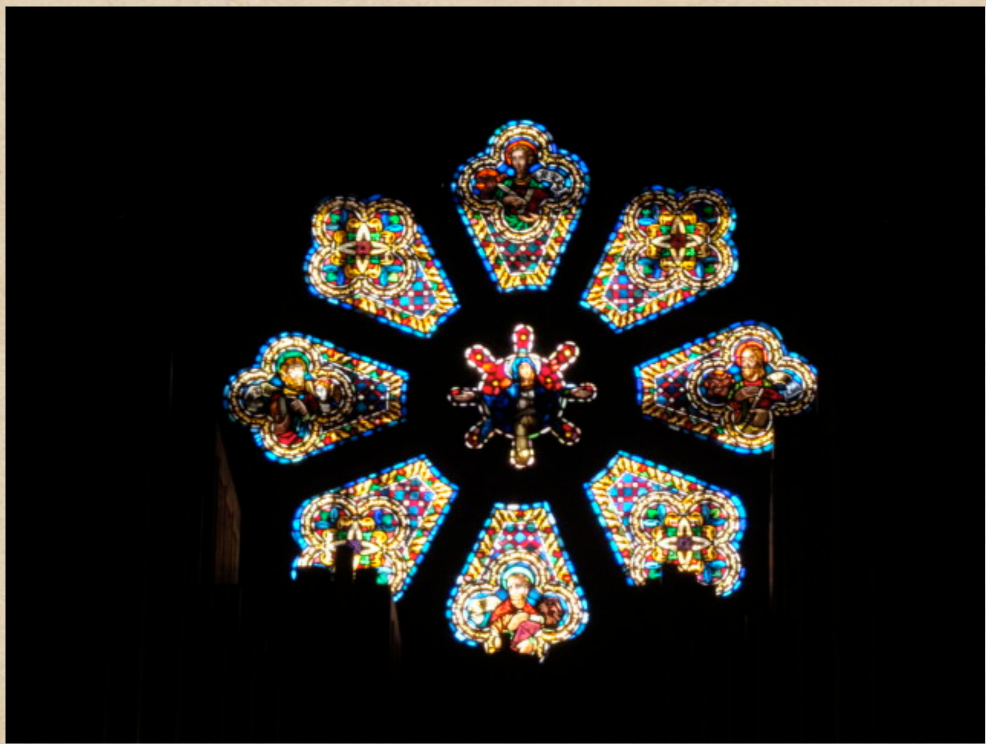
$$\sigma \longrightarrow \big(P(\sigma),\ \underset{\displaystyle P(\sigma^{-1})}{\underset{\shortparallel}{Q(\sigma)}}\big)$$

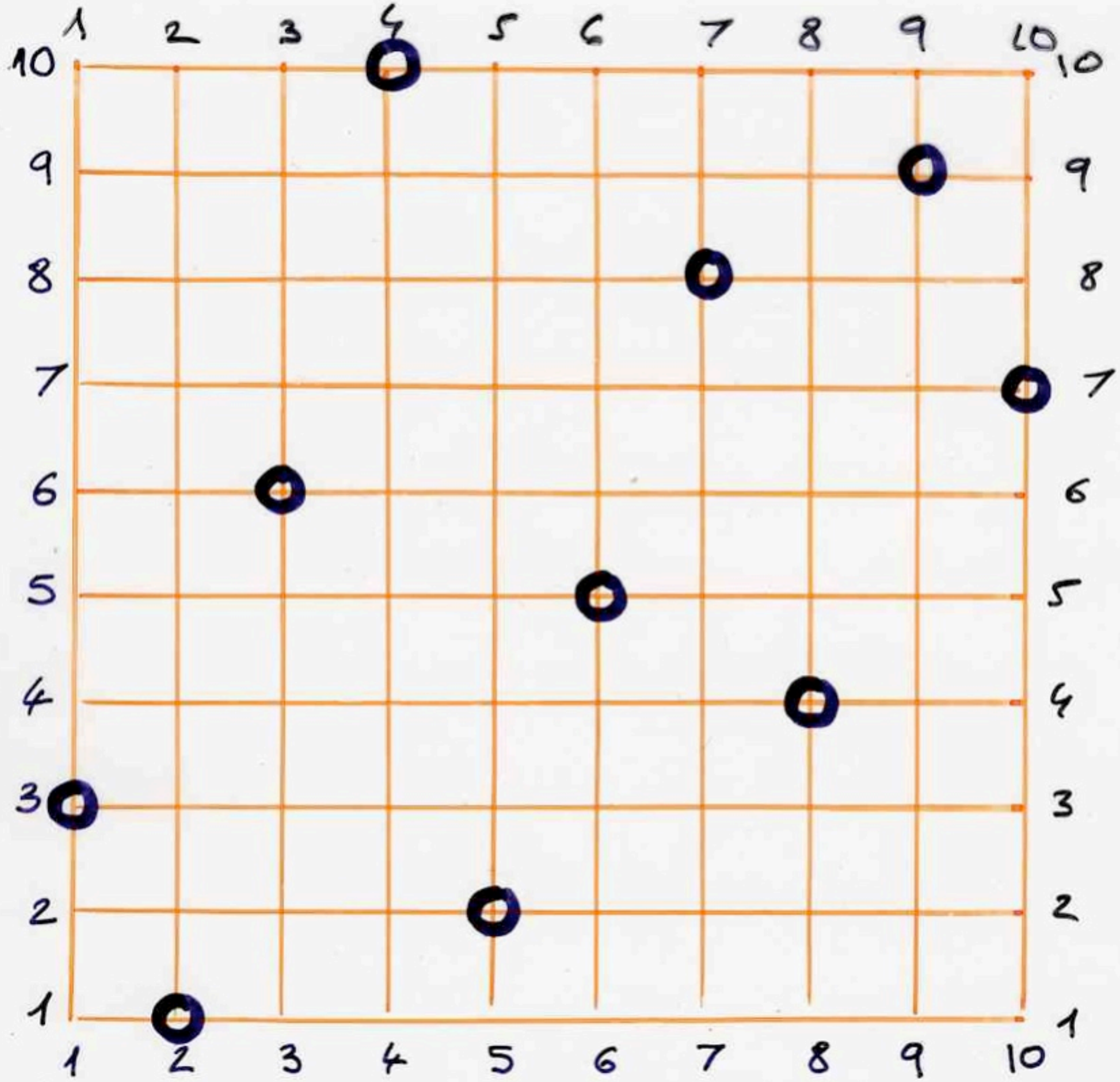
RSK

Robinson-Schensted-Knuth

A geometric version of RSK with "light" and "shadow lines"

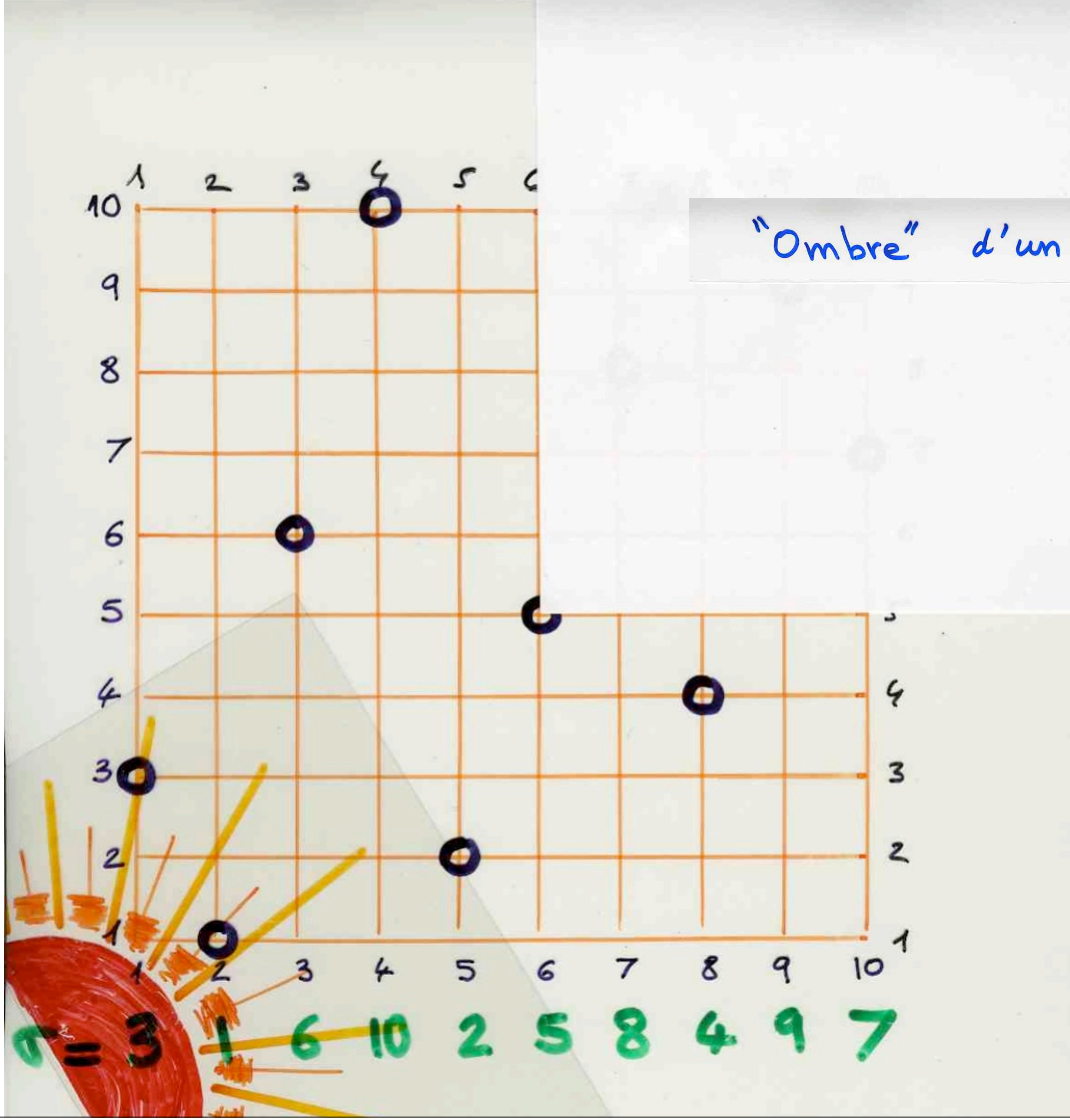
xgv, 1976





$\sigma = 3 \quad 1 \quad 6 \quad 10 \quad 2 \quad 5 \quad 8 \quad 4 \quad 9 \quad 7$

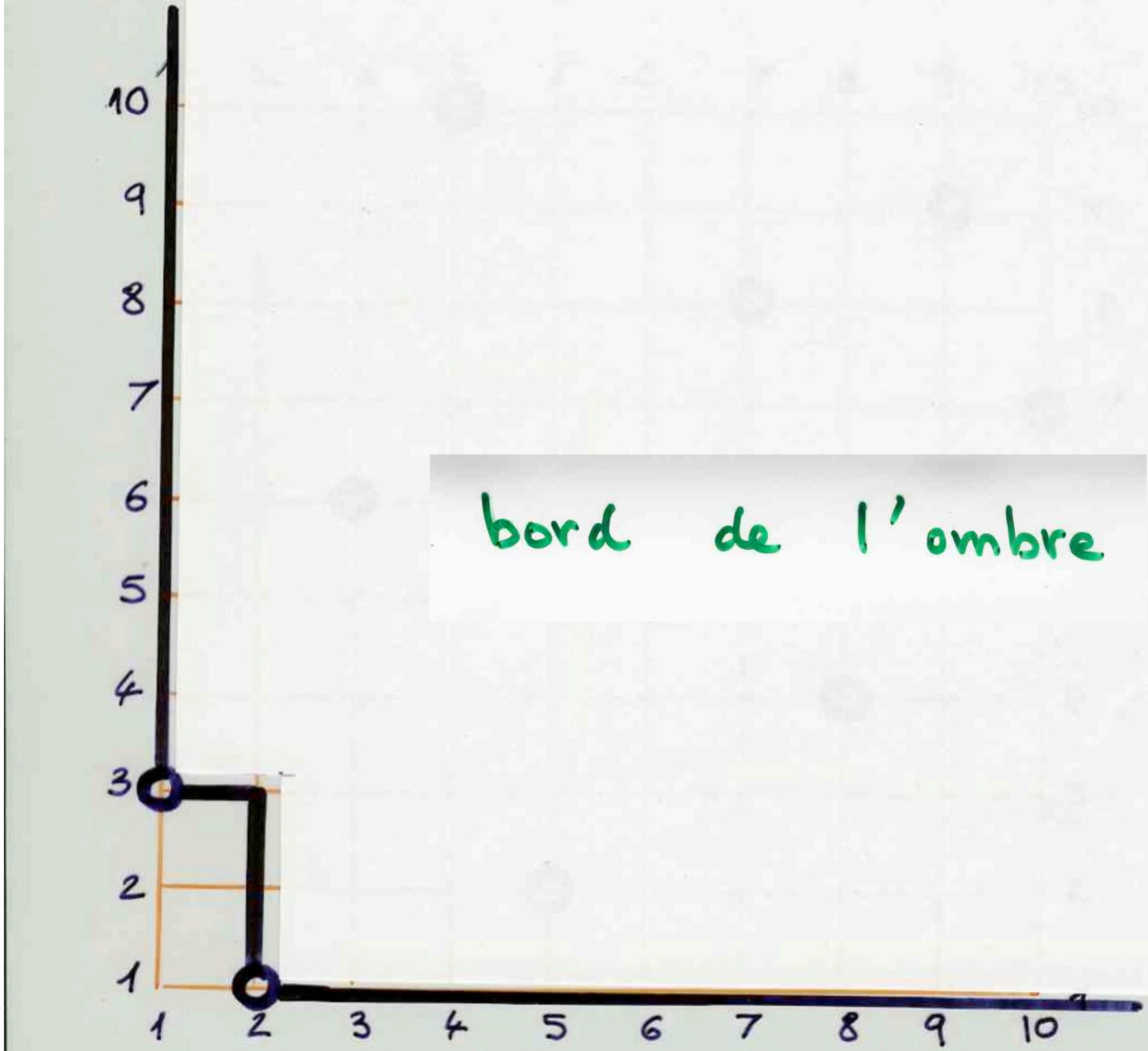
"Ombre" d'un point



"Ombre" de la permutation

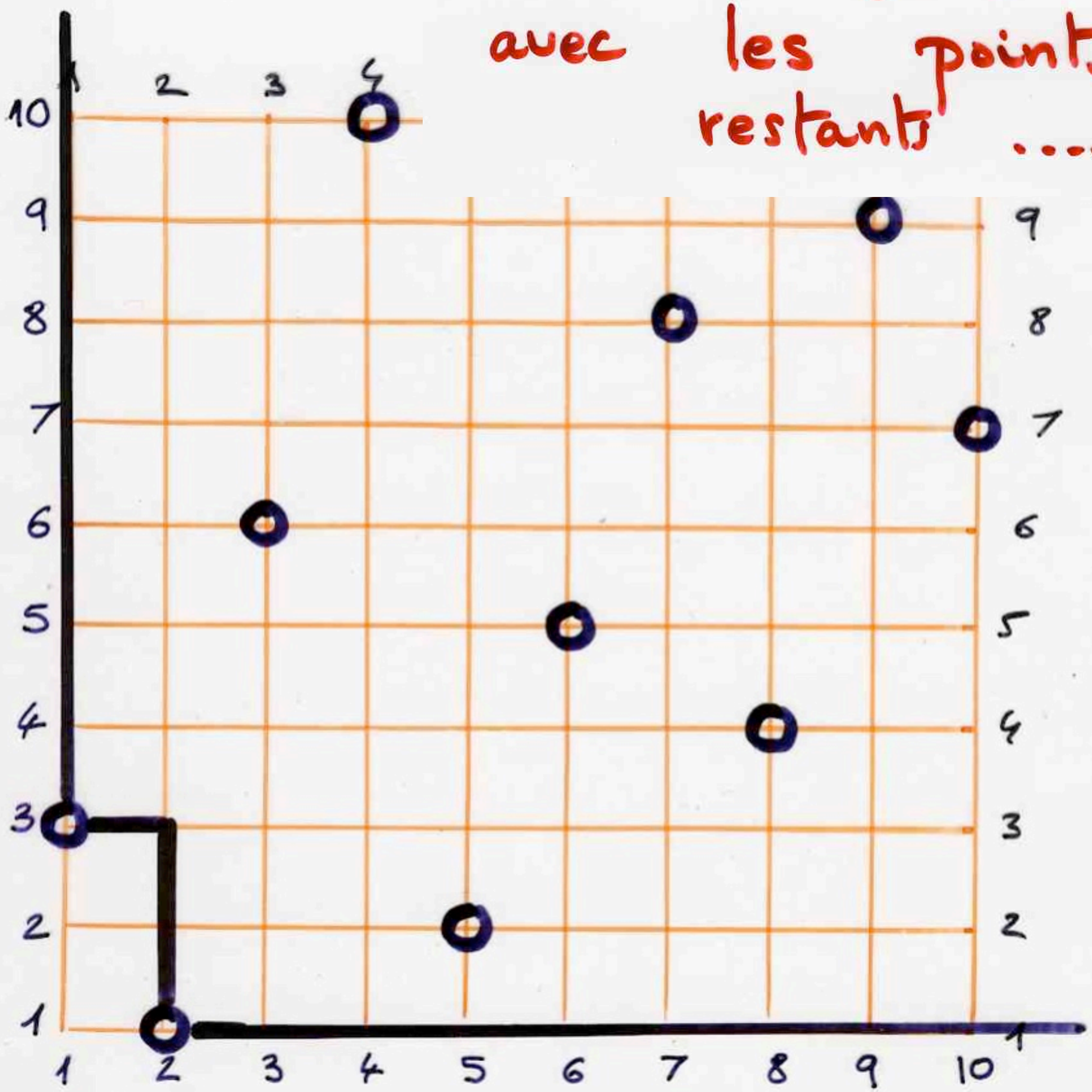
= union des ombres

$\sigma = 3\ 1\ 6\ 10\ 2\ 5\ 8\ 4\ 9\ 7$

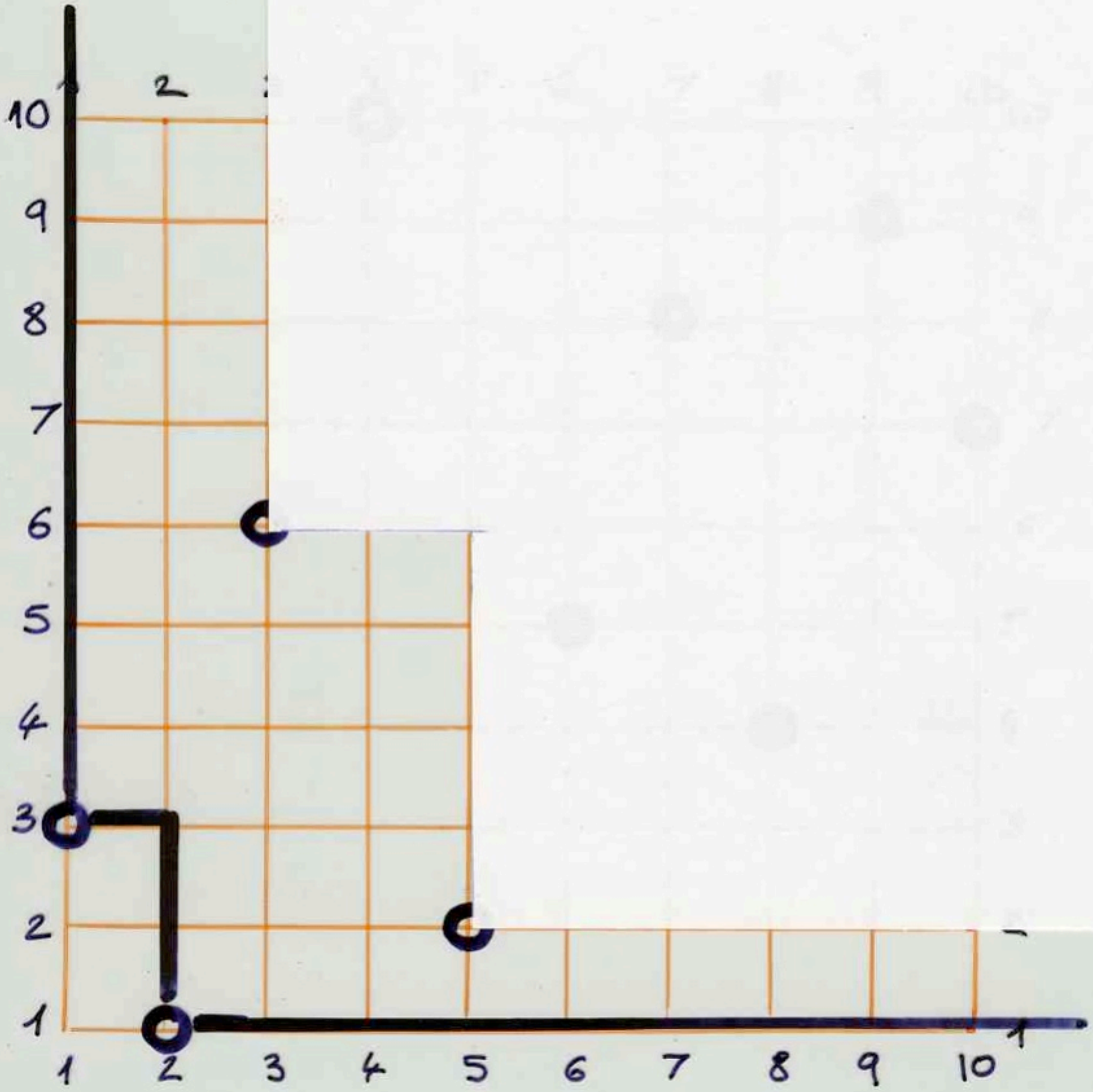


$\sigma = 3\ 1\ 6\ 10\ 2\ 5\ 8\ 4\ 9\ 7$

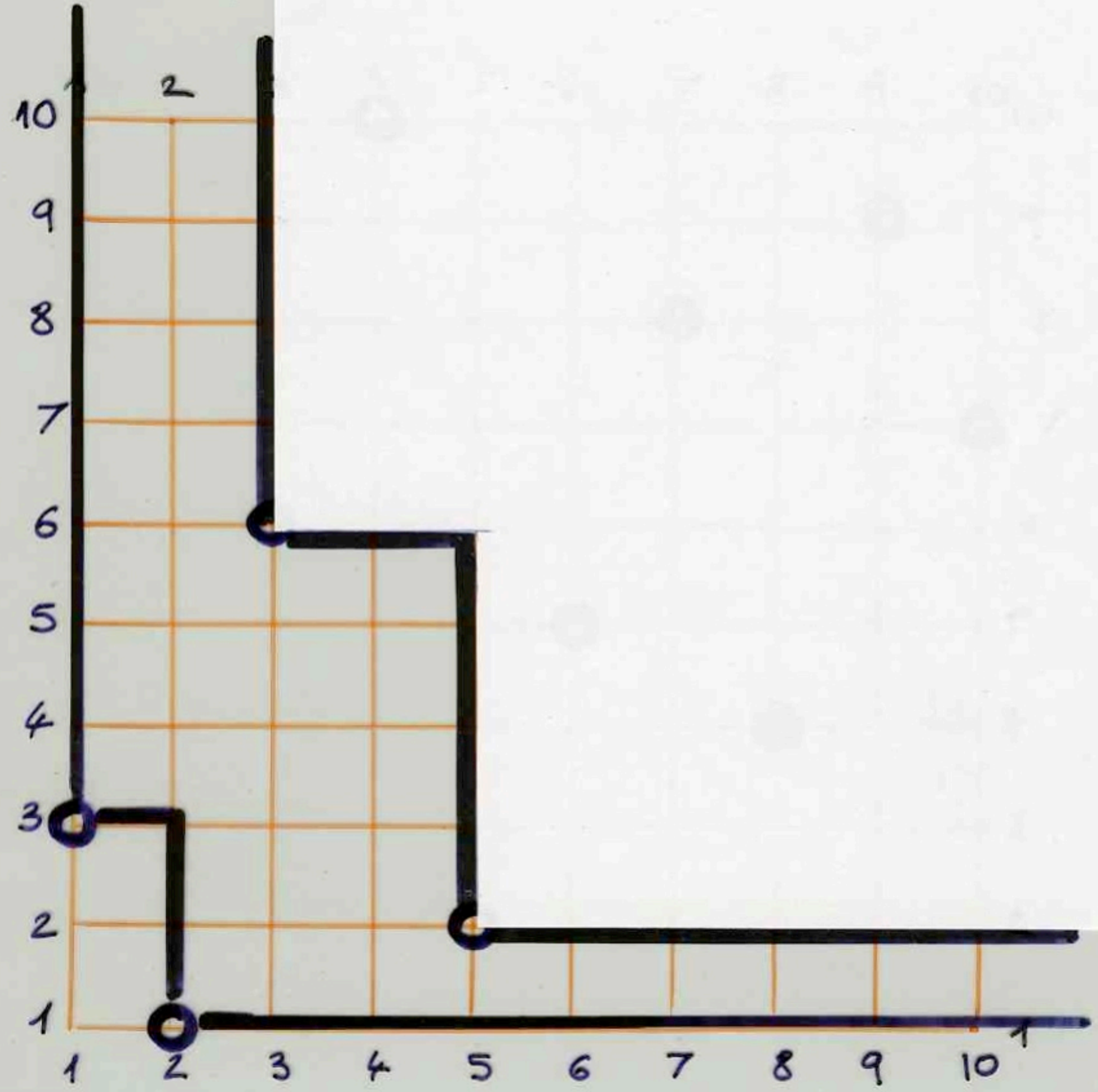
recommençons avec les points restants ....



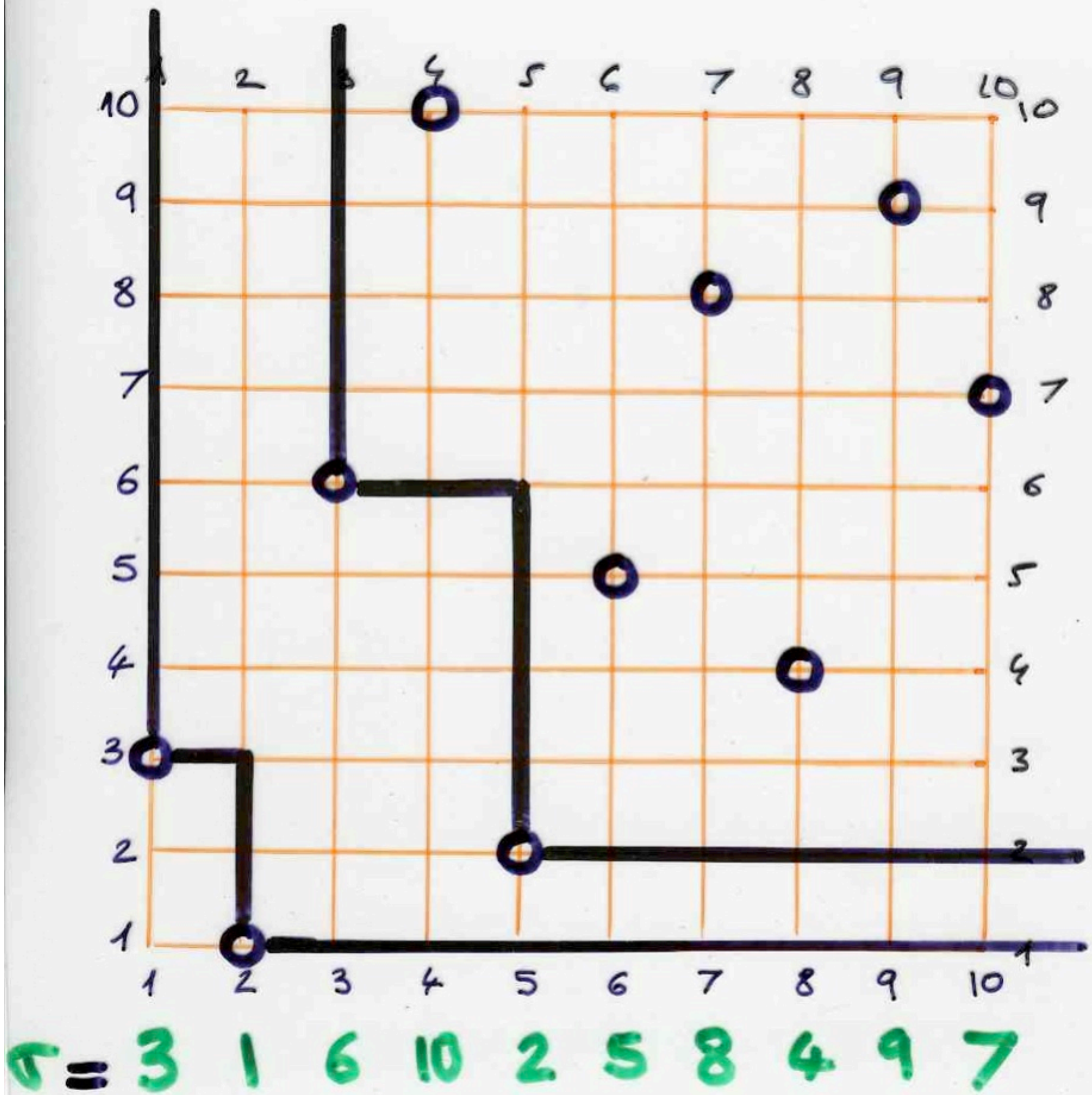
$\sigma = 3\ 1\ 6\ 10\ 2\ 5\ 8\ 4\ 9\ 7$



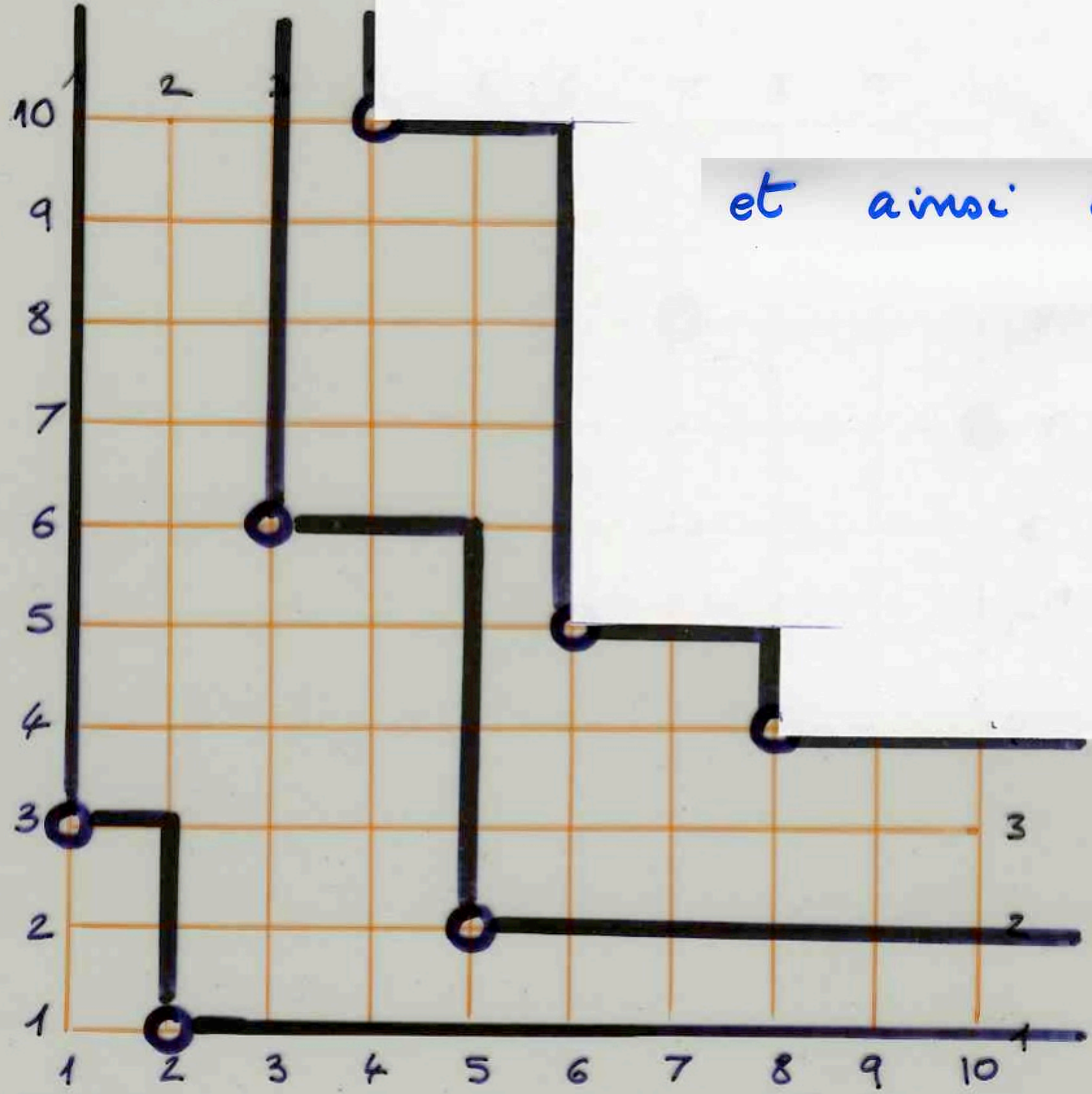
$\sigma = 3\ 1\ 6\ 10\ 2\ 5\ 8\ 4\ 9\ 7$



$\sigma = 3\ 1\ 6\ 10\ 2\ 5\ 8\ 4\ 9\ 7$

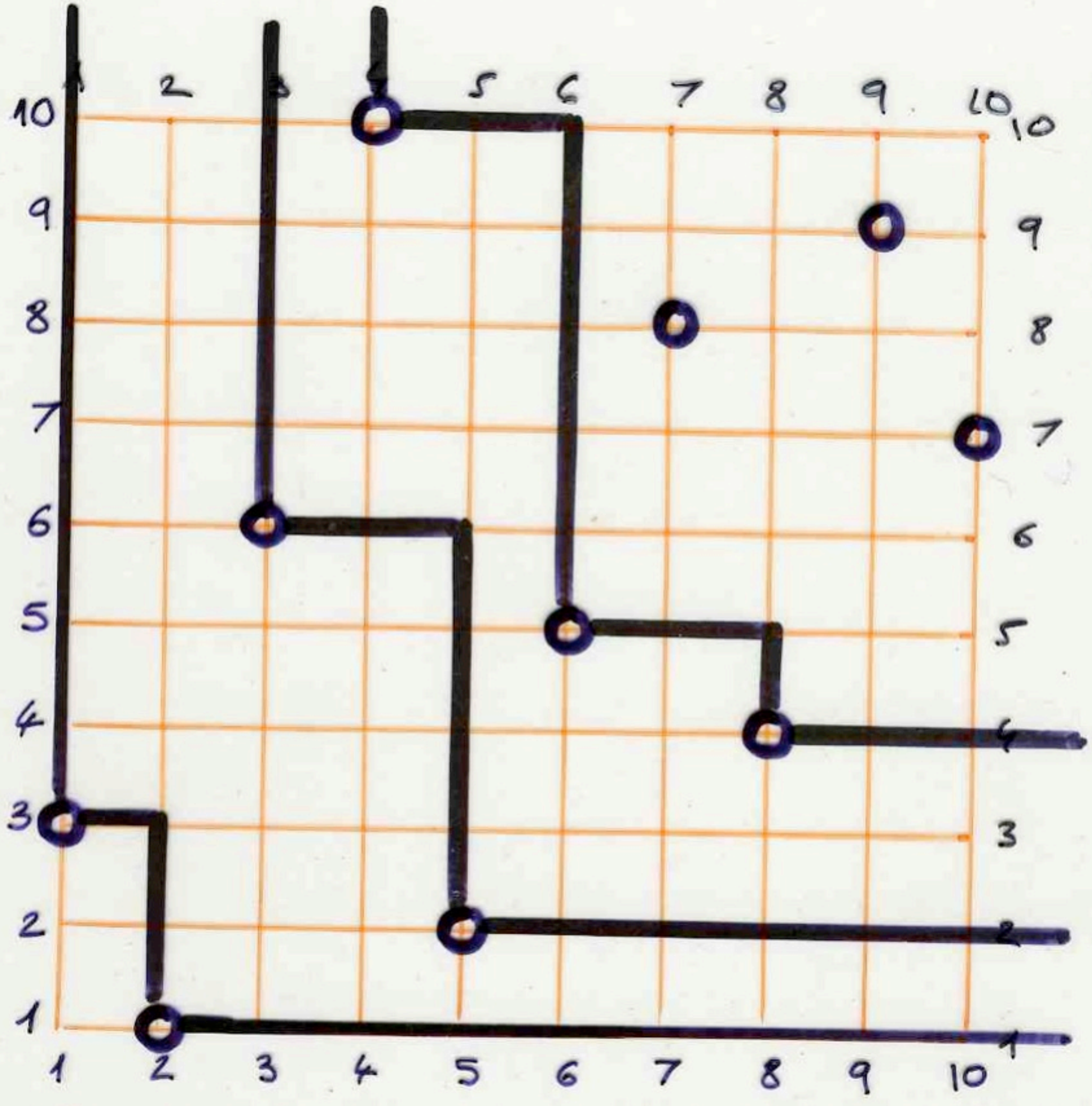


1 2 3 4 5 6 7 8 9 10
σ = 3 1 6 10 2 5 8 4 9 7

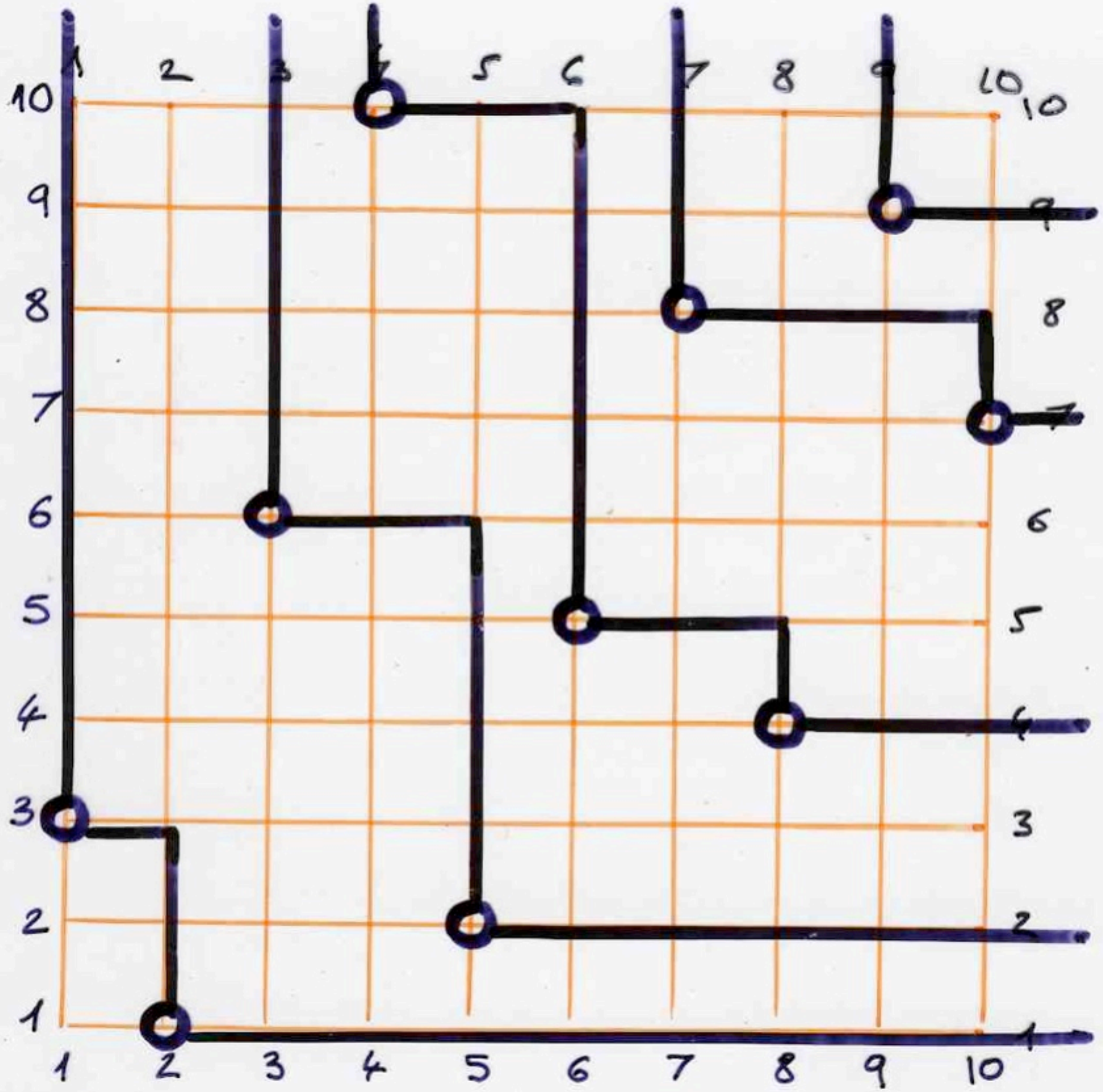


et ainsi de suite ...

$\sigma = 3\ 1\ 6\ 10\ 2\ 5\ 8\ 4\ 9\ 7$

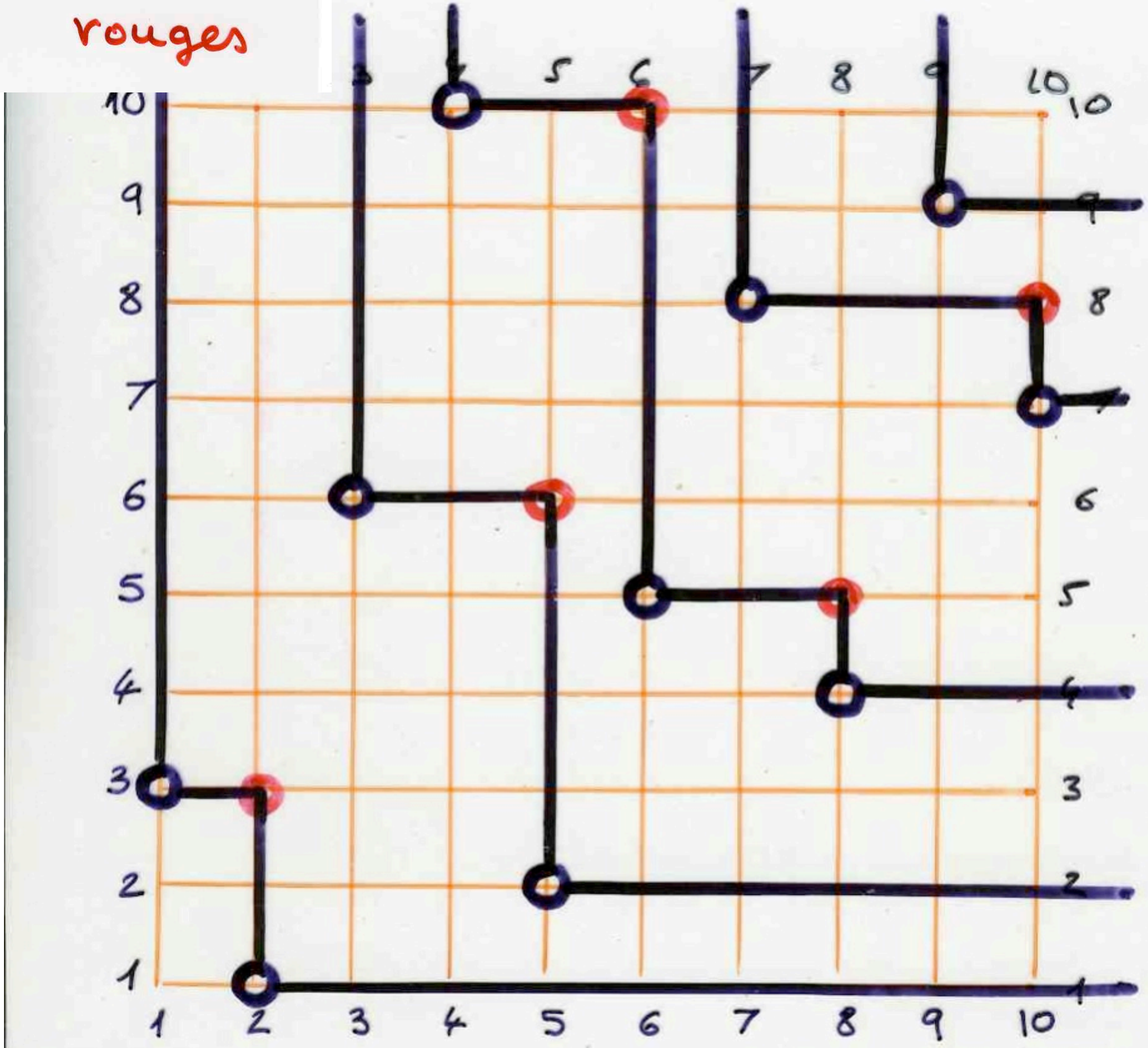


$\sigma = 3\ 1\ 6\ 10\ 2\ 5\ 8\ 4\ 9\ 7$

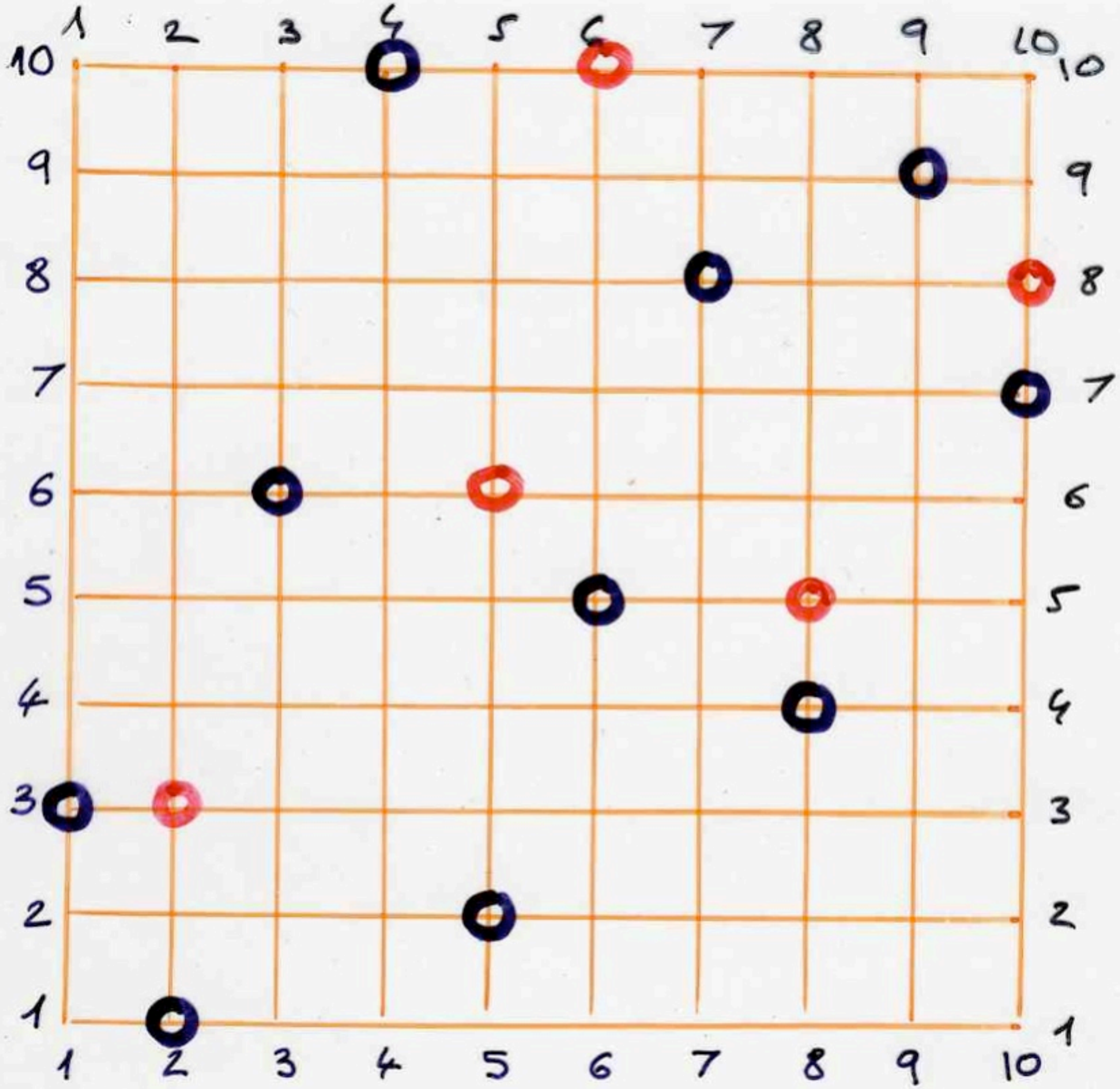


$\sigma = 3\ 1\ 6\ 10\ 2\ 5\ 8\ 4\ 9\ 7$

des nouveaux points
les rouges

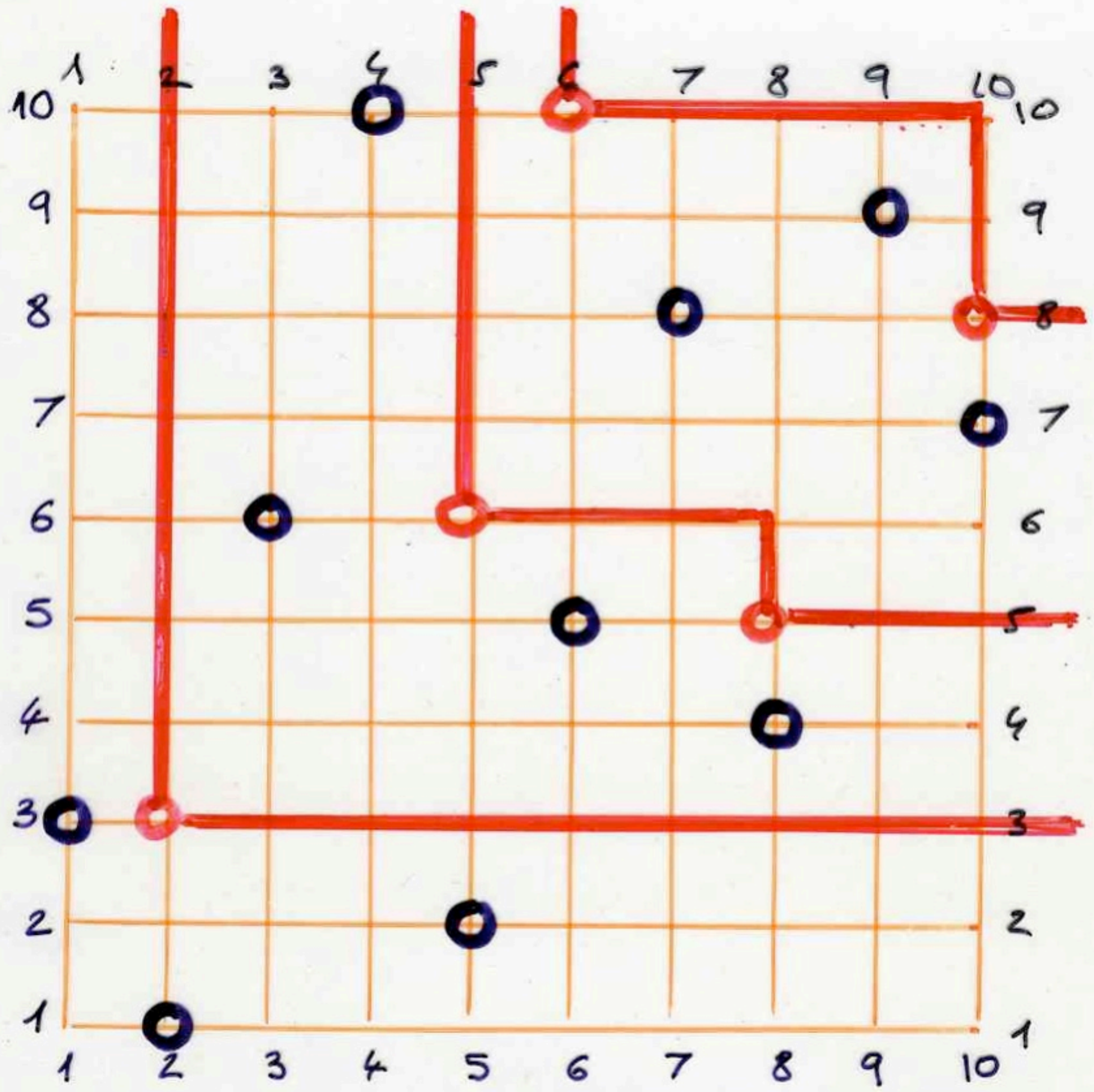


$\sigma = 3 \; 1 \; 6 \; 10 \; 2 \; 5 \; 8 \; 4 \; 9 \; 7$

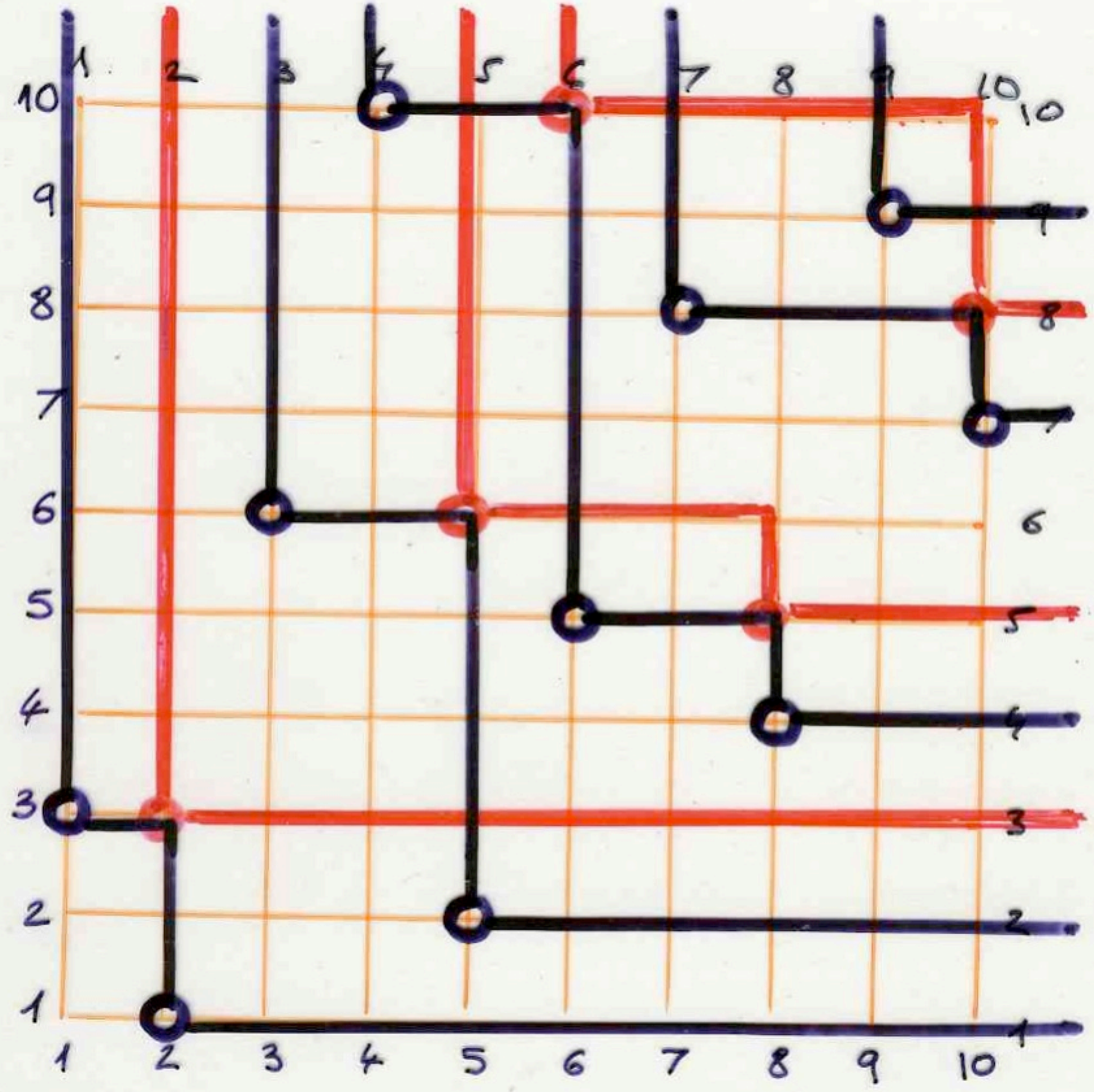


$\sigma = 3 \quad 1 \quad 6 \quad 10 \quad 2 \quad 5 \quad 8 \quad 4 \quad 9 \quad 7$

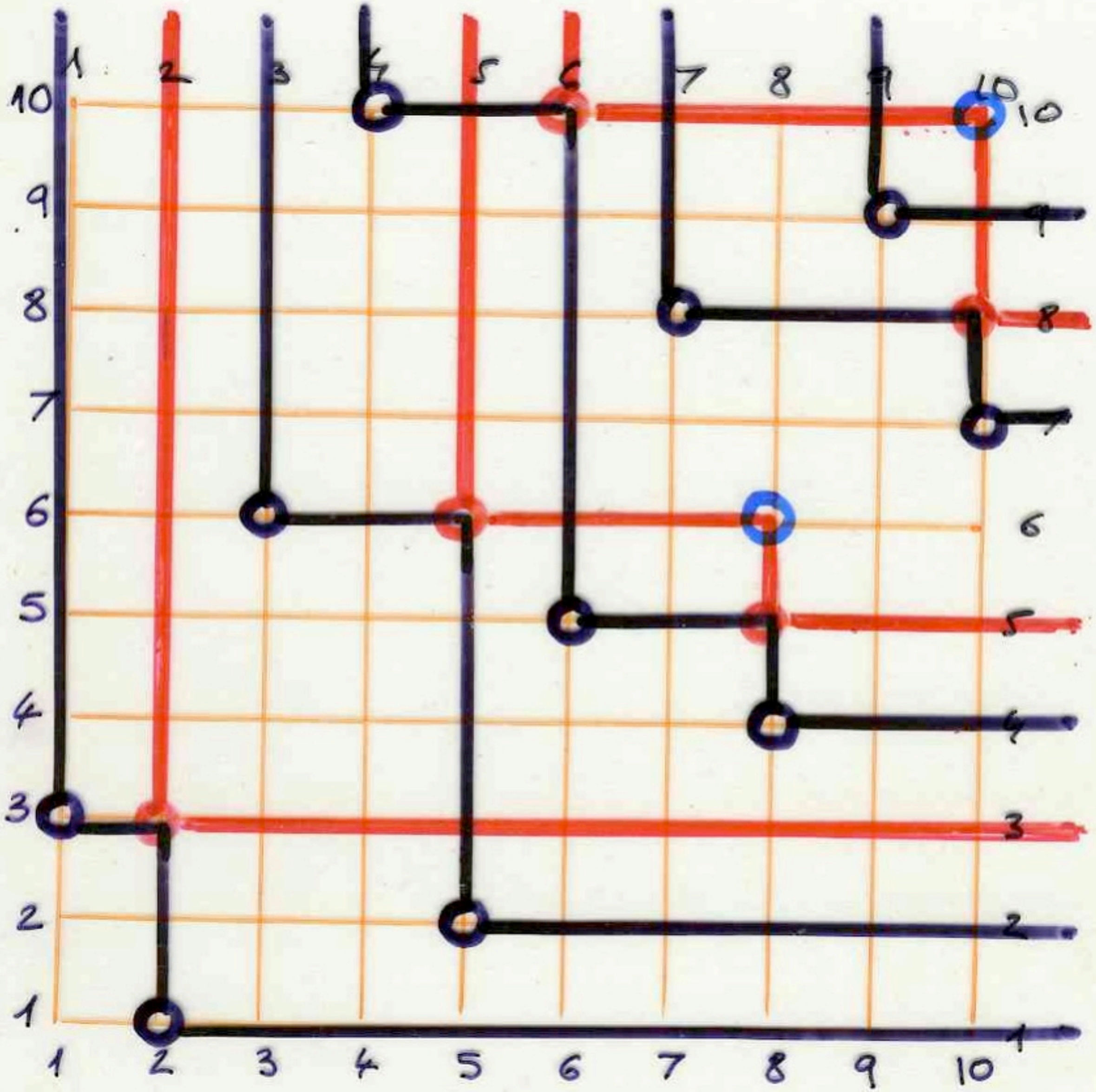
Répétons sur les points rouges la construction des bords d'ombres.



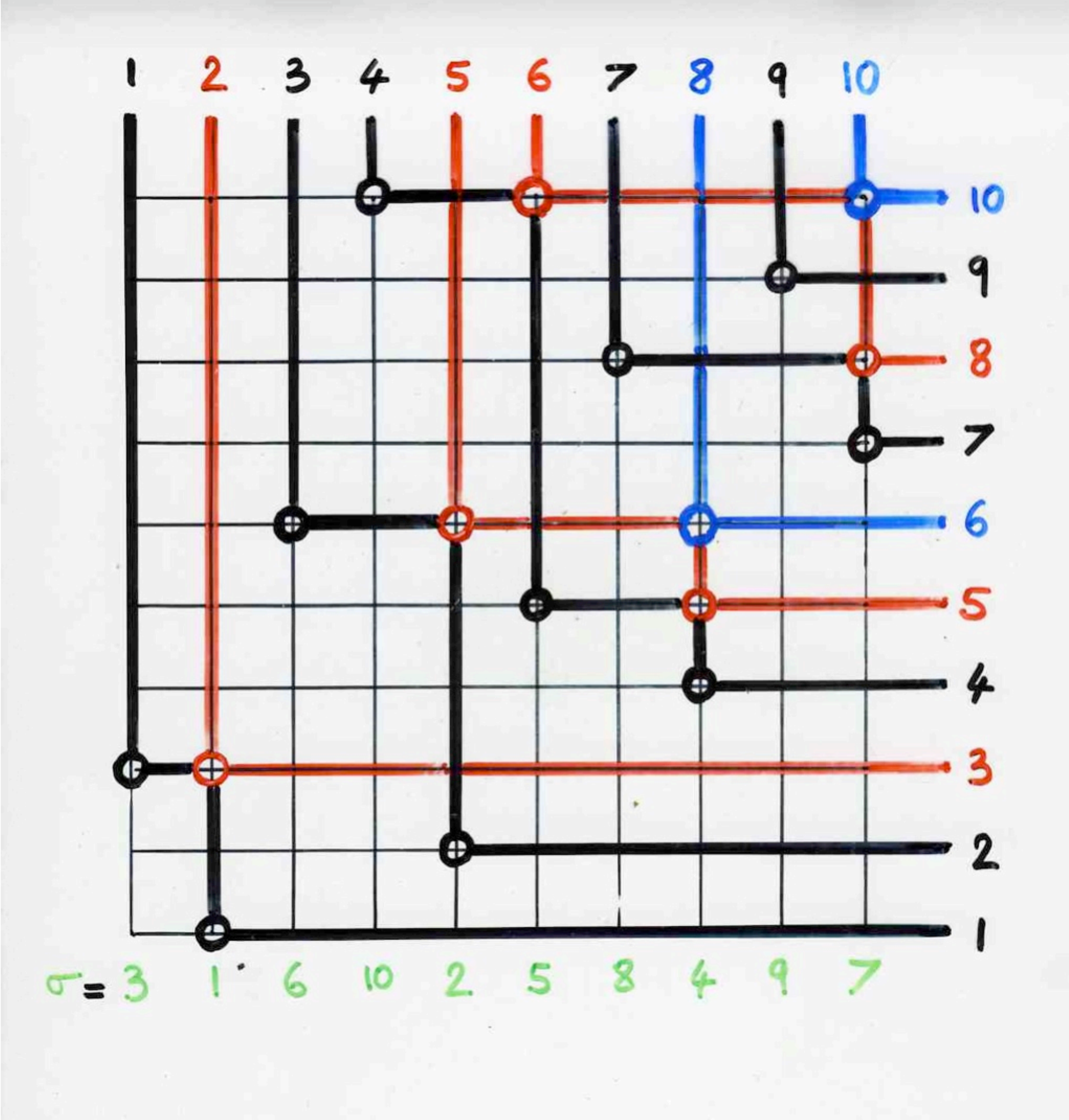
$\sigma = 3\ 1\ 6\ 10\ 2\ 5\ 8\ 4\ 9\ 7$



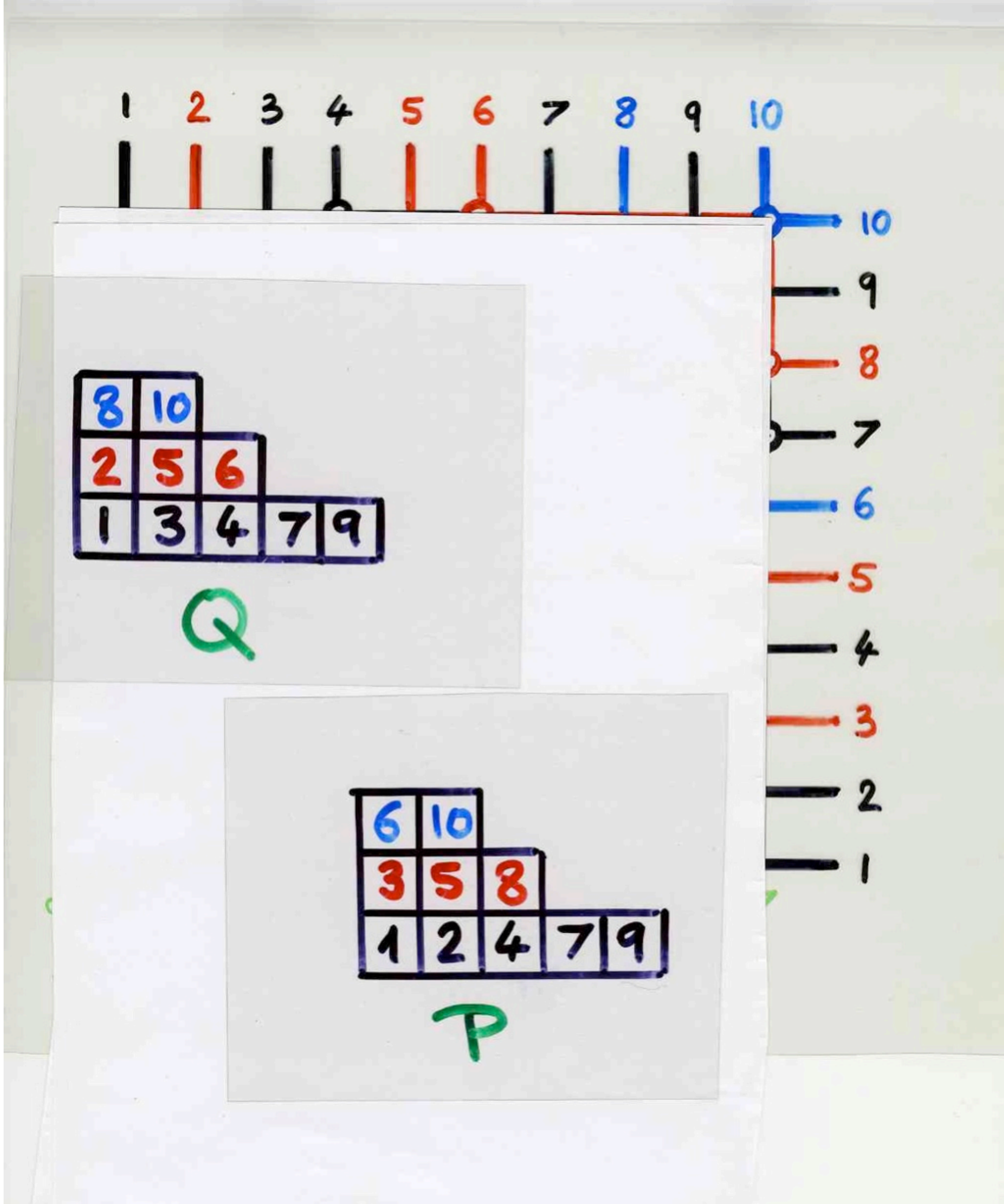
$\sigma = 3\ 1\ 6\ 10\ 2\ 5\ 8\ 4\ 9\ 7$

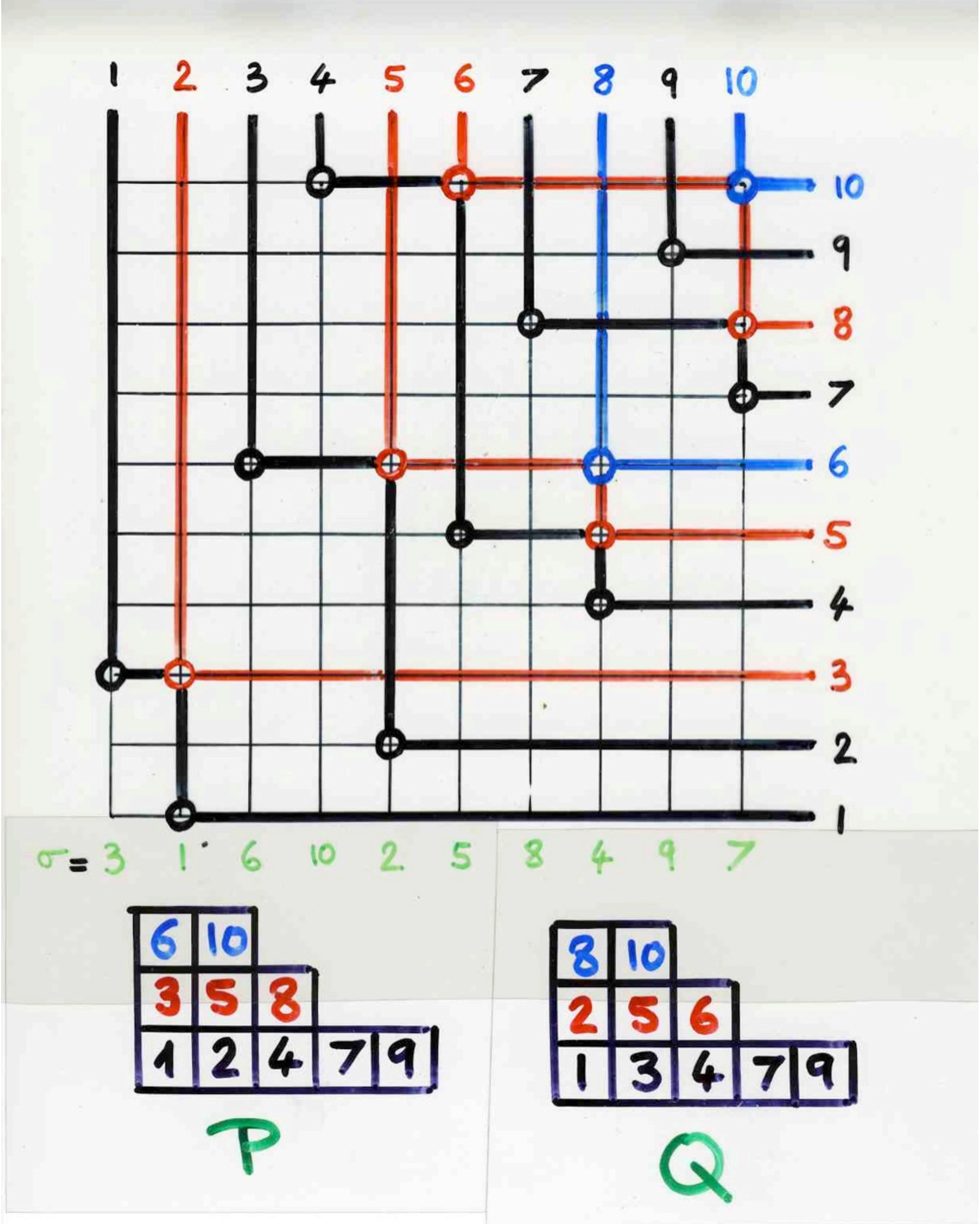


$\sigma = 3\ 1\ 6\ 10\ 2\ 5\ 8\ 4\ 9\ 7$


1 2 3 4 5 6 7 8 9 10
10
9
8
7
6
5
4
3
2
1
σ = 3 1 6 10 2 5 8 4 9 7


1 2 3 4 5 6 7 8 9 10
10
9
8
7
6
5
4
3
2
1

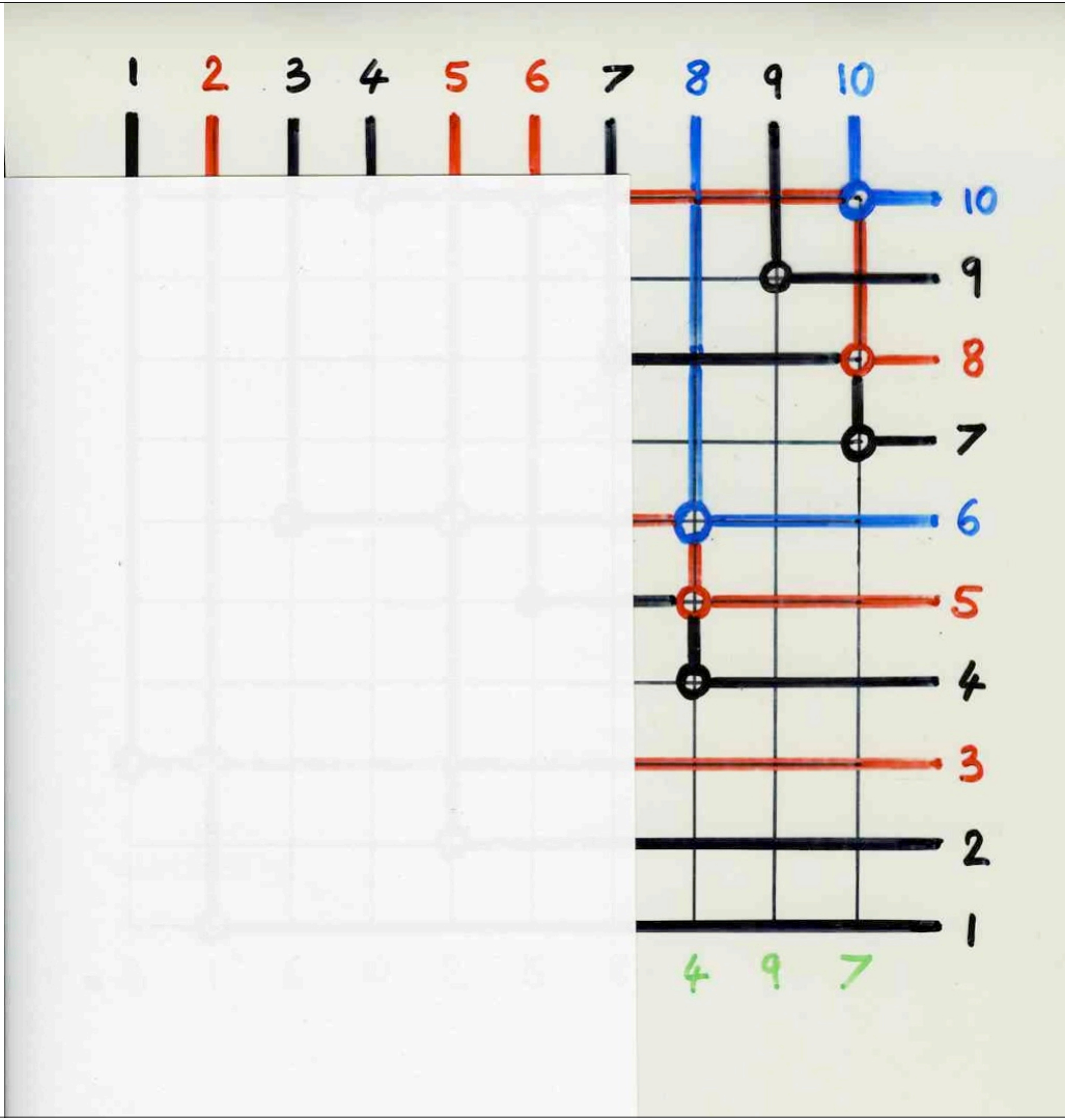

1 2 3 4 5 6 7 8 9 10
10
9
8
7
6
5
4
3
2
1
8 10
2 5 6
1 3 4 7 9
Q
6 10
3 5 8
1 2 4 7 9
P



1 2 3 4 5 6 7 8 9 10
10
9
8
7
6
5
4
3
2
1
σ = 3 1 6 10 2 5 8 4 9 7
6 10
3 5 8
1 2 4 7 9
P
8 10
2 5 6
1 3 4 7 9
Q

$$\sigma \longleftrightarrow (P, Q)$$

$$\sigma^{-1} \longleftrightarrow (Q, P)$$



1 2 3 4 5 6 7 8 9 10
10
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4
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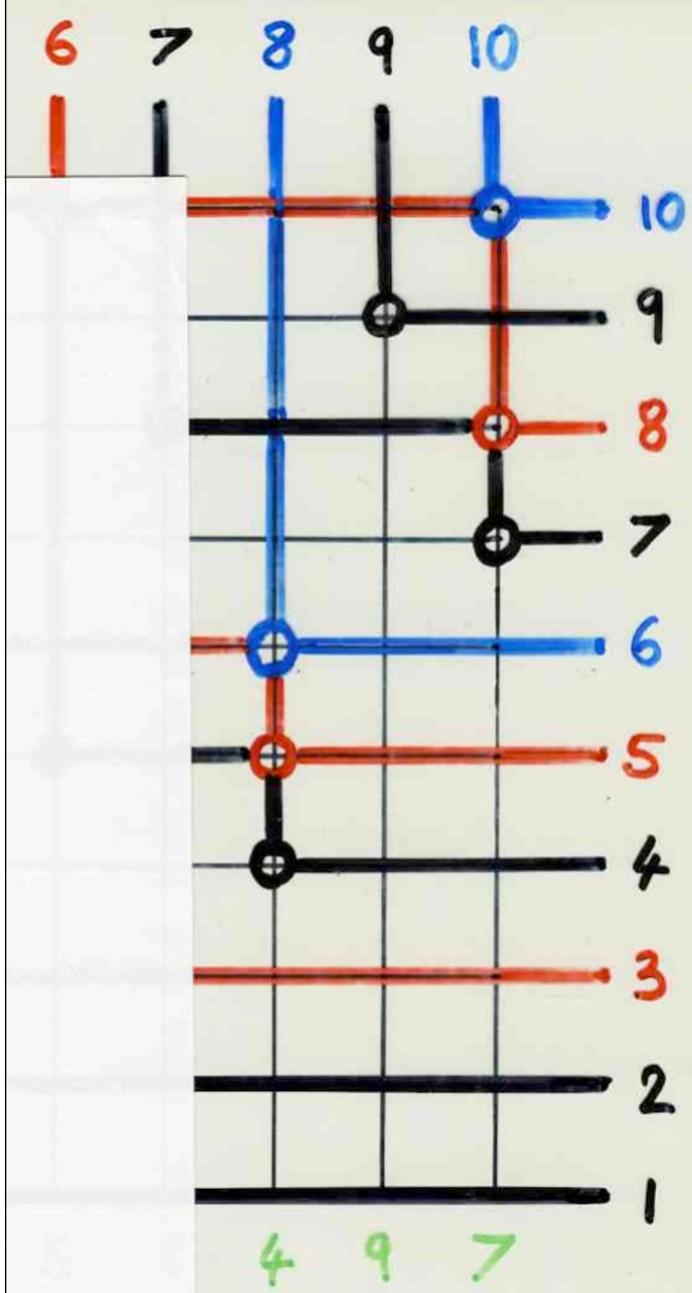
2
1
4 9 7



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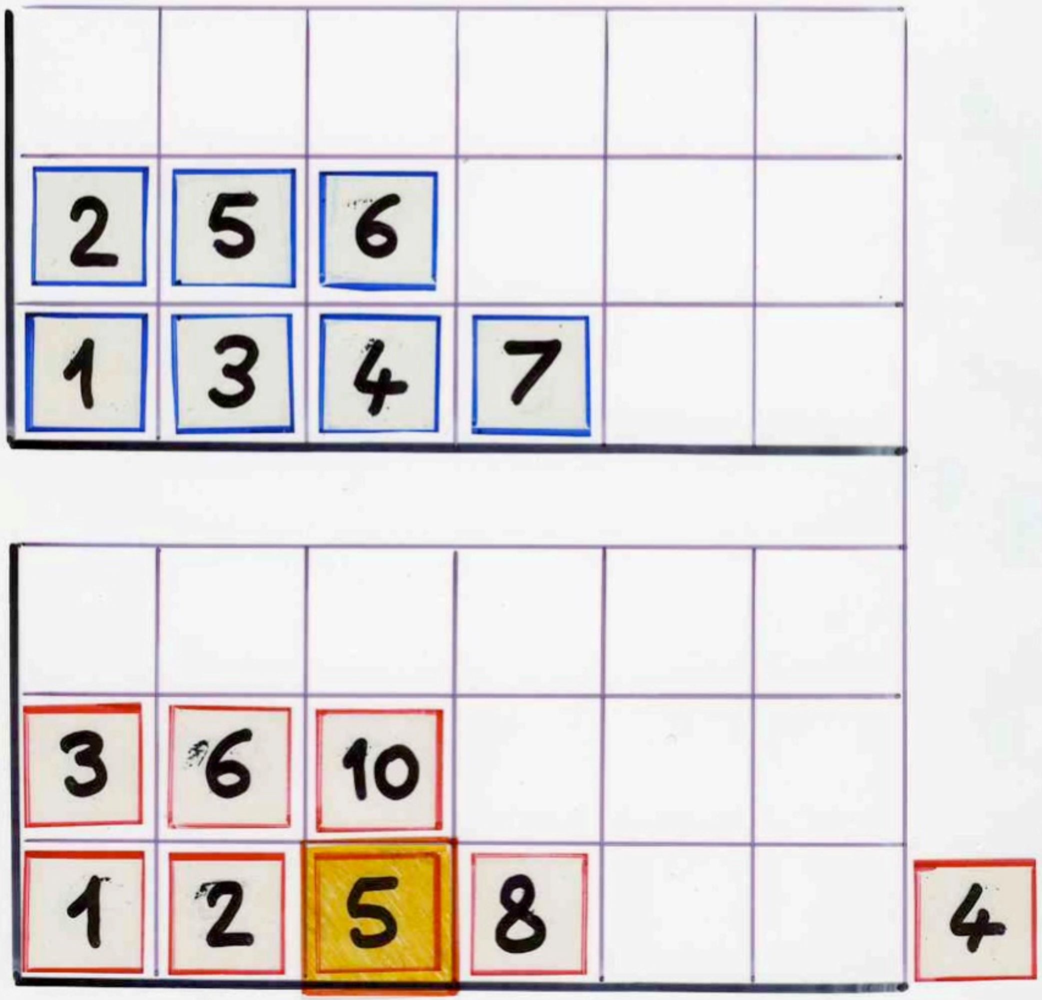
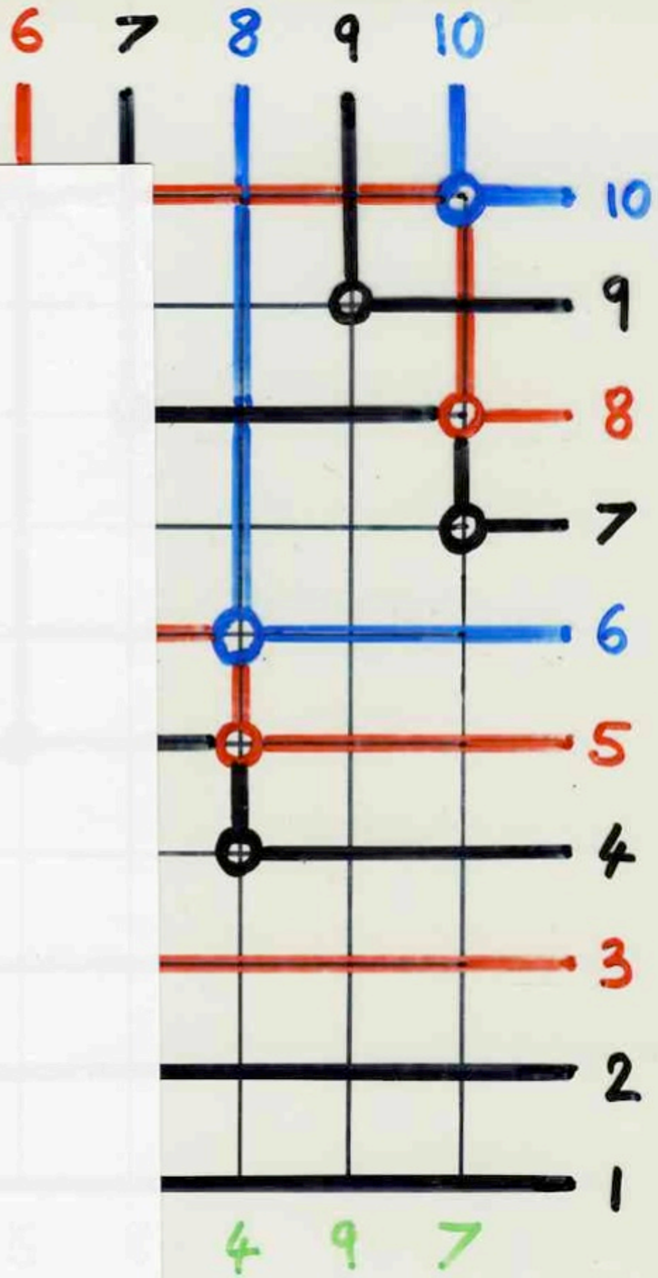
3	1	6	10	2	5	8	4	9	7





1	2	3	4	5	6	7	8	9	10
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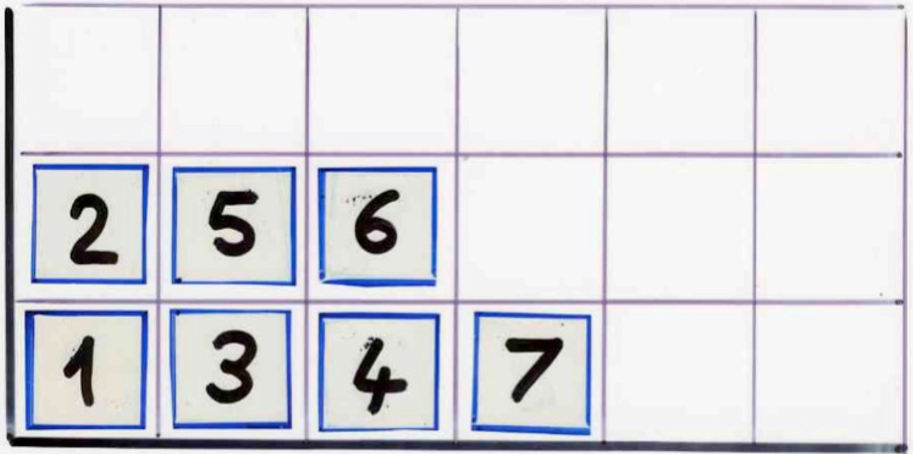

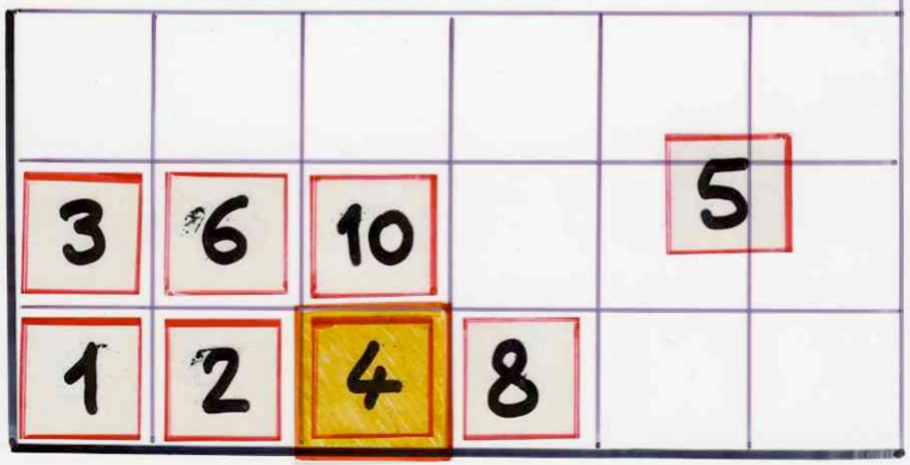

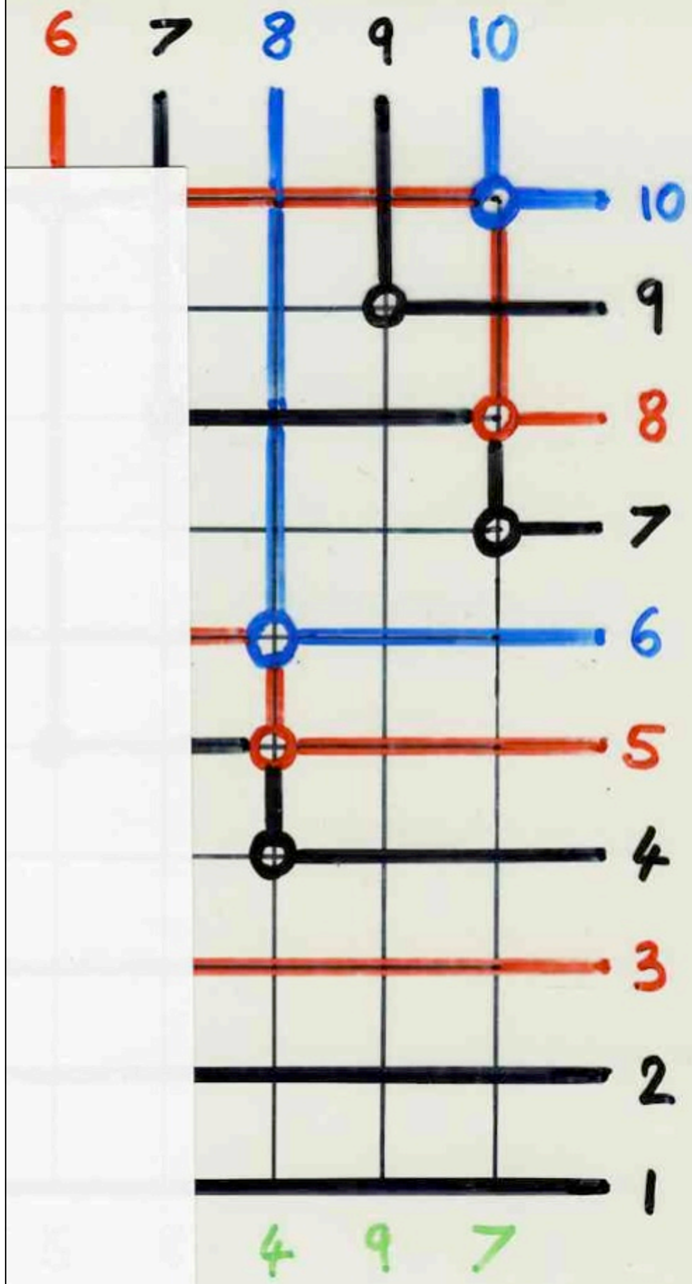

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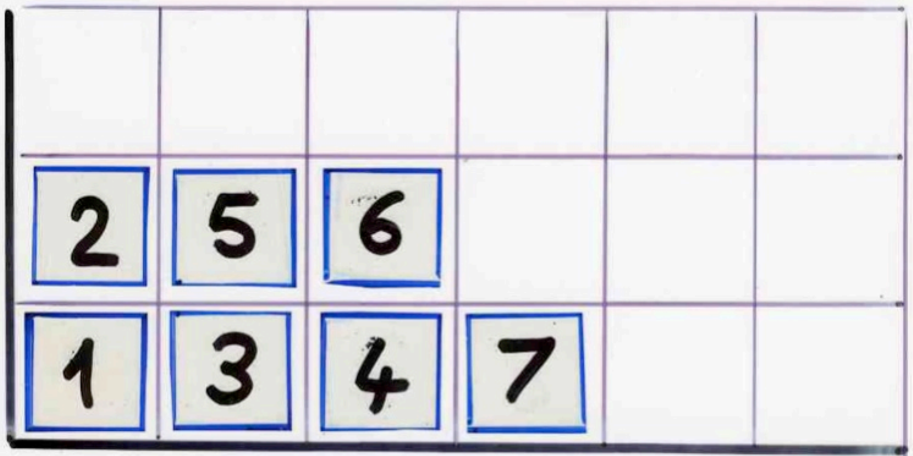
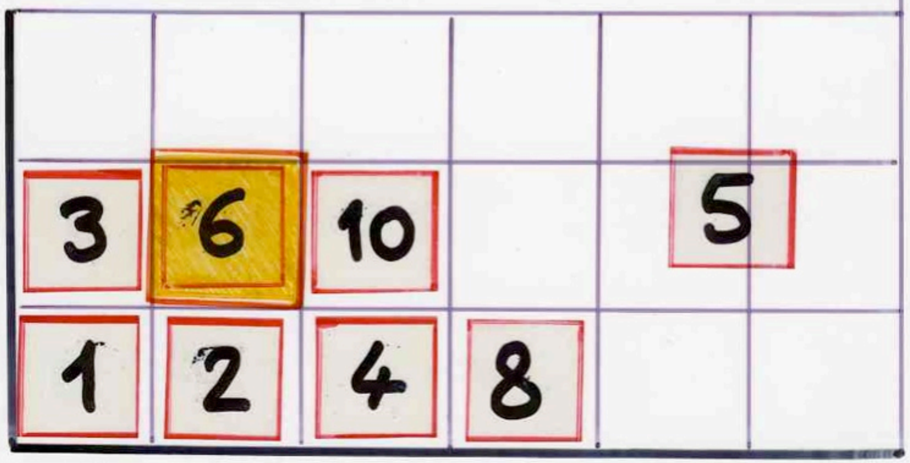
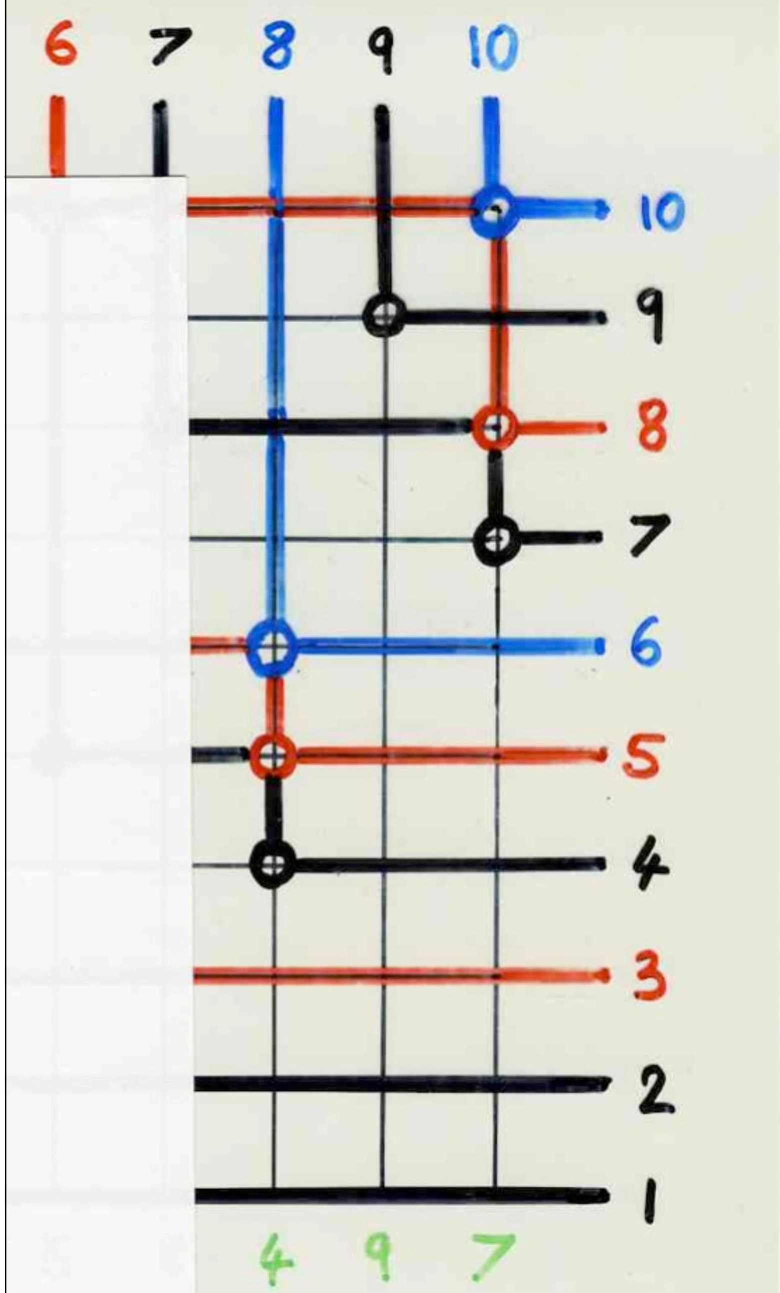
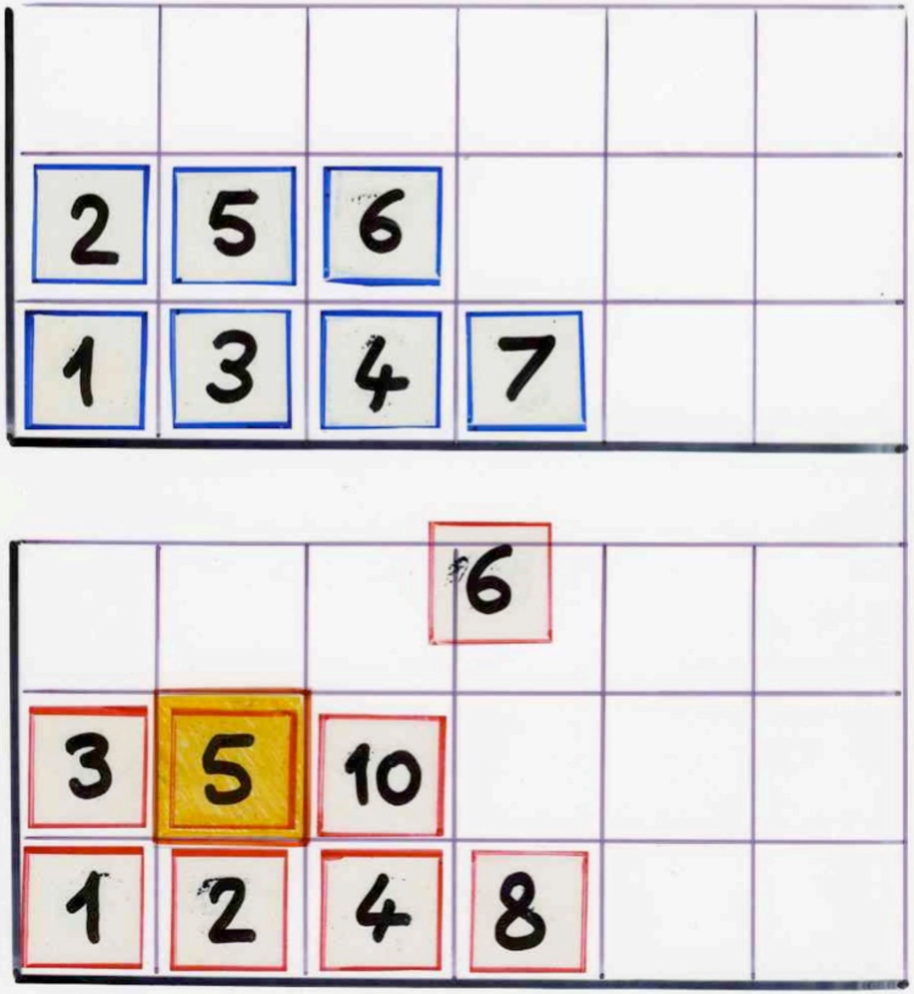
3	1	6	10	2	5	8	4	9	7

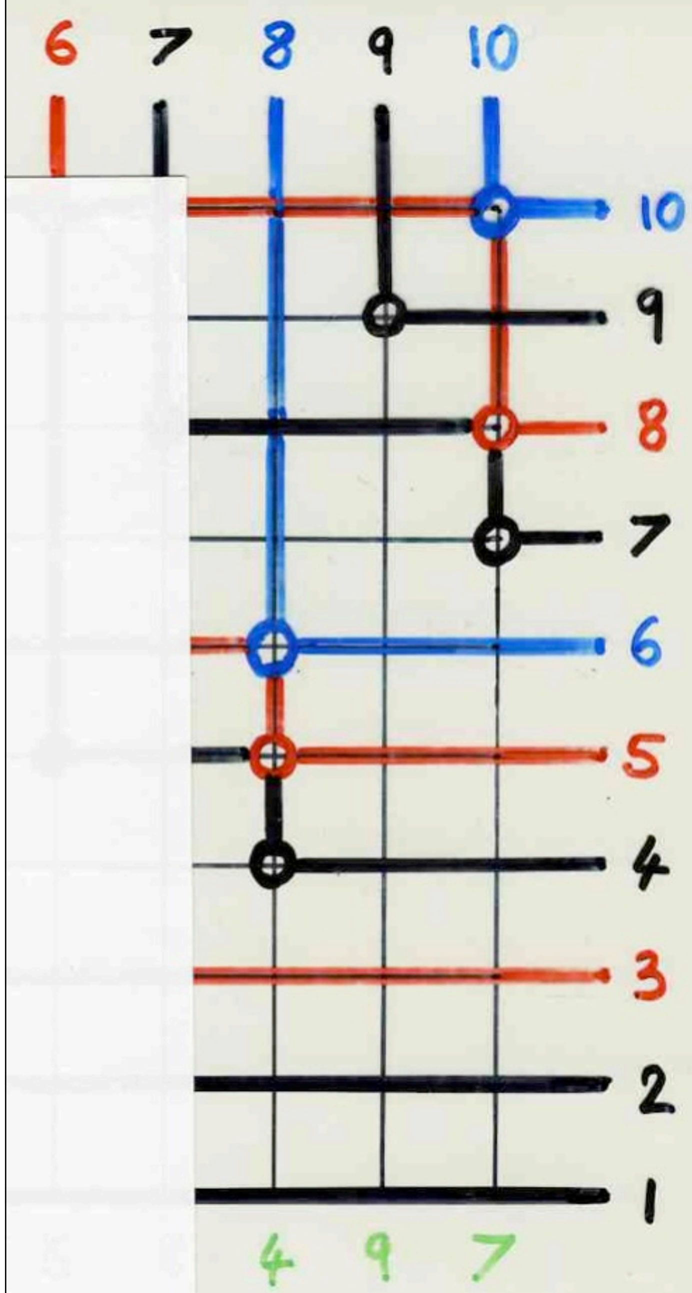






1	2	3	4	5	6	7	8	9	10
3	1	6	10	2	5	8	4	9	7





1	2	3	4	5	6	7	8	9	10
3	1	6	10	2	5	8	4	9	7

8					
2	5	6			
1	3	4	7		

6					
3	5	10			
1	2	4	8		

Young tableaux

An introduction to RSK

RSK with Schensted's insertions

Jeu de taquin

Geometric version of RSK

Cellular Ansatz 1

Cellular Ansatz 2

gâteau