



“Des arbres dans les étoiles,
des arbres dans les grains de lumière”

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avec

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mardi 17 juin 2014

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Association Cont'Science

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Université de Vienne

Arbres de la nature,
des arbres partout

Bäume in der Natur,
Bäume überall

















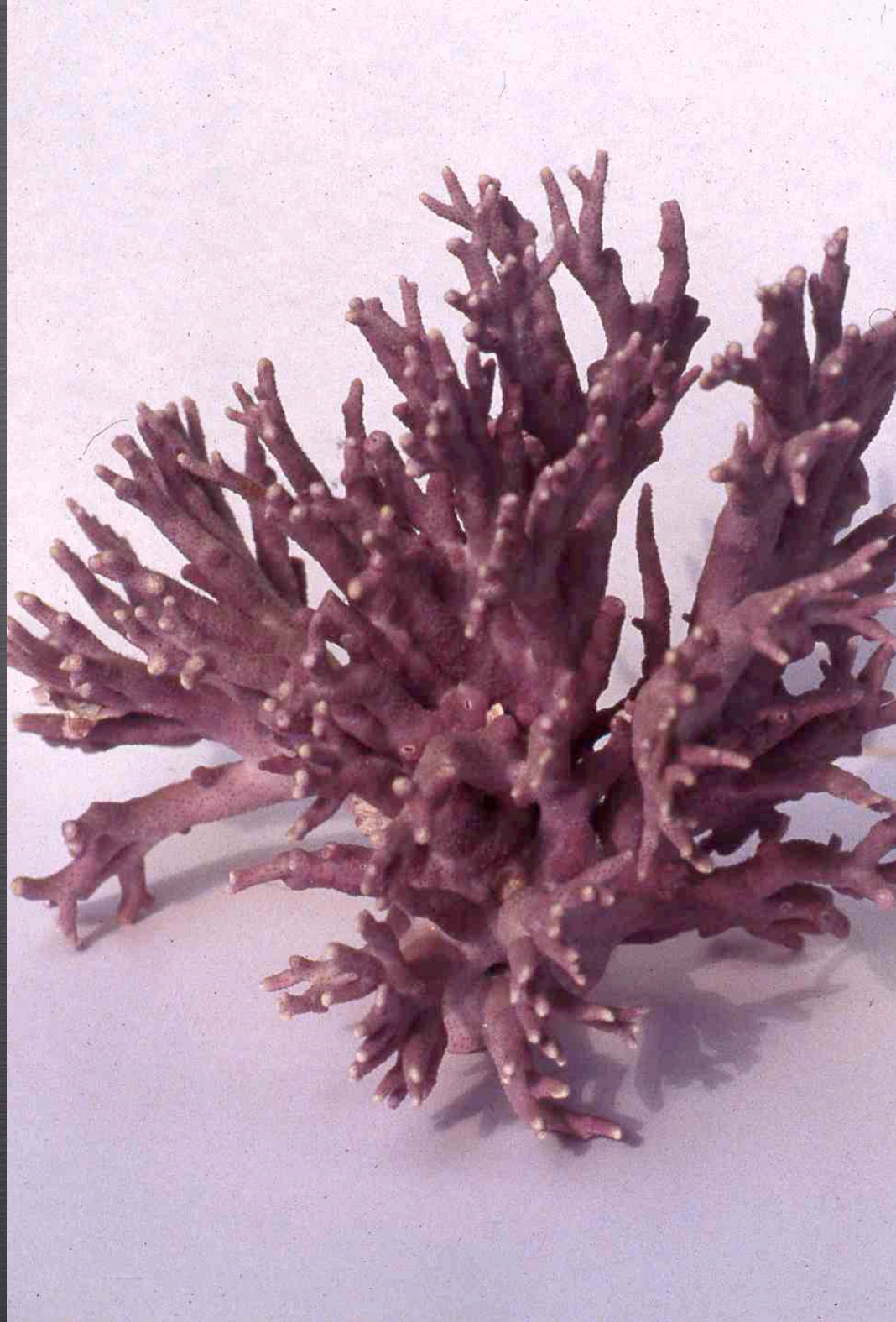






KORALLE

CORAIL





DECHARGE ELECTRIQUE

ELEKTRISCHE ENTLADUNG



VINCENT FLEURY

DEPOTS ELECTROLYTIQUES



ELEKTROLYTISCHE ABSCHIEDUNGEN

DIGITATION VISQUEUSE

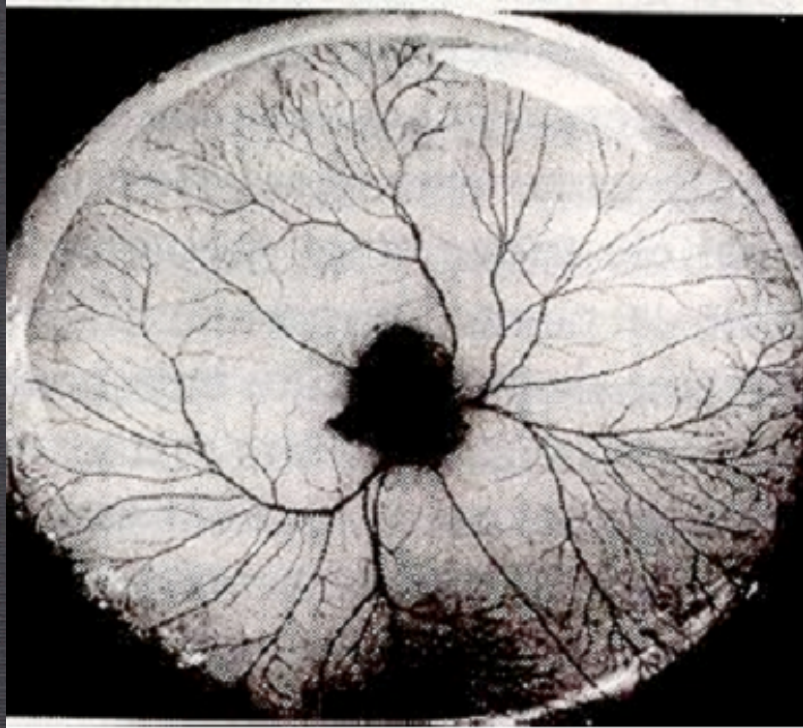
VISKOSE FLÜSSIGKEITSFORMEN

INJECTION D'HUILE ENTRE
DEUX PLAQUES

ÖL ZWISCHEN ZWEI PLATTEN EINGESPRITZT

POUMON

OEUF



Ei

LUNGE

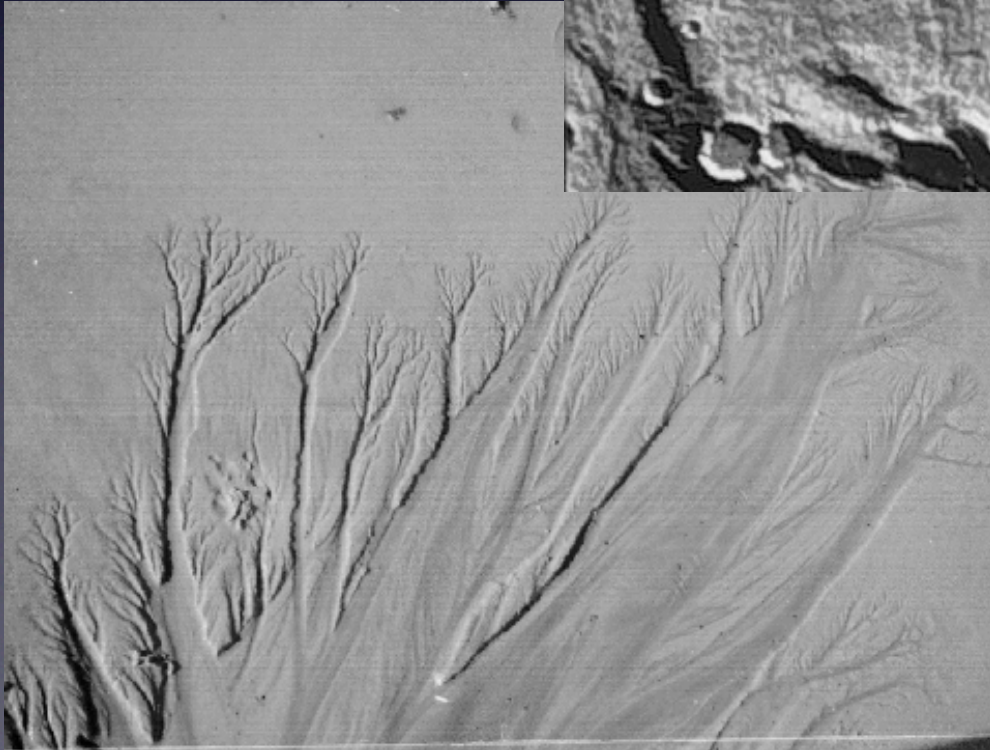
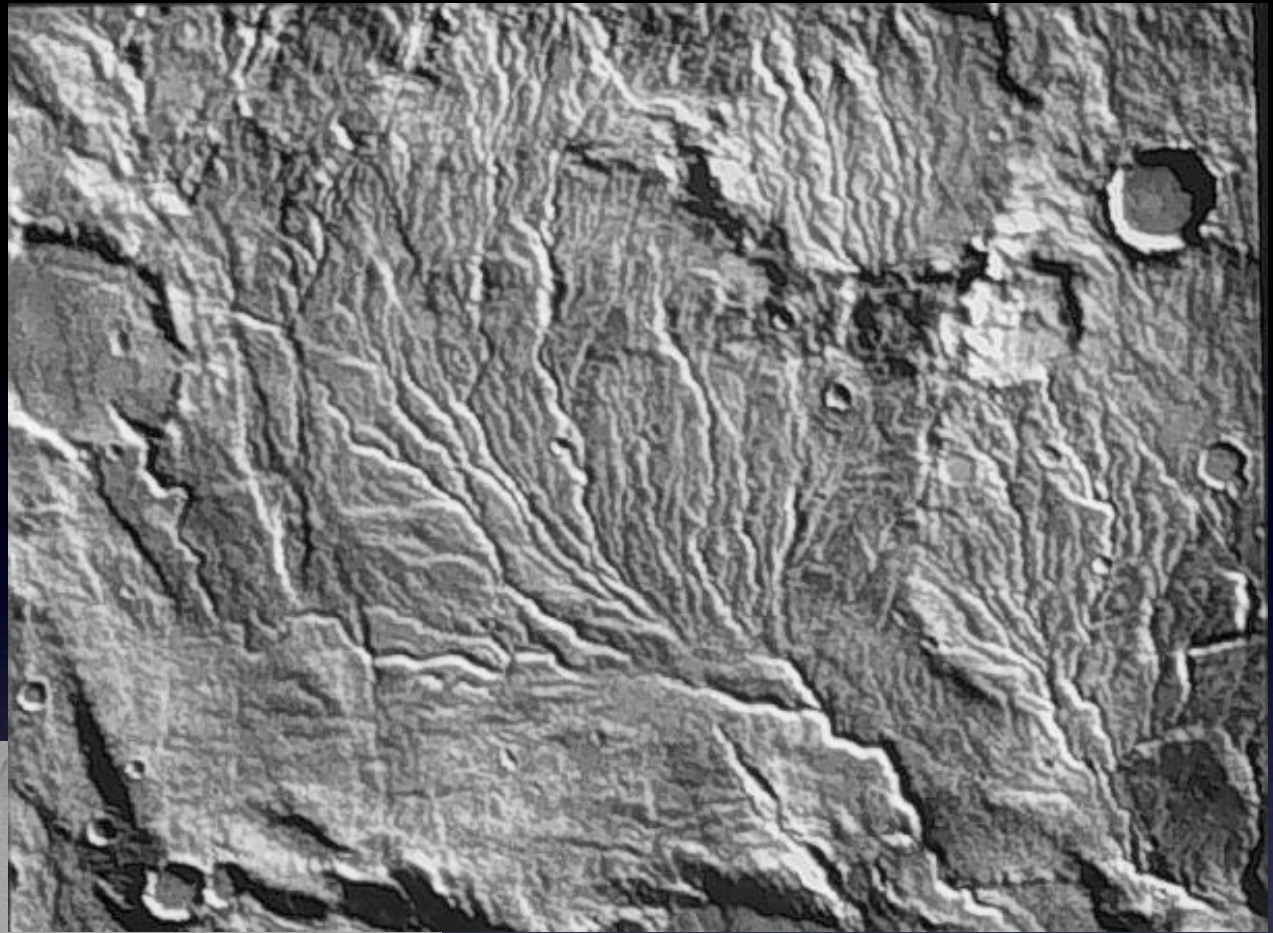




NATIONAL GEOGRAPHIC

SUR MARS

AUF DEM MARS



**AUF DER ERDE –
AUF EINEM STRAND**

SUR TERRE

SUR UNE PLAGE



BÄUME,
BAUMSTRUKTUREN
ÜBERALL ...

DES ARBRES
DES ARBORESCENCES PARTOUT



BÄUME,
BAUMSTRUKTUREN
ÜBERALL ...

DES ARBRES
DES ARBORESCENCES PARTOUT

BERNARD CLAVEL

“La beauté
mathématiques
des arbres ”



Des arbres dans les étoiles ?



L'infiniment grand ...

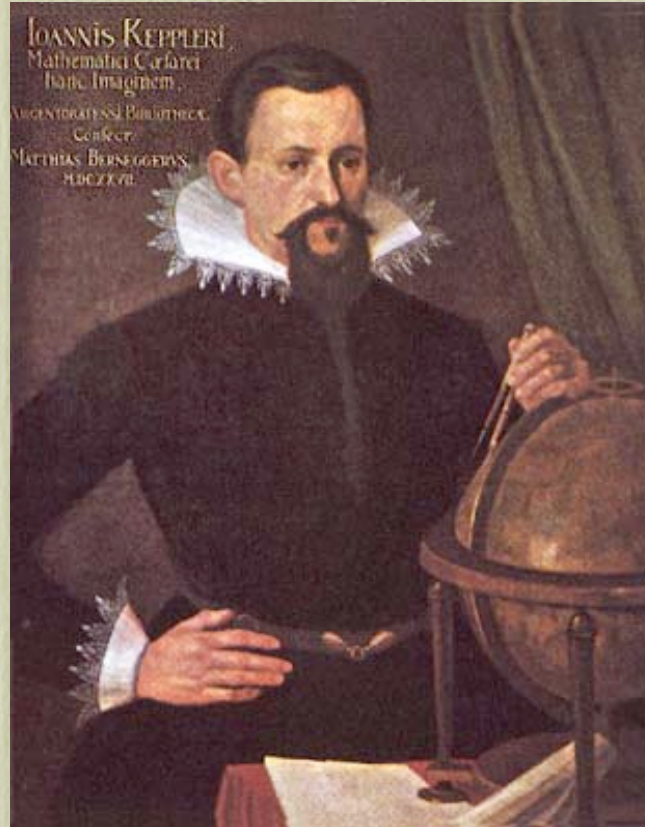
Les étoiles, les planètes, les galaxies,
l'univers, sa naissance et son histoire,
espace, temps, matière

comprendre l'univers avec les mathématiques



Galileo Galilei
1564-1642

géométrie
classique



Johannes Kepler
1571 - 1630

la géométrie euclidienne



Isaac Newton
1643-1727

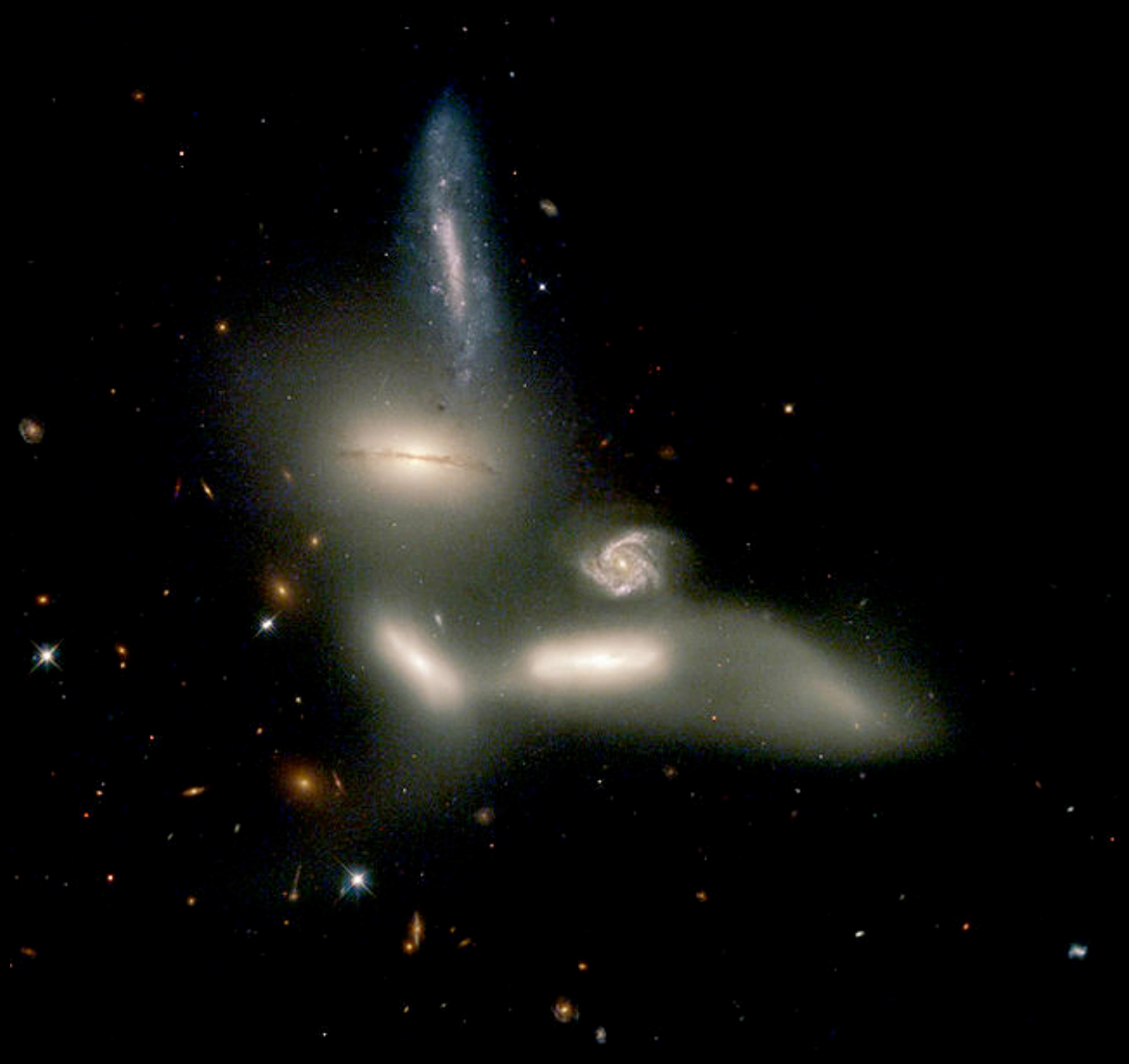
mécanique
classique



Albert Einstein
1879-1955

Théorie de la relativité restreinte générale

gravitation





des arbres dans les
grains de lumière ?





collégiale Notre-Dame Vernon



Daniel B. Holeman

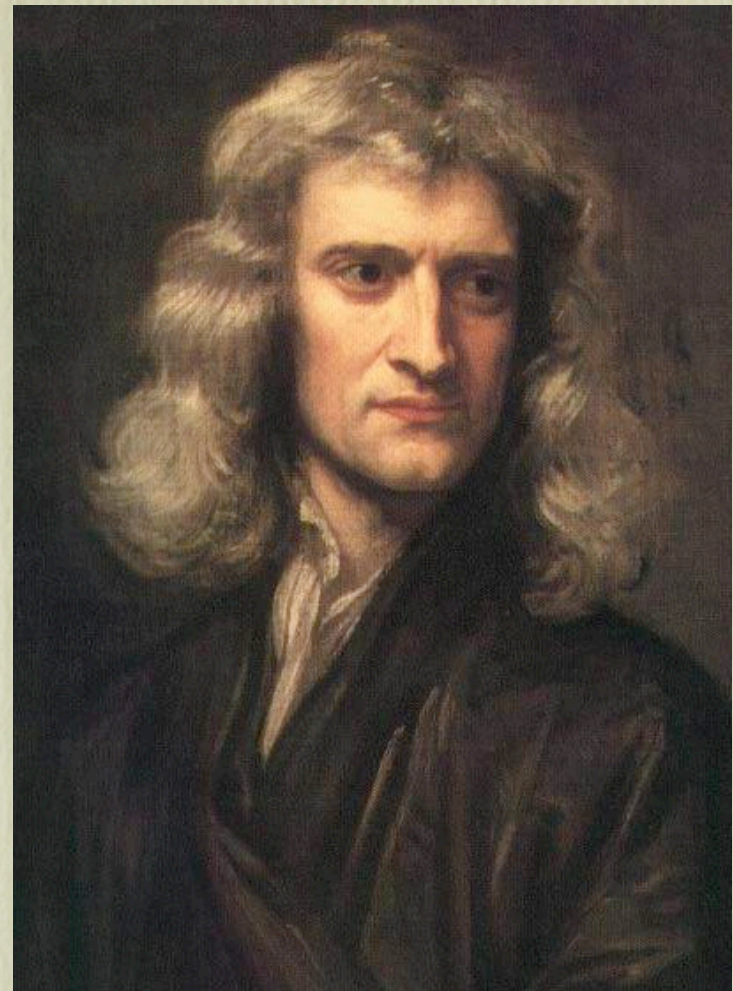
L'infiniment petit ...

les atomes, les électrons,
les particules de matière, de lumière,
les photons, ...





Christian Huygens
1629-1695

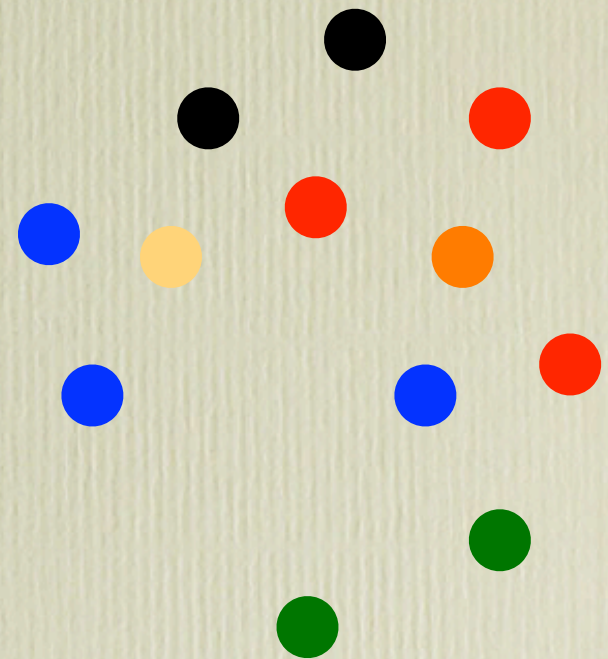


Isaac Newton
1643-1727

la lumière:

vibration ?

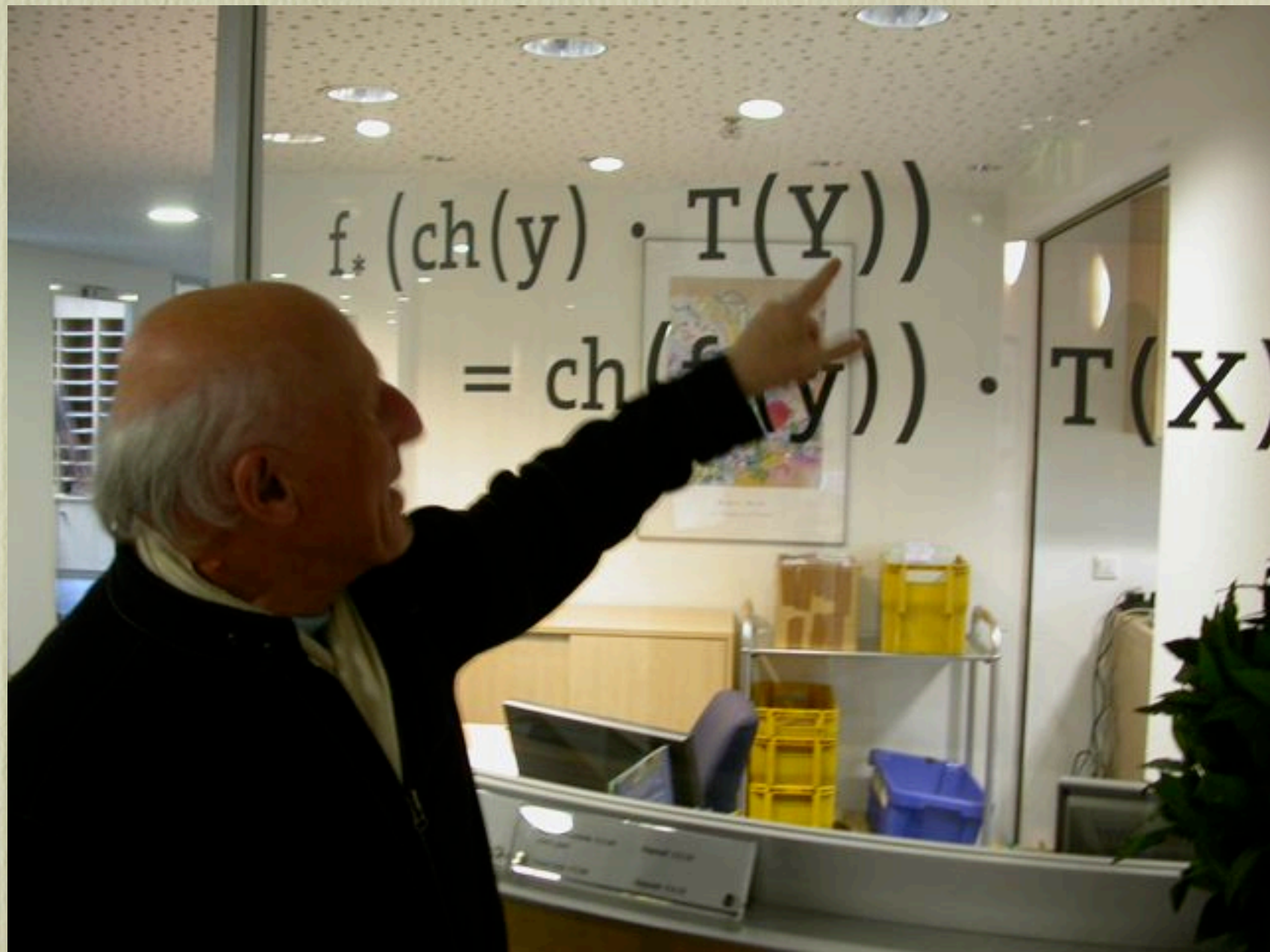
ou particules de matière ?





A photograph of a dense forest. The image is filled with numerous thin, vertical tree trunks, likely birches or aspens, which are light brown and have a slightly textured bark. Some trunks are straight and vertical, while others are slightly leaning. In the foreground, there are several fallen branches and some green foliage, possibly grass or small plants, which are slightly out of focus. The background is a dense canopy of green leaves, suggesting a lush forest environment. The lighting is natural, with some areas being brighter than others, creating a sense of depth and texture.

Si vous êtes perdus dans la forêt
des mathématiques, détendez
vous et regardez les images



regardez une formule mathématique
comme une oeuvre d'art abstrait

Rogers - Ramanujan identities

$$R_I \quad \sum_{n \geq 0} \frac{q^{n^2}}{(1-q)(1-q^2)\dots(1-q^n)} = \prod_{\substack{i \equiv 1, 4 \\ \text{mod } 5}} \frac{1}{(1-q^i)}$$

$$R_{II} \quad \sum_{n \geq 0} \frac{q^{n^2 + n}}{(1-q)(1-q^2)\dots(1-q^n)} = \prod_{\substack{i \equiv 2, 3 \\ \text{mod } 5}} \frac{1}{(1-q^i)}$$

Srinivasan
Ramanujan
(1887-1920)



Le langage des mathématiques, c'est comme le solfège utilisé pour écrire de la musique

Habituellement, à l'école vous apprenez seulement comment écrire des mathématiques, mais il est difficile d'entendre la beauté des mathématiques ...

mais les mathématiques sont la musique !

Subsublemma 1.1.3:

$$\sum_{\pi \in S_k} \text{sgn}(\pi) \cdot \pi \left[\frac{x_1 x_2^2 \dots x_k^k}{(1-x_k)(1-x_k x_{k-1}) \dots (1-x_k x_{k-1} \dots x_1)} \right] = \frac{x_1 \dots x_k \prod_{1 \leq i < j \leq k} (x_j - x_i)}{\prod_{i=1}^k (1-x_i) \prod_{1 \leq i < j \leq k} (1-x_i x_j)} \quad (\text{Issai})$$

[Type 'S113(k)'; in ROBBINS, for specific k.]

Proof : See [PS], problem VII.47. Alternatively, (Issai) is easily seen to be equivalent to Schur's identity that sums all the Schur functions ([Ma], ex I.5.4, p. 45). This takes care of subsublemma 1.1.3. $\square$

Inserting (Issai) into (Stanley), expanding $\prod_{1 \leq i < j \leq k} (x_j - x_i)$ by Vandermonde's expansion,

$$\sum_{\pi \in S_k} \text{sgn}(\pi) \cdot \pi(x_1^0 x_2^1 \dots x_k^{k-1}) ,$$

using the antisymmetry of Δ_k once again, and employing crucial fact $\aleph_4$, we get the following string of equalities:

$$\begin{aligned} b_k(n) &= \frac{1}{k!} CT_{x_1, \dots, x_k} \left\{ \frac{\Delta_k(x_1, \dots, x_k)}{\prod_{i=1}^k (\bar{x}_i)^{k+n} x_i^{n+k-1}} \left(\frac{x_1 \dots x_k \prod_{1 \leq i < j \leq k} (x_j - x_i)}{\prod_{i=1}^k (1-x_i) \prod_{1 \leq i < j \leq k} (1-x_i x_j)} \right) \right\} \\ &= \frac{1}{k!} CT_{x_1, \dots, x_k} \left\{ \frac{\Delta_k(x_1, \dots, x_k)}{\prod_{i=1}^k (\bar{x}_i)^{k+n+1} x_i^{n+k-2} \prod_{1 \leq i < j \leq k} (1-x_i x_j)} \left(\sum_{\pi \in S_k} \text{sgn}(\pi) \cdot \pi(x_1^0 x_2^1 \dots x_k^{k-1}) \right) \right\} \\ &= \frac{1}{k!} \sum_{\pi \in S_k} CT_{x_1, \dots, x_k} \left\{ \pi \left[\frac{\Delta_k(x_1, \dots, x_k)}{\prod_{i=1}^k (\bar{x}_i)^{k+n+1} x_i^{n+k-2} \prod_{1 \leq i < j \leq k} (1-x_i x_j)} \left(\prod_{i=1}^k x_i^{i-1} \right) \right] \right\} \\ &= \frac{1}{k!} \sum_{\pi \in S_k} CT_{x_1, \dots, x_k} \left\{ \pi \left[\frac{\Delta_k(x_1, \dots, x_k)}{\prod_{i=1}^k (\bar{x}_i)^{k+n+1} x_i^{n+k-i-1} \prod_{1 \leq i < j \leq k} (1-x_i x_j)} \right] \right\} \\ &= \frac{1}{k!} \sum_{\pi \in S_k} CT_{x_1, \dots, x_k} \left\{ \frac{\Delta_k(x_1, \dots, x_k)}{\prod_{i=1}^k (\bar{x}_i)^{k+n+1} x_i^{n+k-i-1} \prod_{1 \leq i < j \leq k} (1-x_i x_j)} \right\} \\ &= \frac{1}{k!} \left(\sum_{\pi \in S_k} 1 \right) CT_{x_1, \dots, x_k} \left\{ \frac{\Delta_k(x_1, \dots, x_k)}{\prod_{i=1}^k (\bar{x}_i)^{k+n+1} x_i^{n+k-i-1} \prod_{1 \leq i < j \leq k} (1-x_i x_j)} \right\} \\ &= CT_{x_1, \dots, x_k} \left\{ \frac{\Delta_k(x_1, \dots, x_k)}{\prod_{i=1}^k (\bar{x}_i)^{k+n+1} x_i^{n+k-i-1} \prod_{1 \leq i < j \leq k} (1-x_i x_j)} \right\}, \quad (\text{George''}) \end{aligned}$$

where in the last equality we have used Levi Ben Gerson's celebrated result that the number of elements in S_k (the symmetric group on k elements,) equals $k!$. The extreme right of (George''') is exactly the right side of (MagogTotal). This completes the proof of sublemma 1.1. $\square$



Un exemple d'objet mathématique:
les arbres binaires ou arbres mathématiques

donner une abstraction des arbres
dans le monde qui nous entoure

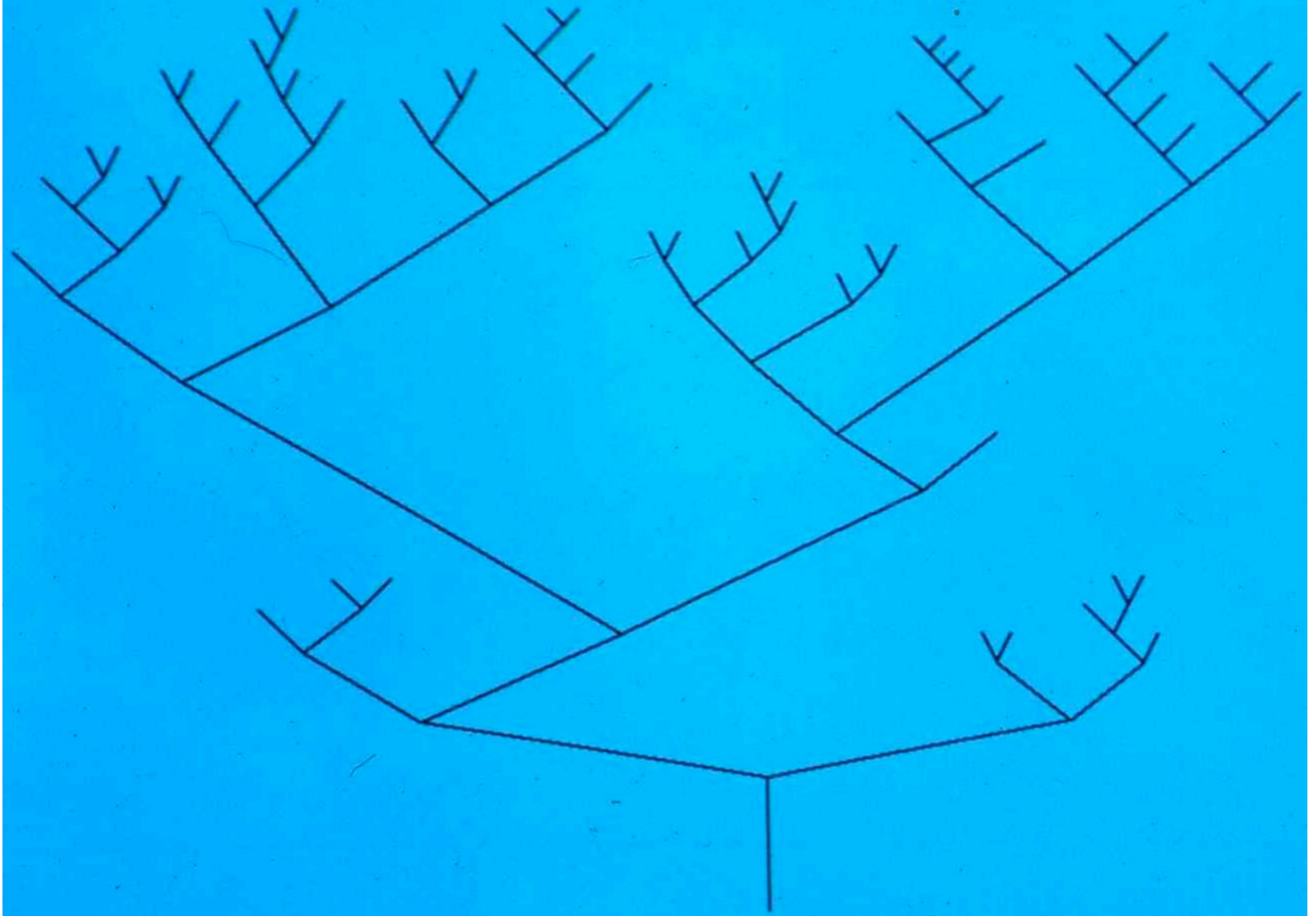
Des arbres naturels ...
aux arbres mathématiques

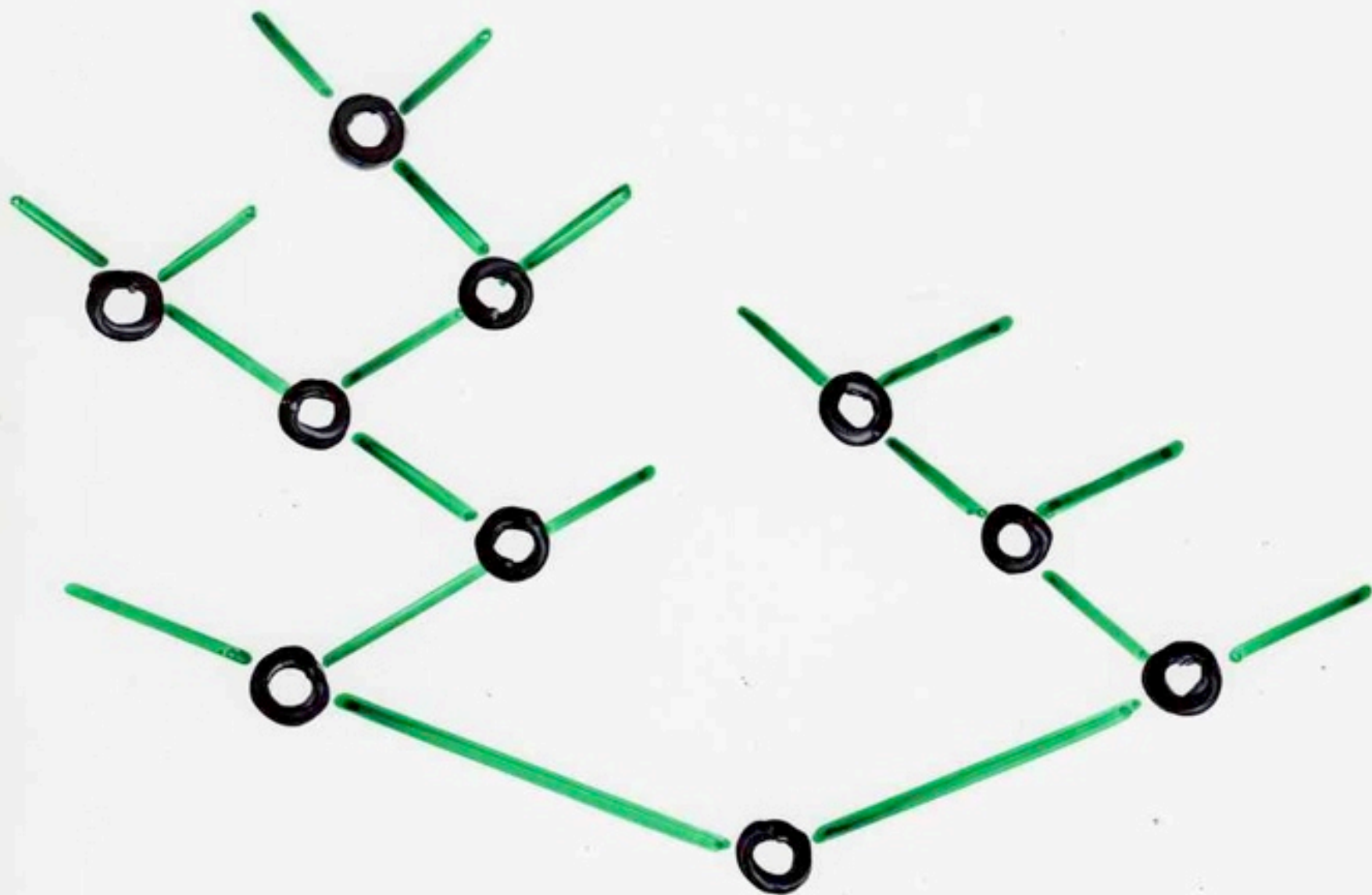






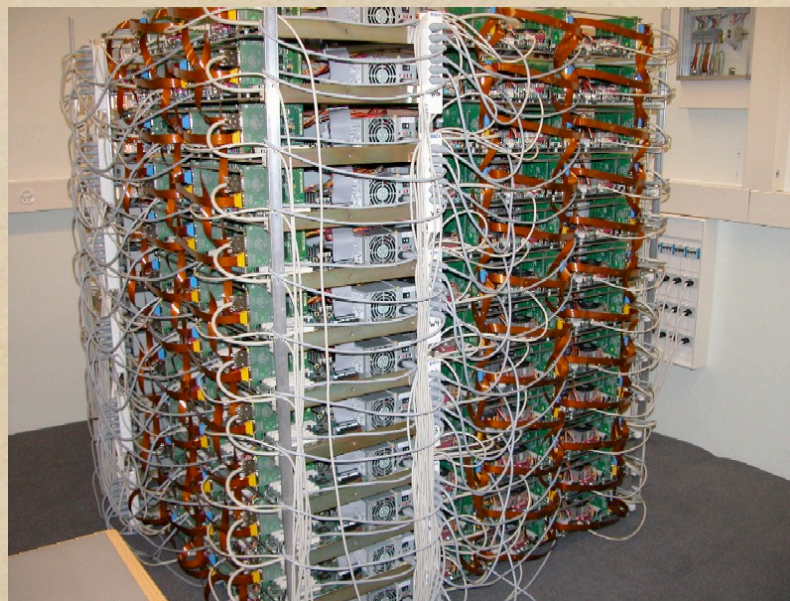






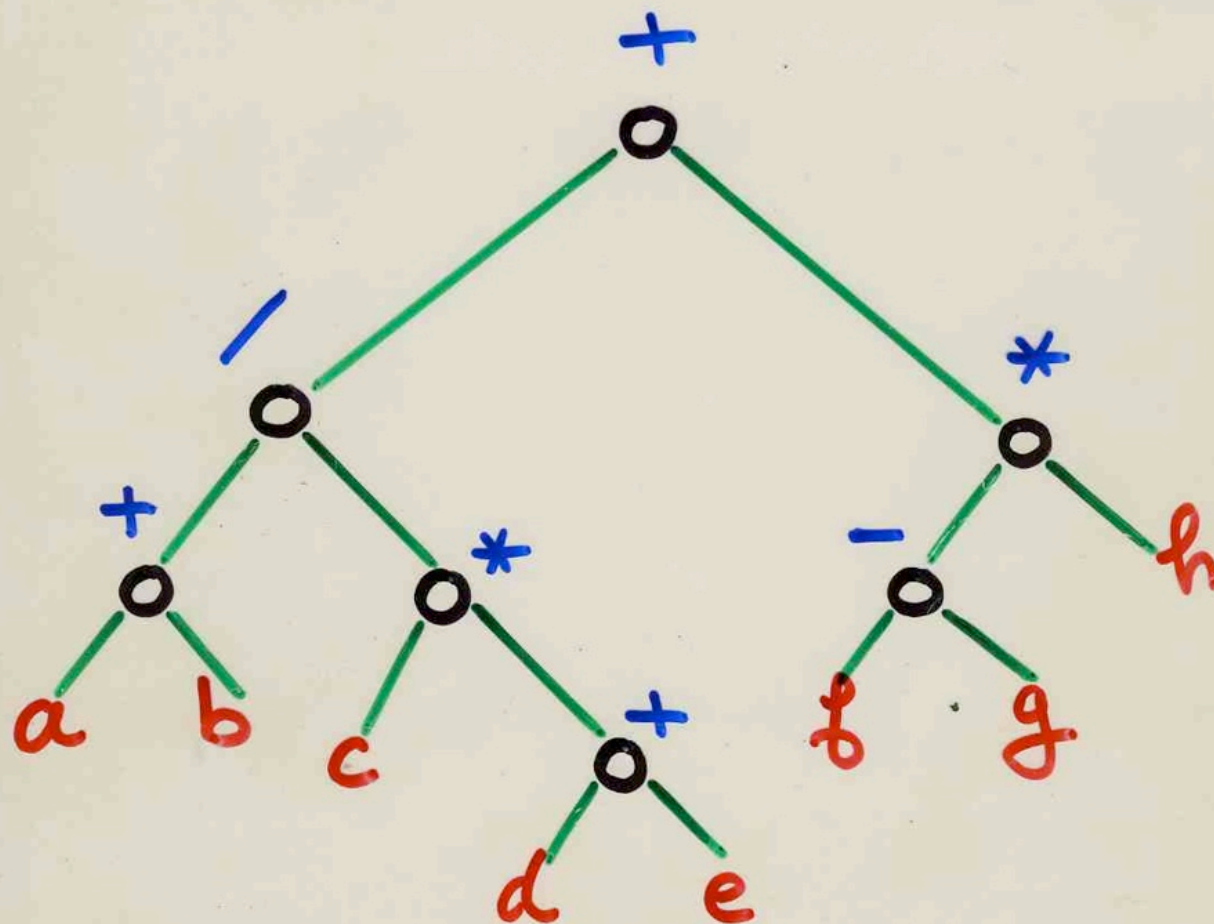
VICTOR HUGO

des arbres dans les ordinateurs ...

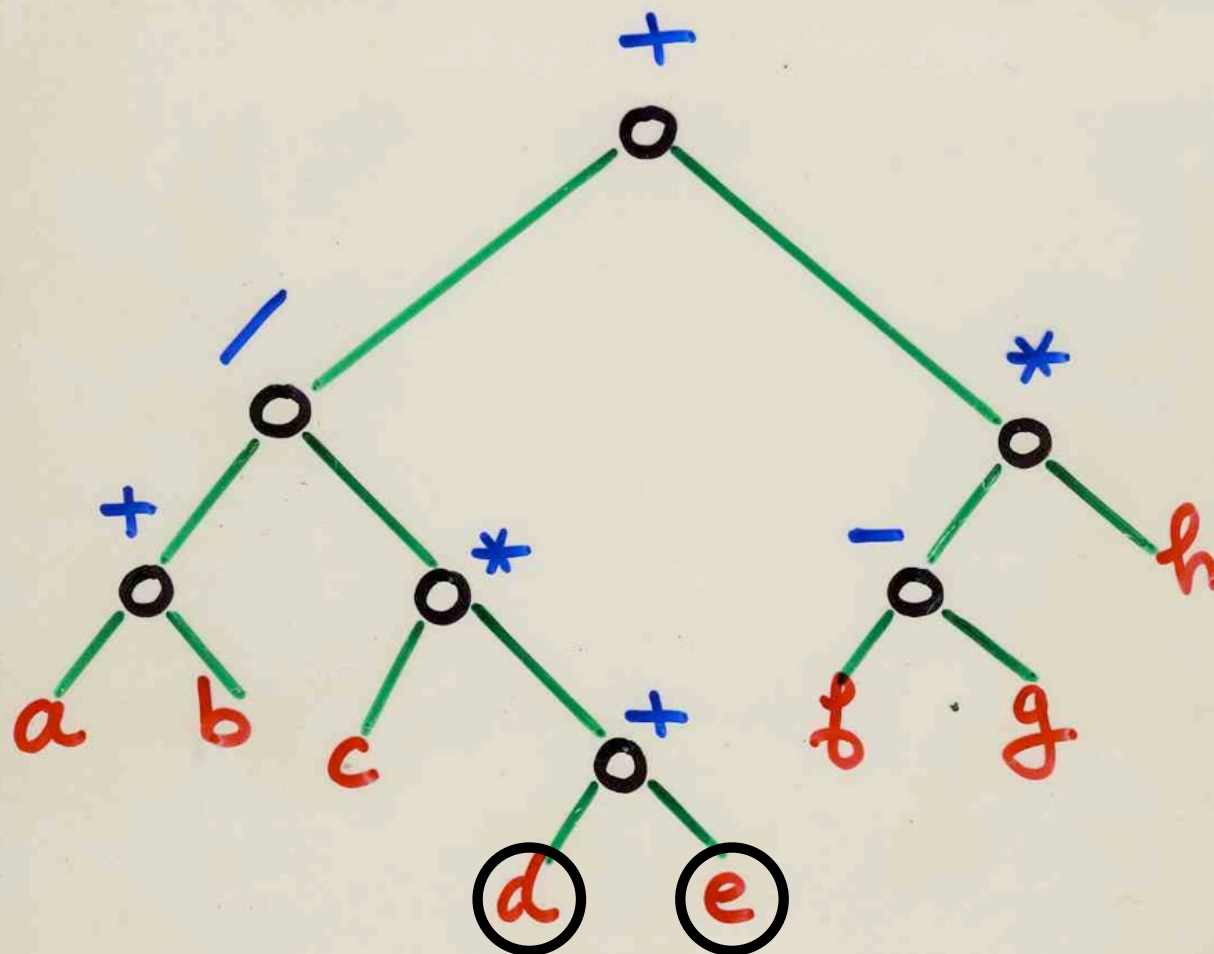


calcul d'une expression arithmétique

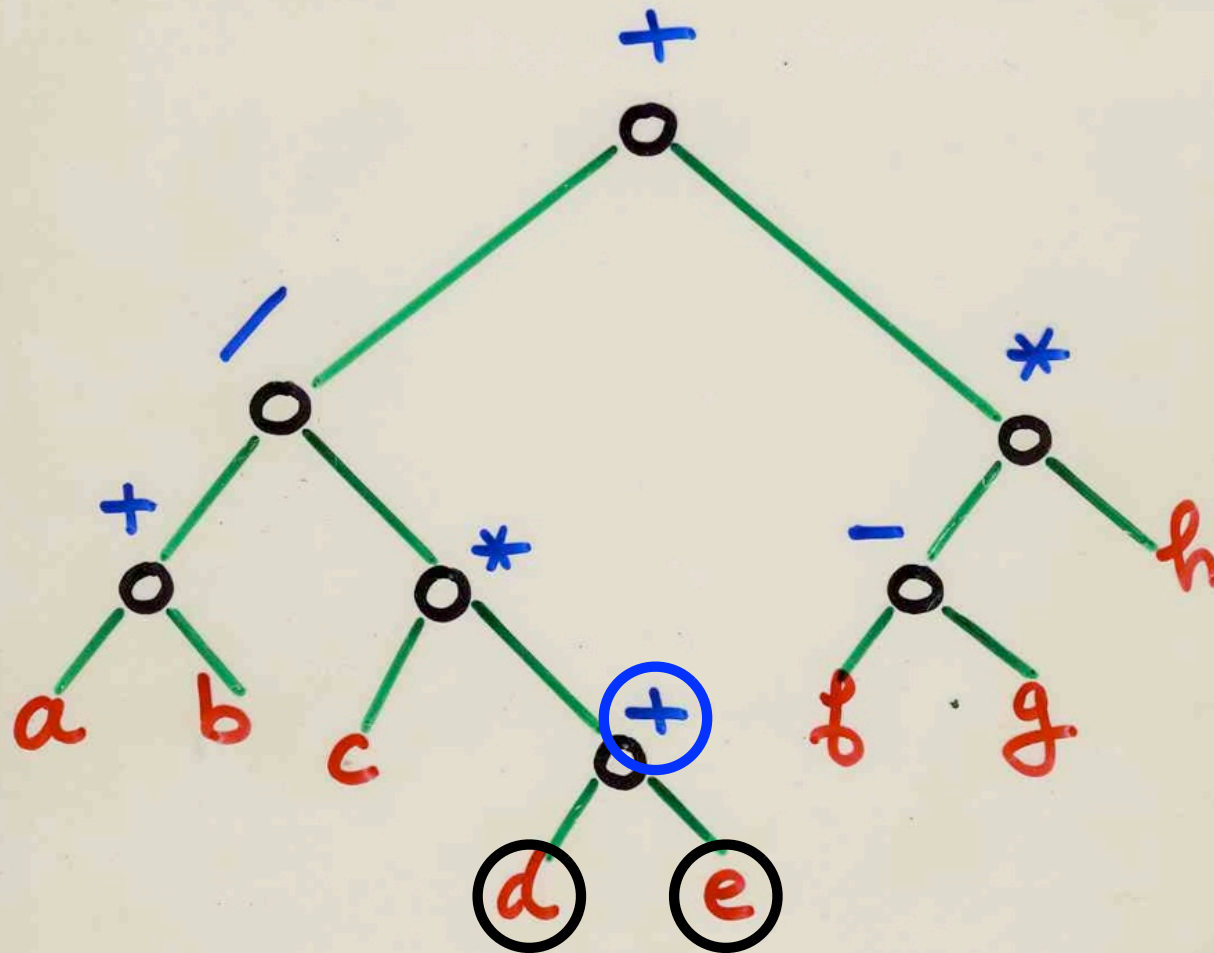
$$\frac{(a+b)}{c(d+e)} + (f-g)h$$



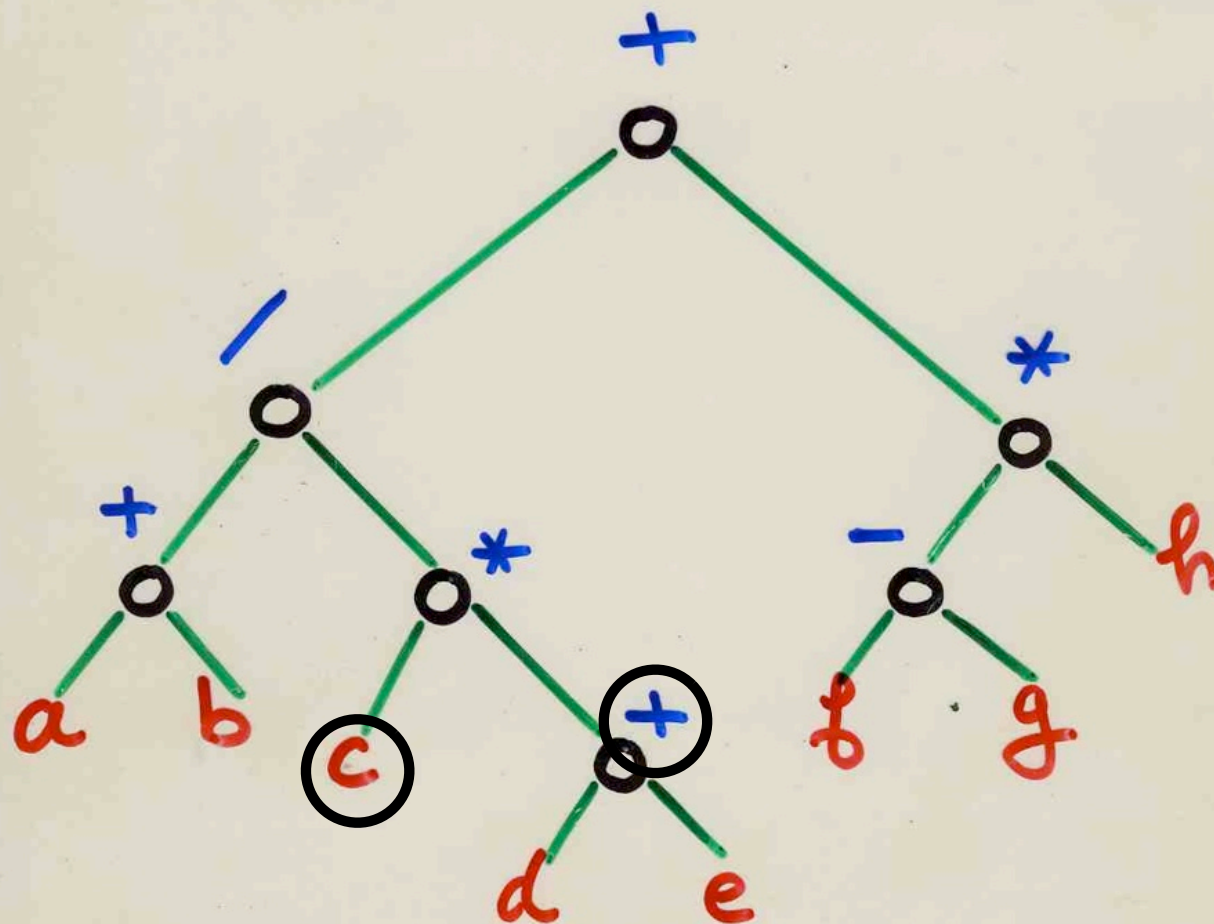
$$\frac{(a+b)}{c(d+e)} + (f-g)h$$



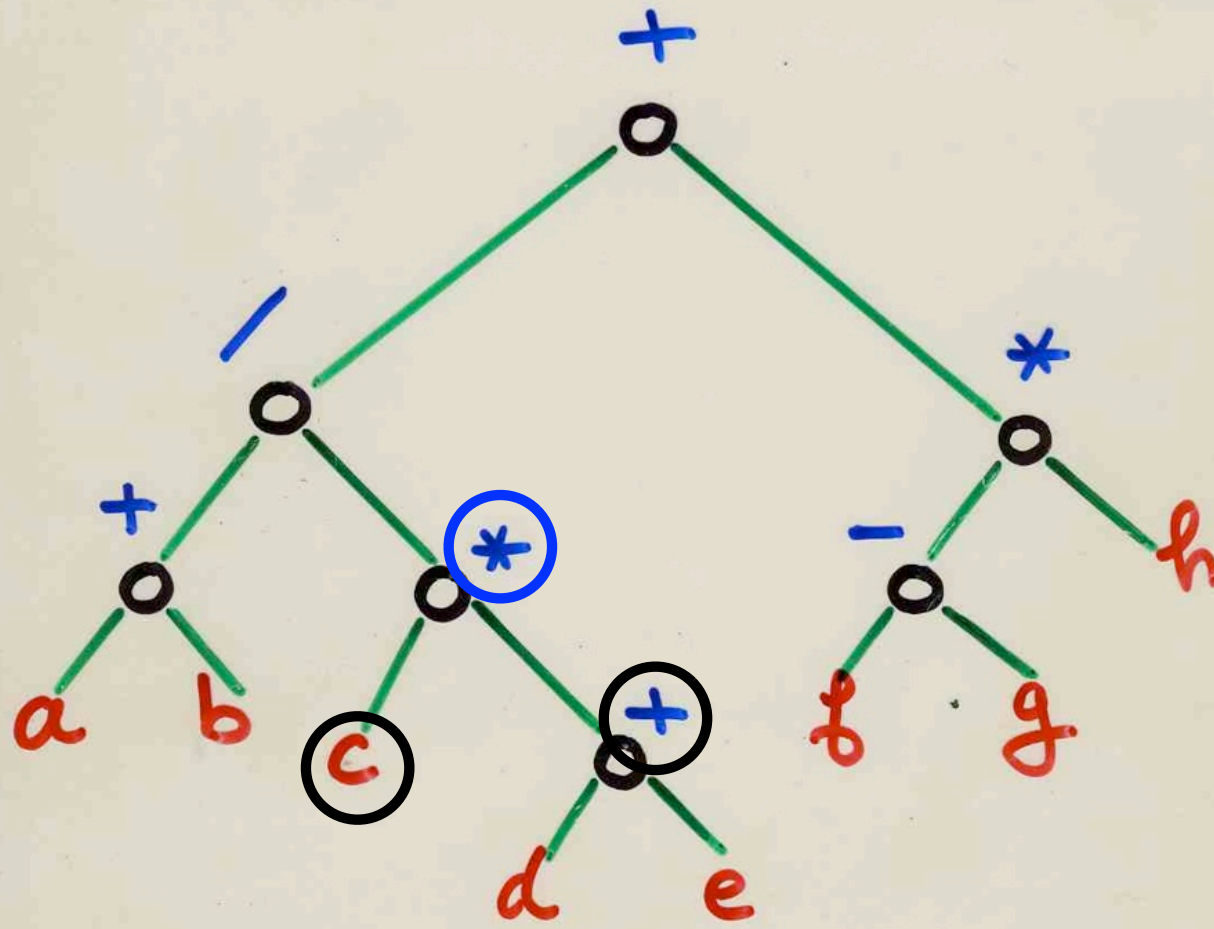
$$\frac{(a+b)}{c(d+e)} + (f-g)h$$



$$\frac{(a+b)}{c(d+e)} + (f-g)h$$



$$\frac{(a+b)}{c(d+e)} + (f-g)h$$



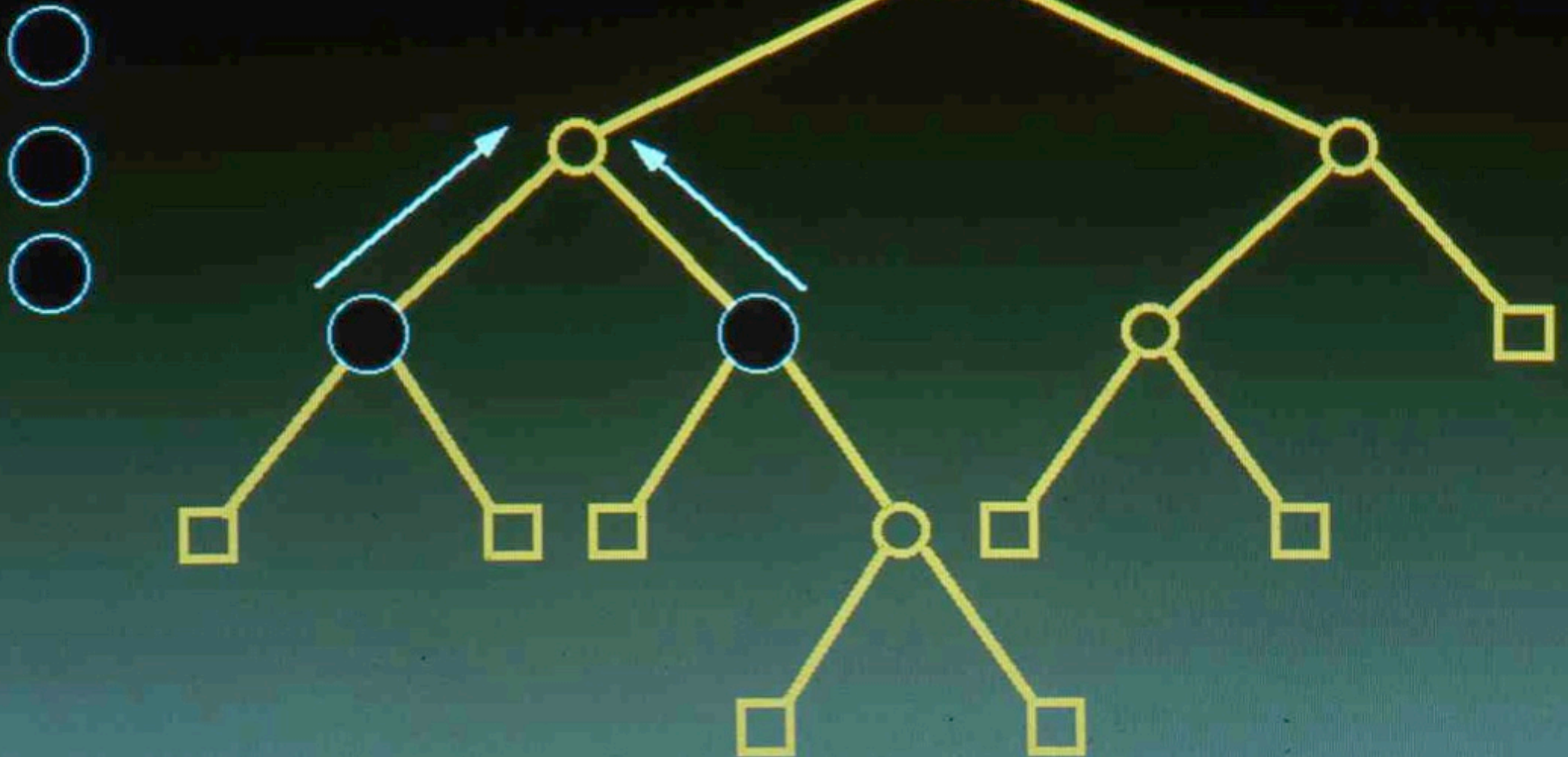
$$\frac{(a+b)}{c(d+e)} + (f-g)h$$

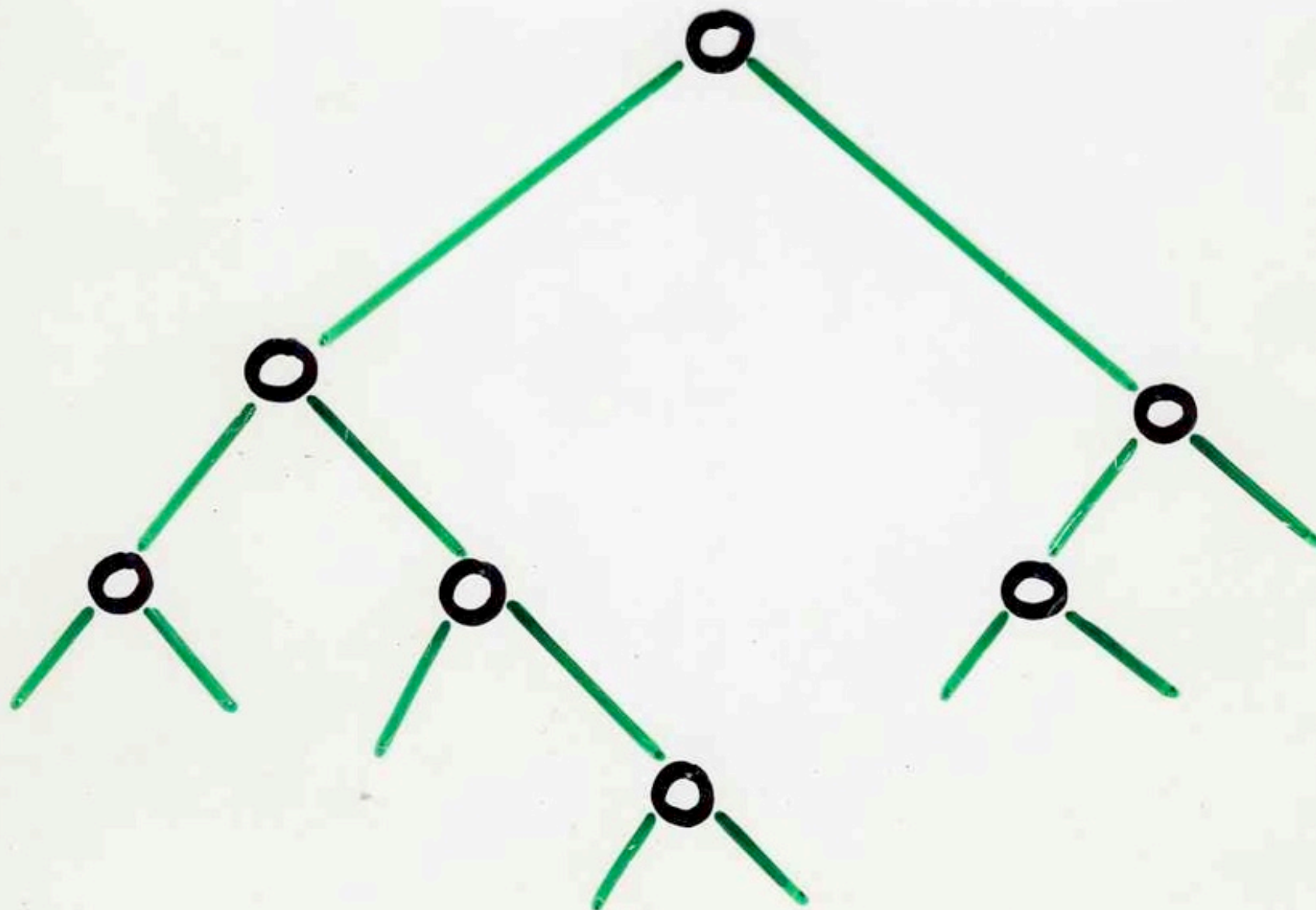
nombre
de
nécessaires
d'une

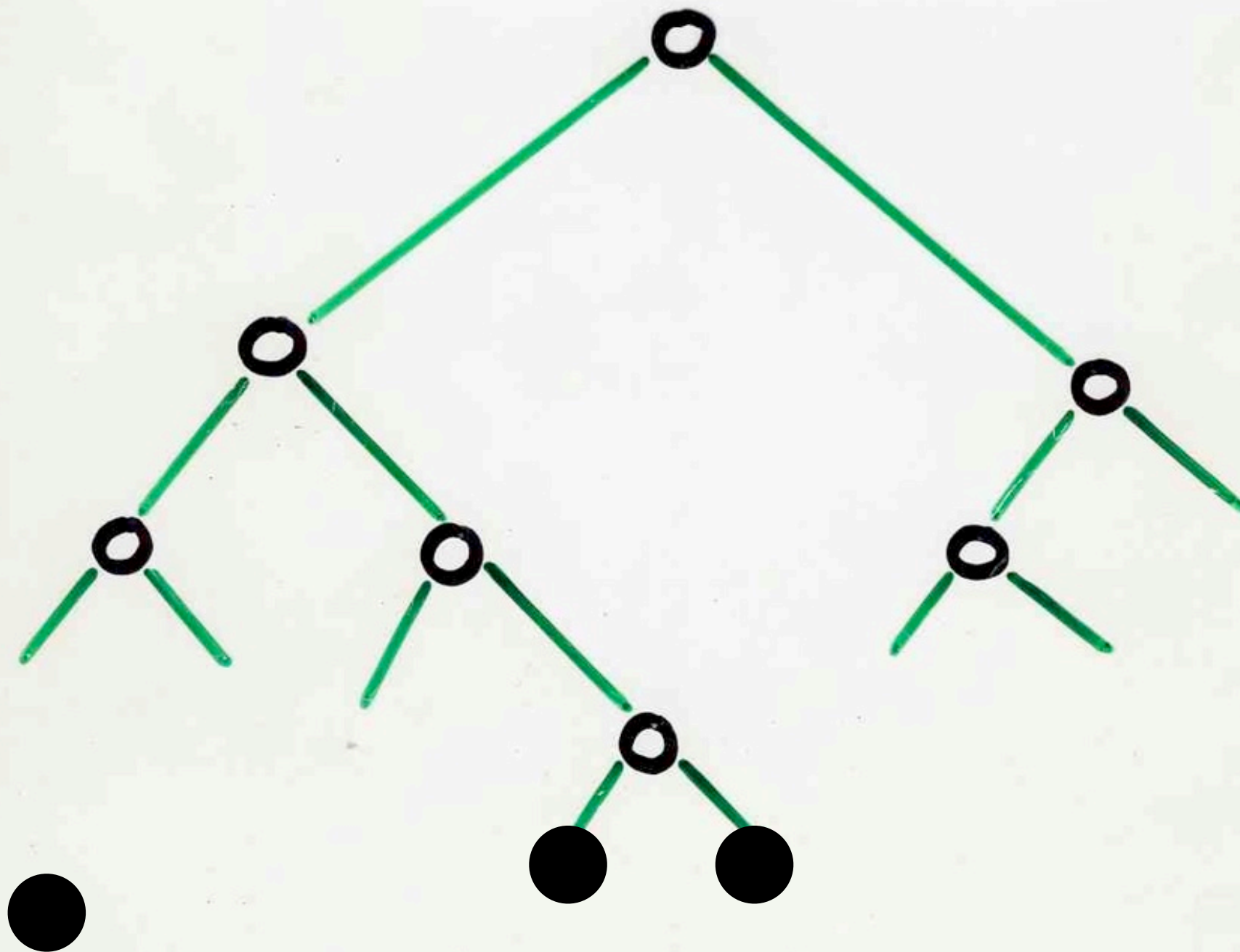
minimum
registres

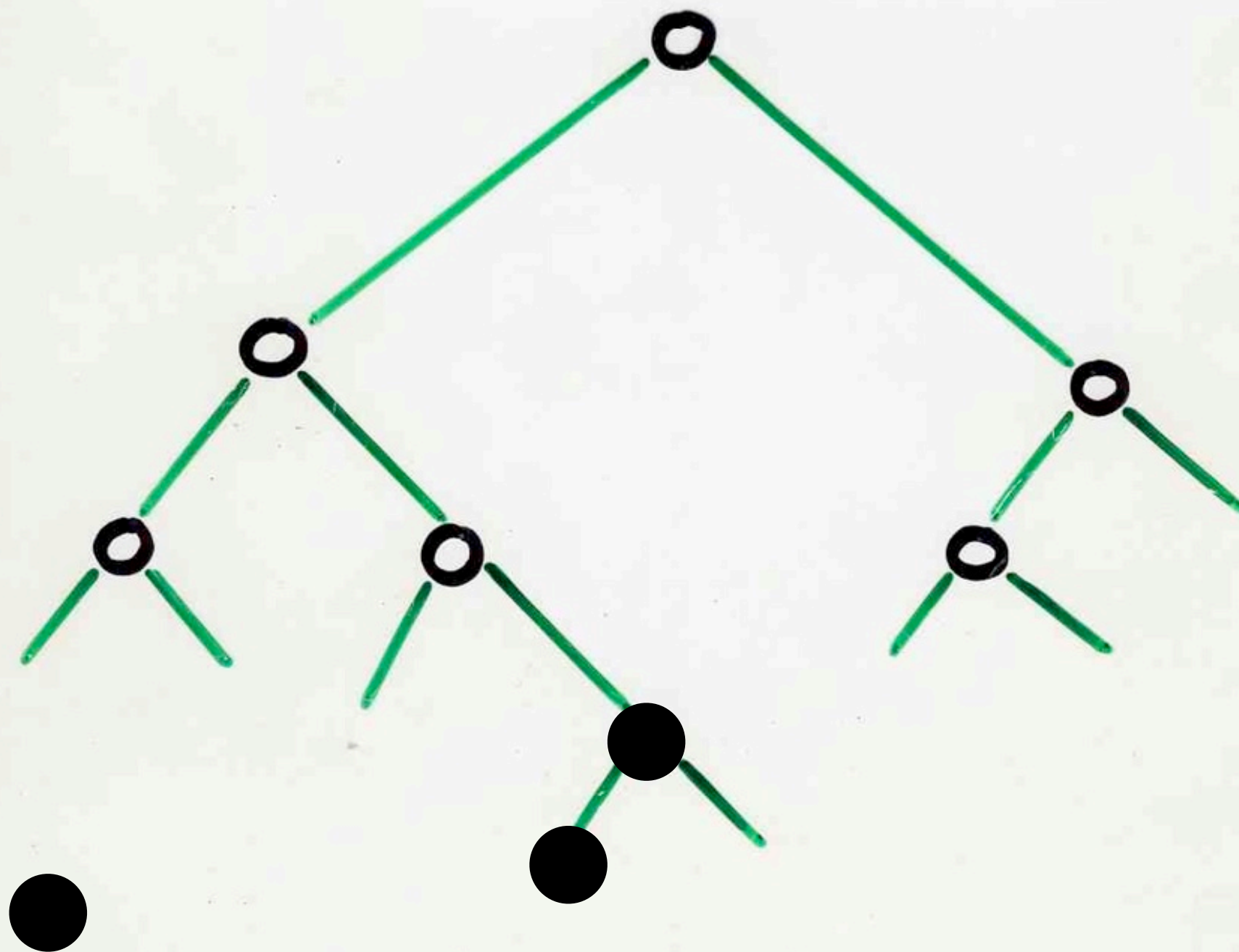
pour le calcul
expression arithmétique

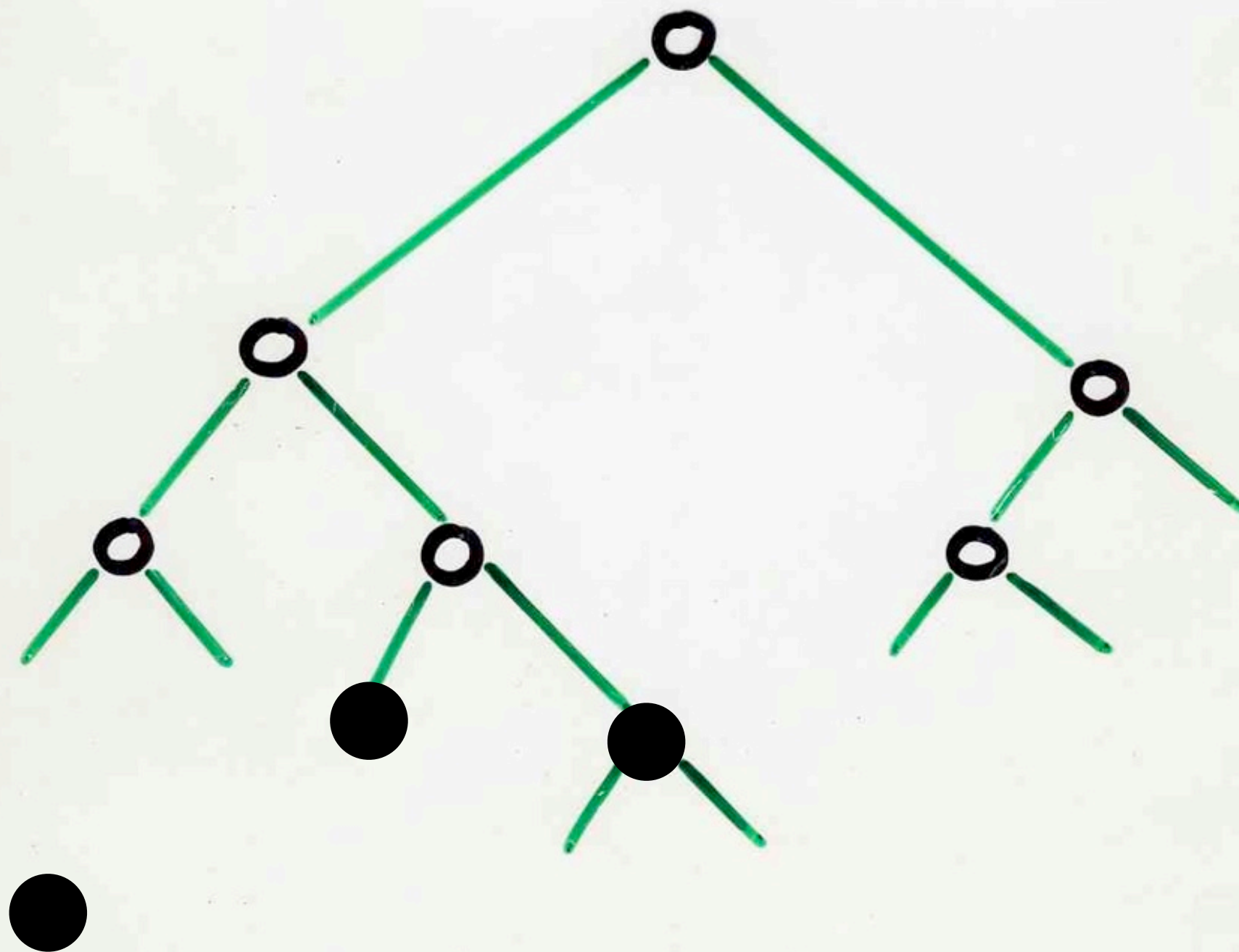
Pebbles problem

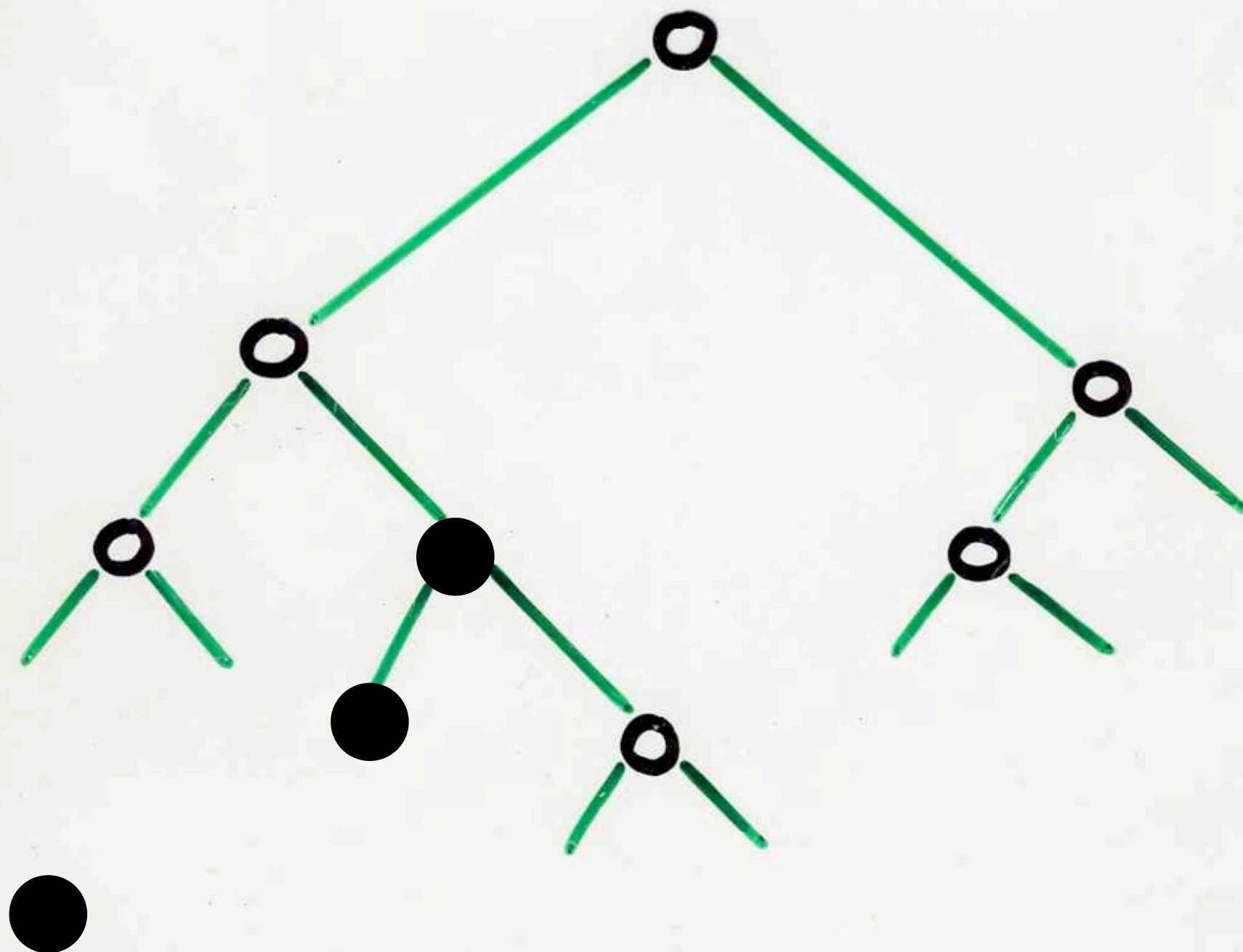


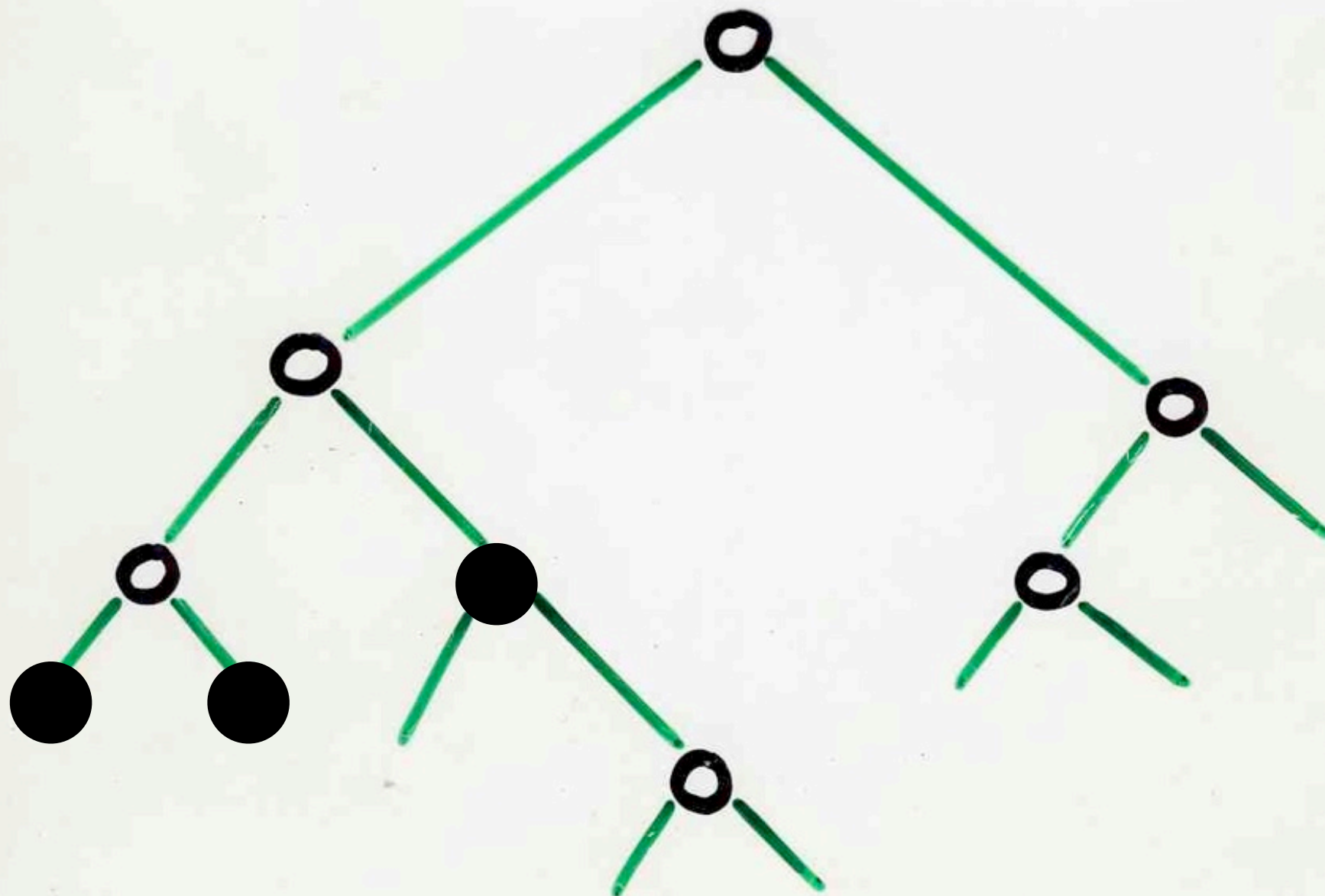


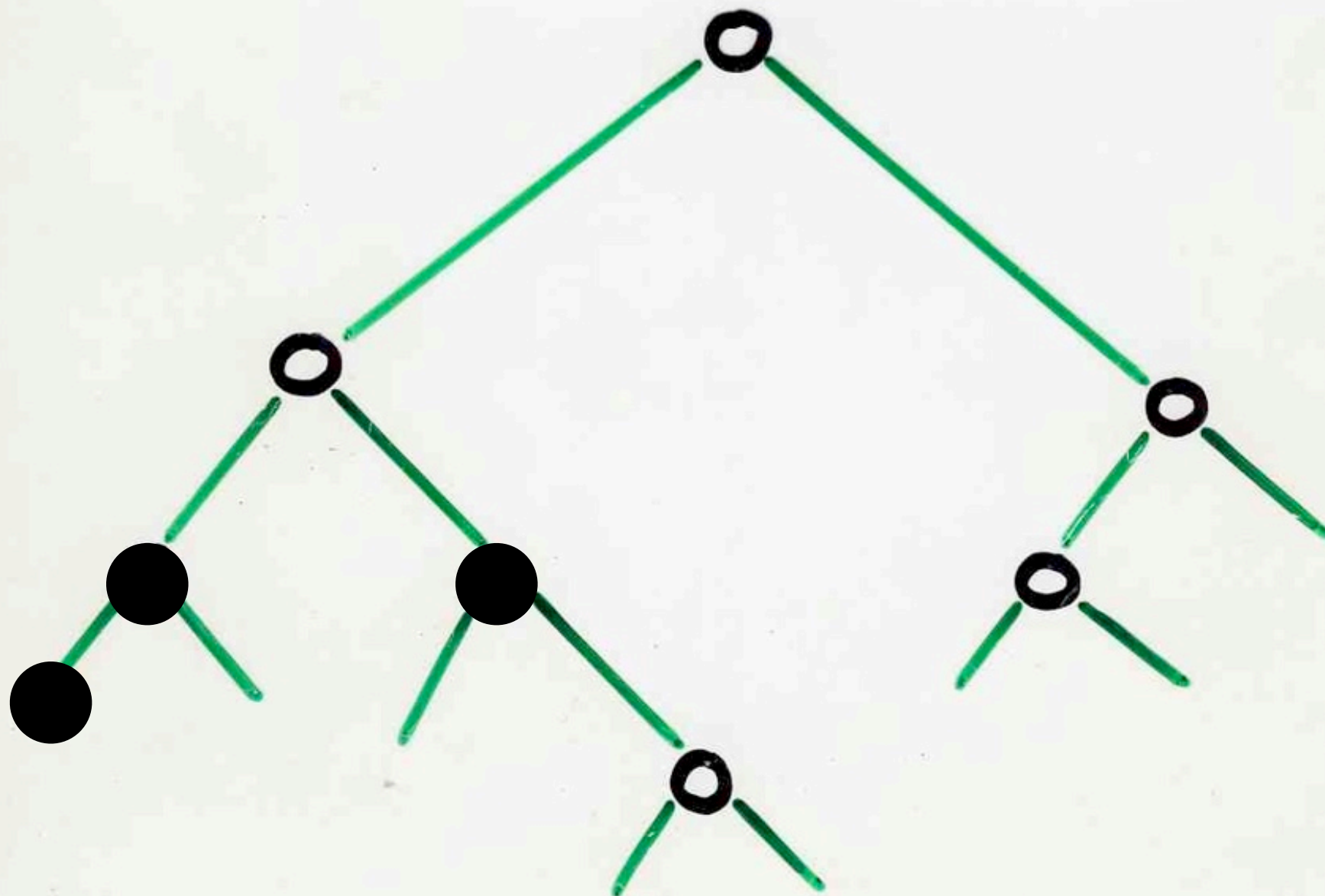


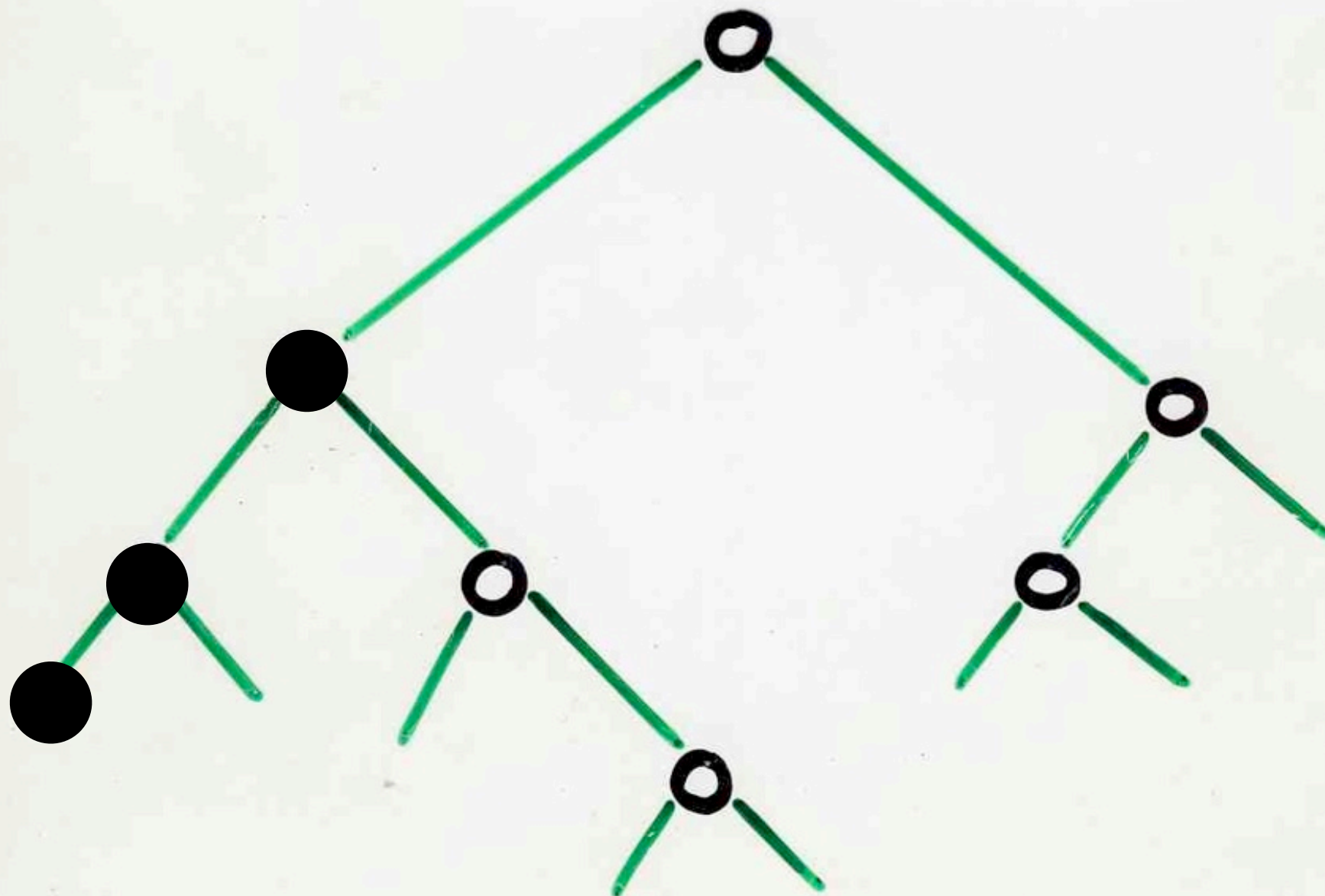


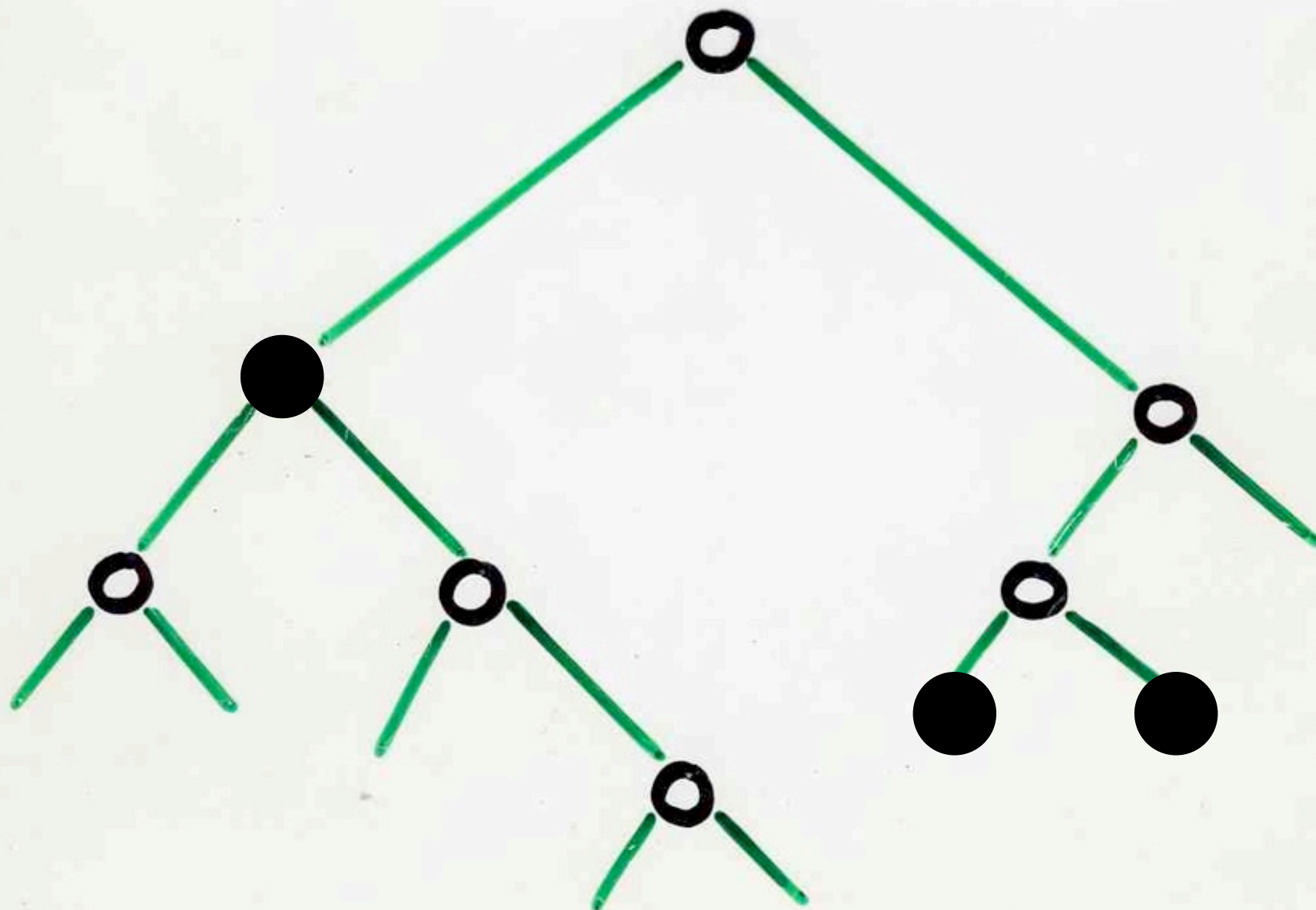


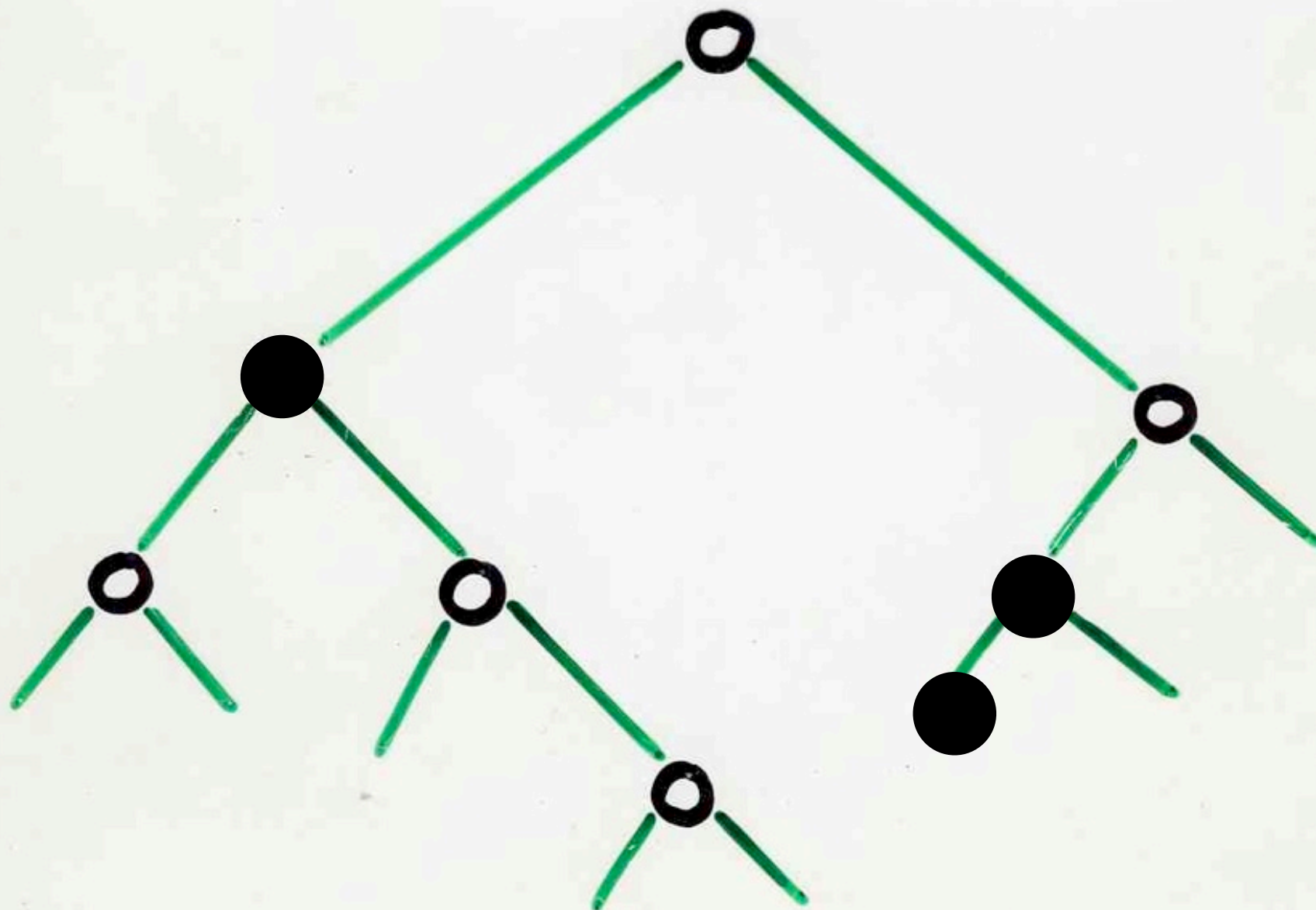


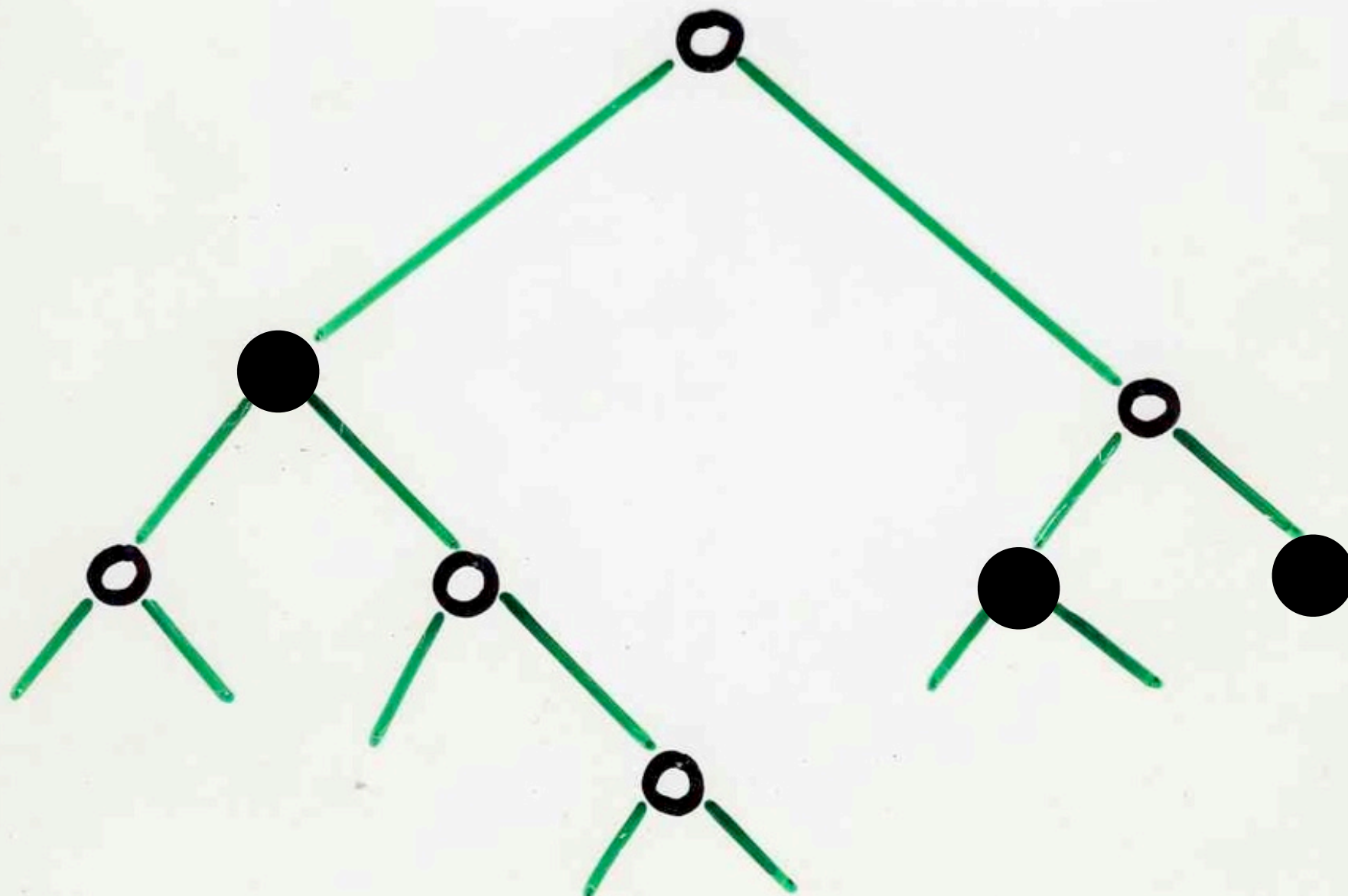


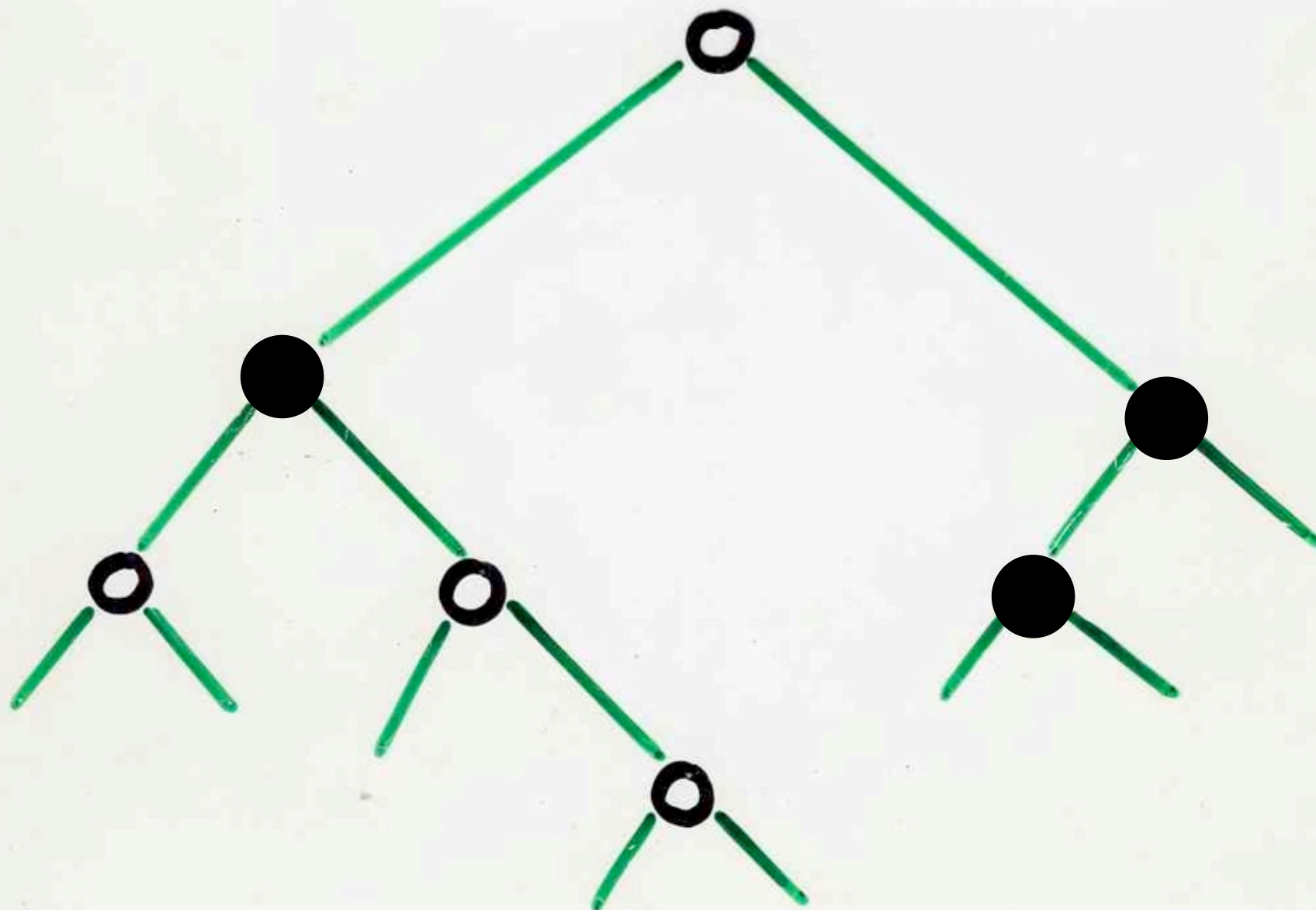


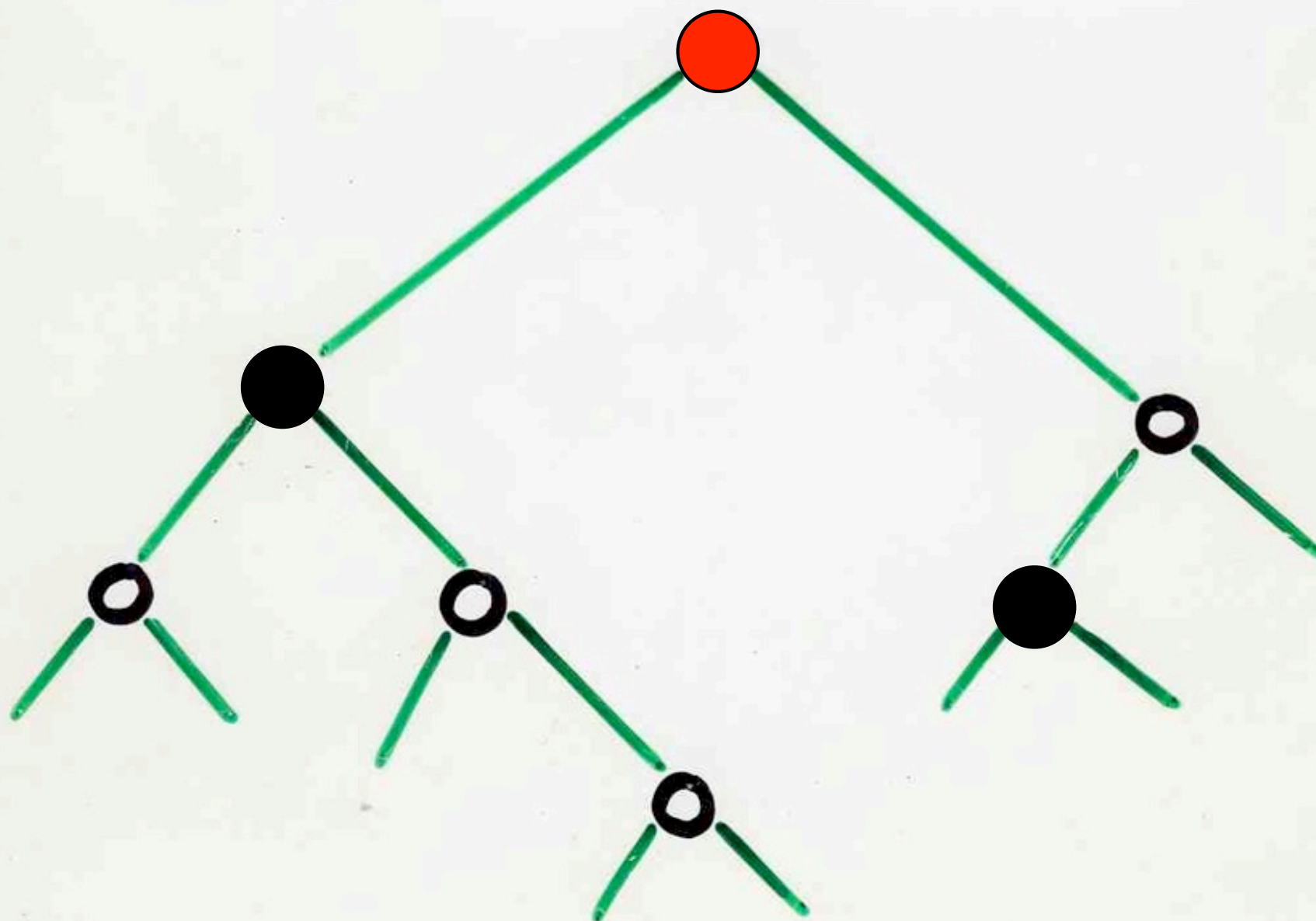












Des arbres aux rivières ...



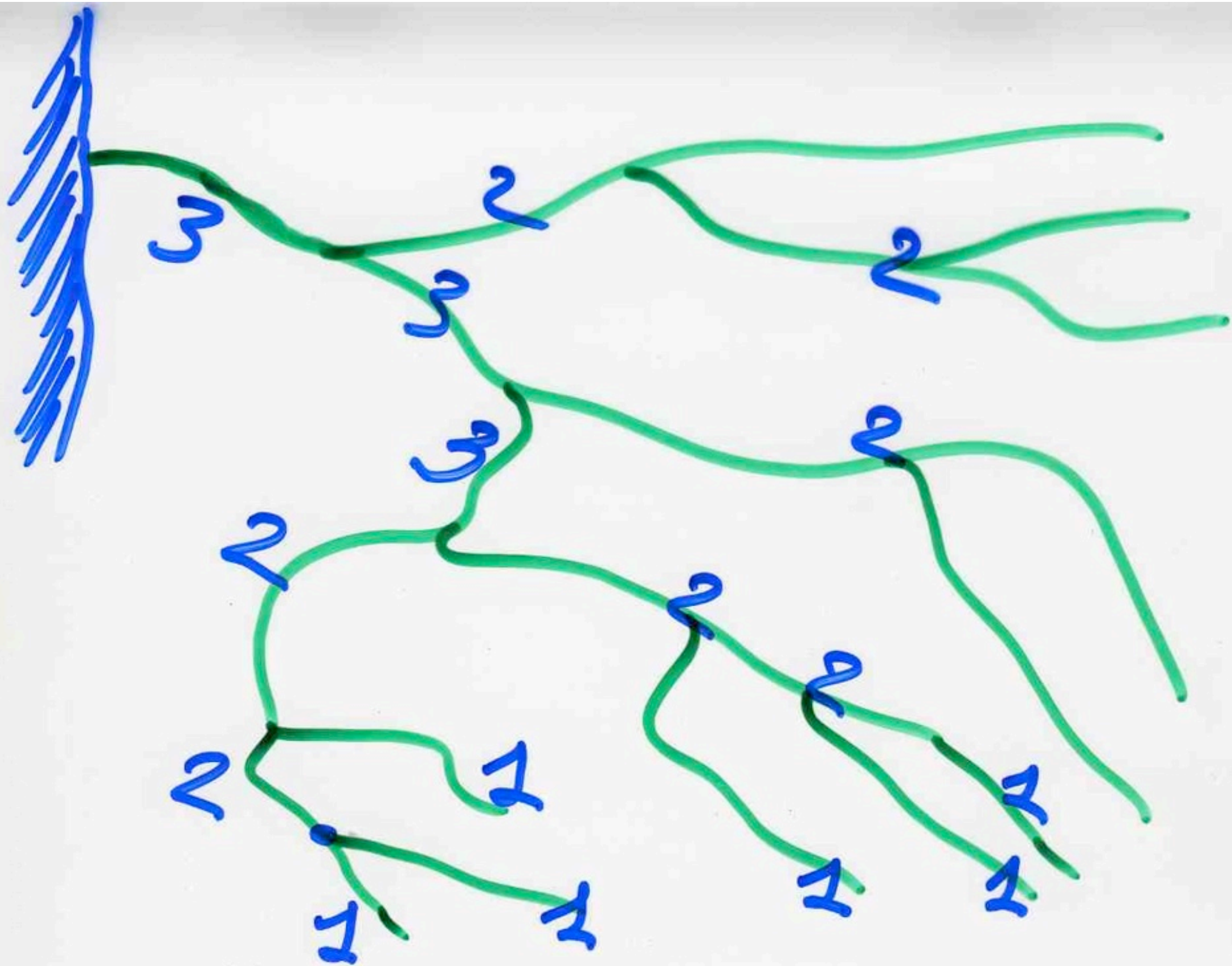


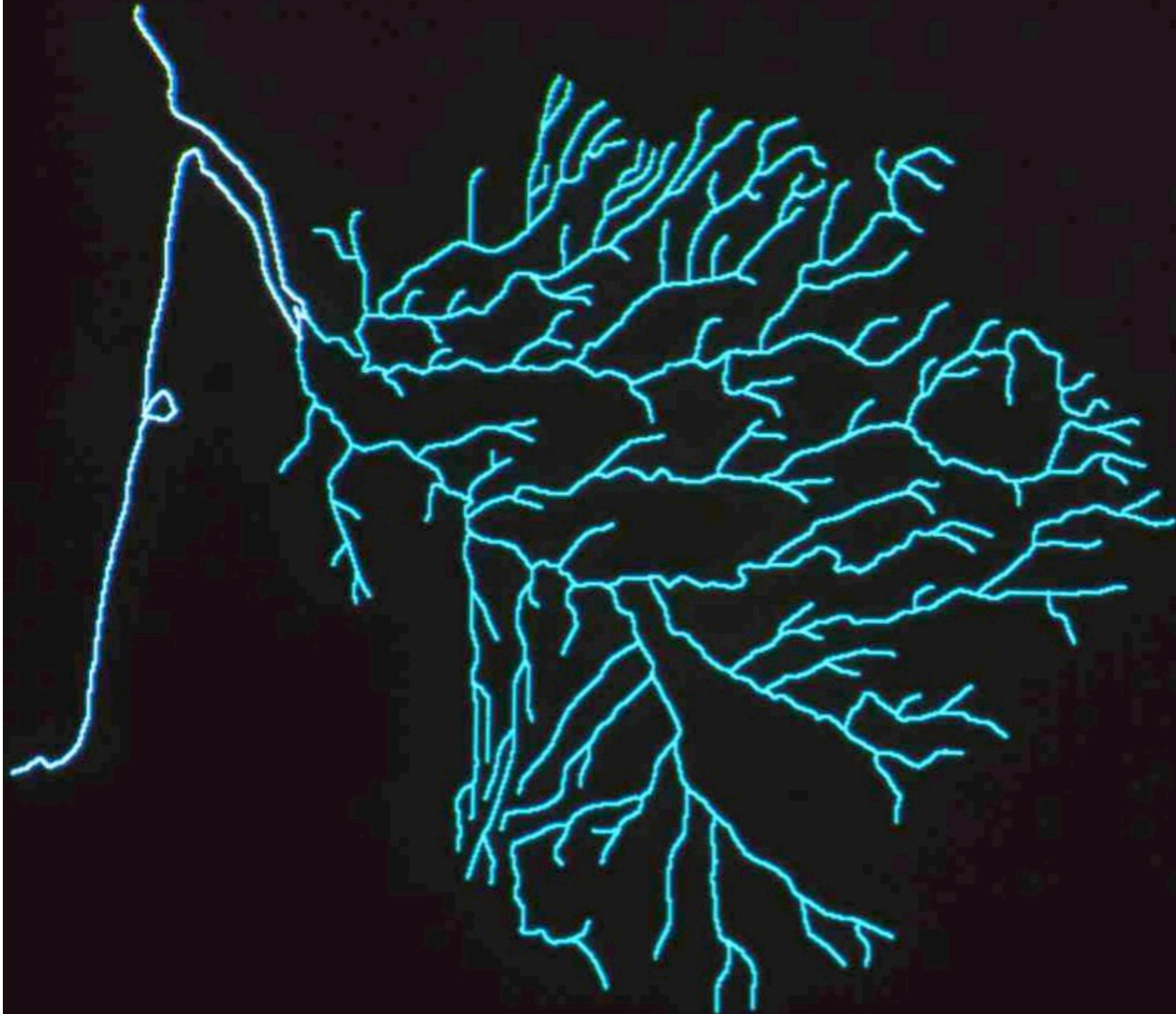
Horton (1945)
Strahler (1952)

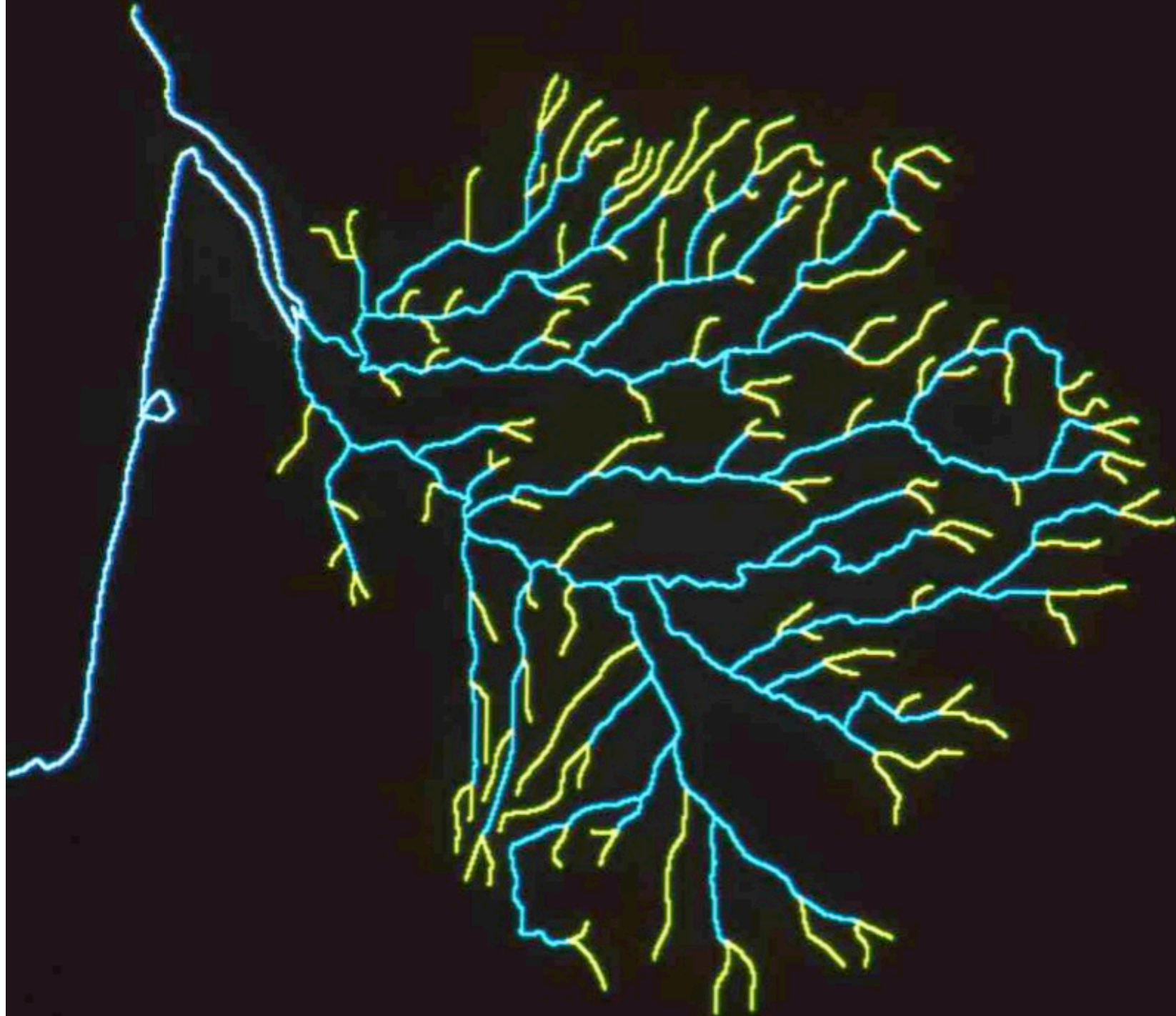
Hydrogéologie

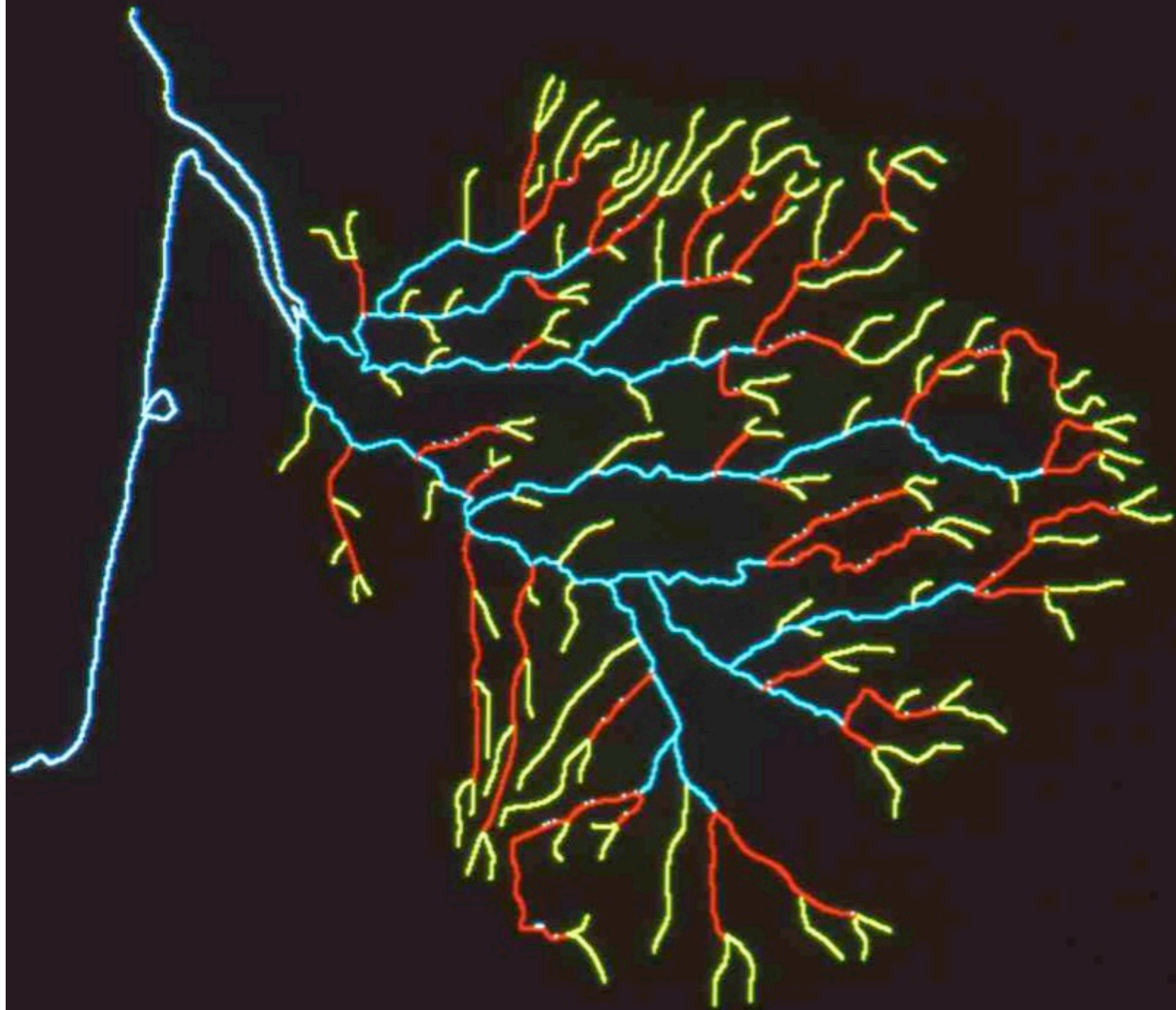
morphologie des
bassins fluviaux

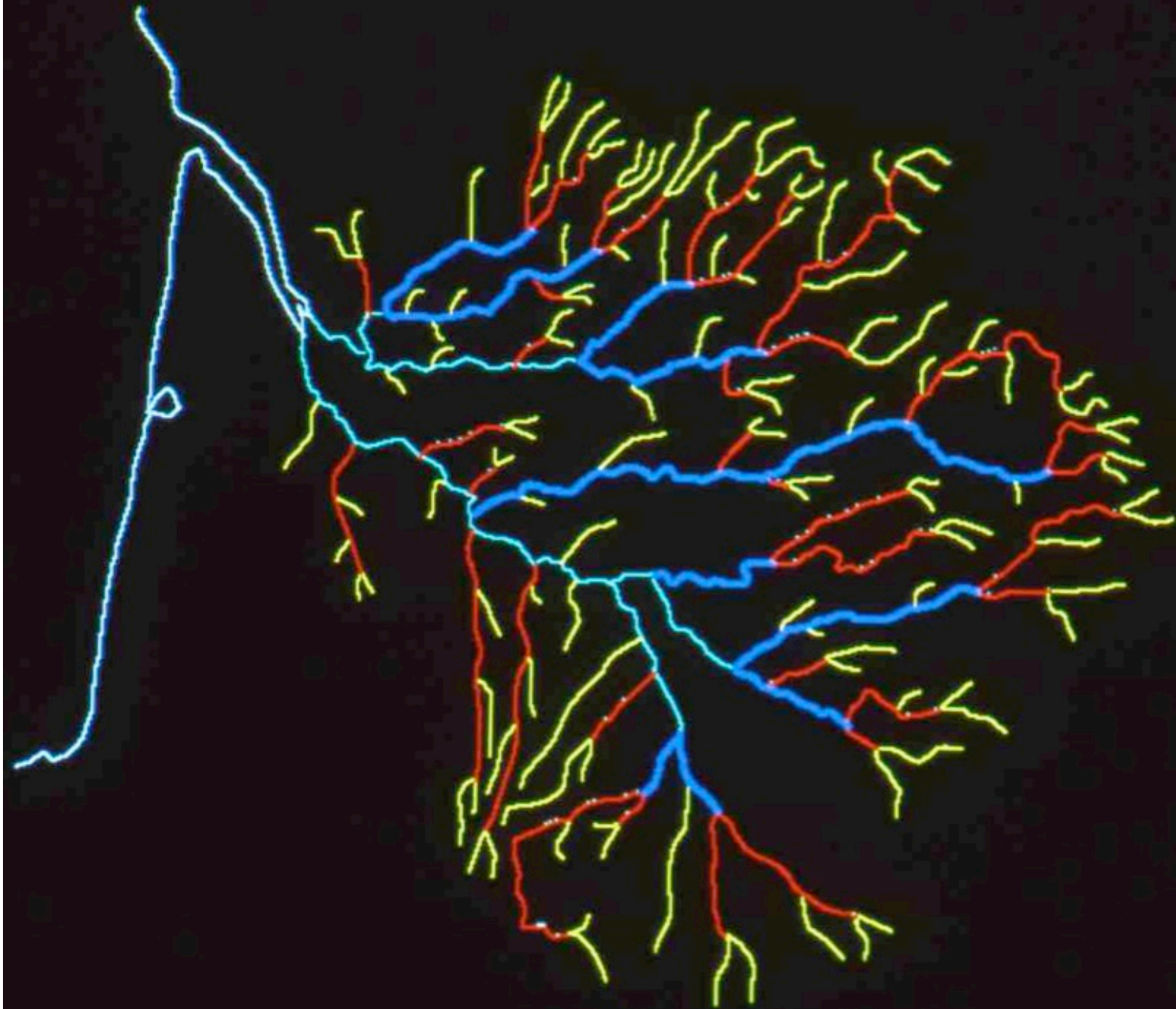
Ordre d'une rivière

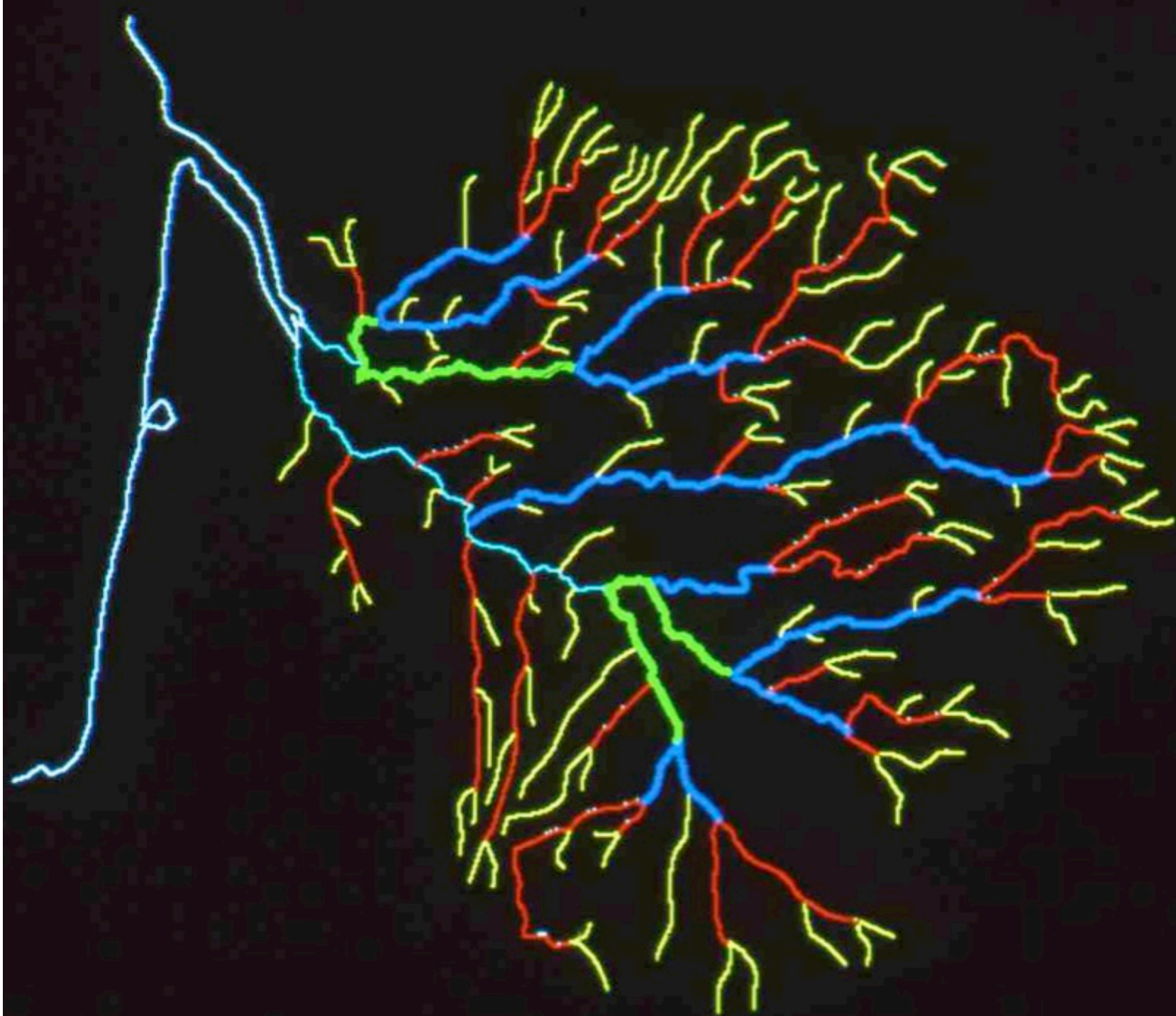


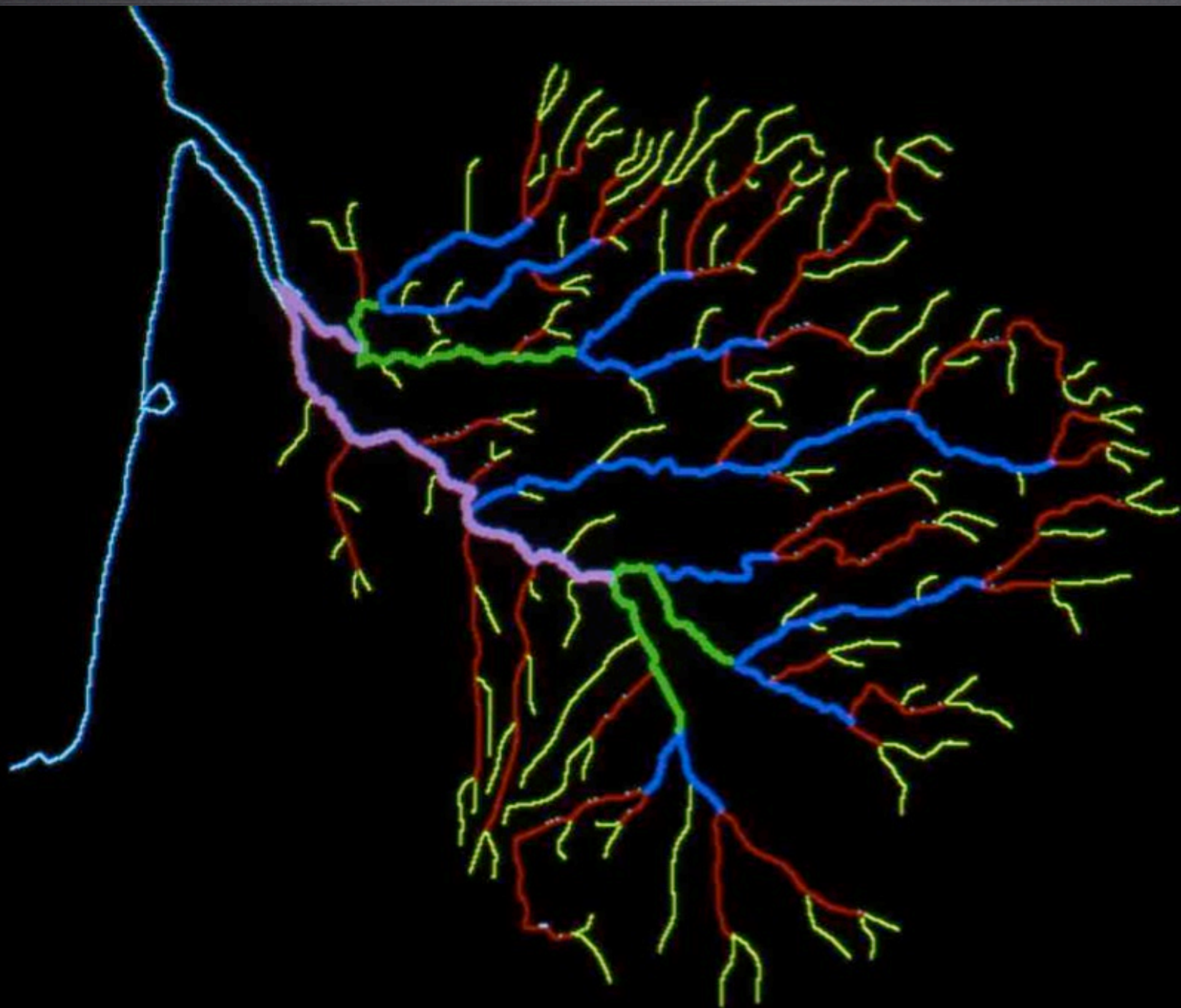


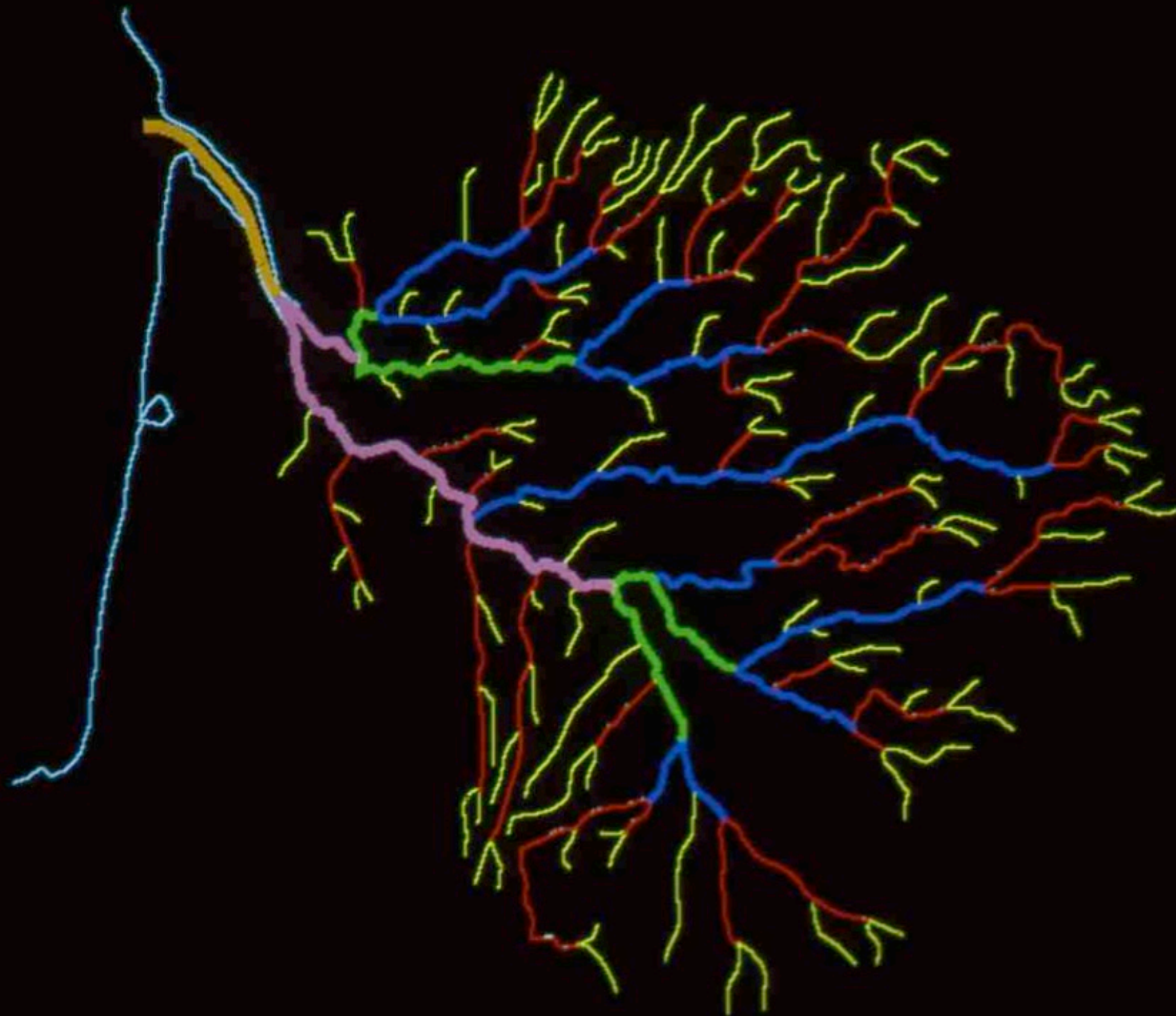


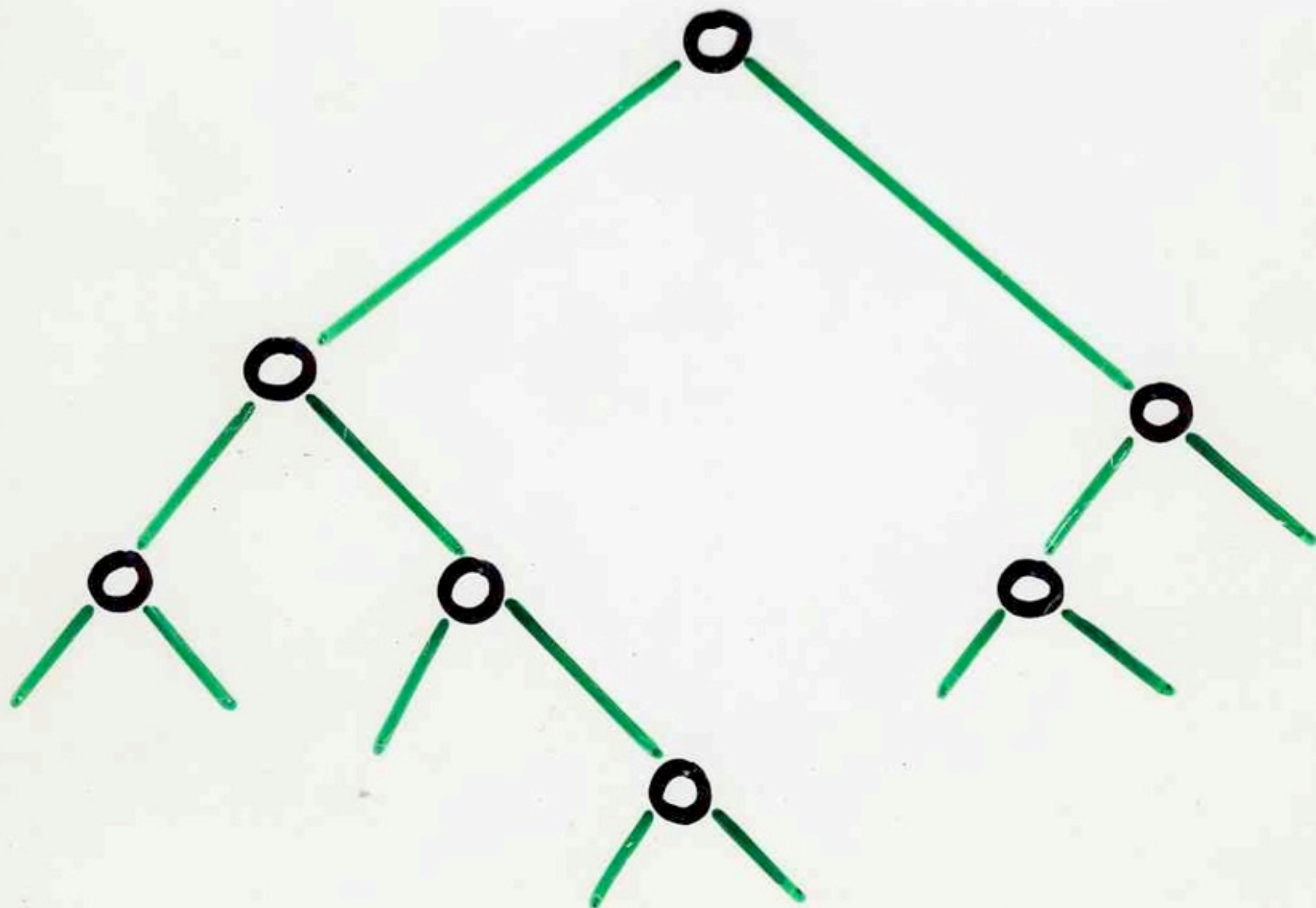




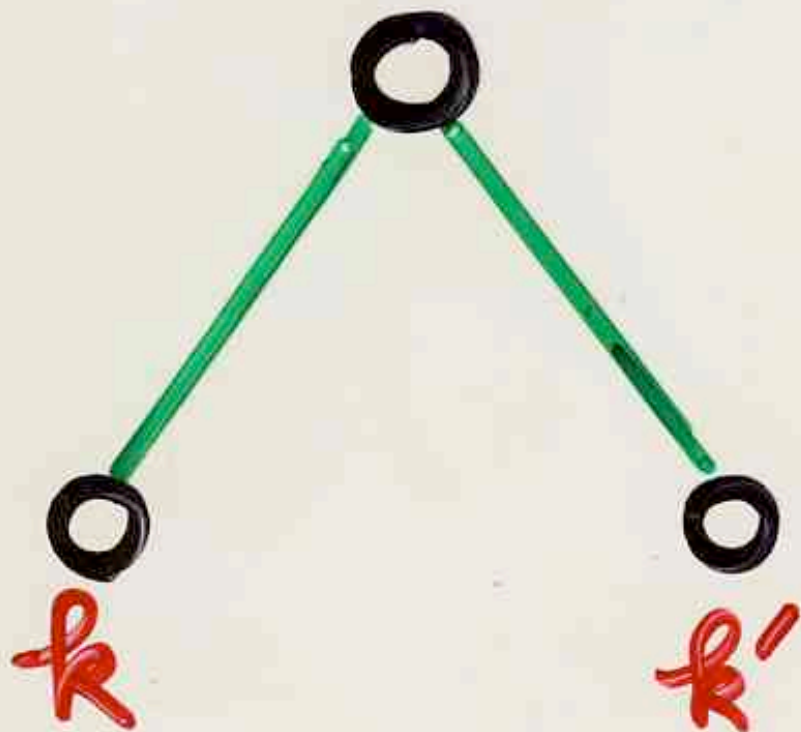




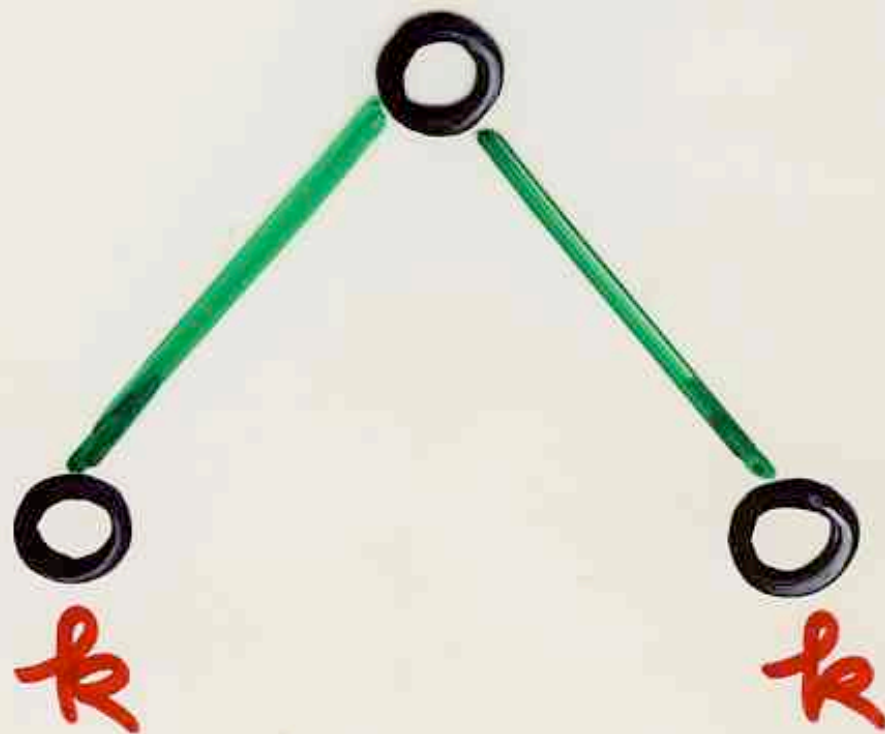




$\max(k, k')$



$k+1$



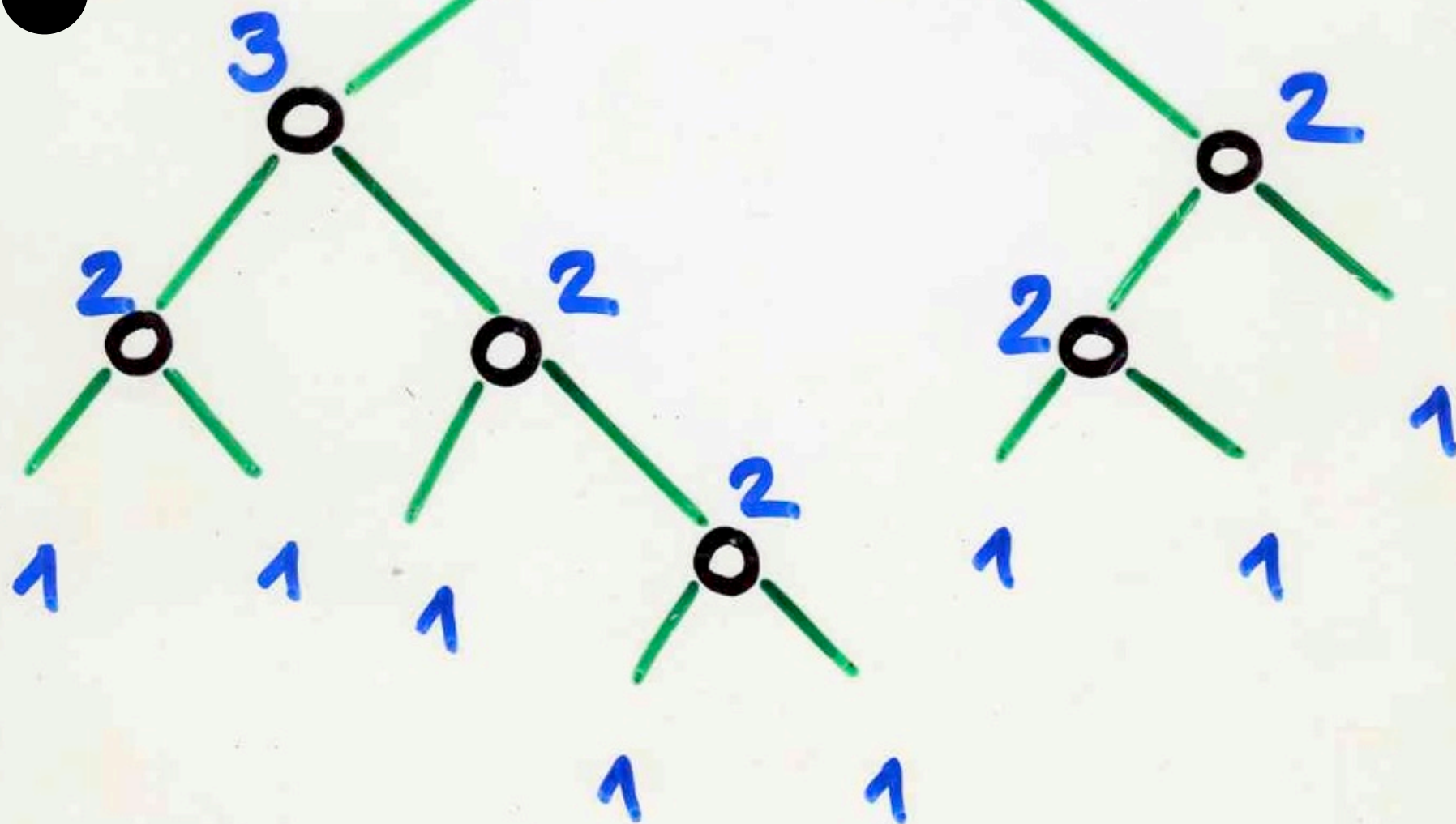


= nombre
minimum
de registres

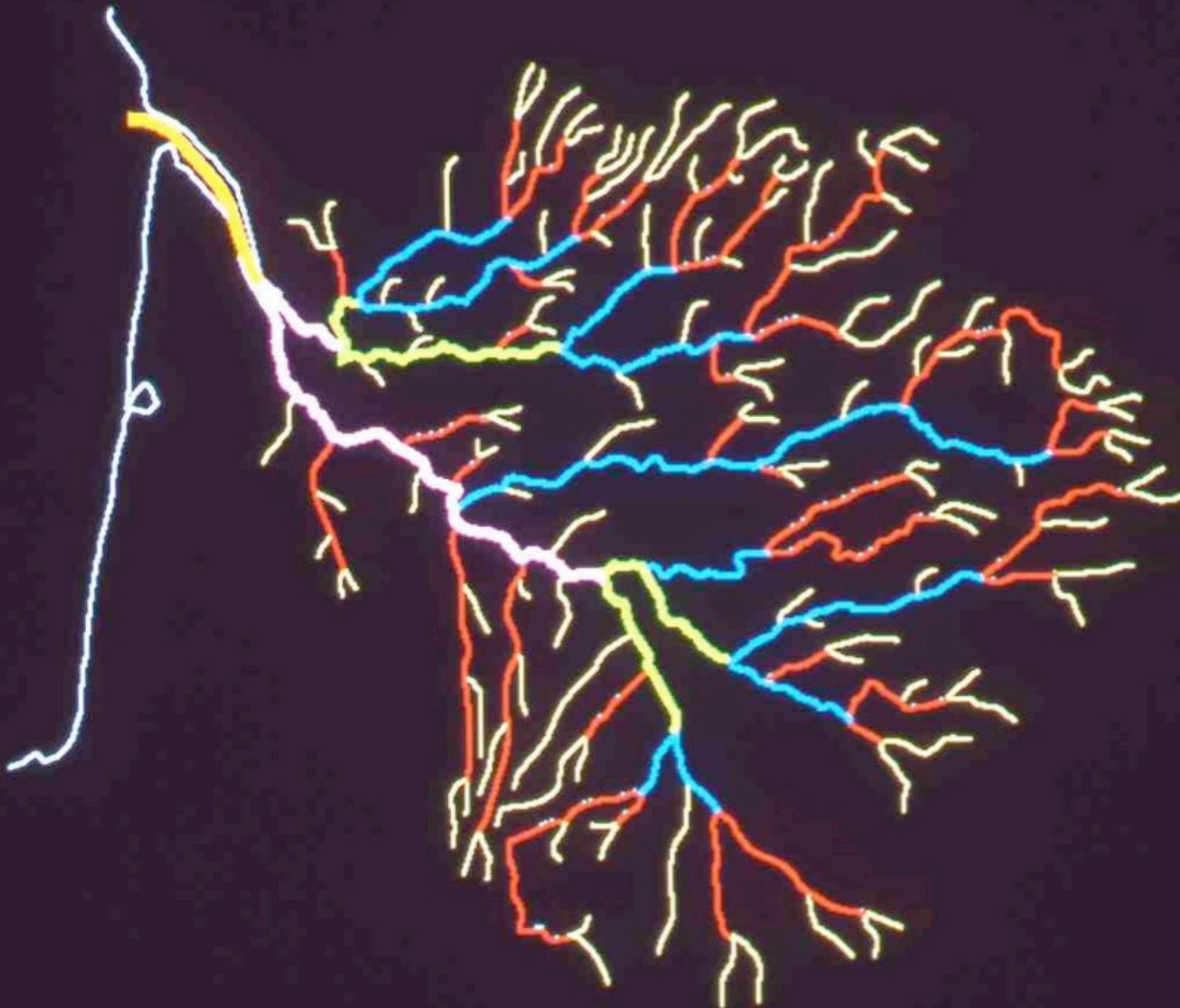
3

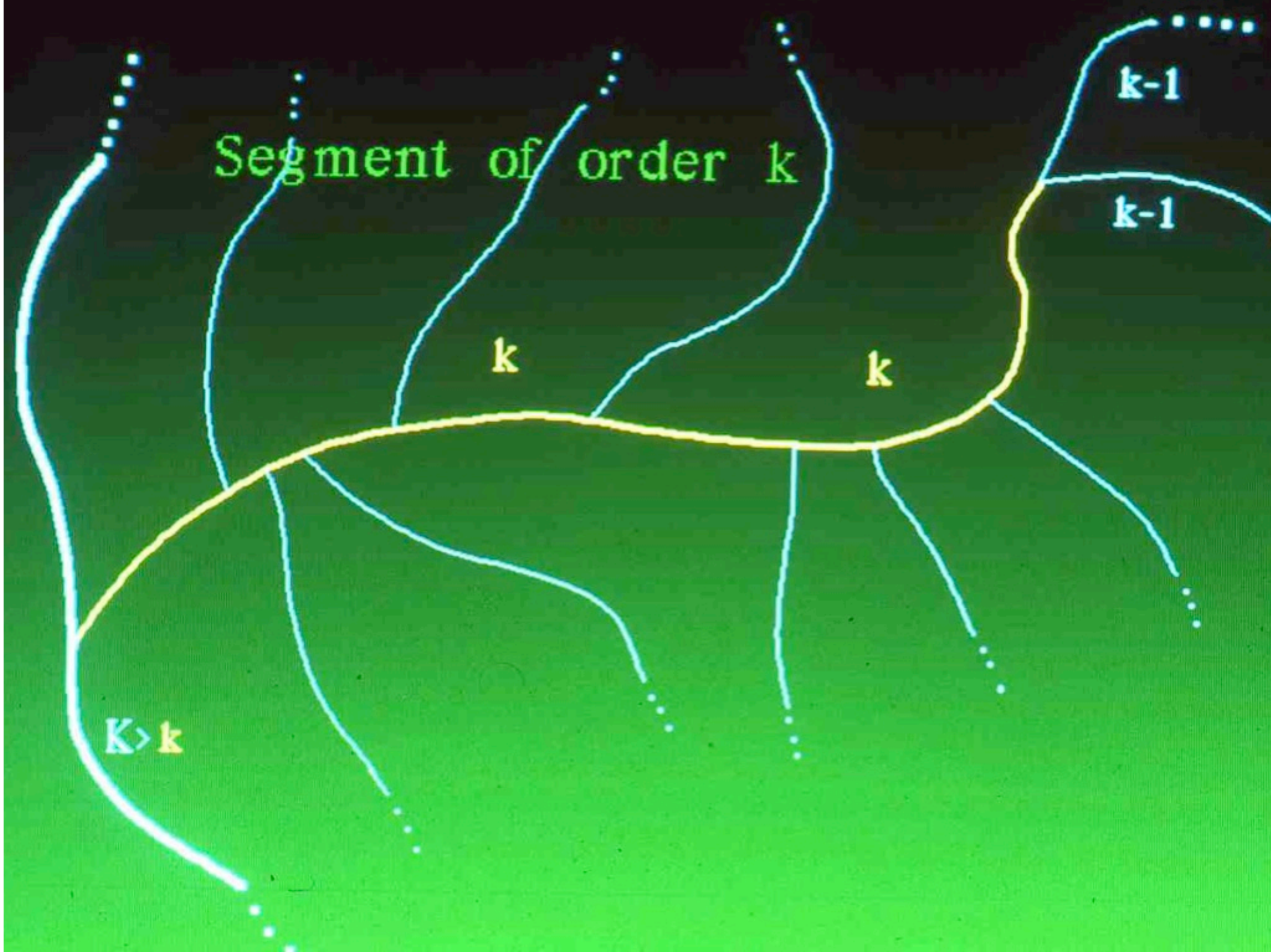
= $St(B)$

nombre de
Strahler



rivière ou segment d'ordre k



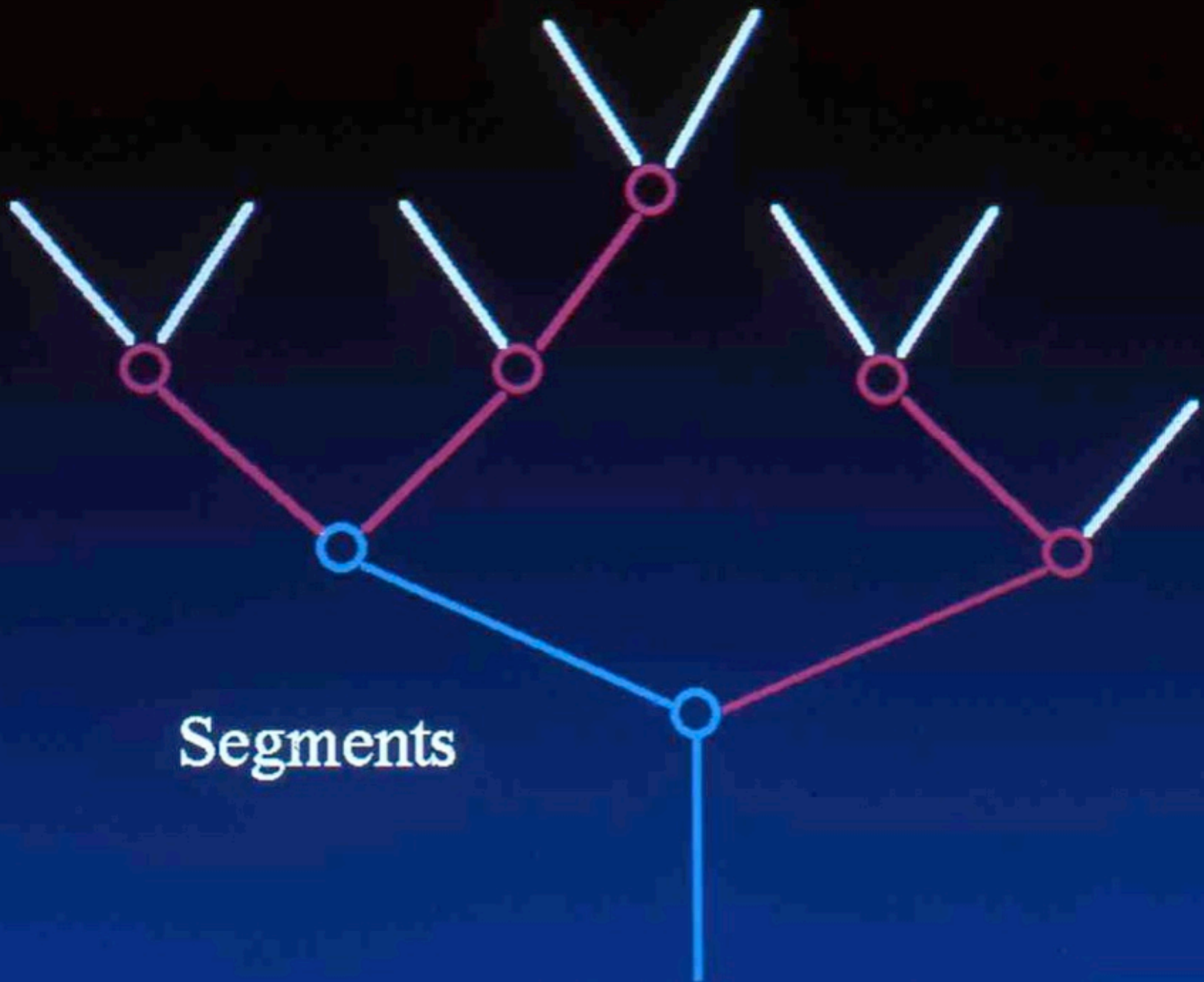


Rapport de branchement

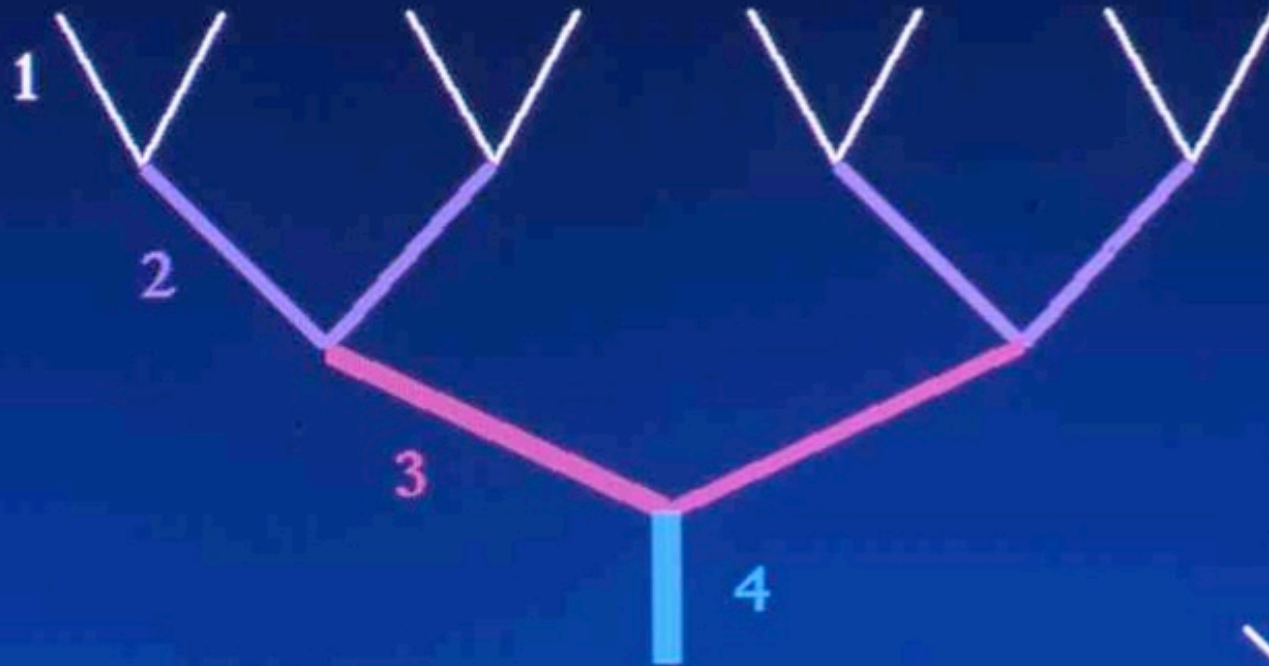
$$\beta_k = \frac{b_k}{b_{k+1}}$$

$$3 < \beta_k < 5$$

b_k = nombre de segments (rivières)
d'ordre k



Segments



arbre binaire parfait



arbre binaire
«très fin»

corrélation entre la «forme» du réseau fluvial
et
la structure du sous-sol profond

Prud'homme, Nadeau, Vigneaux,
1970, 1980

informatique graphique

matrice de ramification
d'un arbre binaire

Arquès, Eyrolles, Janey, X.V.

SIGGRAPH'89, IMAGINA' 90



LE CORBUSIER

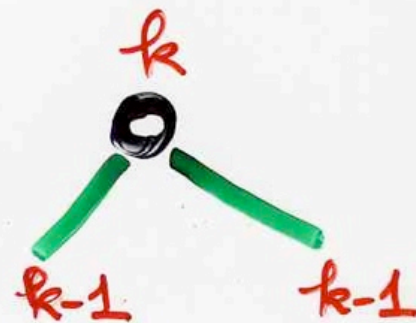
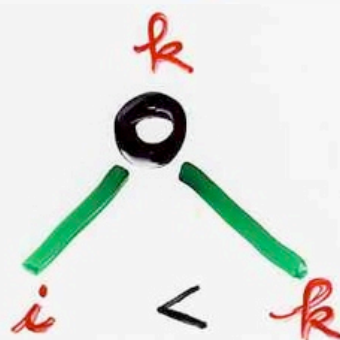
Images de synthèse
d'arbres, feuilles, paysages, ...

Anquetin, Eyrolles, Jamey, X.G.V.

SIGGRAPH '89

AFA

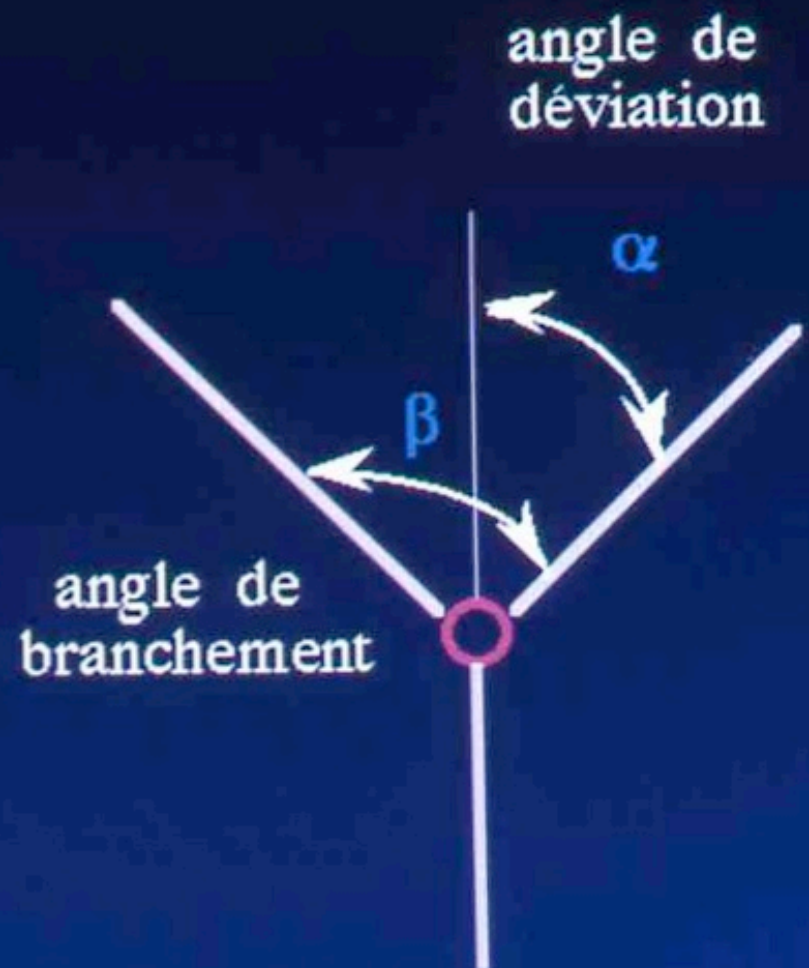
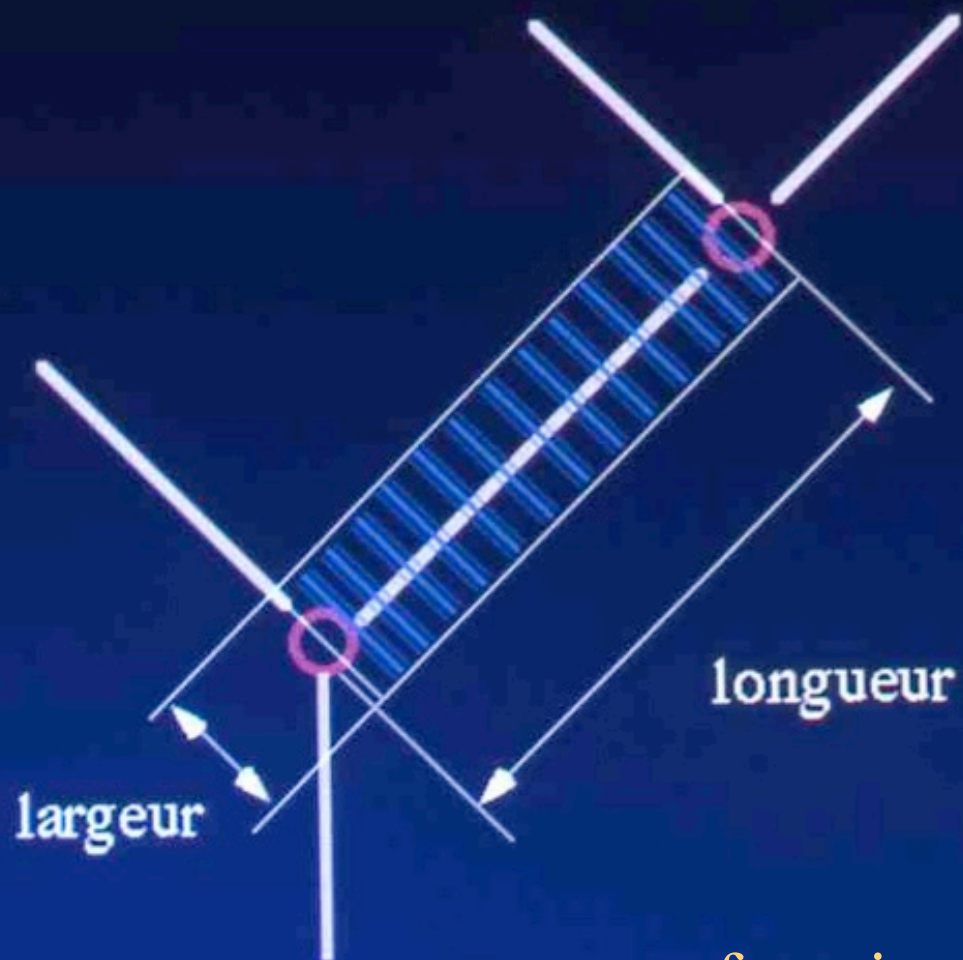
matrice
de
ramification



$$P_{k,i} = \frac{b_{k,i}}{a_k}$$

biordre (k, i)

tableau
de
probabilités



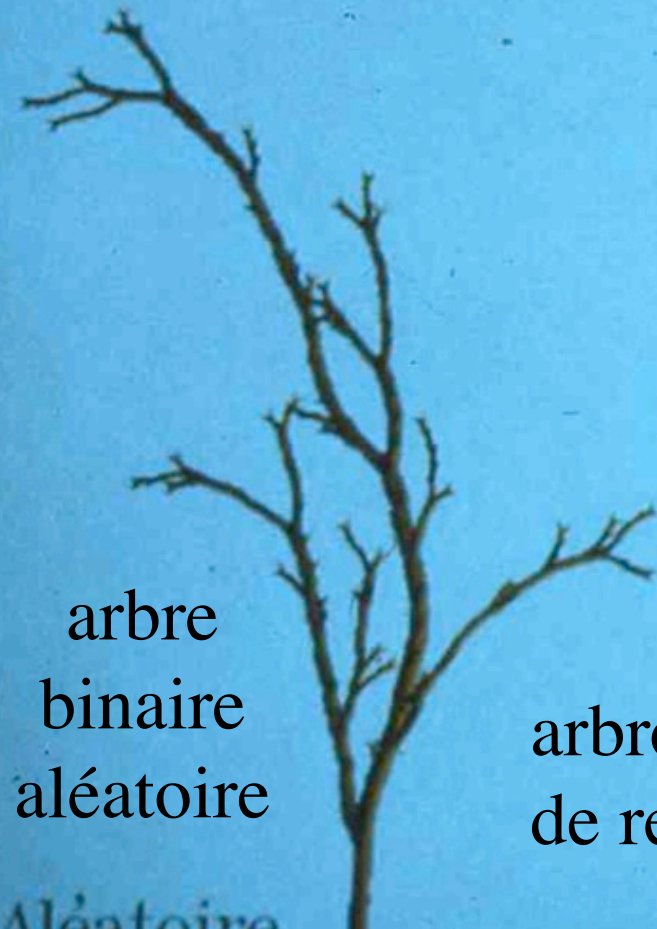
fonctions de l'ordre k
et du biordre (k,i)







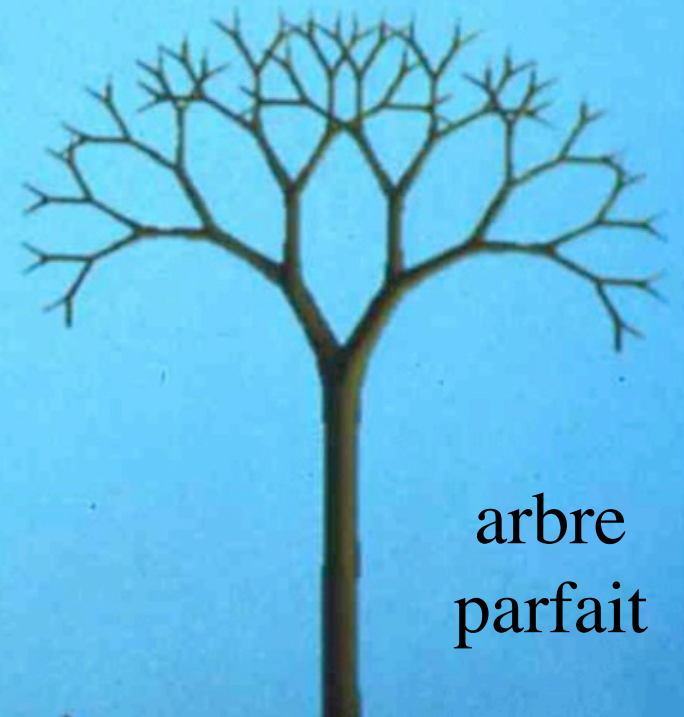
mélange de 3 matrices de
ramification



arbre binaire
de recherche



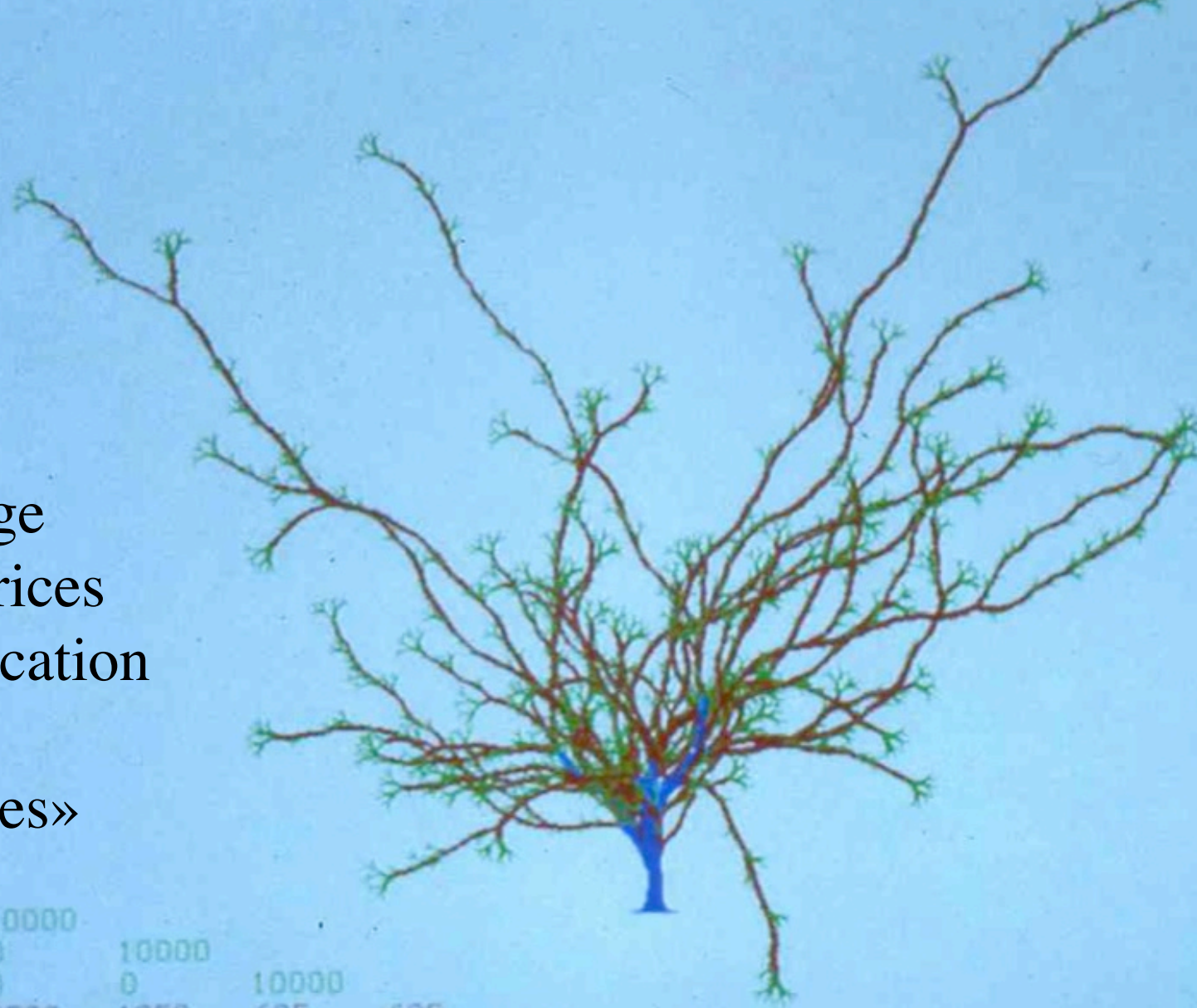
aléatoire



arbre
parfait

mélange
de 3 matrices
de ramification

3 «formes»



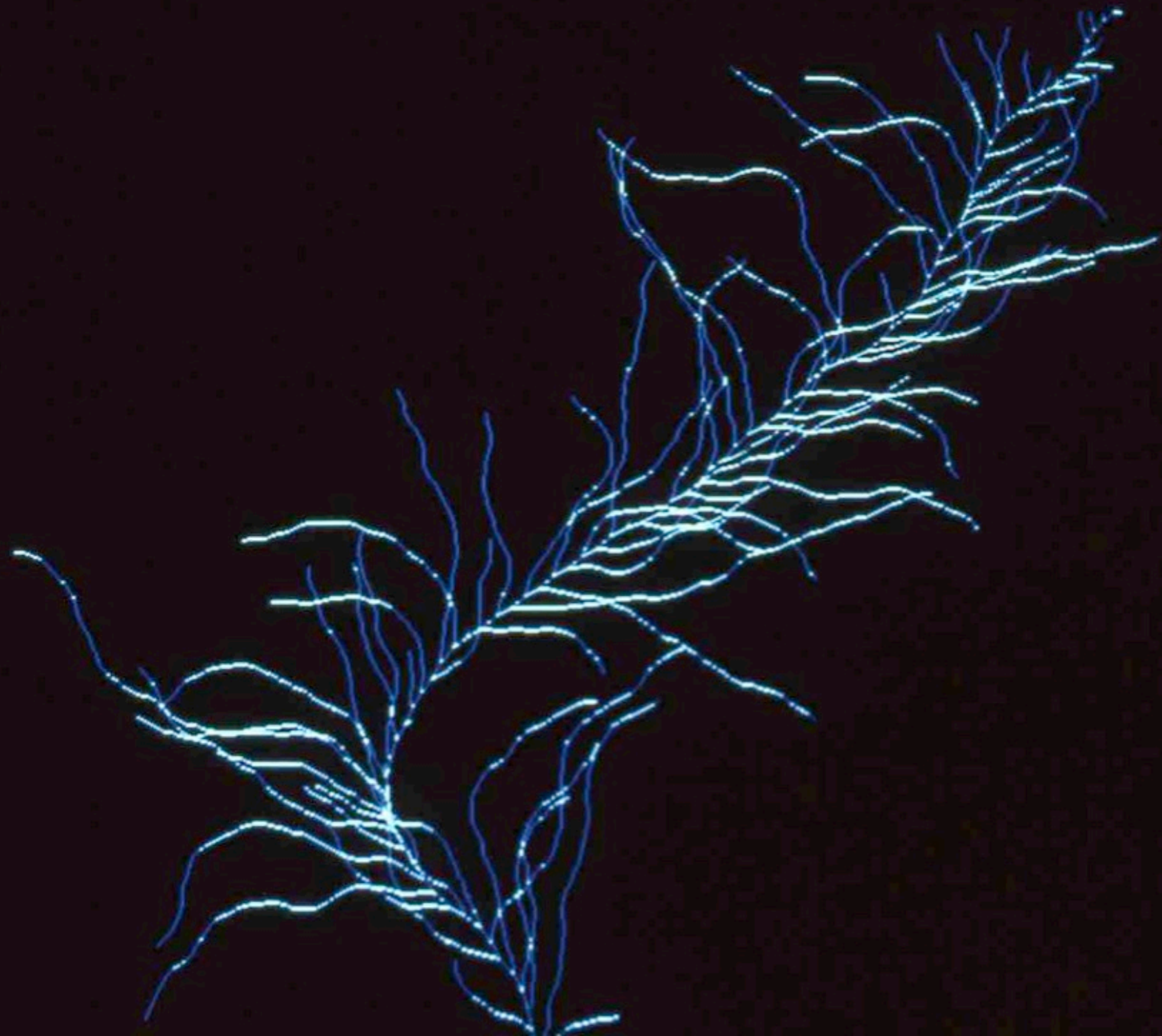
2 : 0	10000										
3 : 0	0	10000									
4 : 0	0	0	10000								
5 : 5000	2500	1250	625	625							
6 : 5000	2500	1250	625	313	312						
7 : 125	250	500	1000	2000	3000	3125					
8 : 63	125	250	500	1000	2000	3000	3062				
9 : 31	63	125	250	500	1000	2000	3000	3031			
10 : 15	31	63	125	250	500	1000	2000	3000	3016		
11 : 7	15	31	63	250	125	500	1000	2000	3000	3009	





A\$A

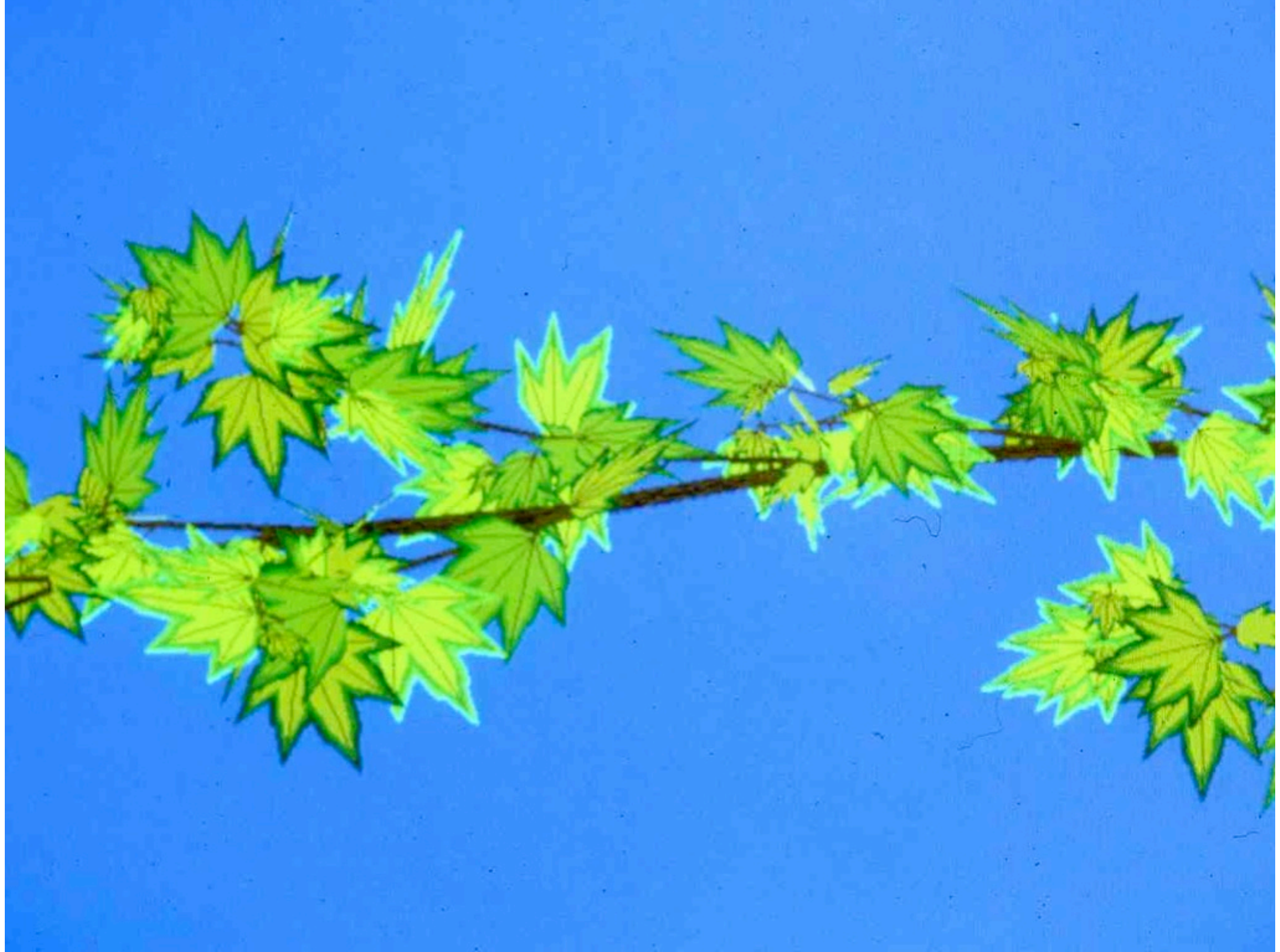














S'il existe une quelconque beauté dans
ces images de synthèse d'arbres, elle
n'est que le pâle reflet de l'extraordinaire
beauté des mathématiques se cachant
derrière les algorithmes générant ces
images.

Nombre de Strahler

moyen

parmi tous les arbres binaires
ayant n sommets

$$St_n = \log_4 n + f(\log_4 n) + o(1)$$

Flajolet, Raoult, Vuillemin (1979)
Kemp (1979)

f fonction
périodique

Théorie des nombres

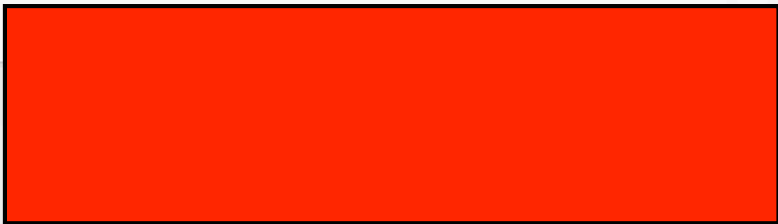
$T(n)$ = nombre total de 1 dans
les écritures binaires
des nombres $1, 2, \dots, (n-1)$.

série génératrice

$S_{n,k}$ = nombre d'arbres binaires B
ayant n sommets (internes)
et nombre de Strahler
 $st(B) = k$

$$S_k(t) = \sum_{n \geq 0} S_{n,k} t^n$$

série formelle


$$S_1 = 1$$

$$S_2 = \frac{t}{1 - 2t}$$

$$S_3 = \frac{t^3}{1 - 6t + 10t^2 - 4t^3}$$

$$S_4 = \frac{t^7}{1 - 14t + 78t^2 - 220t^3 + 330t^4 - 252t^5 + 84t^6 - 8t^7}$$



Pafnuty Tchebychev
(1887-1920)

polynômes de
Tchebychev

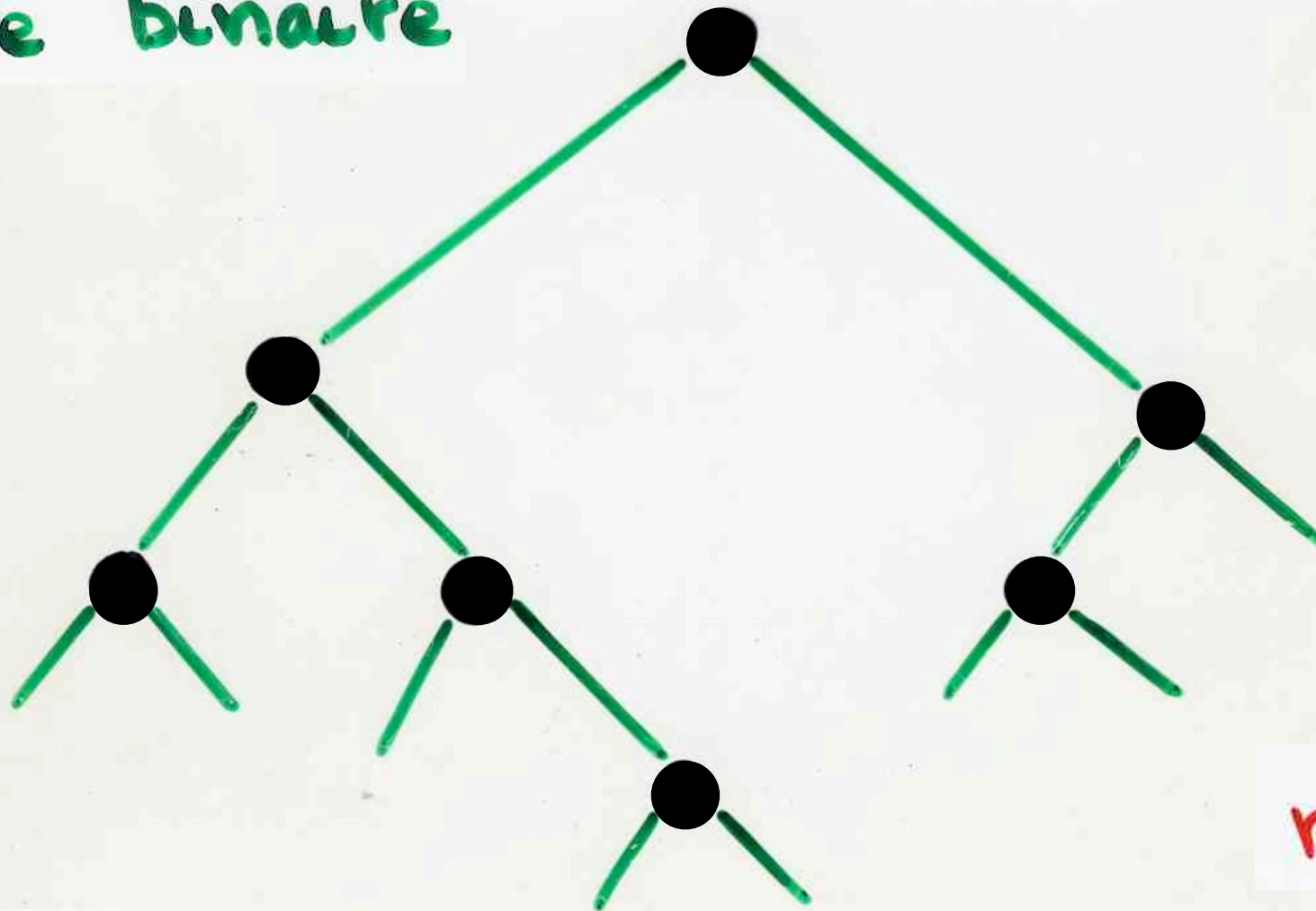
Trigonométrie

$$\sin(n+1)\theta = (\sin\theta) \mathbf{U}_n(\cos\theta)$$

Compter les arbres ...



arbre binaire



$$n = 7$$

C_n

nombre d'arbres binaires
ayant n sommets (internes)

[ou $n+1$ sommets externes]
feuilles

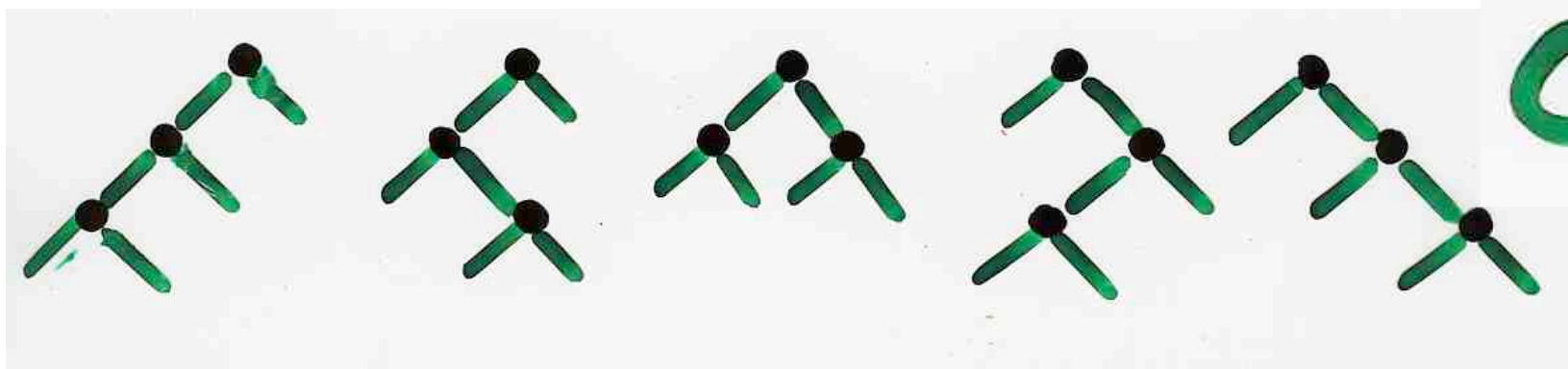




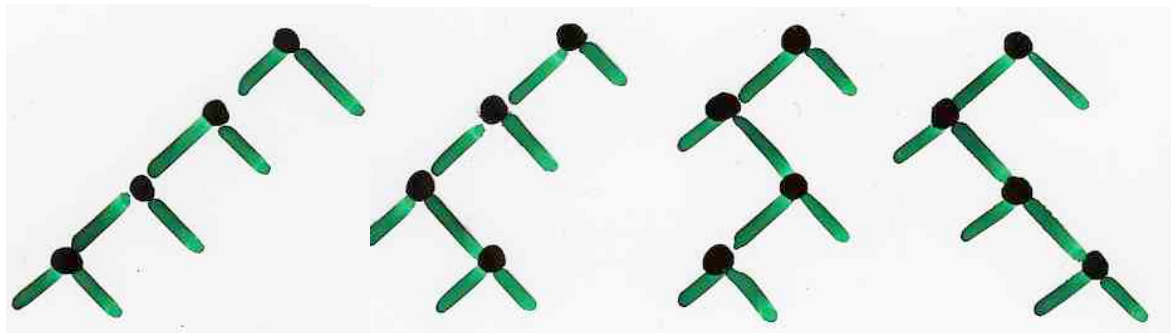
$$C_1 = 1$$



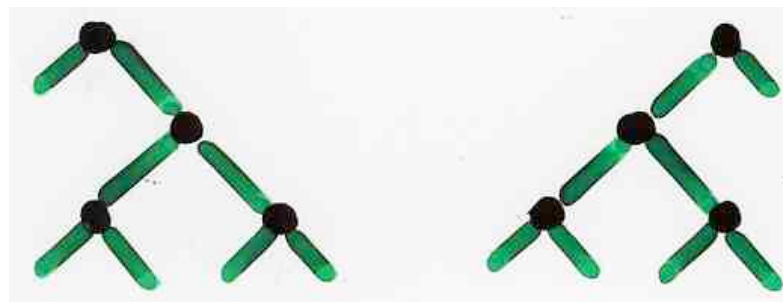
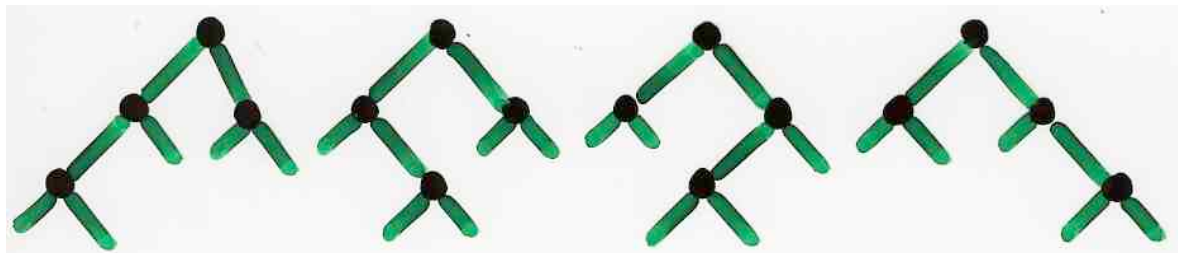
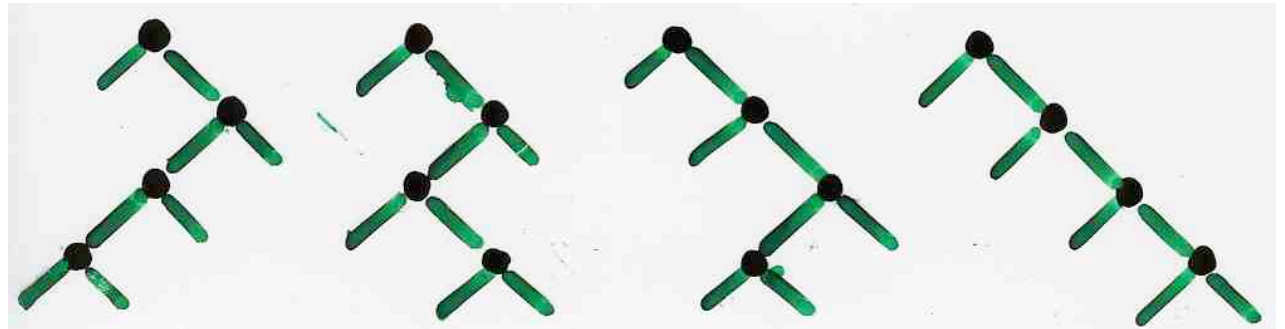
$$C_2 = 2$$



$$C_3 = 5$$



$$C_4 = 14$$



nombre
de
Catalan

$$C_n = \frac{1}{n+1} \binom{2n}{n}$$

$$= \frac{(2n)!}{(n+1)! n!}$$

$$n! = 1 \times 2 \times \dots \times n$$

nombres de Catalan

1 1 2 5 14 42



E. Catalan
(1814 - 1894)

Geht, und steht hier auf 8 nicht liegenden Stellen geschrieben. Auf der Diagonalen I. a_1^1 ; II. a_2^1, a_1^2 ; III. a_3^1, a_2^2, a_1^3 ; IV. $a_4^1, a_3^2, a_2^3, a_1^4$; V. $a_5^1, a_4^2, a_3^3, a_2^4, a_1^5$

Wenn hier ein Punkt hier 3 Diagonalen in 4 Triangula geschrieben, und steht hier auf 14 liegenden Stellen geschrieben.

Man ist hier Generaliter. In ein Polygonum hier n Punkte hier $n-3$ Diagonalen in $n-2$ Triangula geschrieben, auf der letzten liegenden Stellen geschrieben.

Es ist eine in der Folge dieser liegenden Stellen = x

so ist

Wenn $n = 1, 2, 5, 14, 42, 132, 429, 1430, \dots$

so ist $x = 1, 2, 5, 14, 42, 132, 429, 1430, \dots$

Es ist eine in der Folge dieser liegenden Stellen = x

$$x = \frac{2 \cdot 6 \cdot 10 \cdot 14 \cdot 18 \cdot 22 \cdot \dots \cdot (4n-10)}{2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7 \cdot \dots \cdot (n-1)} = \frac{(2n)!}{(n+1)!n!}$$

$$14 = 5 \cdot \frac{14}{5}, 42 = 14 \cdot \frac{3}{2}, 132 = 42 \cdot \frac{4}{3}, 1430 = 132 \cdot \frac{11}{5}$$

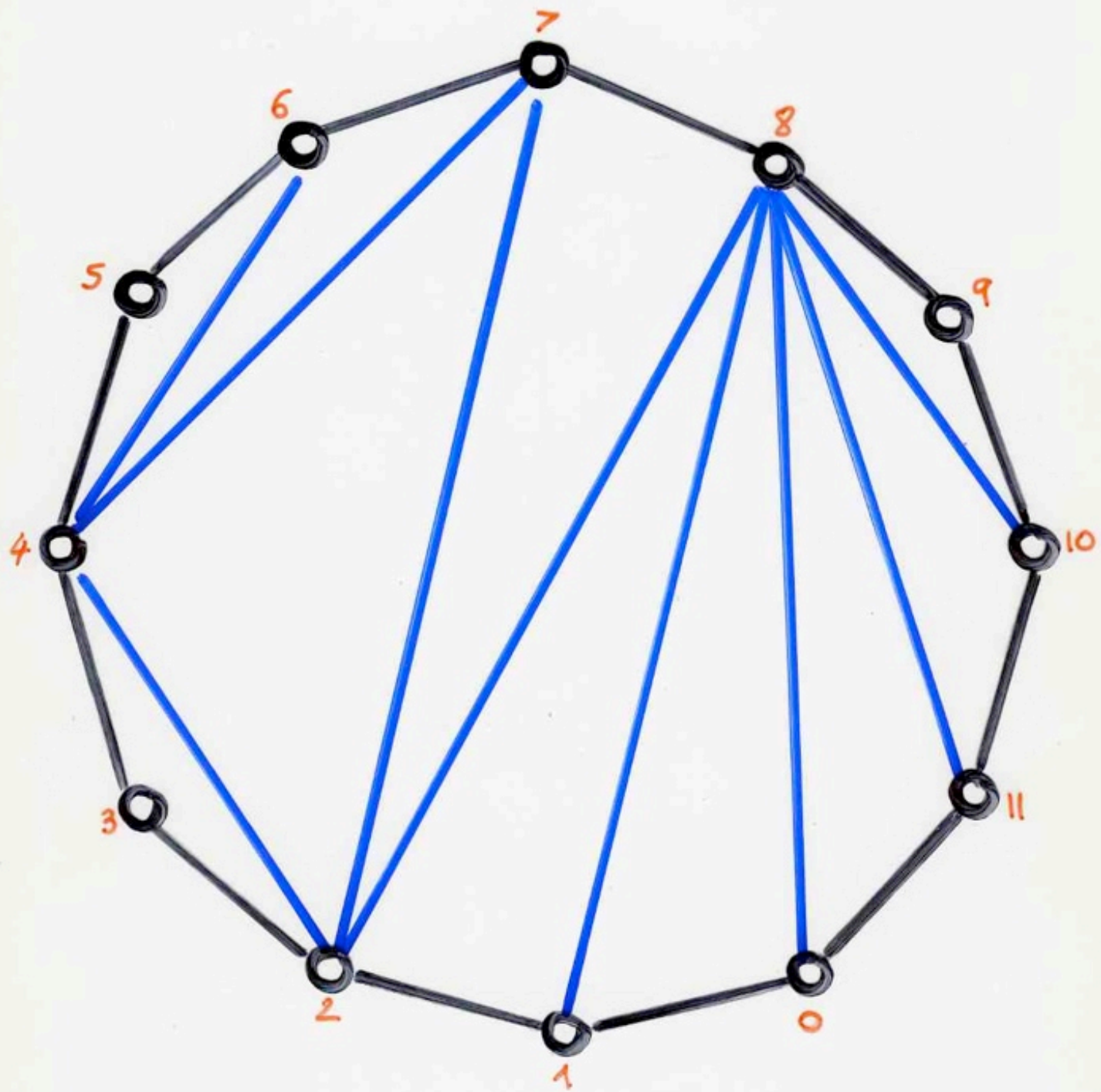
$$C_n = \frac{1}{n+1} \binom{2n}{n} \quad n! = 1 \times 2 \times 3 \times \dots \times n$$

Une lettre de Leonhard Euler
à Christian Goldbach

Berlin, 4 Septembre 1751

Leonhard
Euler
1707 - 1783





vermuthet, dass die Reihe ist unendl. Ist die Reihe unendl. so ist die Summe die Propaganda

$$\frac{1 - 2a - \sqrt{1 - 4a^2}}{2a^2}$$

$$1 + 2a + 5a^2 + 14a^3 + 42a^4 + 132a^5 + \text{etc}$$

gemischte Art

$$1 + 2a + 5a^2 + 14a^3 + 42a^4 + 132a^5 + \text{etc} = \frac{1 - 2a - \sqrt{1 - 4a^2}}{2a^2}$$

$$\text{alle wenn } a = \frac{1}{4} \quad 1 + \frac{2}{4} + \frac{5}{16} + \frac{14}{64} + \frac{42}{256} + \text{etc} = 1$$

Die hier erwähnte Reihe ist für die Propaganda
unendlich und unbestimmt, jedoch nach unendlichkeit, und
es ist die Summe der Reihe die Propaganda
Reihe lang zu bestimmen

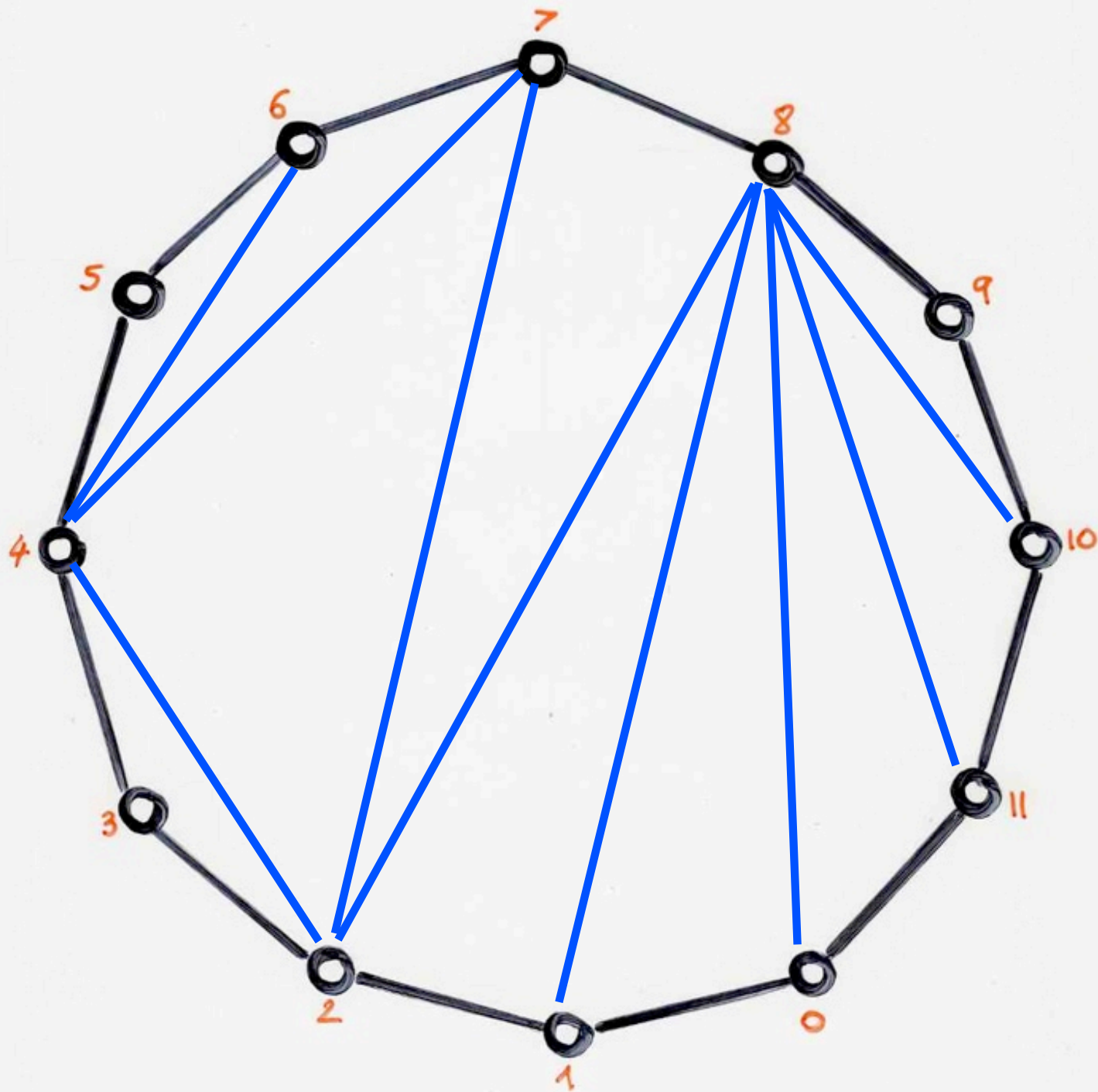
von Joseph Euler

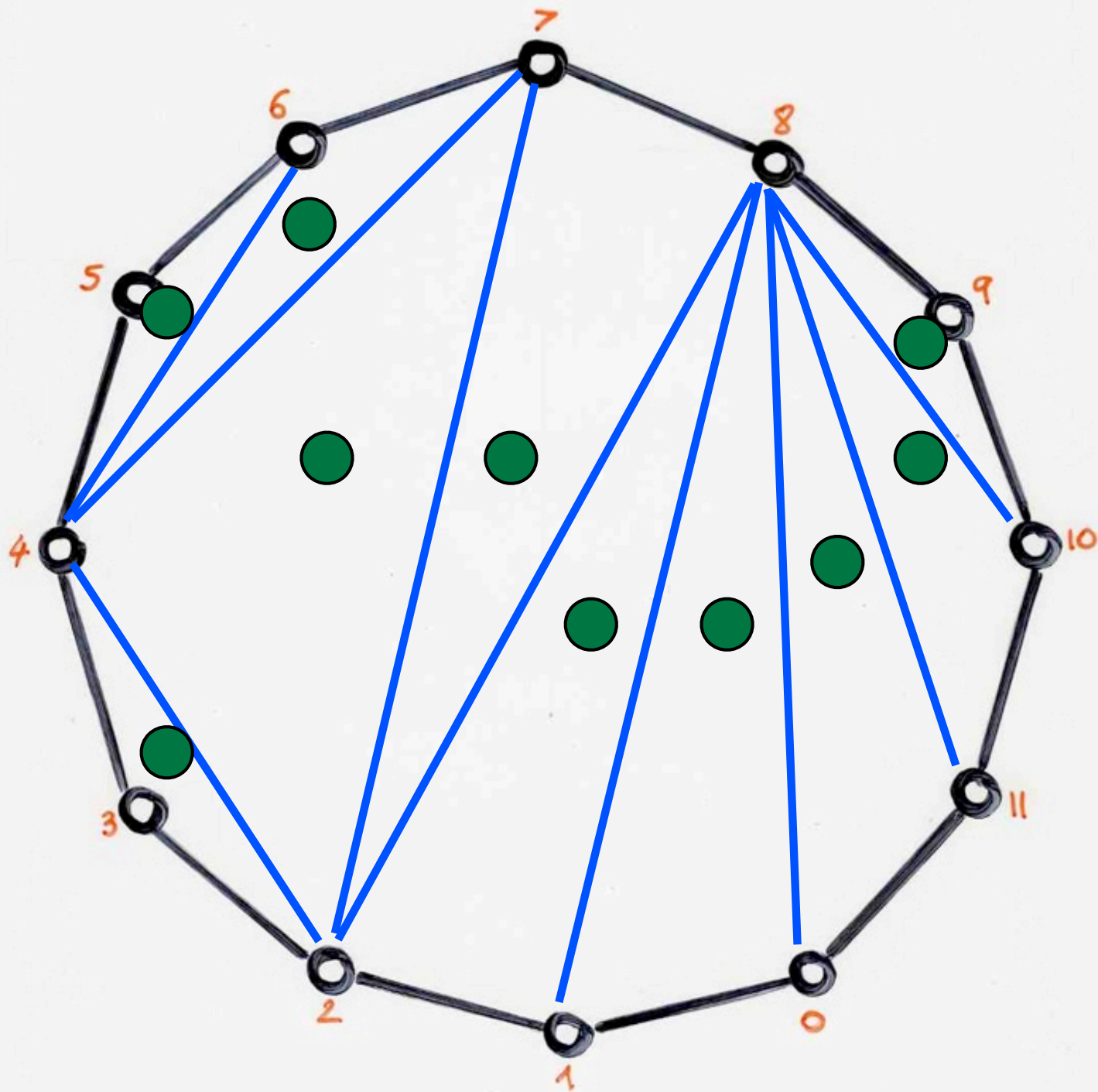
Wien 24^{te} Sept
1751.

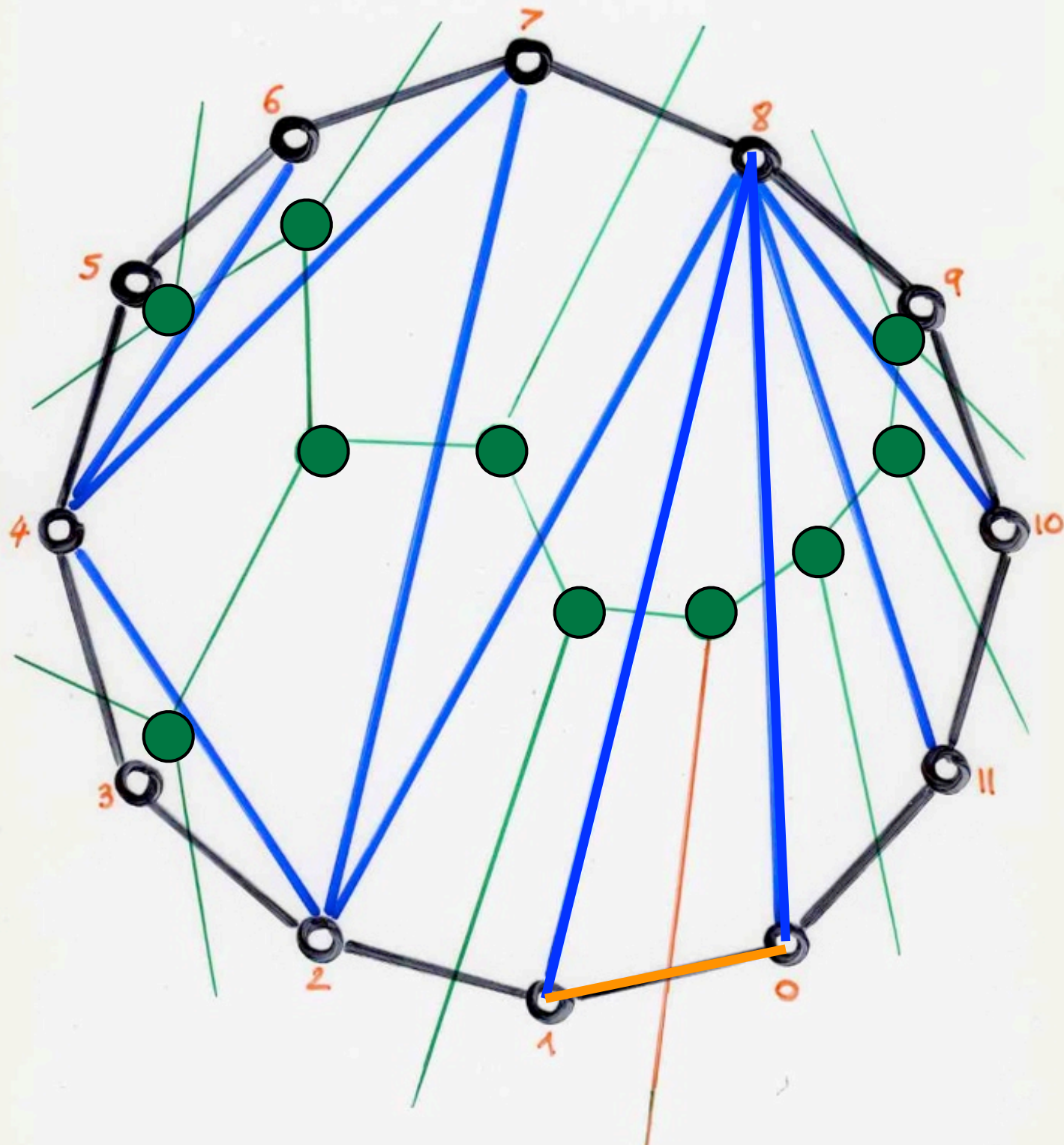
4 Sept 1751
Berlin

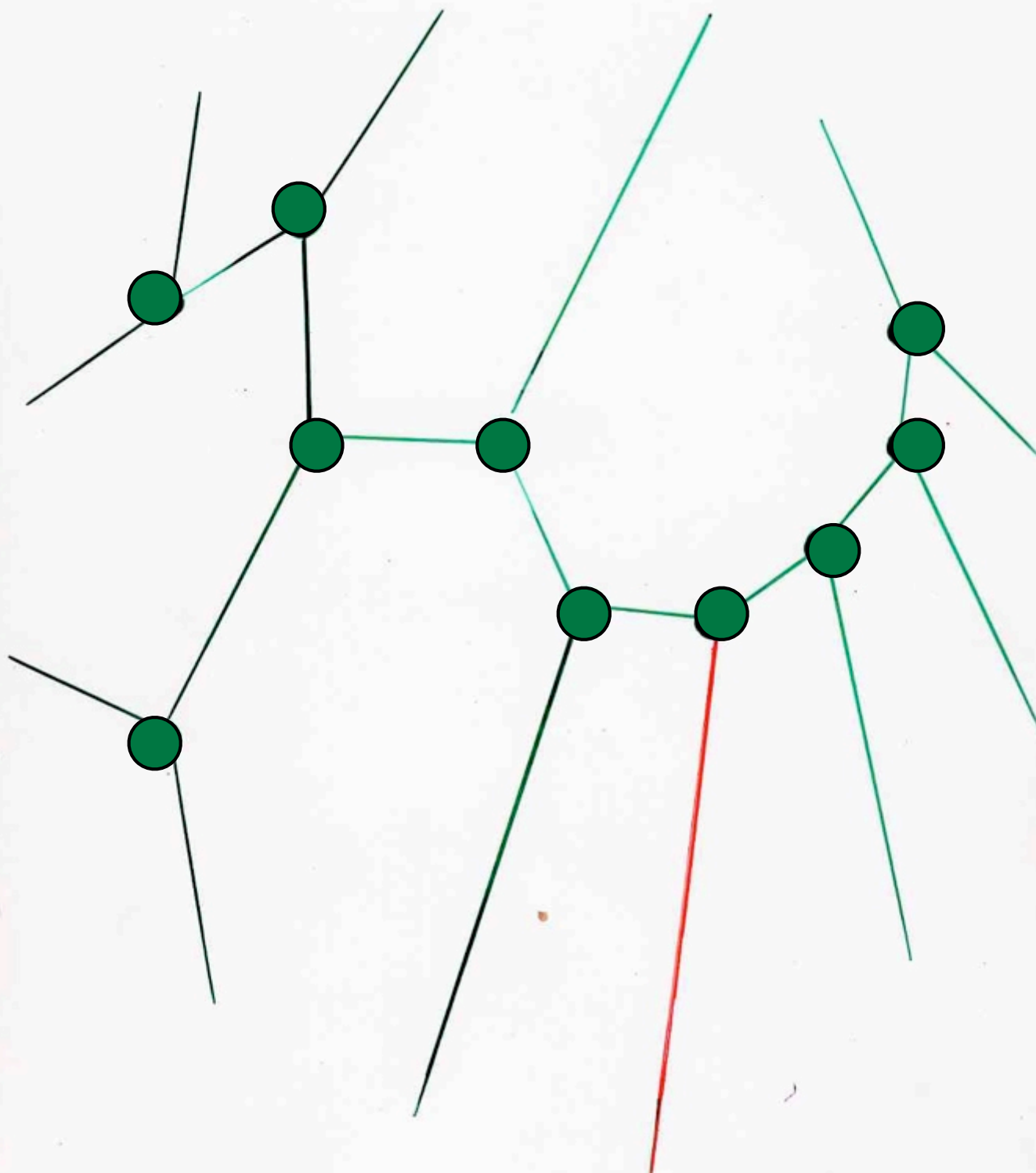
gefragte Summe
Euler

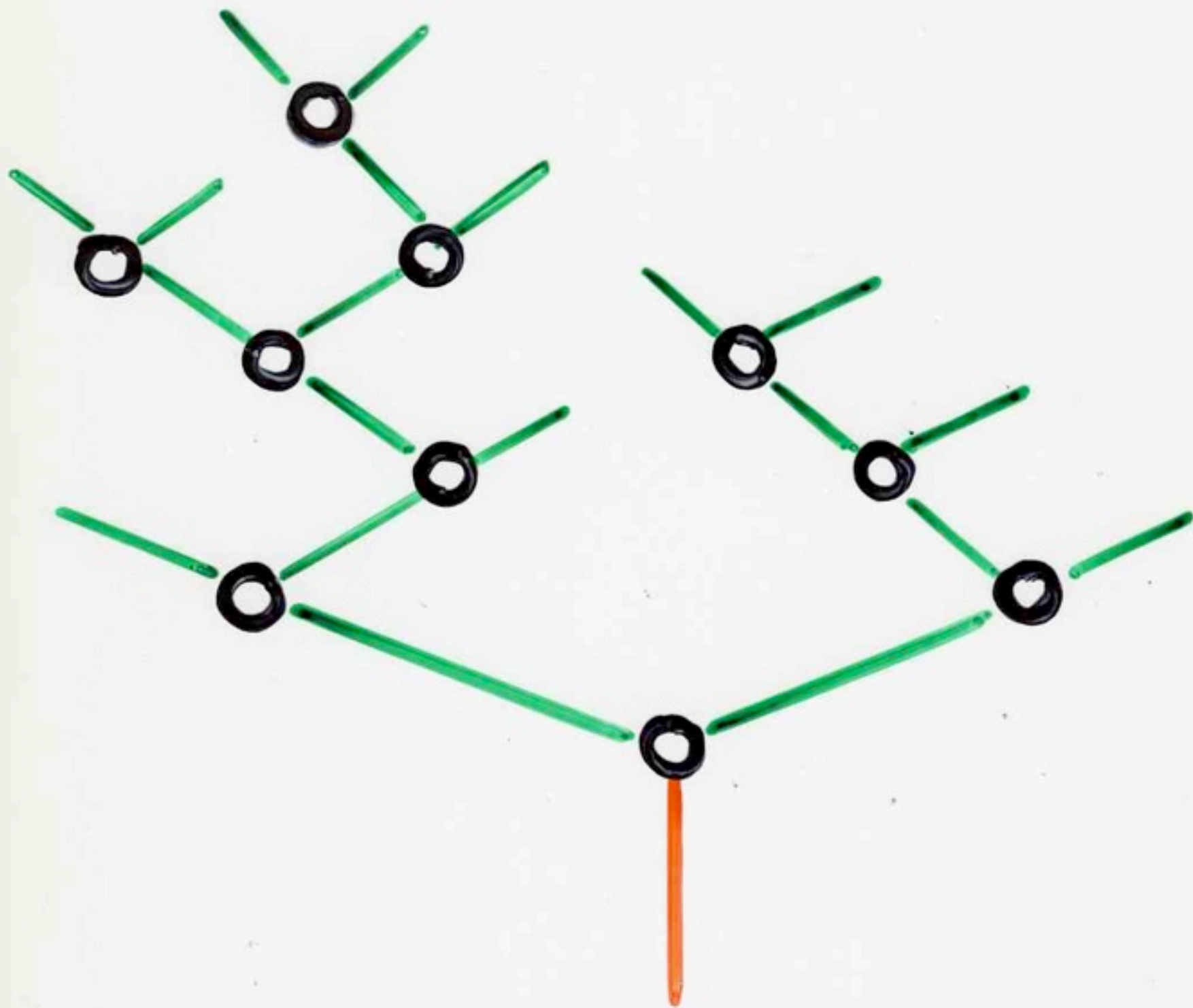
des triangulations
aux arbres binaires

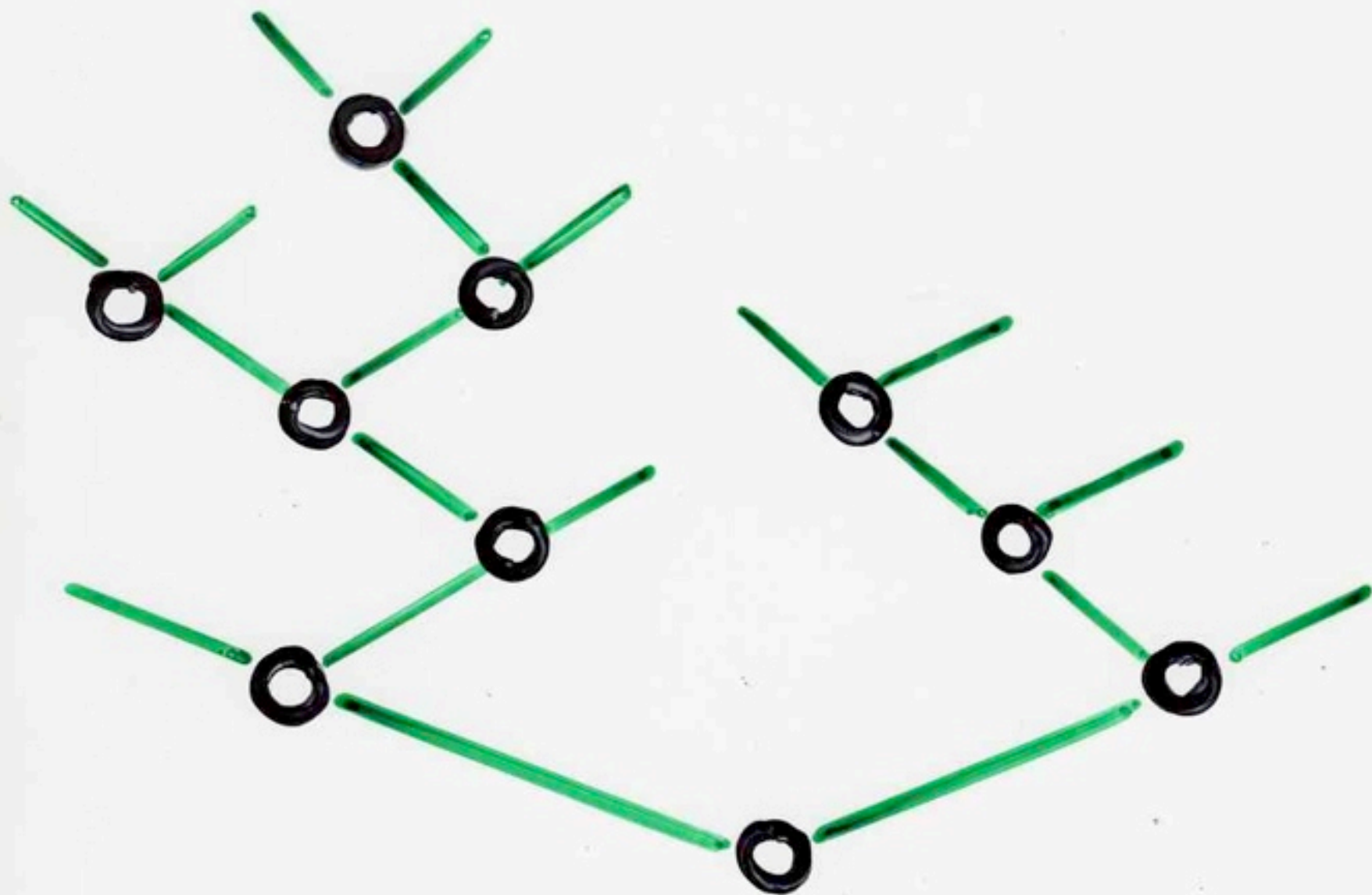






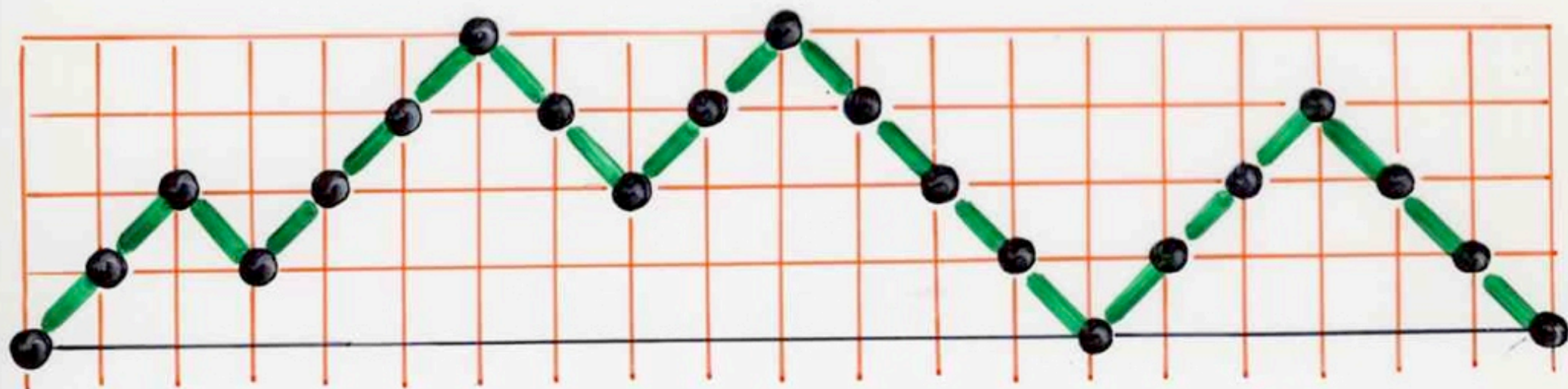






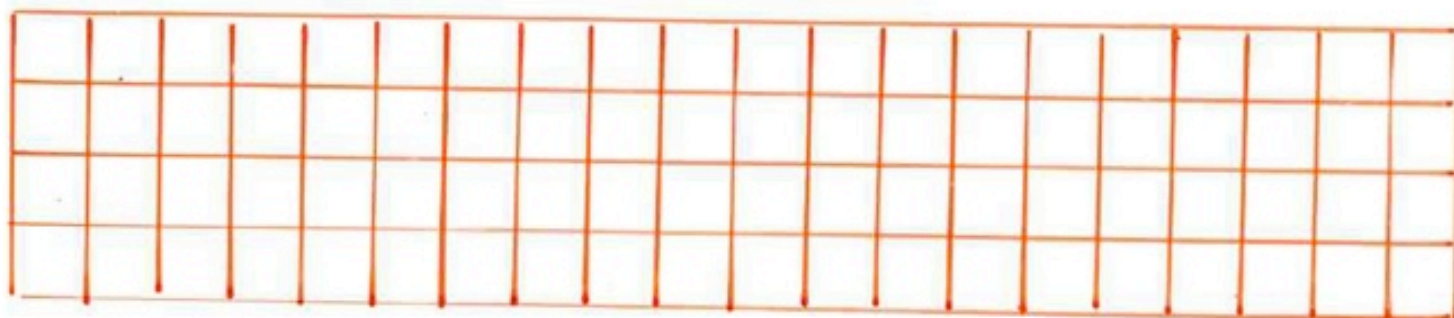
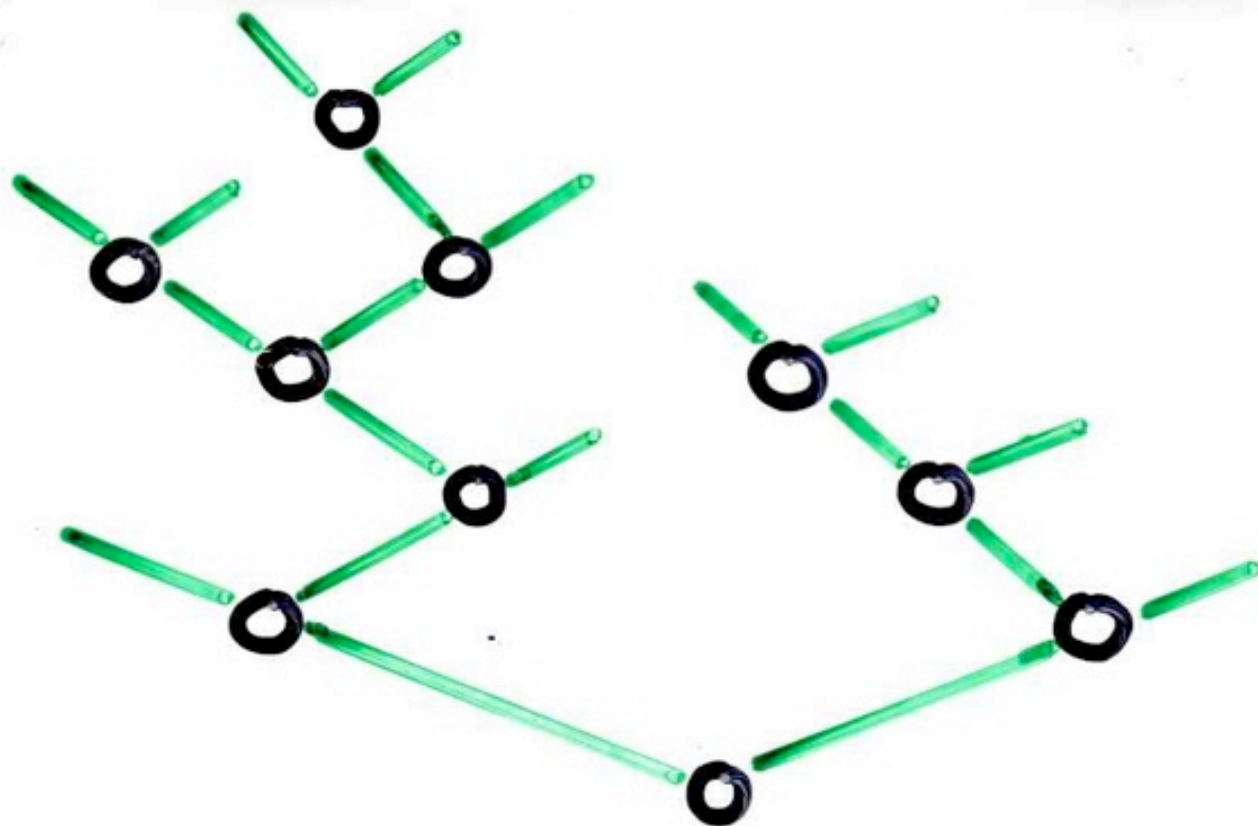
Comment démontrer
la relation existant entre

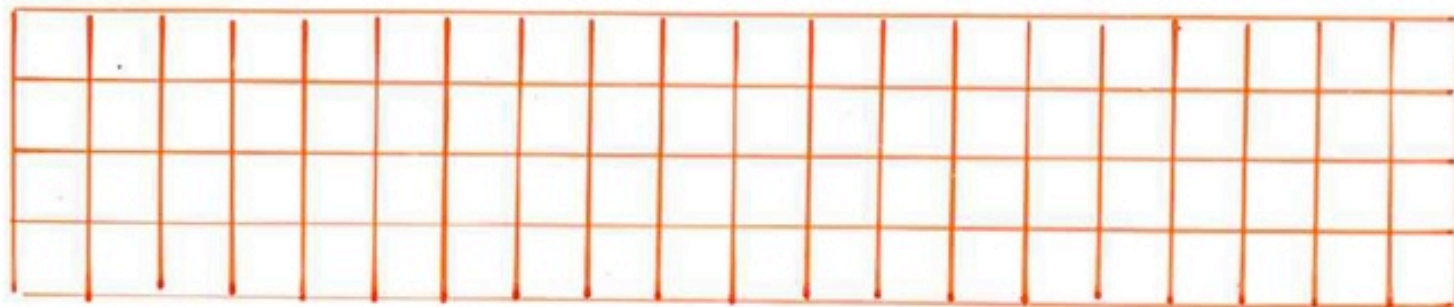
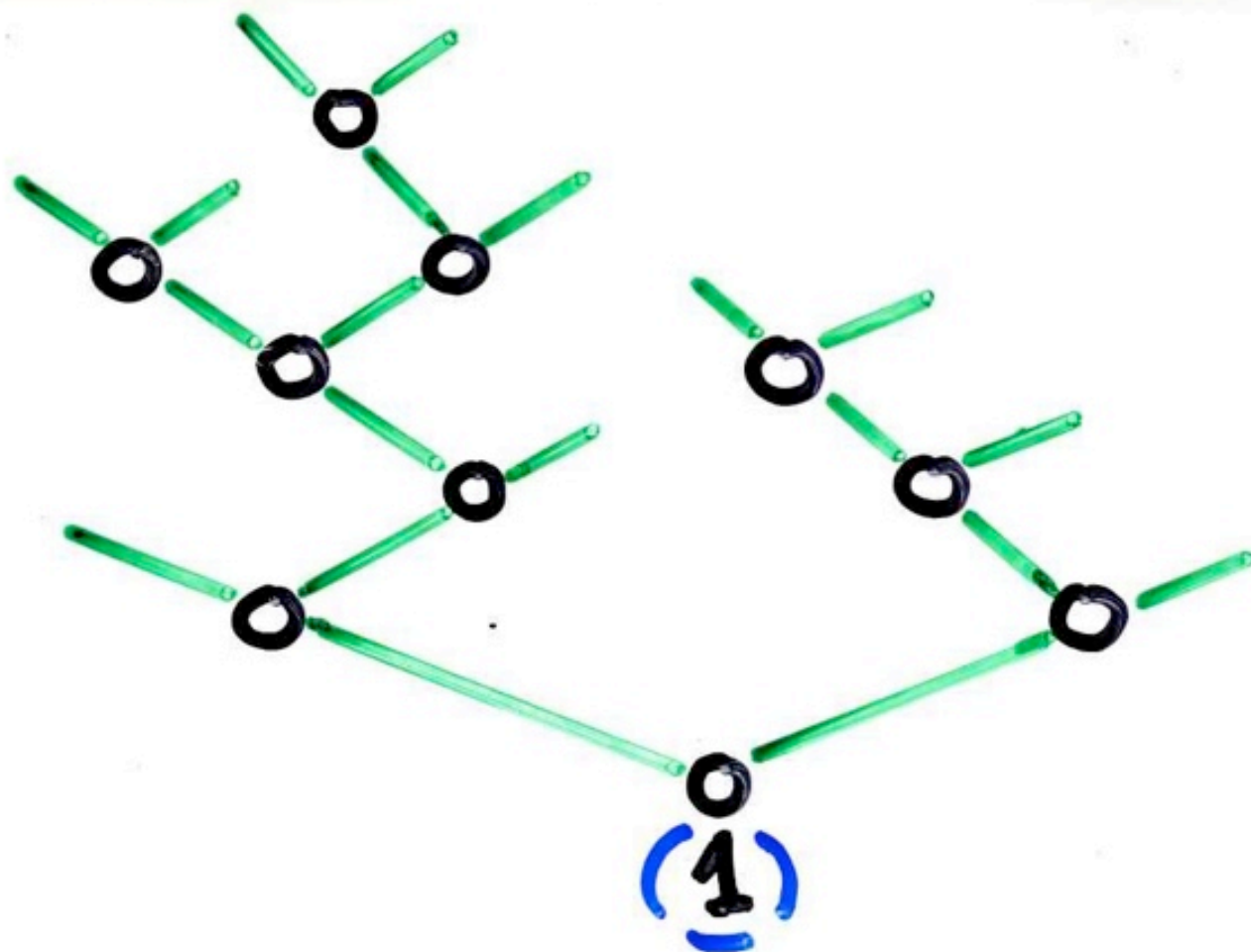
les nombres de Strahler
et
les polynômes de Tchebychev ?

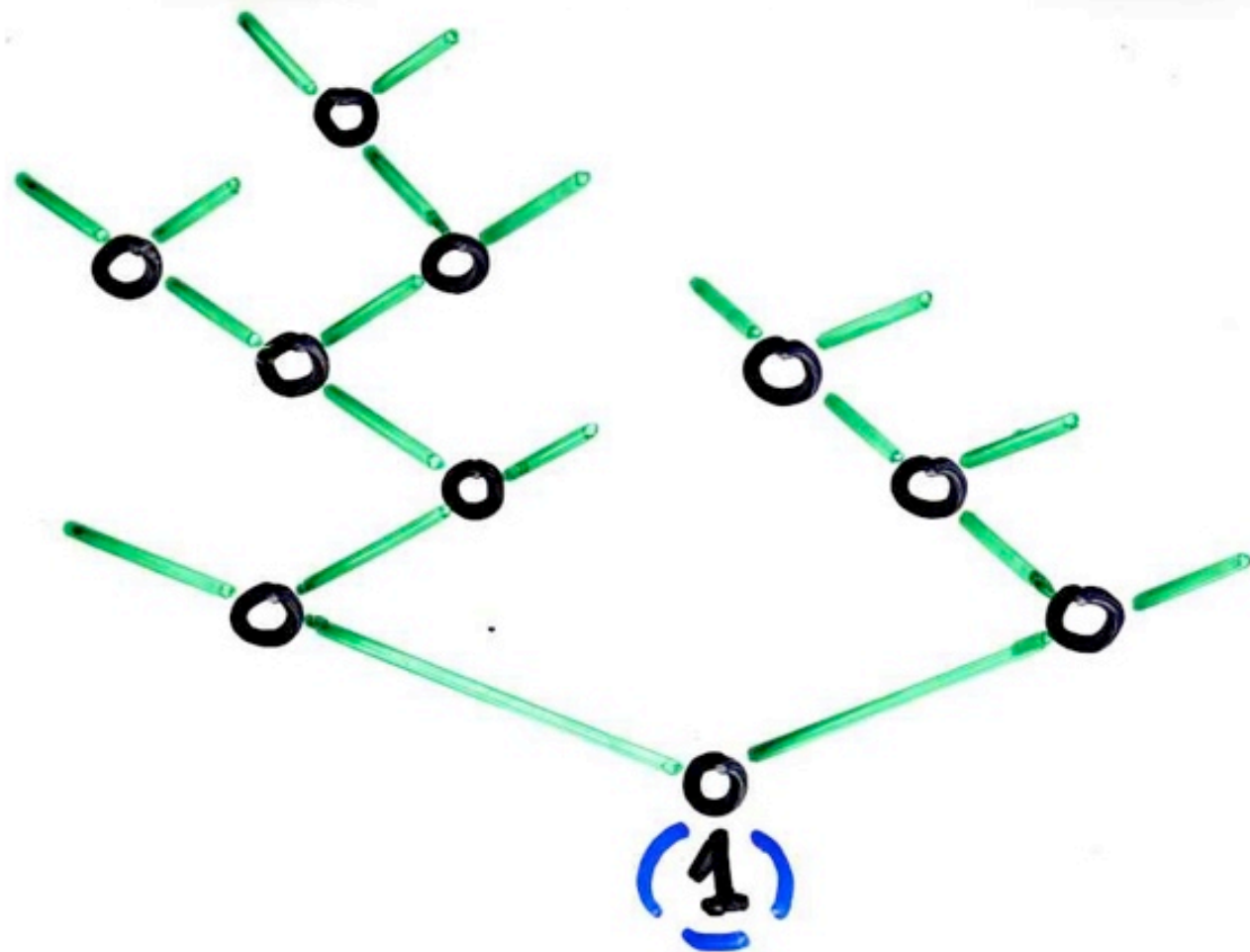


chemin de Dyck

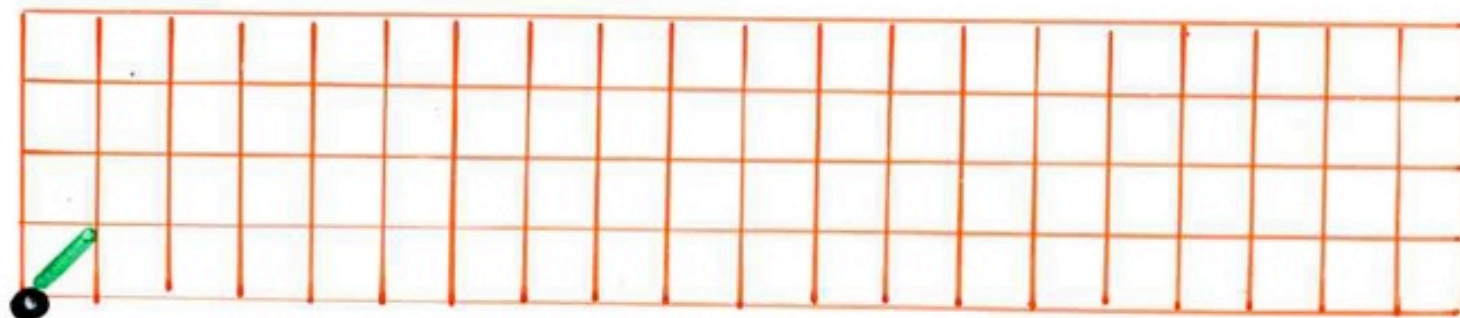
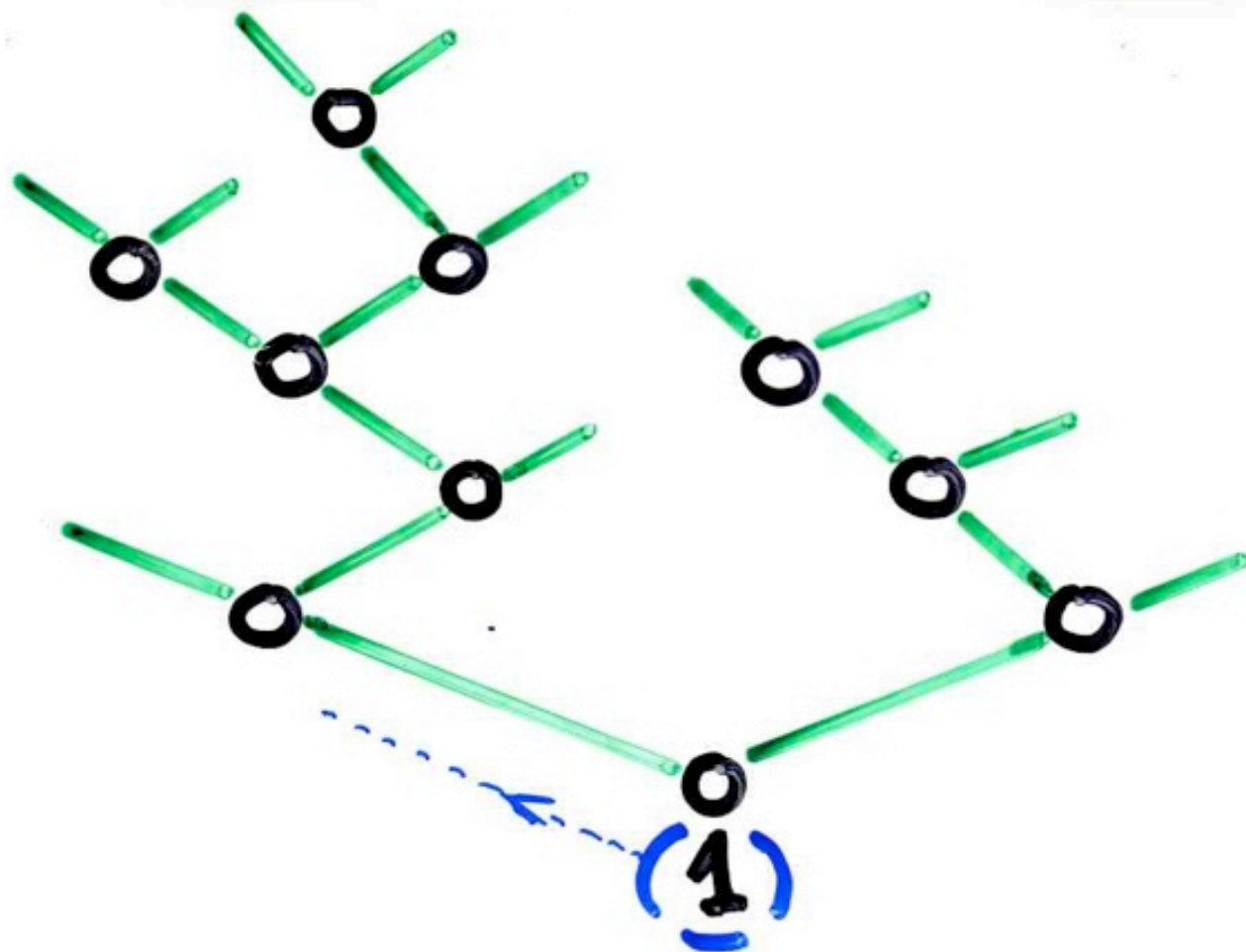
des arbres binaires ...
aux chemins de Dyck

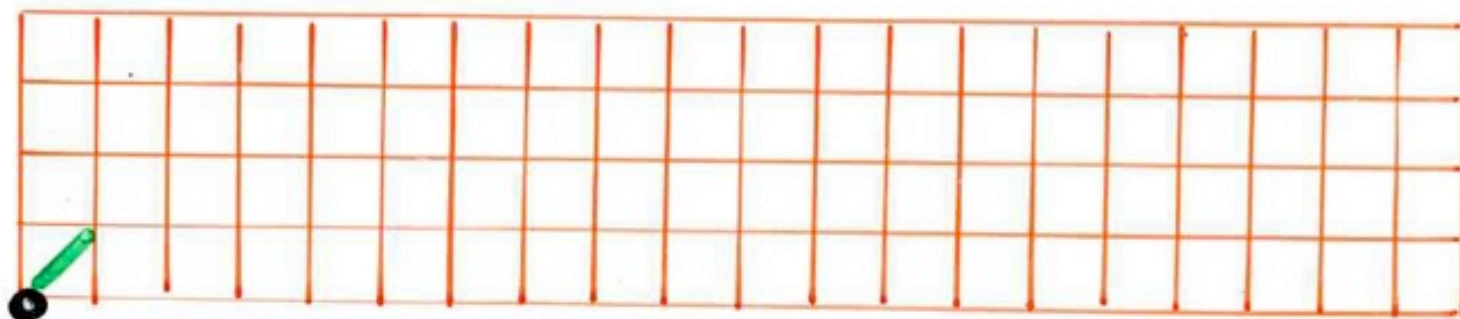
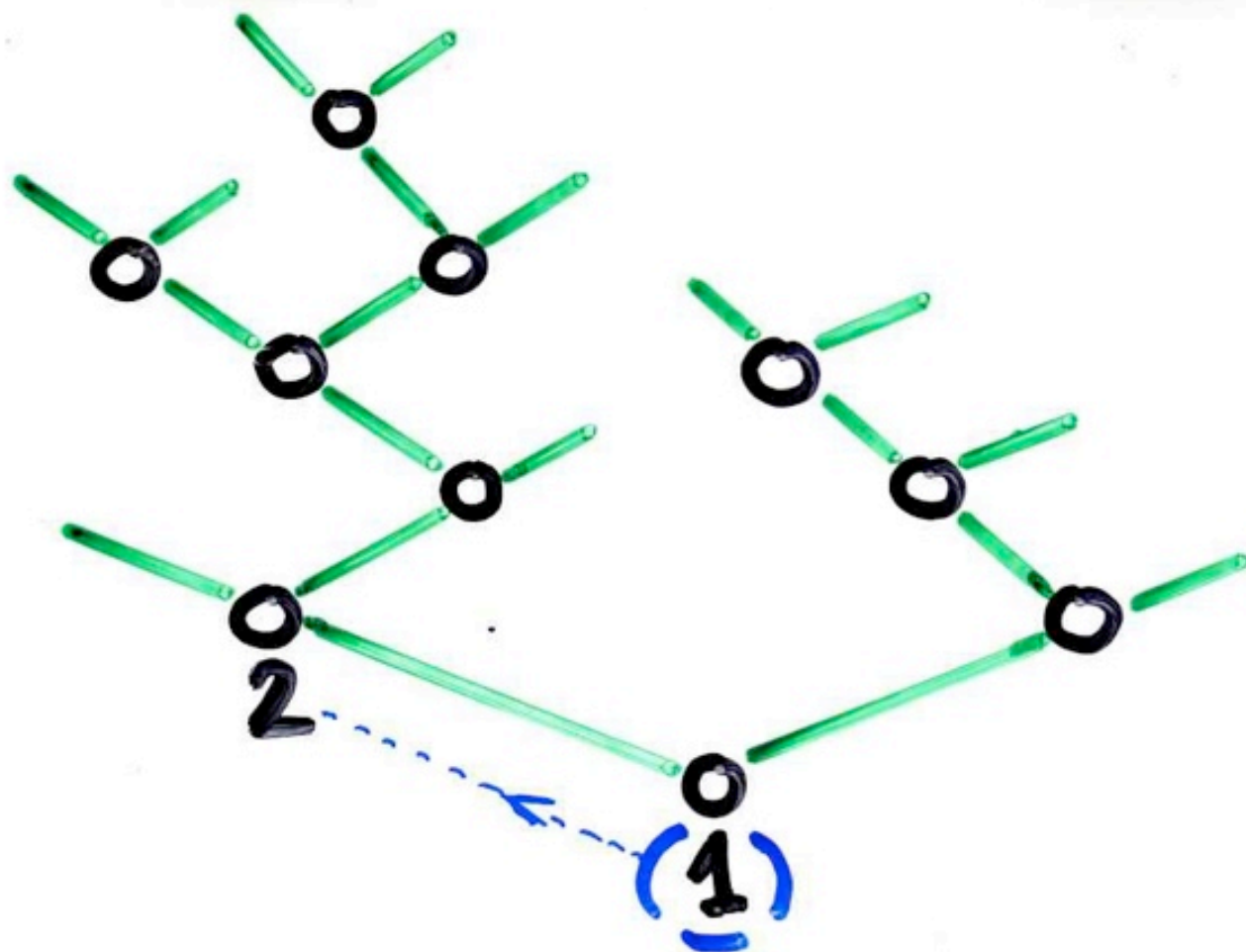


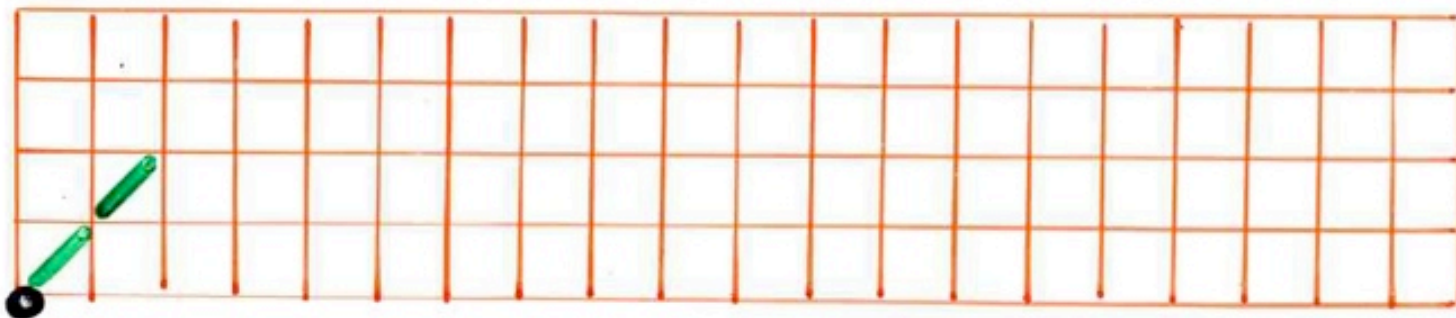
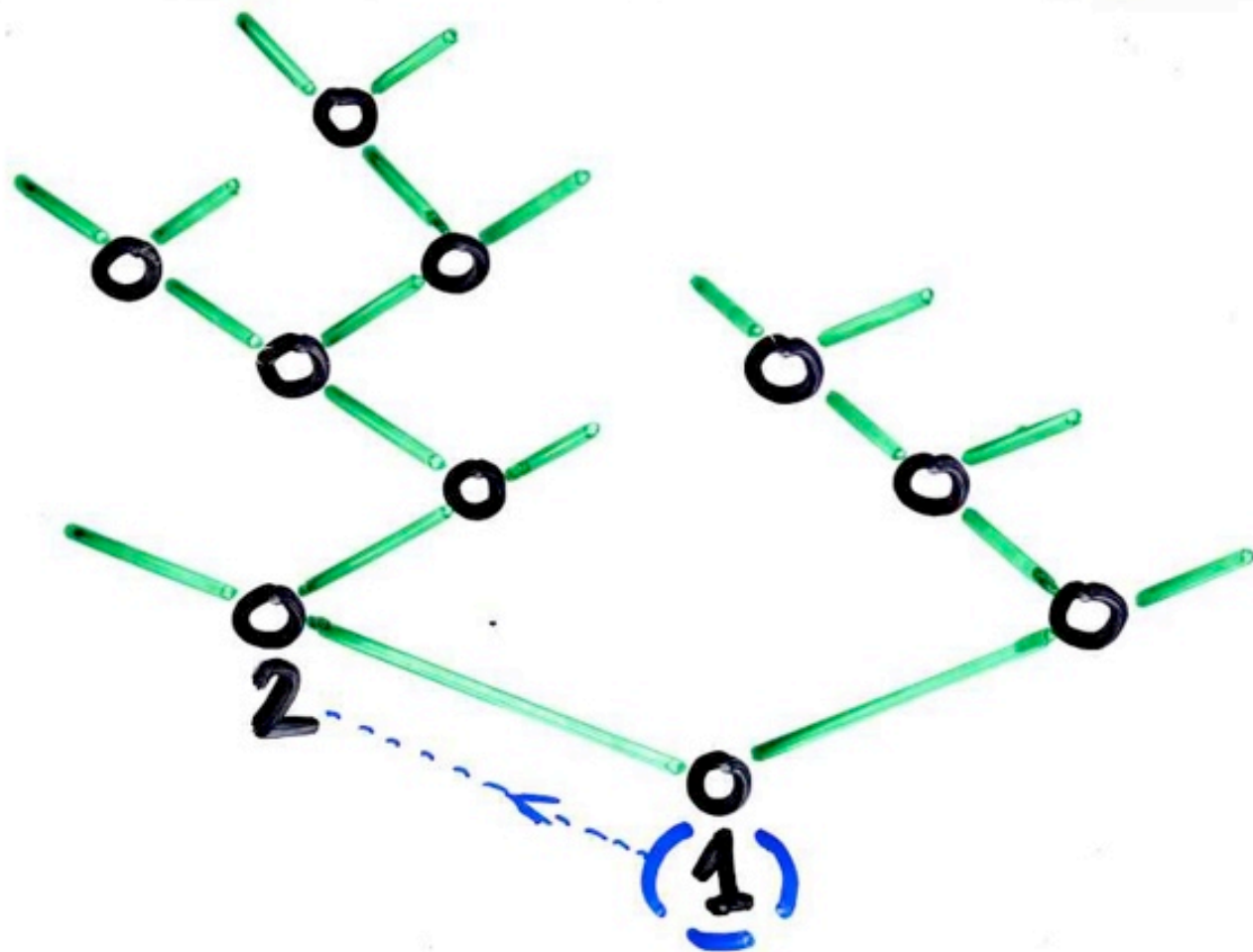


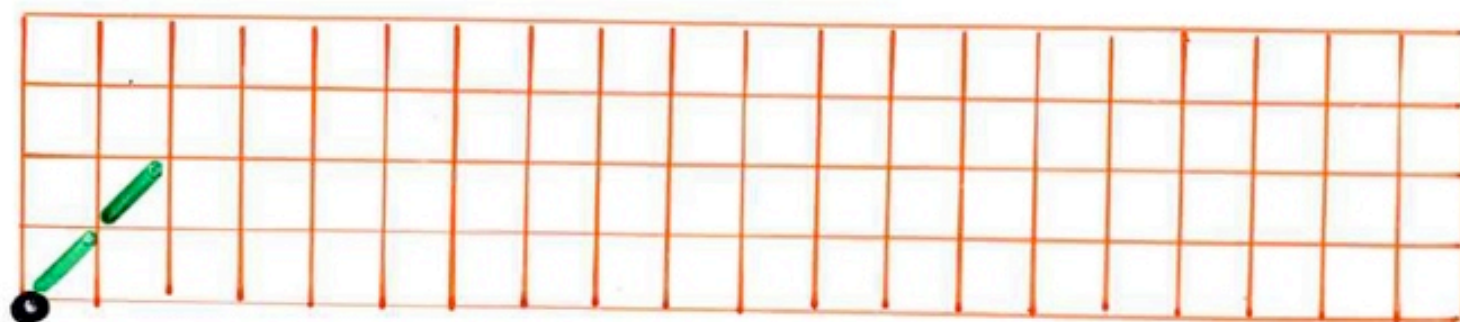
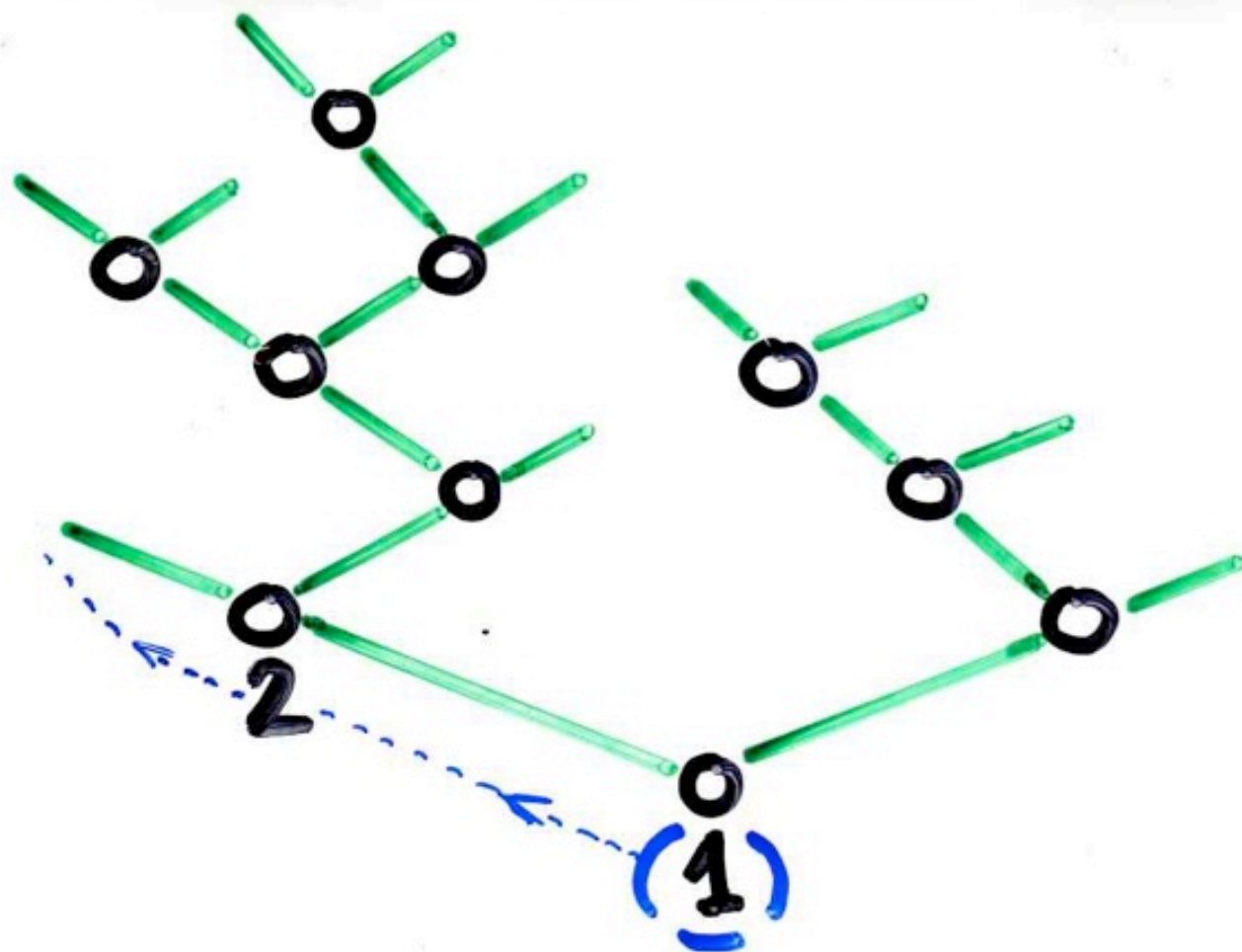


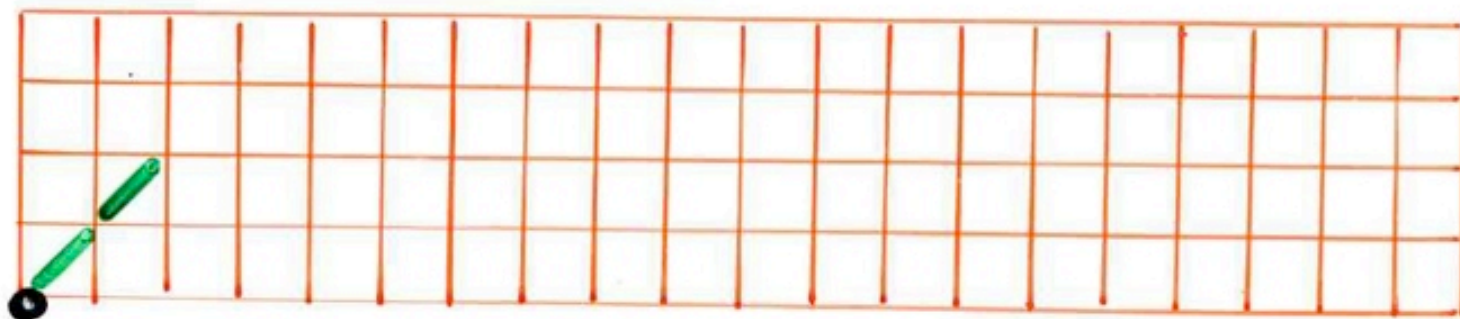
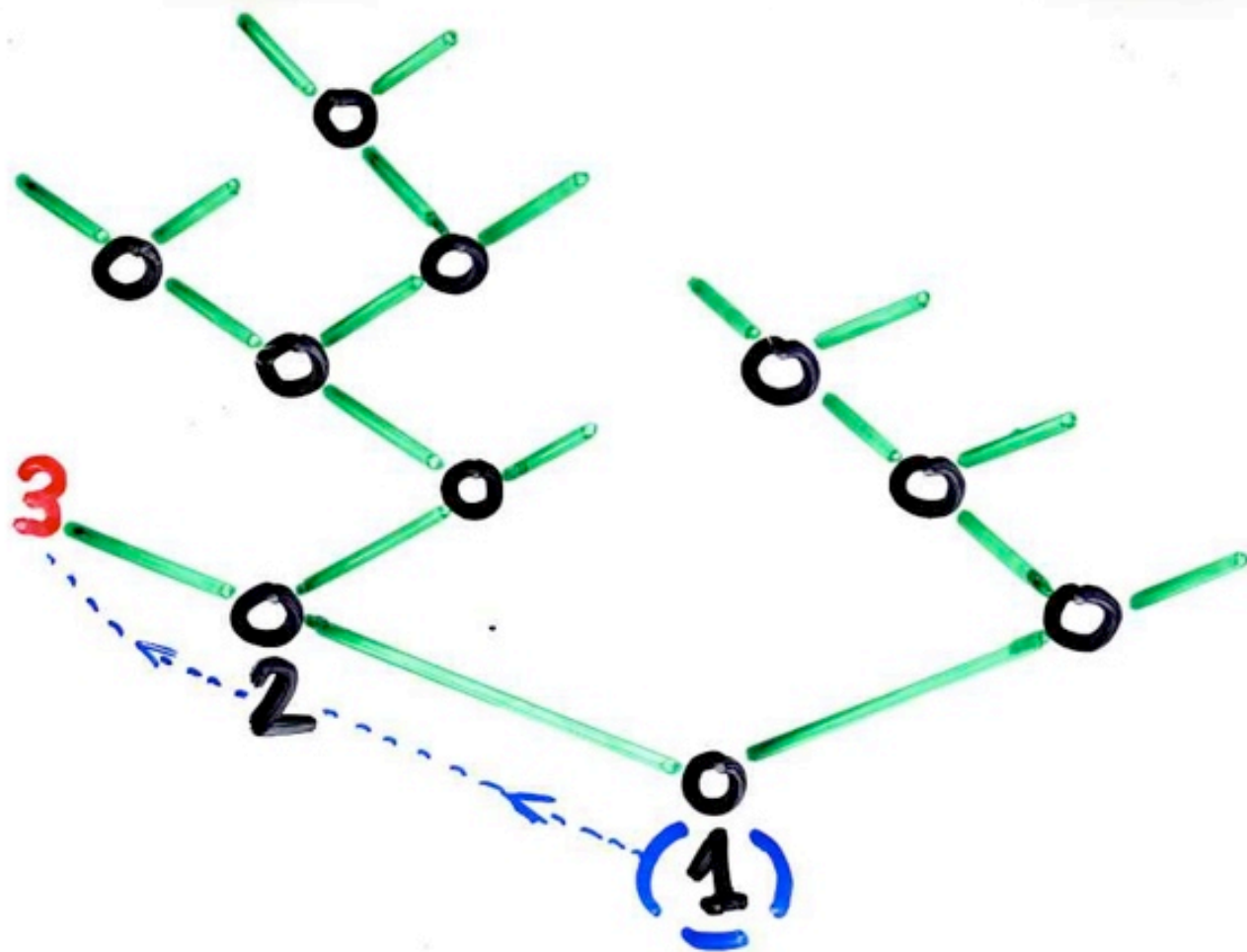
A grid consisting of 20 columns and 5 rows, outlined in orange. A small green line segment is attached to the bottom-left corner of the grid, extending downwards and to the left.

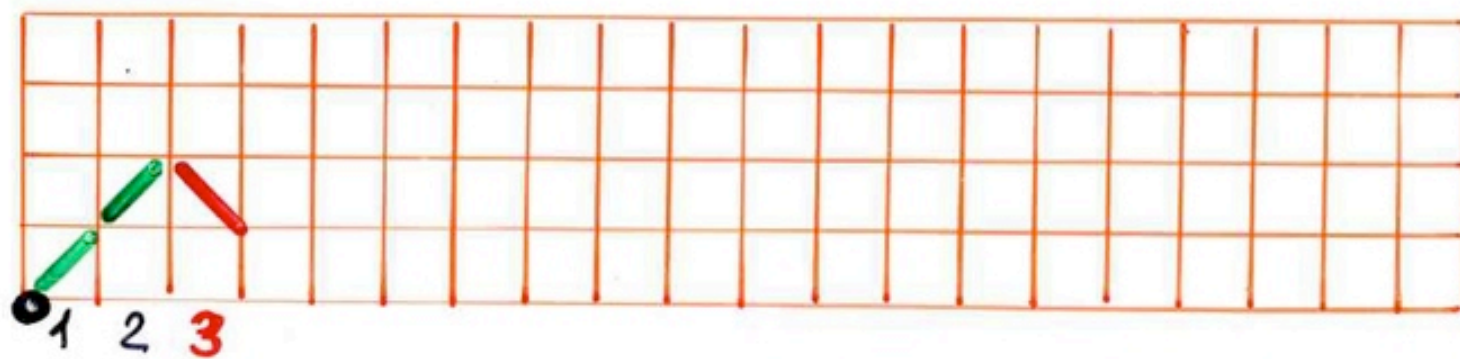
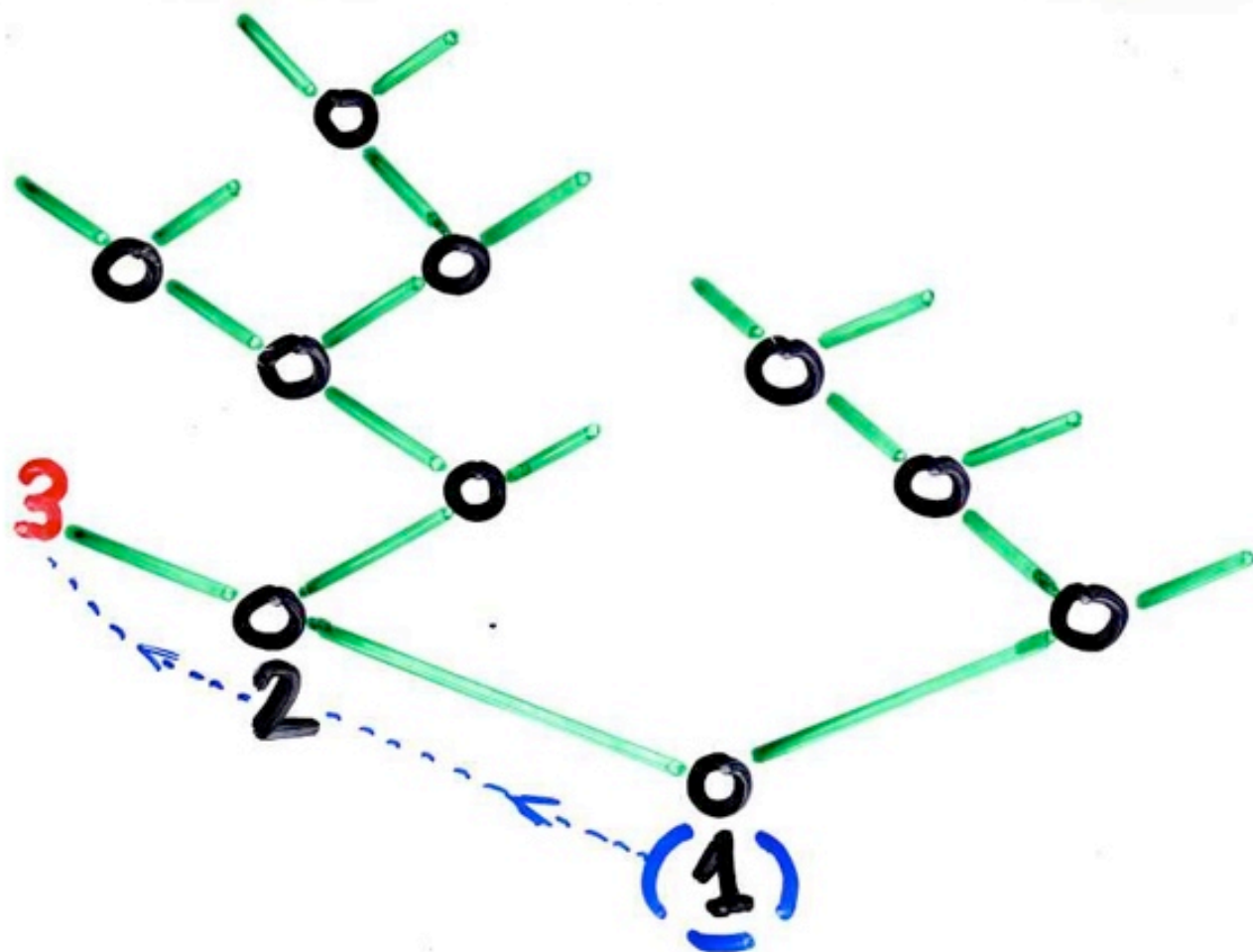


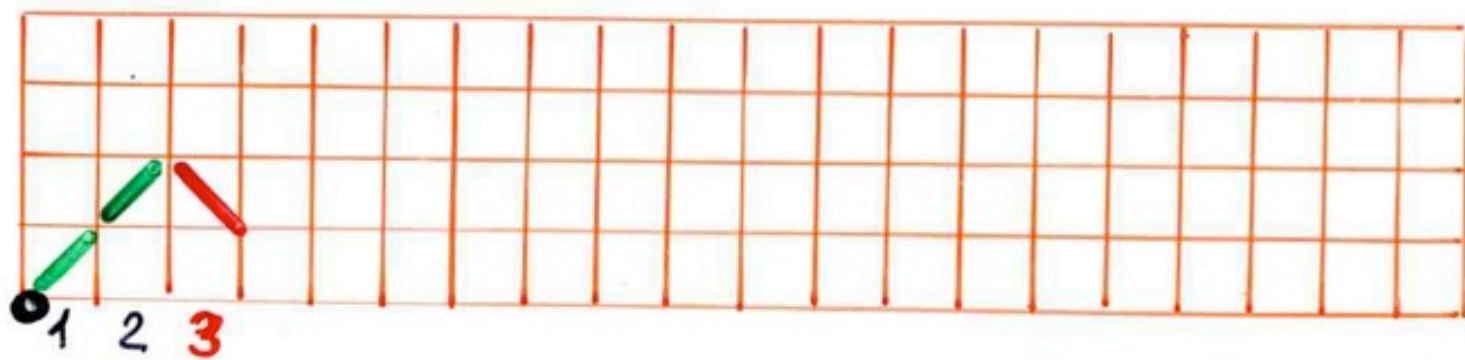
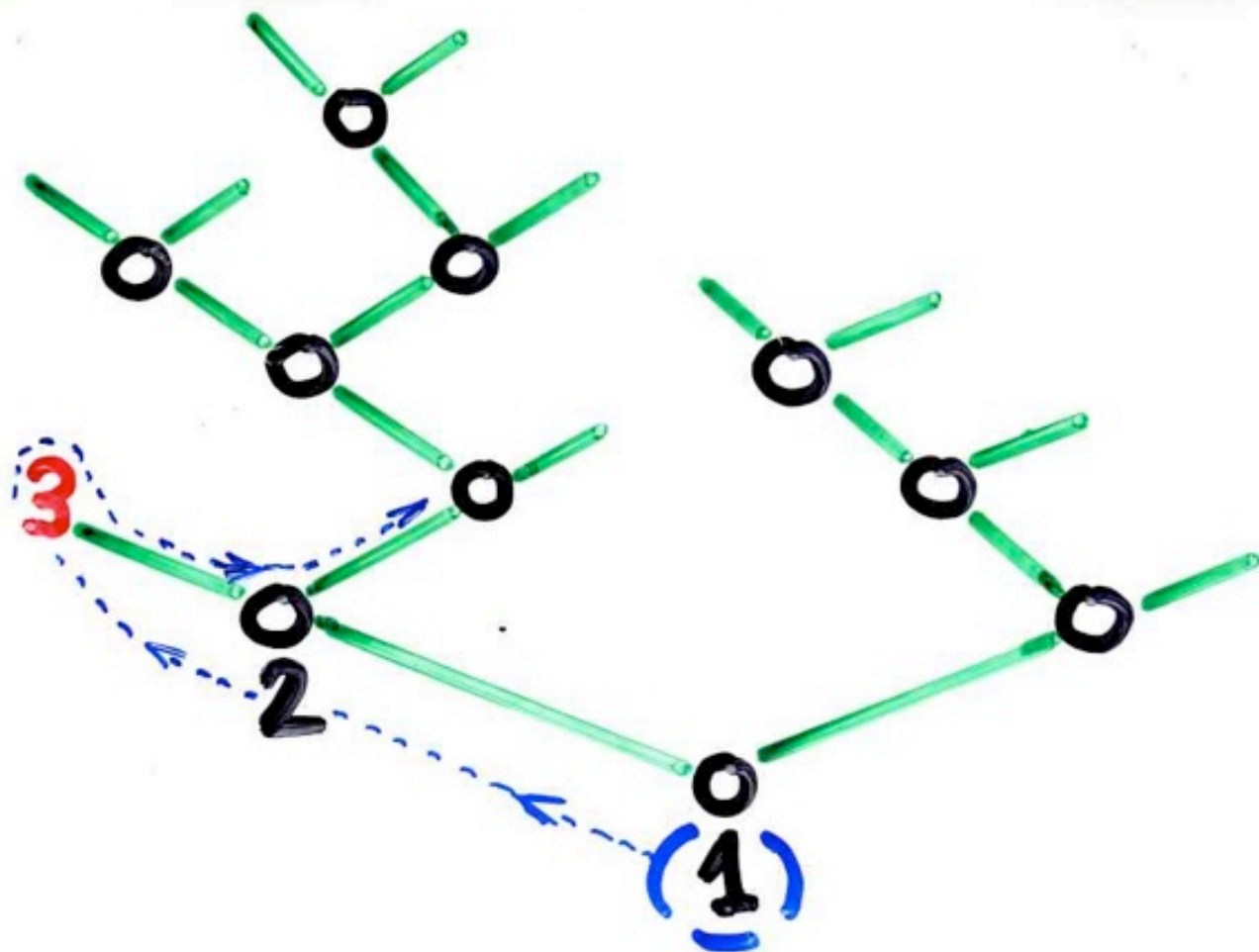


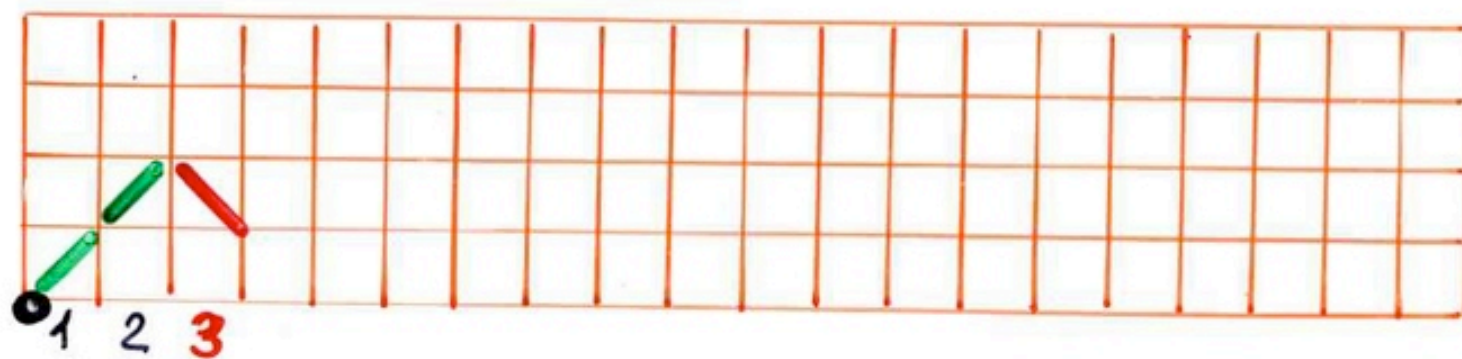
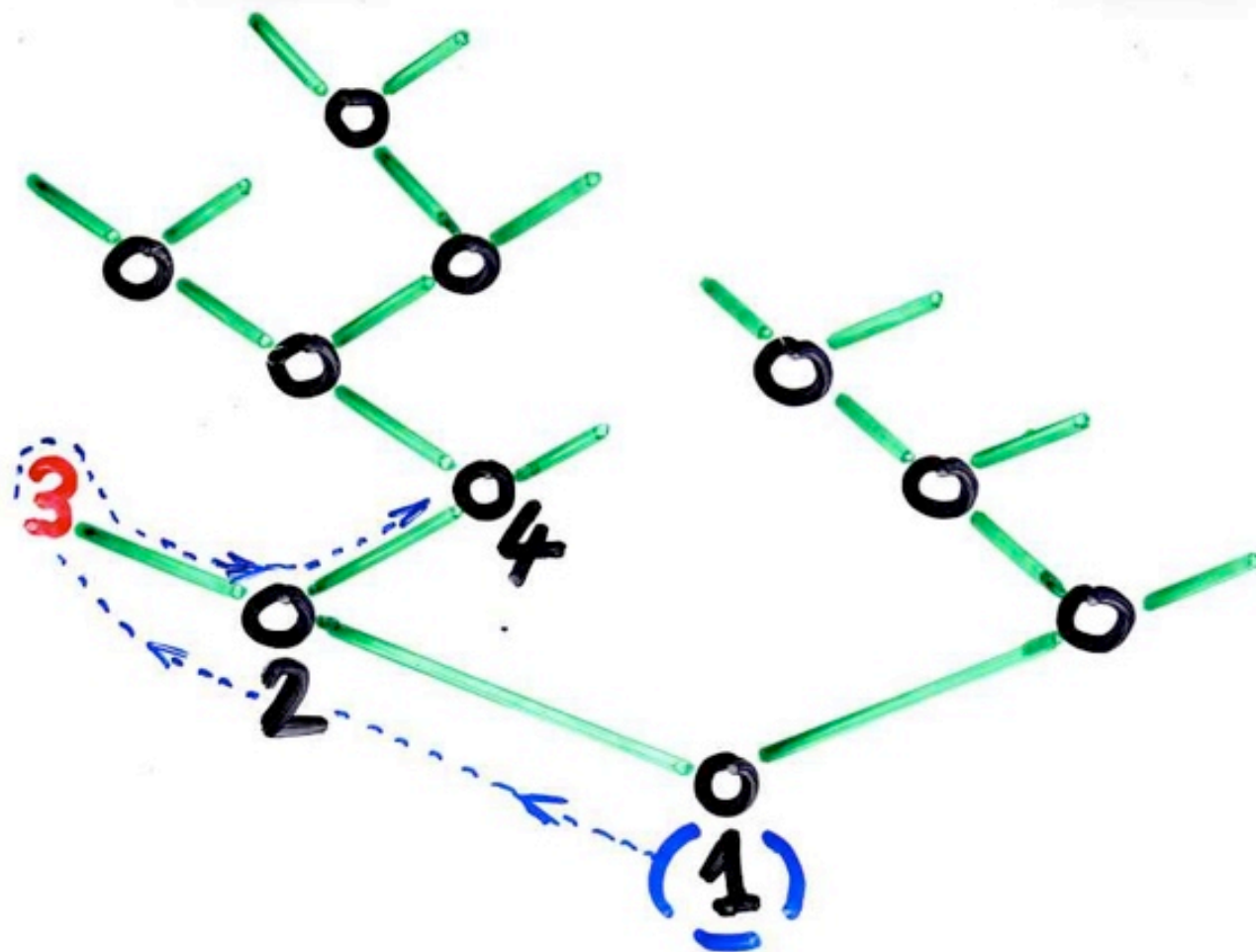


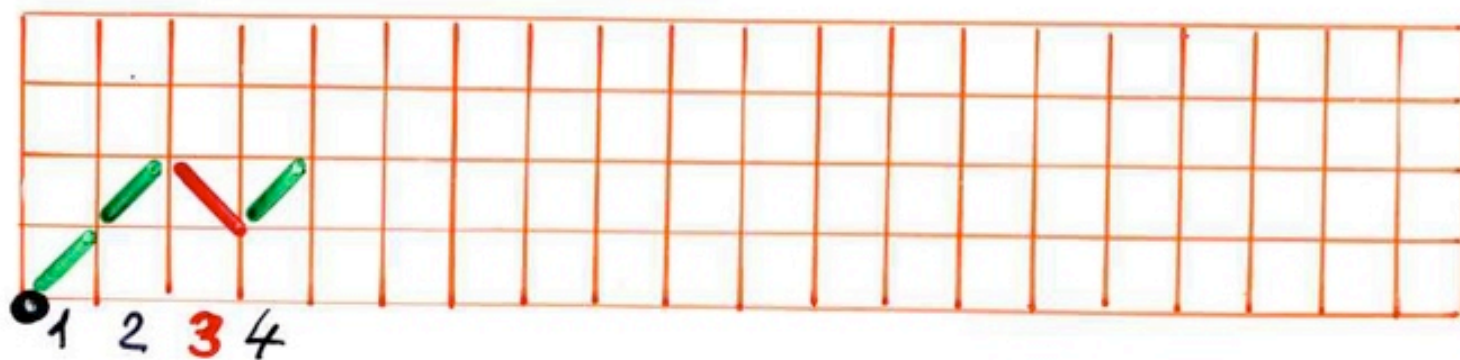
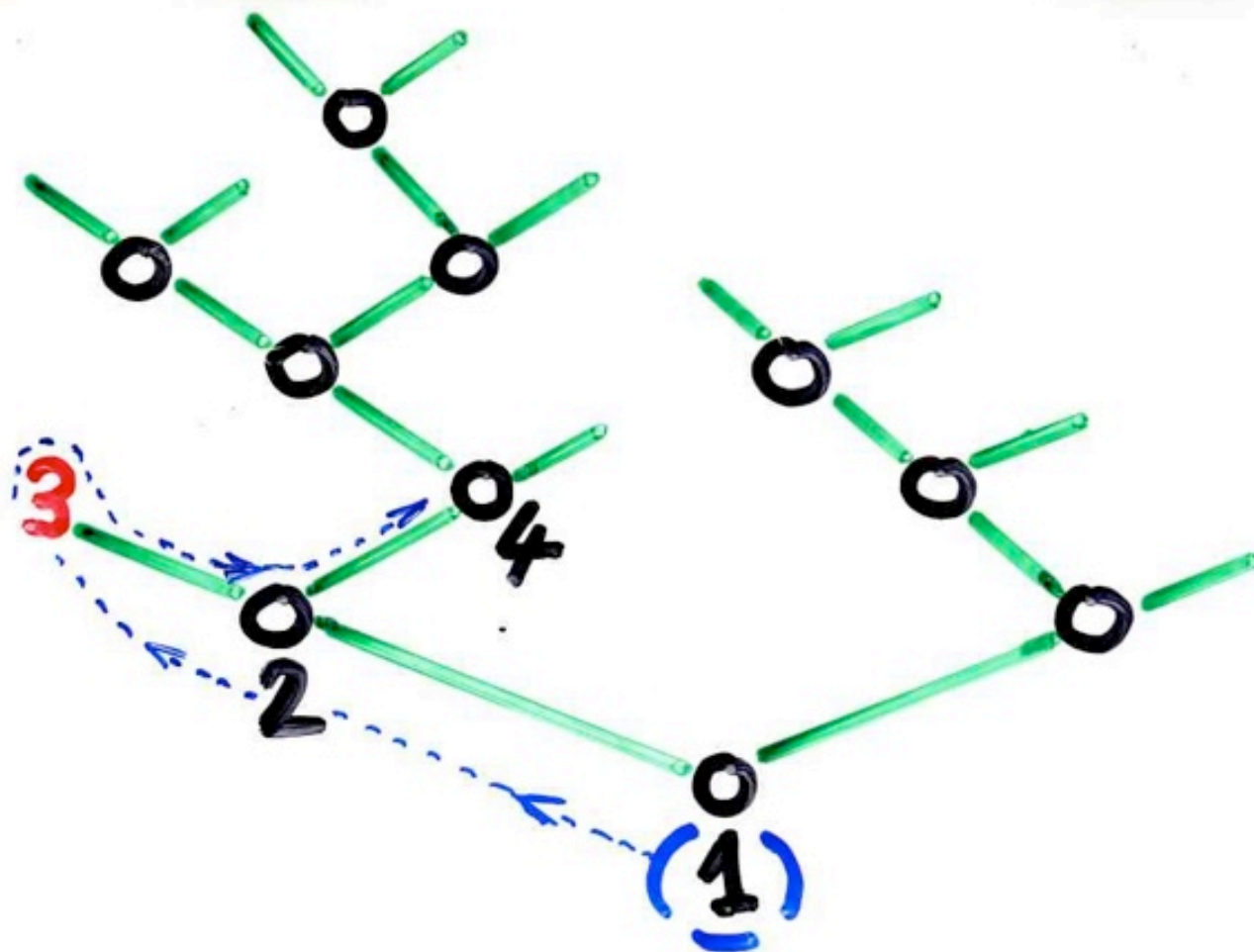


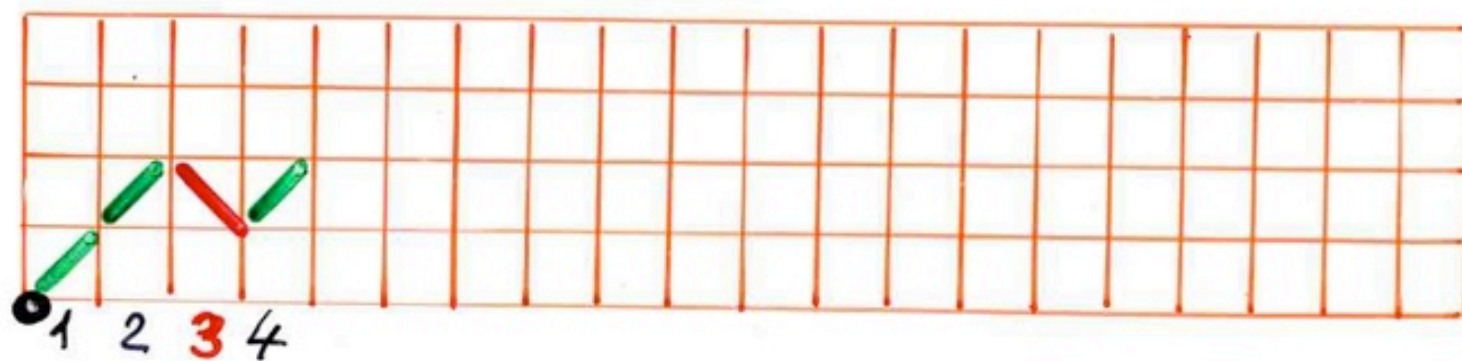
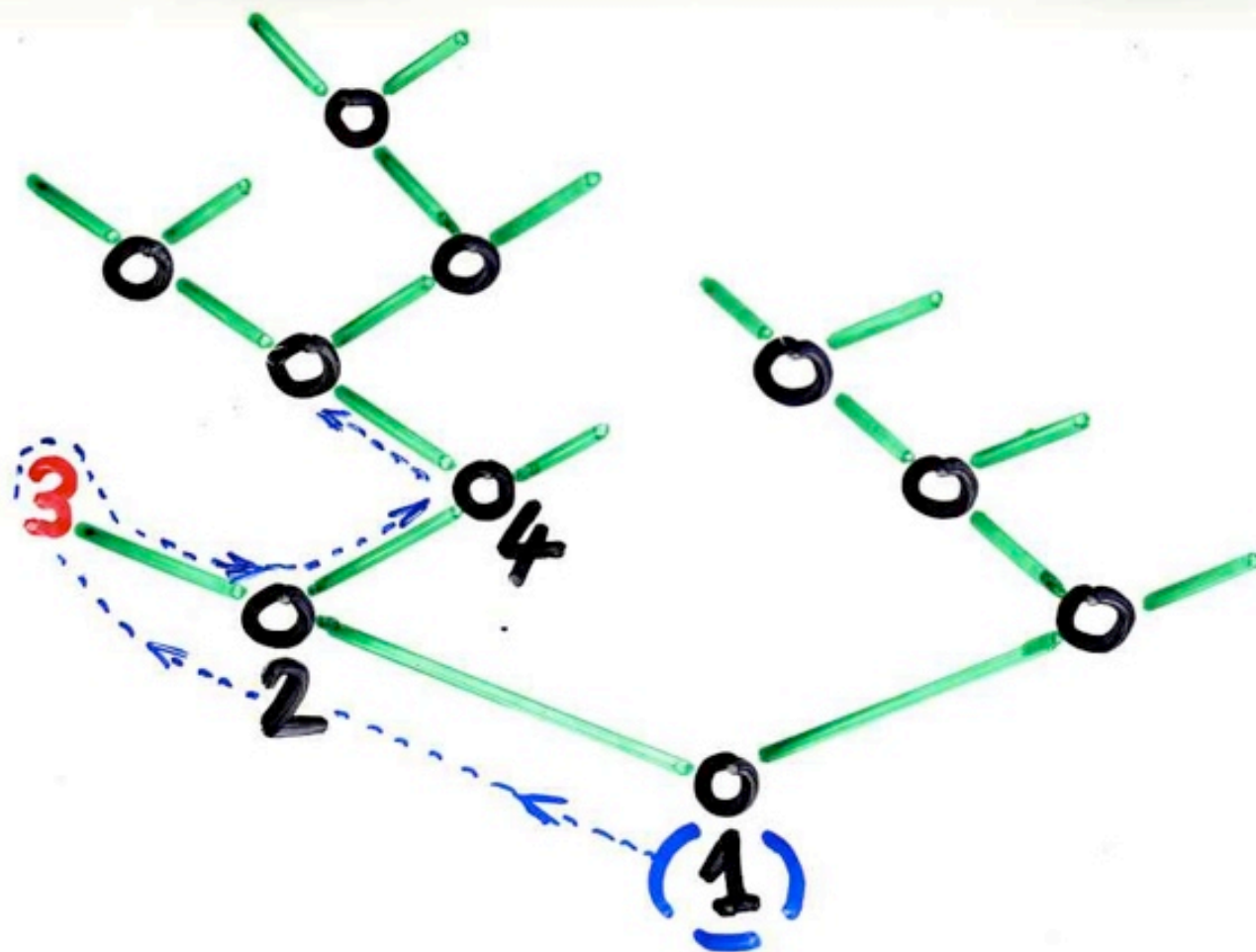


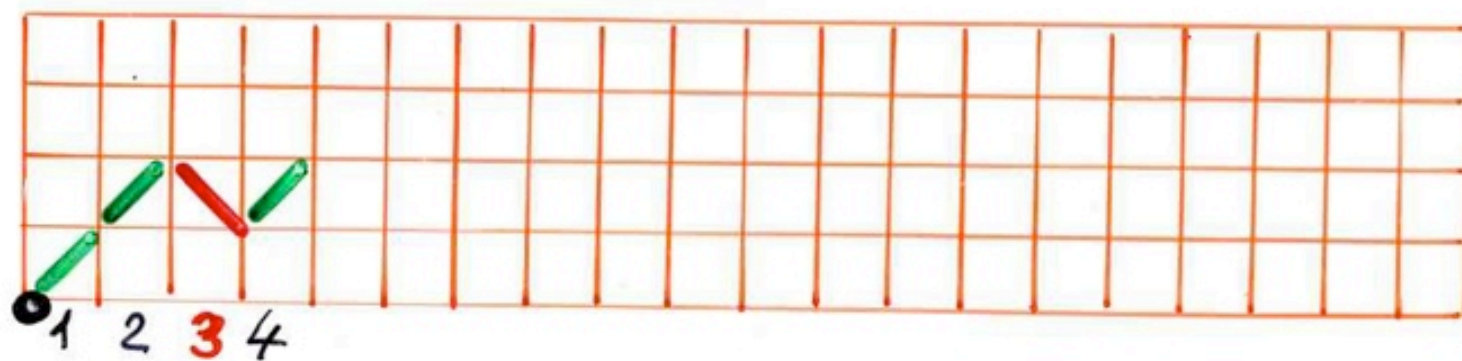
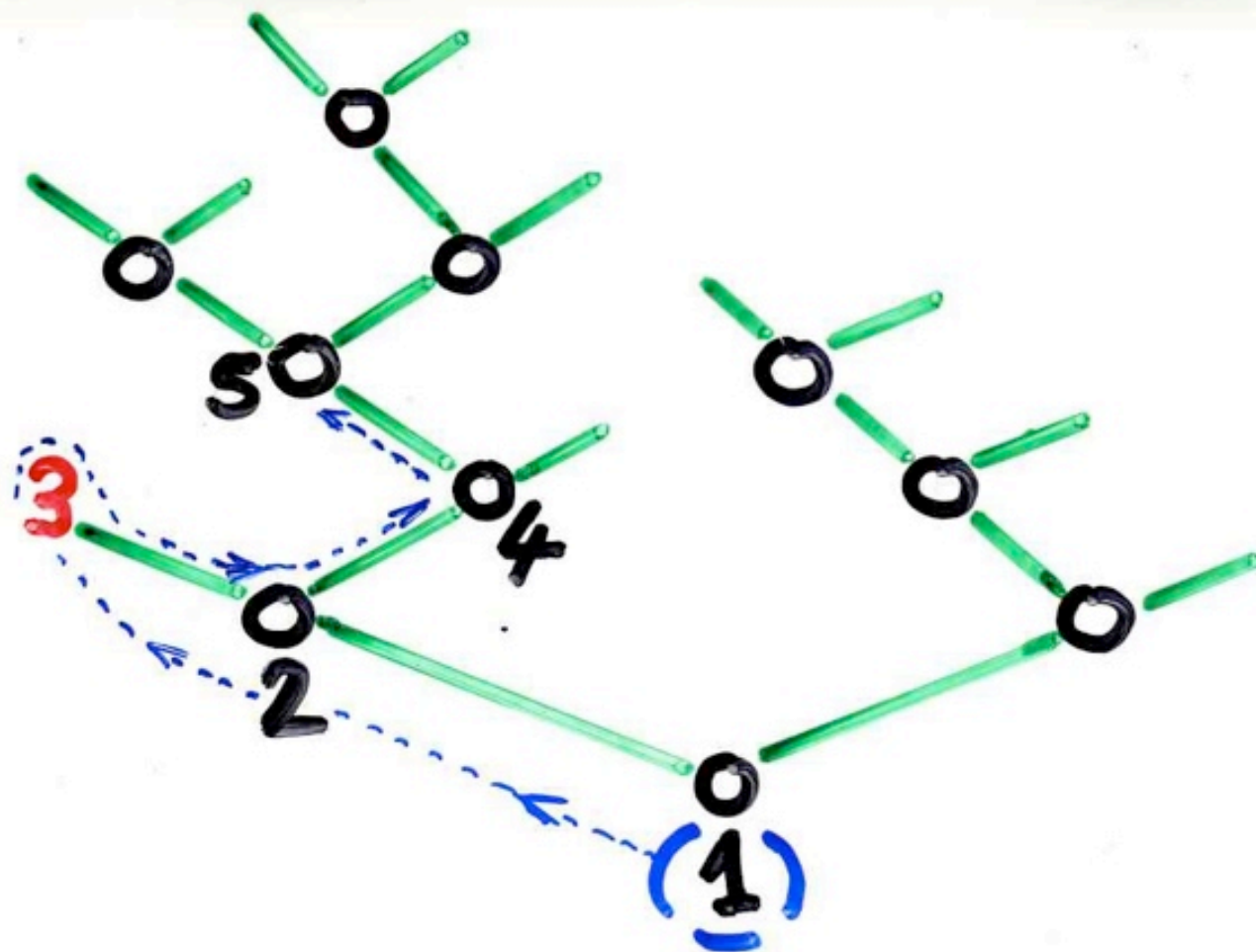


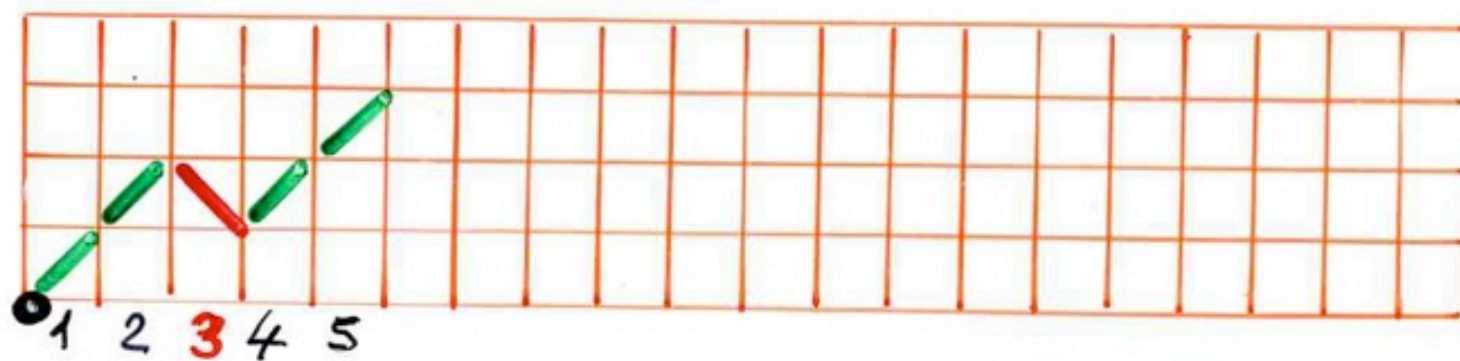
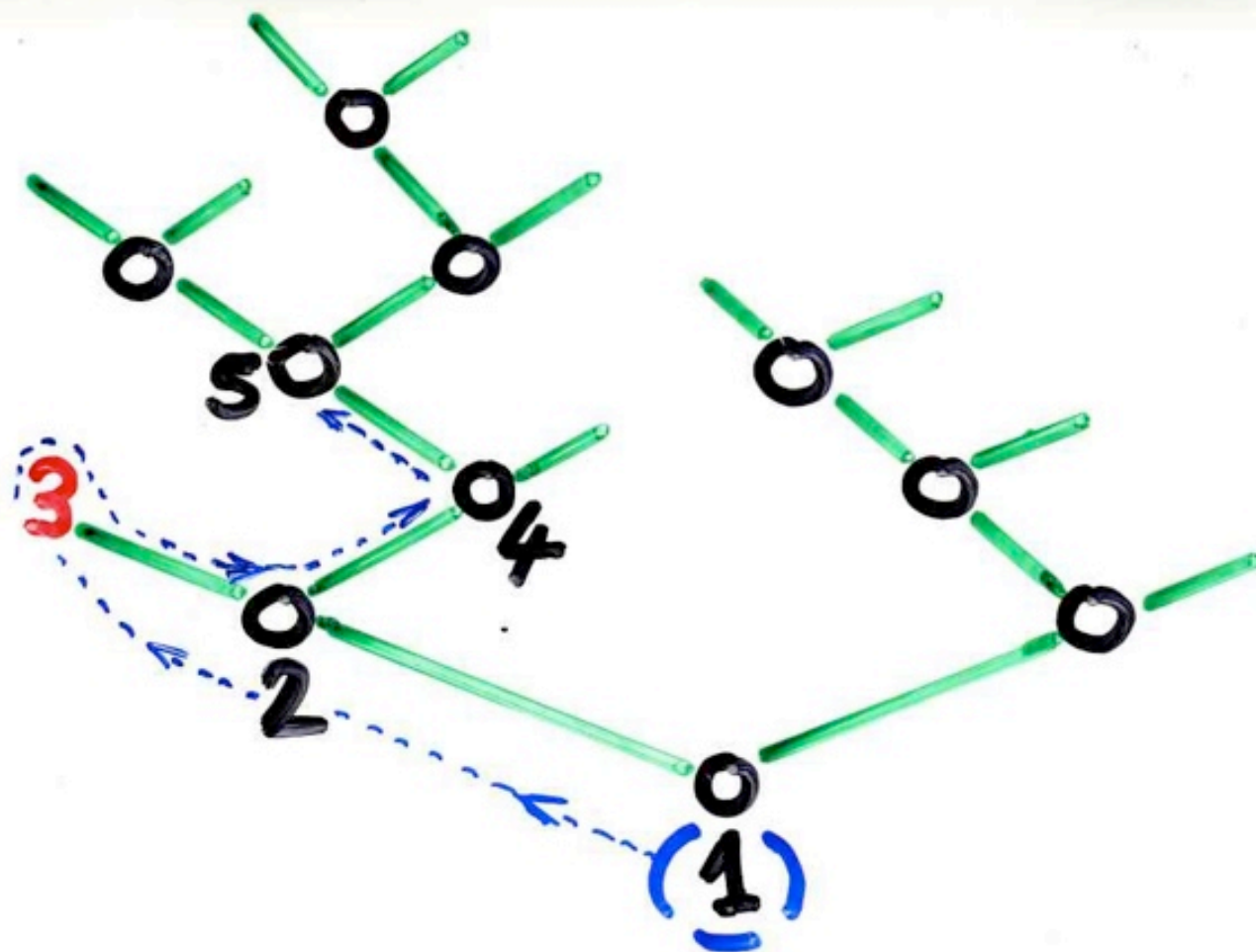


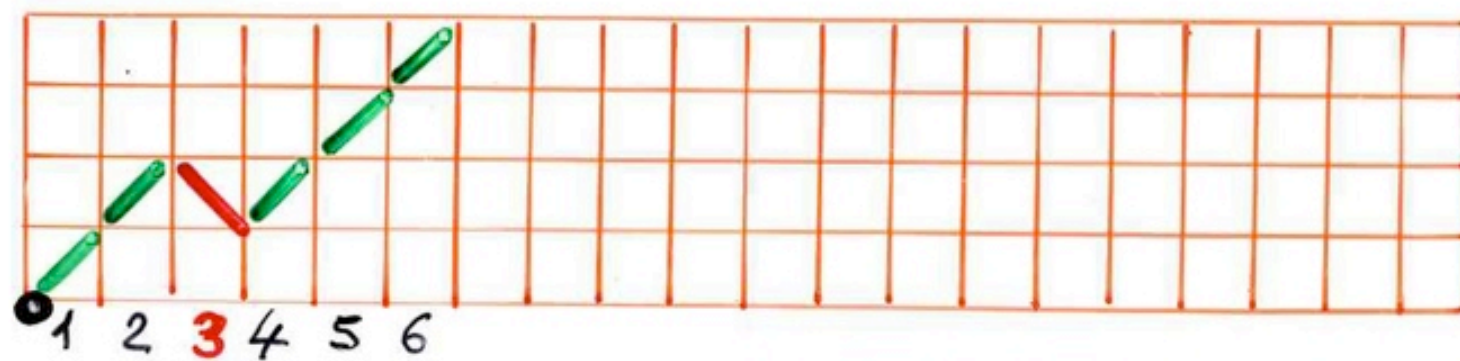
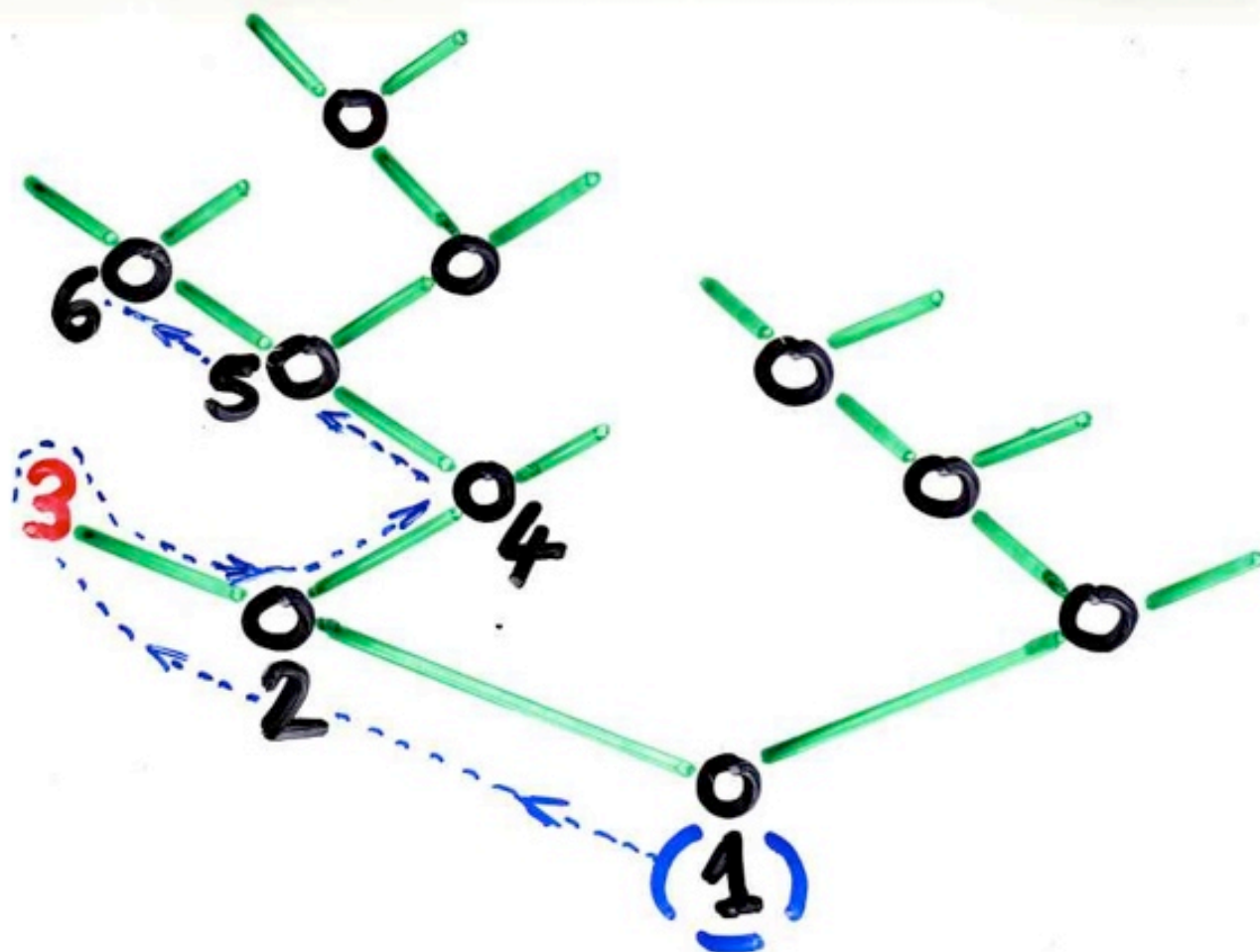


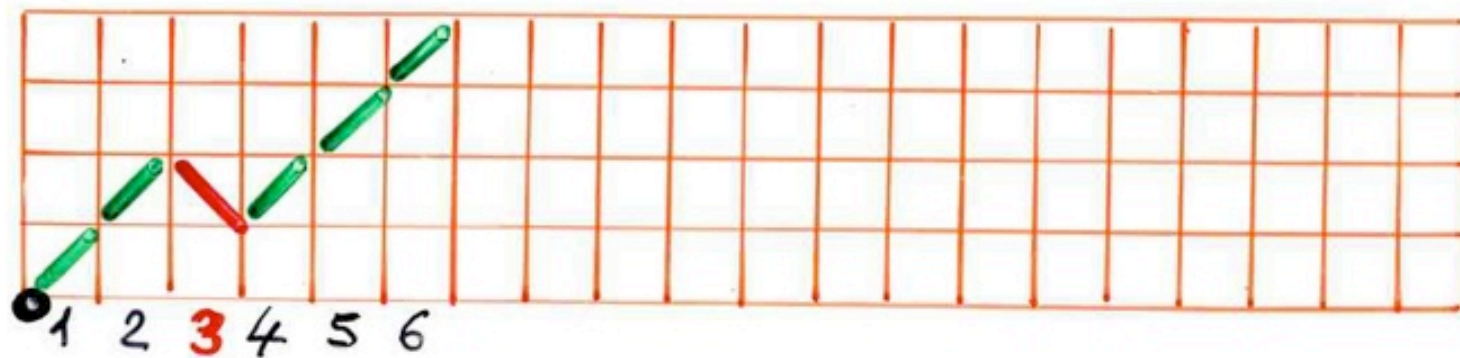
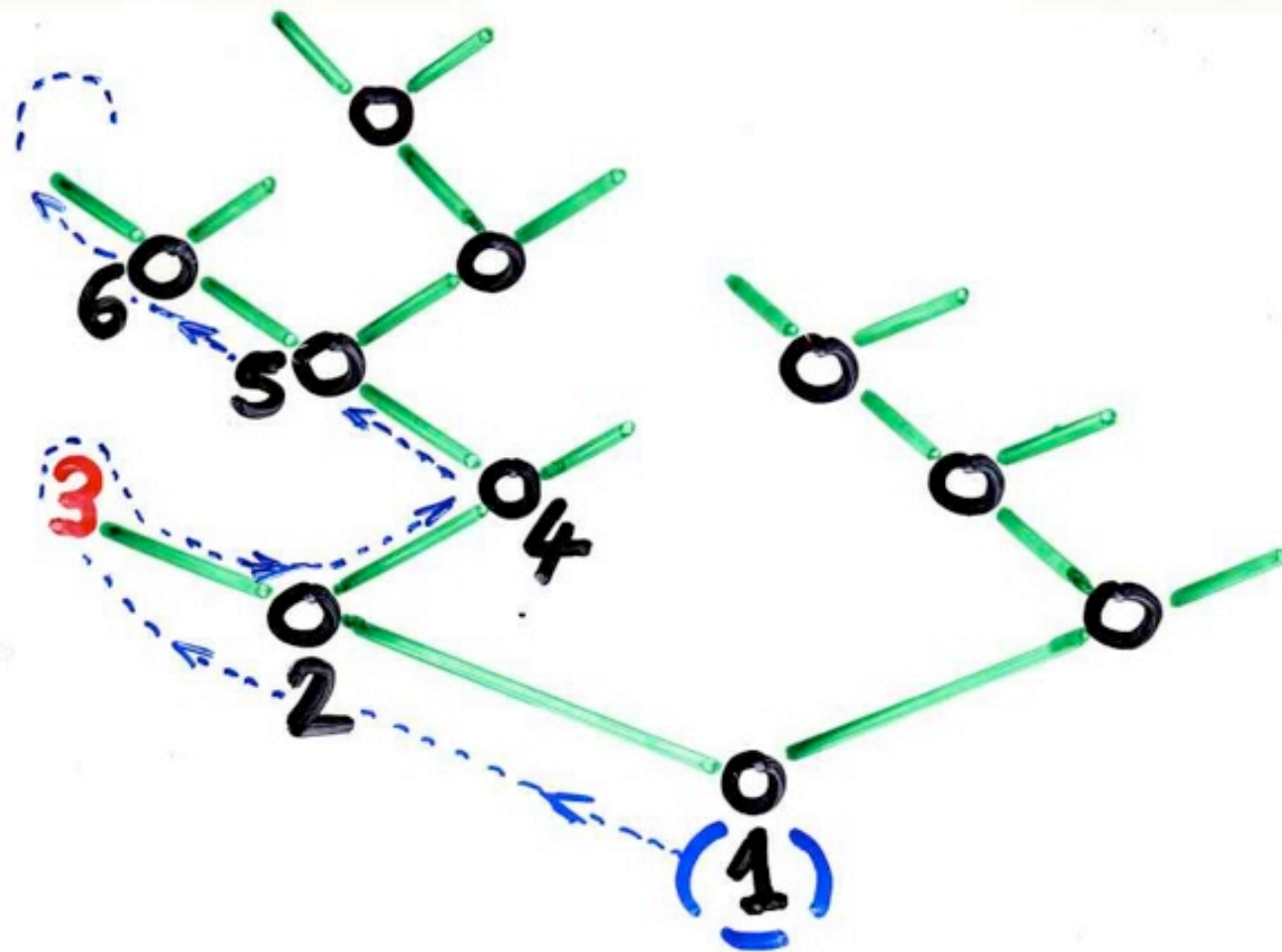


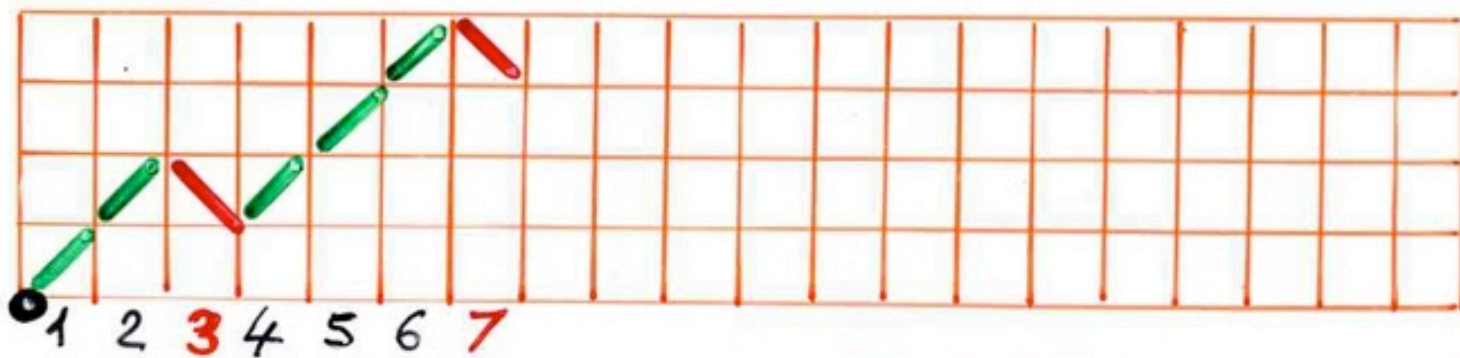
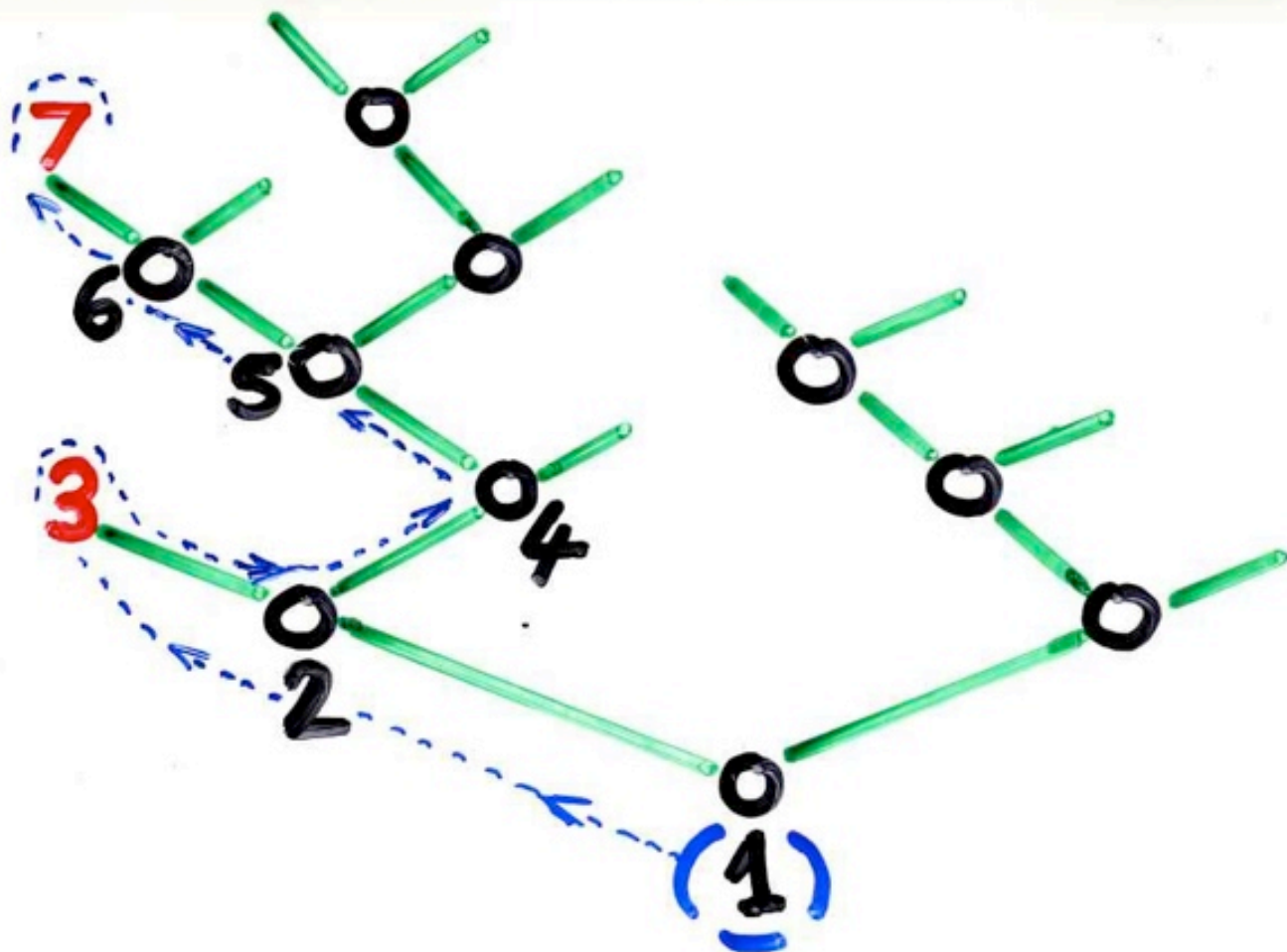


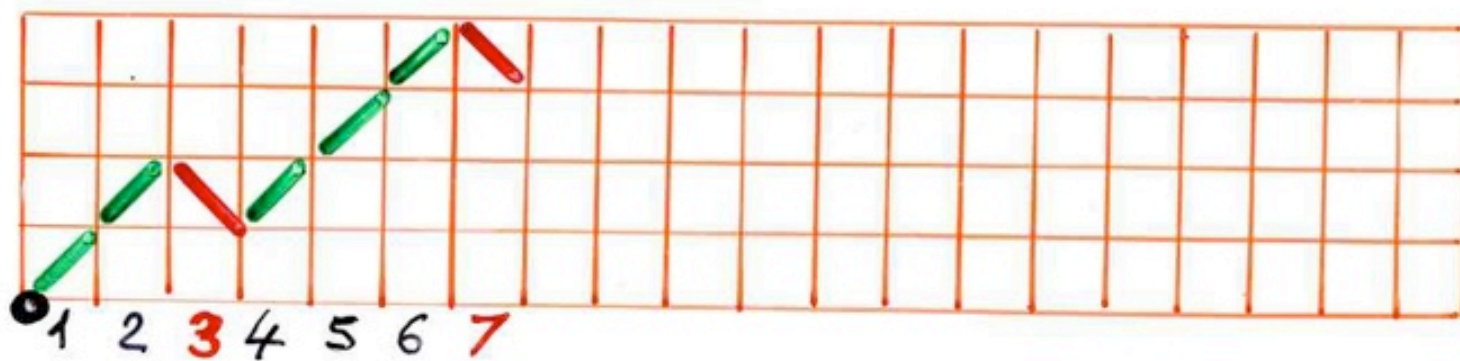
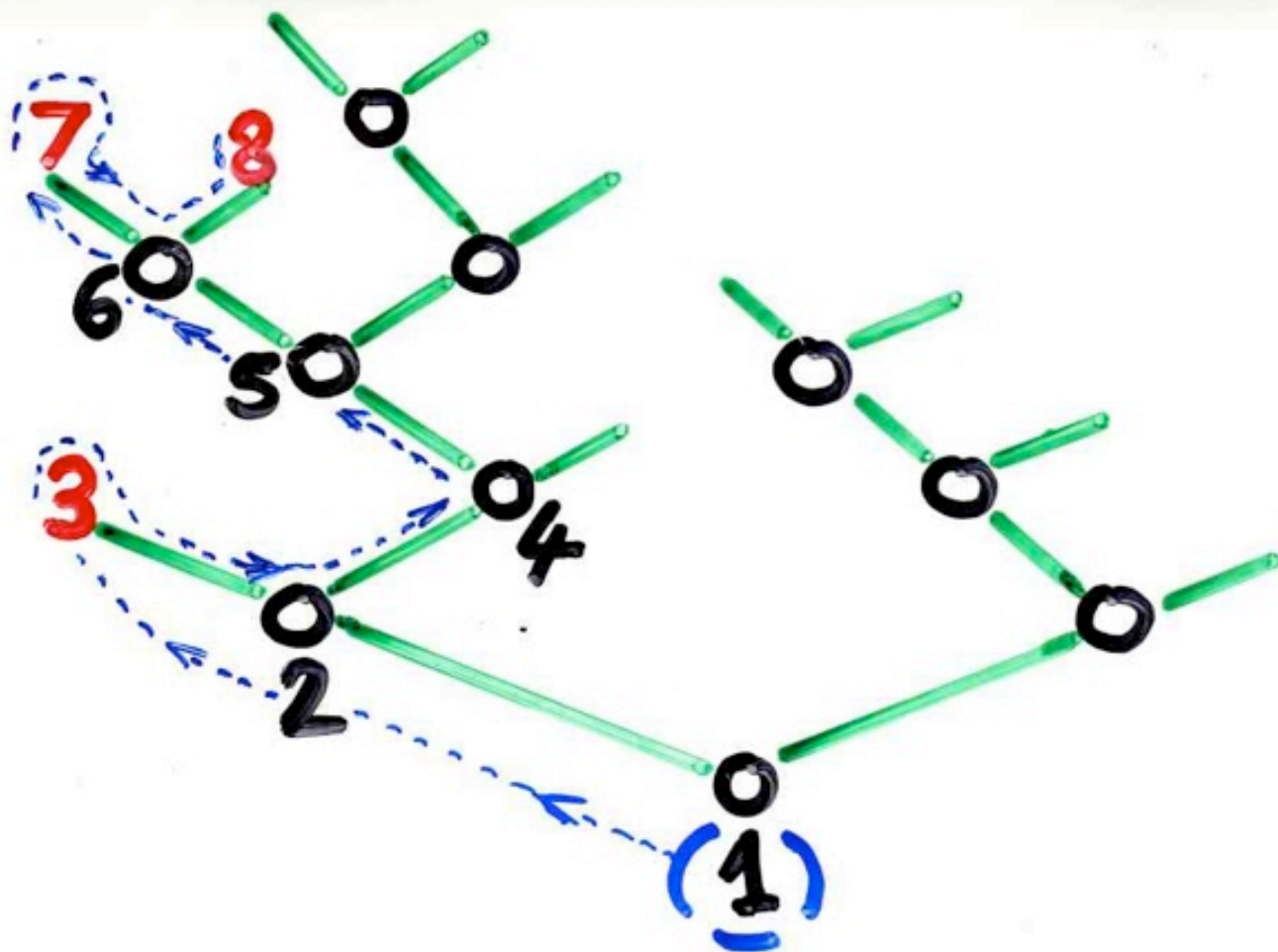


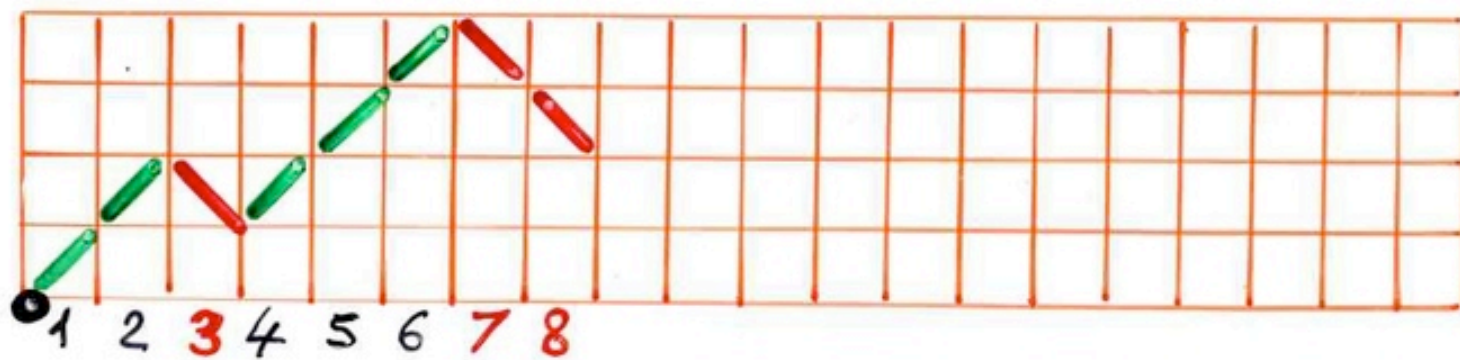
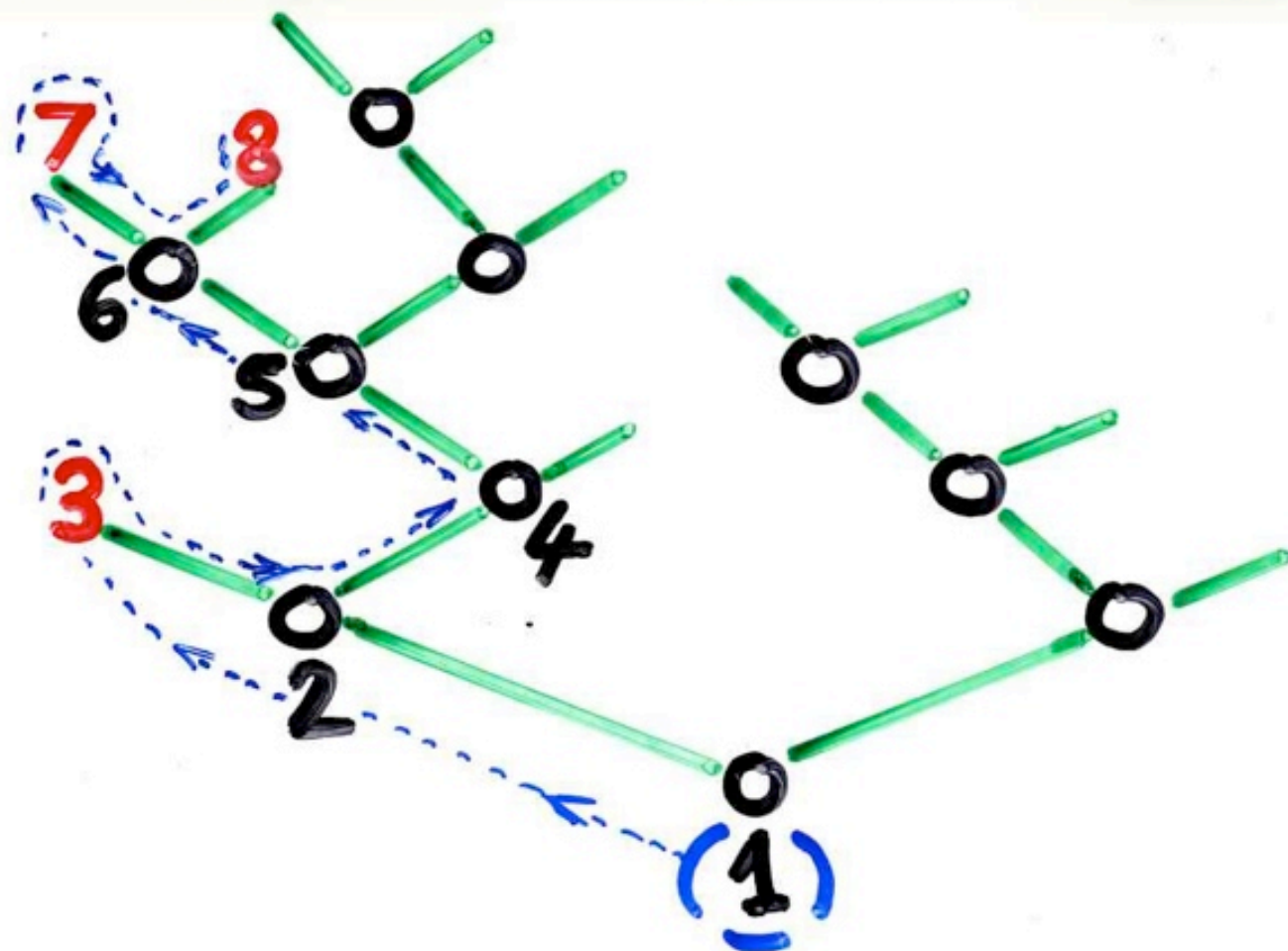


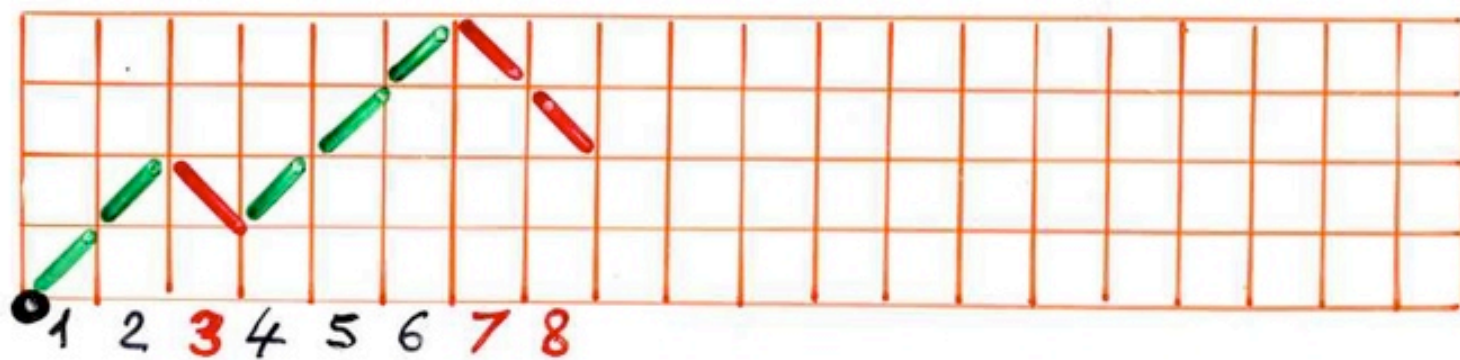


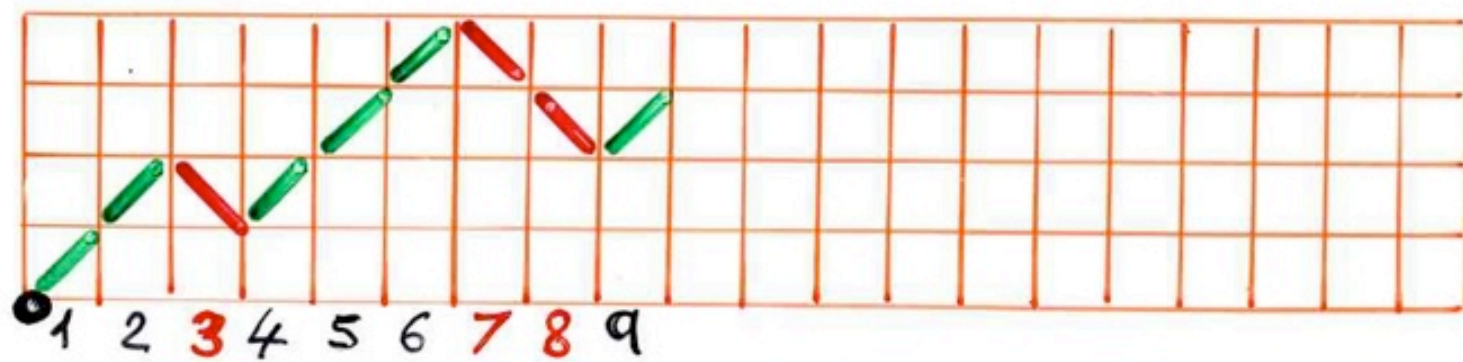
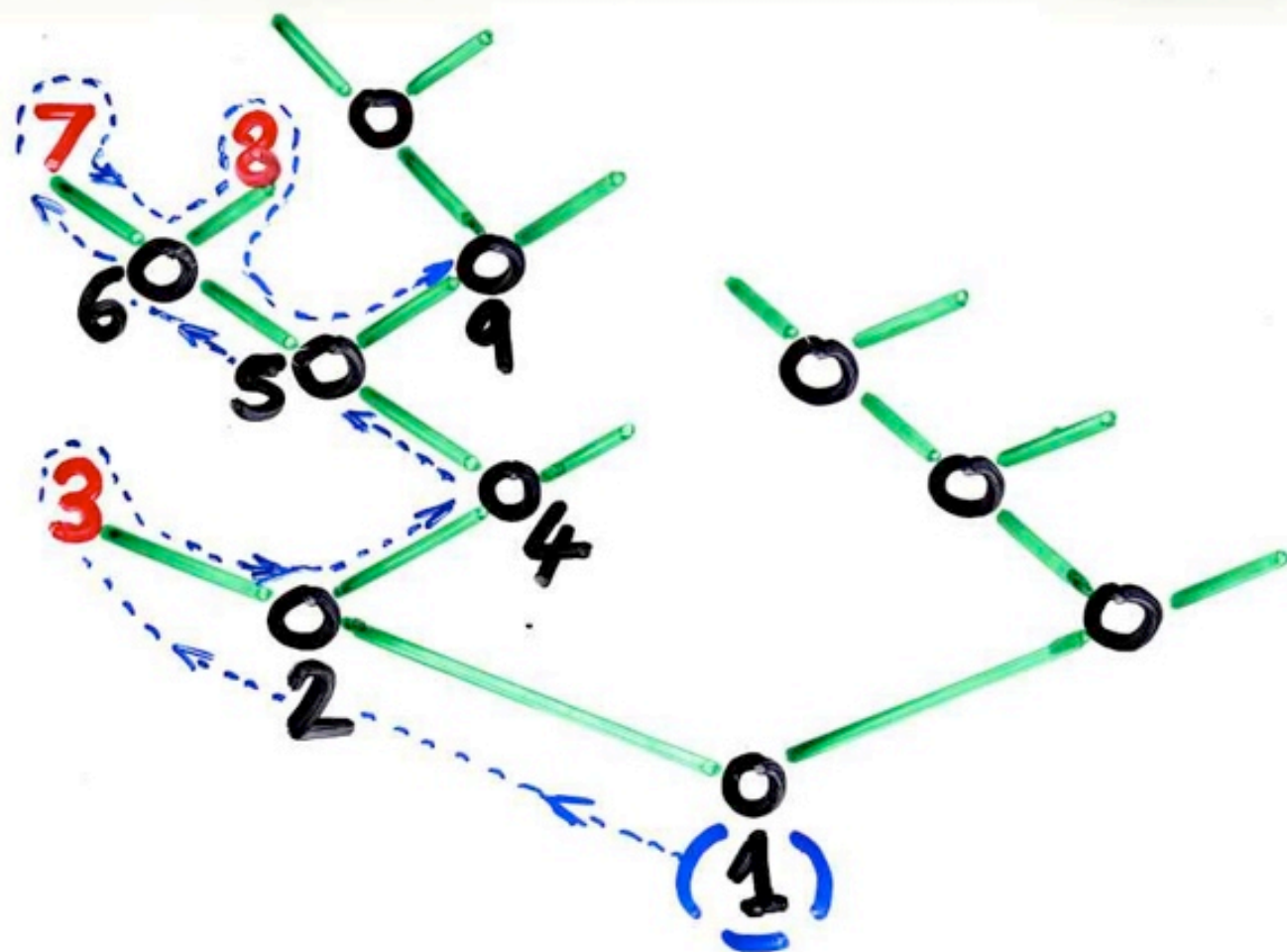


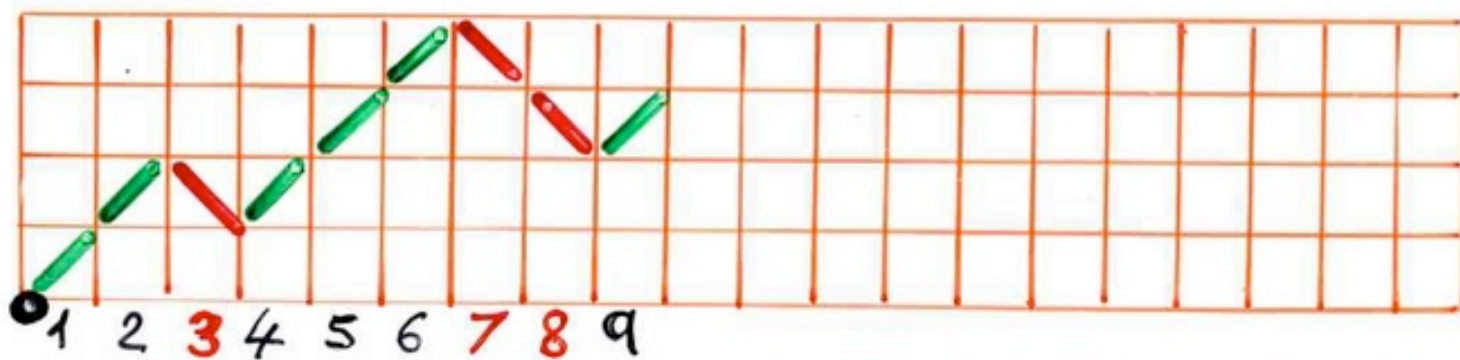
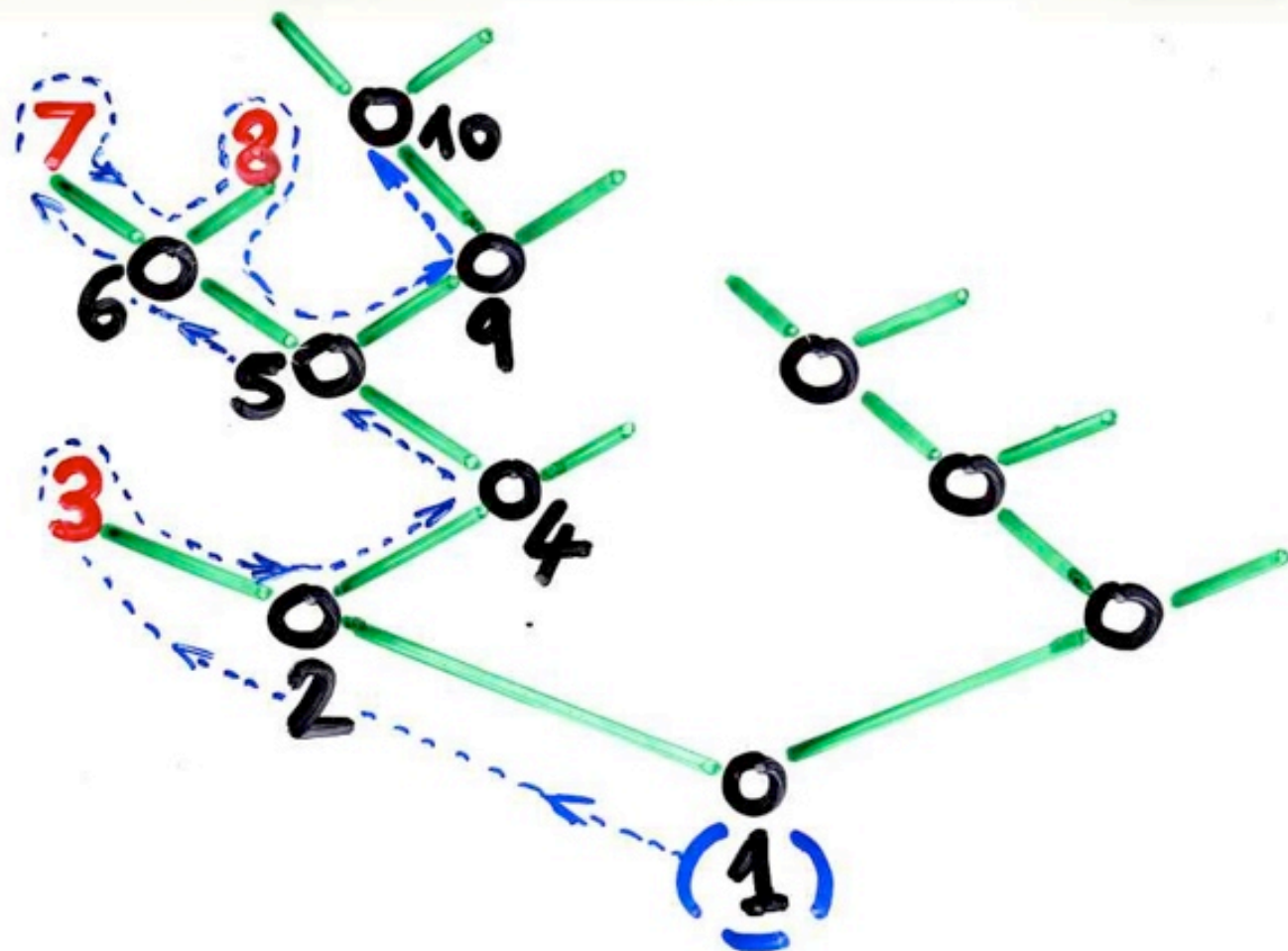


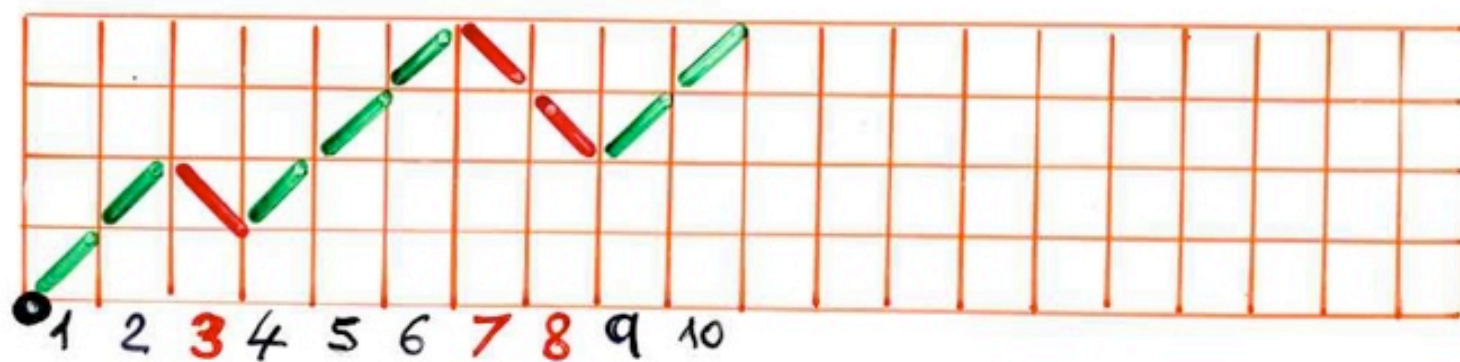
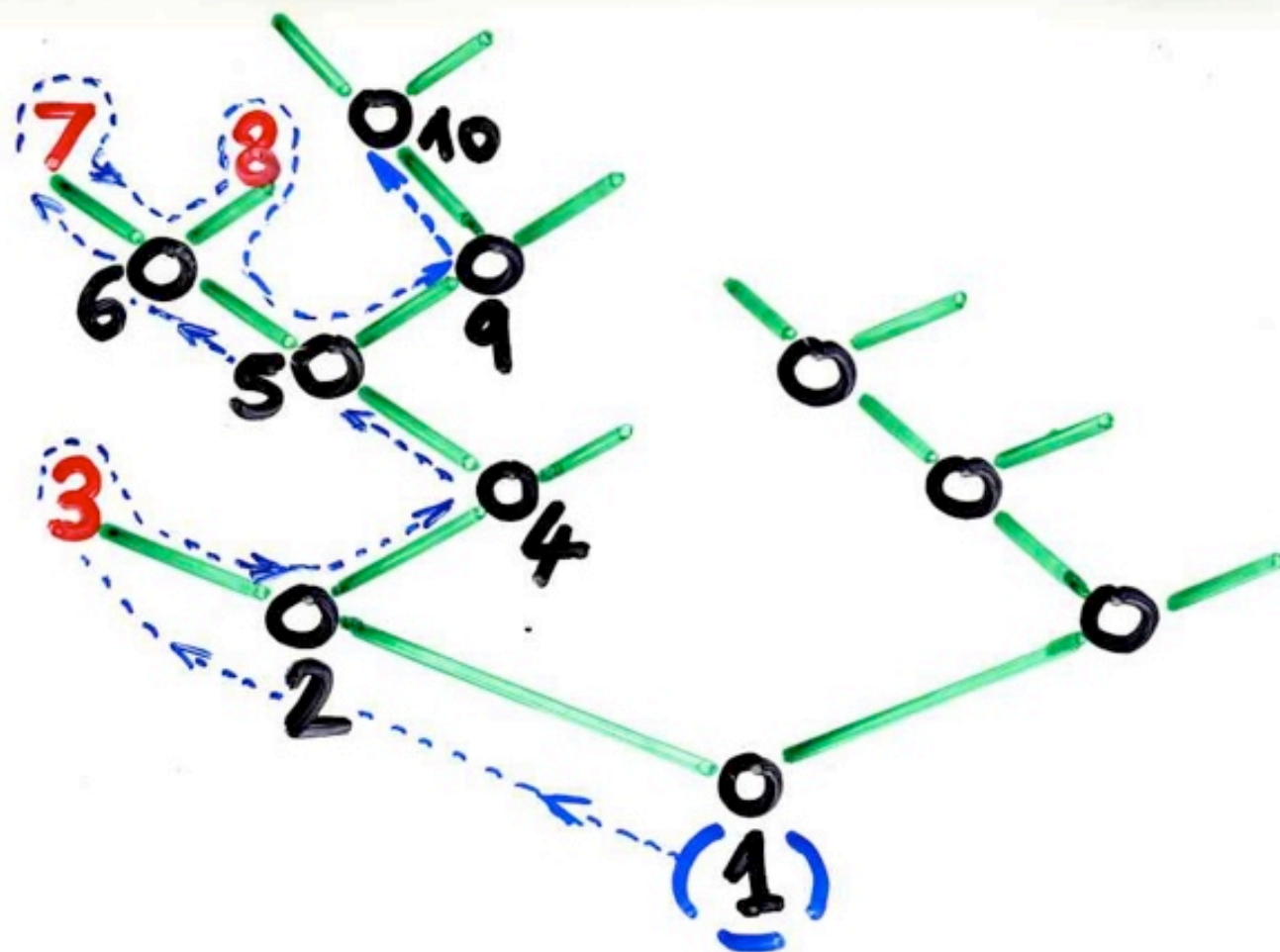


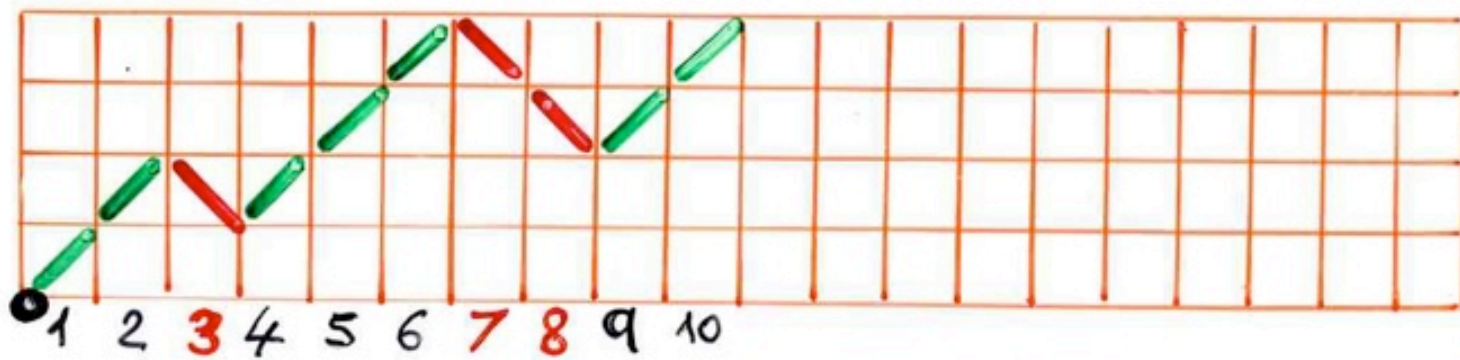
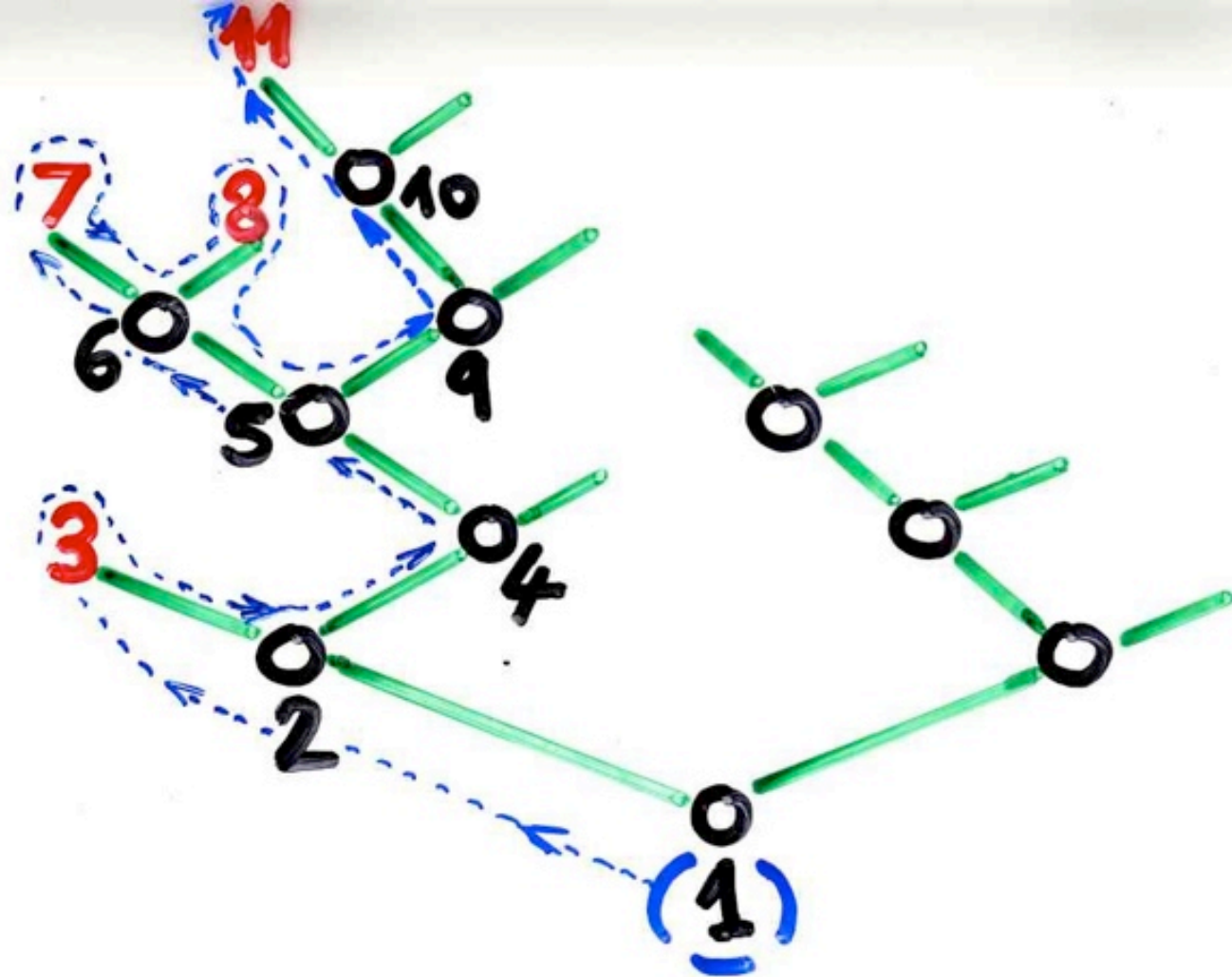


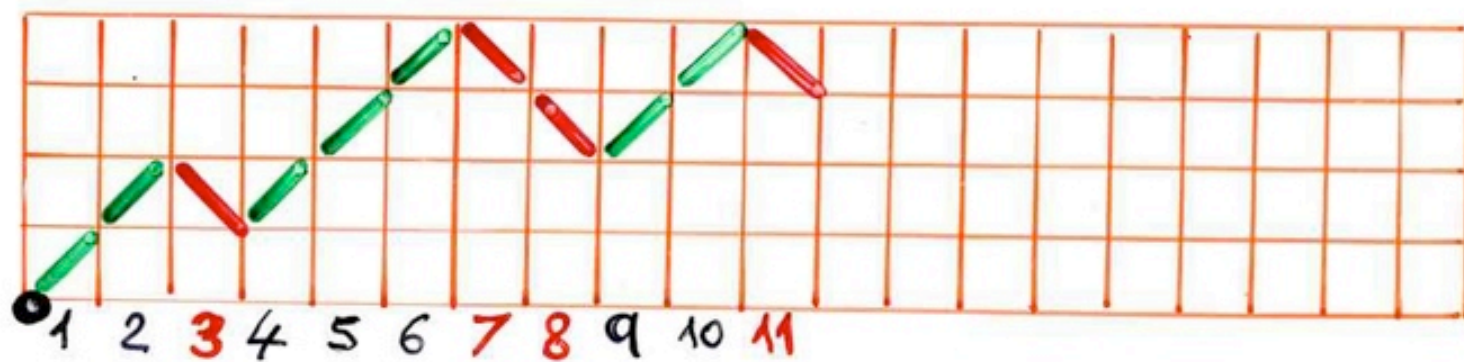
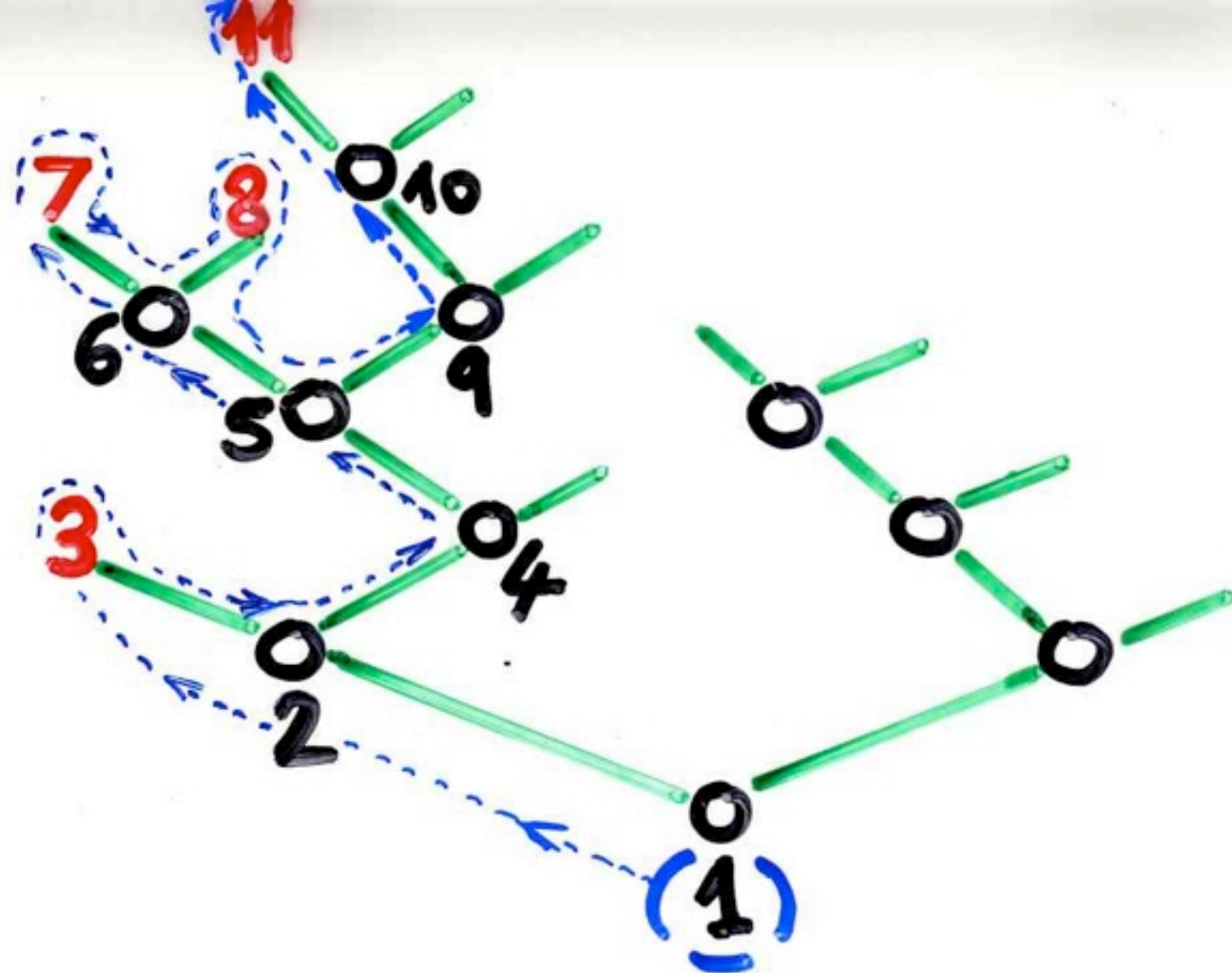


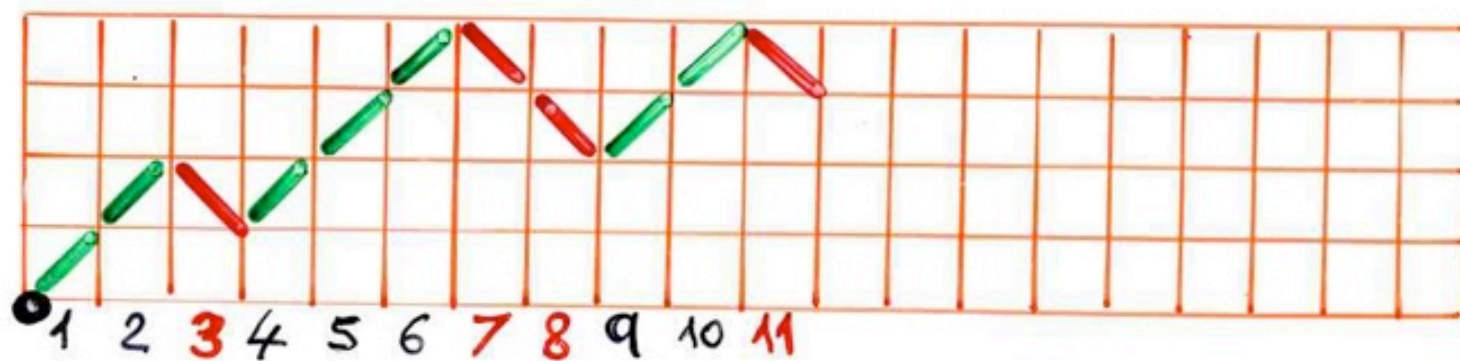
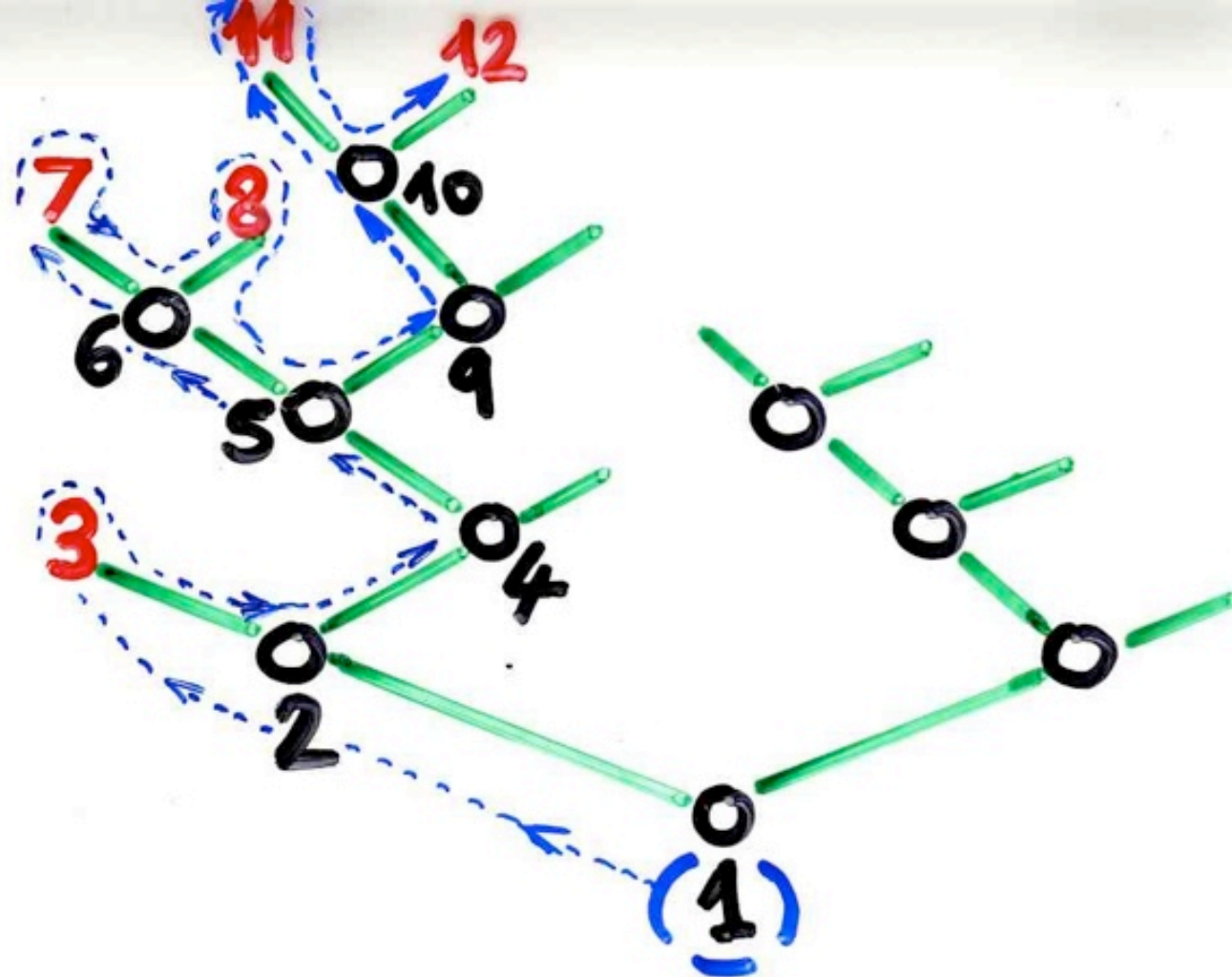


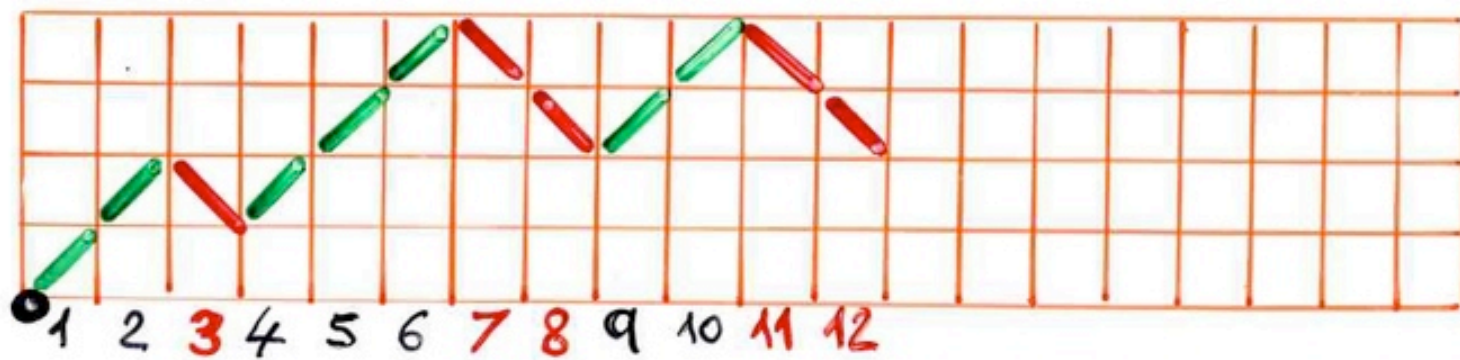
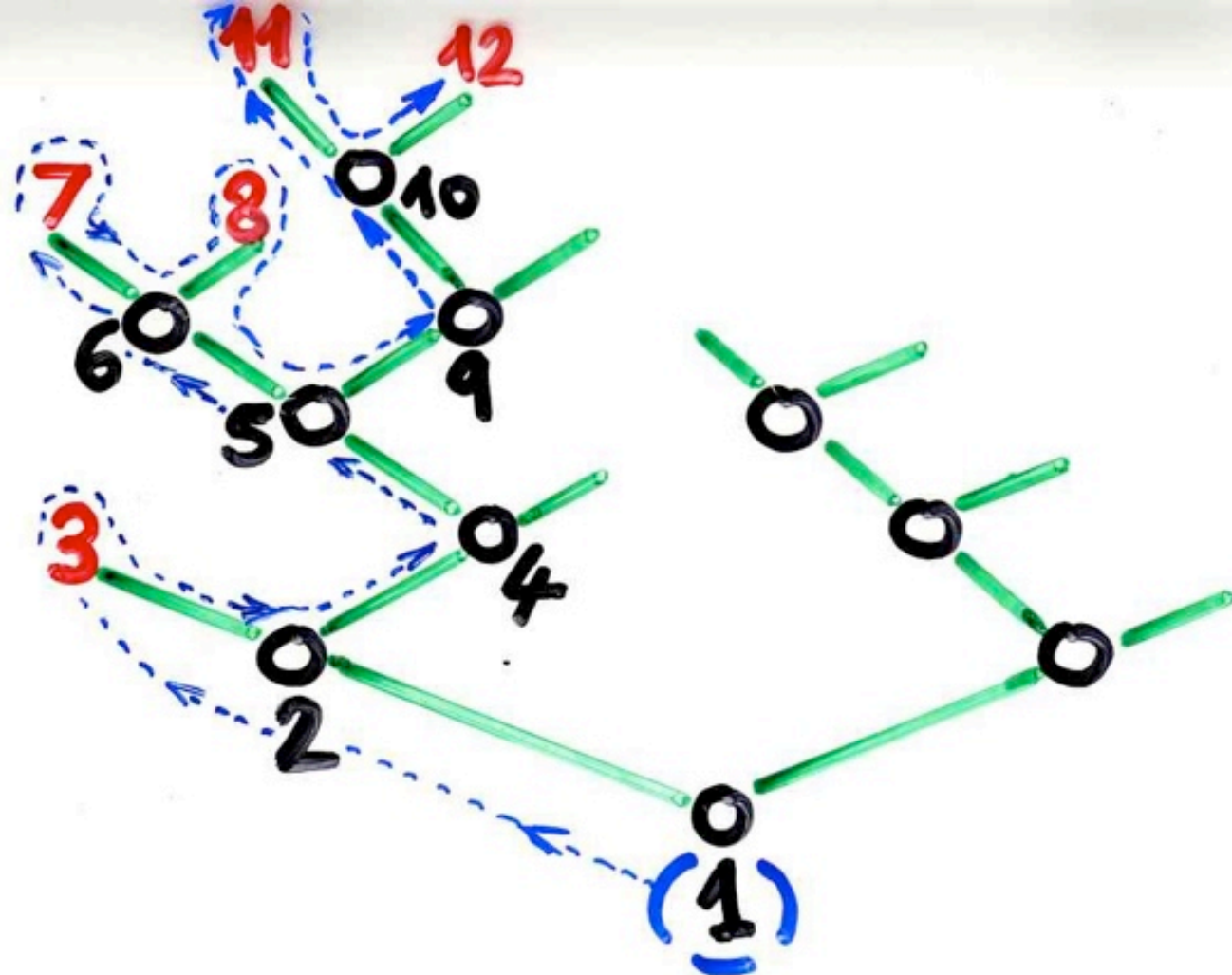


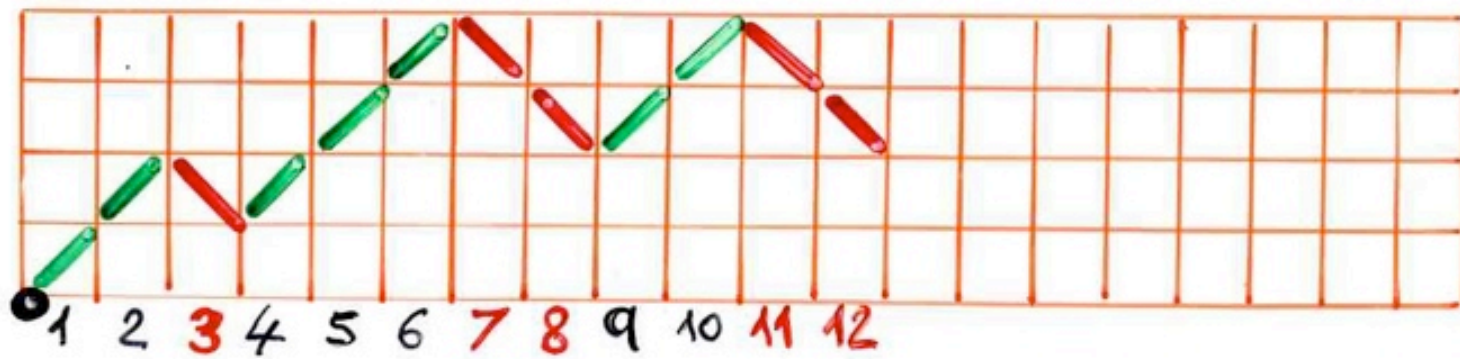
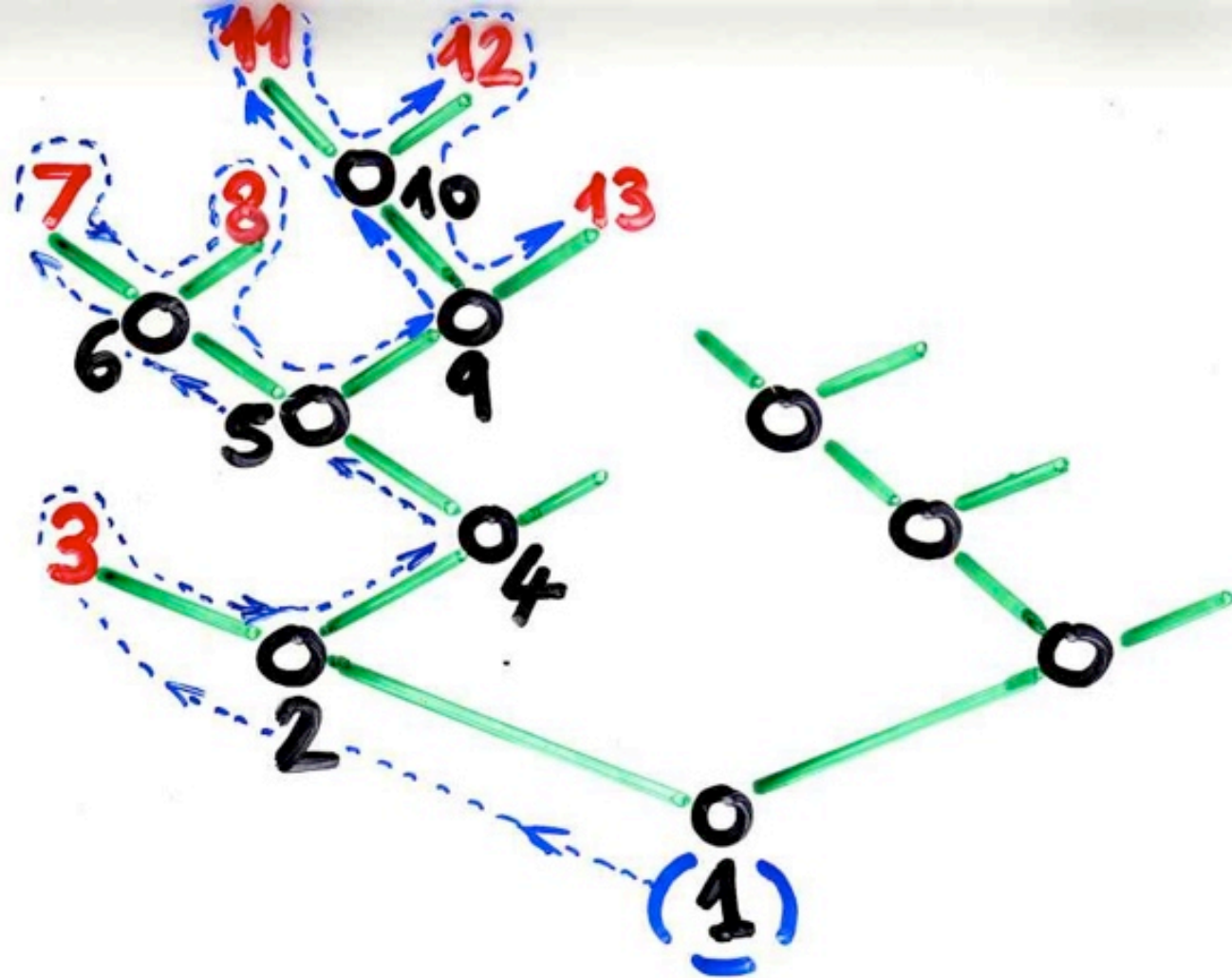


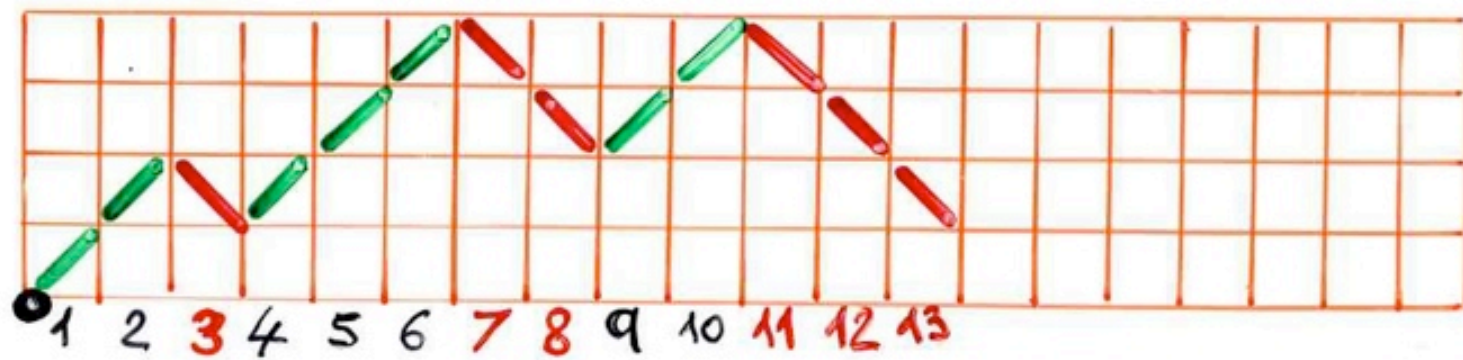
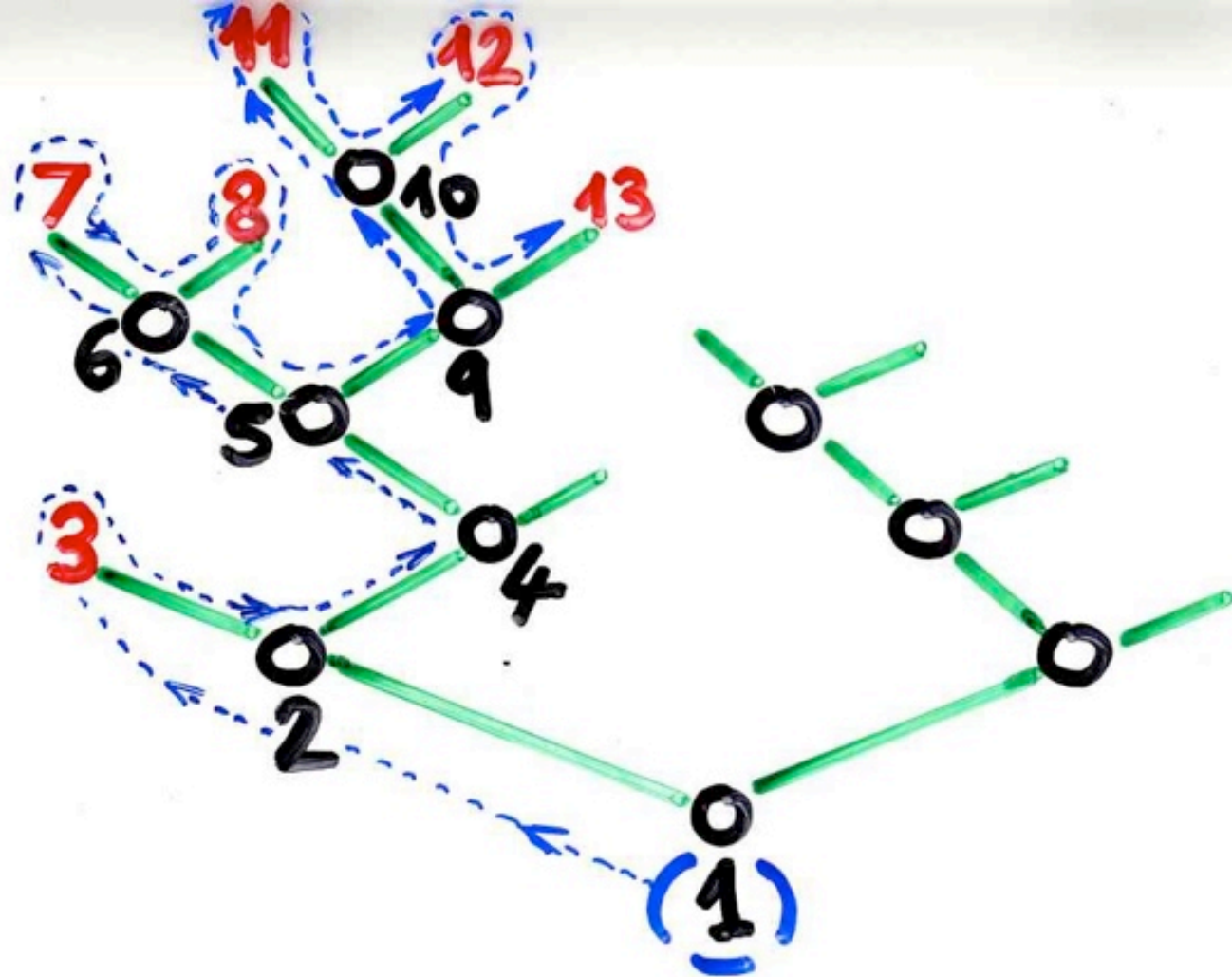


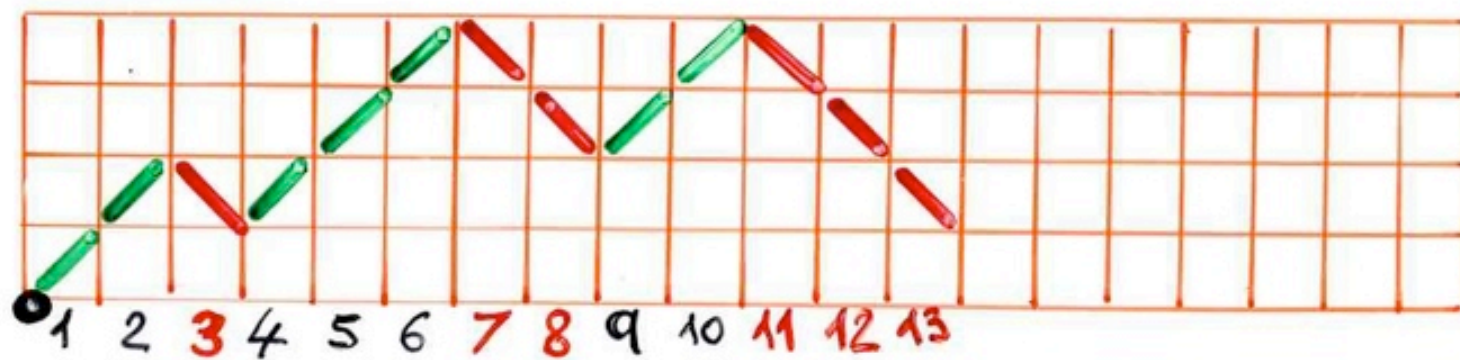
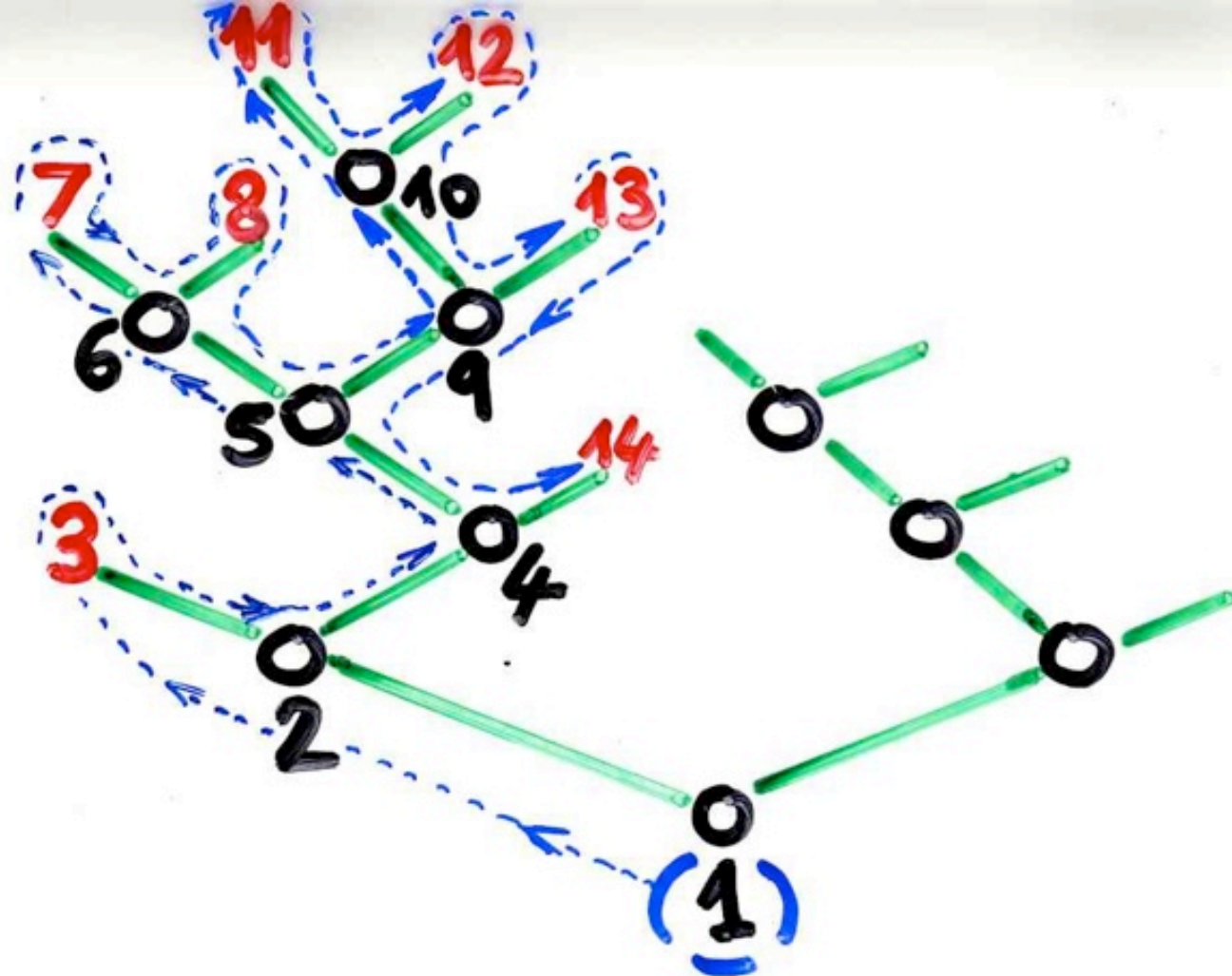


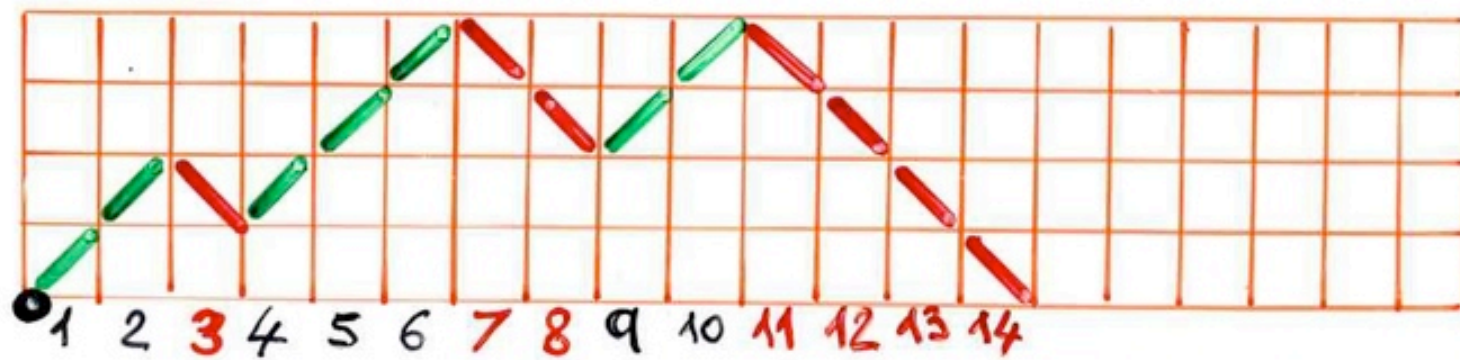
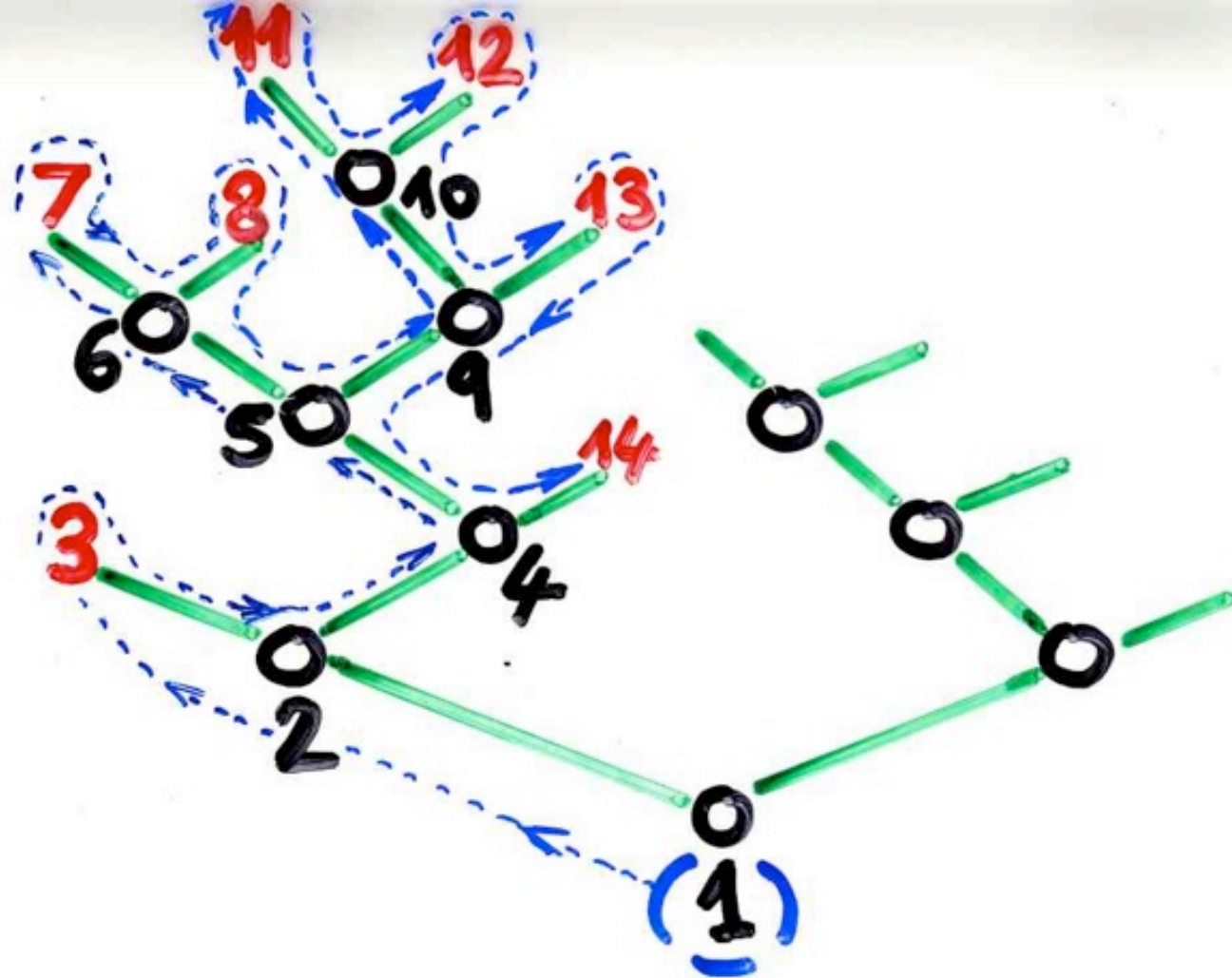


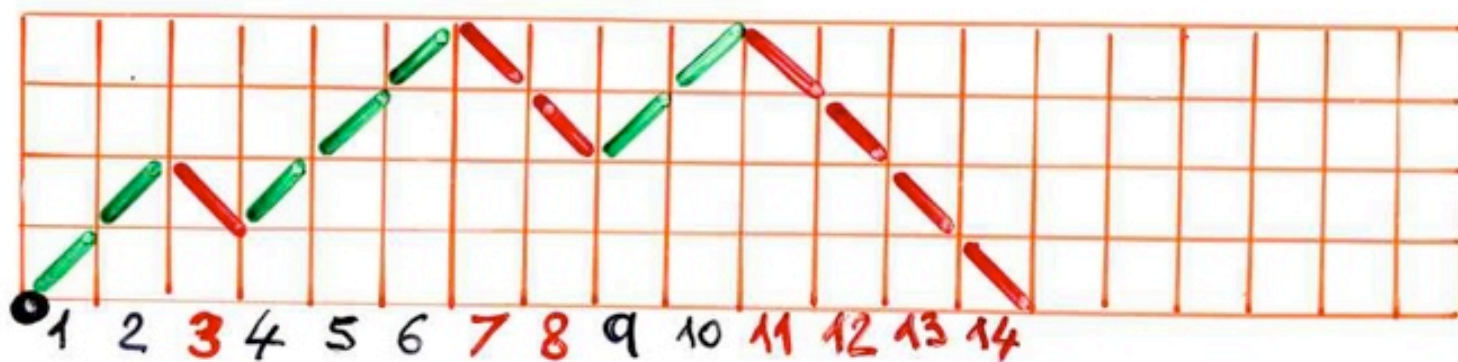
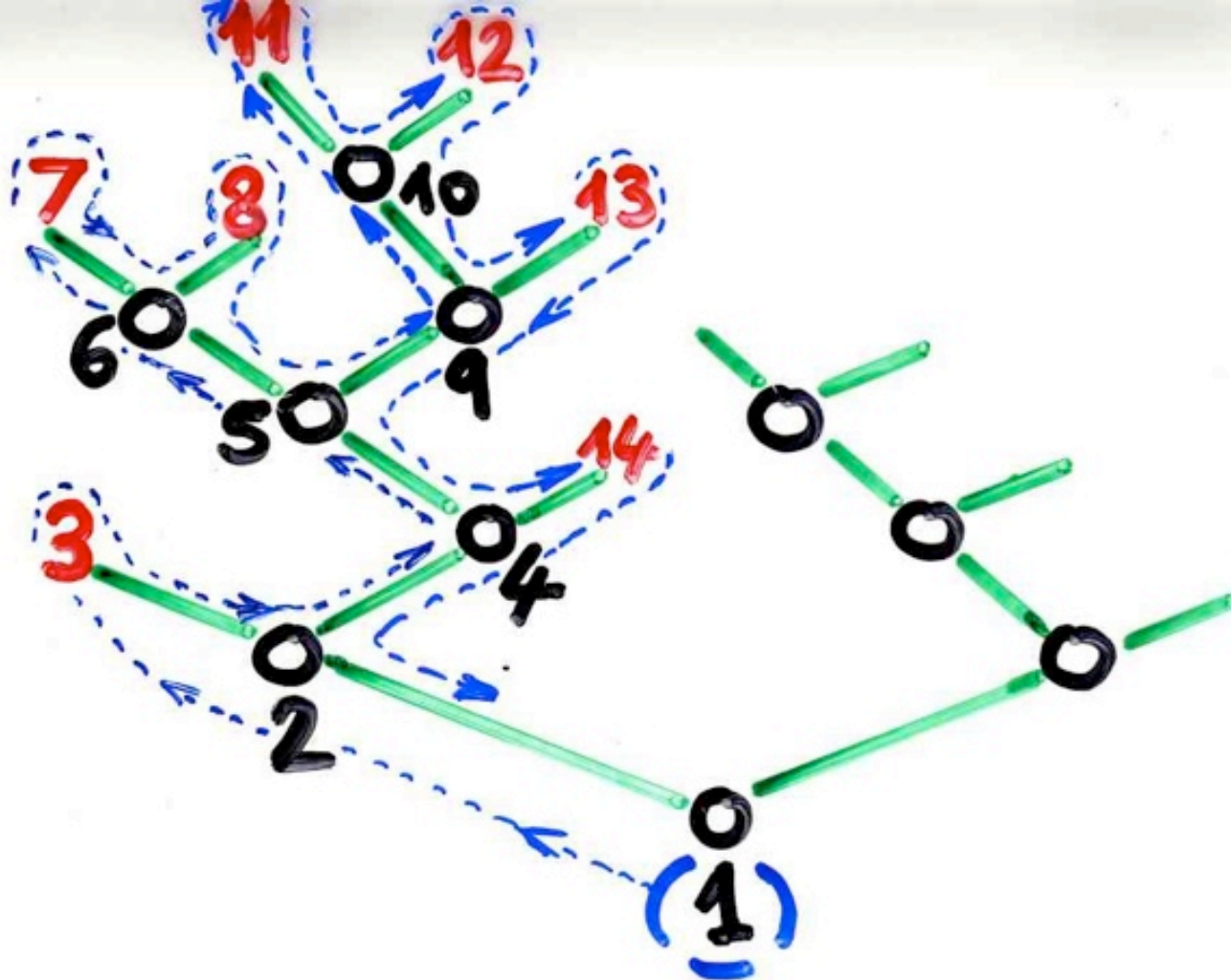


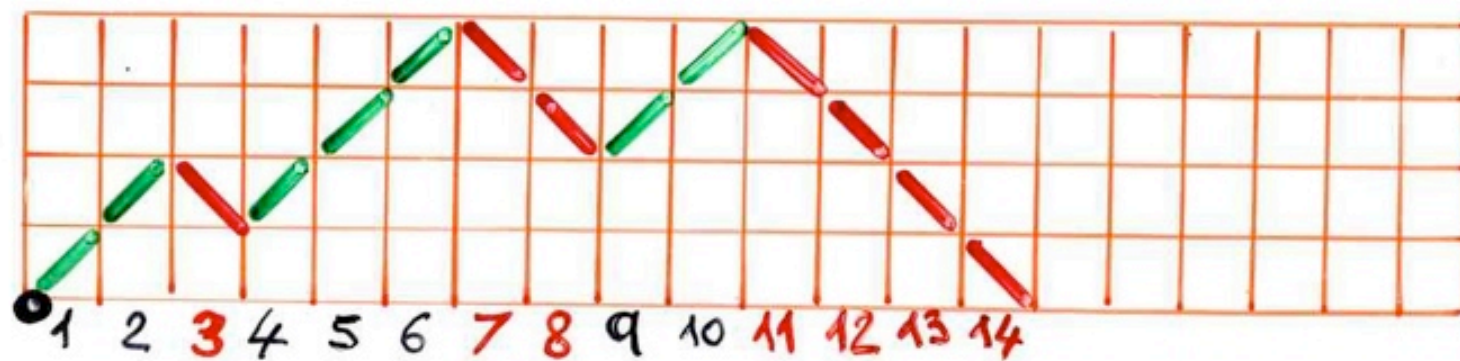
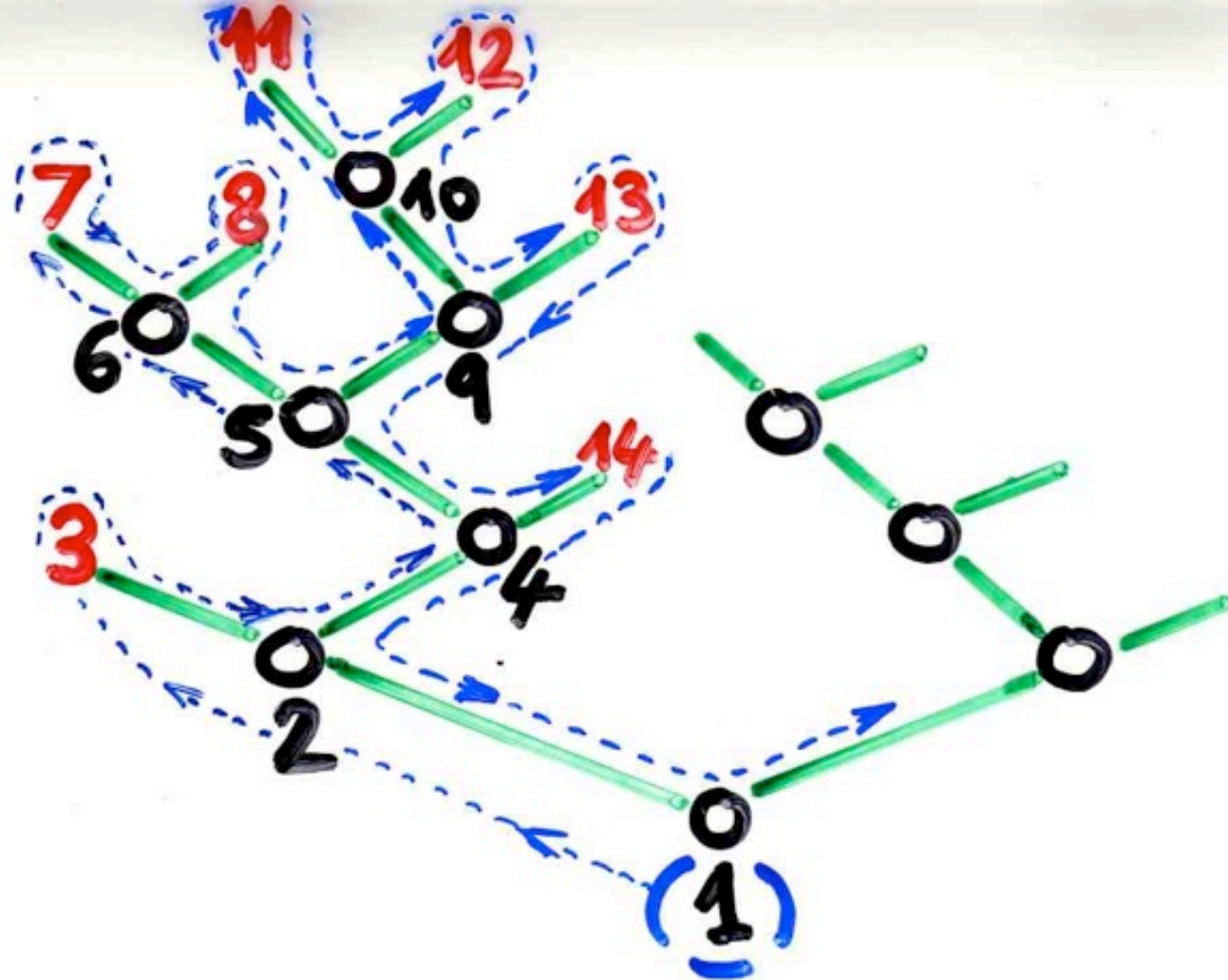


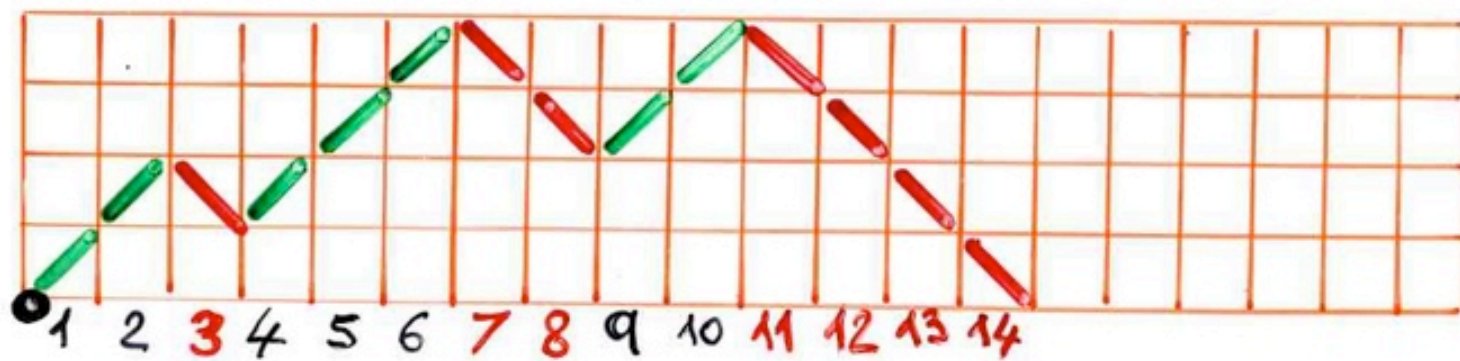
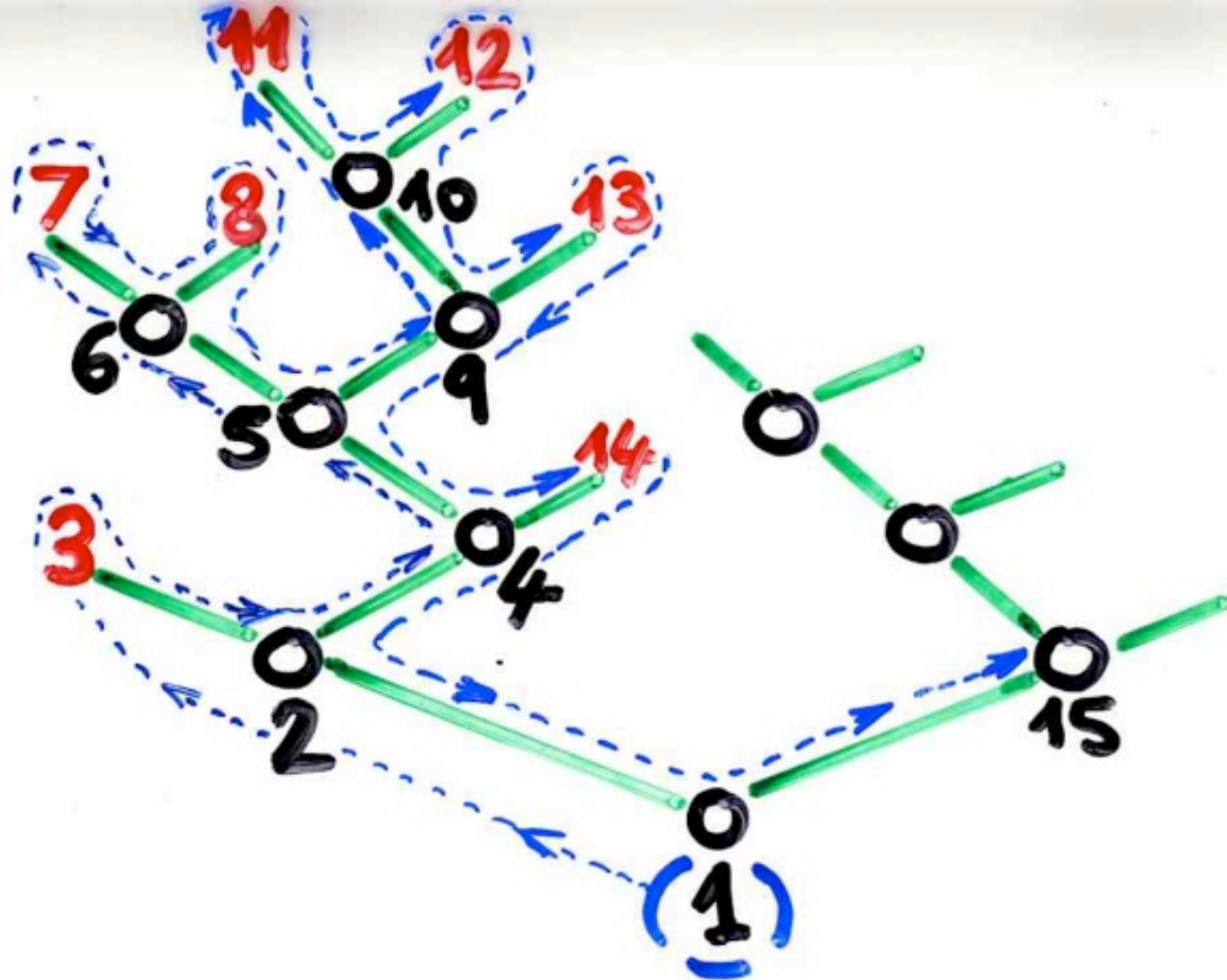


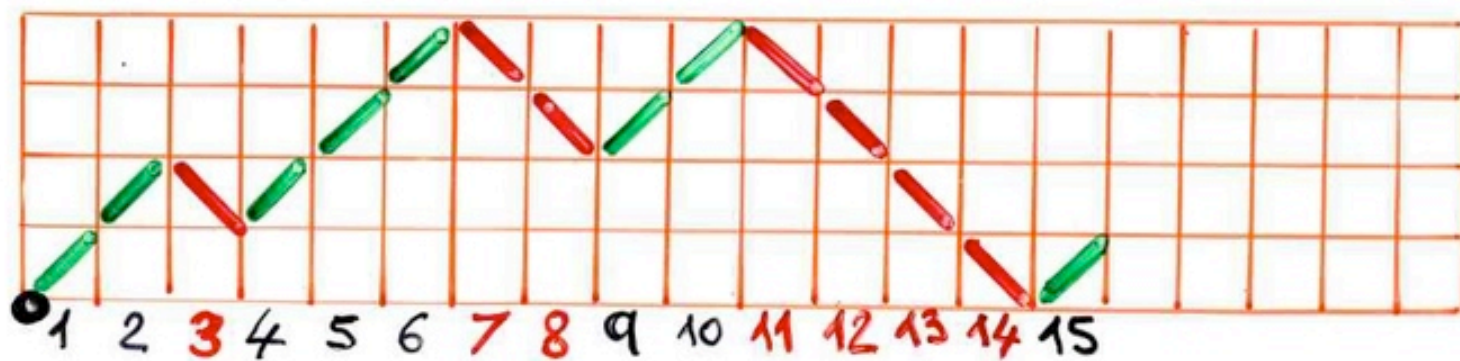
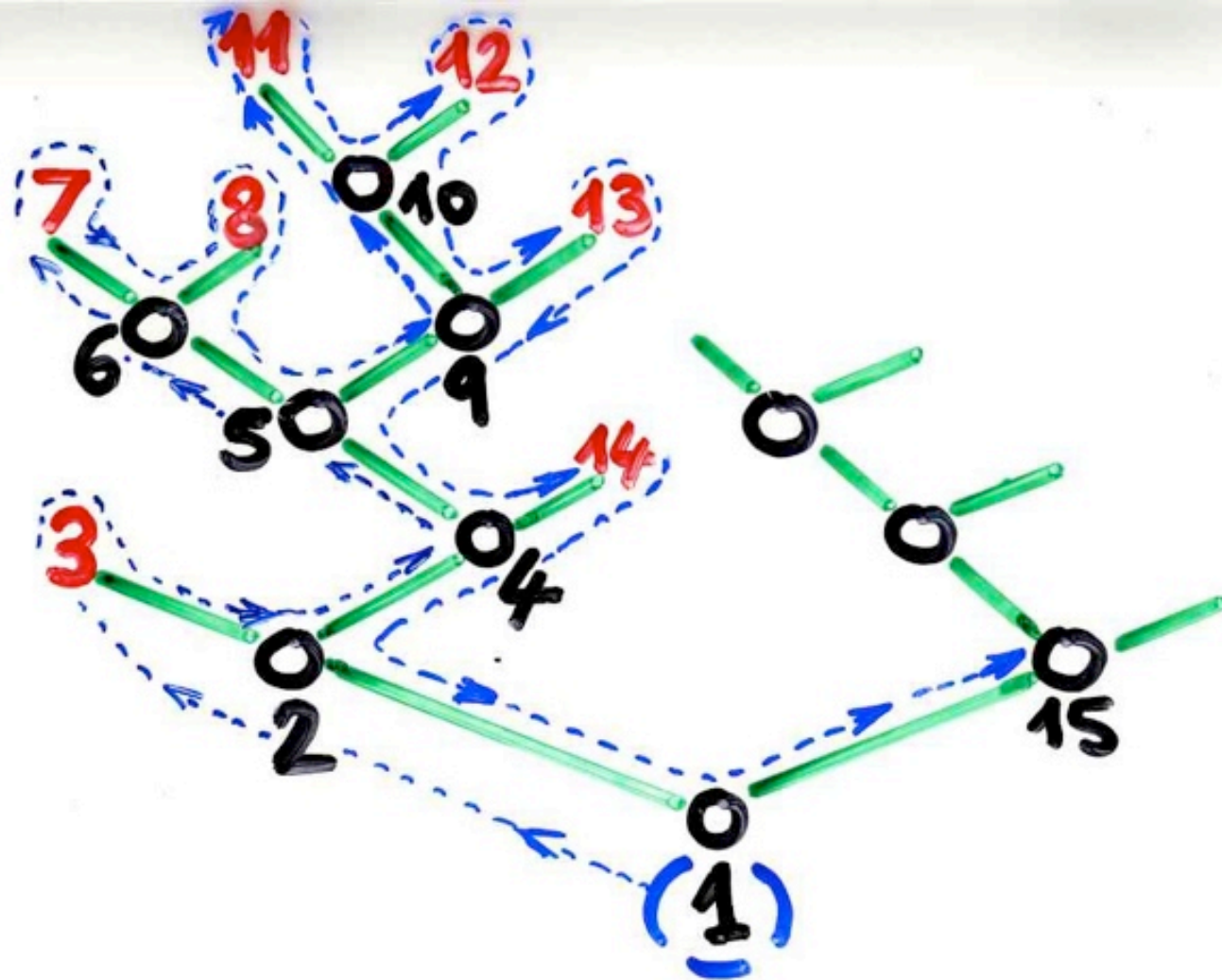


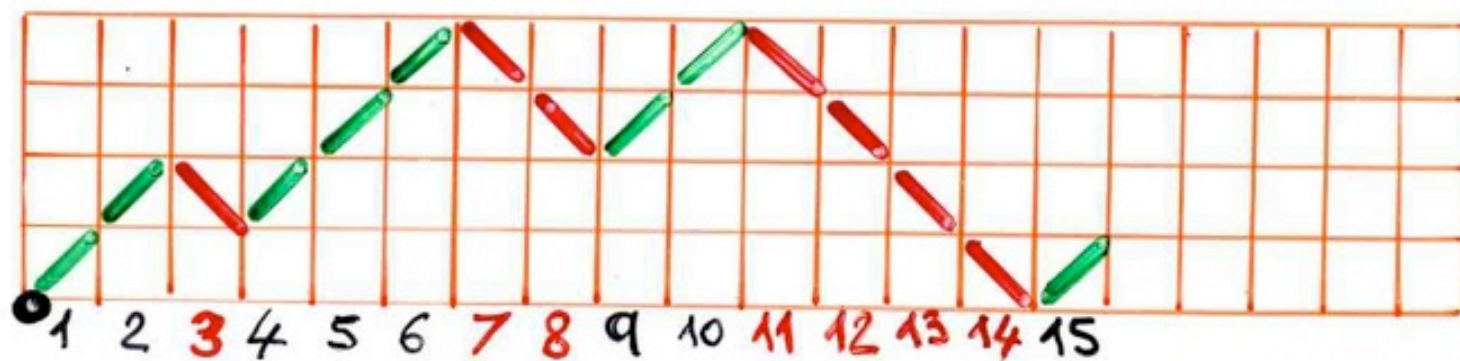
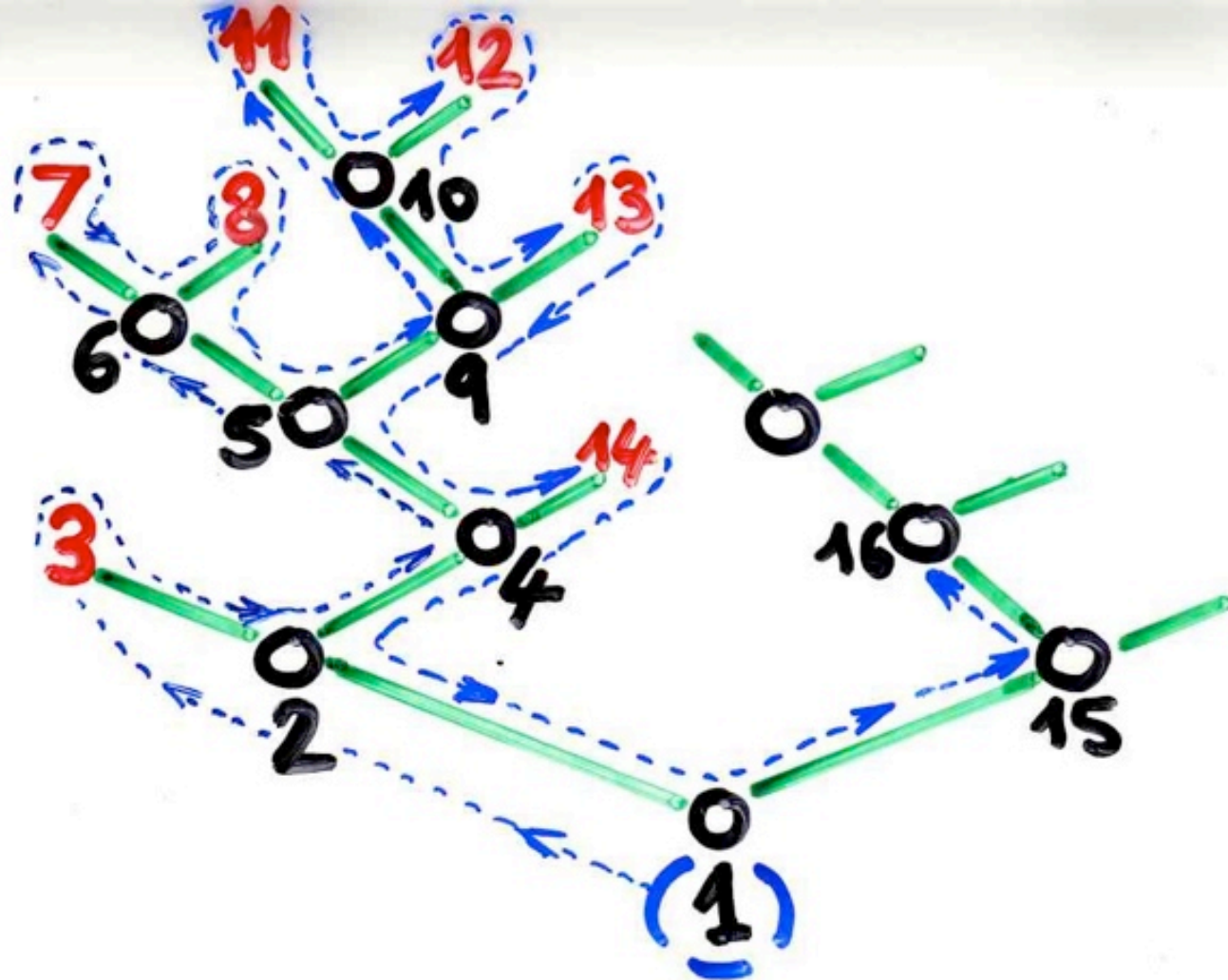


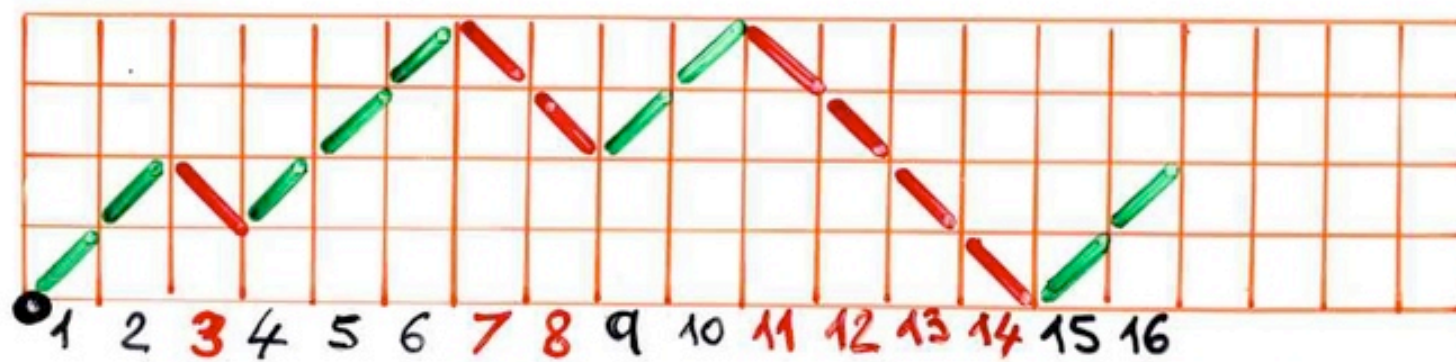
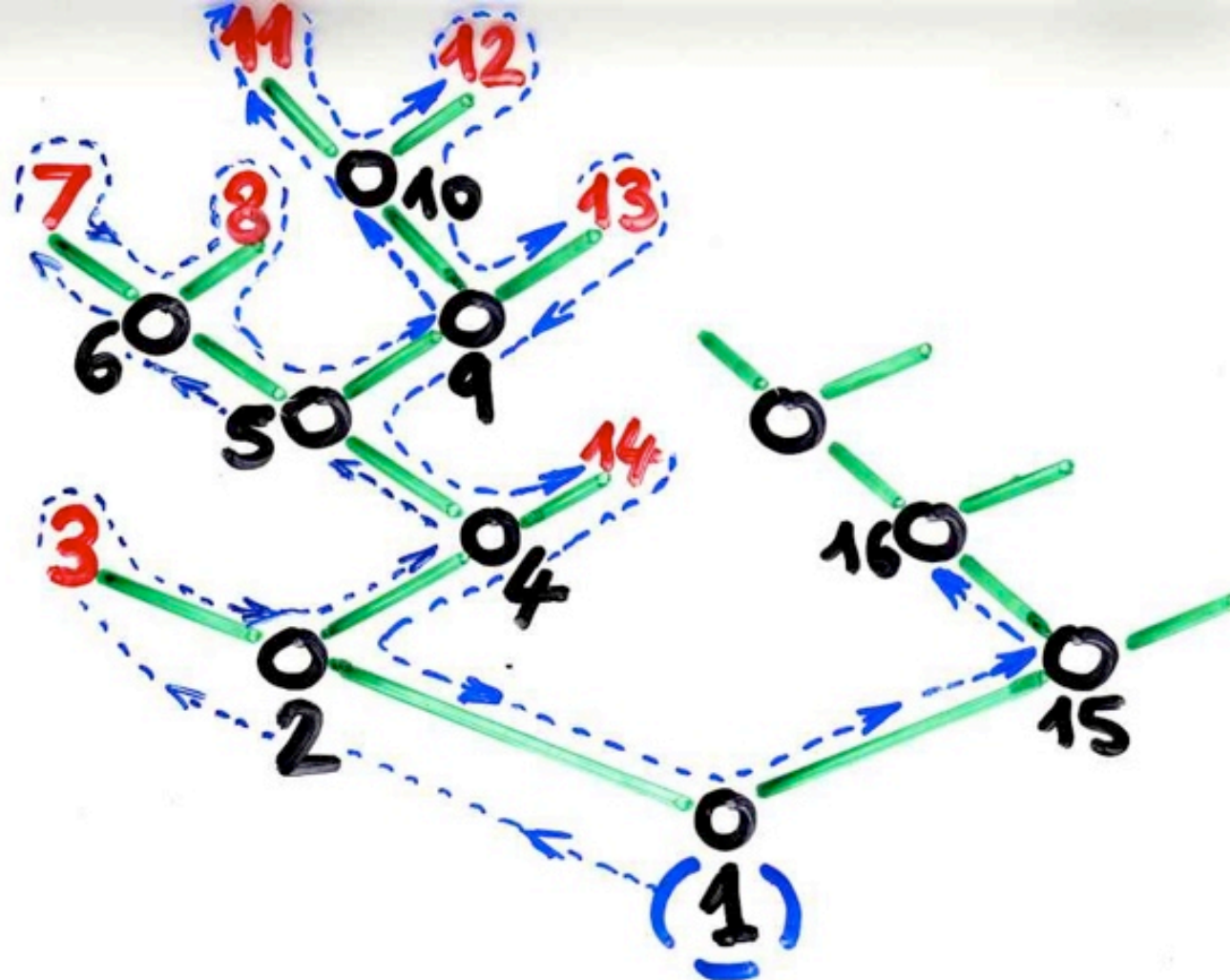


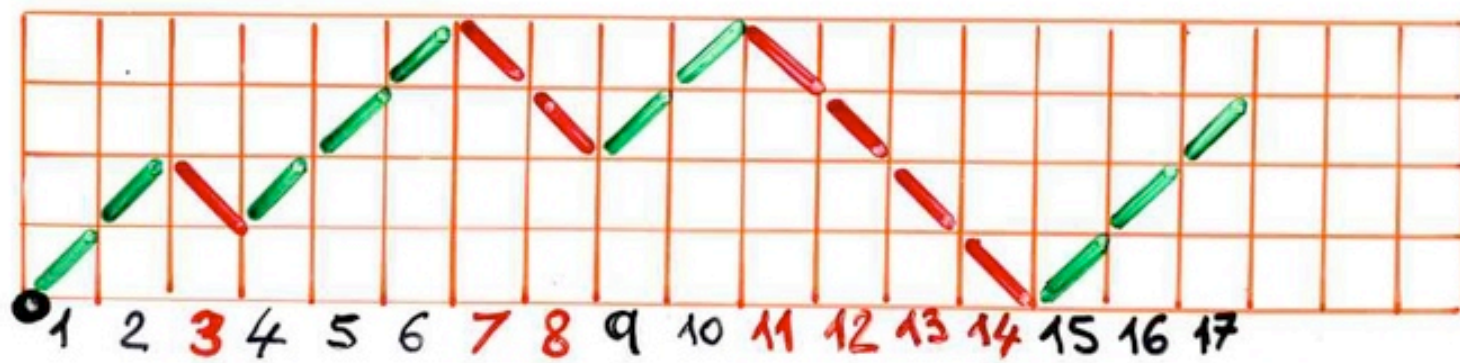
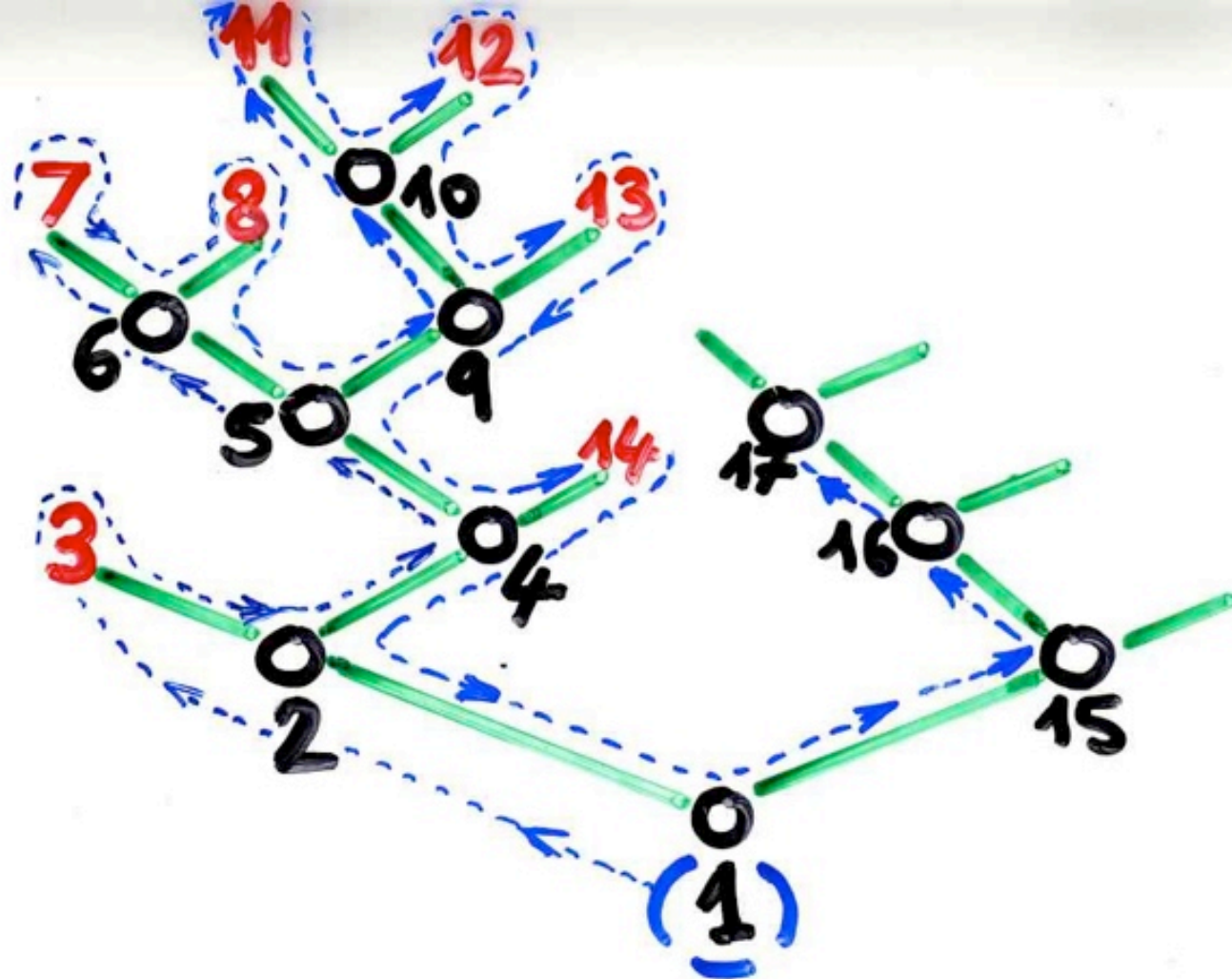


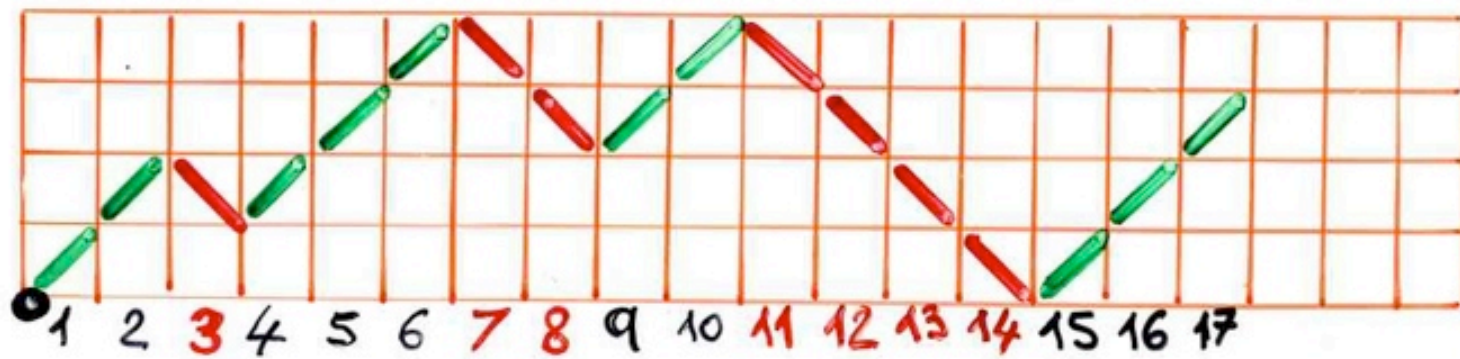
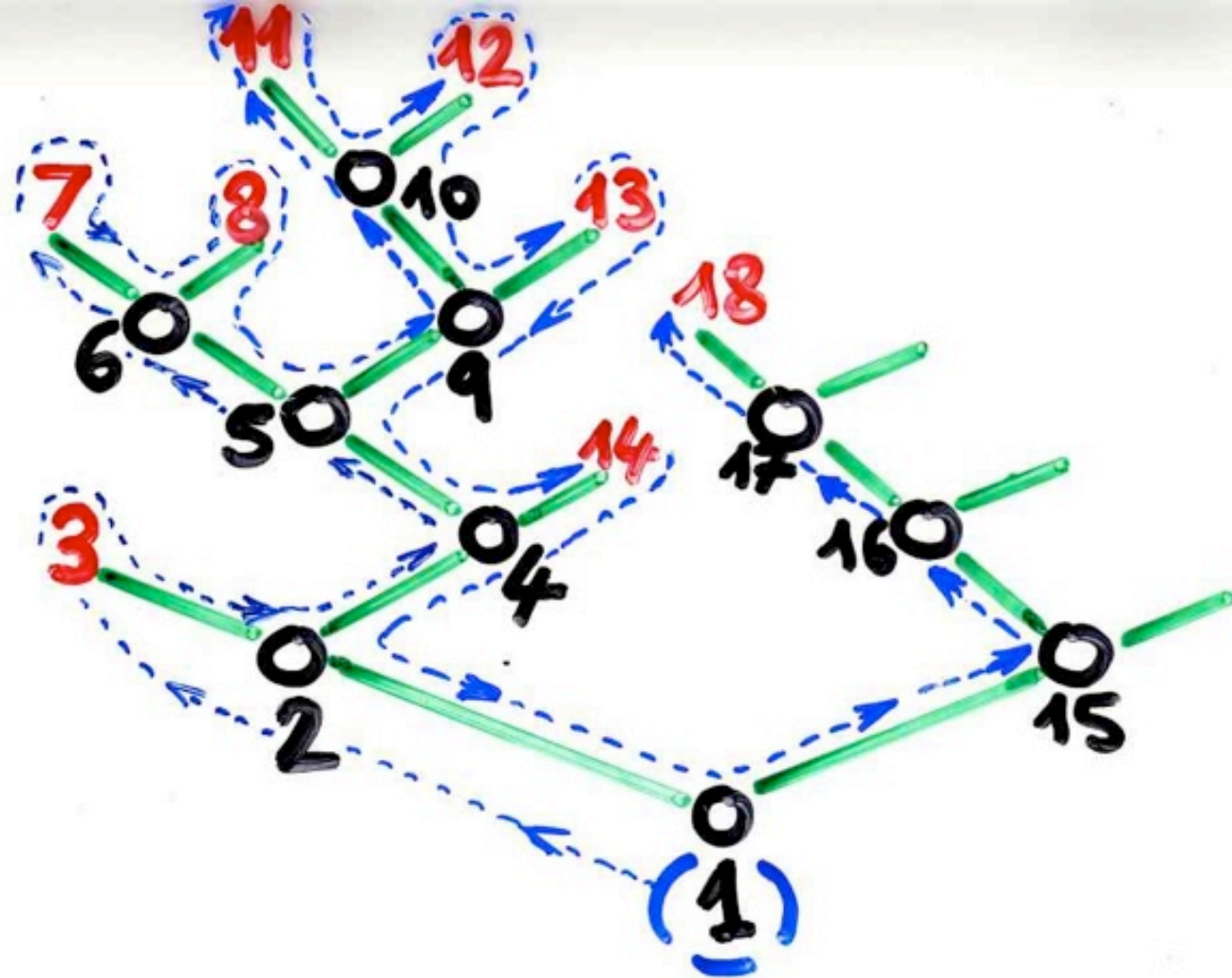


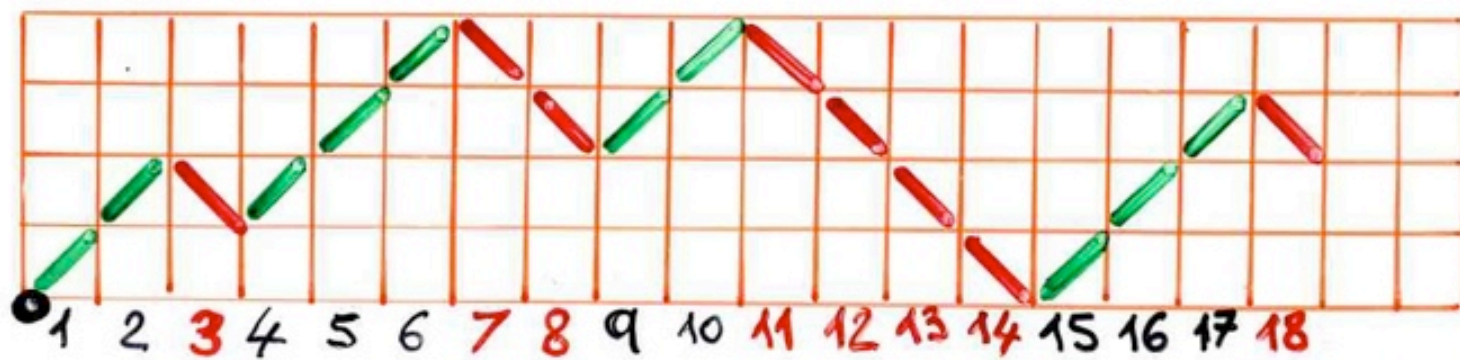
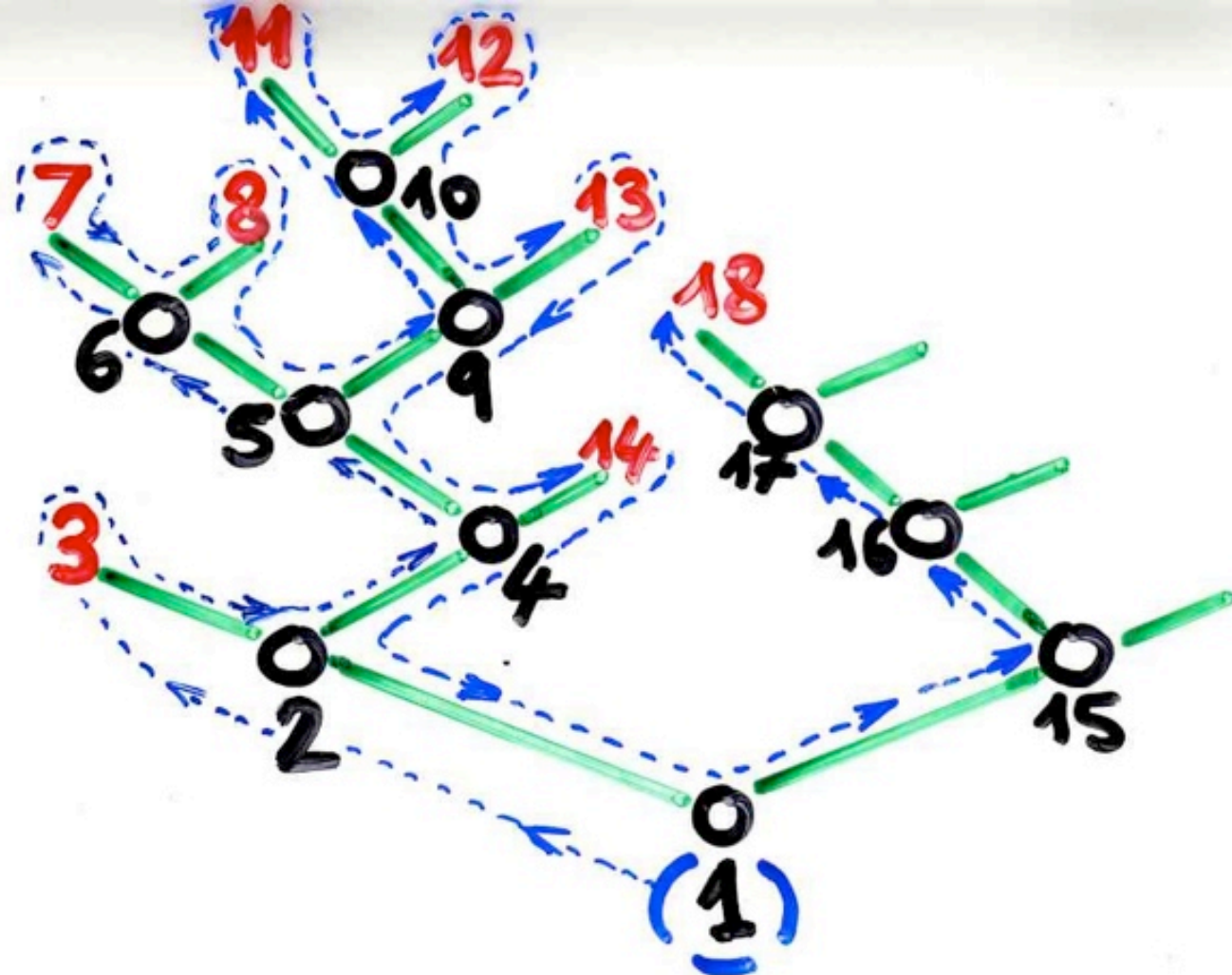


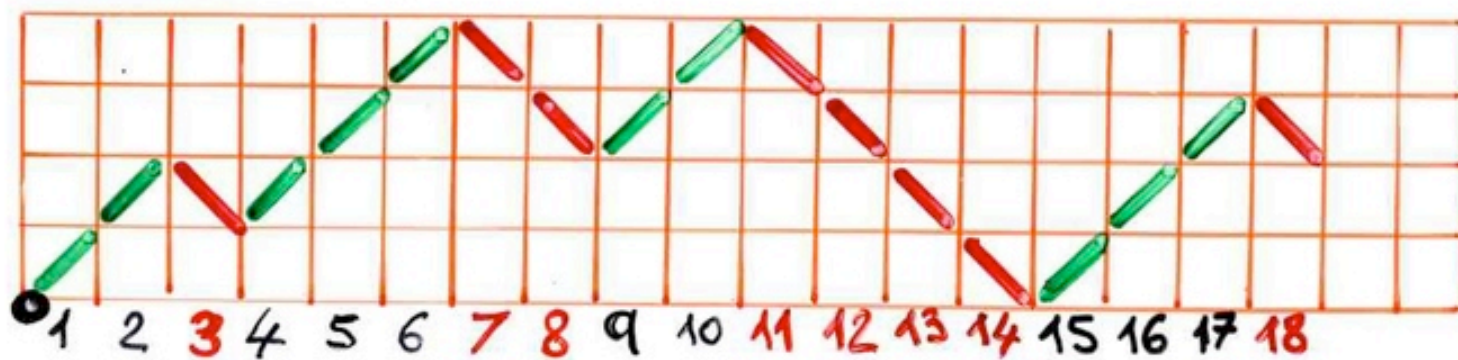
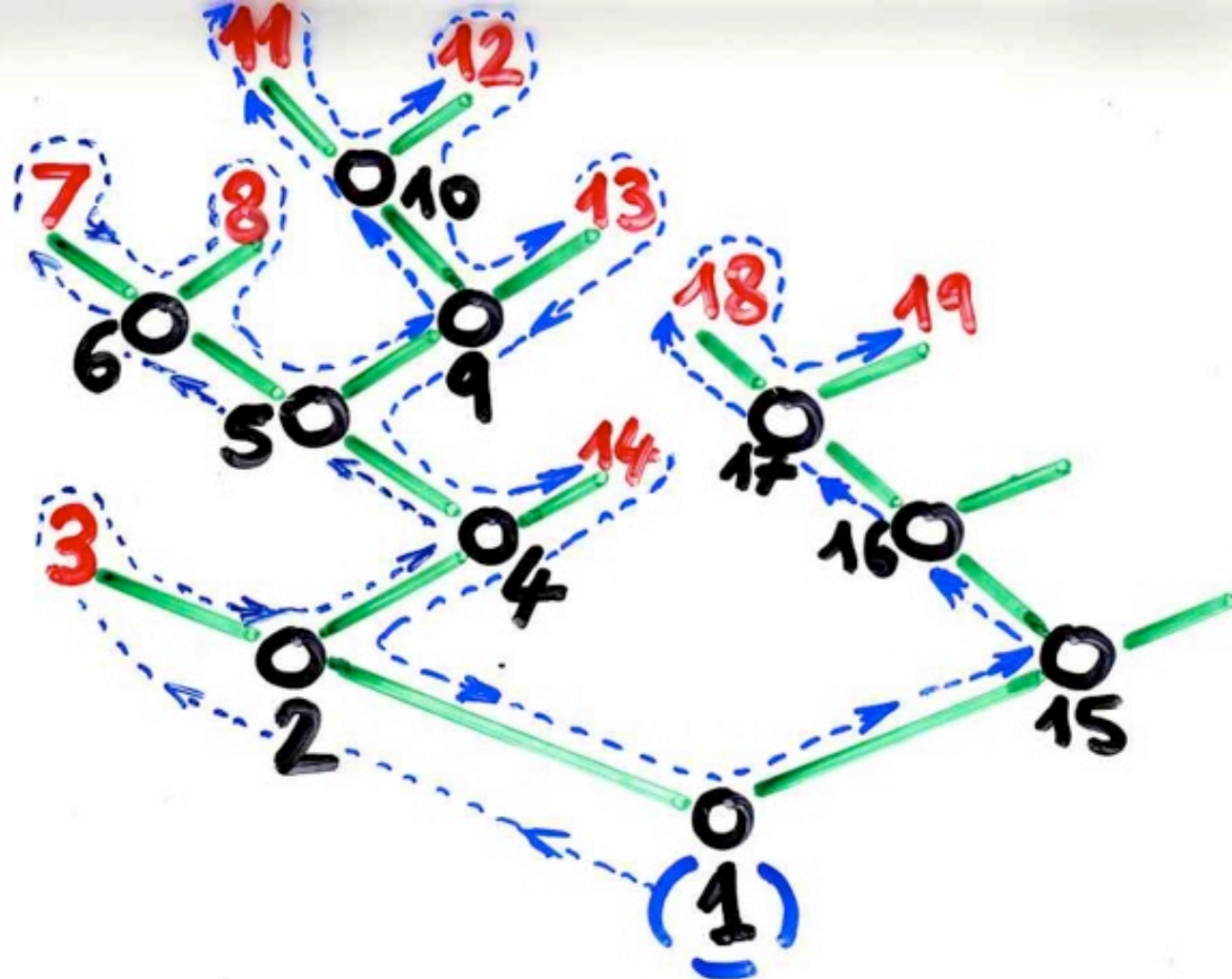


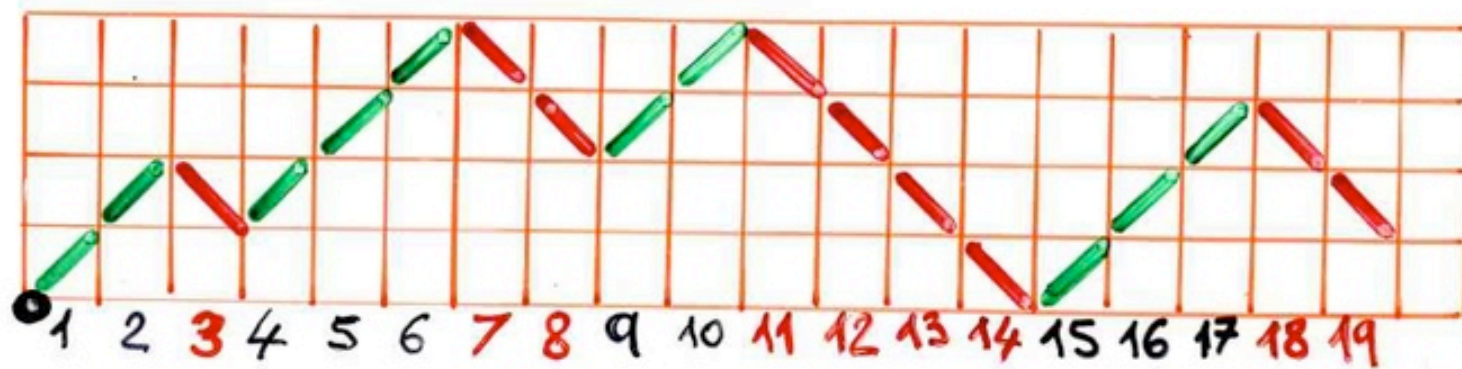
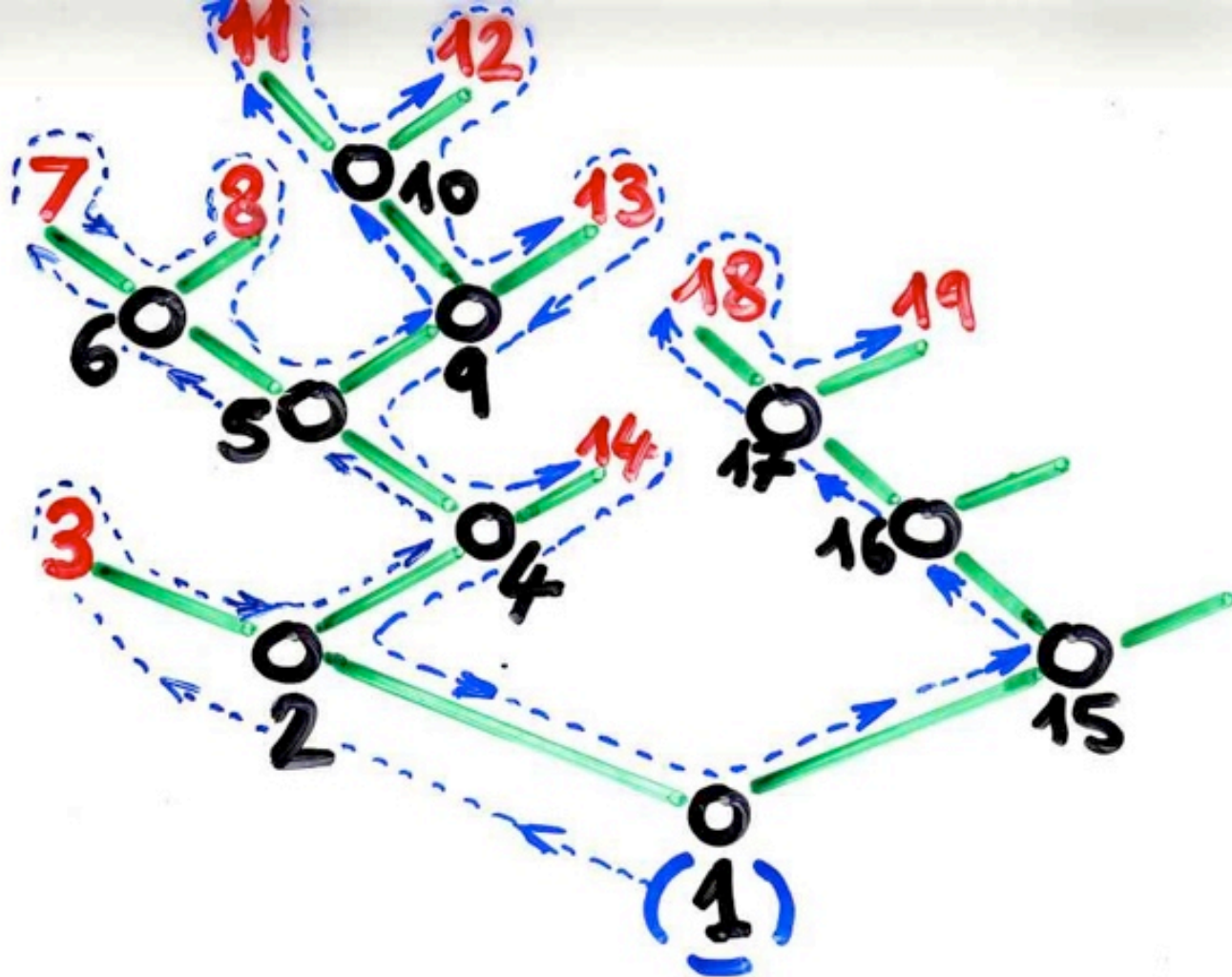


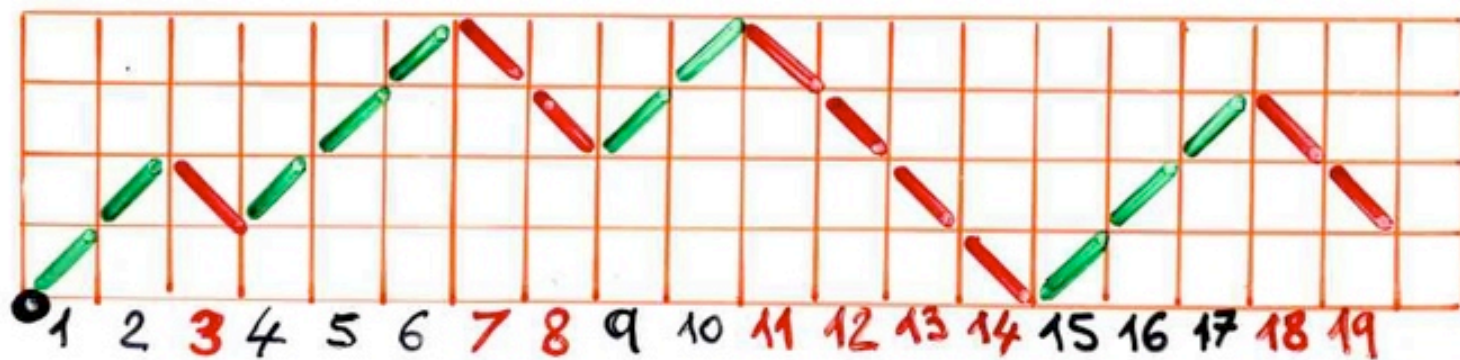
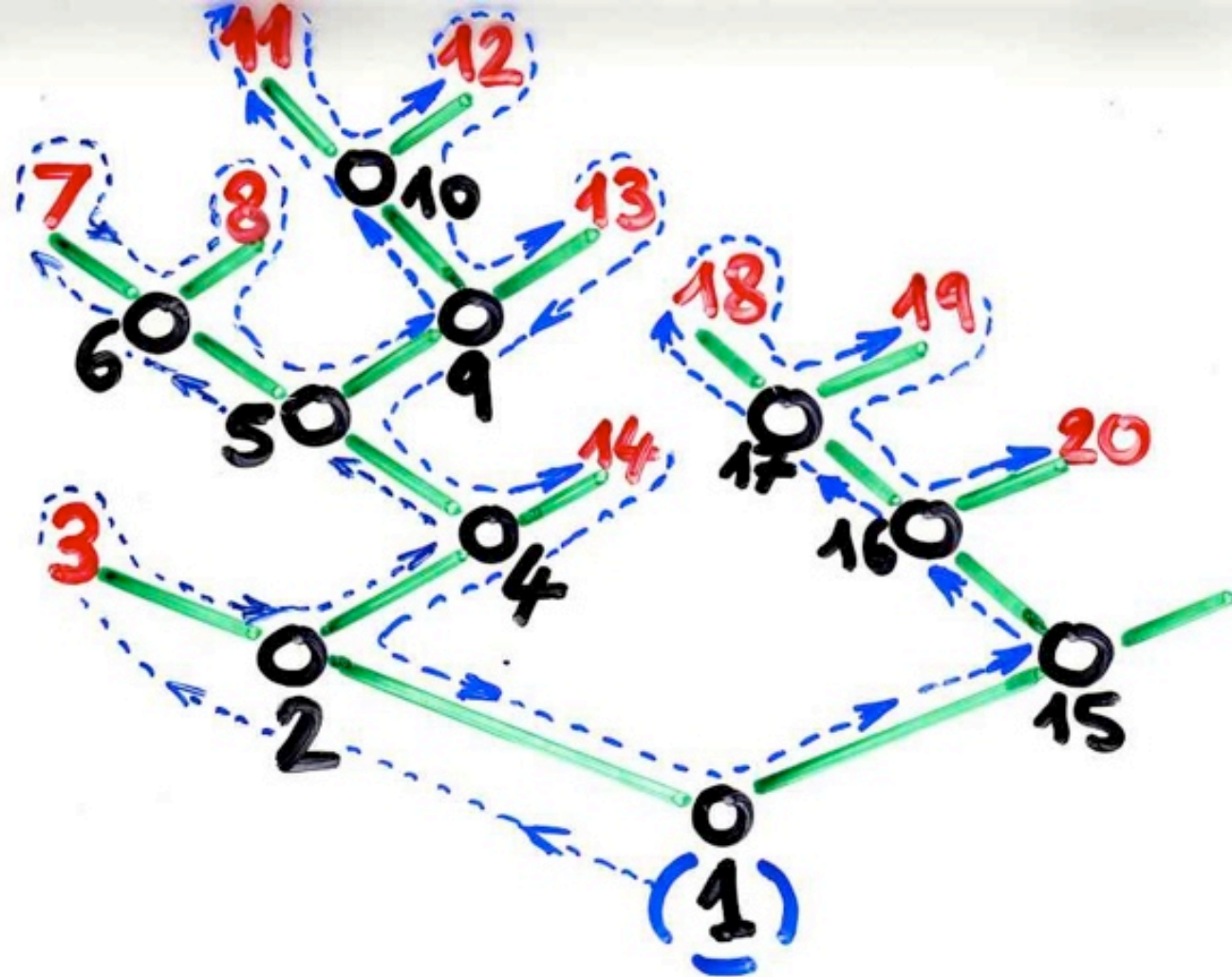


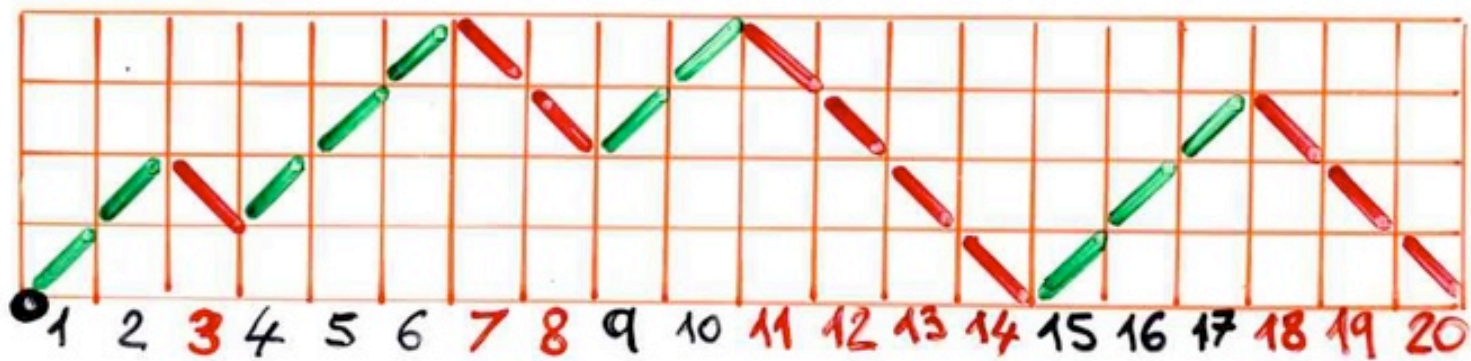
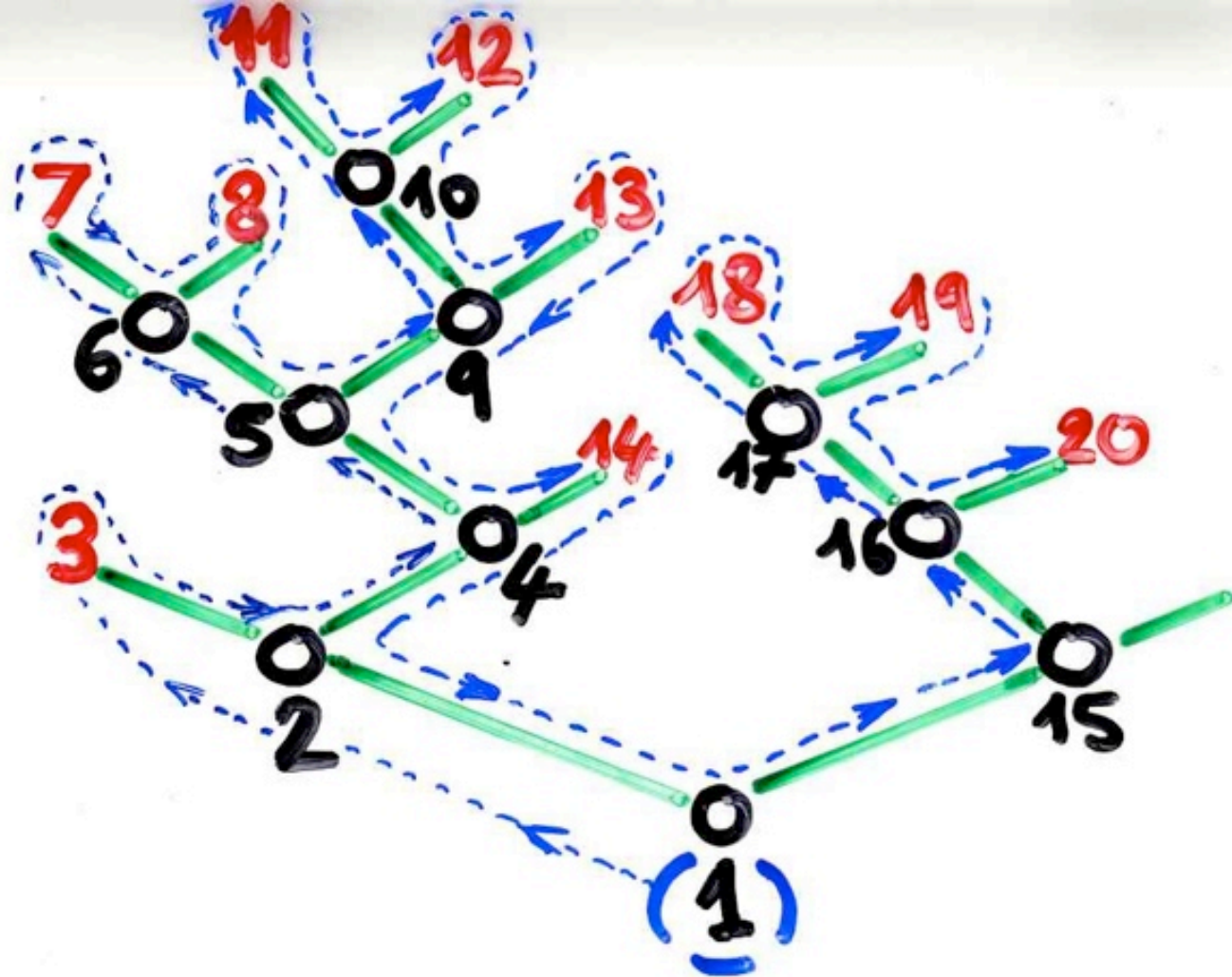


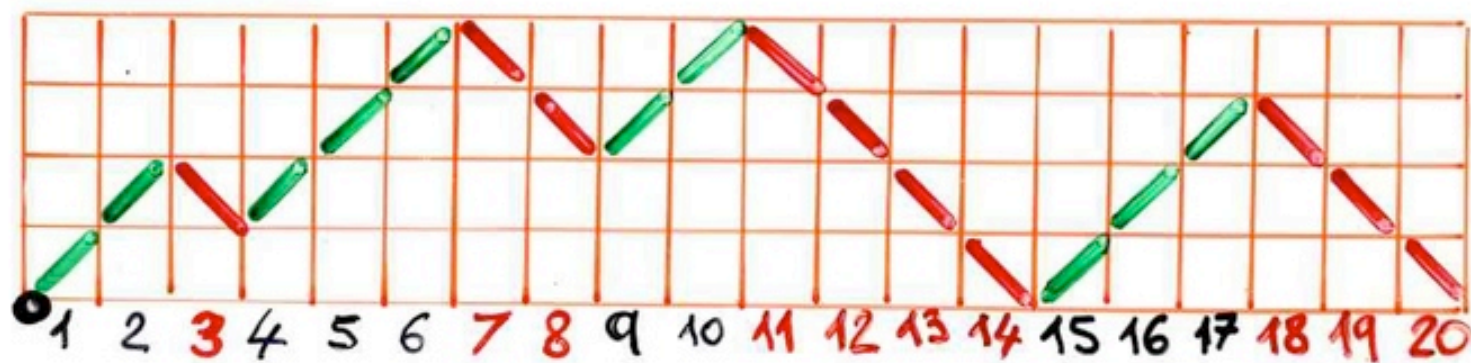




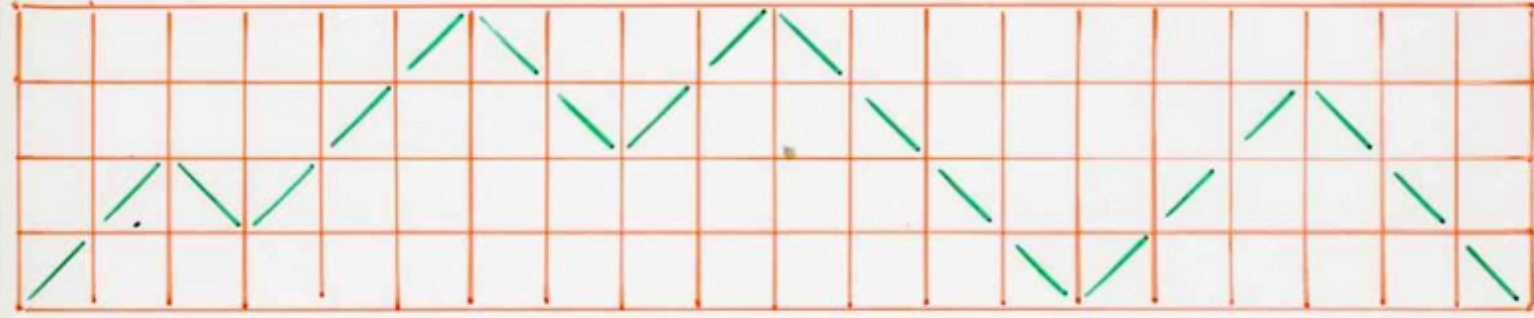


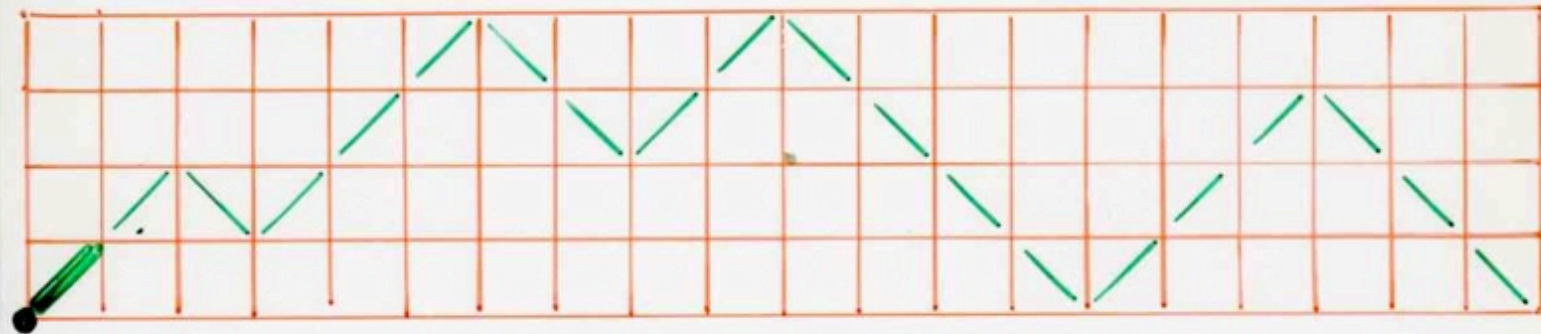


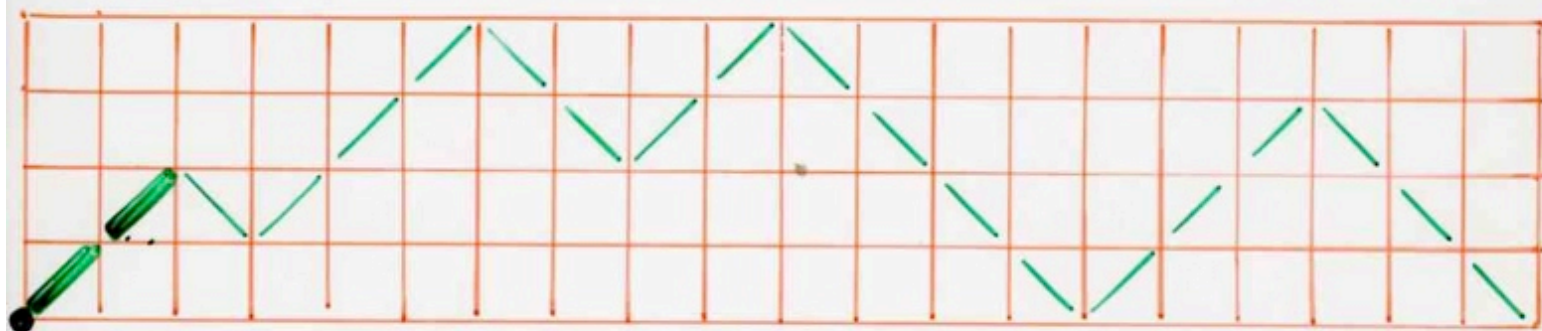
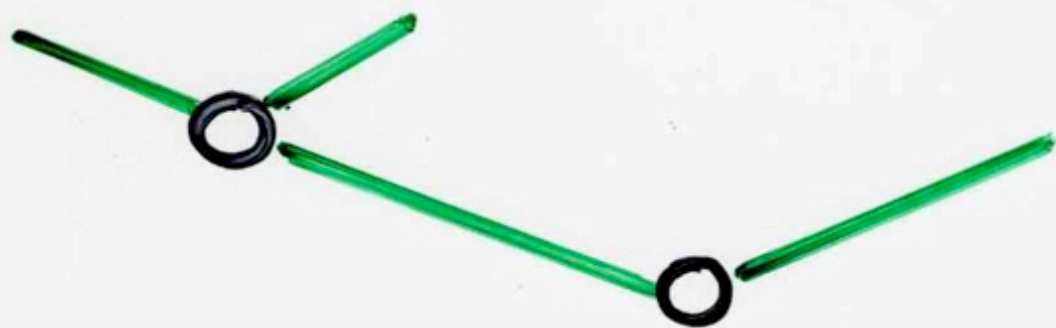


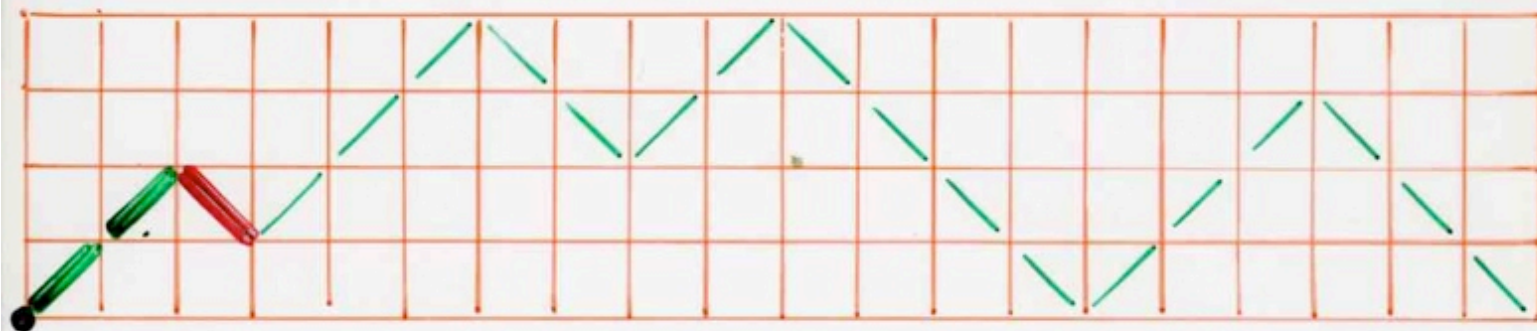
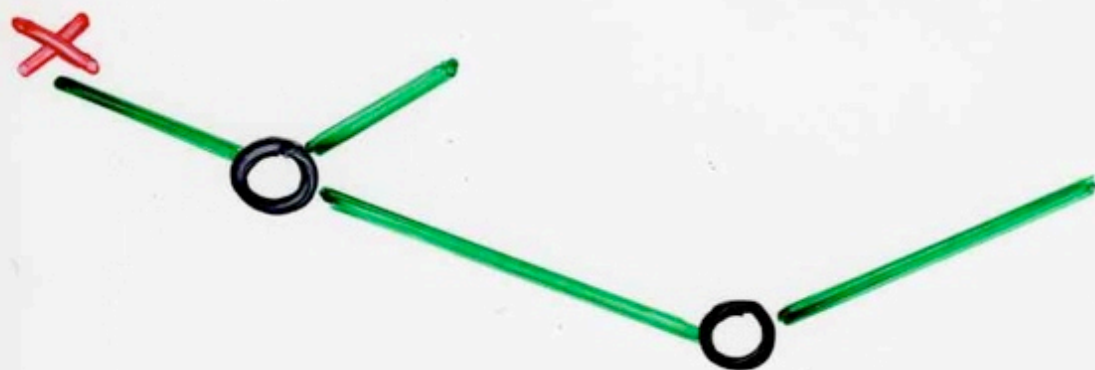


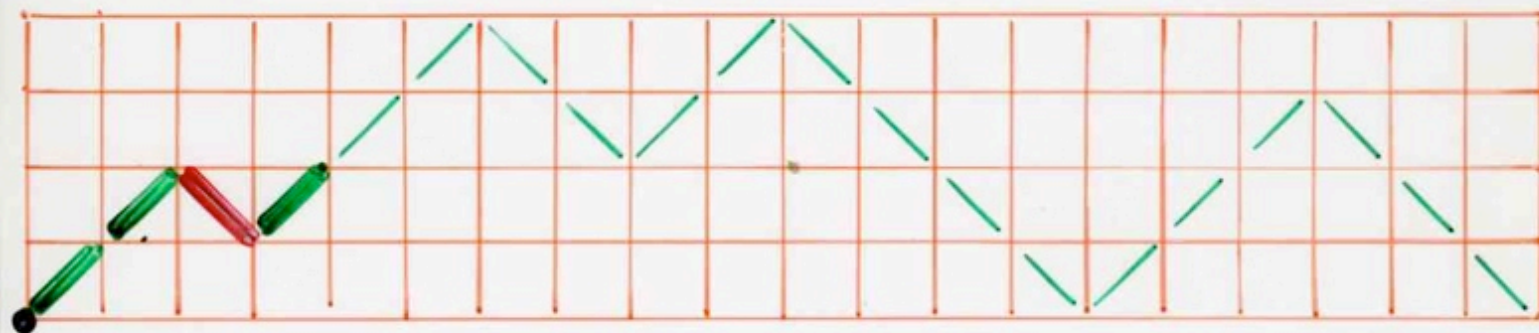
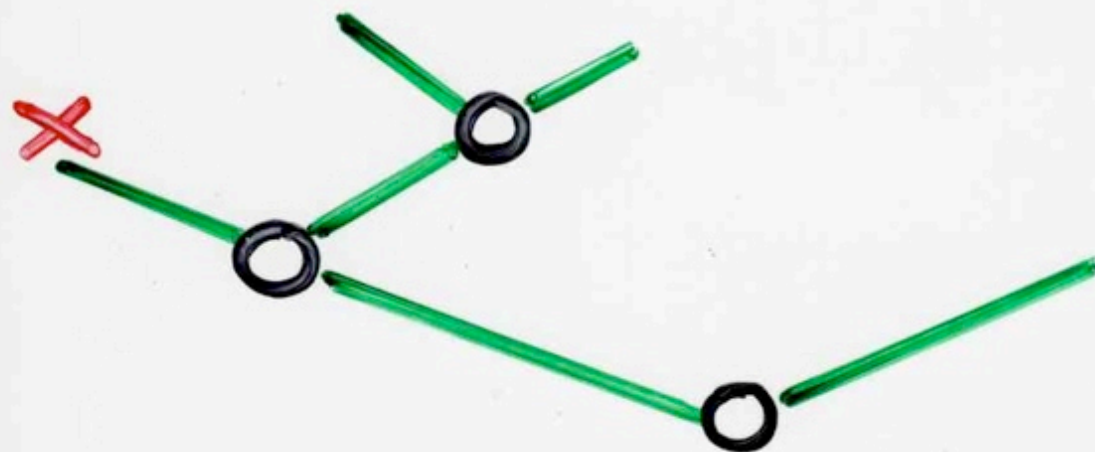
bijection inverse

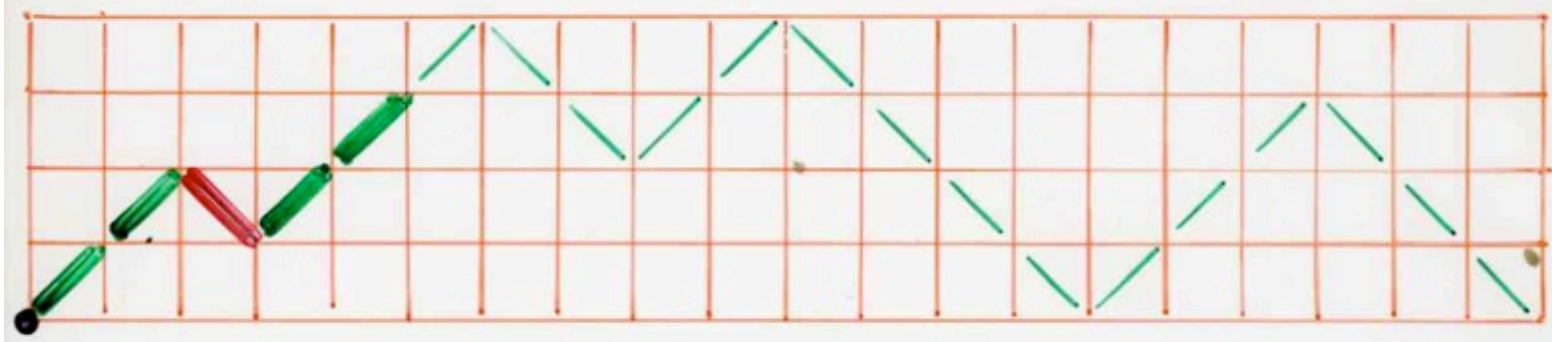
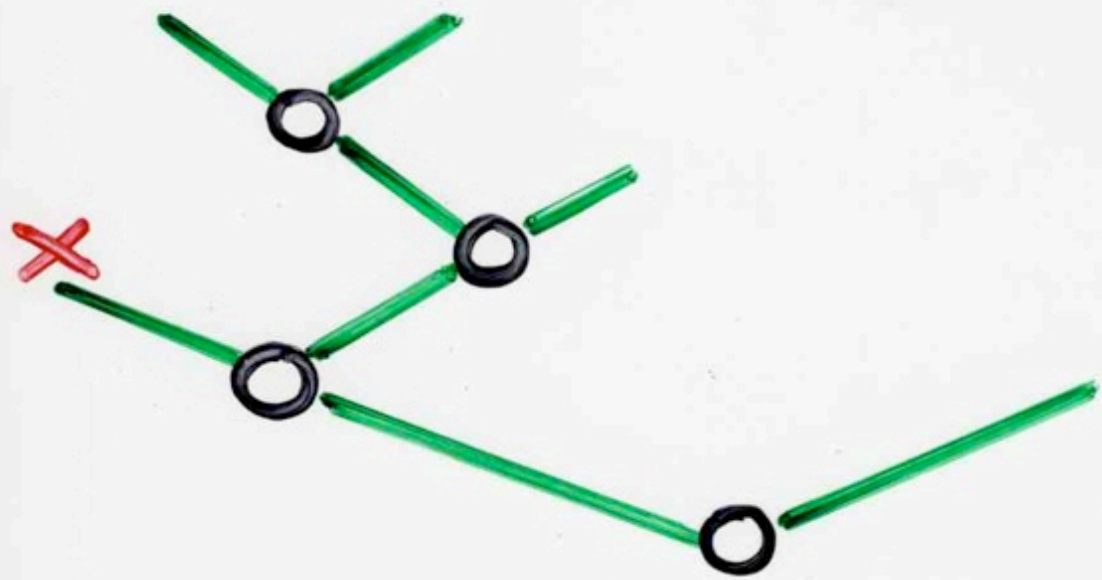


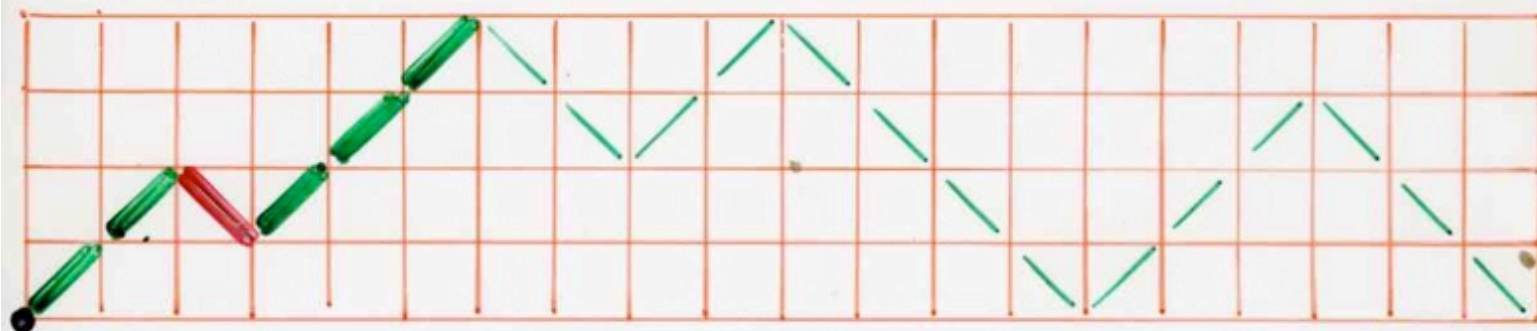


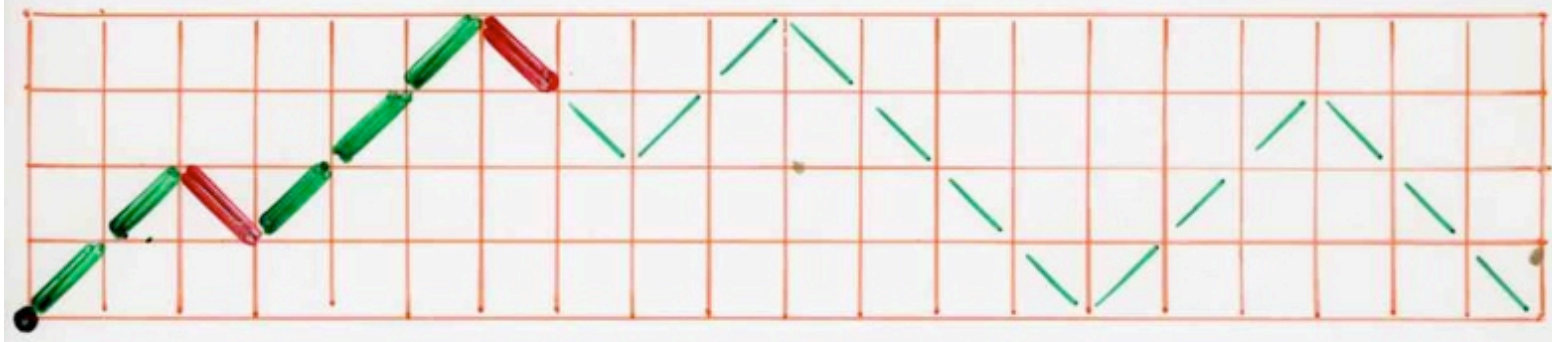
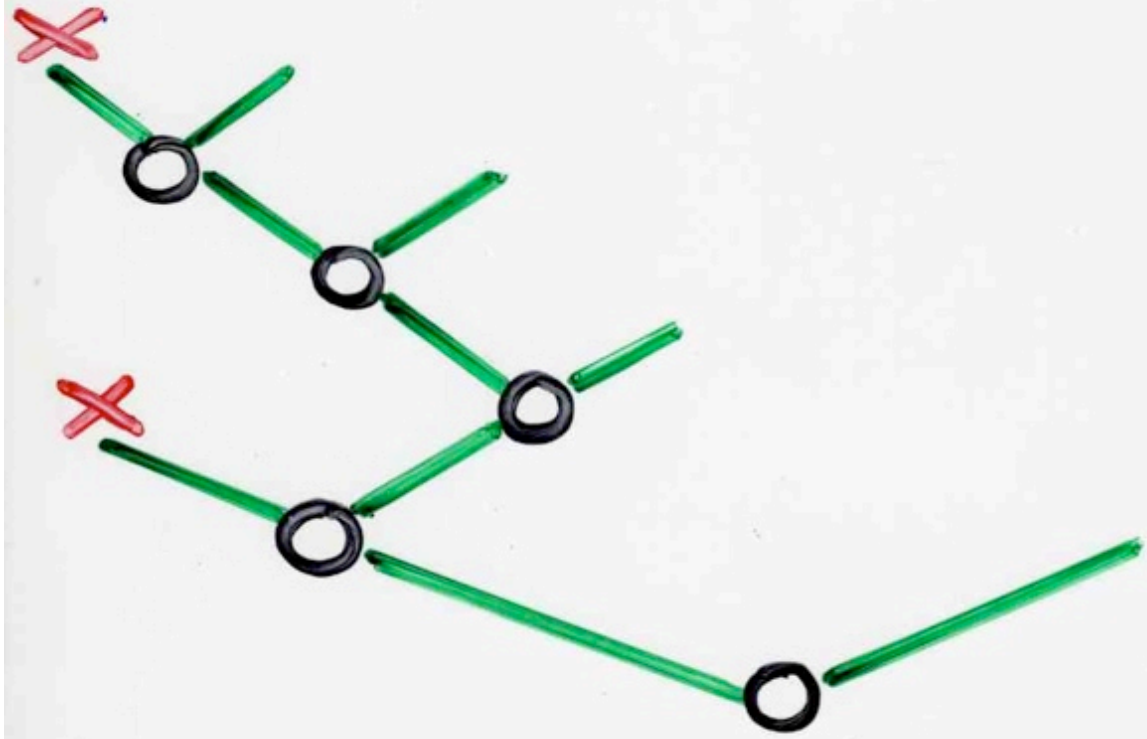


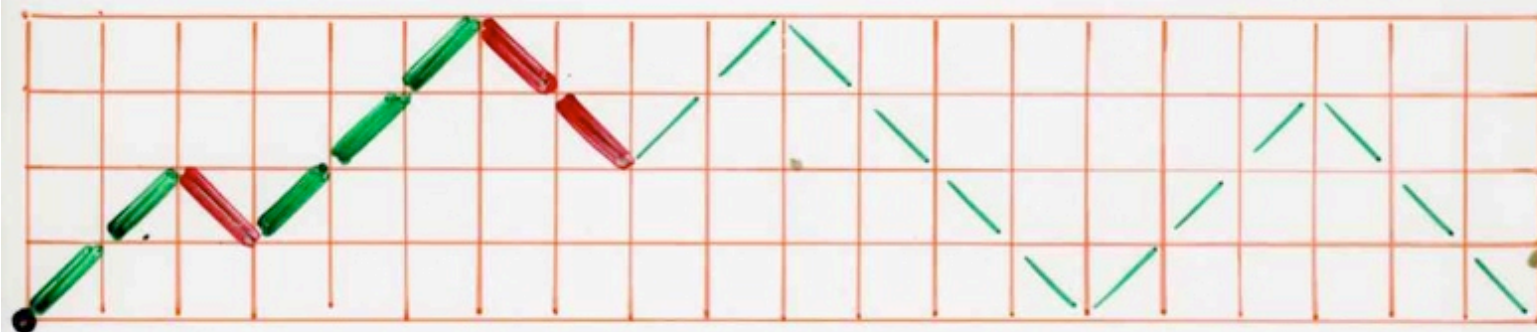
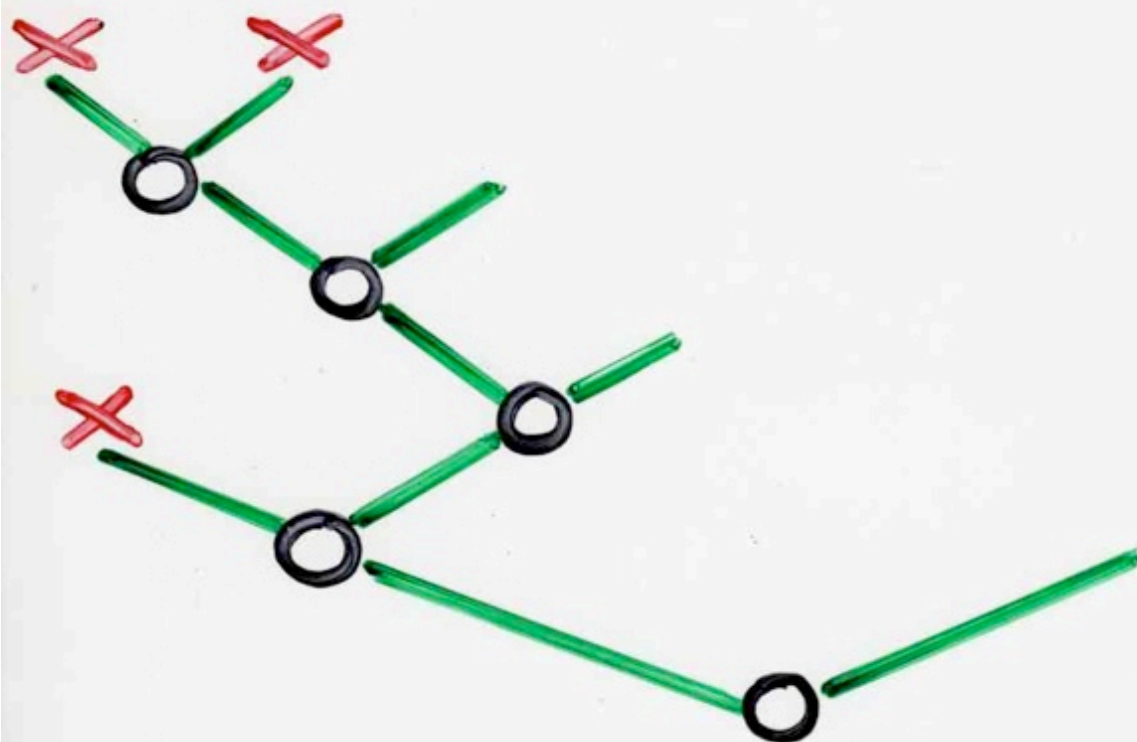


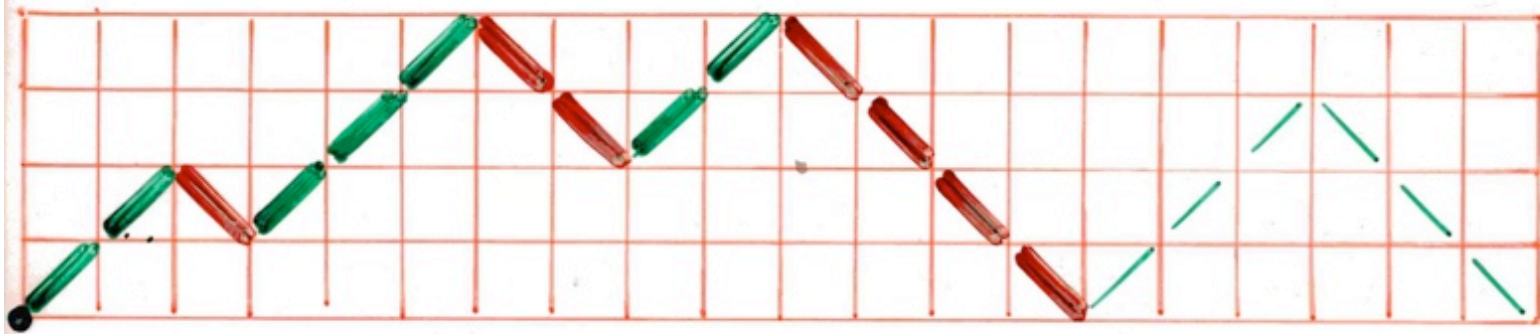
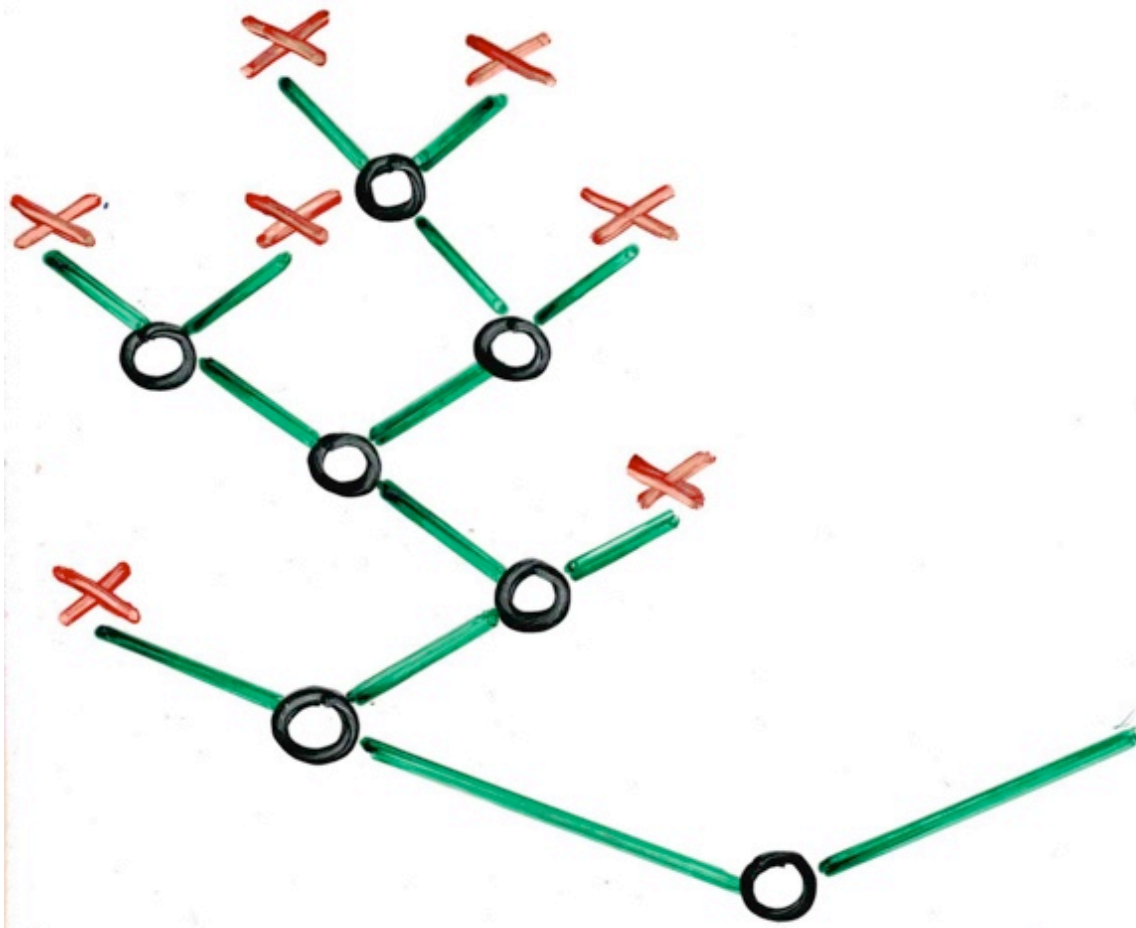


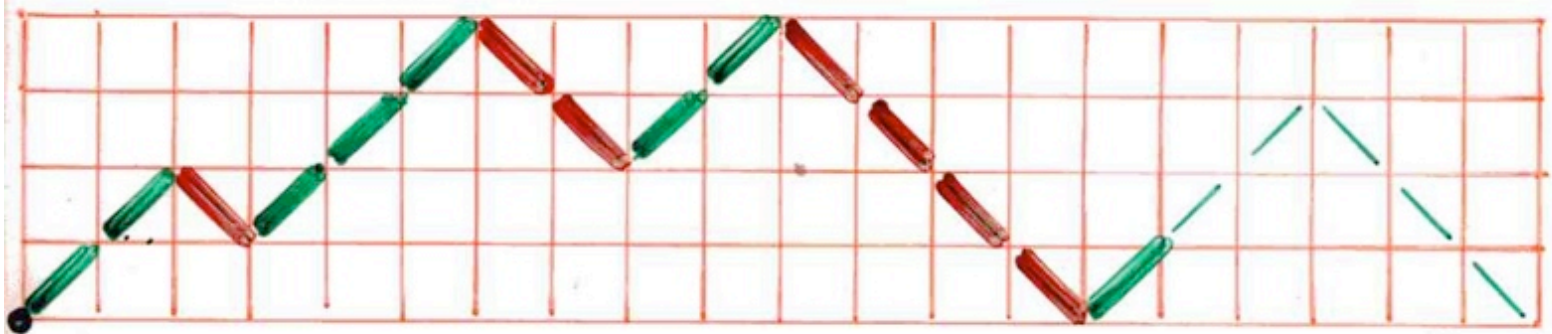
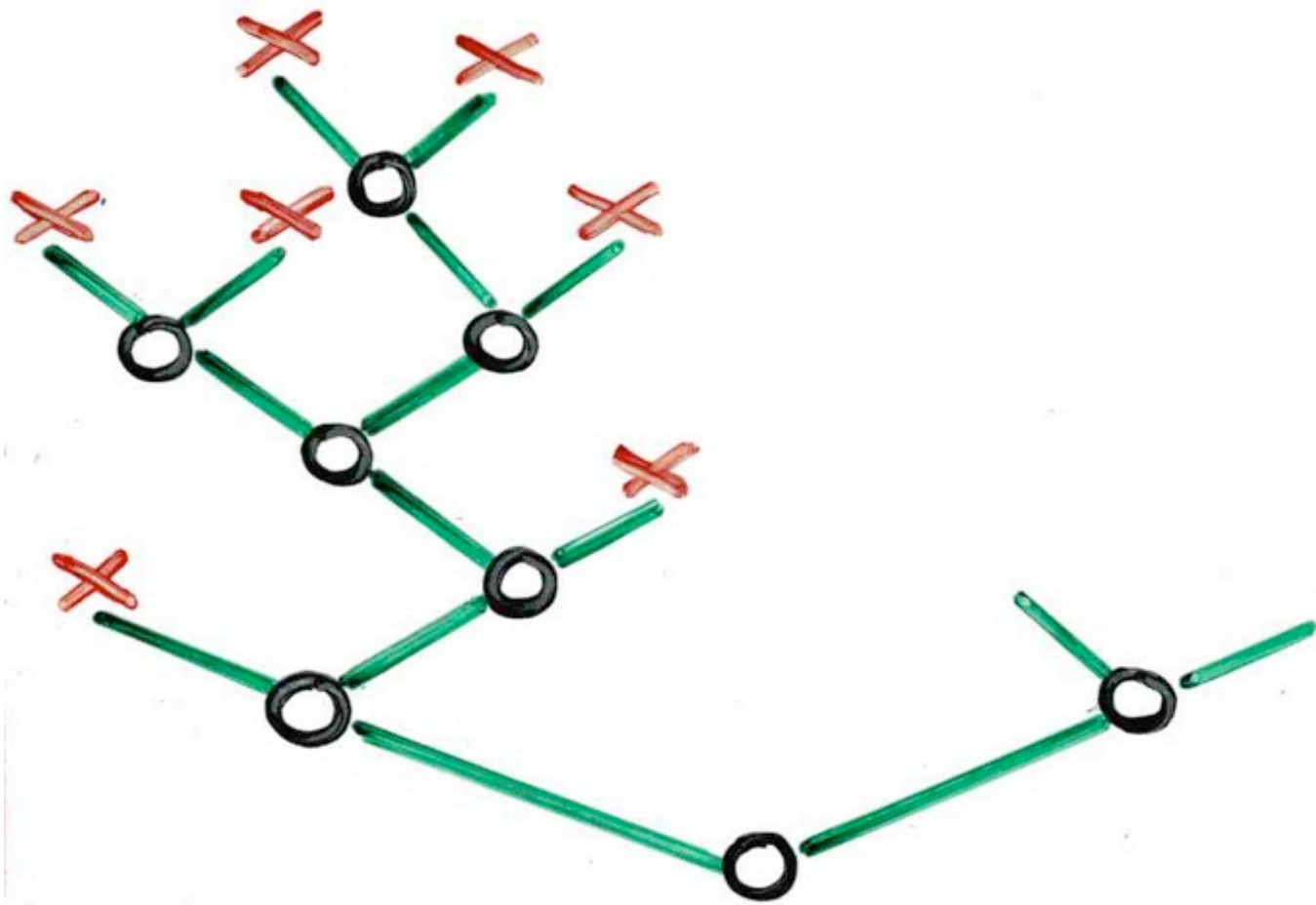


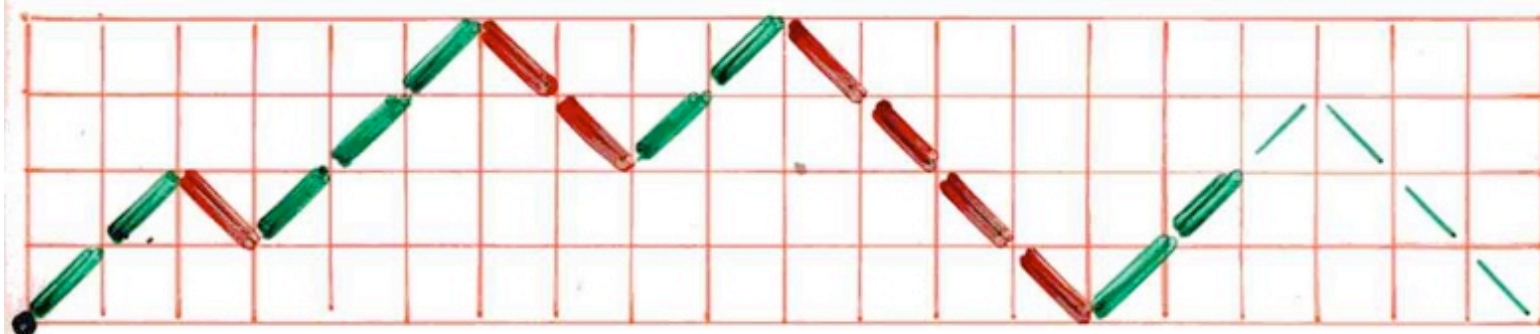
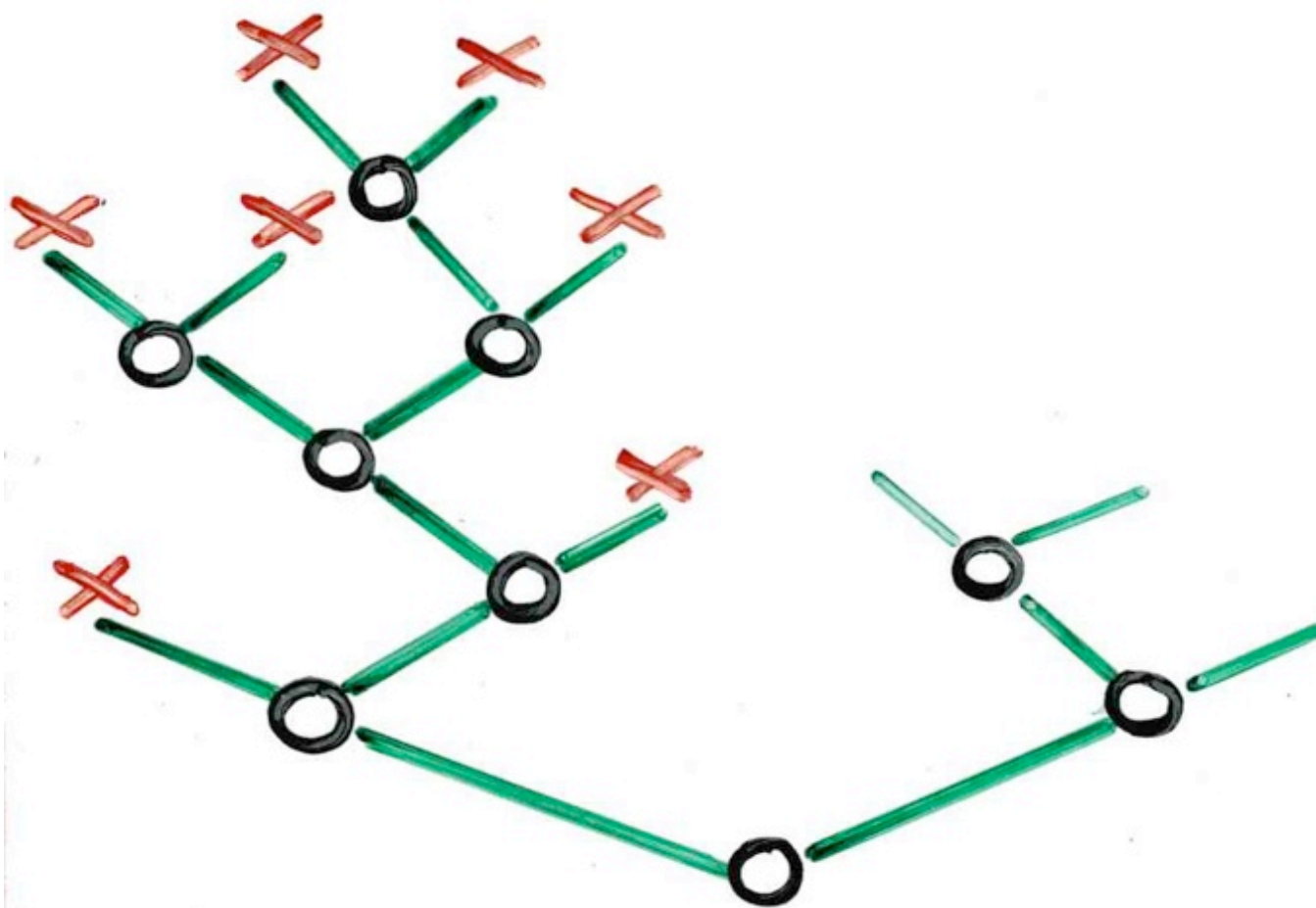


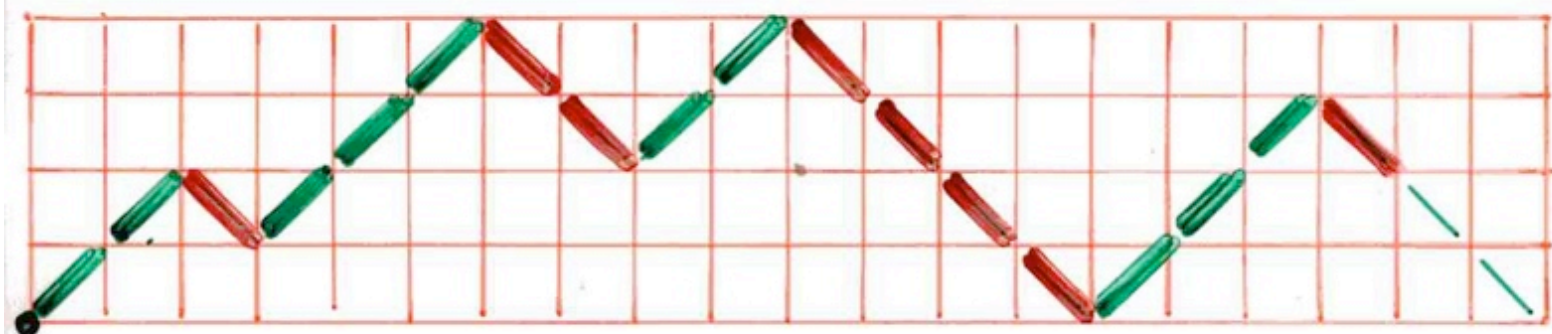
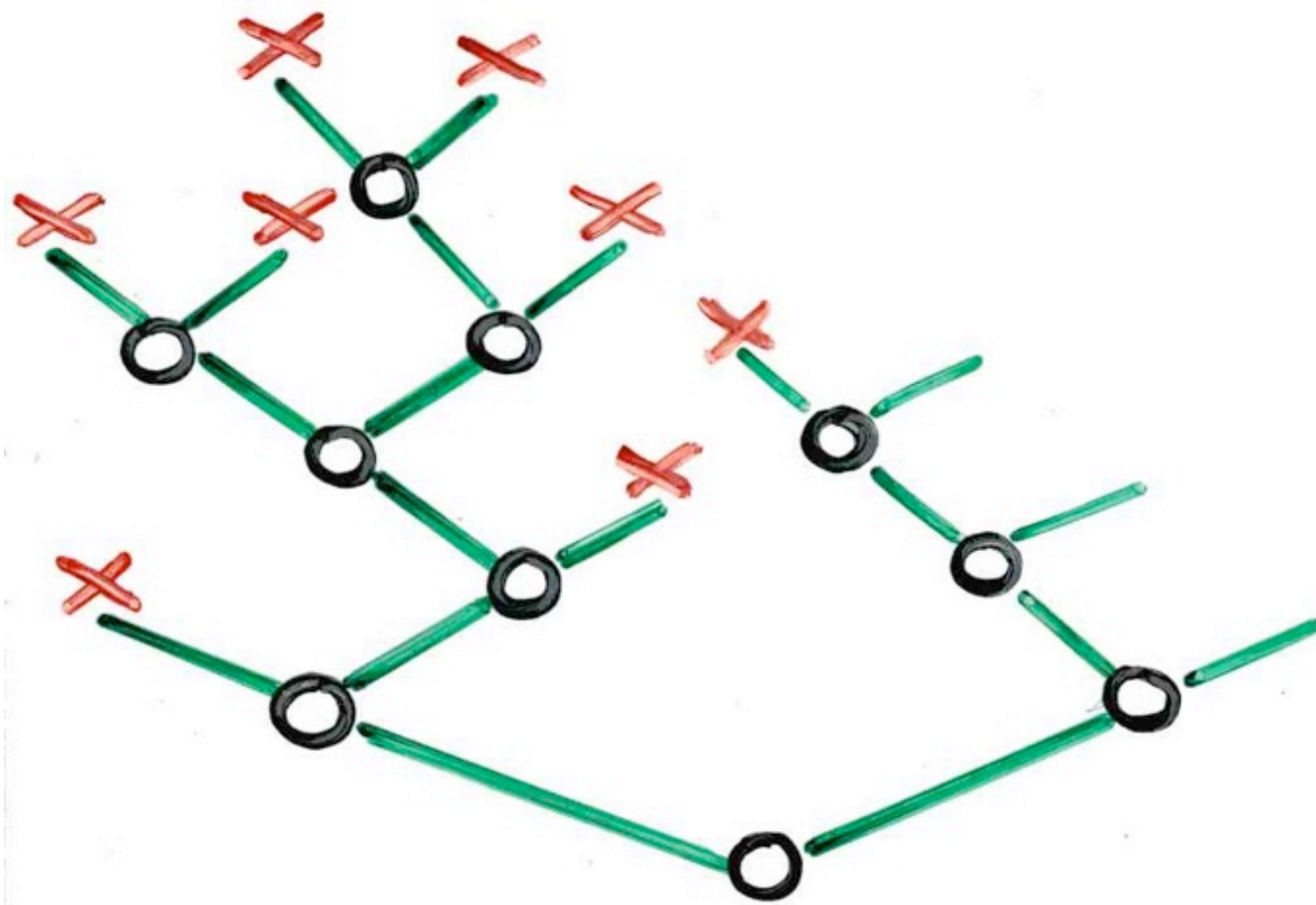


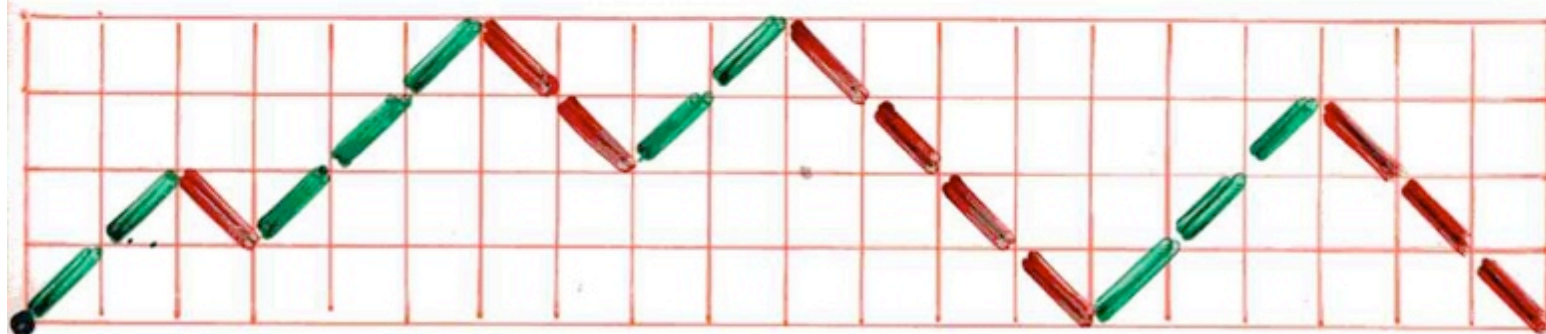
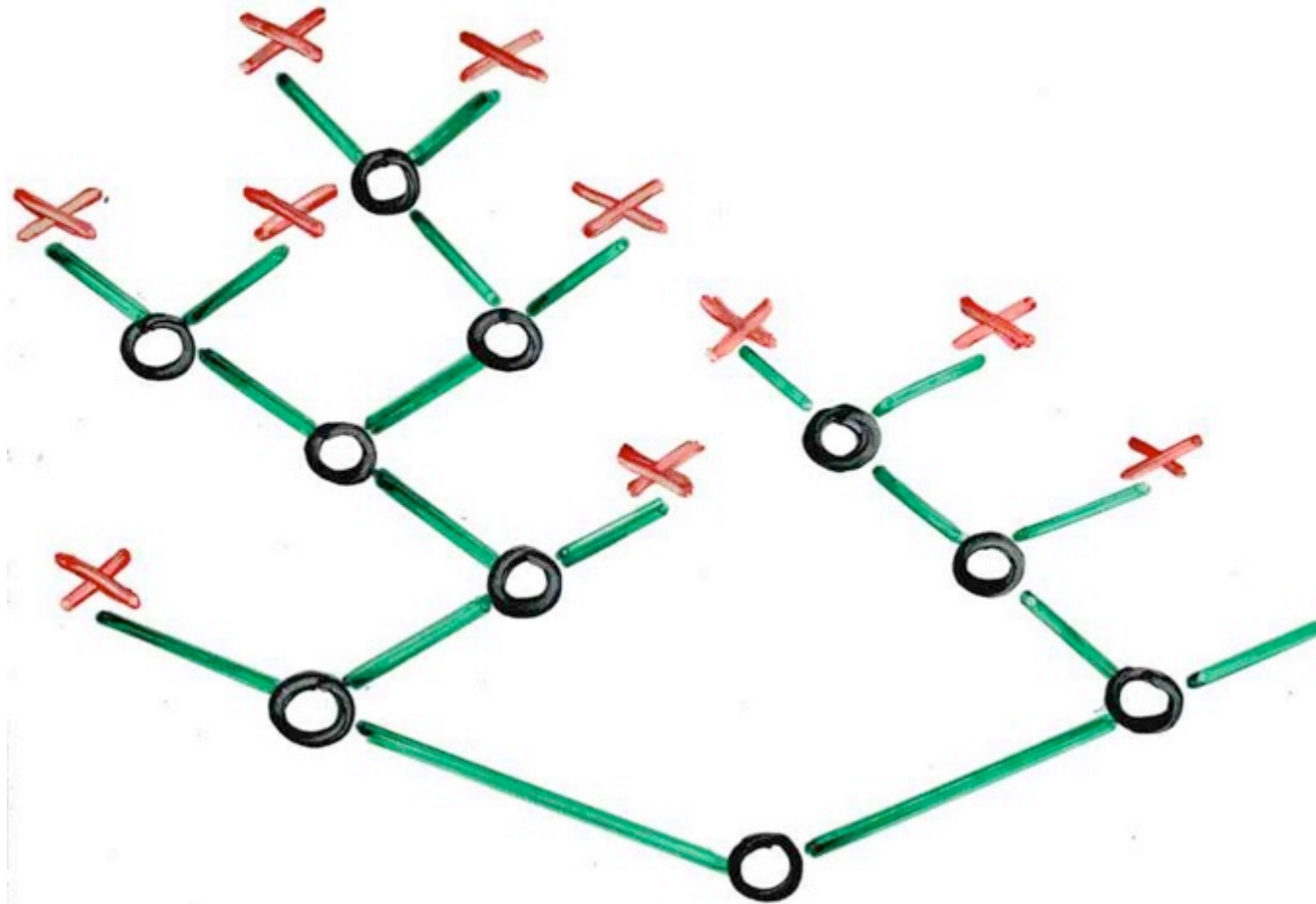


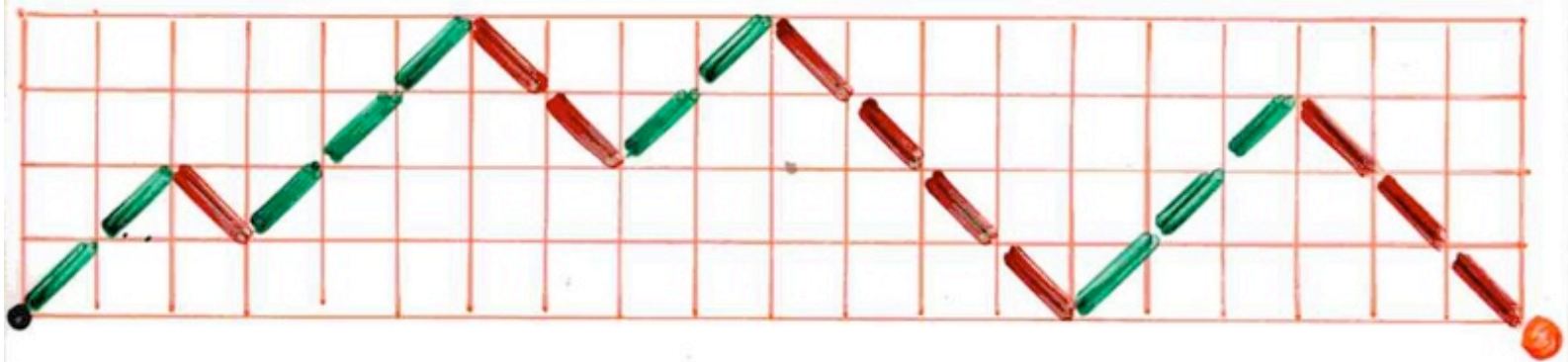
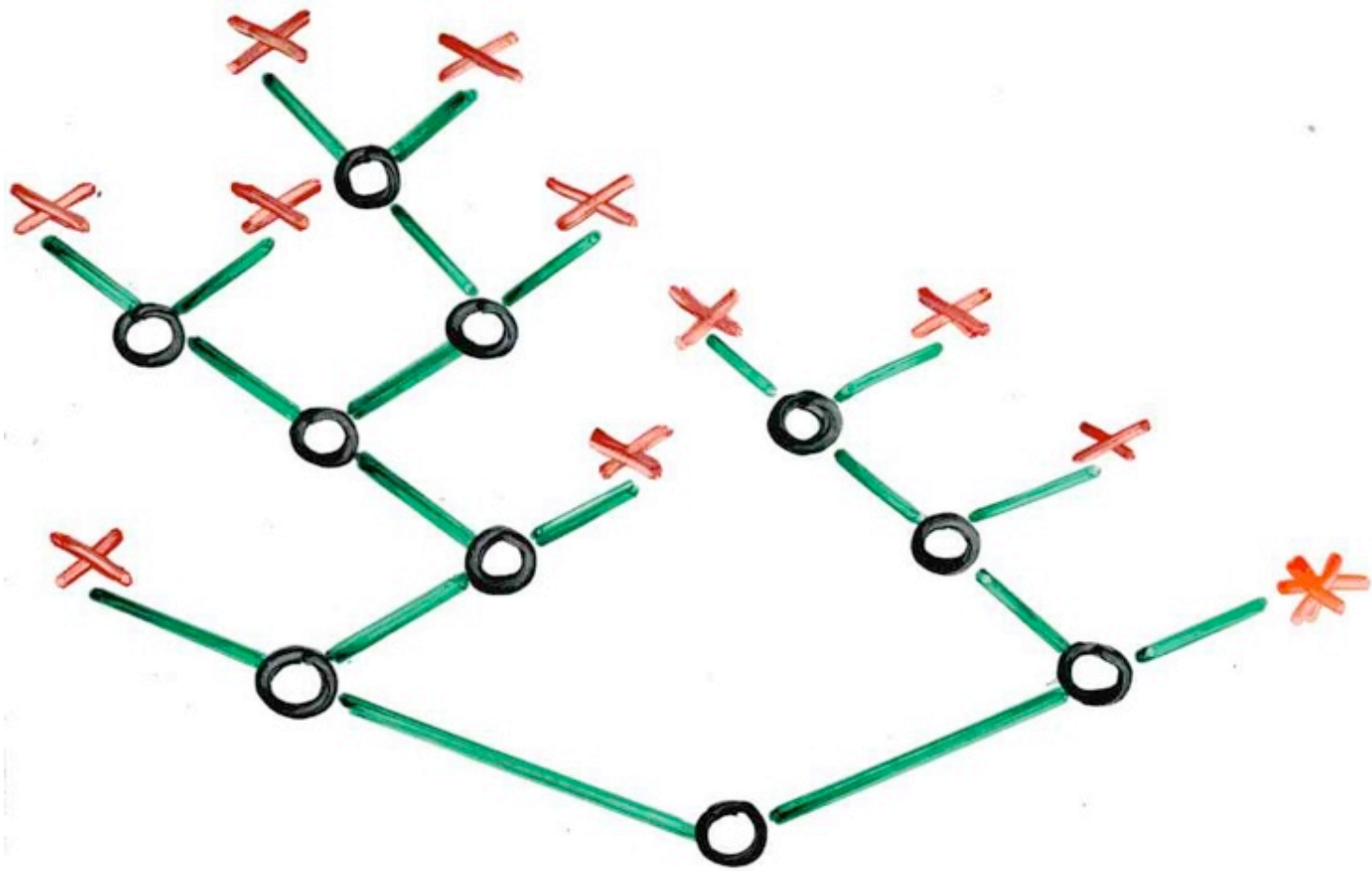












Le majestueux noyer

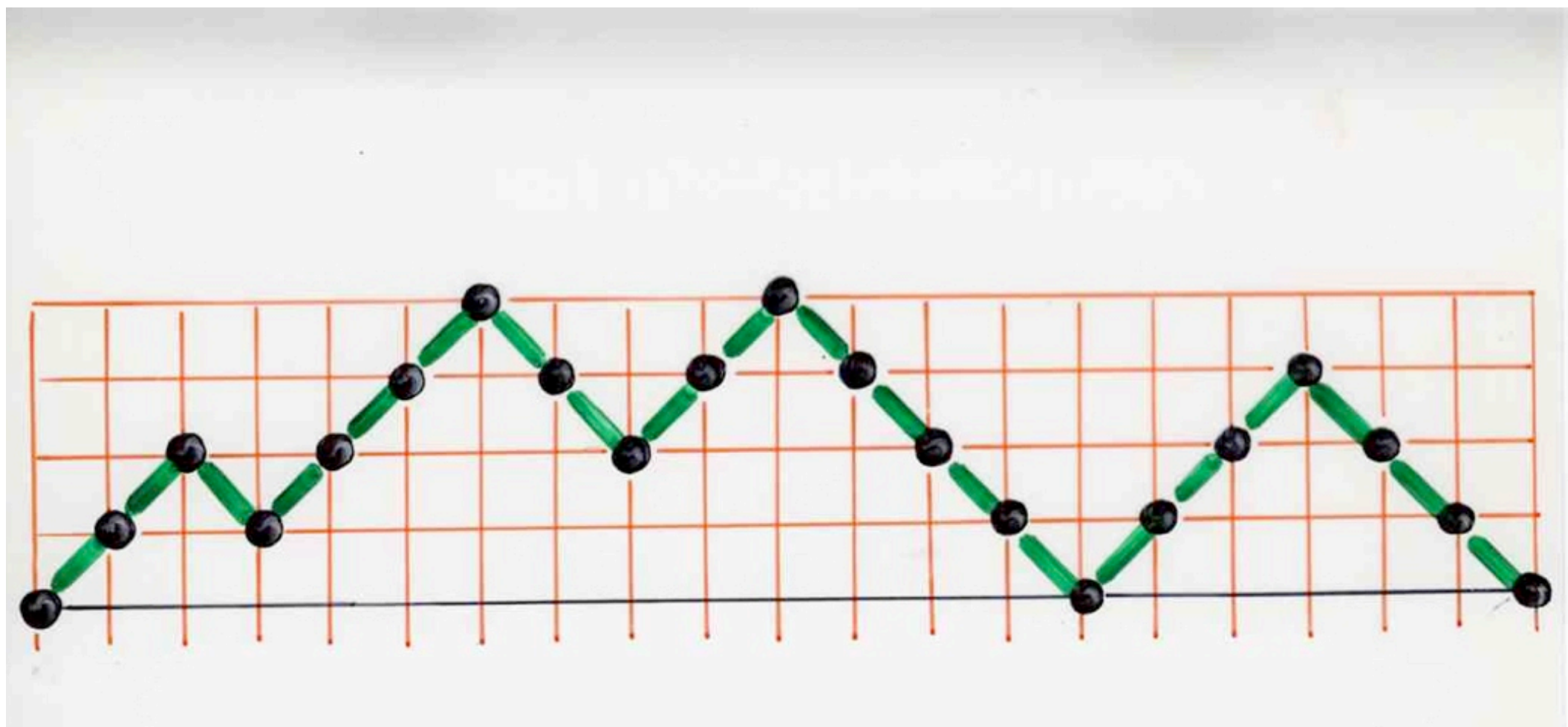


Marcia Pig Lagos
auteur

hauteur logarithmique



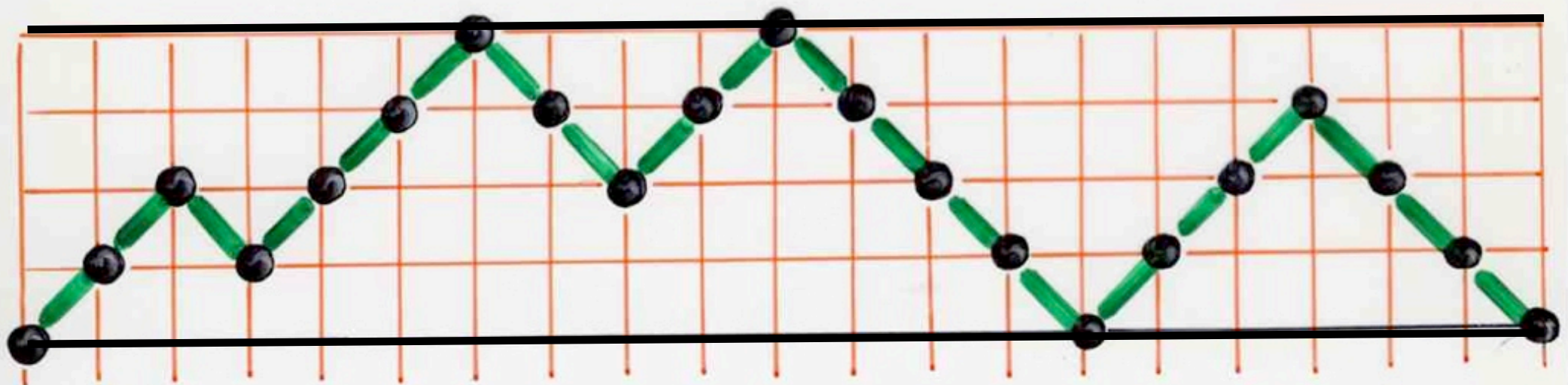
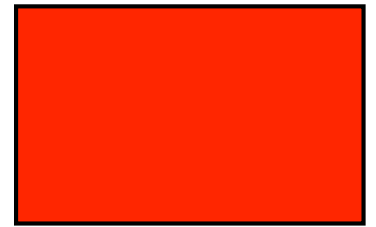
chemin de Dyck w



chemin de Dyck w

hauteur $h(w)$

$= 4$



= 4

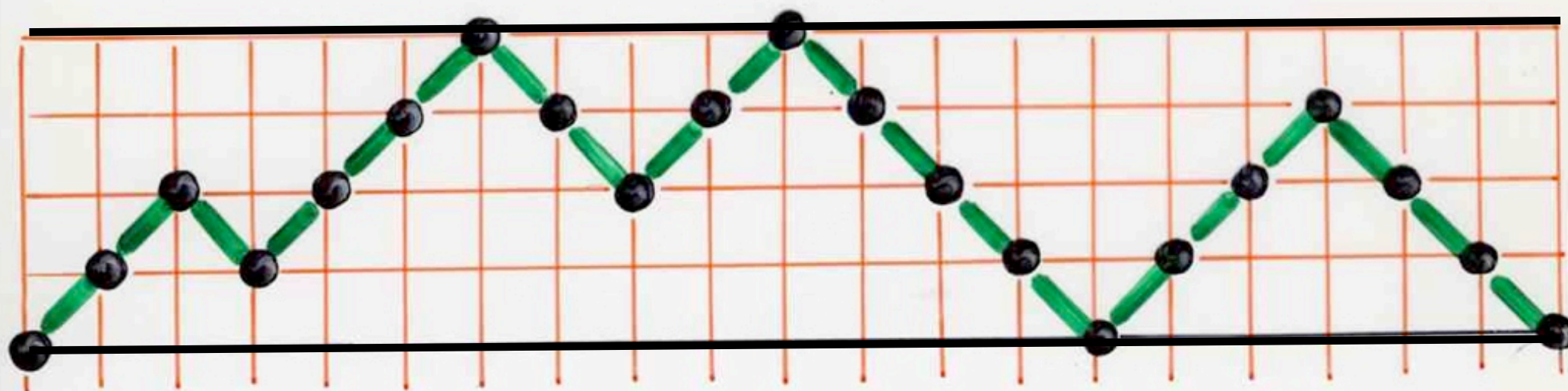
hauteur
logarithmique $lh(w)$

$$= \lfloor \log_2(1 + h(w)) \rfloor$$

$\lfloor \rfloor$ partie entière

$$= \lfloor \log_2(1 + h(w)) \rfloor$$

$L \setminus$ partie entière





arbres binaires B
ayant n sommets
nb de Strahler
 $St(B) = k$



Frangon
(1984)

chemins de Dyck w
de longueur $2n$
hauteur log.
 $lh(w) = k$

même distribution !



Nombre de Strahler

moyen

parmi tous les arbres binaires
ayant n sommets

$$St_n = \log_4 n + f(\log_4 n) + o(1)$$

Flejole, Raoult, Vuillemin (1979)
Kemp (1979)

f fonction
périodique

matrices de ramification
comme
une analyse mathématique de la forme
des structures arborescentes

Comment «mesurer» la forme d'un arbre ?

BERNARD
GANTNER





ARBRES AUX CORBEAUX

MUSÉE DU LOUVRE

Quantifier la forme d'un arbre binaire ...

arbre touffu

effilé

brousailleux

épineux

bien équilibré

matrices
de ramification
en physique

DIGITATION VISQUEUSE

VISKOSE FLÜSSIGKEITSFORMEN



DLA

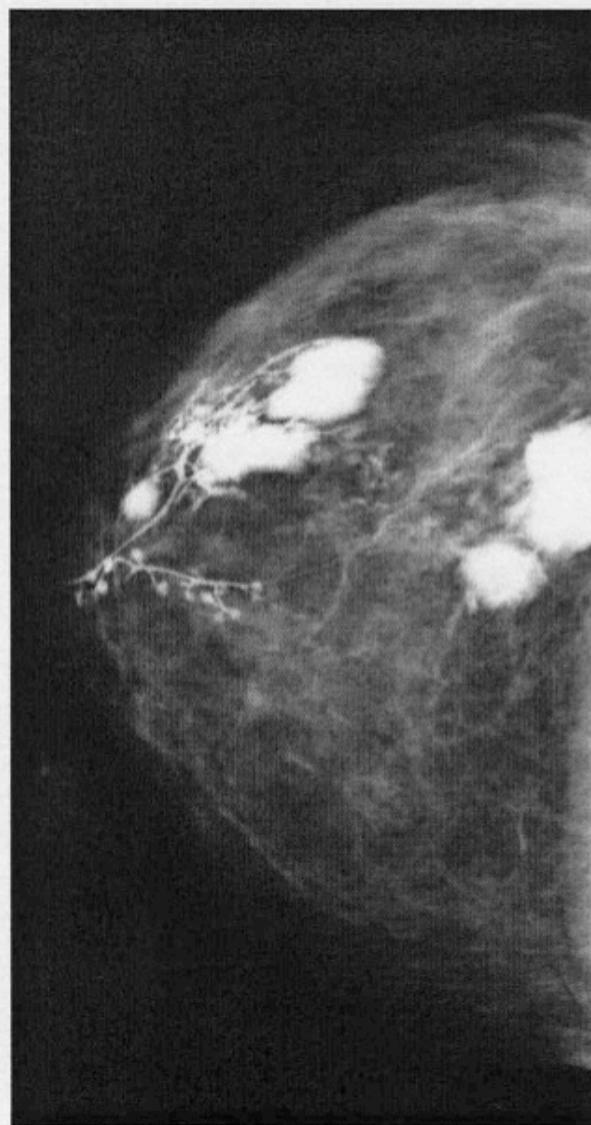
Diffusion
Limited
Aggregation



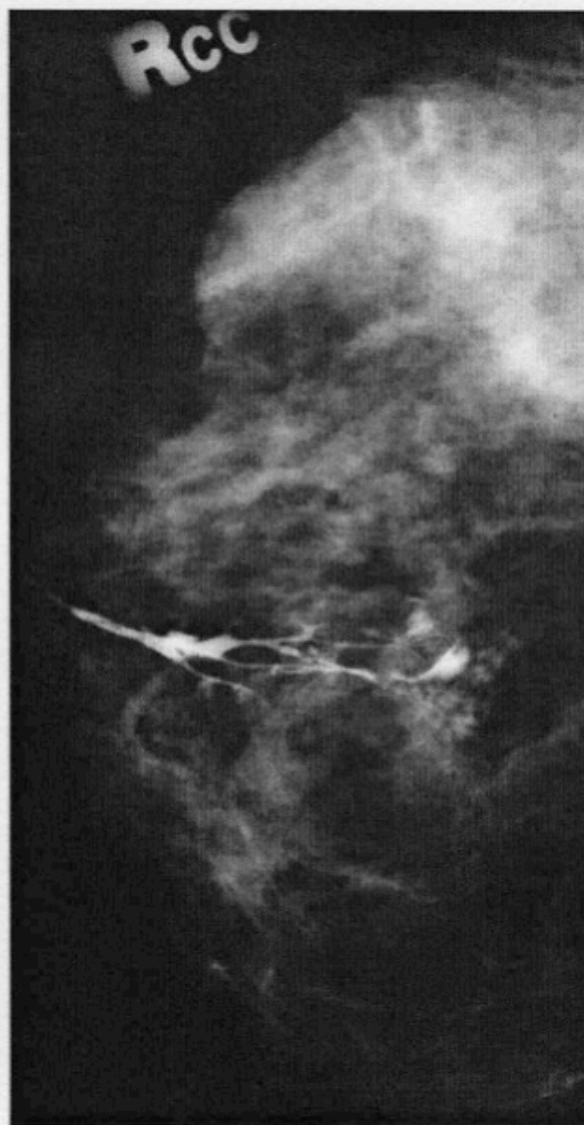
classification des galactogrammes
avec des matrices de ramification

P. Bakic, M. Albert, A. Maidment (2003)

galactographie
mammographie digitale



a.



b.

Figure 4. Two examples of galactograms that have been correctly classified by means of R matrices. **(a)** Galactogram with no reported findings (patient age, 45 years; right CC view; $r_{3,2} = 0.5$ and $r_{3,3} = 0.19$). (Large bright regions seen in this galactogram are due to extravasation, which did not affect the segmentation of the ductal tree.) **(b)** Galactogram with a reported finding of cysts (patient age, 55 years; right CC view; $r_{3,2} = 0.33$ and $r_{3,3} = 0.67$).

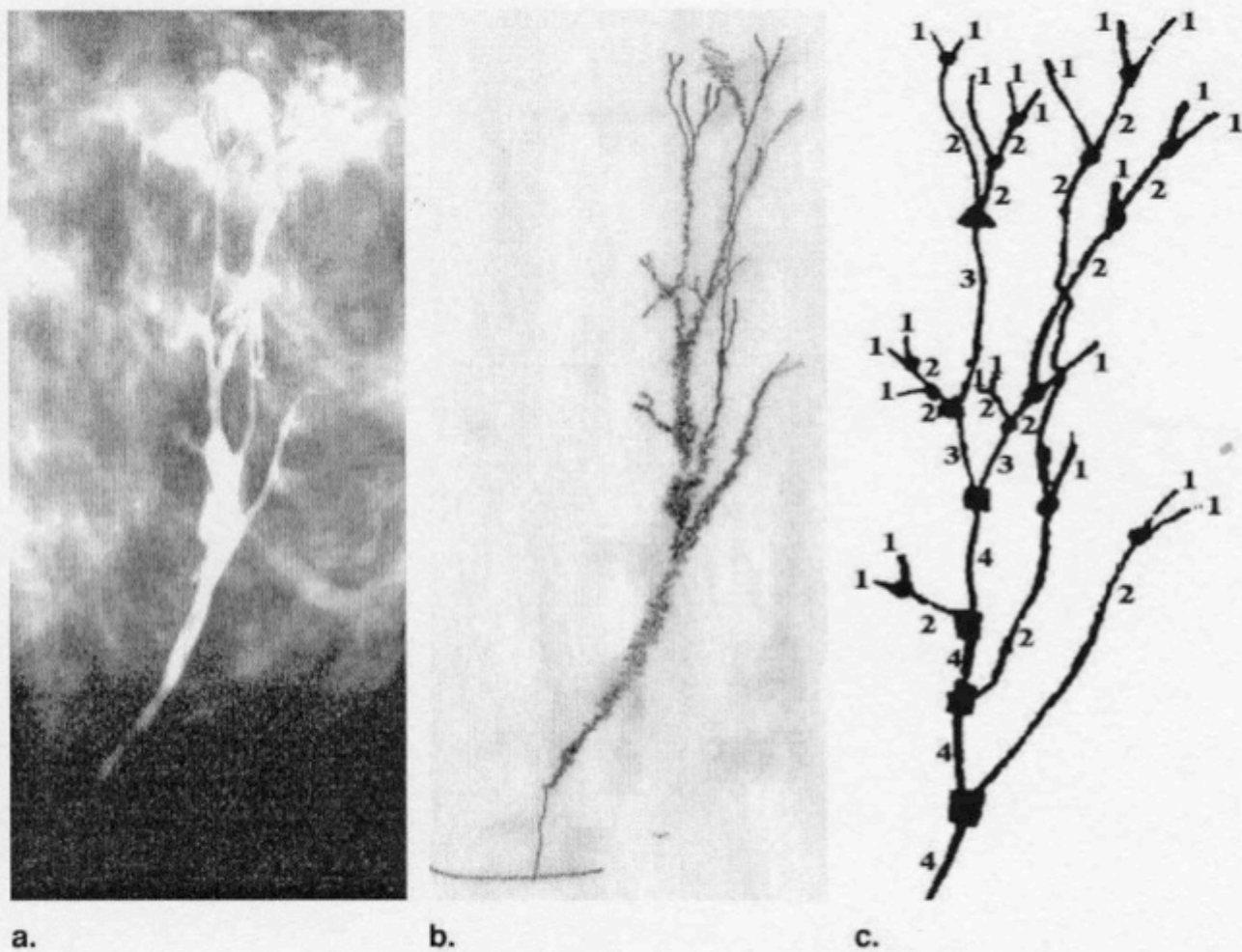


Figure 1. Segmentation of a ductal tree, showing (a) part of a galactogram with a contrast-enhanced ductal network, (b) the manually traced network of larger ducts from the contrast-enhanced portion of the galactogram, (c) numeric labeling of branches in the ductal network, and (d) the R matrix computed from the branching pattern. The dots, triangles, and squares denote branching points of different levels of the tree.

$$R = \begin{bmatrix} r_{2,1} & r_{2,2} & . & . \\ r_{3,1} & r_{3,2} & r_{3,3} & . \\ r_{4,1} & r_{4,2} & r_{4,3} & r_{4,4} \end{bmatrix} = \begin{bmatrix} 0.43 & 0.57 & . & . \\ 0 & 0.33 & 0.67 & . \\ 0 & 0.75 & 0 & 0.25 \end{bmatrix}$$

d.

Ce que l'oeil humain ne peut pas voir à la radiographie,
l'analyse mathématique de la forme de la structure
aroborescente,
elle peut le voir ...

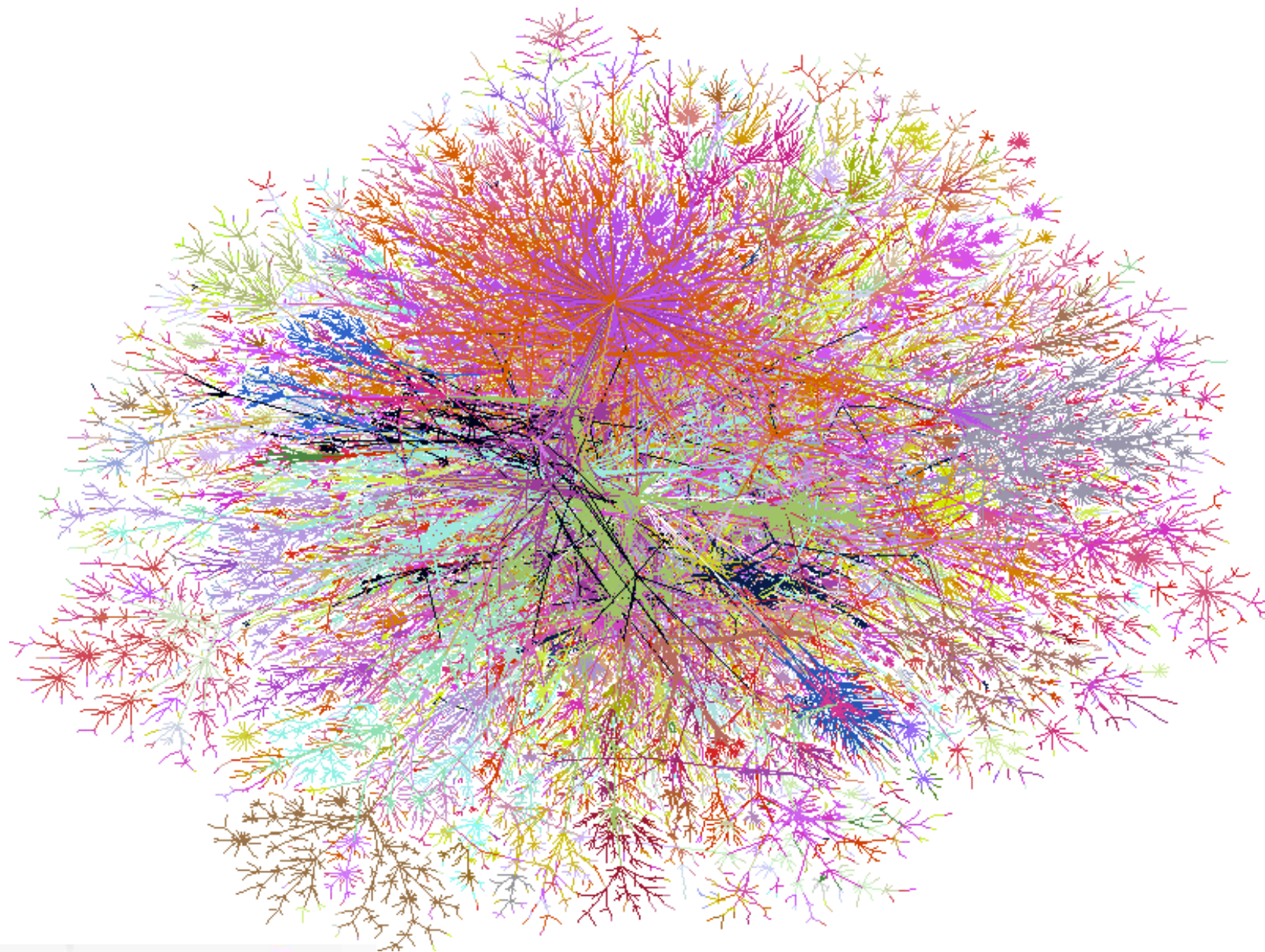
visualisation de l'information

- Visualisation de l'information
très grands graphes

D. Auber, M. Delest

Y. Chénicota, G. Melançon, J.M. Fedou

extension de l'analyse de Horton-Strahler
des arbres aux graphes

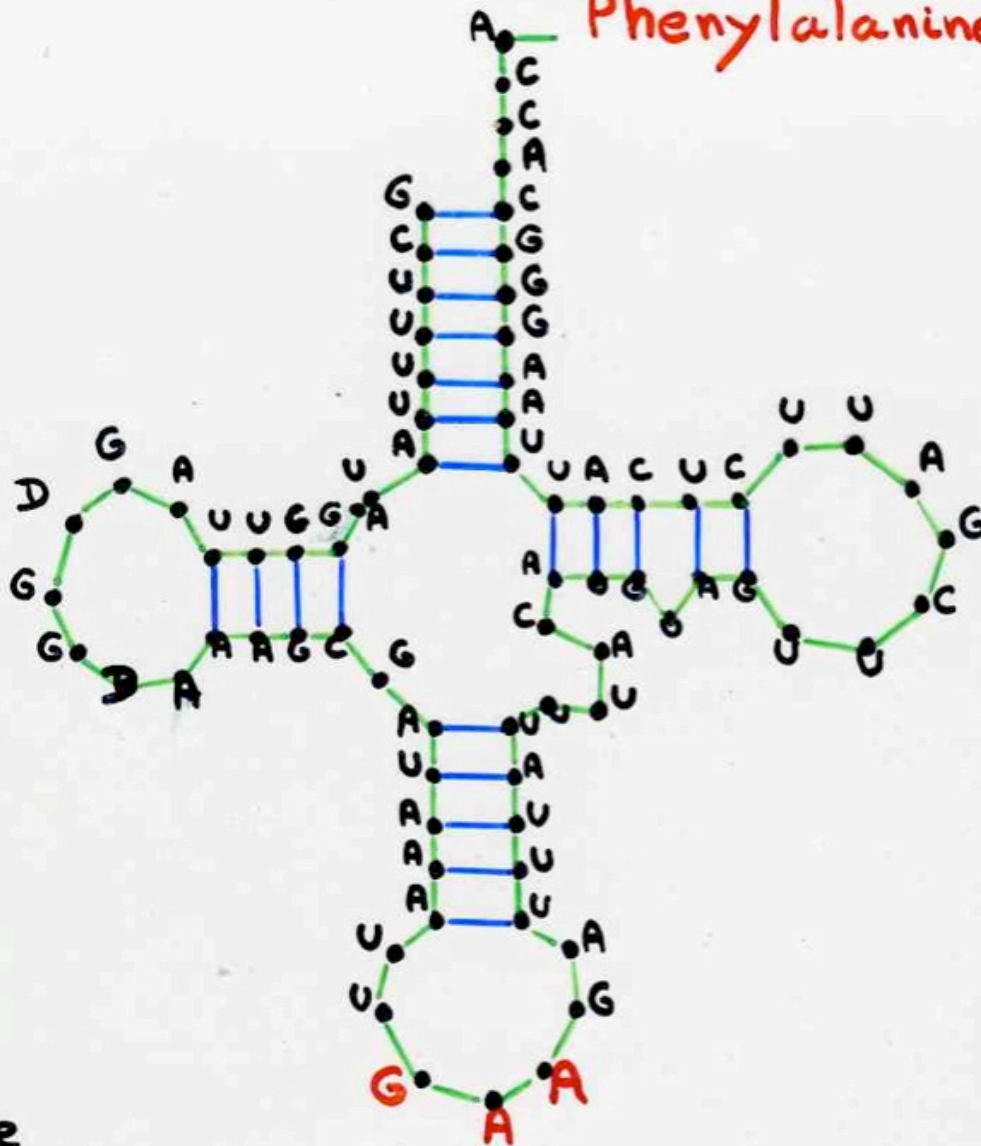


bgiciel Tulip
D. Auber

des arbres dans les molécules ...

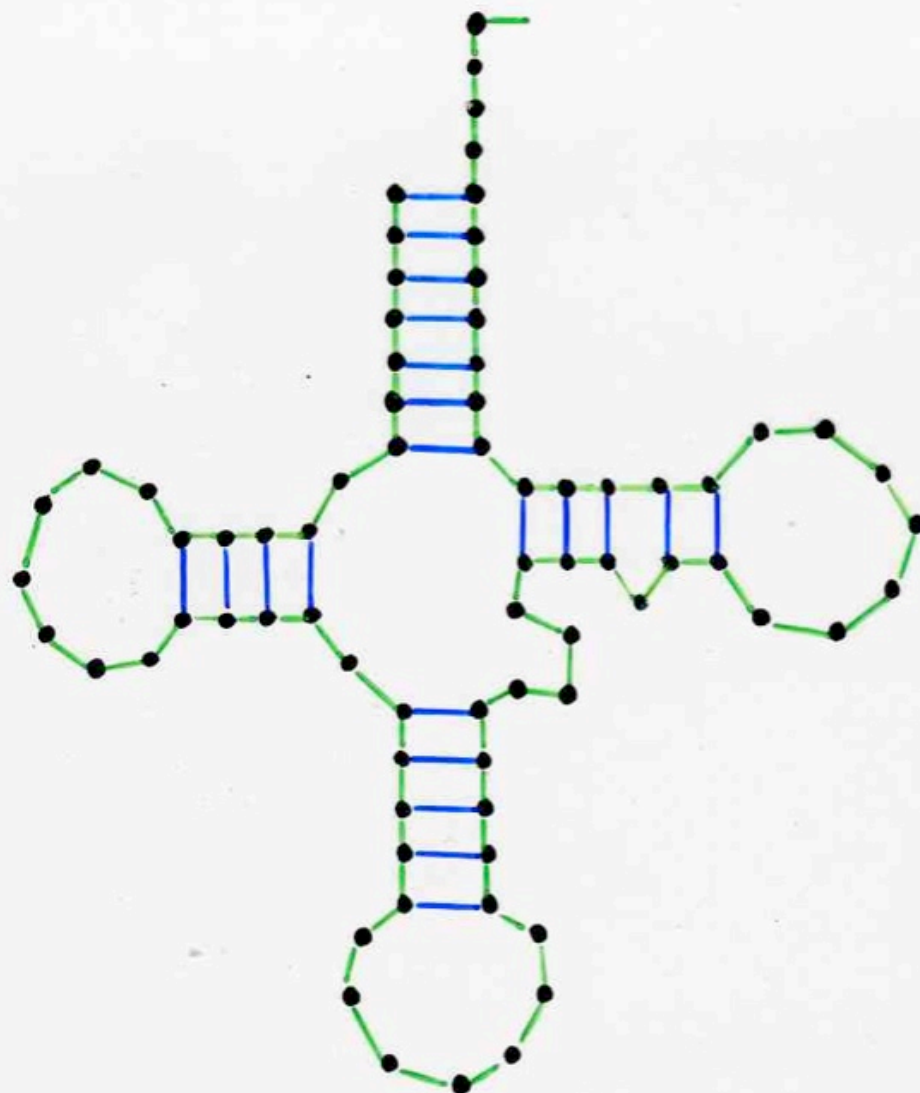


Phenylalanine

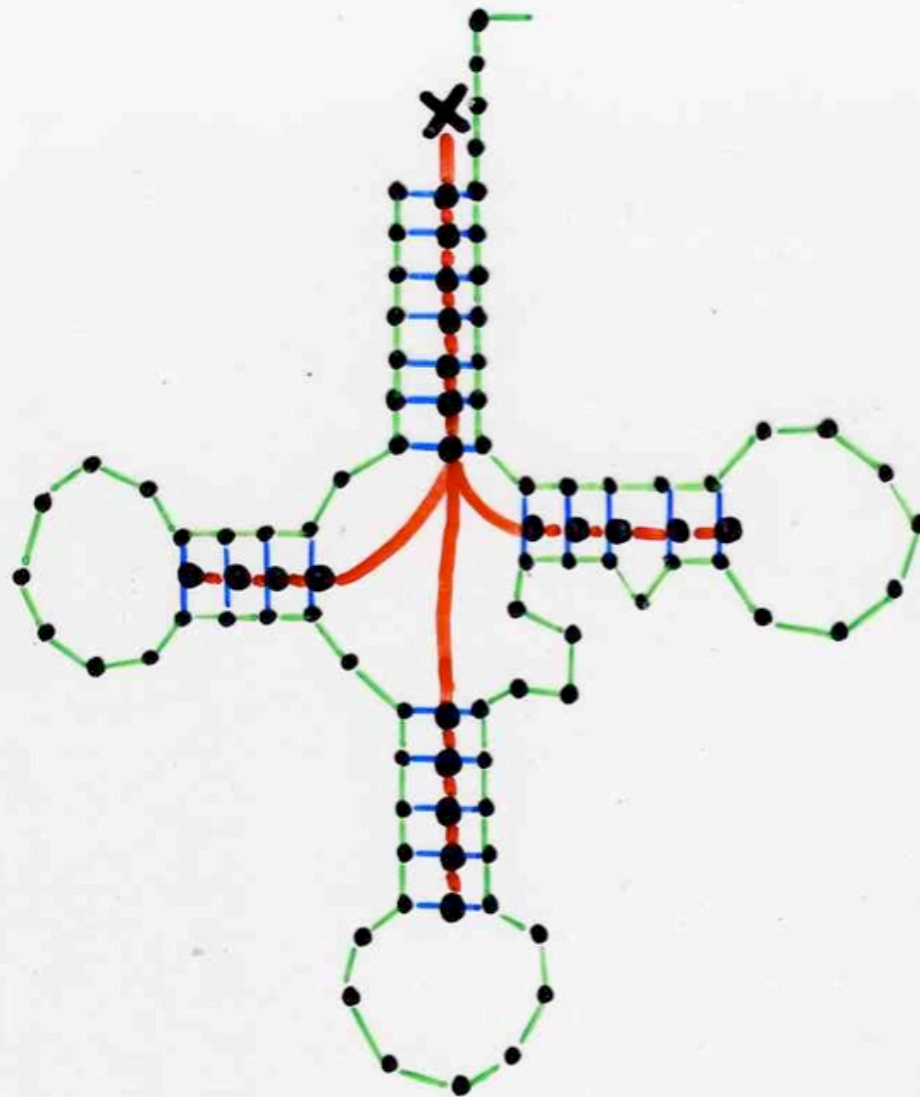


A denine
U racyle
G uanine
C ytosine

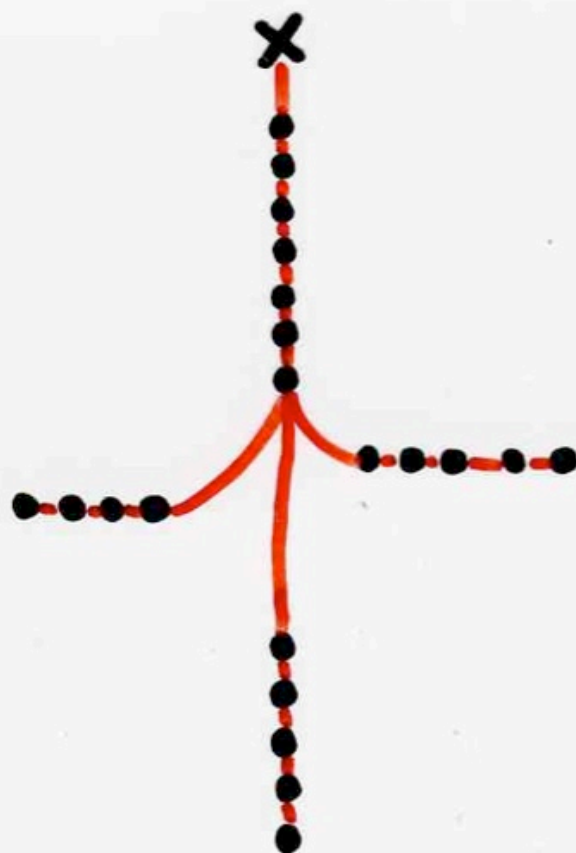
tARN^{Phe}



tARN^{Phe}



tARN^{Phe}



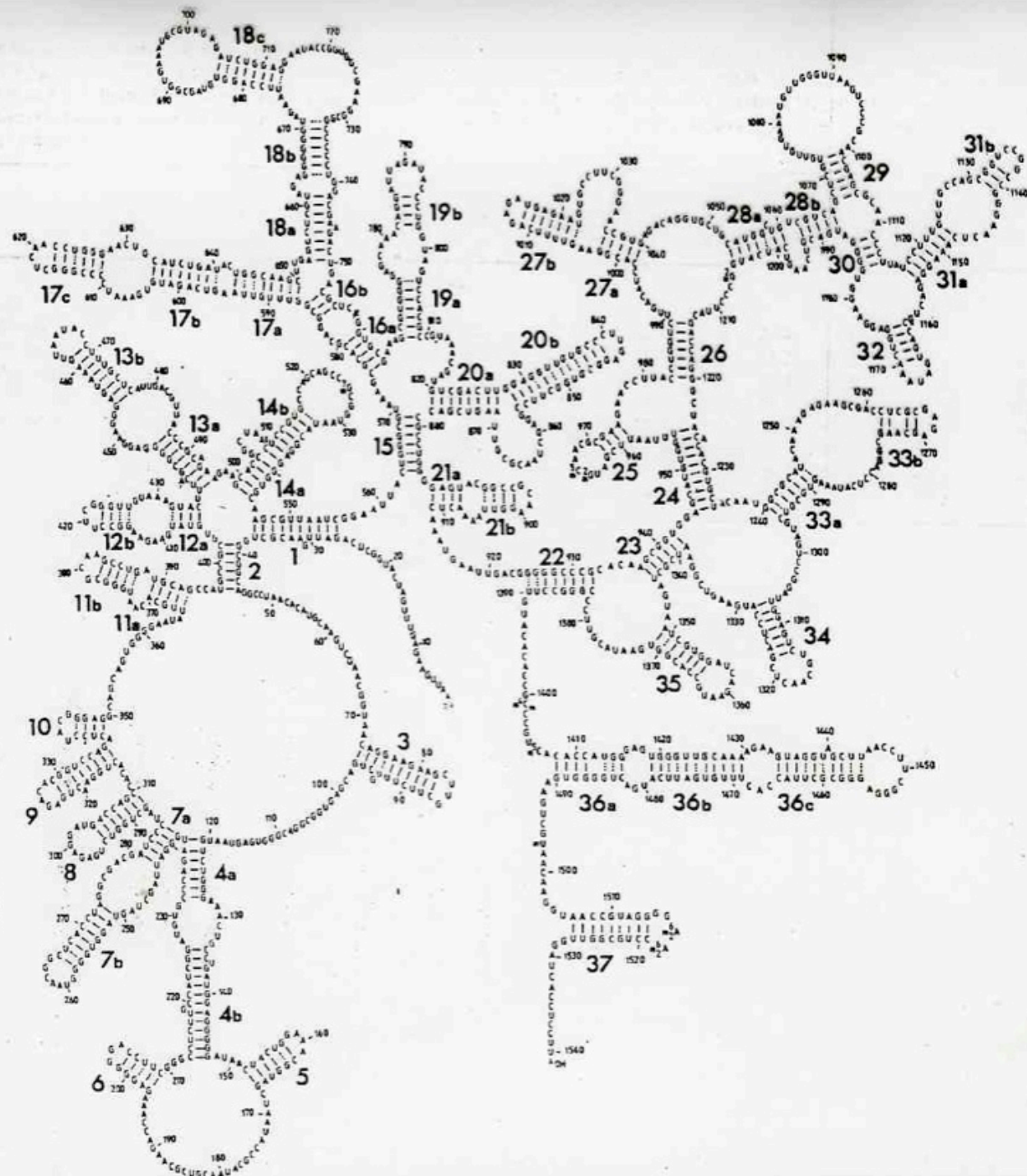


Fig. 1. Secondary structure model of the 16-S RNA from *E. coli*. This model has been fully described elsewhere [18]. The various secondary structure motifs are numbered for reference. Base-pairings 2 and 23 are included in this up-dated scheme and slight modifications have been introduced into helices 18b and 33b

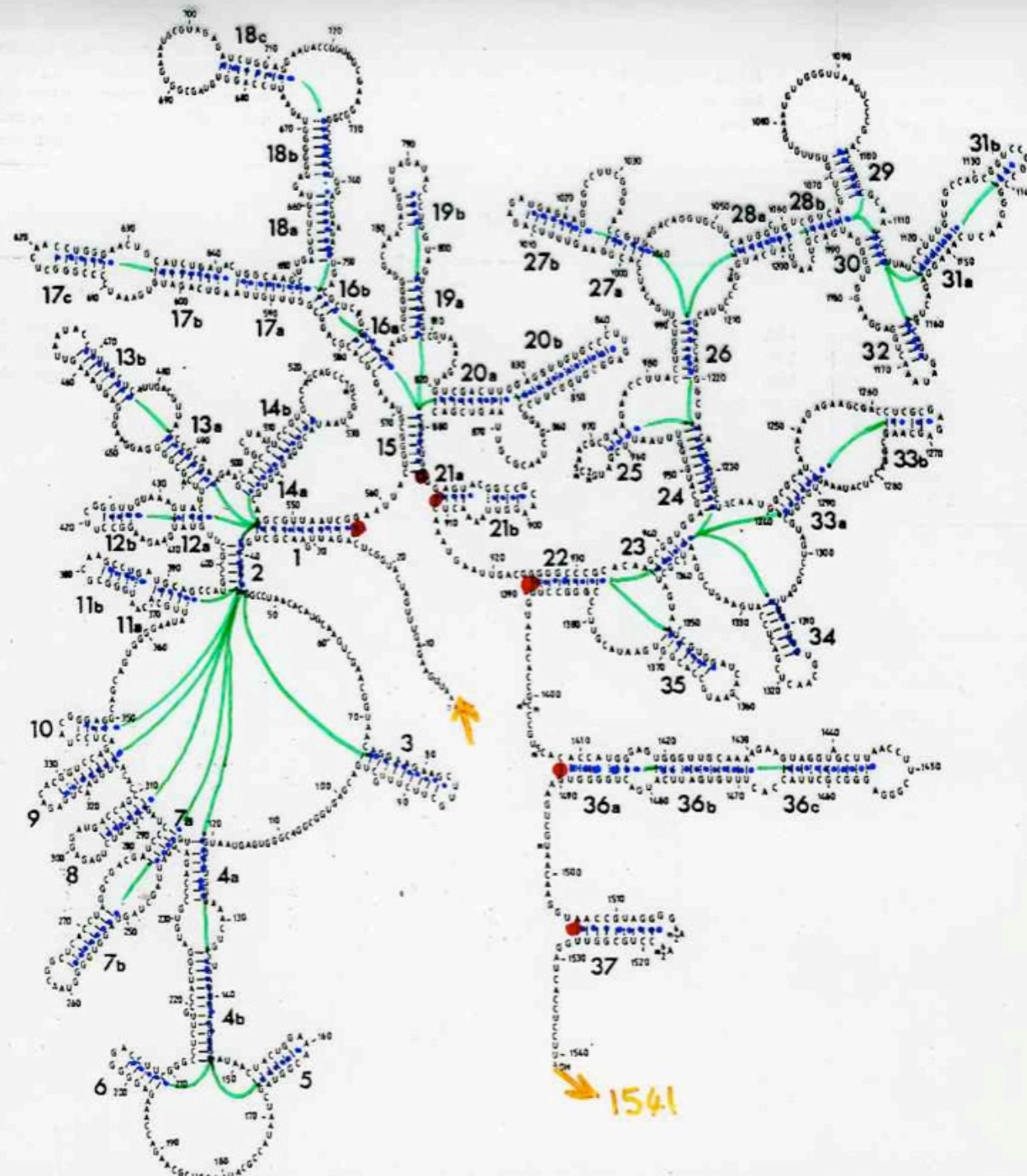
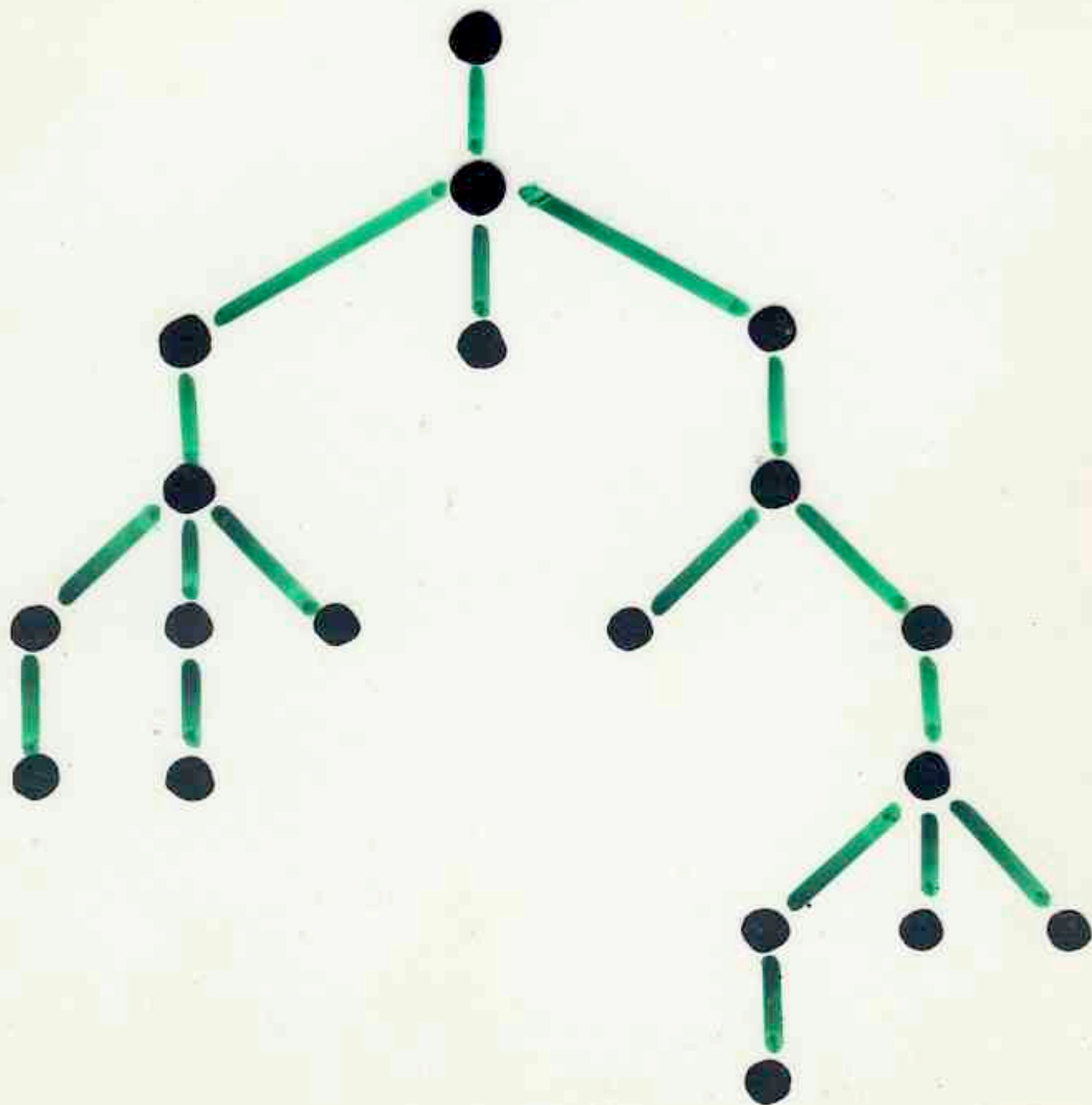
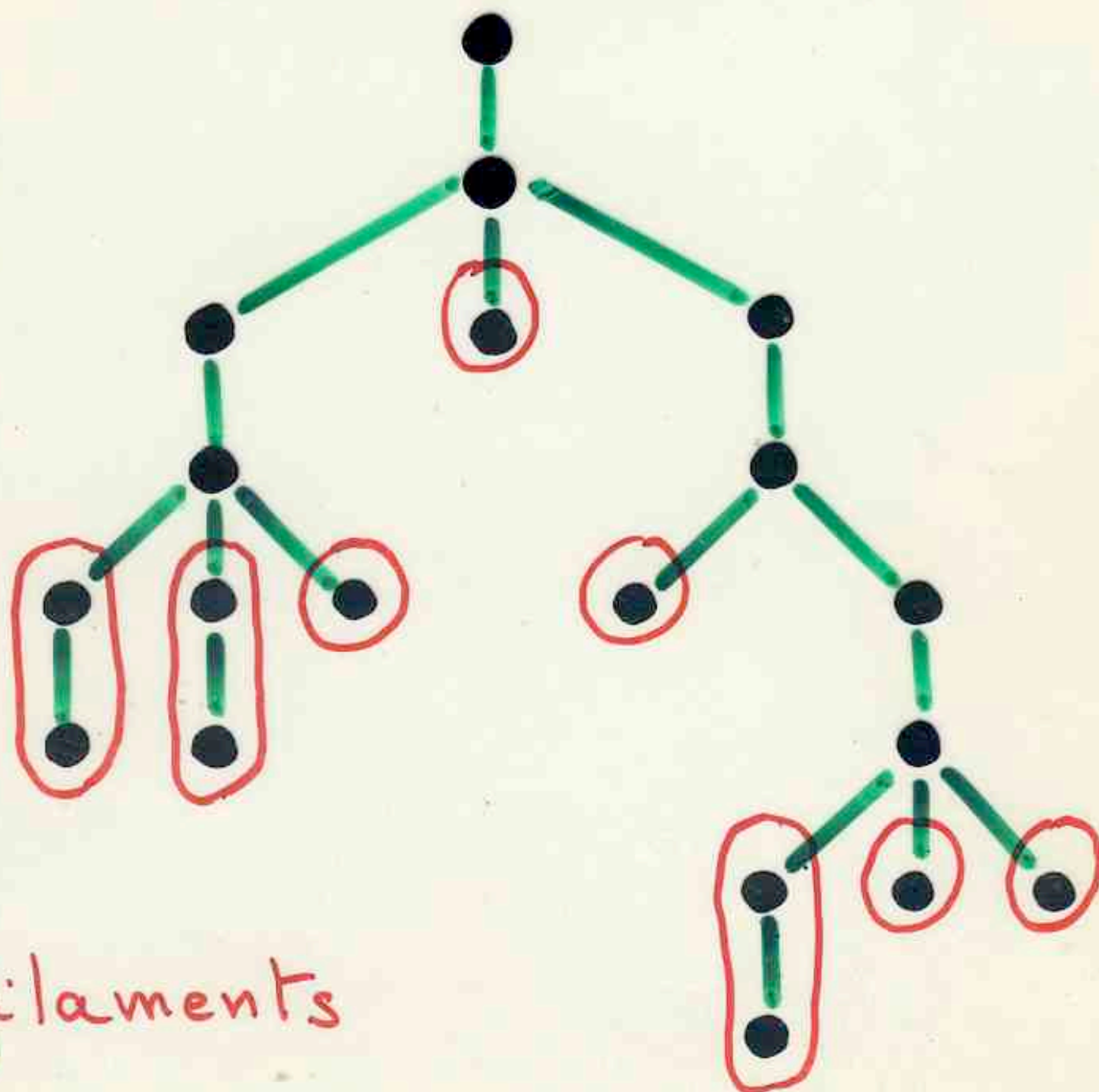


Fig. 1. Secondary structure model of the 16S RNA from *E. coli*. This model has been fully described elsewhere [18]. The various secondary structure motifs are numbered for reference. Base-pairings 2 and 23 are included in this up-dated scheme and slight modifications have been introduced into helices 18b and 33b

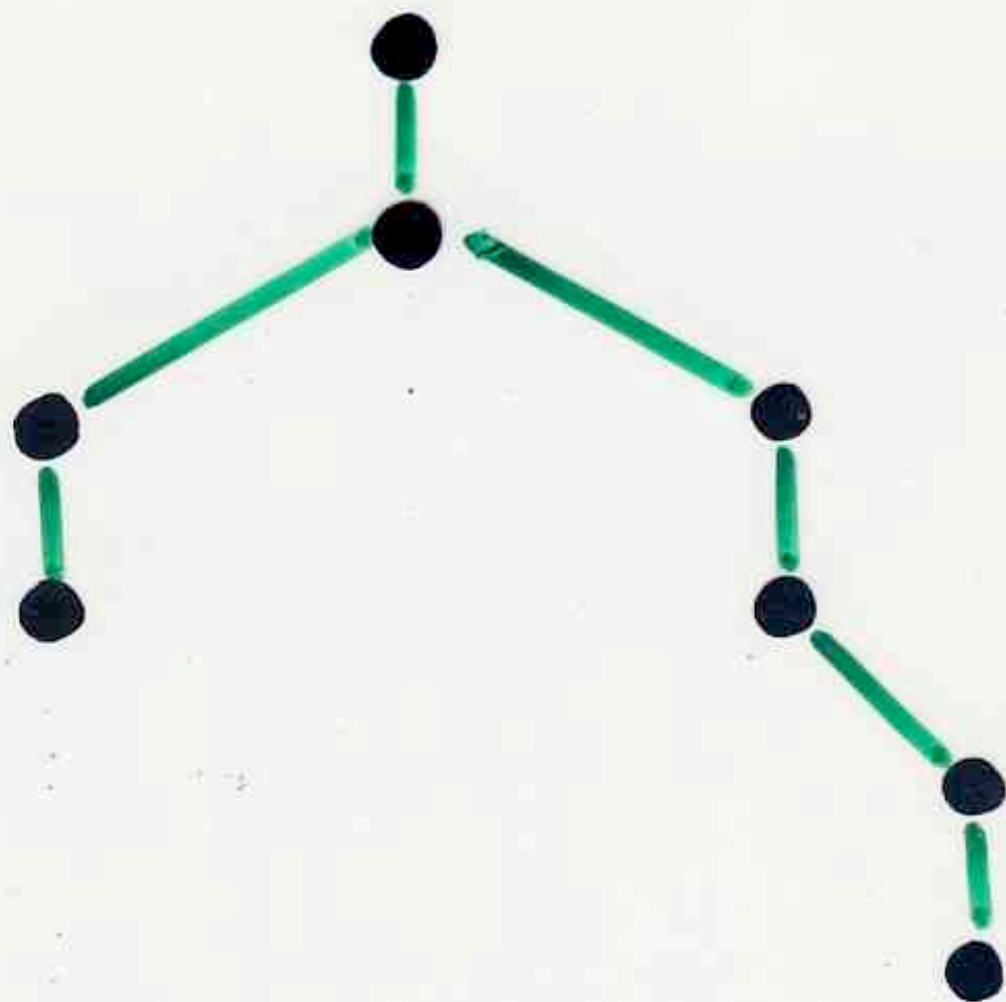
«complexité» ou «ordre»
d'une molécule

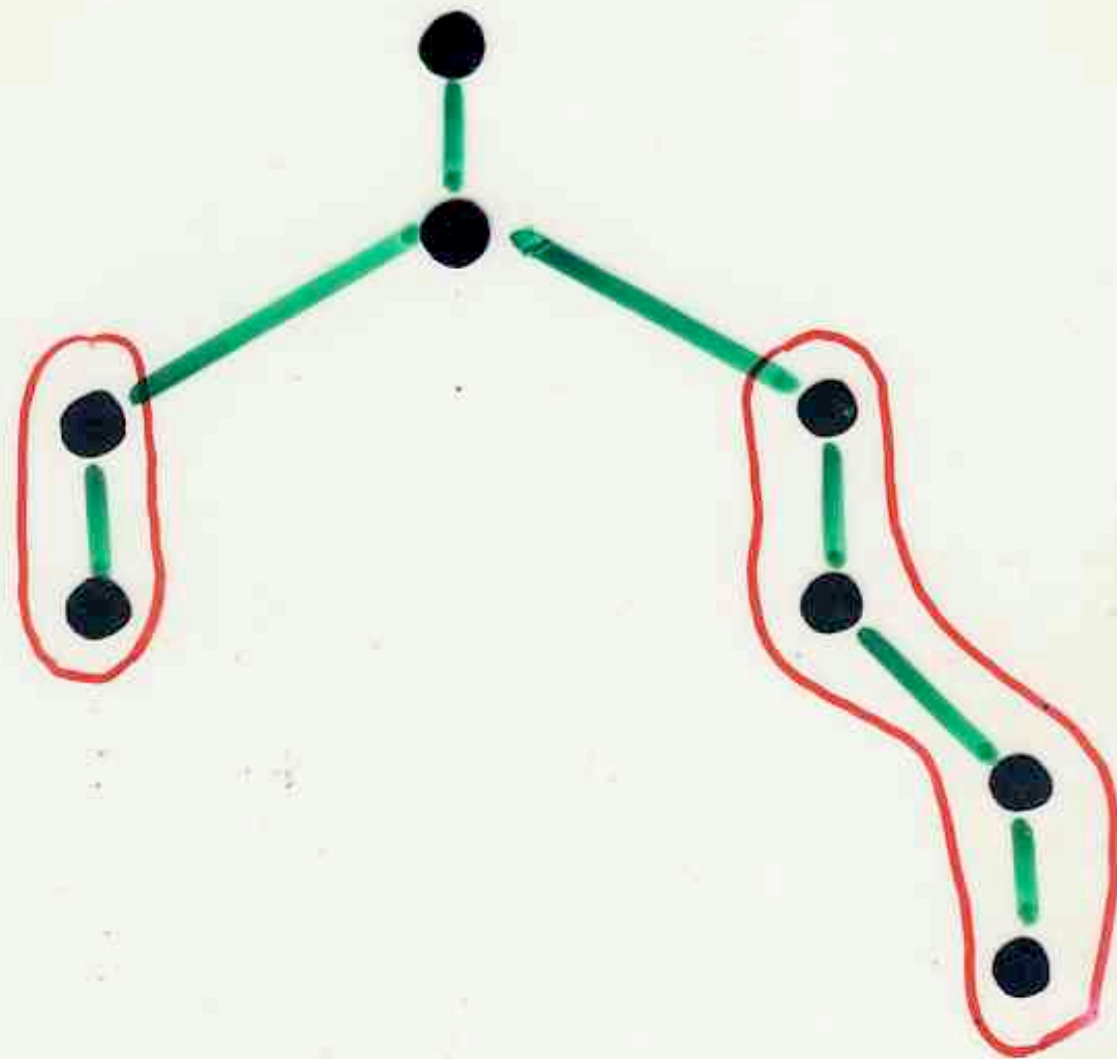
M. Waterman





filaments



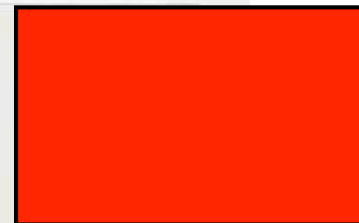








$F_{n,k}$ = nombre de
forêts d'arbres
ayant n sommets
et d'ordre k



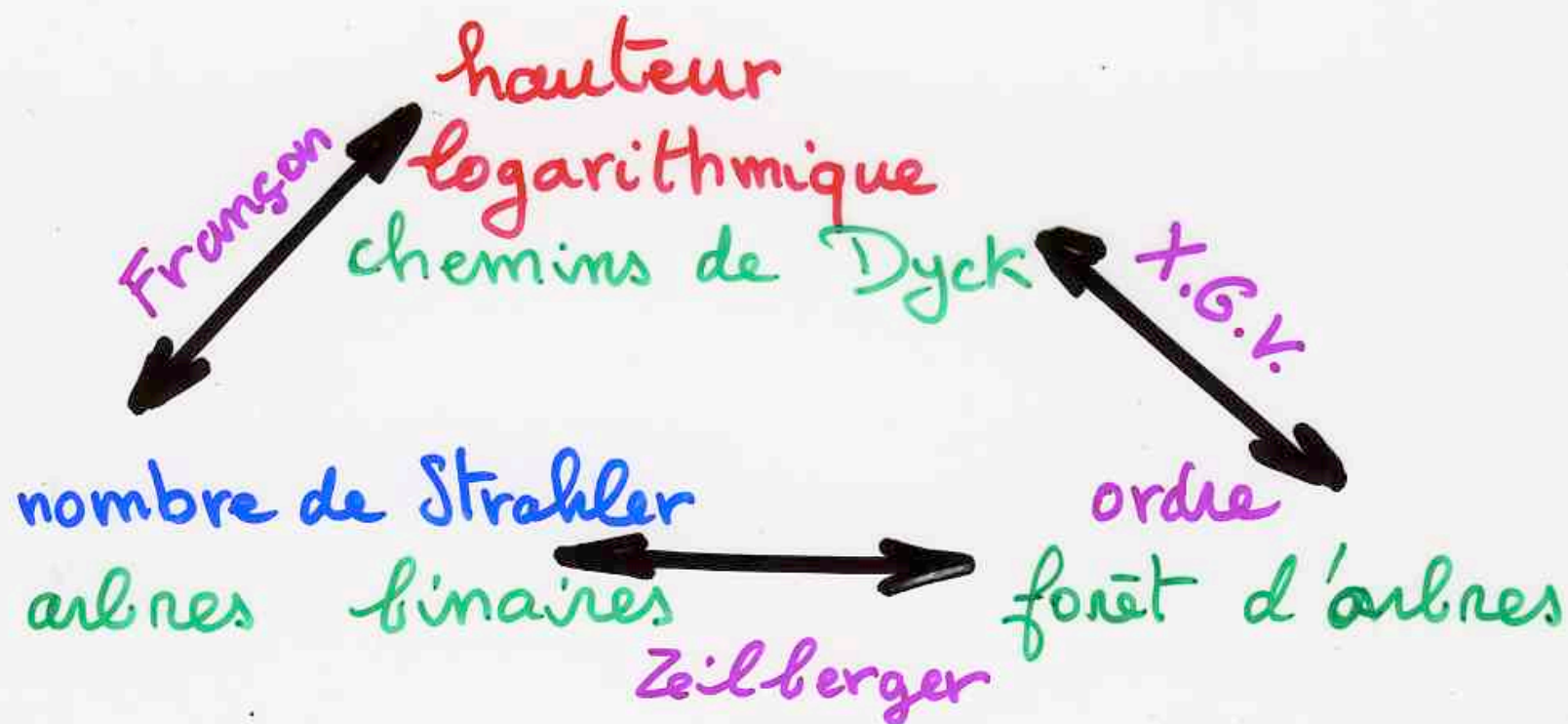
$F_{n,k}$ = nombre de
forêts d'arbres
ayant n sommets
et d'ordre k

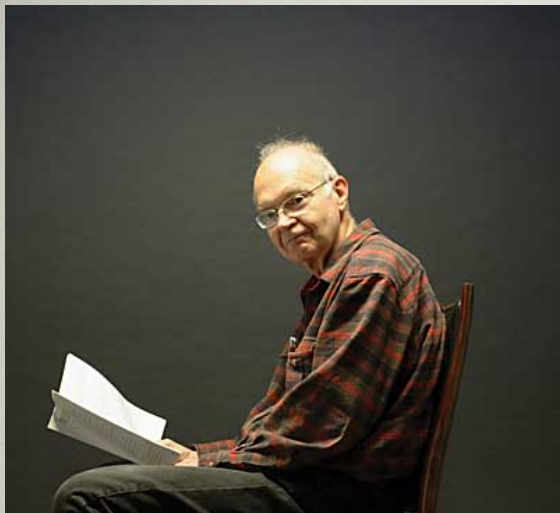
= $S_{n,k}$

à nouveau
même
distribution !!

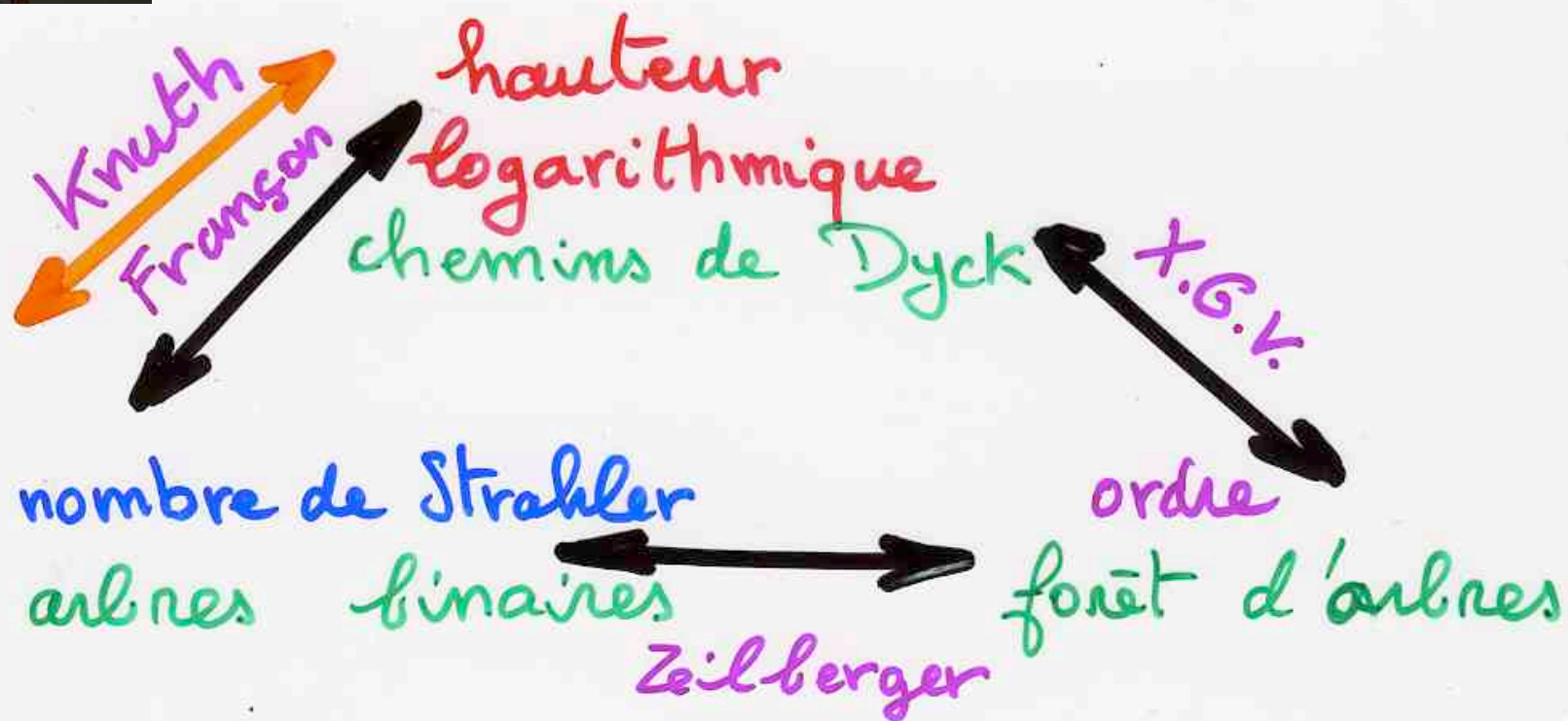
Vauchassade de Chaumont
X. V. (1985) (2001)

D. Zeilberger (1985)





Donald Knuth



LE JARDIN ZEN

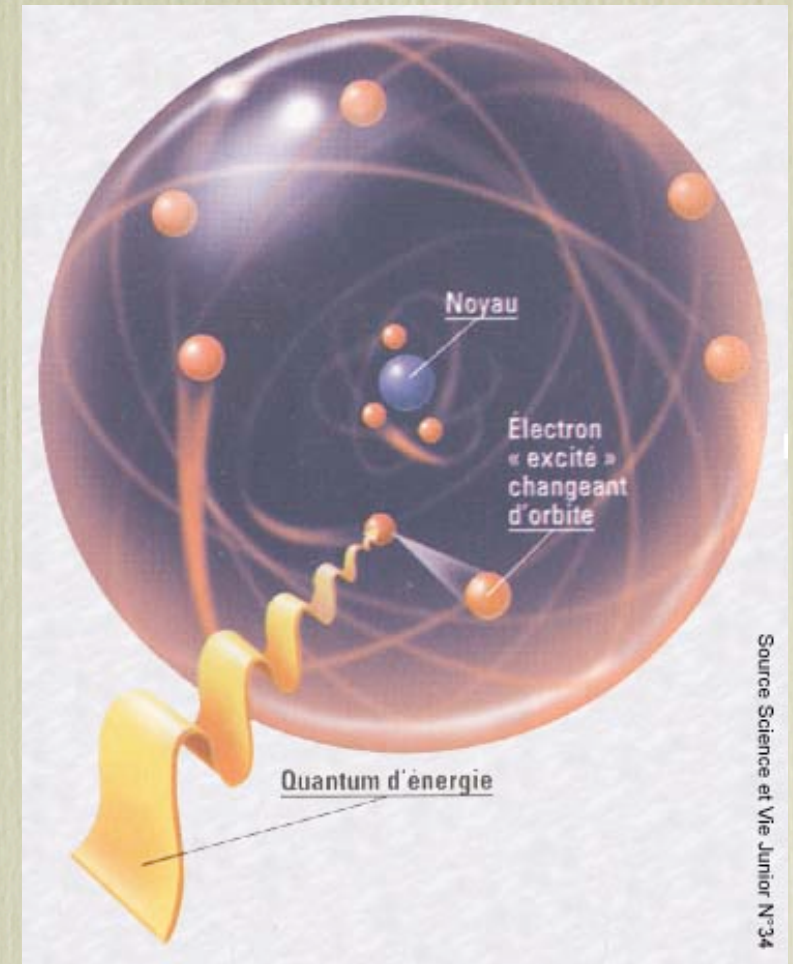
L'infinitement petit

Des arbres dans les grains de lumière ?

le monde quantique

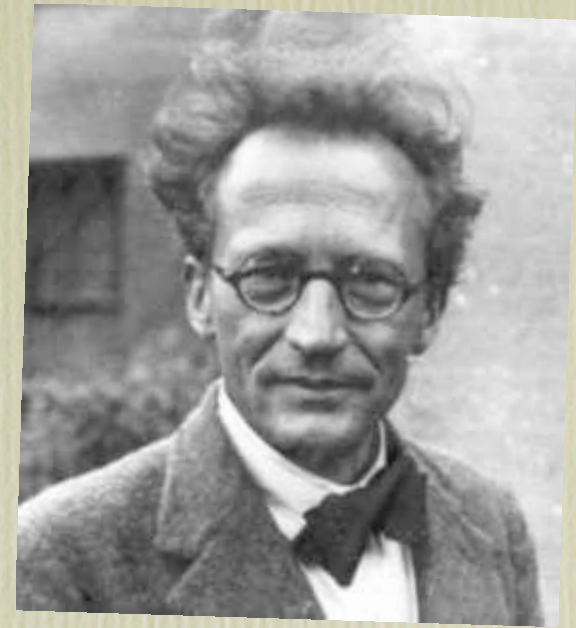


Serge Haroche
Nobel de physique 2012



Source Science et Vie Junior N°34

mécanique quantique
très loin du bon sens usuel

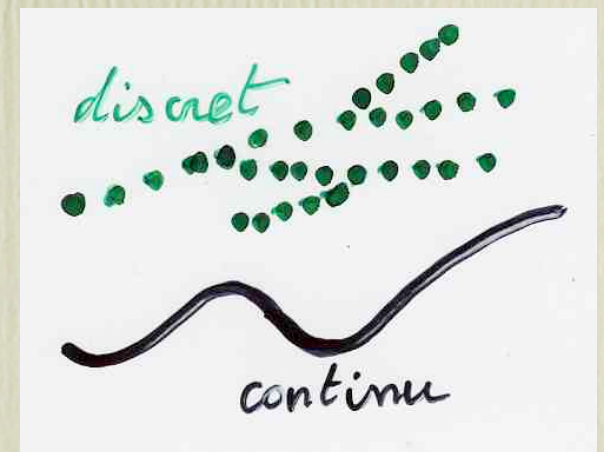


les particules ont une tendance à exister

le fameux chat de Schrödinger, mort et vivant en même temps

incertitude quantique

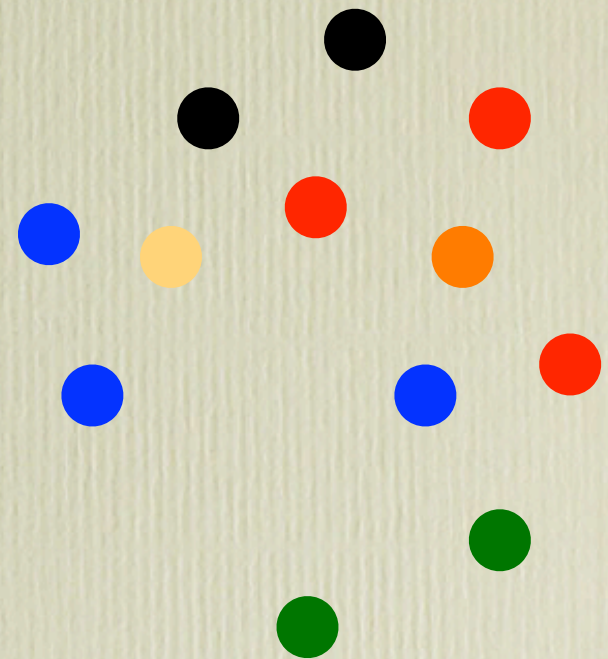
espace, temps, matière, énergie:
continu ou discret ?



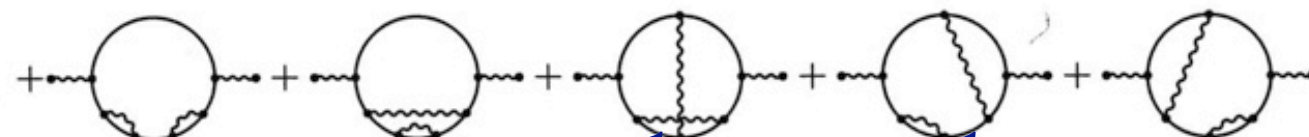
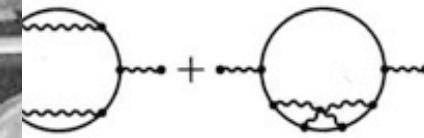
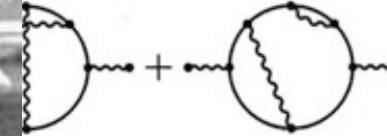
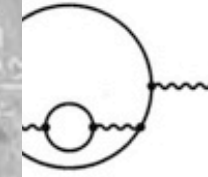
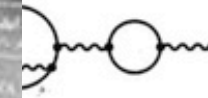
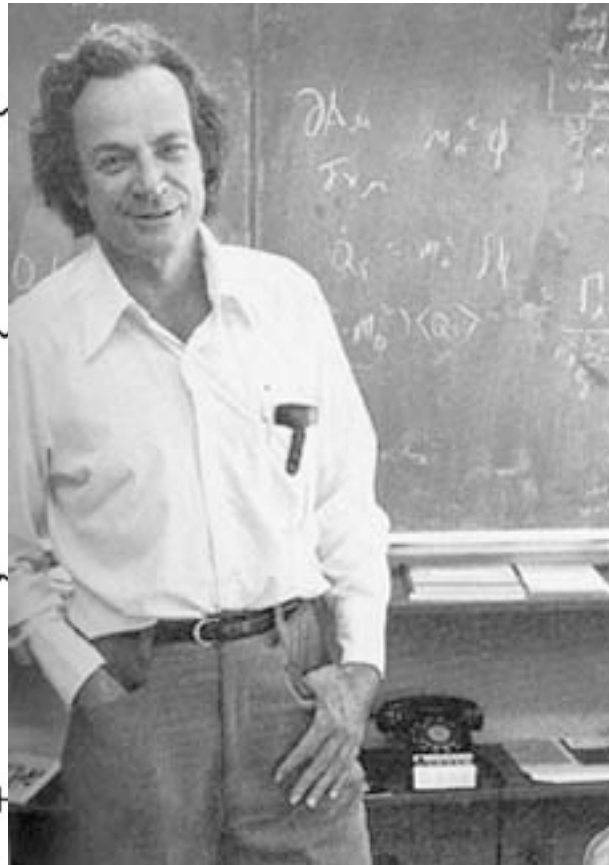
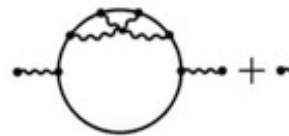
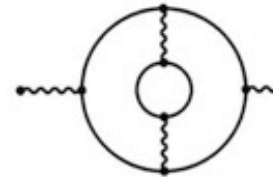
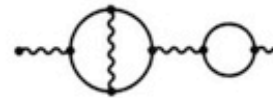
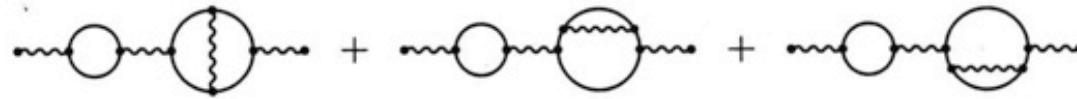
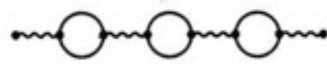
la lumière:

vibration ?

ou particules de matière ?



diagrammes
de
Feynman



interactions entre particules, photons

somme infinie de quantités infinies ?!?

supprimer le double infini ...

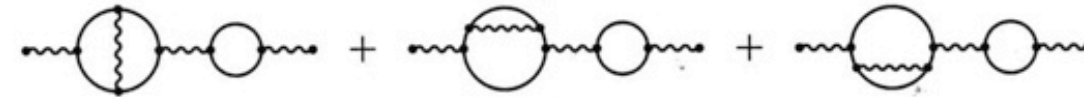
renormalisation quantique

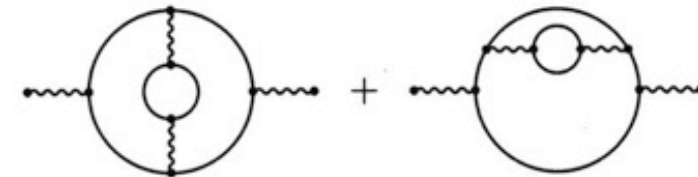
recette de cuisine

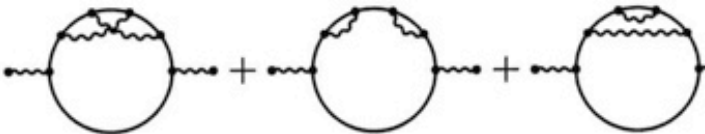
Diagrammes de Feynman

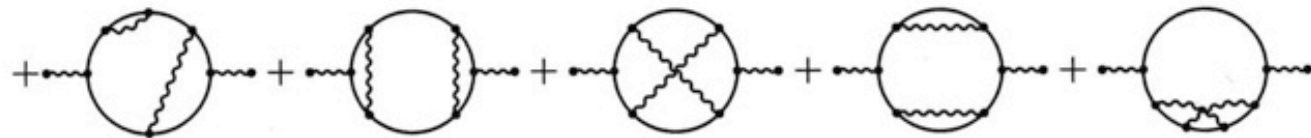
$$\sigma^\gamma(\Upsilon) =$$

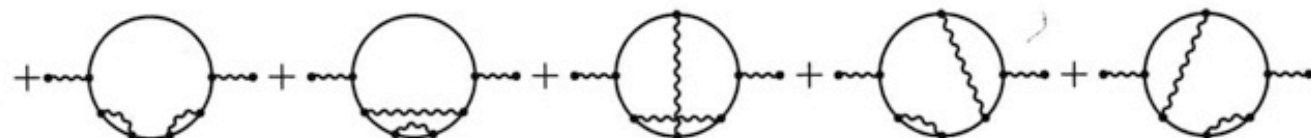

$$\sigma^\gamma(\Upsilon) =$$


$$\sigma^\gamma(\Upsilon\Upsilon) =$$


$$\sigma^\gamma(\Upsilon\Upsilon) =$$


$$\sigma^\gamma(\Upsilon\Upsilon) =$$






explication avec
les mathématiques
des arbres

les mathématiques d'Euclide , beaucoup de figures
jusqu'à Newton

après, élimination des figures

Lagrange, traité de mécanique:
pas une seule figure

équations, identités, abstraction pure



Joseph-Louis Lagrange

1736 - 1813

AVERTISSEMENT

DE LA DEUXIÈME ÉDITION.

On a déjà plusieurs Traités de Mécanique, mais le plan de celui-ci est entièrement neuf. Je me suis proposé de réduire la théorie de cette Science, et l'art de résoudre les problèmes qui s'y rapportent, à des formules générales, dont le simple développement donne toutes les équations nécessaires pour la solution de chaque problème.

Cet Ouvrage aura d'ailleurs une autre utilité : il réunira et présentera sous un même point de vue les différents principes trouvés jusqu'ici pour faciliter la solution des questions de Mécanique, en montrera la liaison et la dépendance mutuelle, et mettra à portée de juger de leur justesse et de leur étendue.

Je le divise en deux Parties : la Statique ou la Théorie de l'Équilibre, et la Dynamique ou la Théorie du Mouvement ; et, dans chacune de ces Parties, je traite séparément des corps solides et des fluides.

On ne trouvera point de Figures dans cet Ouvrage. Les méthodes que j'y expose ne demandent ni constructions, ni raisonnements géométriques ou mécaniques, mais seulement des opérations algébriques, assujetties à une marche régulière et uniforme. Ceux qui aiment l'Analyse verront avec plaisir la Mécanique en devenir une nouvelle branche, et me sauront gré d'en avoir étendu ainsi le domaine.

aujourd'hui, apparition des «figures»,
mais à un autre niveau

$$\sigma^\gamma(\Upsilon) = \text{diagram 1}$$

$$\sigma^\gamma(\Upsilon) = \text{diagram 2} + \text{diagram 3} + \text{diagram 4}$$

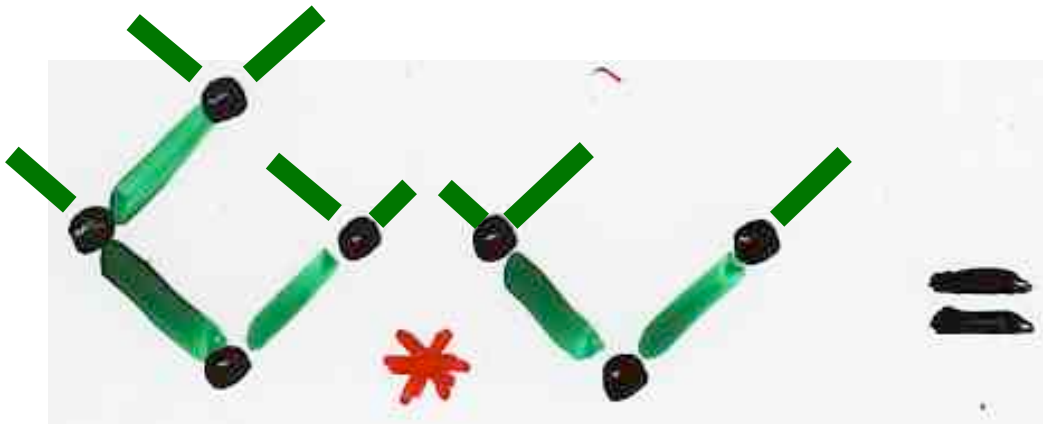
$$\sigma^\gamma(\Upsilon\Upsilon) = \text{diagram 5} + \text{diagram 6} + \text{diagram 7}$$

$$\sigma^\gamma(\Upsilon) = \text{diagram 8} + \text{diagram 9} + \text{diagram 10}$$

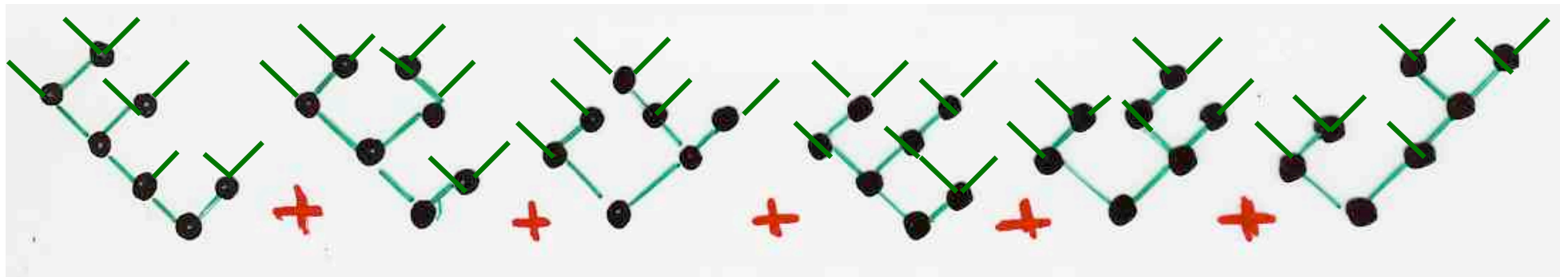
$$\sigma^\gamma(\Upsilon\Upsilon) = \text{diagram 11} + \text{diagram 12} + \text{diagram 13} + \text{diagram 14} + \text{diagram 15}$$

$$+ \text{diagram 16} + \text{diagram 17} + \text{diagram 18} + \text{diagram 19} + \text{diagram 20}$$

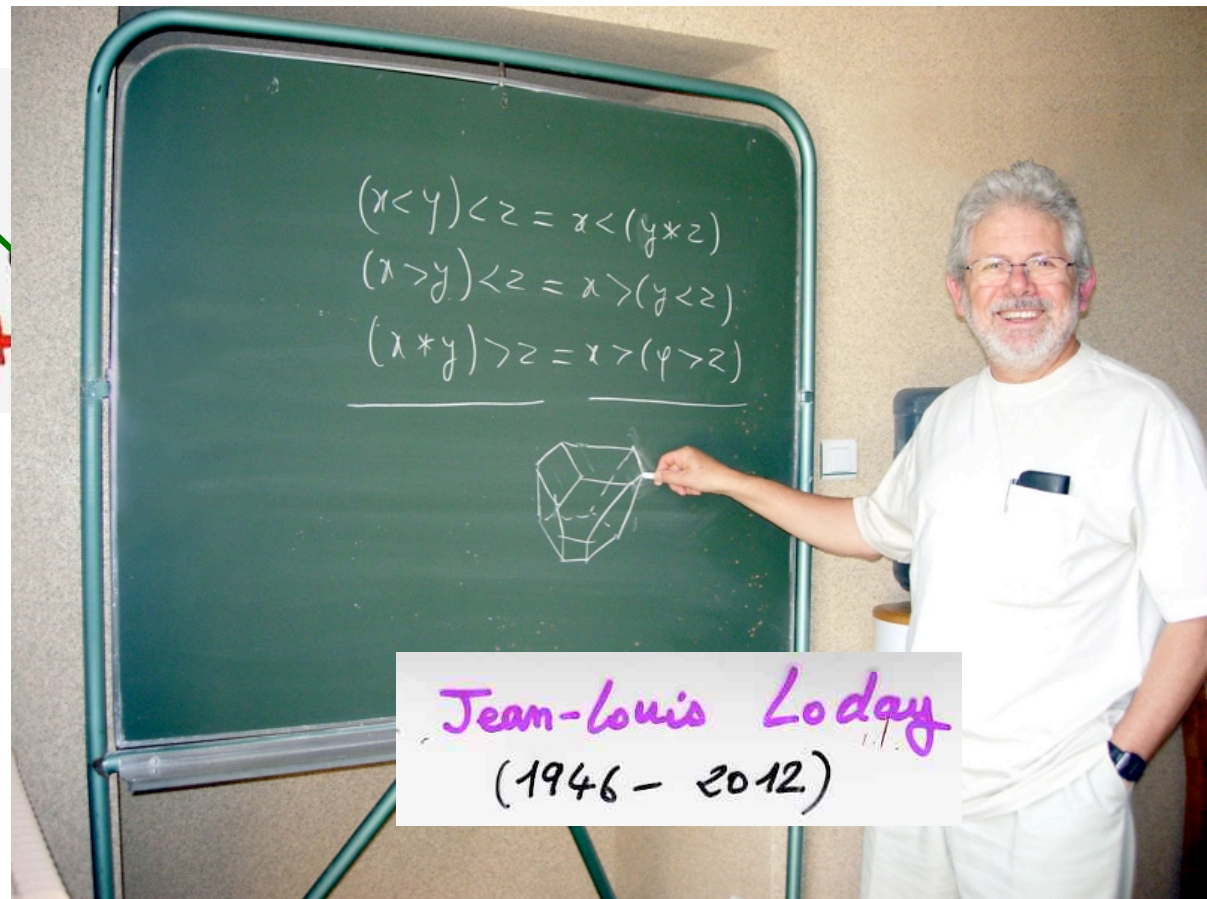
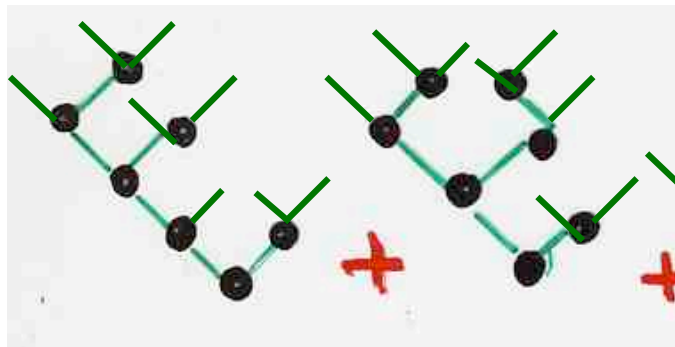
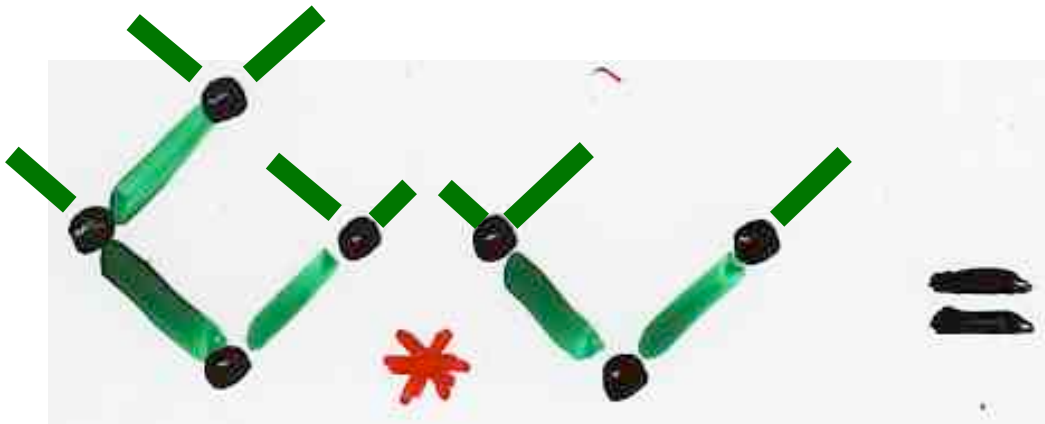
$$+ \text{diagram 21} + \text{diagram 22} + \text{diagram 23} + \text{diagram 24} + \text{diagram 25}$$



produit de deux arbres binaires



physique
combinatoire



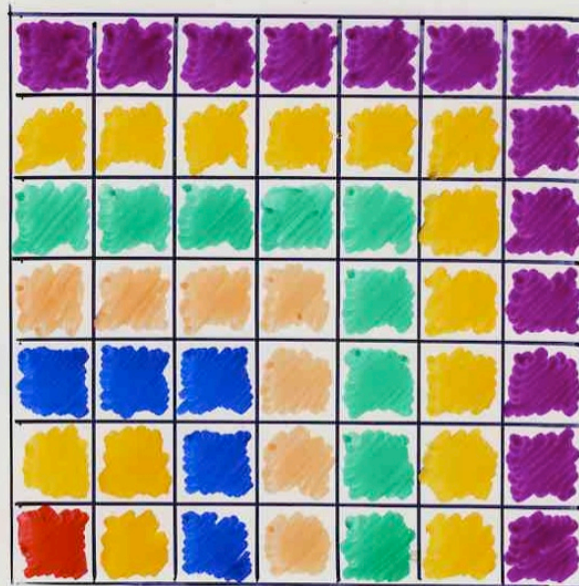
A photograph of Alain Connes, an older man with a grey beard, wearing a white shirt and dark trousers. He is standing in a lecture hall, gesturing with his hands while speaking. Behind him is a large arched window with a brick wall visible outside. To the right, there is a chalkboard with some mathematical diagrams and text, including "D. KREIMER" and a diagram of a circle with a cross. A green exit sign is visible above a doorway in the background. The text "Alain Connes", "renormalisation quantique", and "algèbre d'arbres" is overlaid in white serif font at the bottom right of the image.

Alain Connes
renormalisation quantique
algèbre d'arbres

preuves avec des «figures»

preuves
combinatoires





$$n^2 = 1 + 3 + \dots + (2n-1)$$

interprétations combinatoires

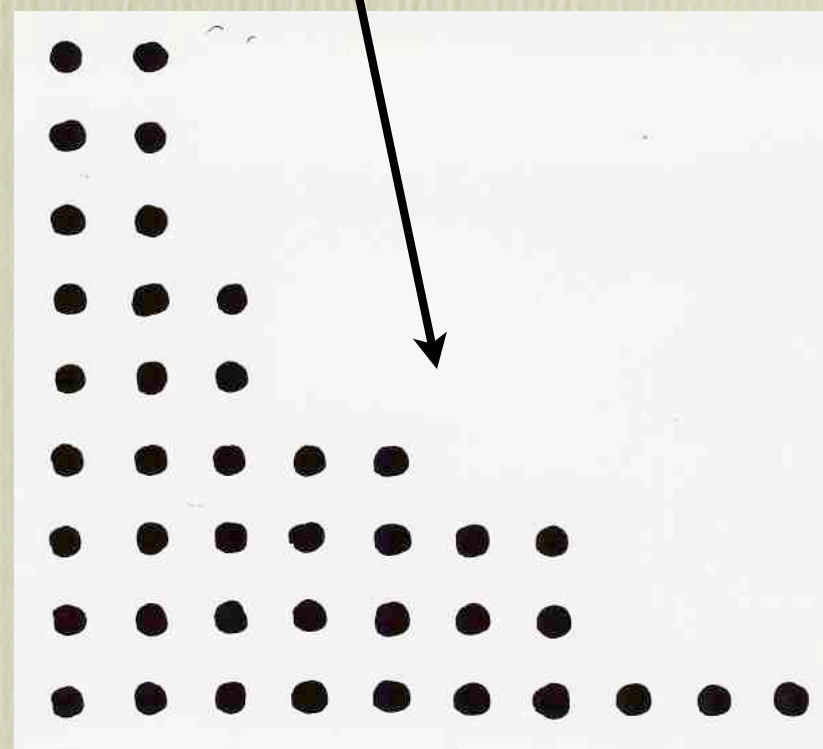
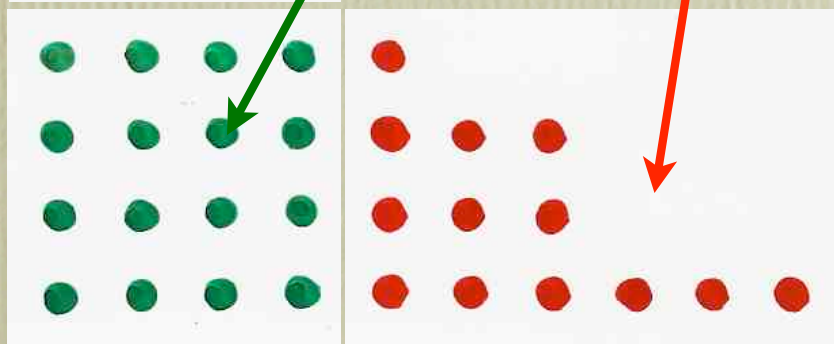
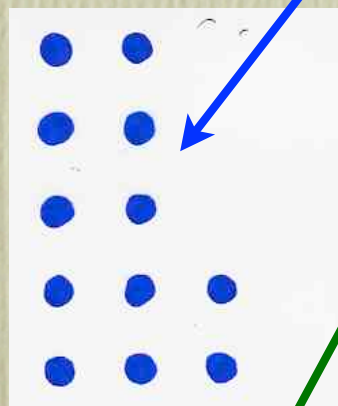


$$\sum_{m \geq 1} \frac{q^{m^2}}{[(1-q)(1-q^2) \cdots (1-q^m)]^2} = \prod_{i \geq 1} \frac{1}{(1-q^i)}$$

preuves combinatoires de certaines identités
avec des constructions
de bijections, de correspondances



$$\sum_{m \geq 1} \frac{q^{m^2}}{[(1-q)(1-q^2) \cdots (1-q^m)]^2} = \prod_{i \geq 1} \frac{1}{(1-q^i)}$$



"dessiner des calculs"

à chaque "morceau" d'une identité



objet combinatoire

correspondances

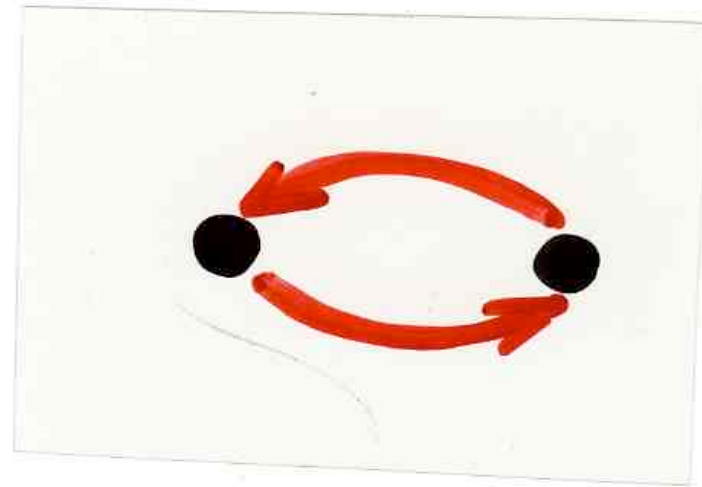
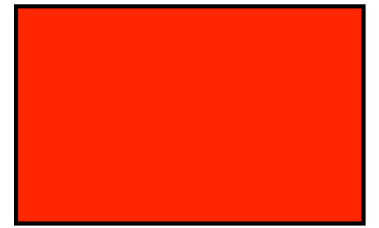
identité $\longleftrightarrow$ constructions combinatoires
bijections

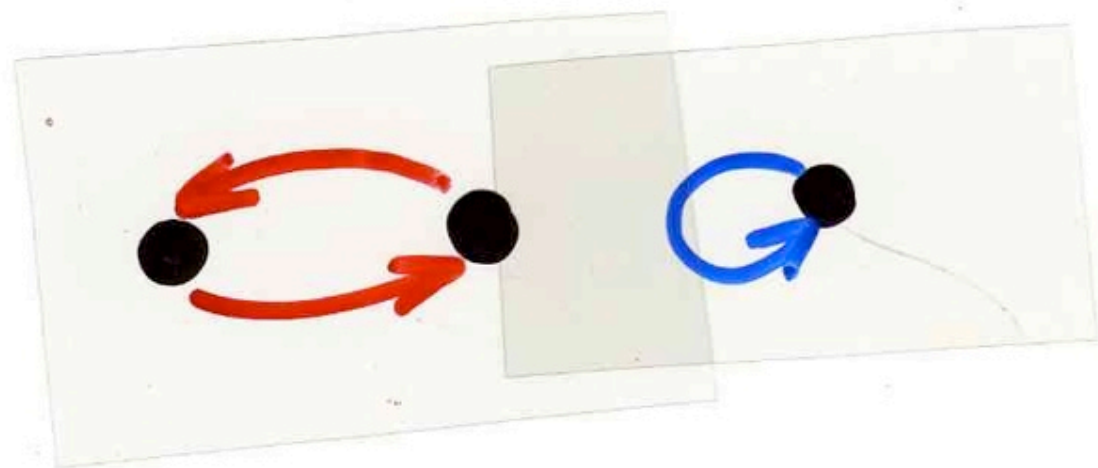


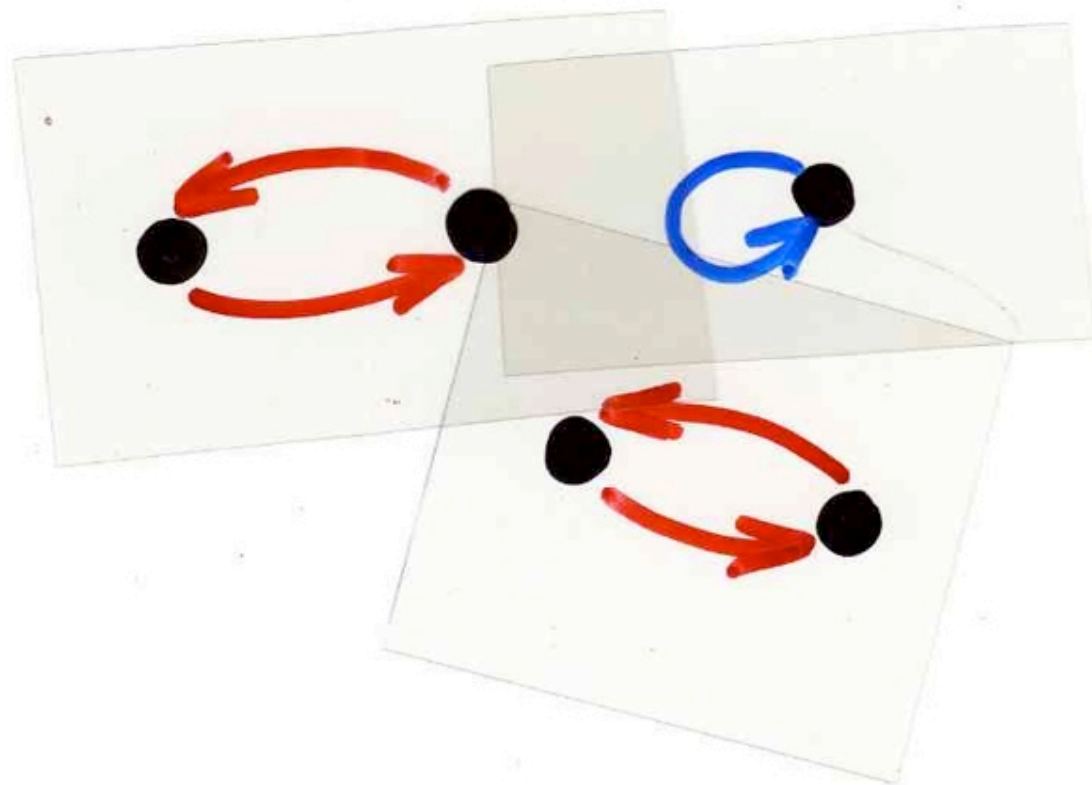
$$\sum_{n \geq 0} H_n(x) H(y) \frac{t^n}{n!} = (1-4t^2)^{-1/2} \exp \left[\frac{4xyt - 4(x^2 + y^2)t^2}{1-4t^2} \right]$$

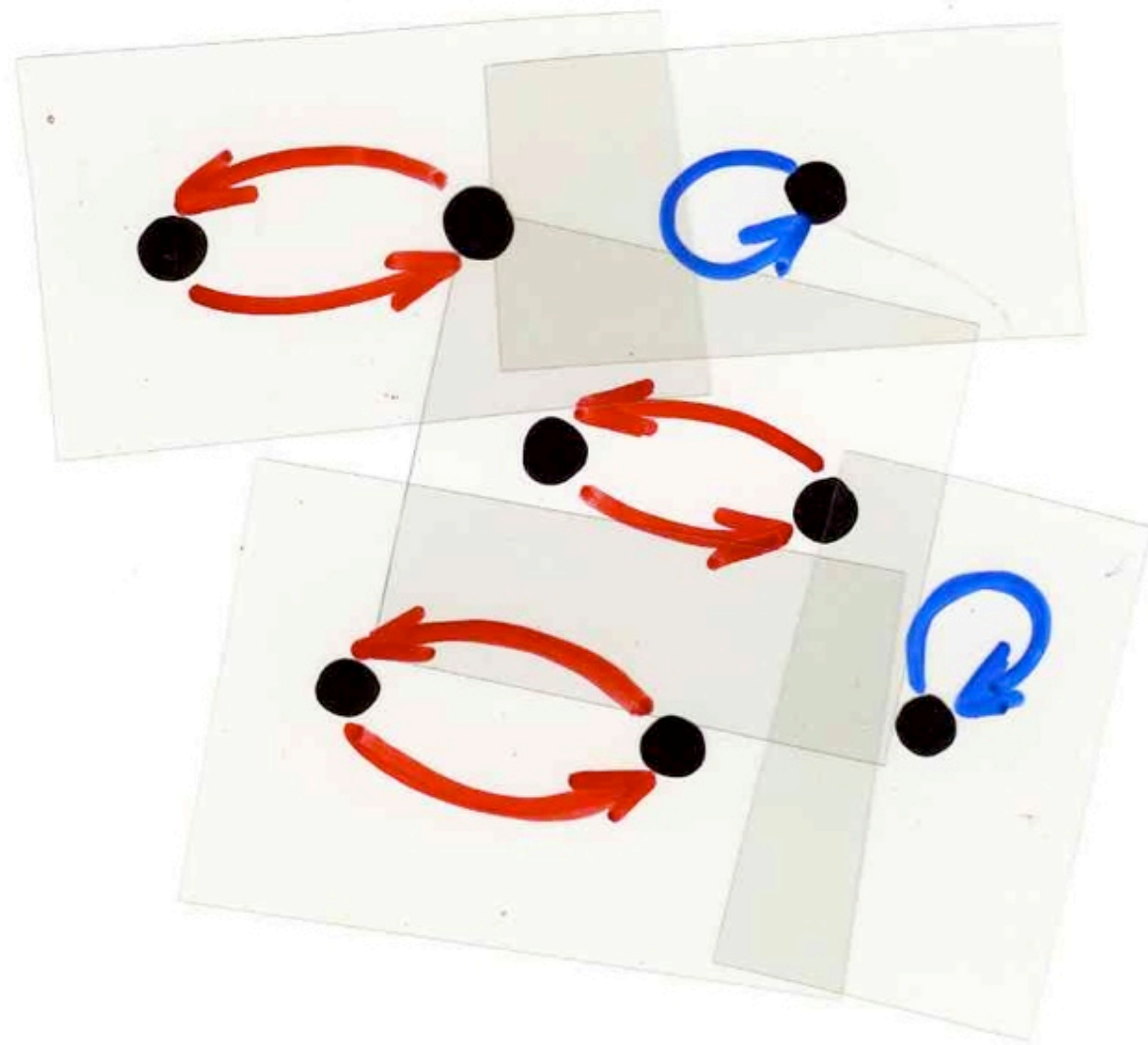
$$\exp \left(\underbrace{\bullet}_{(x)} + \underbrace{\bullet \bullet}_{(-1)} \right)$$

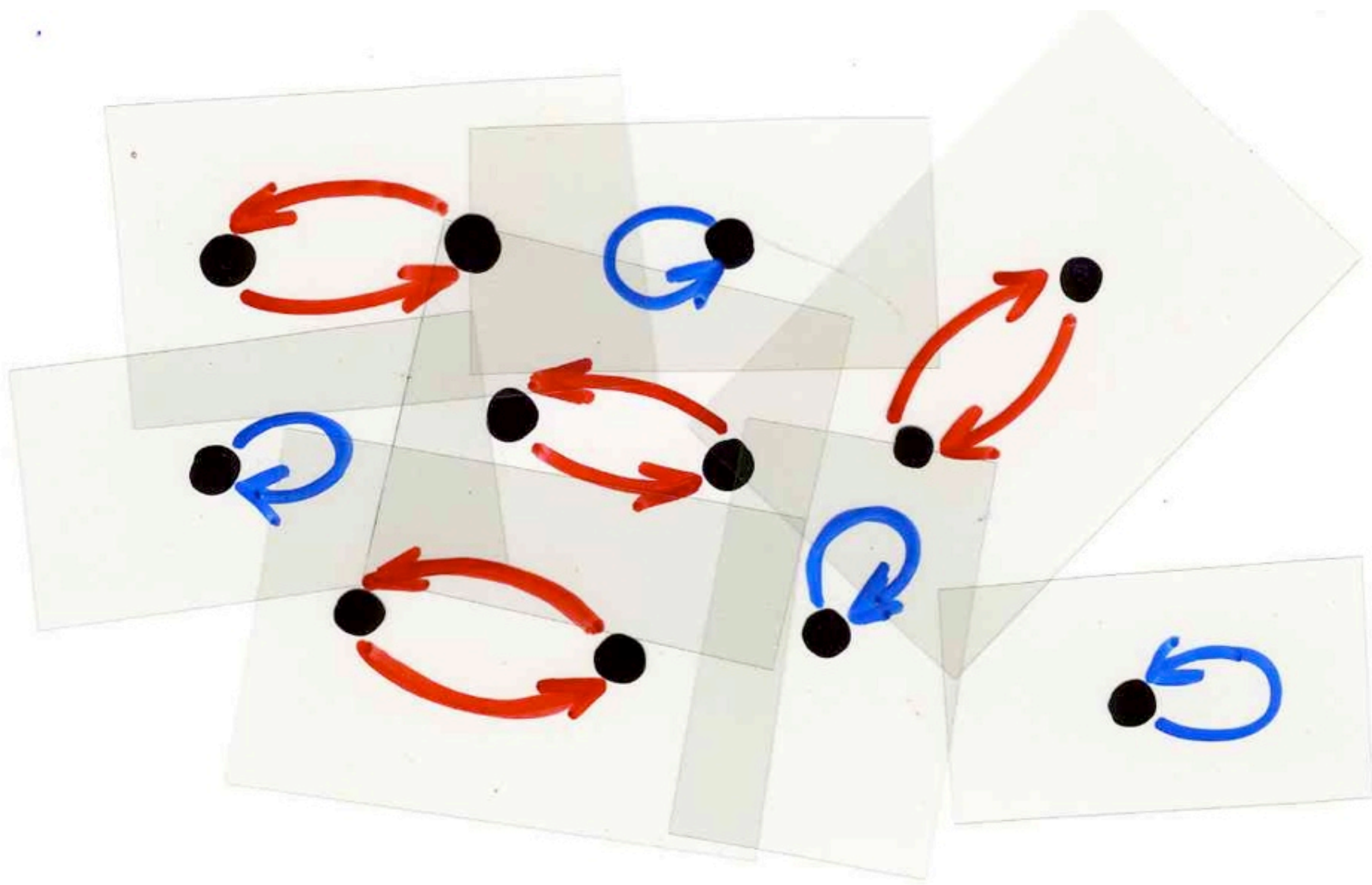
$$\sum_{n \geq 0} H_n(x) \frac{t^n}{n!} = \exp\left(xt - \frac{t^2}{2}\right)$$

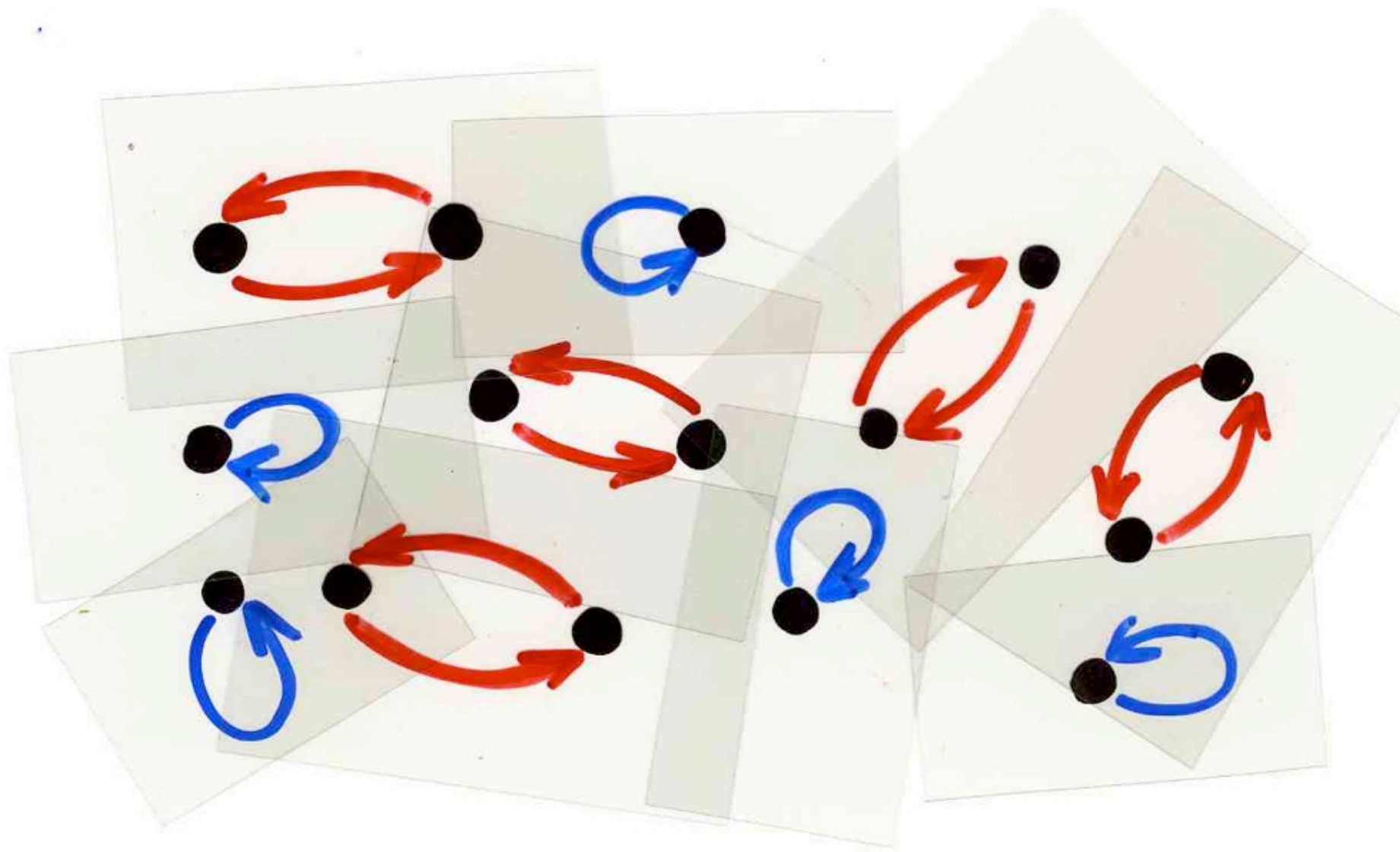






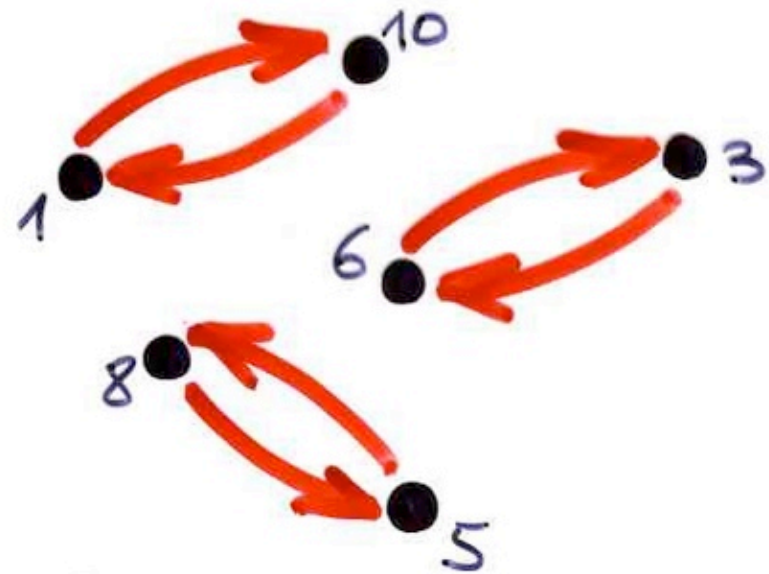
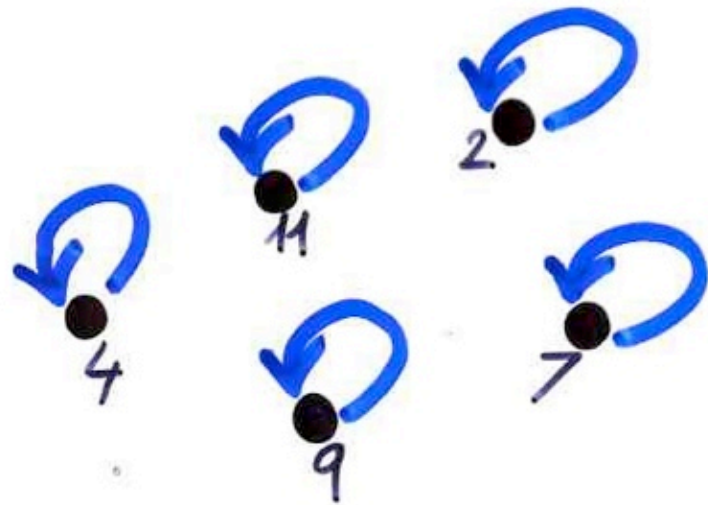






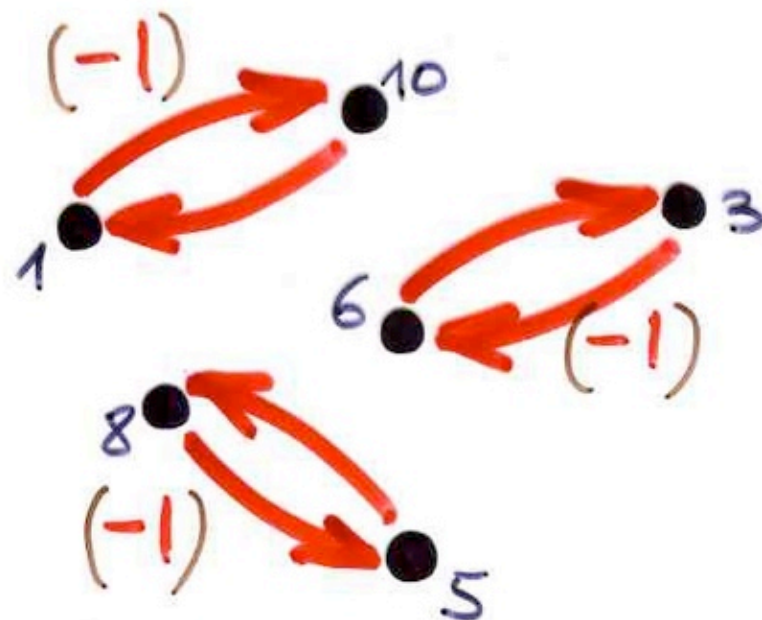
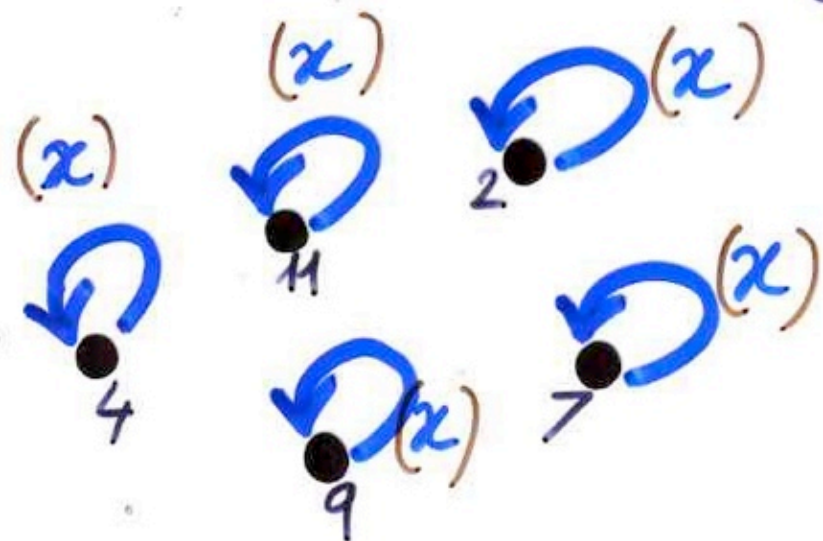
Hermite

configurations



Hermite

configurations

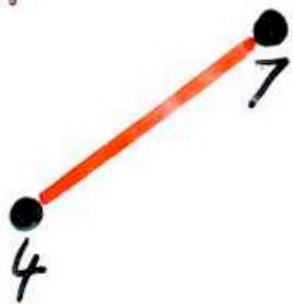


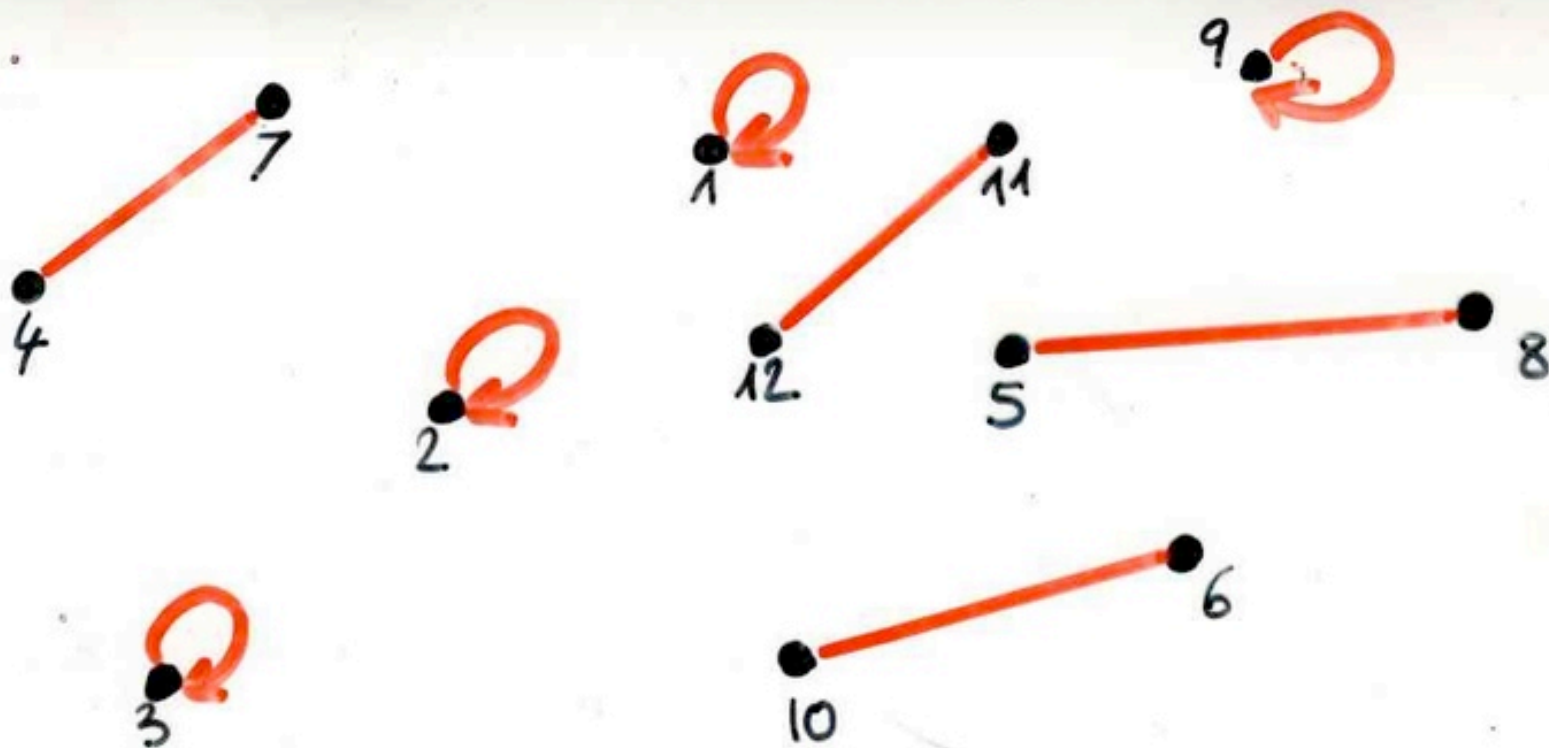
weight

(x)
 (-1)

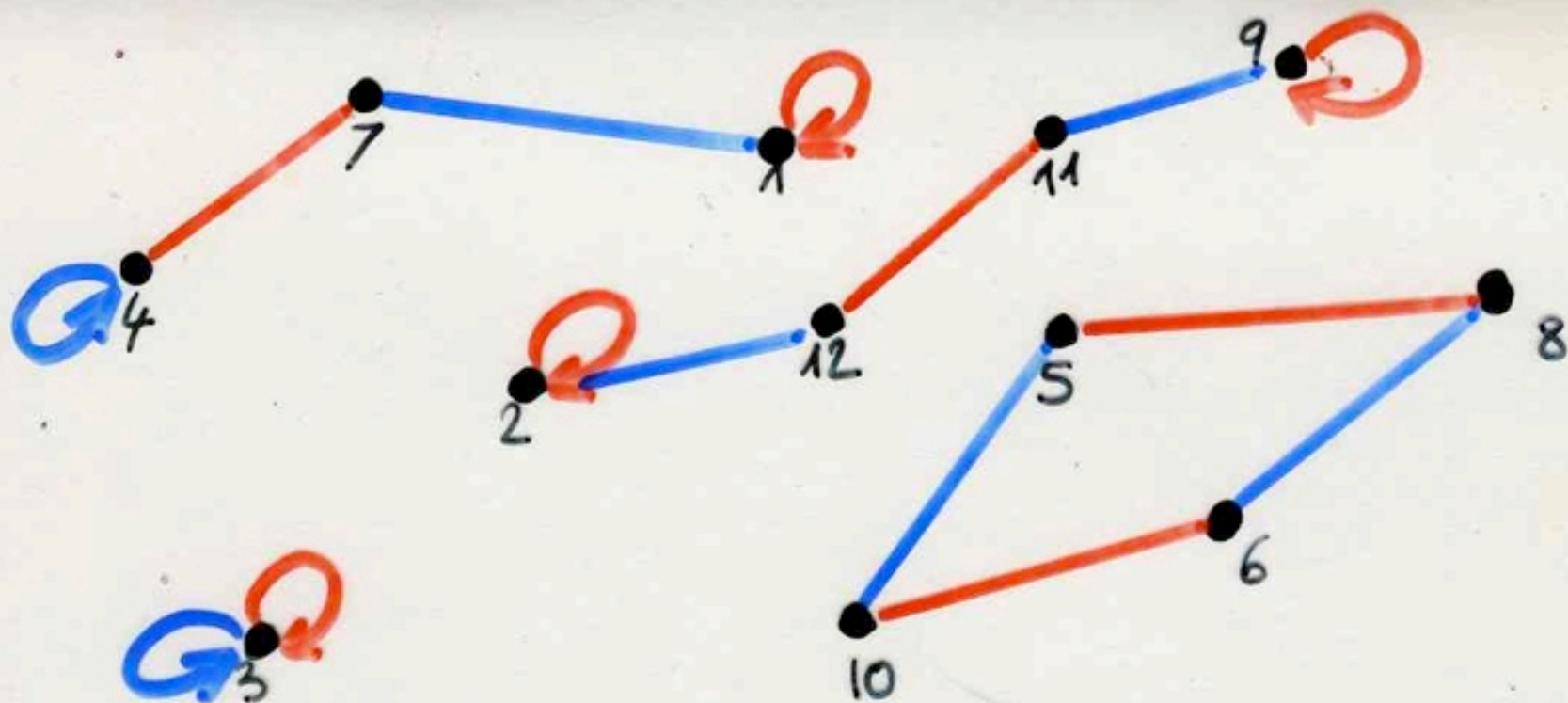
$$\sum_{n \geq 0} H_n(x) H_n(y) \frac{t^n}{n!} = (1-4t^2)^{-1/2} \exp \left[\frac{4xyt - 4(x^2+y^2)t^2}{1-4t^2} \right]$$

$$\sum_{n \geq 0} H_n(x) H_n(y) \frac{t^n}{n!} =$$





$$\sum_{n \geq 0} H_n(x) \frac{t^n}{n!} =$$



$$\sum_{n \geq 0} H_n(x) H_n(y) \frac{t^n}{n!} =$$

$$(1-4t^2)^{-\frac{1}{2}} \exp \left[\frac{4xyt - 4(x^2+y^2)t^2}{1-4t^2} \right]$$

$$\exp \left[\frac{1}{2} \log \frac{1}{(1-4t^2)} + \frac{4xyt - 4(x^2+y^2)t^2}{1-4t^2} \right]$$

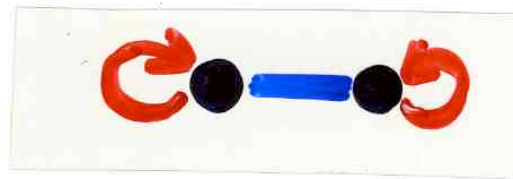
$$\exp \left[\frac{1}{2} \log \frac{1}{(1-4t^2)} + \frac{4xyt - 4(x^2 + y^2)t^2}{1-4t^2} \right]$$

$$\frac{1}{2} \log \frac{1}{(1-4t^2)}$$

$$\frac{-4y^2t^2}{1-4t^2}$$

$$\frac{-4x^2t^2}{1-4t^2}$$

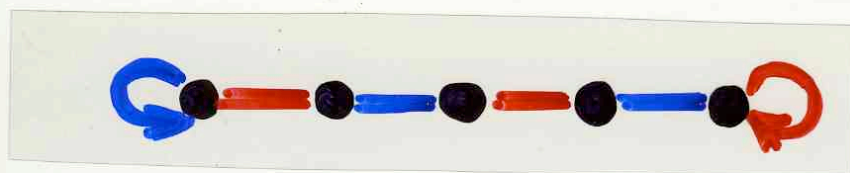
$$\frac{4xyt}{1-4t^2}$$



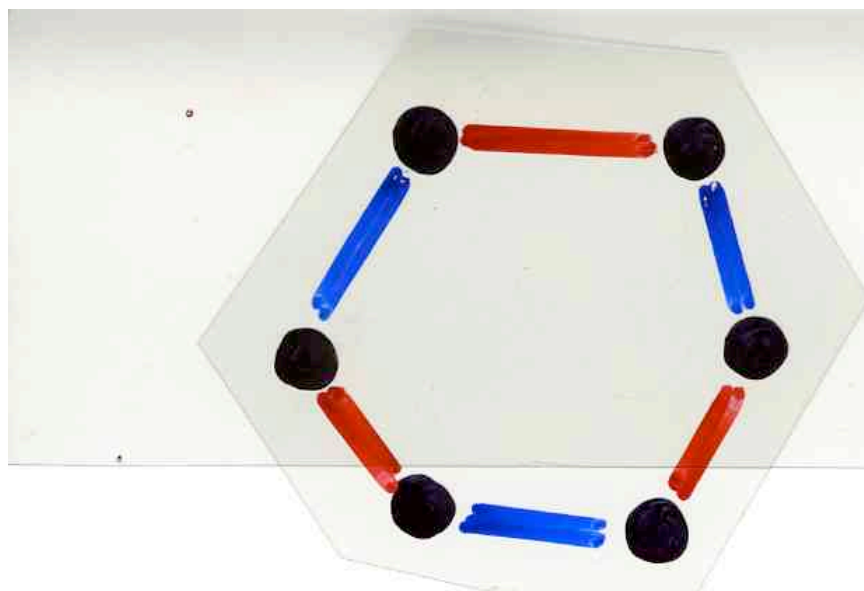
$$\frac{-4x^2 t^2}{1-4t^2}$$



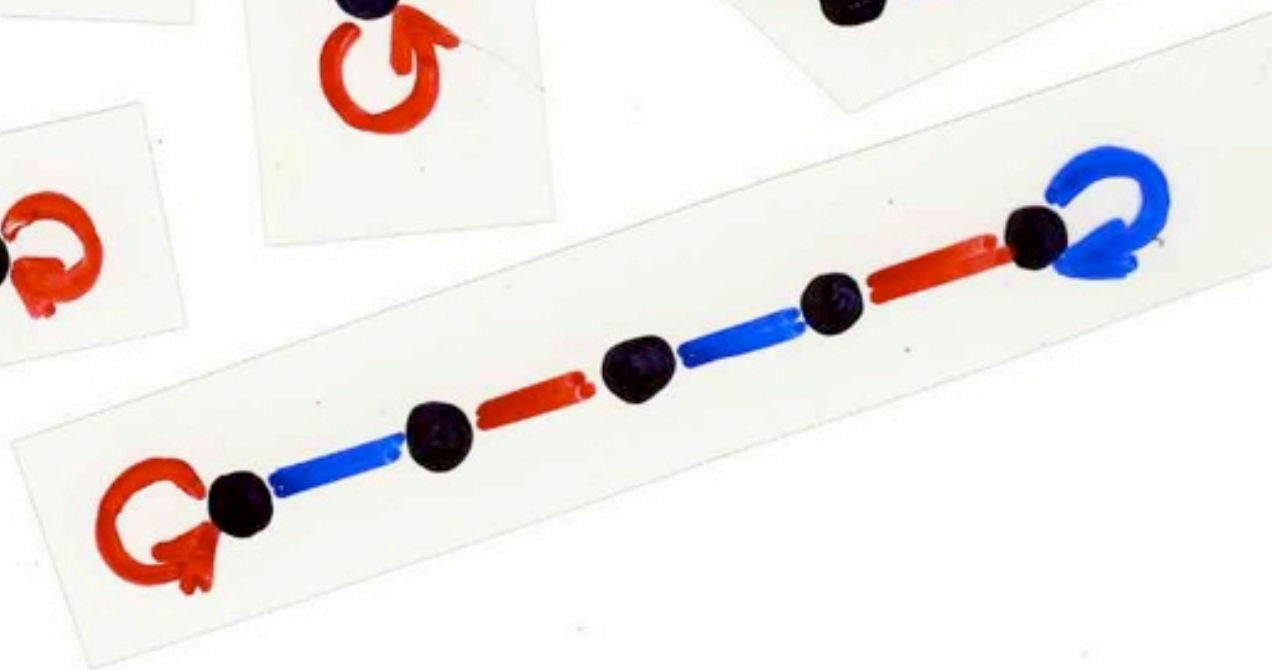
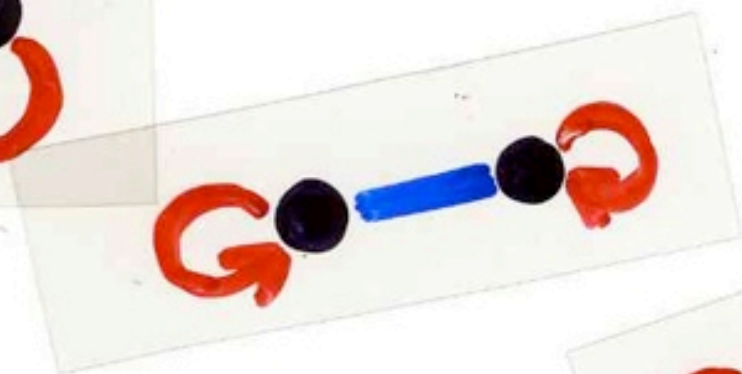
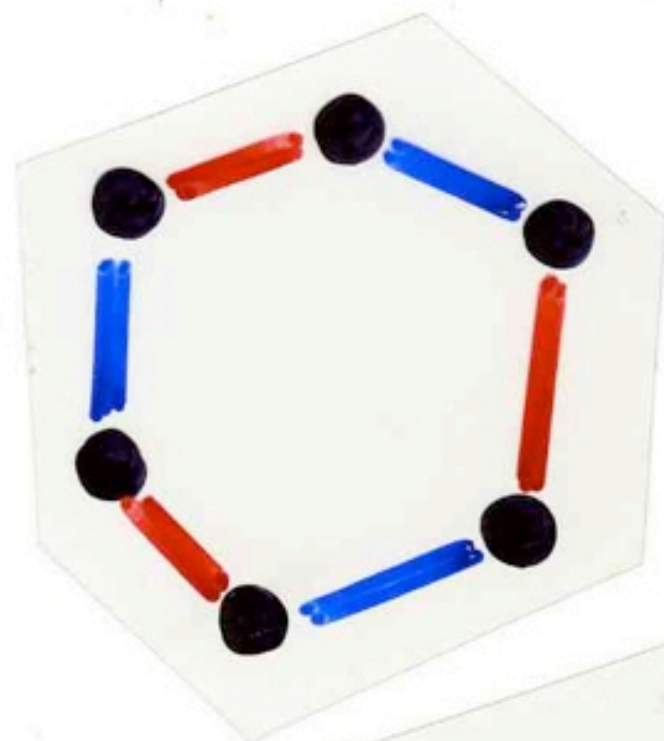
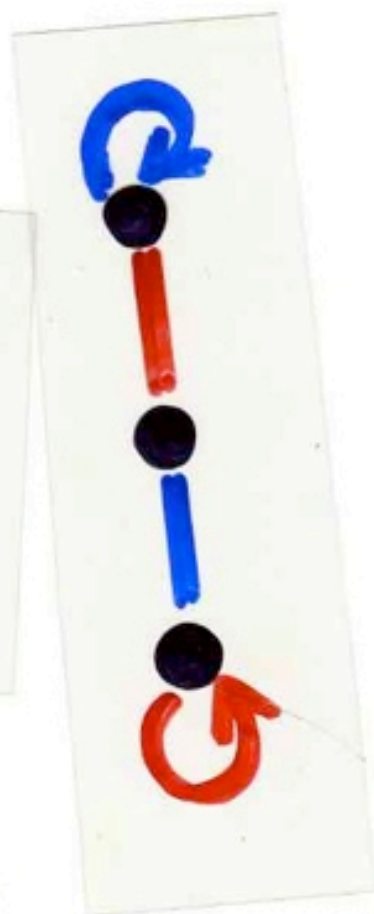
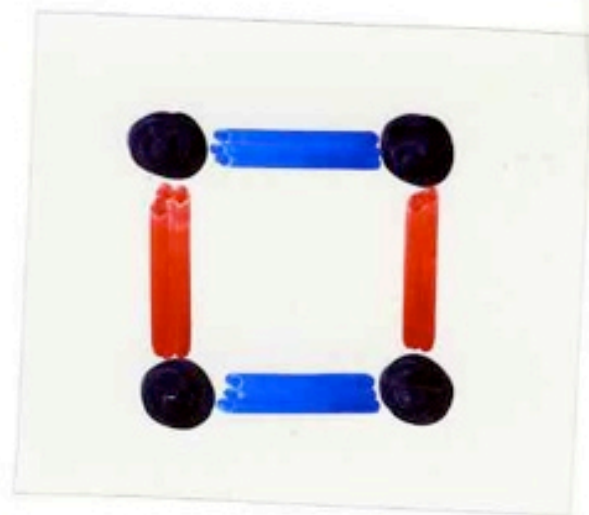
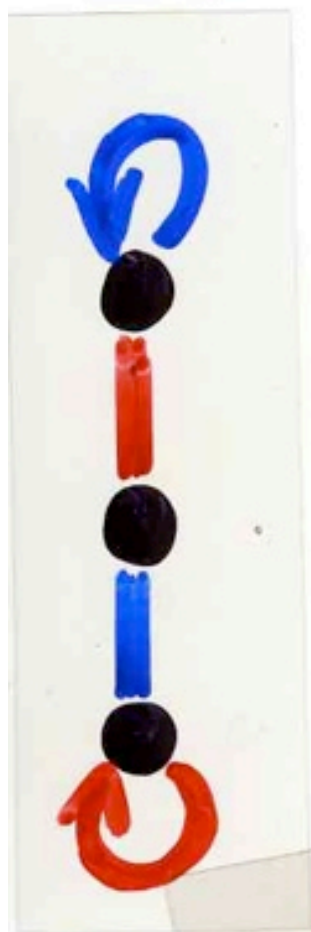
$$\frac{-4y^2 t^2}{1-4t^2}$$

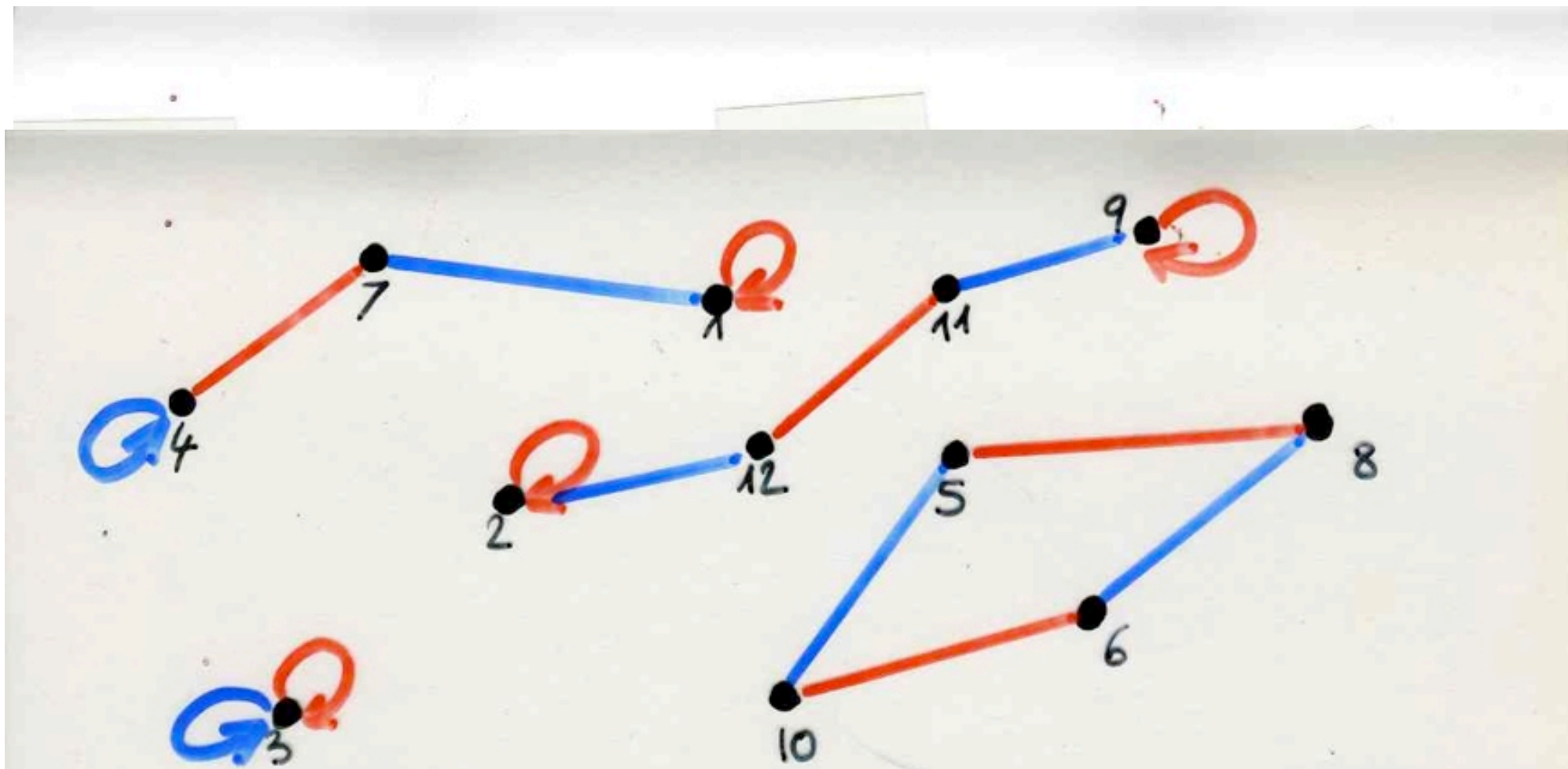


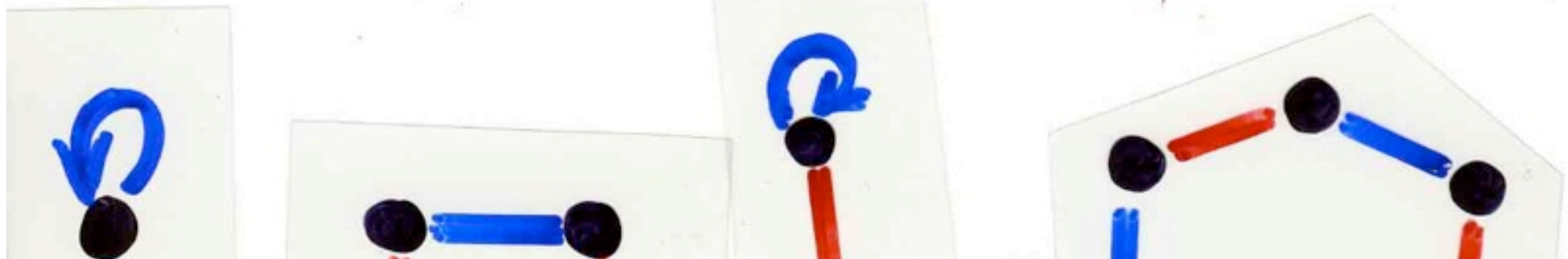
$$\frac{4xyt}{1-4t^2}$$



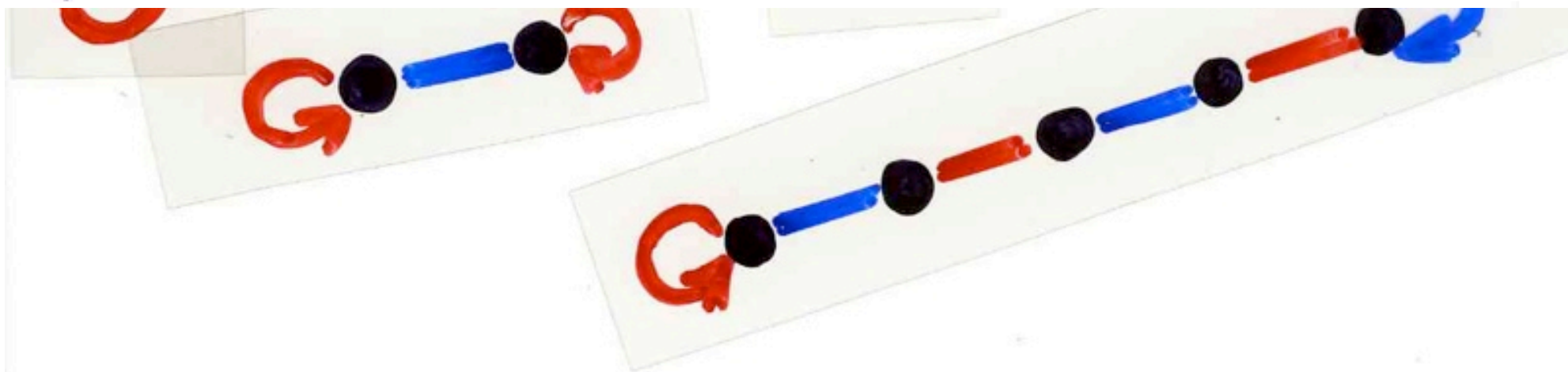
$$\frac{1}{2} \log \frac{1}{(1-4t^2)}$$

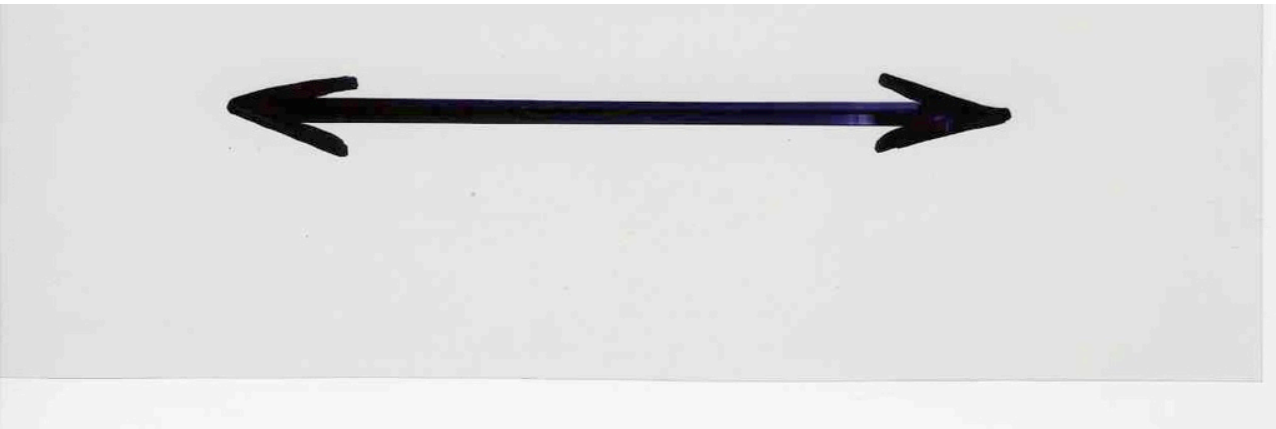






$$\sum_{n \geq 0} H_n(x) H(y) \frac{t^n}{n!} = (1 - 4t^2)^{-1/2} \exp \left[\frac{4xyt - 4(x^2 + y^2)t^2}{1 - 4t^2} \right]$$





==

meilleure
compréhension



Le «paradigme bijectif»

Rogers - Ramanujan identities

$$R_I \quad \sum_{n \geq 0} \frac{q^{n^2}}{(1-q)(1-q^2)\dots(1-q^n)} = \prod_{\substack{i \equiv 1, 4 \\ \text{mod } 5}} \frac{1}{(1-q^i)}$$

$$R_{II} \quad \sum_{n \geq 0} \frac{q^{n^2 + n}}{(1-q)(1-q^2)\dots(1-q^n)} = \prod_{\substack{i \equiv 2, 3 \\ \text{mod } 5}} \frac{1}{(1-q^i)}$$

Srinivasan
Ramanujan
(1887-1920)



quelques formules ...

"La fraction continue" de Ramanujan

$$\frac{1}{1 + \frac{q}{1 + \frac{q^2}{1 + \frac{q^3}{\ddots \frac{1 + q^k}{\dots}}}}}$$

$$\frac{\sum_{n \geq 0} \frac{q^{n^2+n}}{(1-q)(1-q^2) \dots (1-q^n)}}{\sum_{n \geq 0} \frac{q^{n^2}}{(1-q)(1-q^2) \dots (1-q^n)}}$$

$$R(q) = \prod_{n \geq 0} \frac{(1 - q^{5n+1})(1 - q^{5n+4})}{(1 - q^{5n+3})(1 - q^{5n+2})} = \frac{R_{II}}{R_I}$$

$$R(q) = \prod_{n \geq 0} \frac{(1 - q^{5n+1})(1 - q^{5n+4})}{(1 - q^{5n+3})(1 - q^{5n+2})} = \frac{R_{II}}{R_I}$$

$$t = -q [R(q)]^5$$

$$R(q) = \prod_{n \geq 0} \frac{(1-q^{5n+1})(1-q^{5n+4})}{(1-q^{5n+3})(1-q^{5n+2})} = \frac{R_{II}}{R_I}$$

$$t = -q [R(q)]^5$$

$$Y(q) = \prod_{n \geq 0} \frac{(1-q^{6n+2})(1-q^{6n+3})^2(1-q^{6n+4})(1-q^{5n+1})^2(1-q^{5n+4})^2(1-q^{5n})^2}{(1-q^{6n+1})(1-q^{6n+5})(1-q^{6n})^2(1-q^{5n+2})^3(1-q^{5n+3})^3}$$

$$R(q) = \prod_{n \geq 0} \frac{(1-q^{5n+1})(1-q^{5n+4})}{(1-q^{5n+3})(1-q^{5n+2})} = \frac{R_{II}}{R_I}$$

$$t = -q [R(q)]^5$$

$$\gamma(q) = \prod_{n \geq 0} \frac{(1-q^{6n+2})(1-q^{6n+3})^2(1-q^{6n+4})(1-q^{5n+1})^2(1-q^{5n+4})^2(1-q^{5n})^2}{(1-q^{6n+1})(1-q^{6n+5})(1-q^{6n})^2(1-q^{5n+2})^3(1-q^{5n+3})^3}$$

$$Z(t) = \gamma(q(t))$$

$$y(1 + 14t + 97t^2 + 415t^3 + 1180t^4 + 2321t^5 + 3247t^6 + 3300t^7 + 2475t^8 + 1375t^9 + 550t^{10} + 143t^{11} + 18t^{12}) +$$

$$y^2(1 + 17t + 83t^2 + 601t^3 + 1647t^4 + 4606t^5 + 7809t^6 + 710t^7 + 124t^8 - 608t^9 - 440t^{10} - 92t^{11} - 36t^{12}) +$$

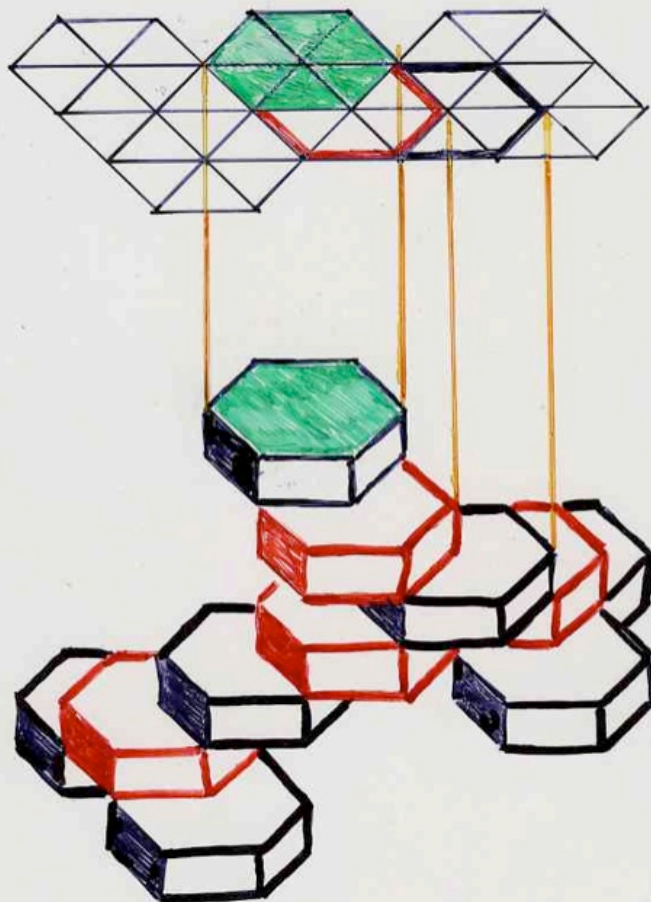
$$y^3(3 + 50t + 381t^2 + 1715t^3 + 5040t^4 + 10130t^5 + 14062t^6 + 13002t^7 + 6930t^8 + 715t^9 - 1595t^{10} - 488t^{11} - 198t^{12}) +$$

$$y^4(1 + 17t + 131t^2 + 595t^3 + 1765t^4 + 3574t^5 + 4939t^6 + 4356t^7 + 1815t^8 - 605t^9 - 1210t^{10} - 616t^{11} - 126t^{12})$$

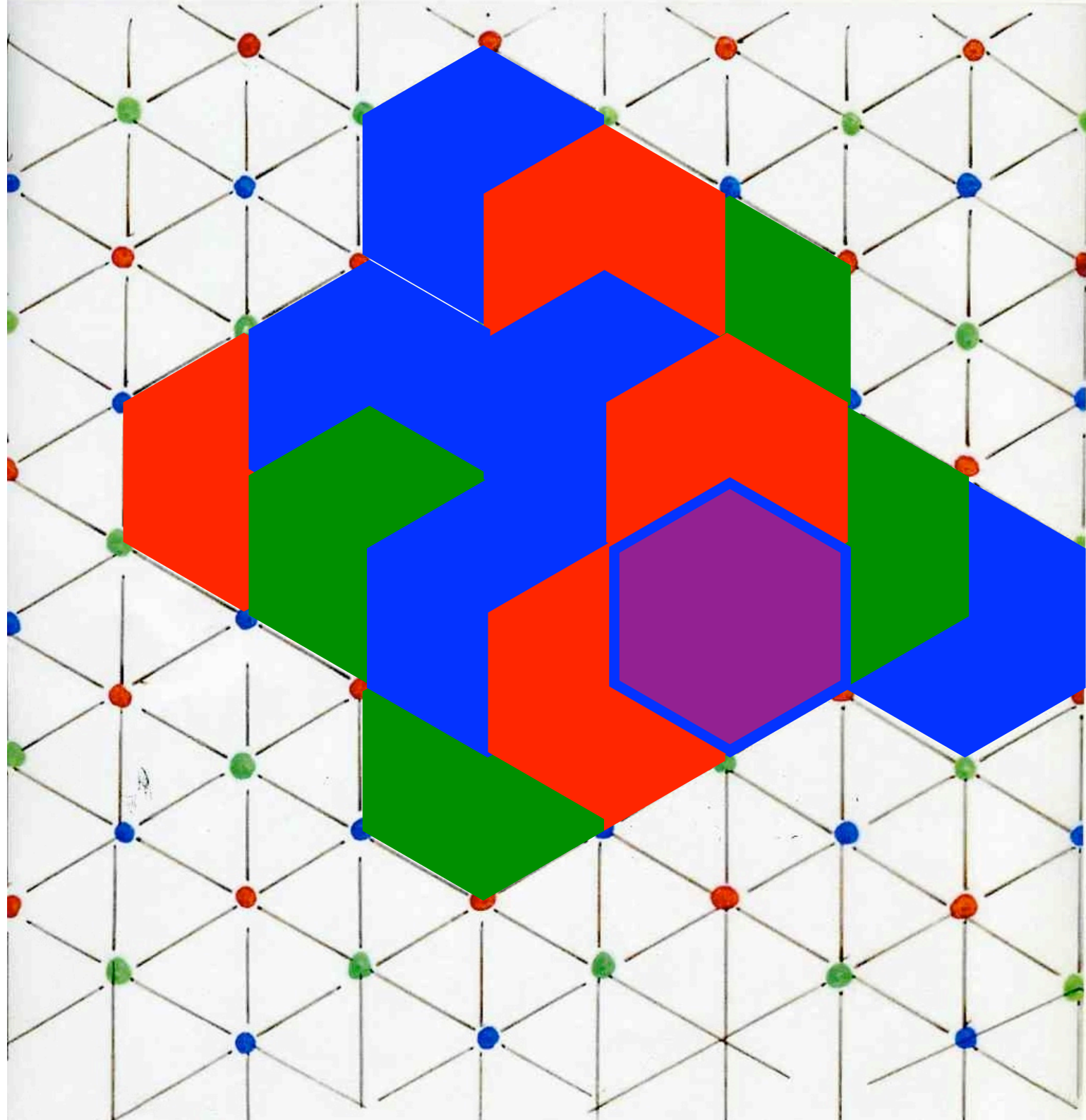
=

$$(t + 11t^2 + 55t^3 + 165t^4 + 330t^5 + 462t^6 + 462t^7 + 330t^8 + 165t^9 + 55t^{10} + 11t^{11} + t^{12})$$

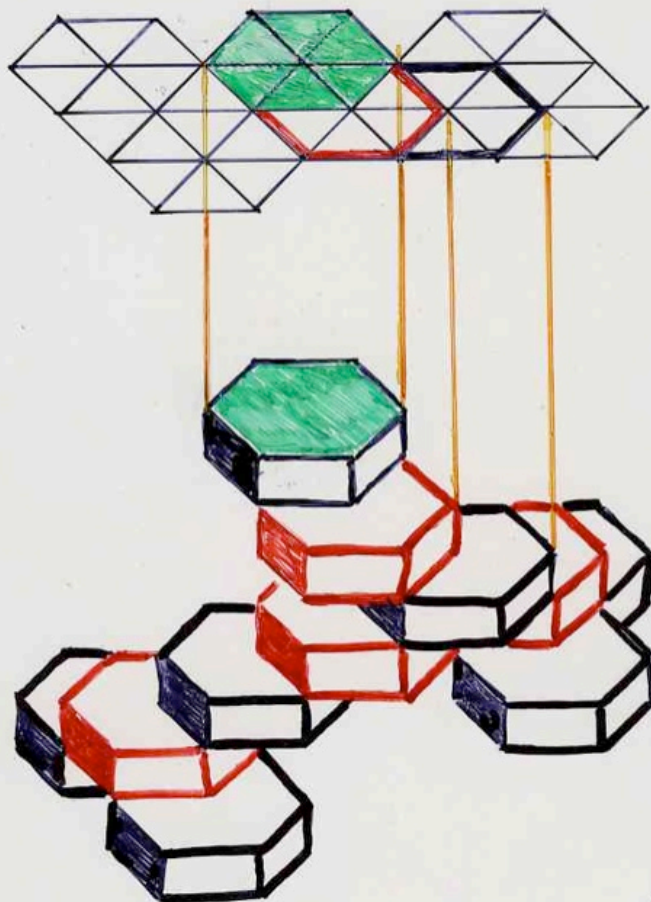
$$-p(-t) = y$$



Le concept d'empilements de pièces



$$-p(-t) = y$$



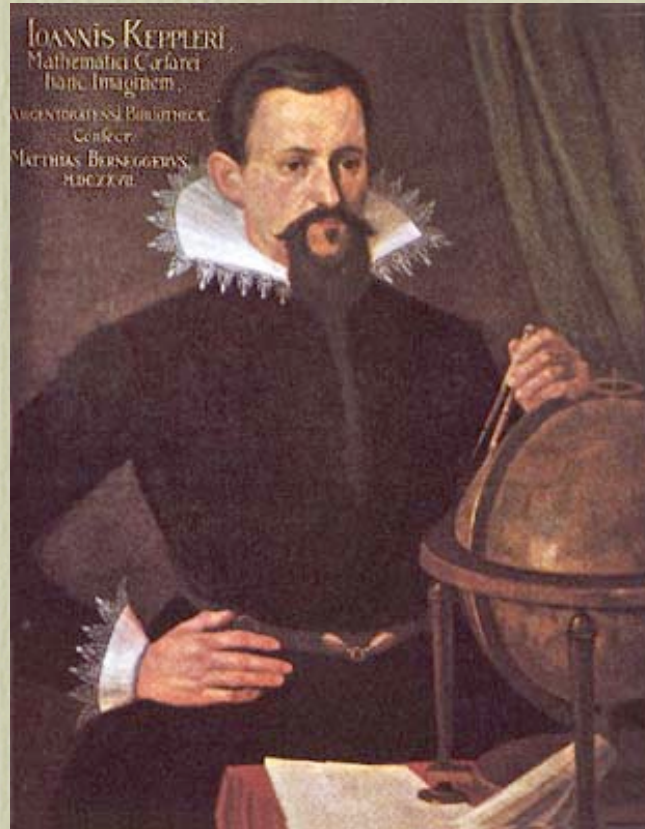
Physique combinatoire

SAINT-EXUPÉRY

L'infinitement grand

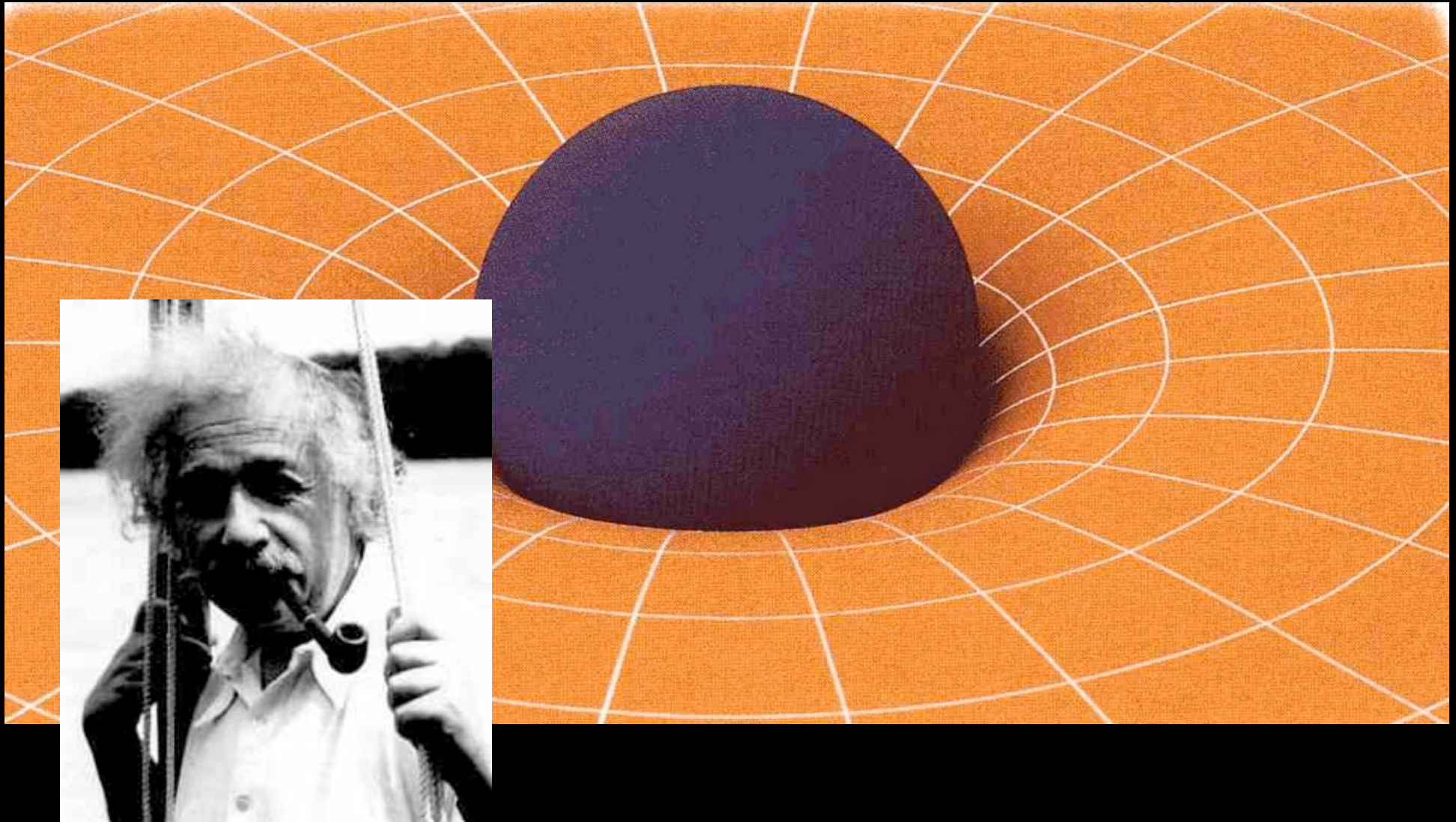
Des arbres dans les étoiles ?





géométrie et mécanique classique
Galileo, Kepler, Newton,...

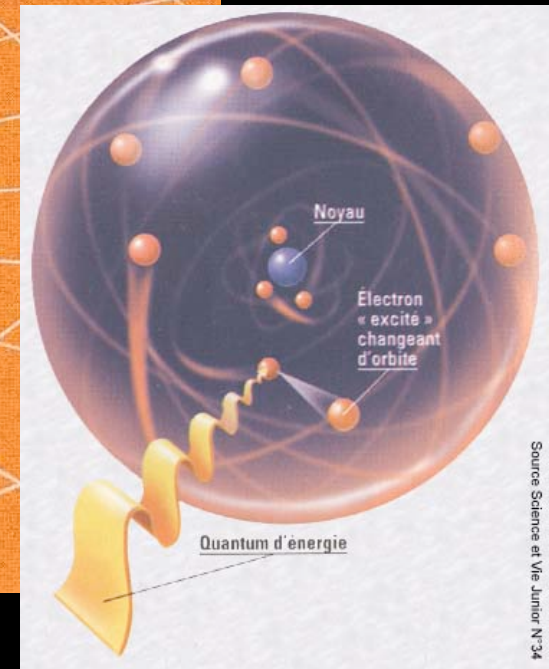
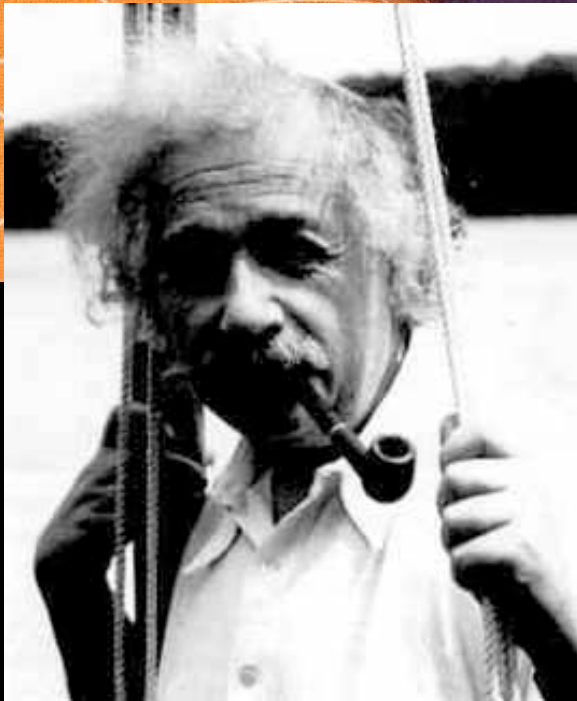
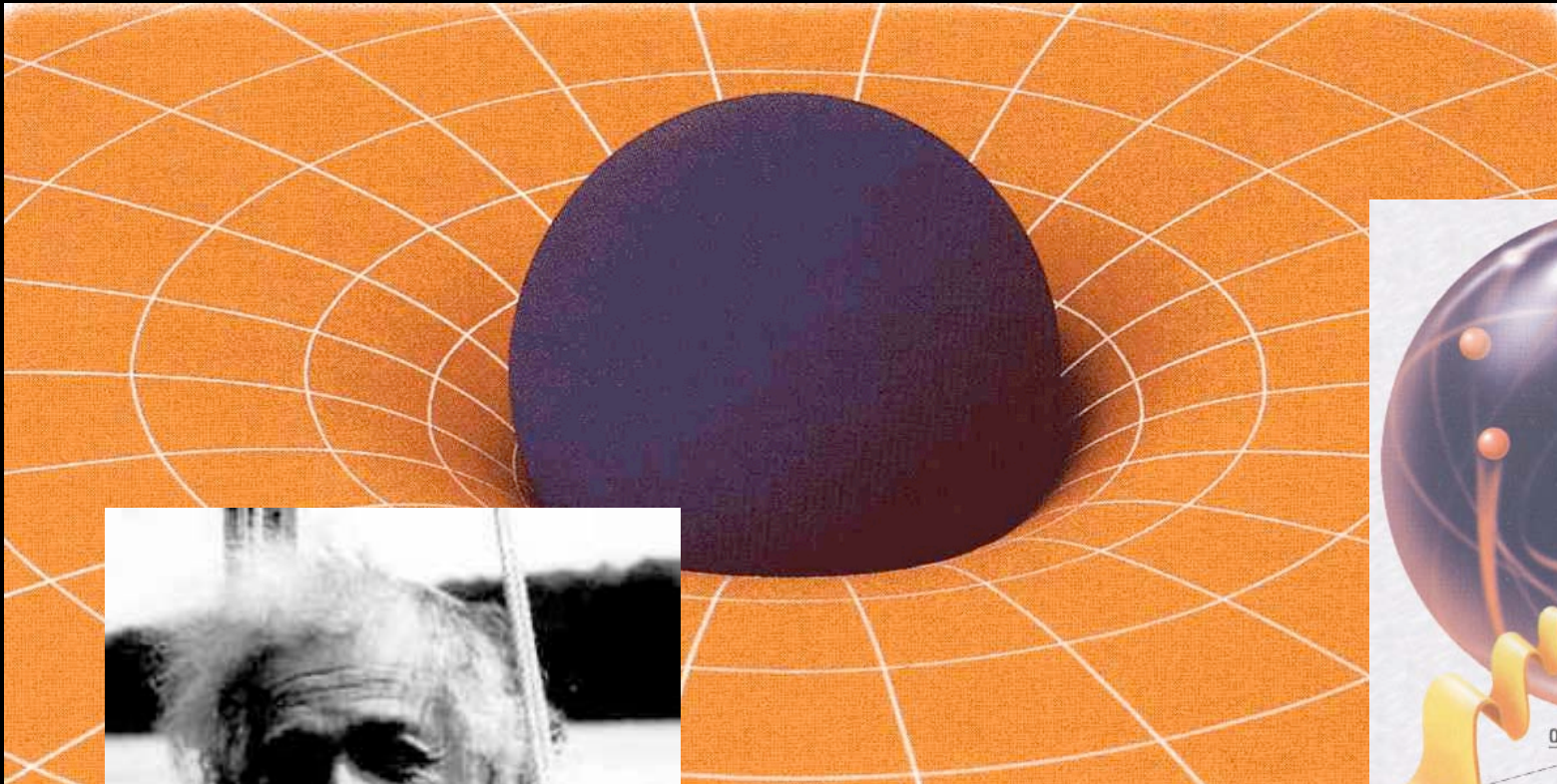
relativité générale





relativité générale

mécanique quantique



gravitation quantique

théorie des cordes

particule comme une corde de violon ... ?
chaque fréquence correspond à une particule.... ?

nombre de Catalan ?

Alain Connes

$$\int e^{i \frac{S(A)}{\hbar}}$$

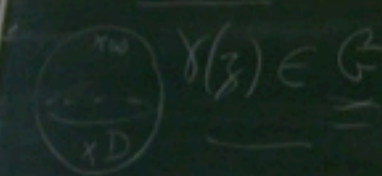
$$\frac{1}{k^2} = \int_0^\infty e^{-\lambda k^2} d\lambda$$

$$\int e^{-\lambda k^2} d^D k = \left(\frac{\pi}{\lambda}\right)^{D/2}$$

DIM-REG

D. KREIMER

Brk Dec



$$\chi(z) \in G$$

$$\chi(z) = \chi(z)$$

(D)

$$D_{||}(\bar{z}) =$$

M. MARCOLLI

$$\chi_N$$

Universal Singular Frac

$$\gamma_W(z, v) = T e^{-\frac{1}{z} \int_0^v u^Y(e) \frac{du}{u}}$$

$$\gamma_W(-z, v) = \sum_{n \geq 0} \sum_{k_j > 0}$$

$$\frac{e(-k_1)e(-k_2) \cdots e(-k_n)}{k_1(k_1+k_2) \cdots (k_1+k_2+\cdots +$$

Same coefficients as

Local Index Formula in NCC

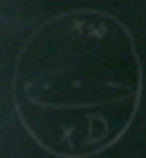
géométrie non-commutative

$$\int e^{-\frac{\lambda k^2}{2}} \quad \int e^{-\lambda k^2} d^D k = \left(\frac{\pi}{\lambda}\right)^{D/2}$$

$\mathcal{D}H\text{-REG}$

D. KREIMER

Brkhl // Dec



$\gamma(z) \in \mathbb{G}$

$\gamma(z) = \gamma_-(z)$

$\rightarrow \gamma_+(D)$

$D: // (\gamma)$

M. MARCOLLI

γ_N

Universal Singular Fra

$$\gamma_U(z, v) = T e^{-\frac{1}{2} \int_0^v u^Y(e) \frac{du}{u}}$$

$$\gamma_U(-z, v) = \sum_{n \geq 0} \sum_{k_j > 0}$$

$$\frac{e(-k_1)e(-k_2) \cdots e(-k_n)}{k_1(k_1+k_2) \cdots (k_1+k_2+\cdots)}$$

Same coefficients as

Local Index Formula in NC

géométrie non-commutative

plus de points ...

$$e^{\frac{i\hbar}{2m} \Delta} = e^{\frac{i\hbar}{2m} \frac{\Delta}{\hbar^2}} = e^{\frac{i}{2m} \Delta}$$

$$\int e^{-\lambda k^2} d^D k = \left(\frac{\pi}{\lambda}\right)^{D/2}$$

$D, KREIMER$ $Birkhoff$ Dec

$$\gamma(z) \in G \quad \gamma(z) = \gamma_-(z)^{-1} \gamma_+(z)$$

$$\xrightarrow{\quad} \gamma_+(D)$$

Universal Singular Fra

$$\gamma_U(z, v) = T e^{-\frac{1}{2} \int_0^v u^Y(e) dt}$$

$$\gamma_U(-z, v) = \sum_{\eta \geq 0} \sum_{k_j > 0}$$

$$\frac{e(-k_1)e(-k_2) \cdots e(-k_n)}{k_1(k_1+k_2) \cdots (k_1+k_2+\cdots+k_n)}$$

Same coefficients as

Local Index Formula in NC

$$D_{i//}(\bar{z}) \rightarrow$$

M. MARCOLLI

$$\gamma_n$$

géométrie non-commutative

plus de droites ...

Universal Singular Frame

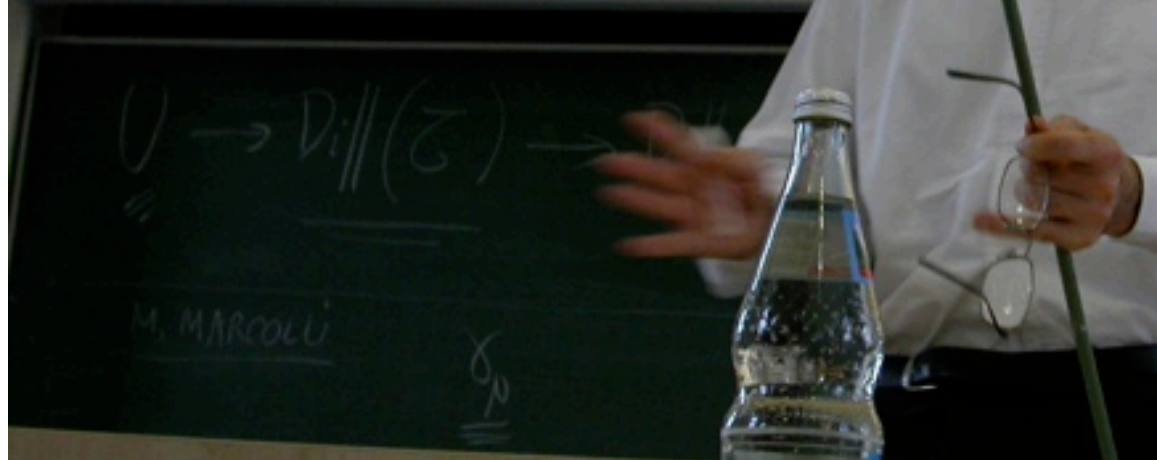
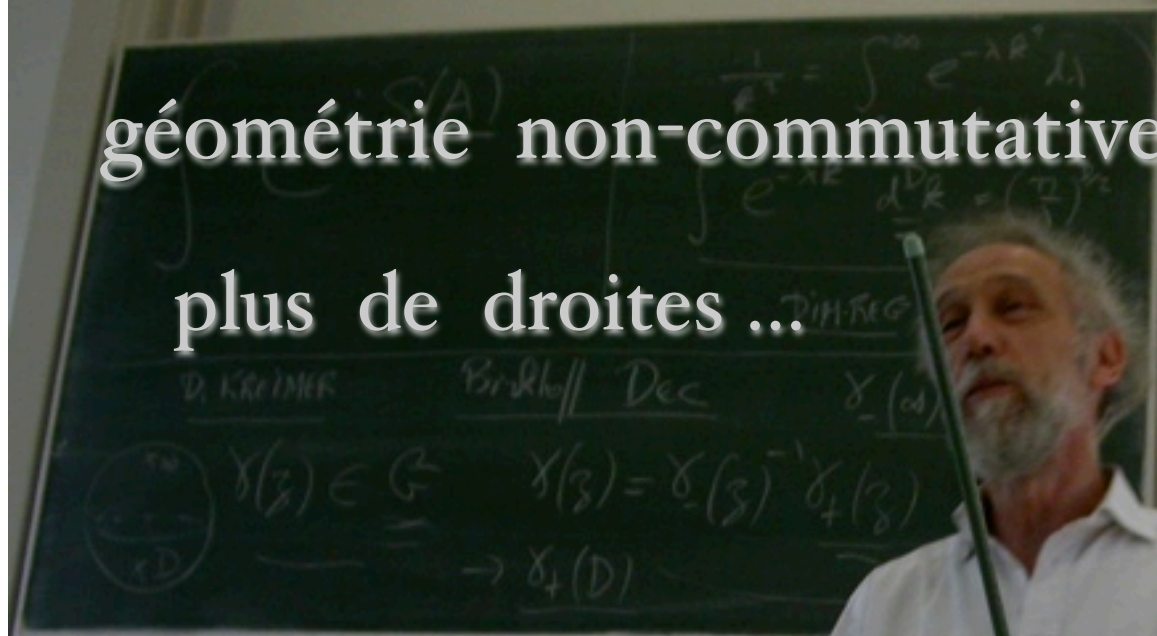
$$\gamma_U(z, v) = T e^{-\frac{1}{z} \int_0^v u^Y(e) \frac{du}{u}} \in U$$

$$\gamma_U(-z, v) = \sum_{n \geq 0} \sum_{k_j > 0} \dots$$

$$\frac{e(-k_1)e(-k_2)\cdots e(-k_n)}{k_1(k_1+k_2)\cdots(k_1+k_2+\cdots+k_n)} v^{\sum k_j}$$

Same coefficients as in

Local Index Formula in NCG (ac + i



géométrie non-commutative ni d'espace ...

Universal Singular Frame

$$\gamma_U(z, v) = \text{Te}^{-\frac{1}{z} \int_0^v u^Y(e) \frac{du}{u}} \in U$$

$$\gamma_U(-z, v) = \sum_{n \geq 0} \sum_{k_j > 0} \frac{e(-k_1) e(-k_2) \cdots e(-k_n)}{k_1 (k_1 + k_2) \cdots (k_1 + k_2 + \cdots + k_n)} v^{\sum k_j}$$

$$\frac{e(-k_1) e(-k_2) \cdots e(-k_n)}{k_1 (k_1 + k_2) \cdots (k_1 + k_2 + \cdots + k_n)} v^{\sum k_j}$$

Same coefficients as in

Local Index Formula in NCG (ac + 1)

$$\frac{1}{x} = \int_0^\infty e^{-xk} dk$$

$\gamma(z) \in G$ $\gamma(z) = \gamma_-(z)$
 $\rightarrow \gamma_+(D)$

$$U \rightarrow \text{Diff}(\mathbb{C})$$

M. MARCOLLI

géométrie non-commutative

$$AB \neq BA$$

Universal Singular Frame

$$\gamma_U(z, v) = \text{Tr} e^{-\frac{1}{z} \int_0^v u^Y(e) \frac{du}{u}} \in U$$

$$\gamma_U(-z, v) = \sum_{n \geq 0} \sum_{k_j > 0} \frac{e(-k_1) e(-k_2) \cdots e(-k_n)}{k_1 (k_1 + k_2) \cdots (k_1 + k_2 + \cdots + k_n)} v^{\sum k_j}$$

$$\frac{e(-k_1) e(-k_2) \cdots e(-k_n)}{k_1 (k_1 + k_2) \cdots (k_1 + k_2 + \cdots + k_n)} v^{\sum k_j}$$

Same coefficients as in

Local Index Formula in NCG (ac

D. KREIMER

Brakhoff Dec

γ



$$\gamma(\gamma) \in G$$

$$\gamma(\gamma) = \gamma(\gamma^{-1})$$

$$\rightarrow \gamma_+(D)$$

$$U \rightarrow \text{Diff}(\mathbb{C})$$

M. MARCOLLI

γ_n

géométrie non-commutative

$AB \neq BA$

et le temps ?

«Le théâtre quantique»

«L'incertitude quantique est le tic-tac de l'horloge divine»

Universal Singular Frame

$$\gamma_U(z, v) = \tau e^{-\frac{1}{z} \int_0^v u^Y(e) \frac{du}{u}} \in U$$

$$\gamma_U(-z, v) = \sum_{\eta \geq 0} \sum_{k_j > 0} \frac{e(-k_1) e(-k_2) \cdots e(-k_n)}{k_1 (k_1 + k_2) \cdots (k_1 + k_2 + \cdots + k_n)} v^\Sigma$$

$$\frac{e(-k_1) e(-k_2) \cdots e(-k_n)}{k_1 (k_1 + k_2) \cdots (k_1 + k_2 + \cdots + k_n)} v^\Sigma$$

Same coefficients as in

Local Index Formula in NCG (ac +

gravitation quantique à boucles



Carlo Rovelli

Et si le temps n'existait pas ?

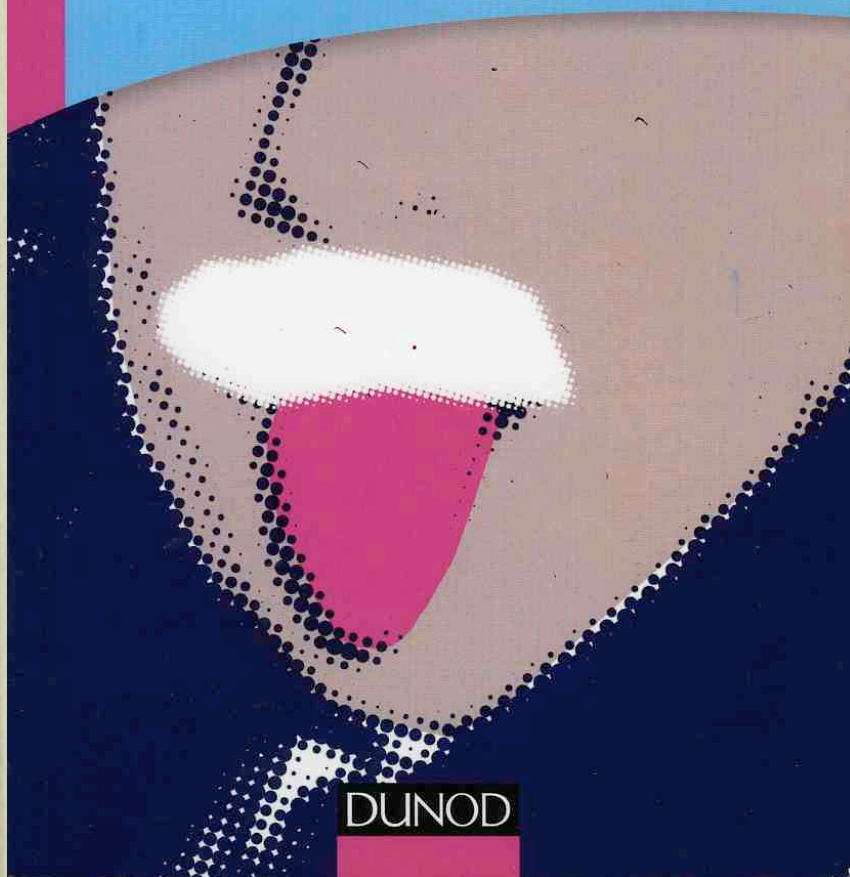
QUAI DES SCIENCES

CARLO ROVELLI

ET SI LE TEMPS N'EXISTAIT PAS ?

UN PEU DE SCIENCE
SUBVERSIVE

DUNOD





écume de
l'espace-temps

particules
d'espace-temps

Dessin
S. Numazawa
Ciel & Espace

bébé
univers

WILLIAM BLAKE

gravitation quantique

triangulations causales dynamiques





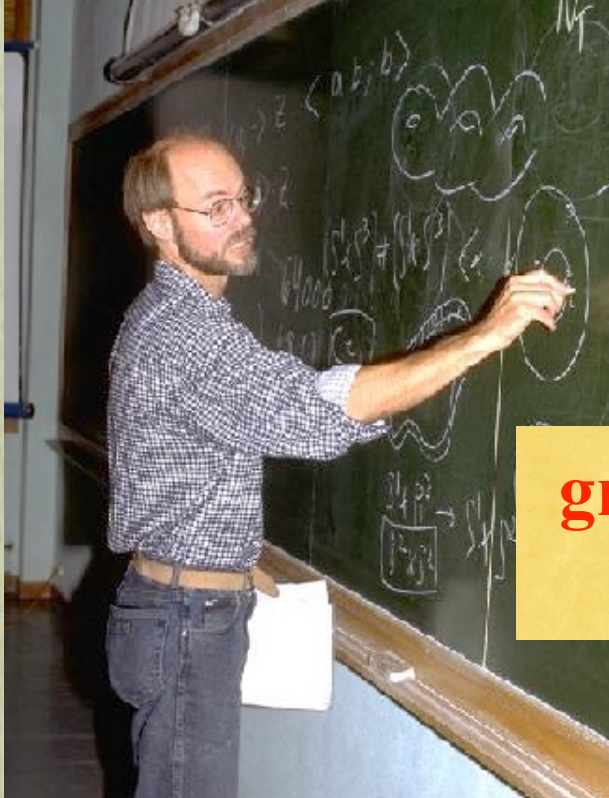
Deepak Dhar
TIFR Bombay

Xavier, you should have
a look at these papers:

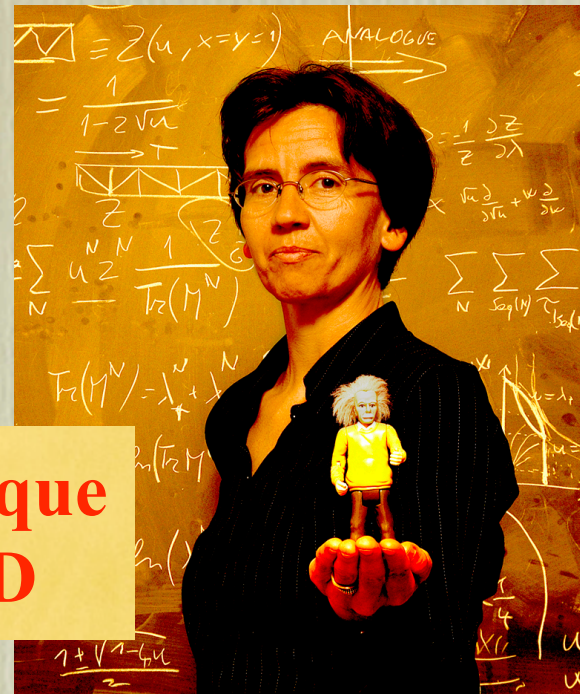
- J. Ambjørn, R. Loll, "Non-perturbative Lorentzian quantum gravity and topology change", Nucl. Phys. B 536 (1998) 407-434
arXiv: hep-th / 9805108

- P. Di Francesco, E. Guilteer, C. Kristjansen, "Integrale 2D Lorentzian gravity and random walks", Nucl. Phys. B 567 (2000) 515-553
arXiv: hep-th / 9907084

gravitation quantique



J. Ambjørn



R. Loll

**gravitation quatique
Lorentzienne 2D**



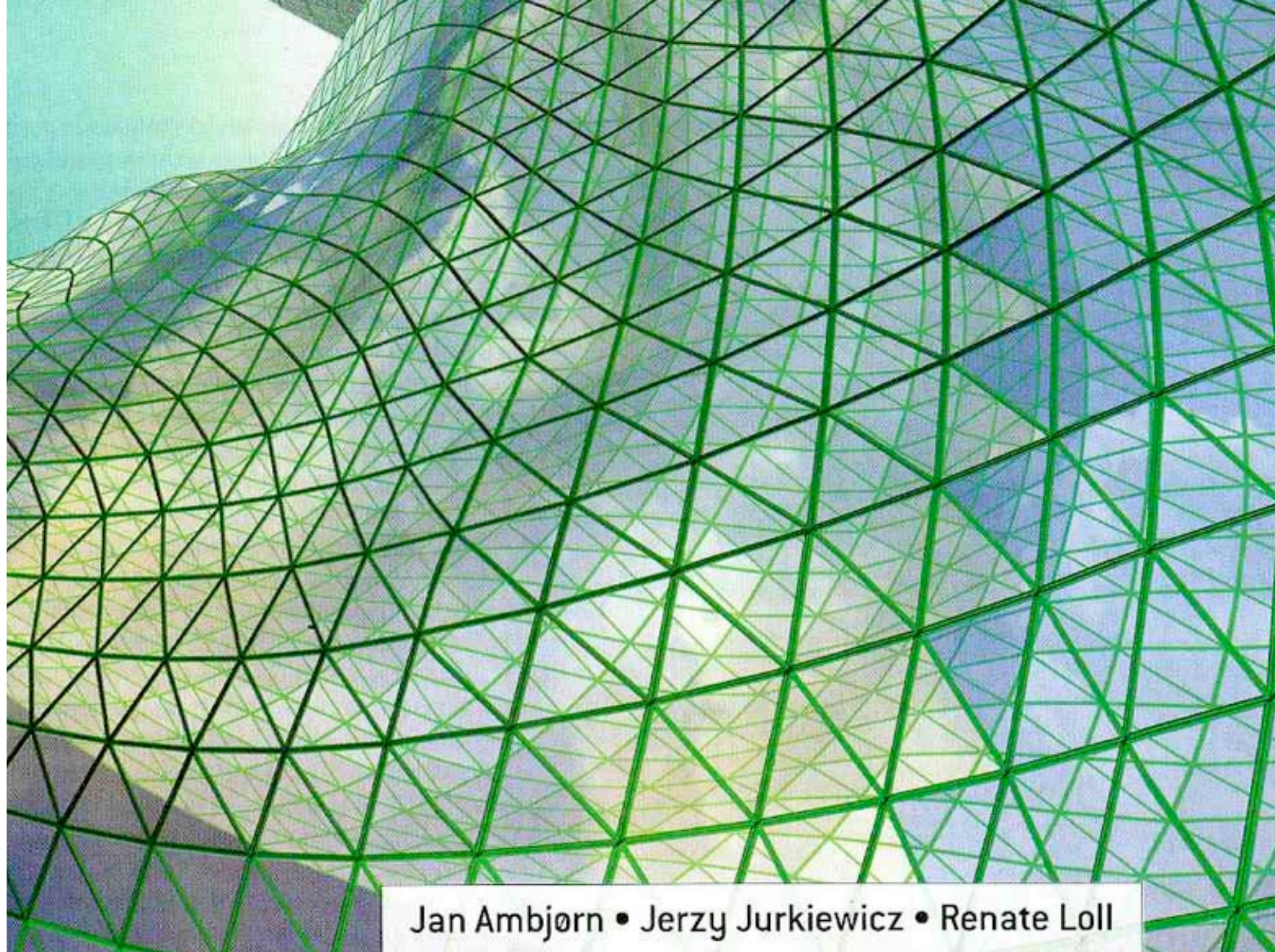
P. Di Francesco



E. Guitter



C. Kristjansen



Jan Ambjørn • Jerzy Jurkiewicz • Renate Loll

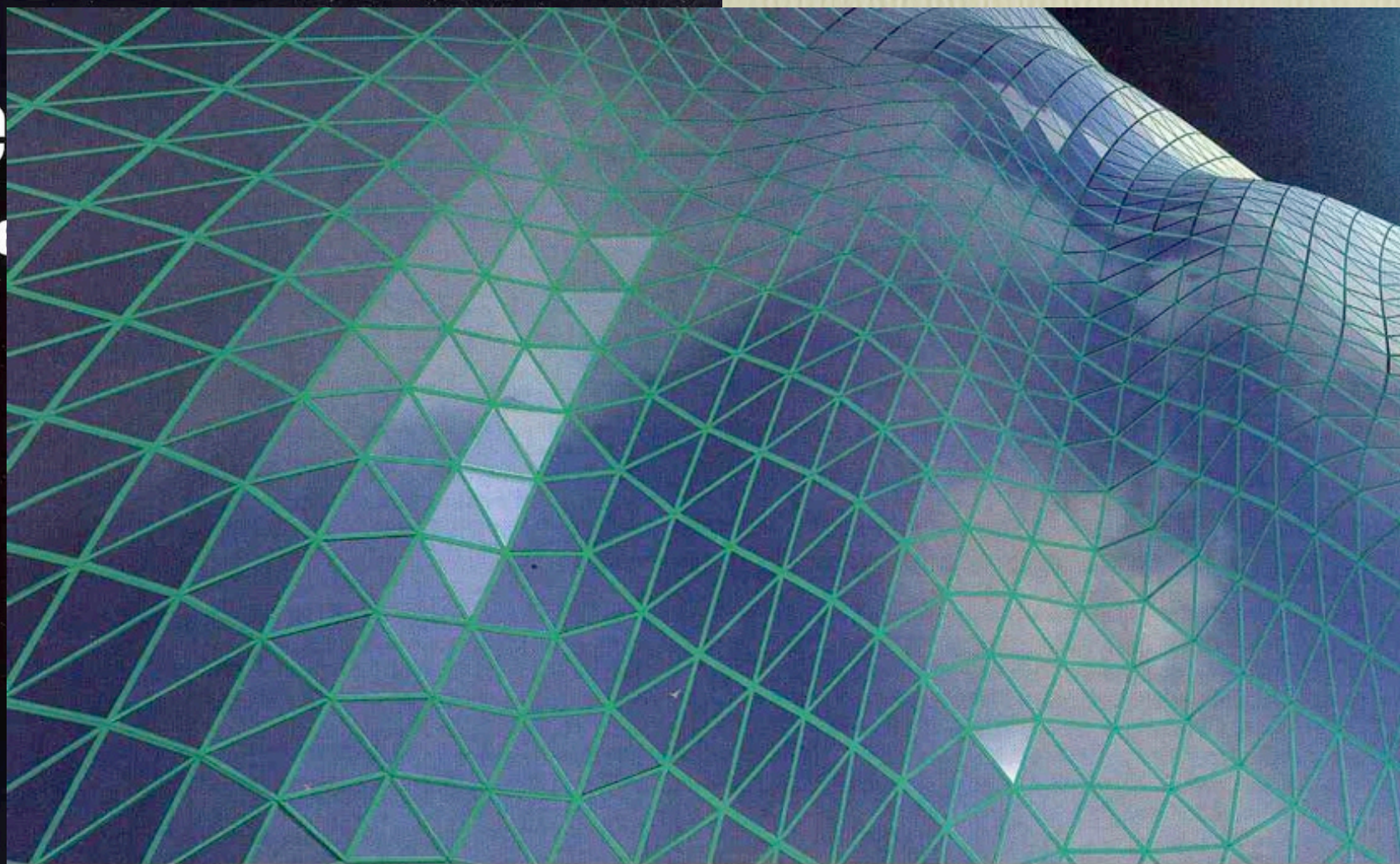
Le ve Des algu

- L'Univers quantique auto-organisé
- Que s'est-il passé à Tougouska il y a 100 ans ?
- Comment détecter les images truquées
- D'où viennent les larves ?

M 02687 - 371 - F: 5,95 €

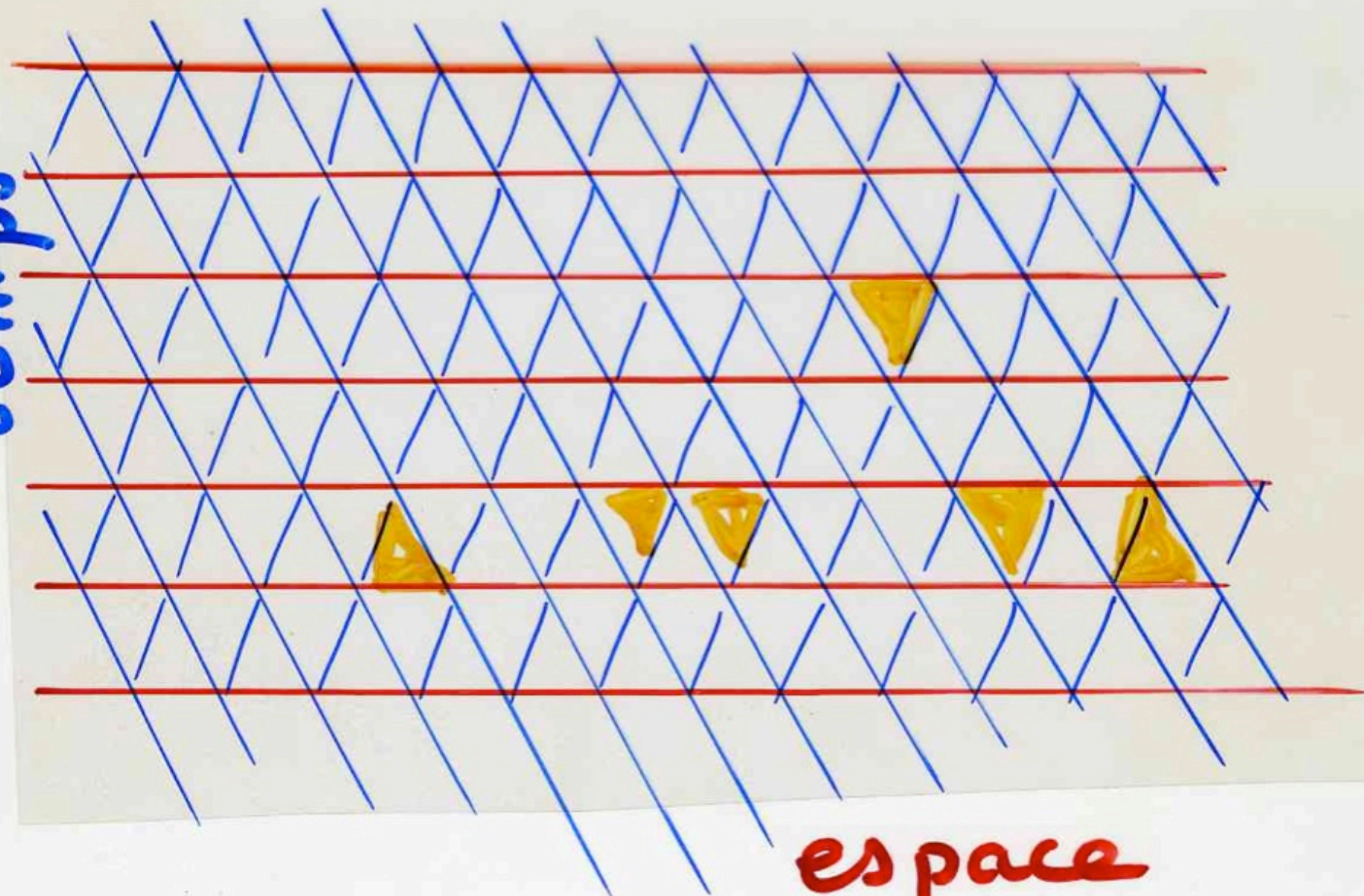


And : 5,95 € - Bel/Lux : 7,10 € - Ita/Port Cont. : 7,10 € - D : 8,90 €
DOM surf : 7,15 € - DOM avion : 8,95 € - CH : 11,80 FS -
CAN : 8,95 \$ CAN - TOM surf : 940 F CFP - TOM avion : 1700 F CFP
- MAY : 8,95 € - MAR : 55 DH.

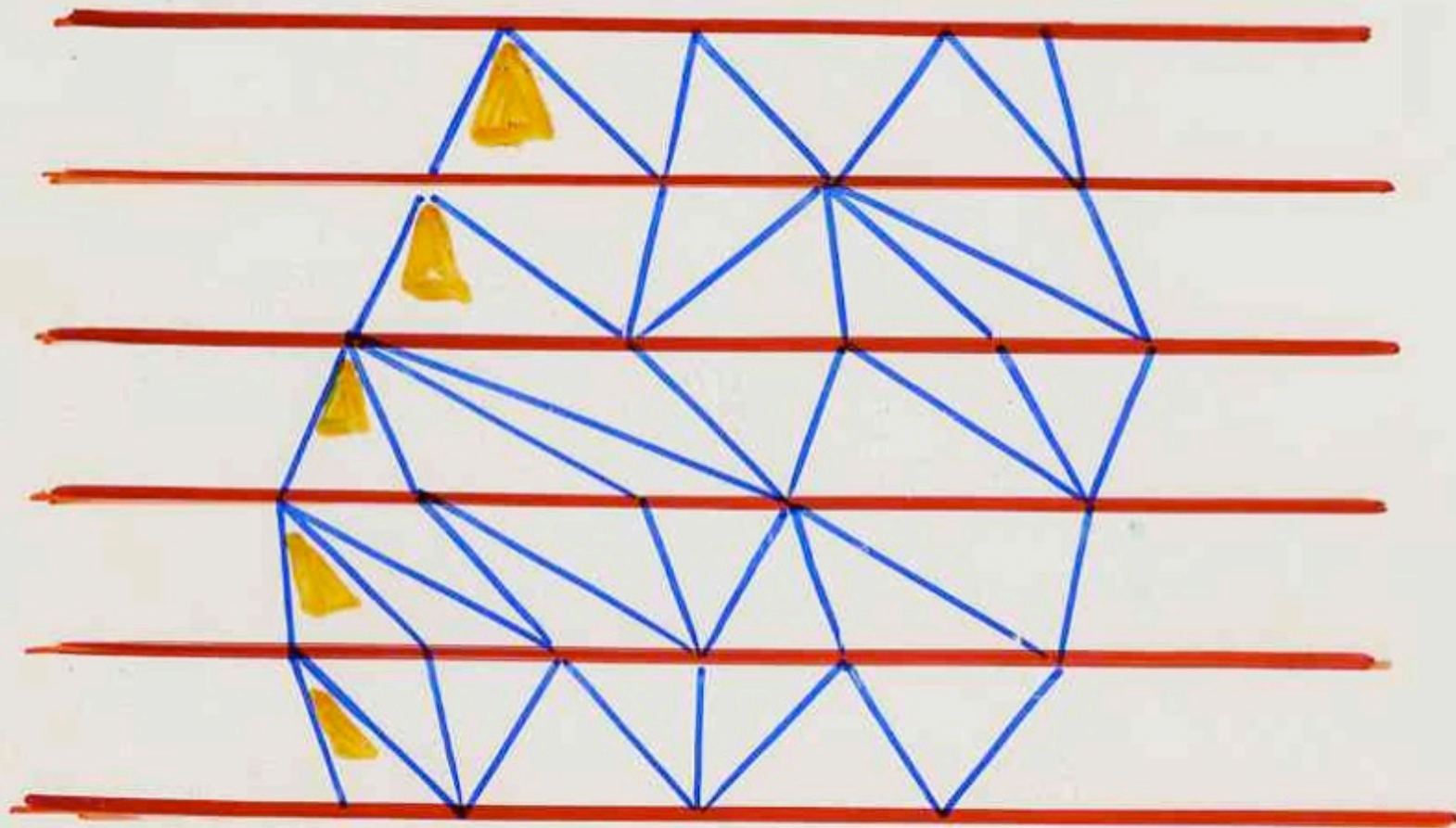


L'univers quantique auto-organisé

temps



espace



triangulation Lorentzienne

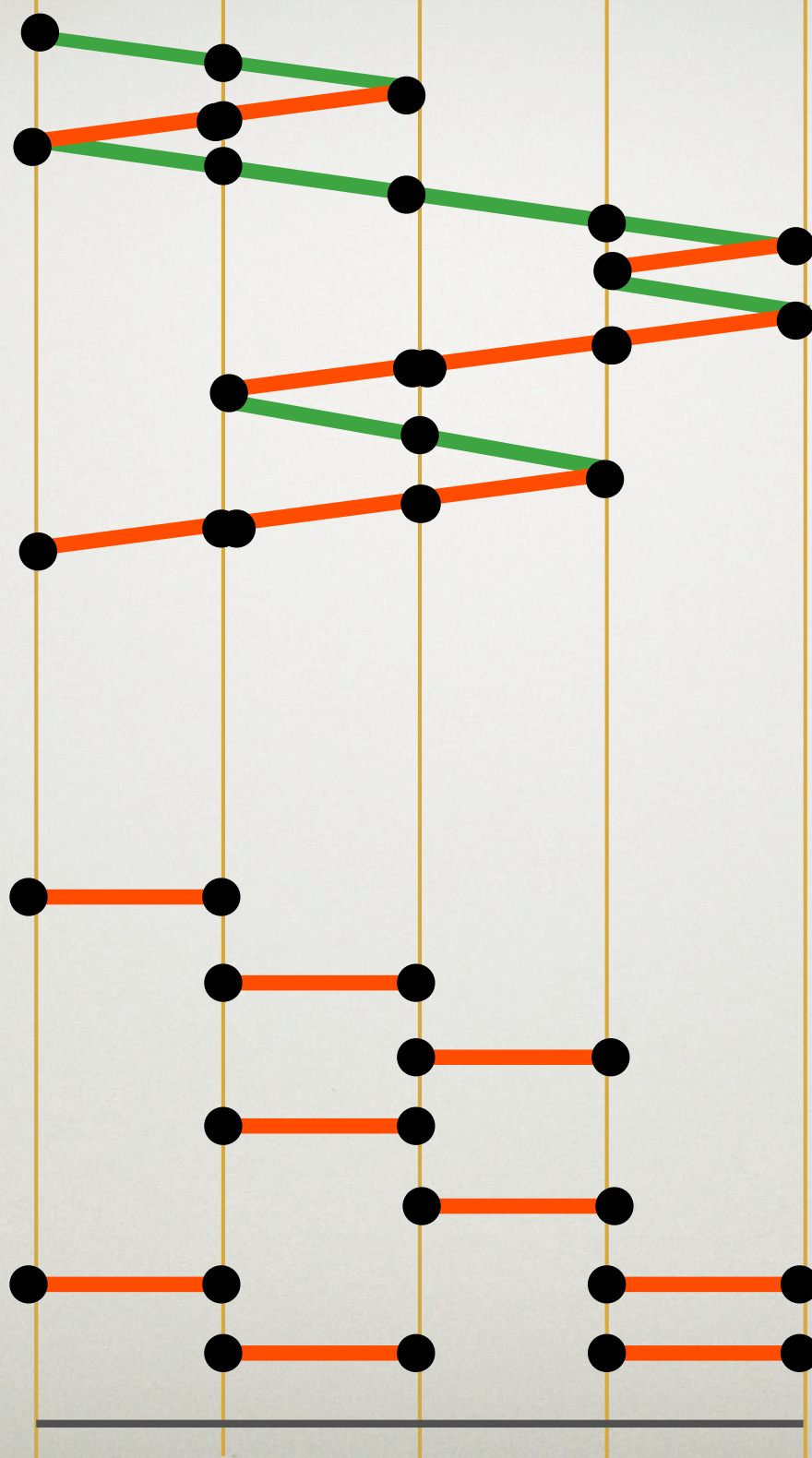
Catalan

number

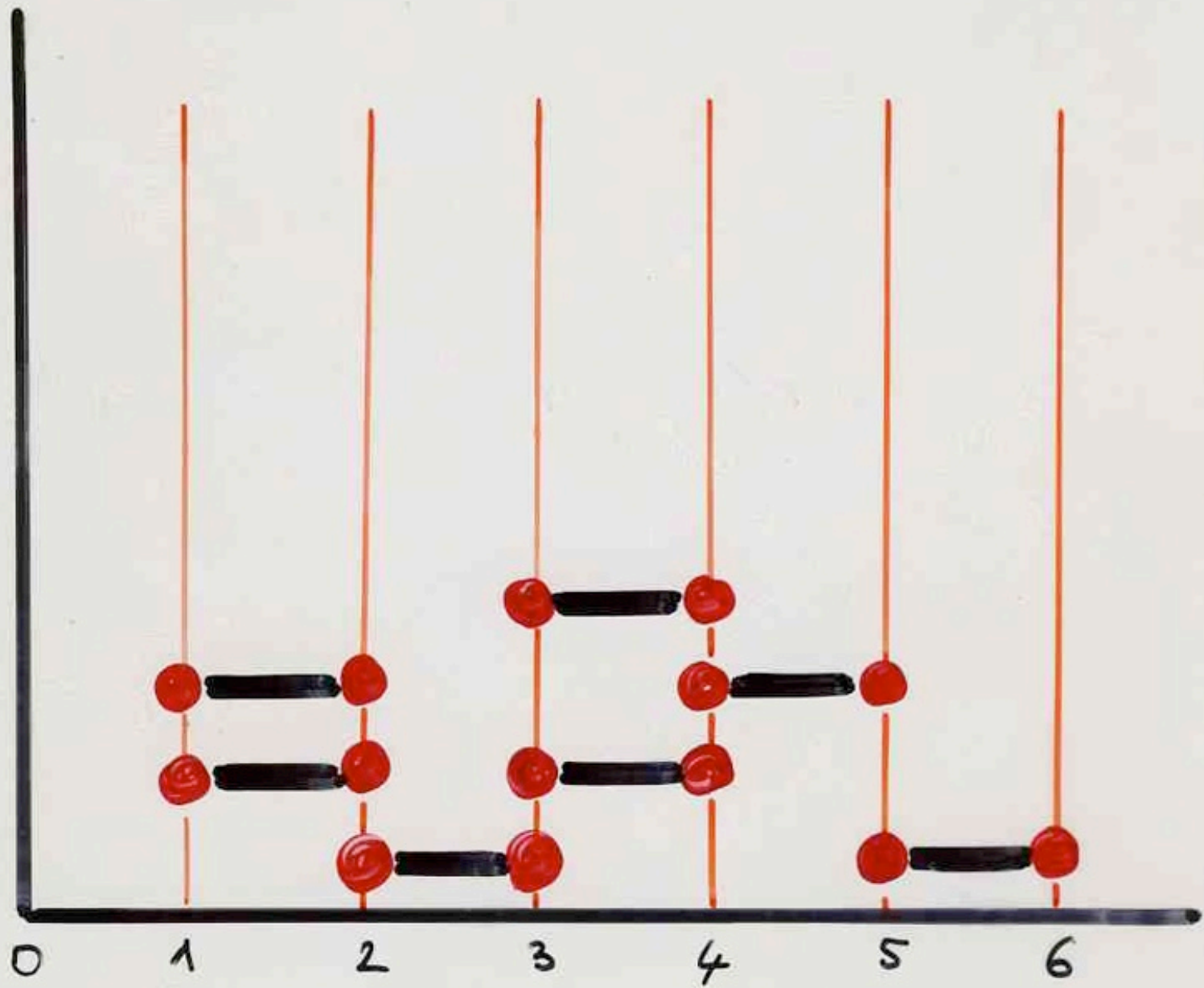


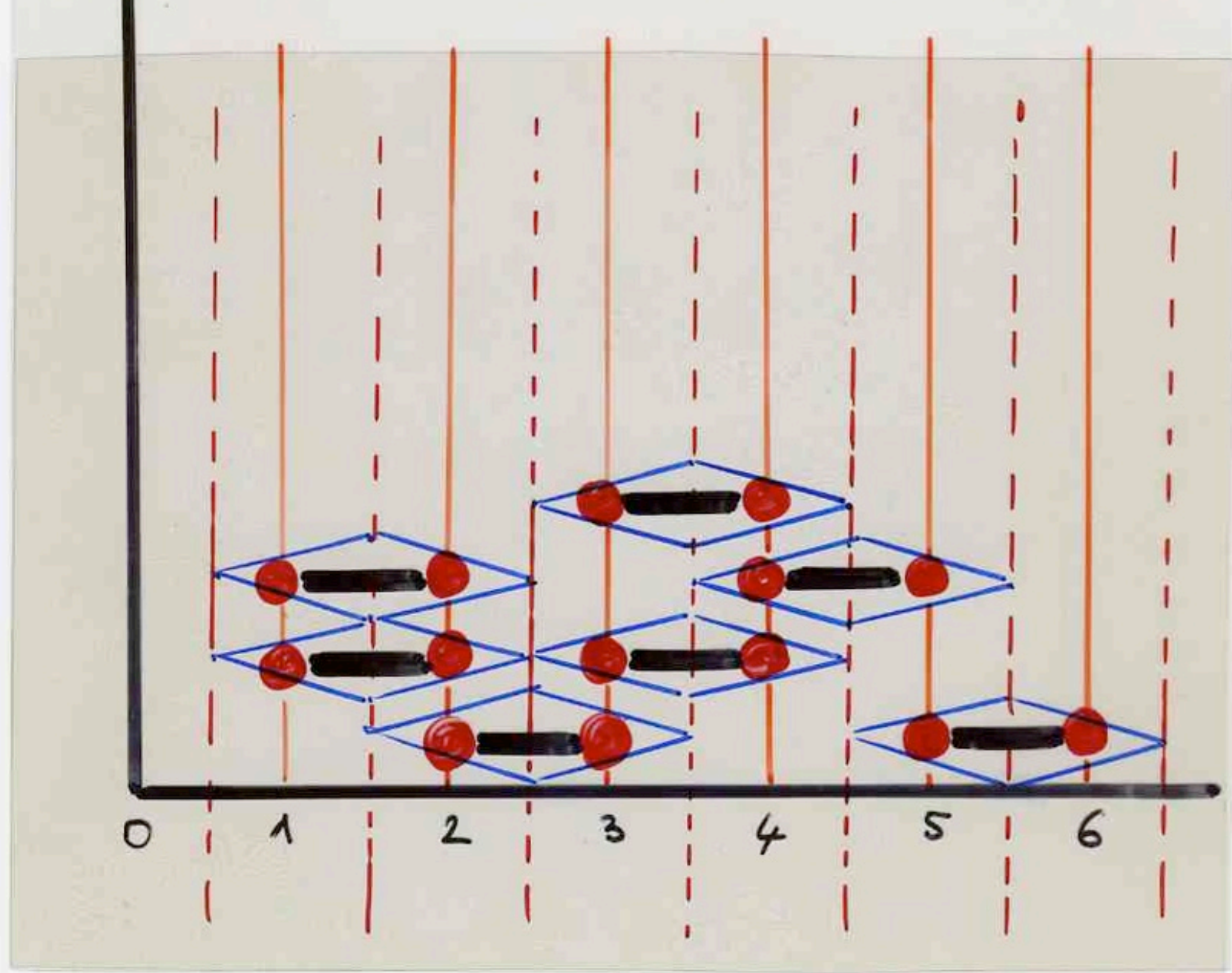
$$C_n = \frac{1}{(n+1)} \binom{2n}{n}$$

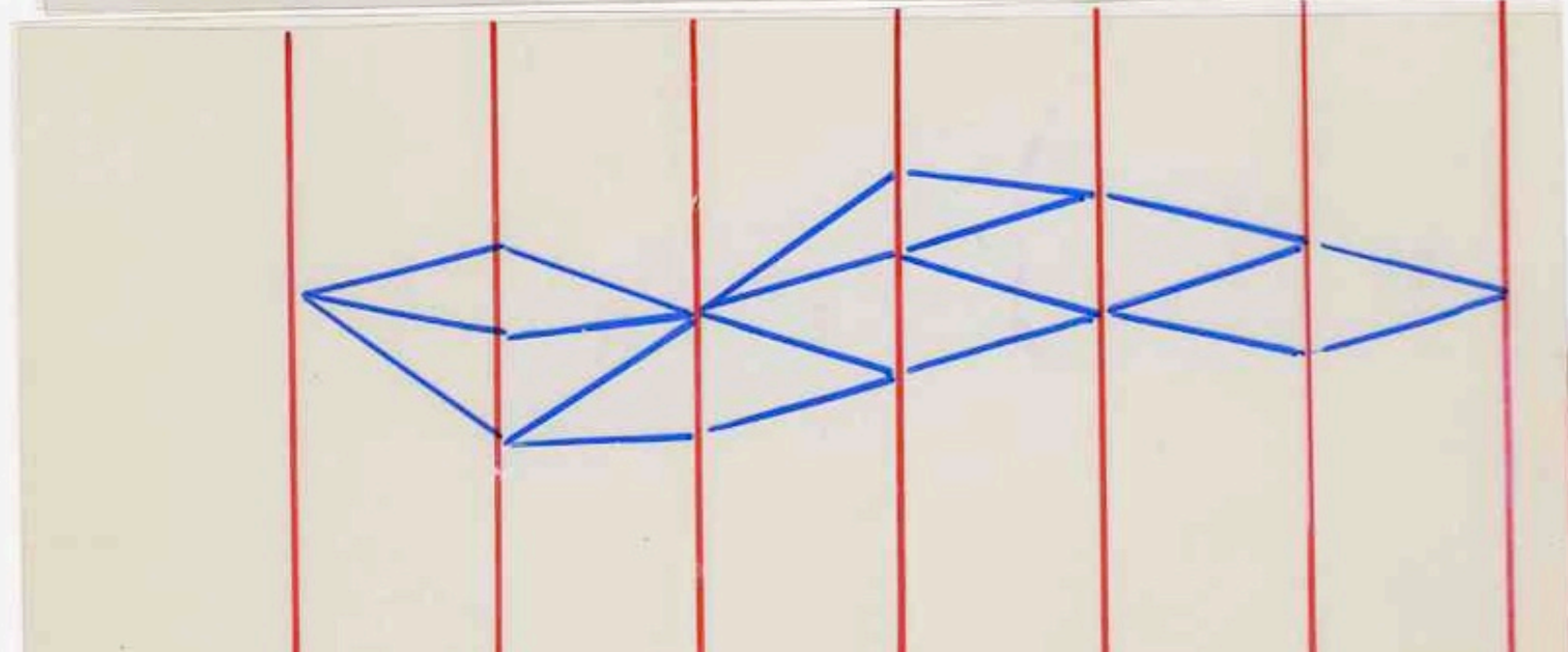
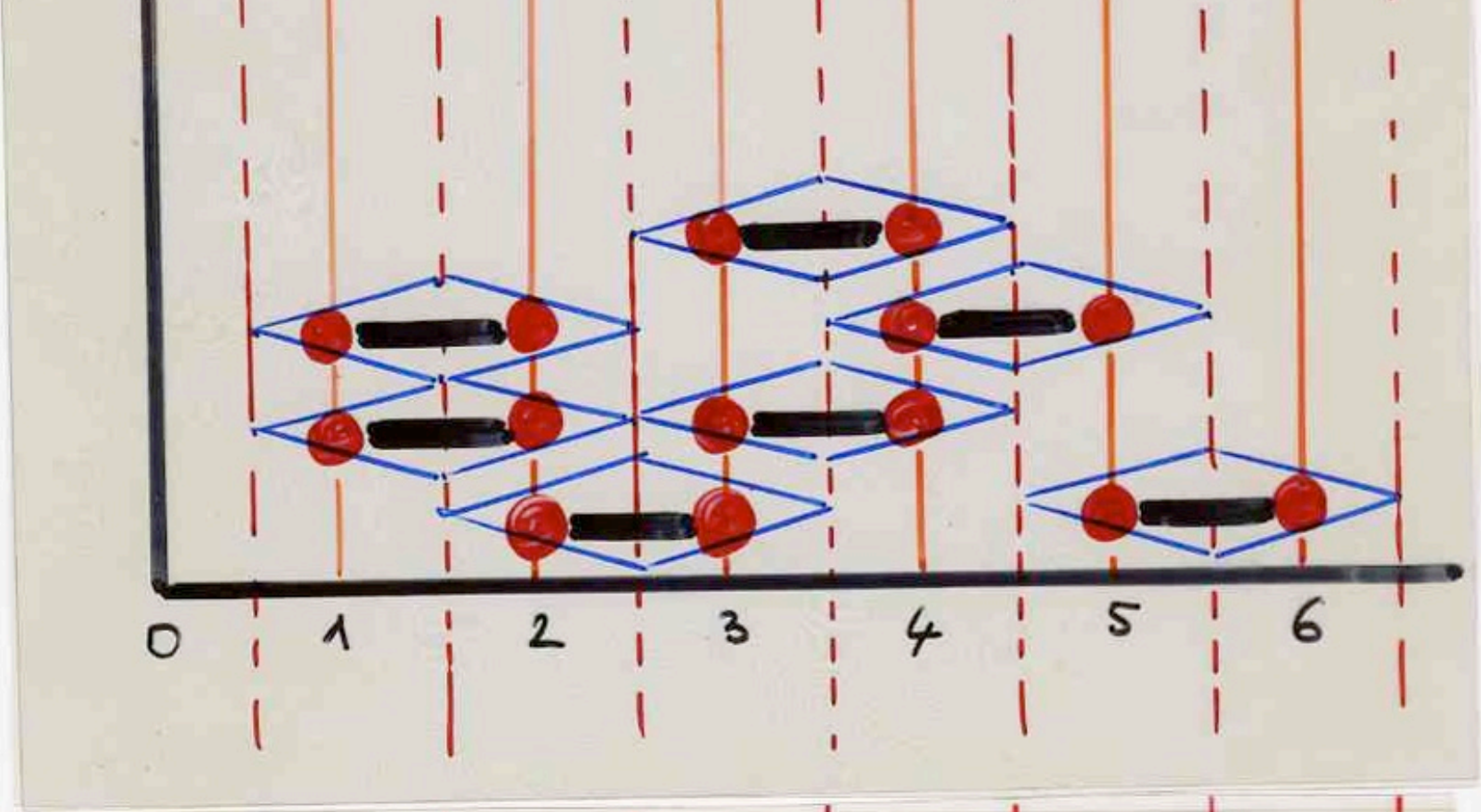
des chemins de Dyck
aux empilements de dominos

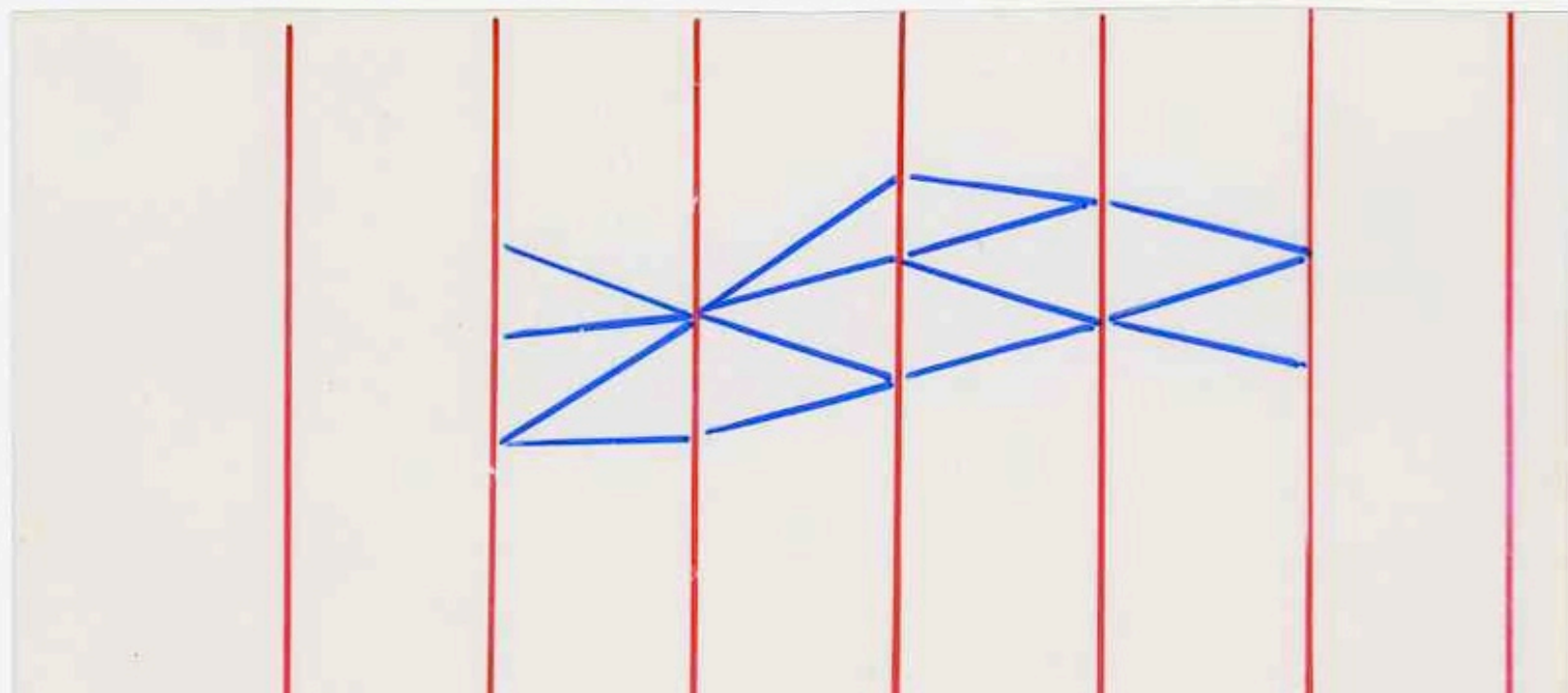
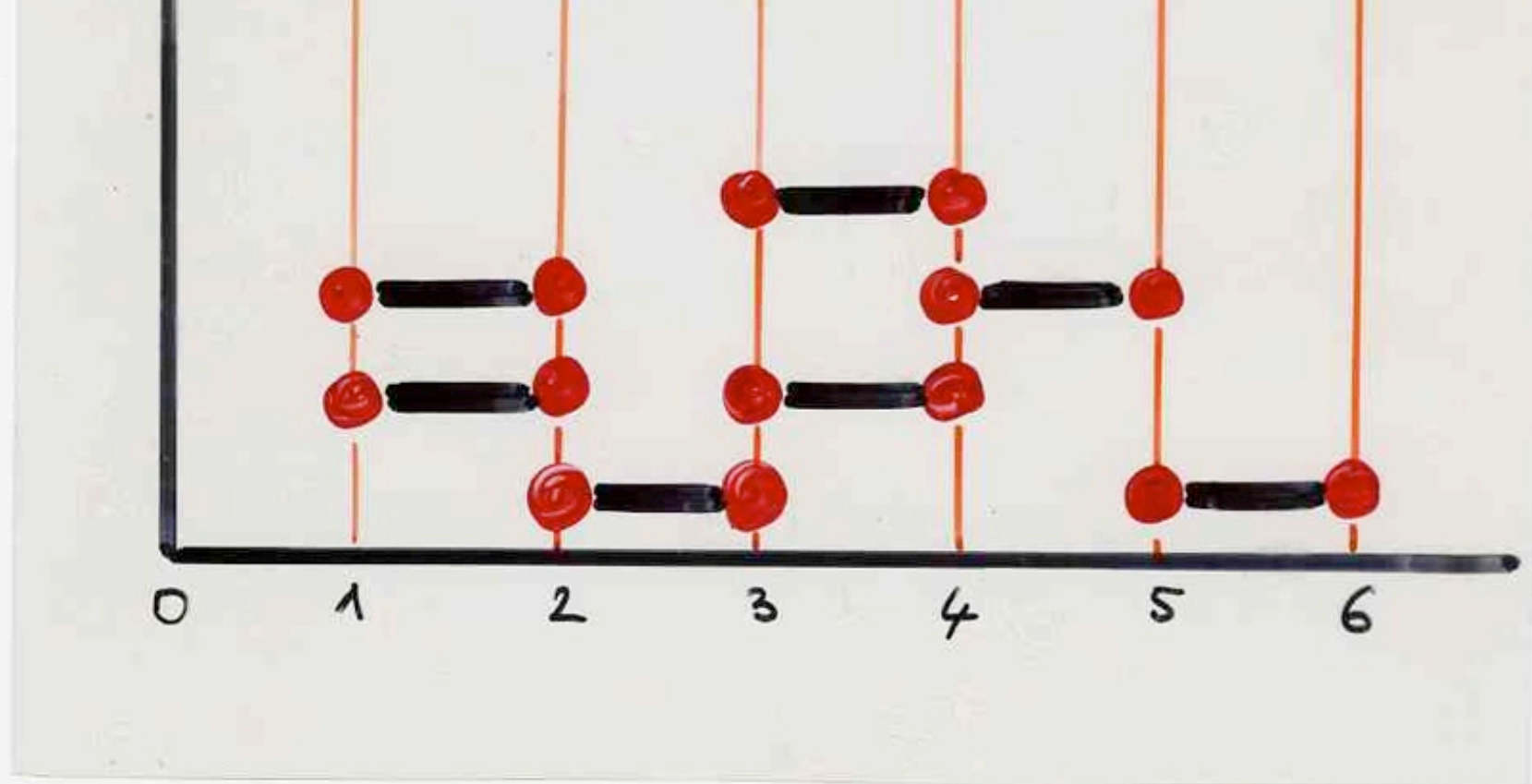


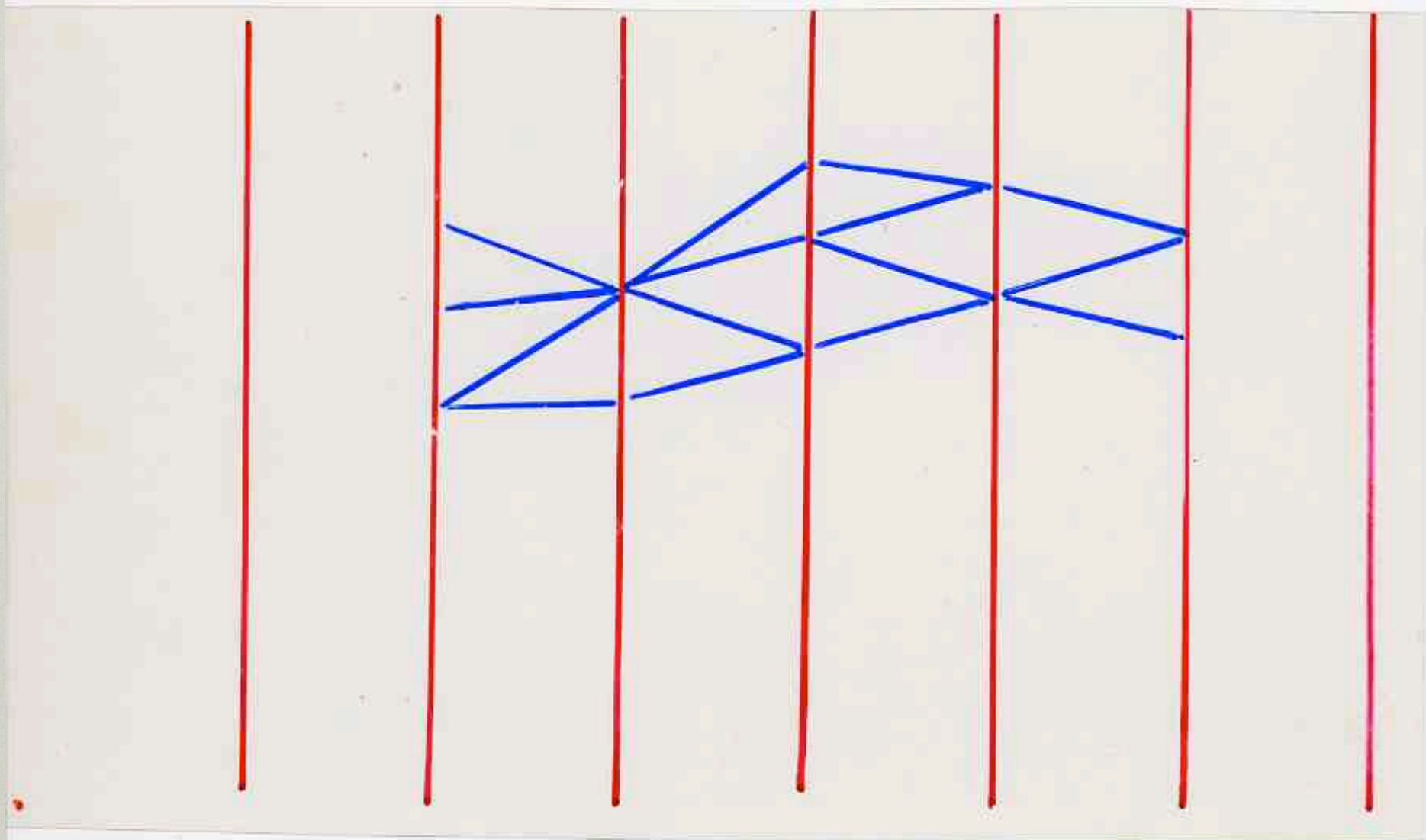
Des empilements de dominos
aux triangulations Lorentziennes

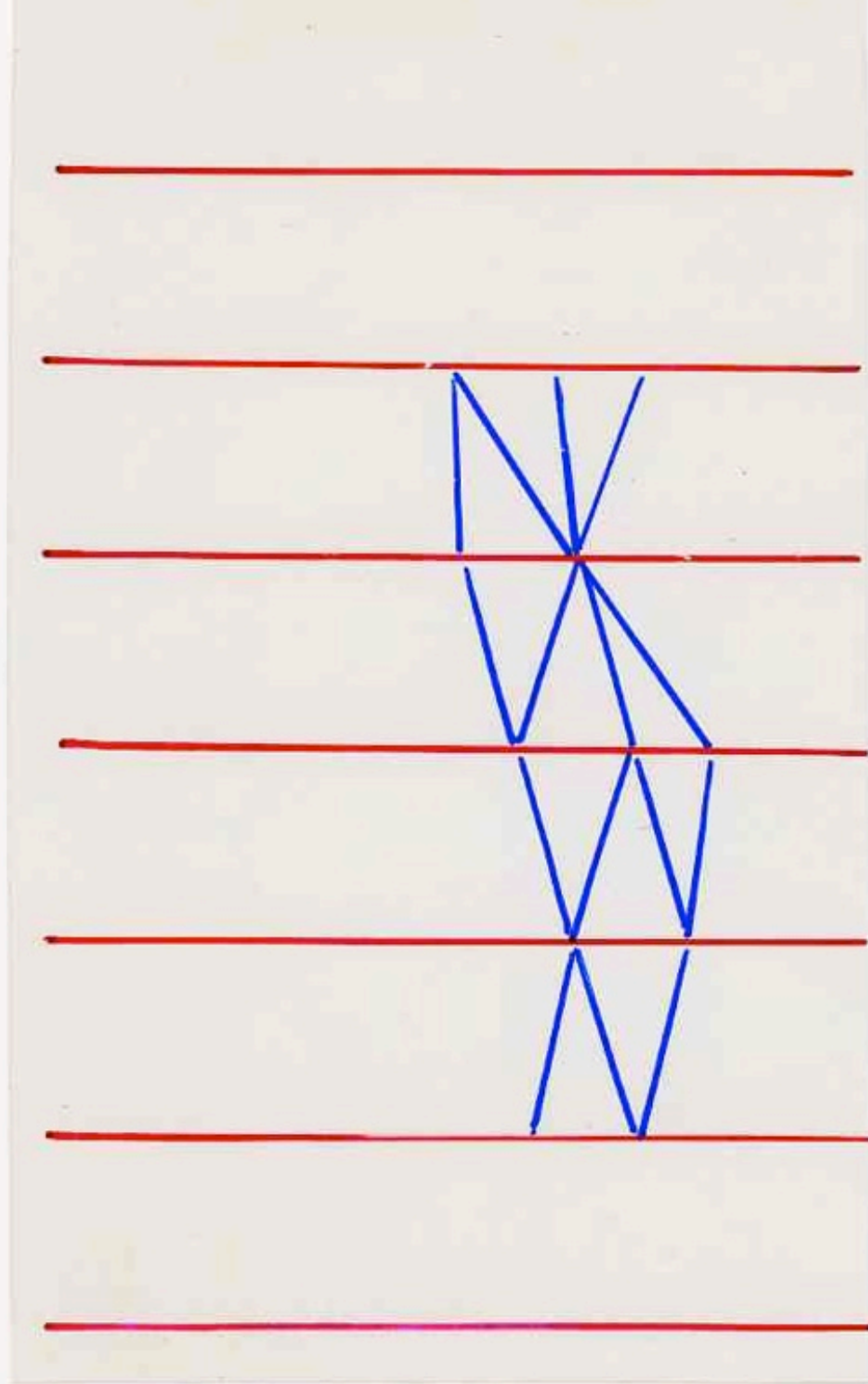




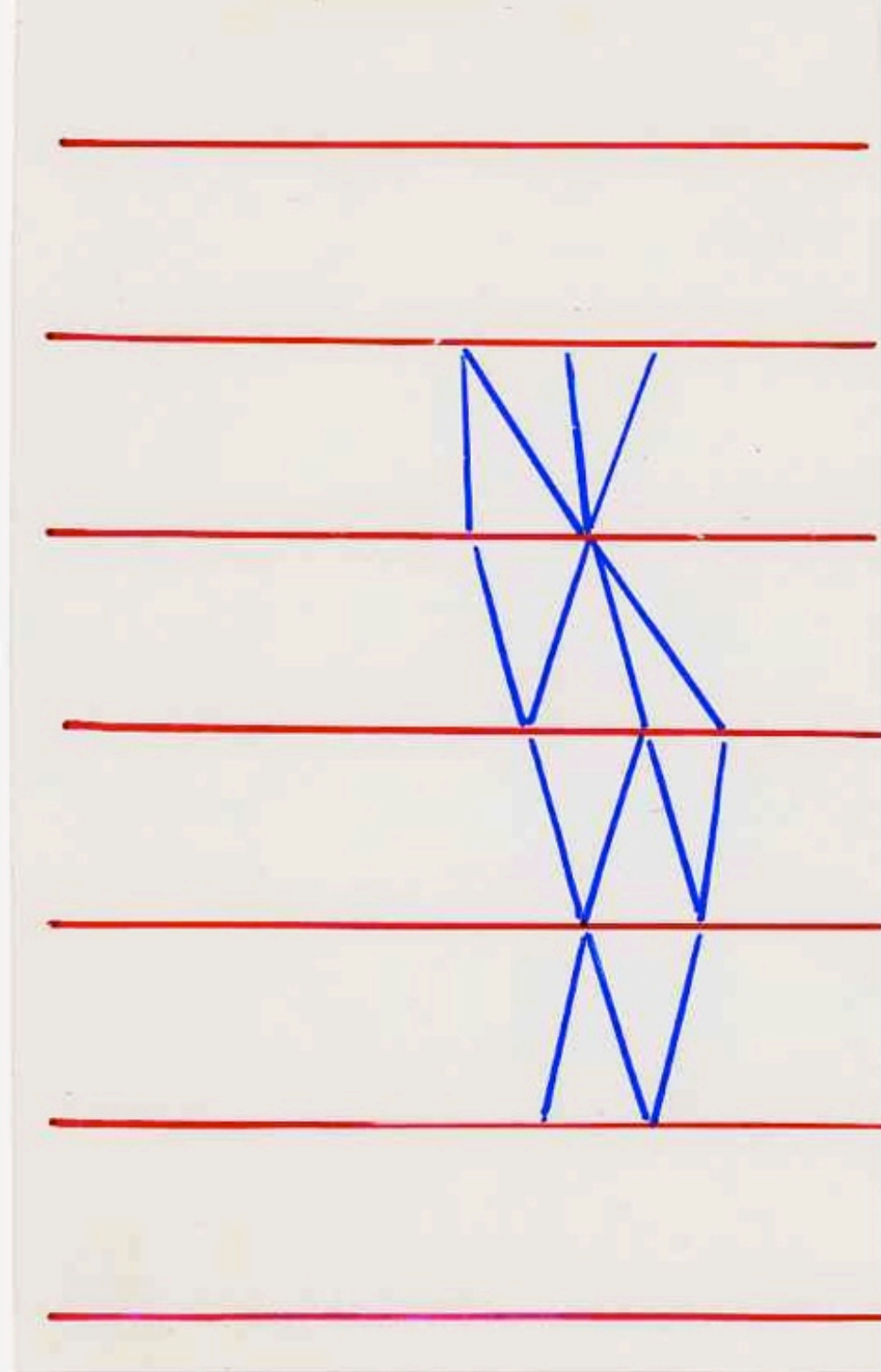


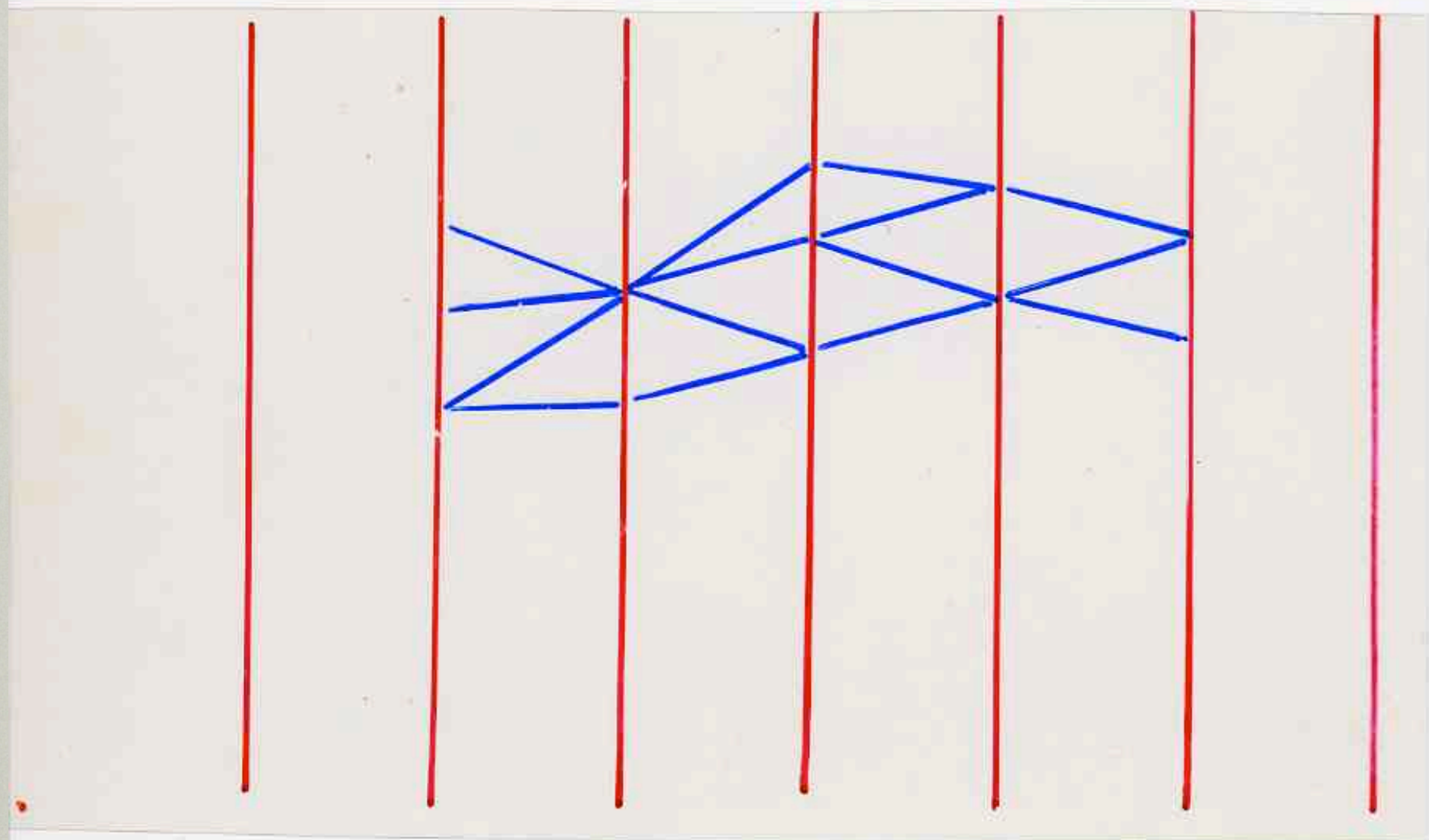


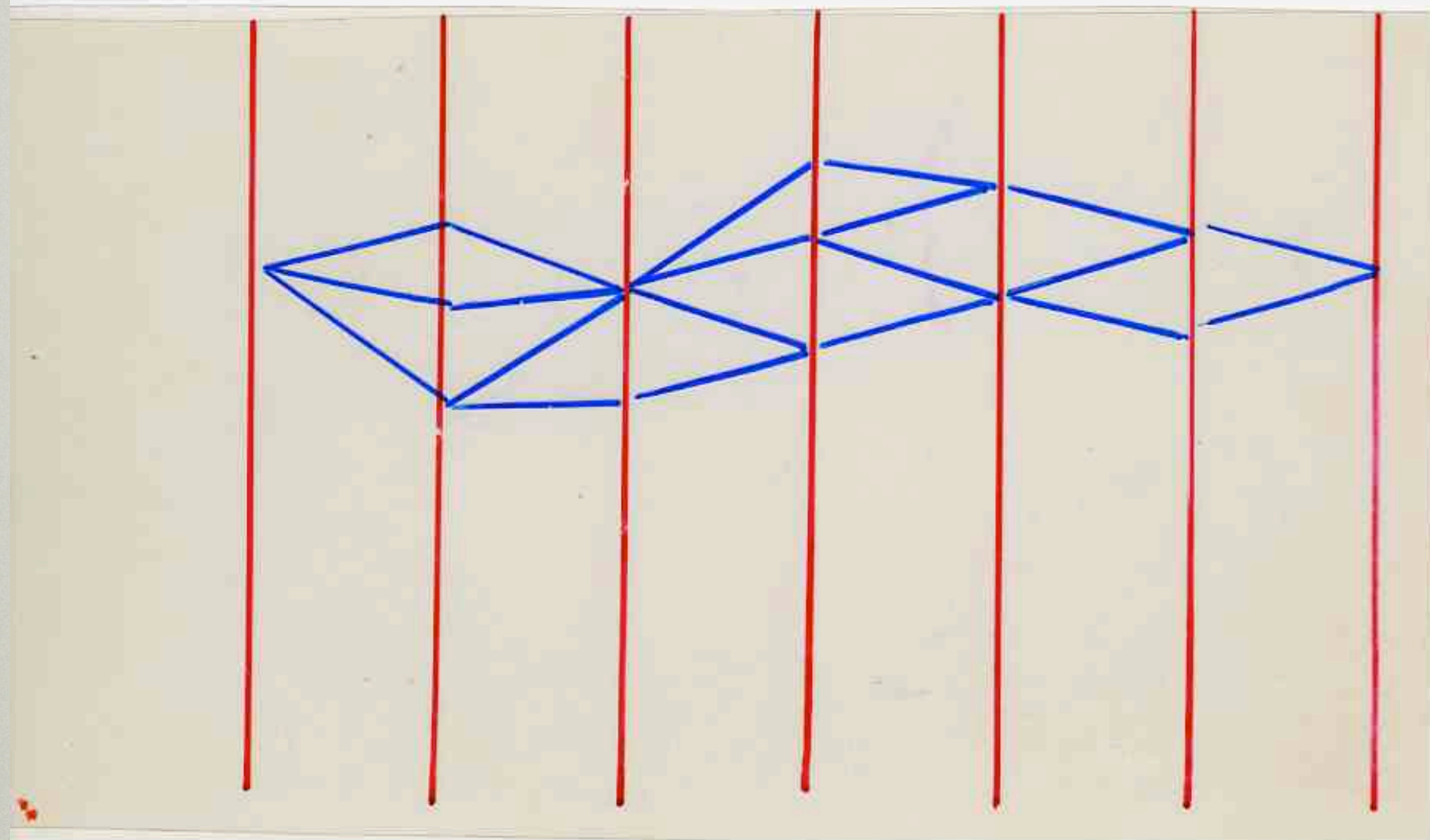


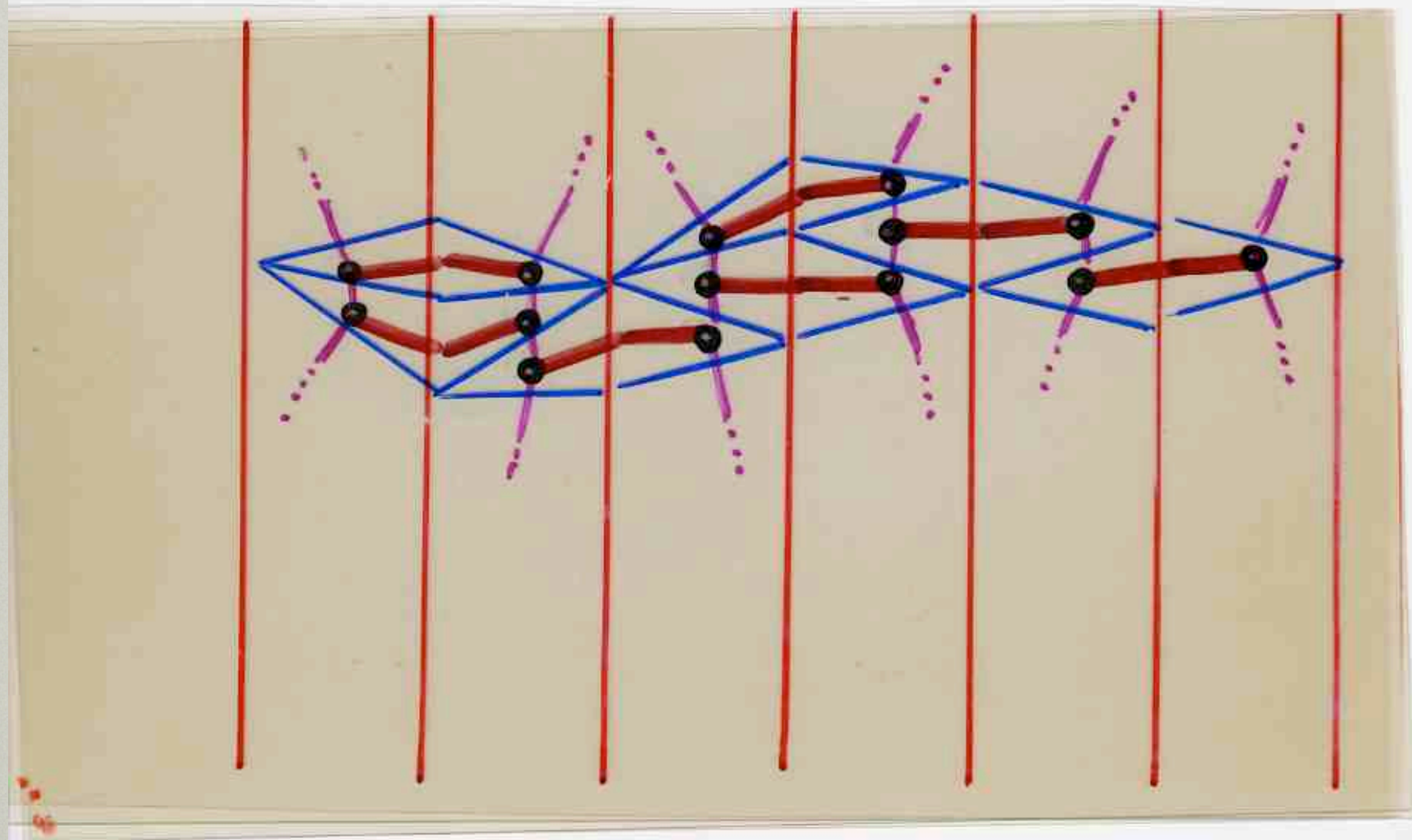


bijection réciproque

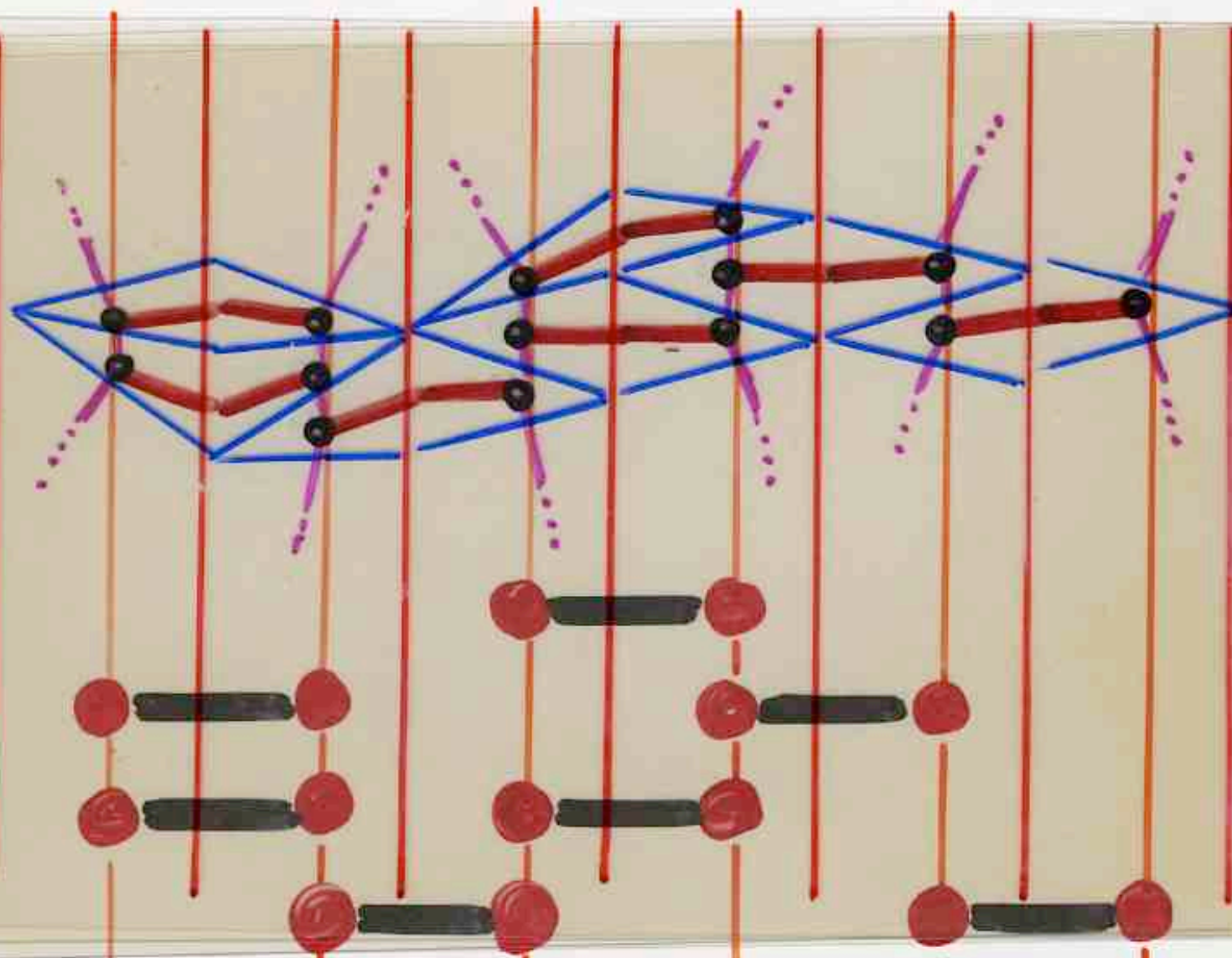


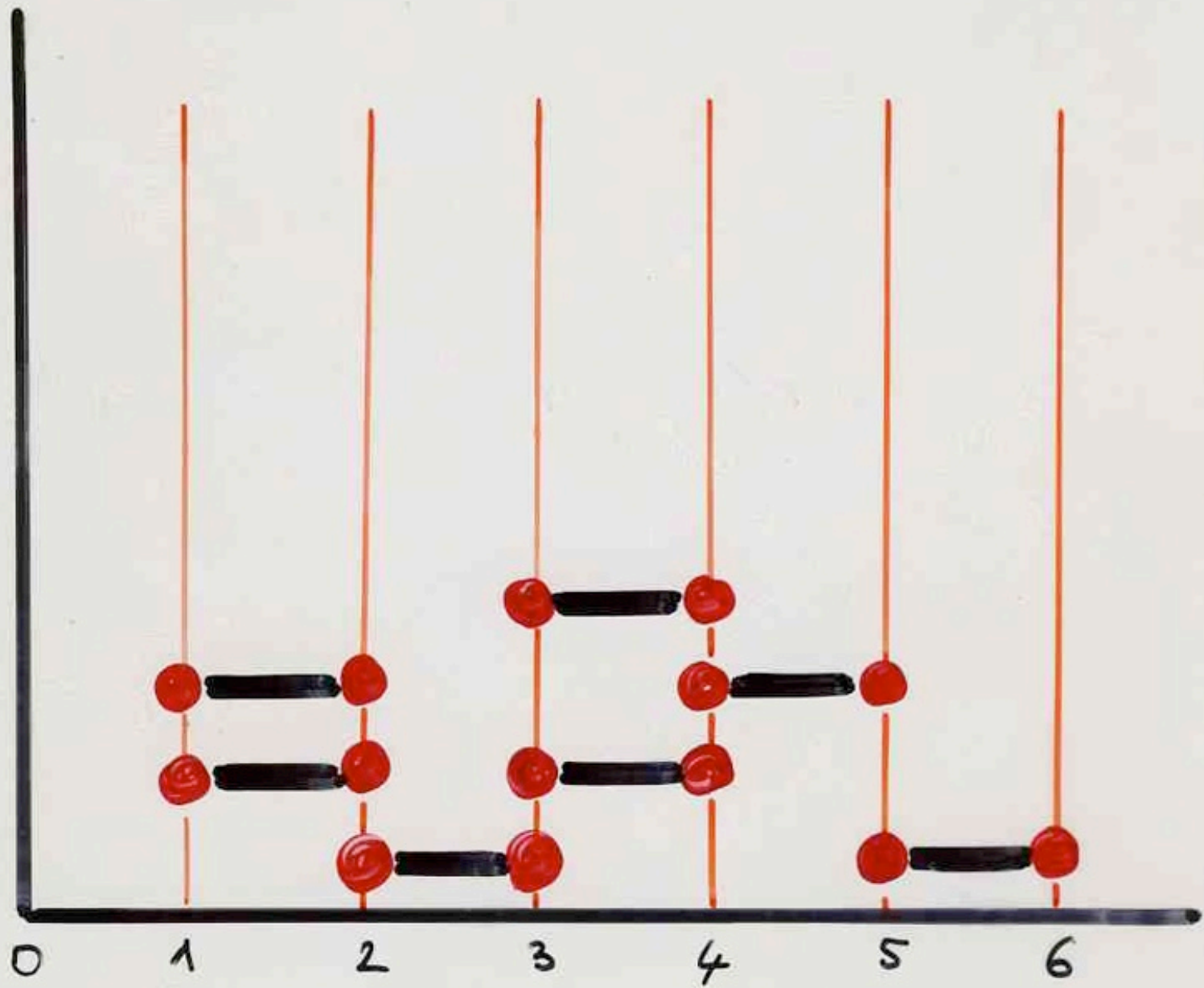






0 1 2 3 4 5 6





métamorphose:

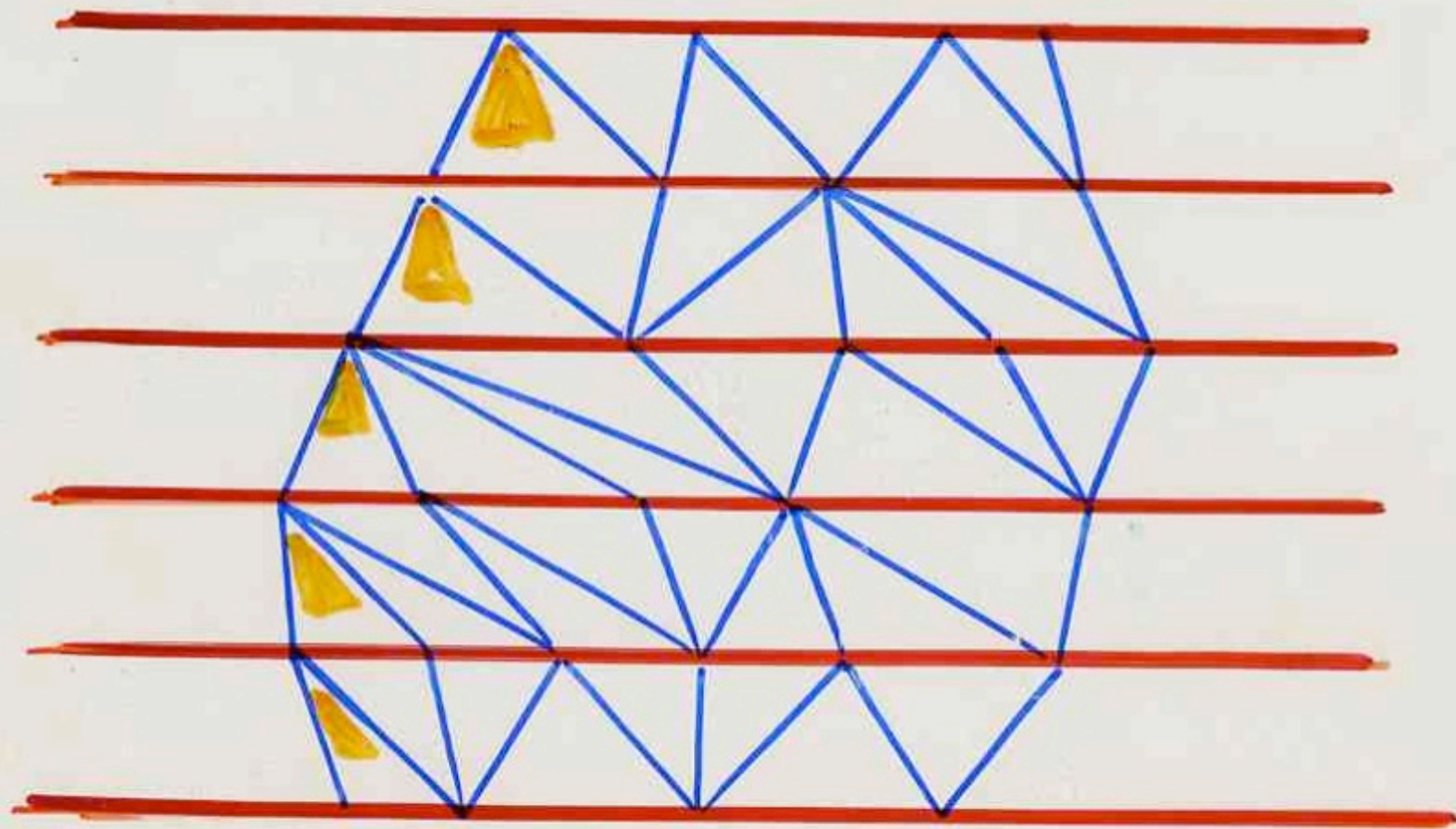
triangulation

arbres binaire

chemin de Dyck

pyramide de dominos

triangulation Lorentzienne



L'ESPACE-TEMPS

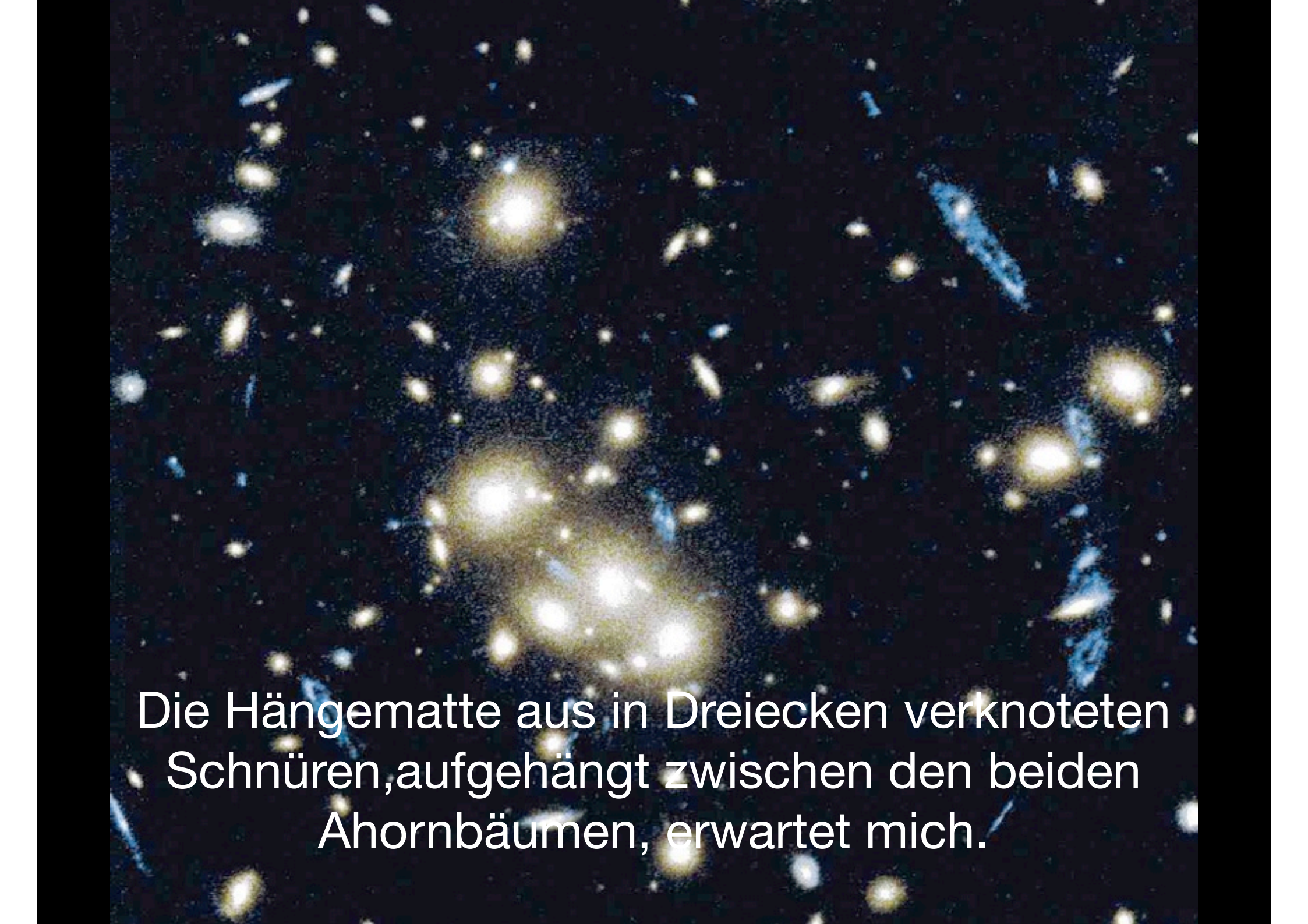
MARCIA PIG LAGOS

AUTEUR





Es ist Sommer, die Nacht ist klar und ruhig.



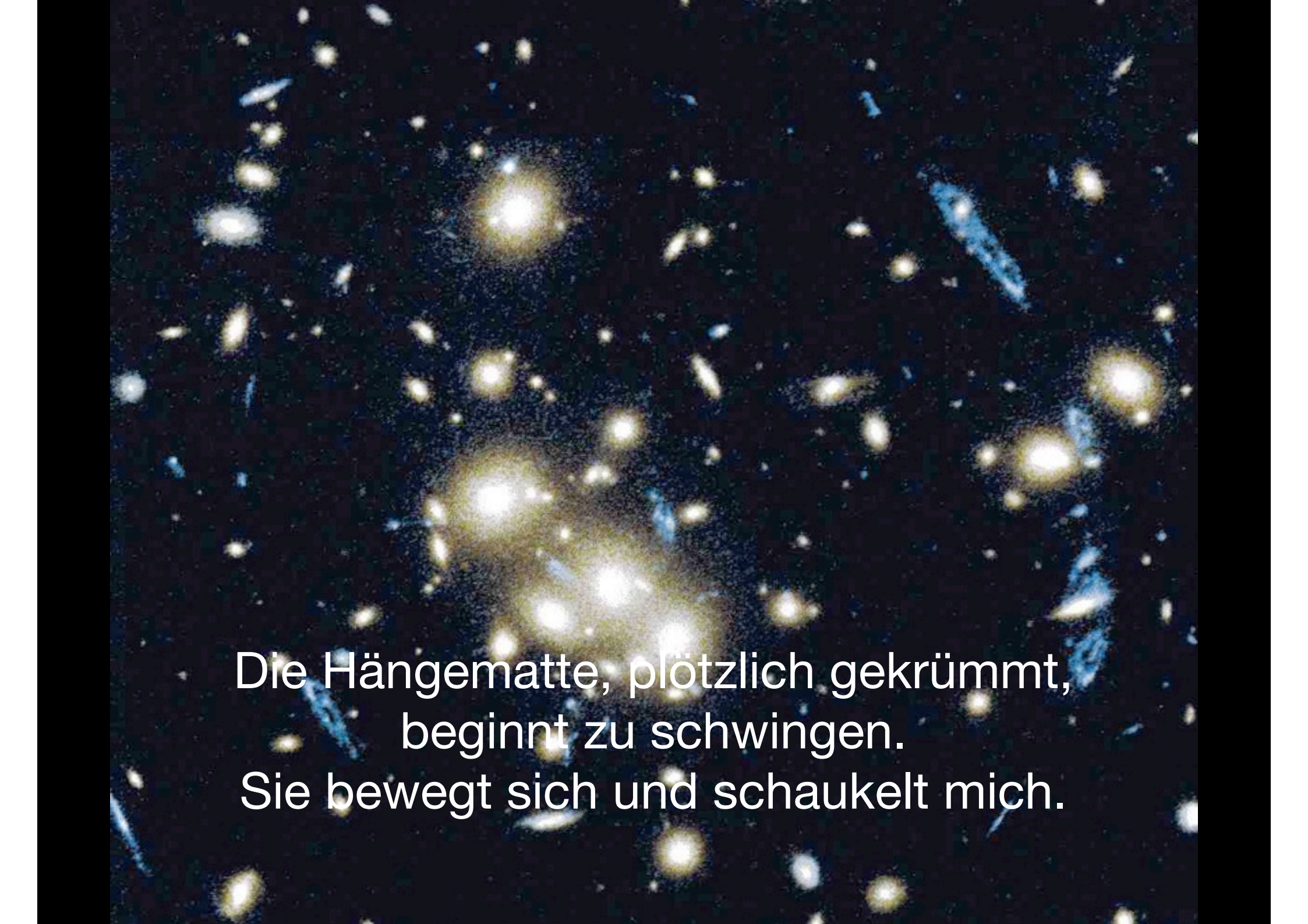
Die Hängematte aus in Dreiecken verknoteten
Schnüren, aufgehängt zwischen den beiden
Ahornbäumen, erwartet mich.



Ich lege mich hin in dem Raum,
den mir das Netz gewährt.



Das Gewicht meines Körpers drückt darauf,
verformt die Dreiecke.



Die Hängematte, plötzlich gekrümmt,
beginnt zu schwingen.
Sie bewegt sich und schaukelt mich.

A deep-field astronomical image showing a vast field of galaxies and stars against a black background. The image is filled with numerous celestial objects, including bright yellowish-white stars and a variety of galaxies. Some galaxies are large and diffuse, while others are small and compact. There are also several blueish-white objects, possibly distant galaxies or star-forming regions. The overall scene is a dense collection of cosmic bodies.

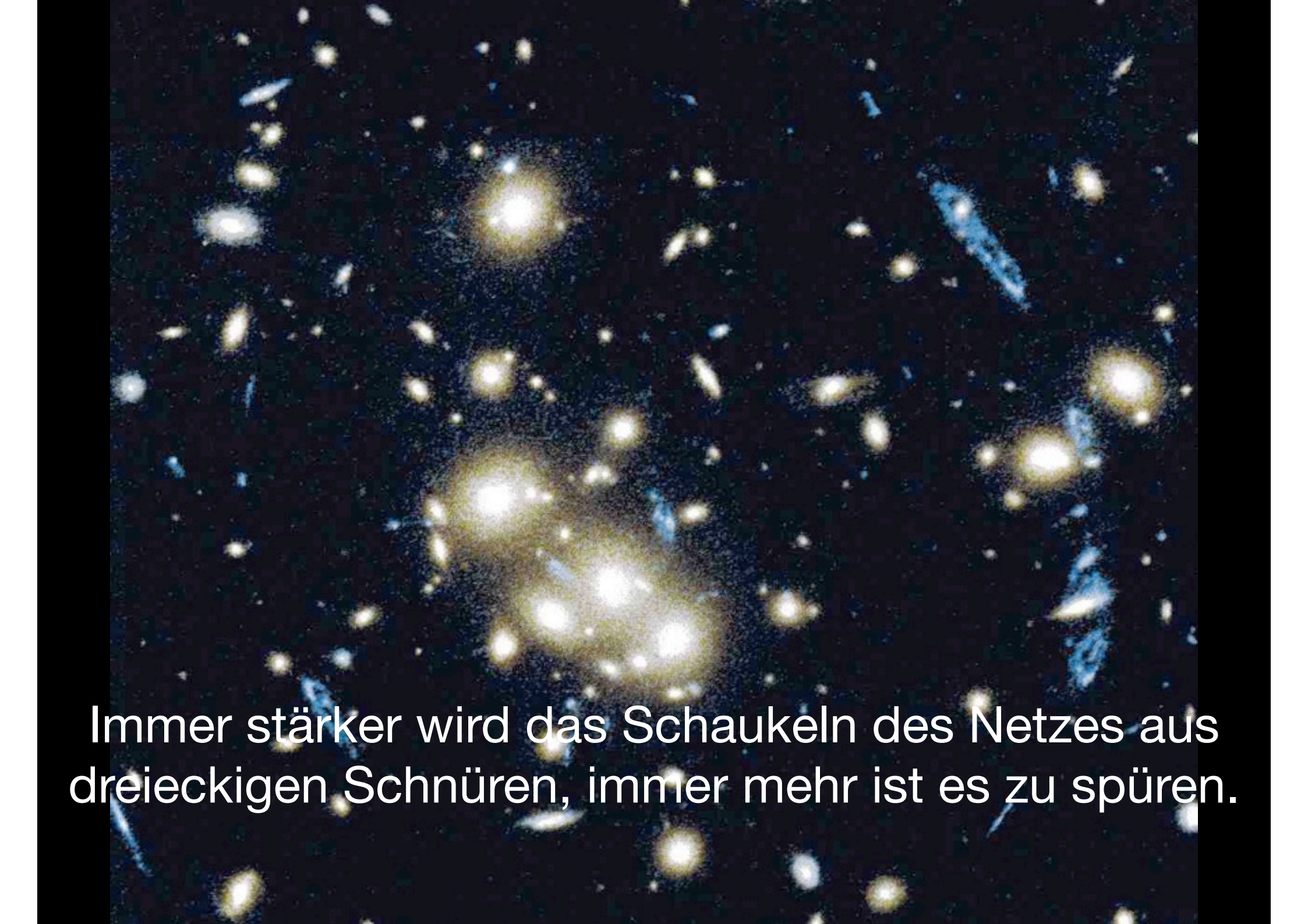
In meinem Blick fangen sich Hummel und Sterne.



Ich schaue hindurch, um mich im Raum zu verlieren.



Ist er blau? Ist er schwarz?
Bewegt er sich in der Zeit?



Immer stärker wird das Schaukeln des Netzes aus dreieckigen Schnüren, immer mehr ist es zu spüren.



Mein Körper beginnt, mitzuschaukeln.
Jedes Organ, jede Zelle.



Das Gefühl, hier zu sein und gleichzeitig fort.

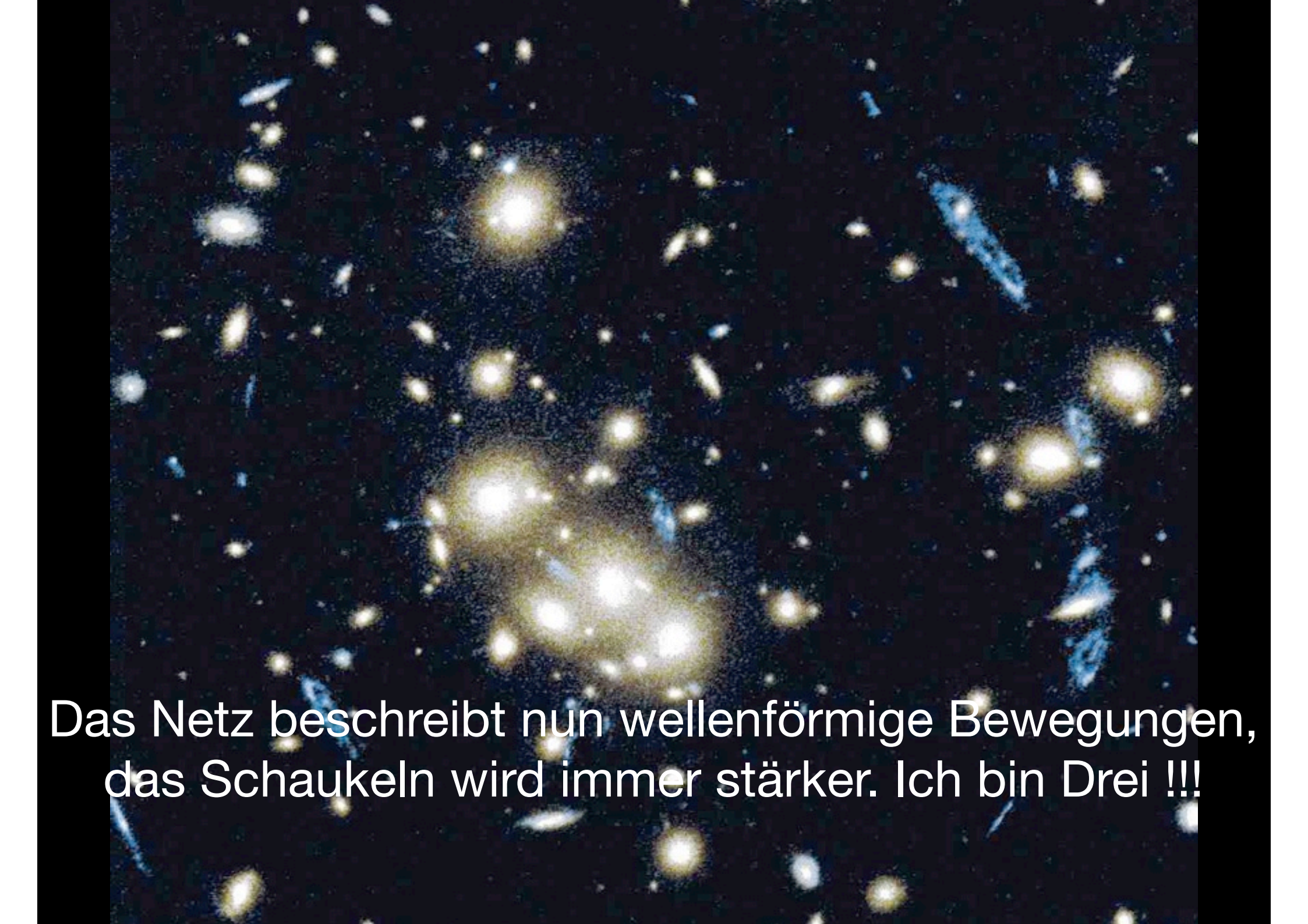




Plötzlich finde ich mich mit meinen vier
Dimensionen in der unendlichen Raumzeit wieder.



Ich erkenne eine gekrümmte Linie und möchte
am liebsten darüber springen.



Das Netz beschreibt nun wellenförmige Bewegungen,
das Schaukeln wird immer stärker. Ich bin Drei !!!

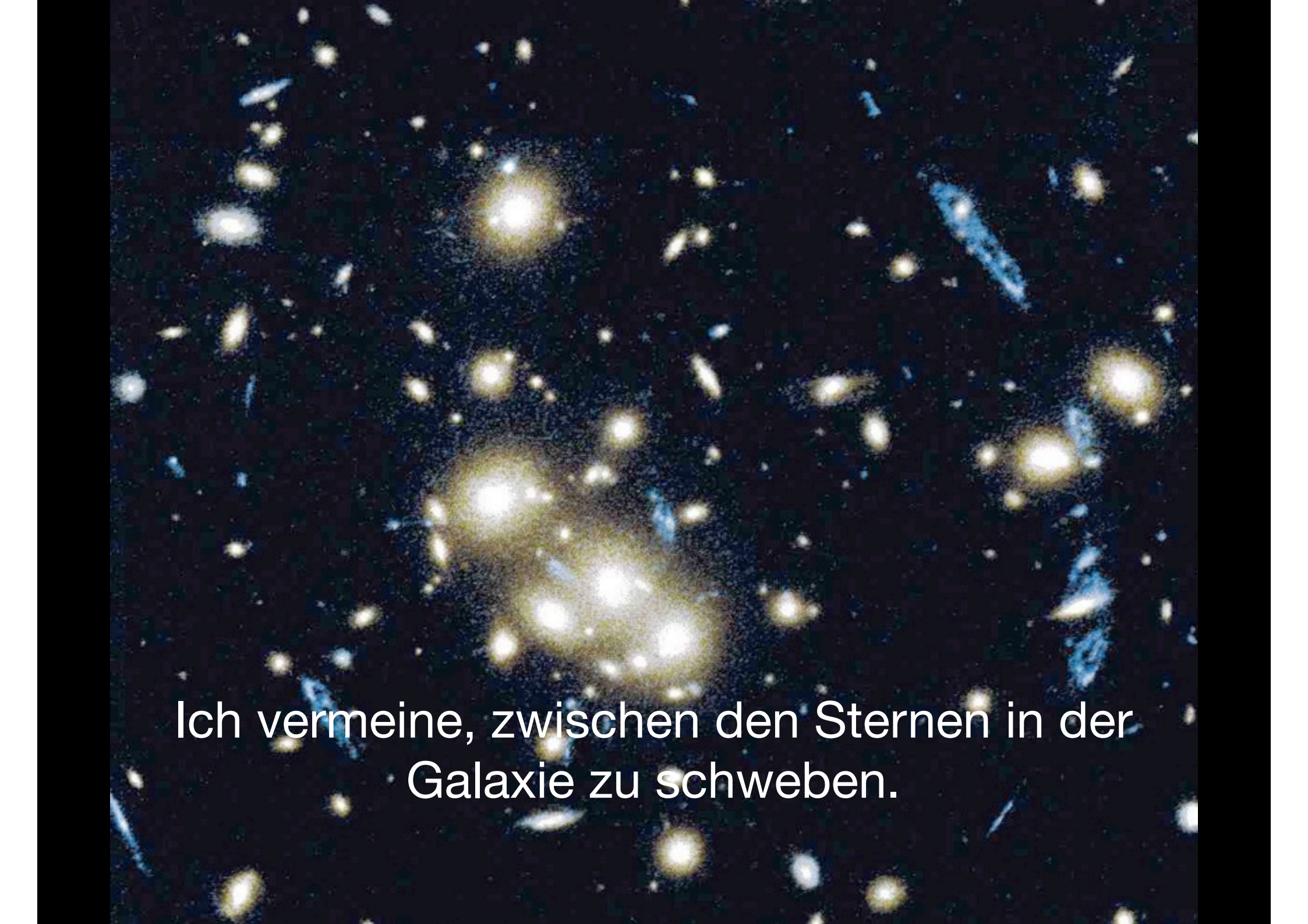


Ich frage mich: Wo ist die Zeit?
Verbirgt sie sich im Raum?



Sie sind zwei. Auf in den himmlischen Schaum...





Ich vermeine, zwischen den Sternen in der
Galaxie zu schweben.



Ich bin überall gleichzeitig. In Ausdehnung
und in völliger Einheit mit dem Universum.



Das, was vorher mein Körper war, besteht nun
aus kleinen Teilchen, die stetig schaukeln.



Lichtpunkte heben sich ab, hier und dort.



Sie verschwinden jedoch gleich wieder,
vom Vakuum aufgesogen.



Alles ist nämlich da: die Materie, das Licht, der
Schaum, das Vakuum, die Raumzeit – alles ist Eins.



Den vibrierenden Atem des Universums spüren.



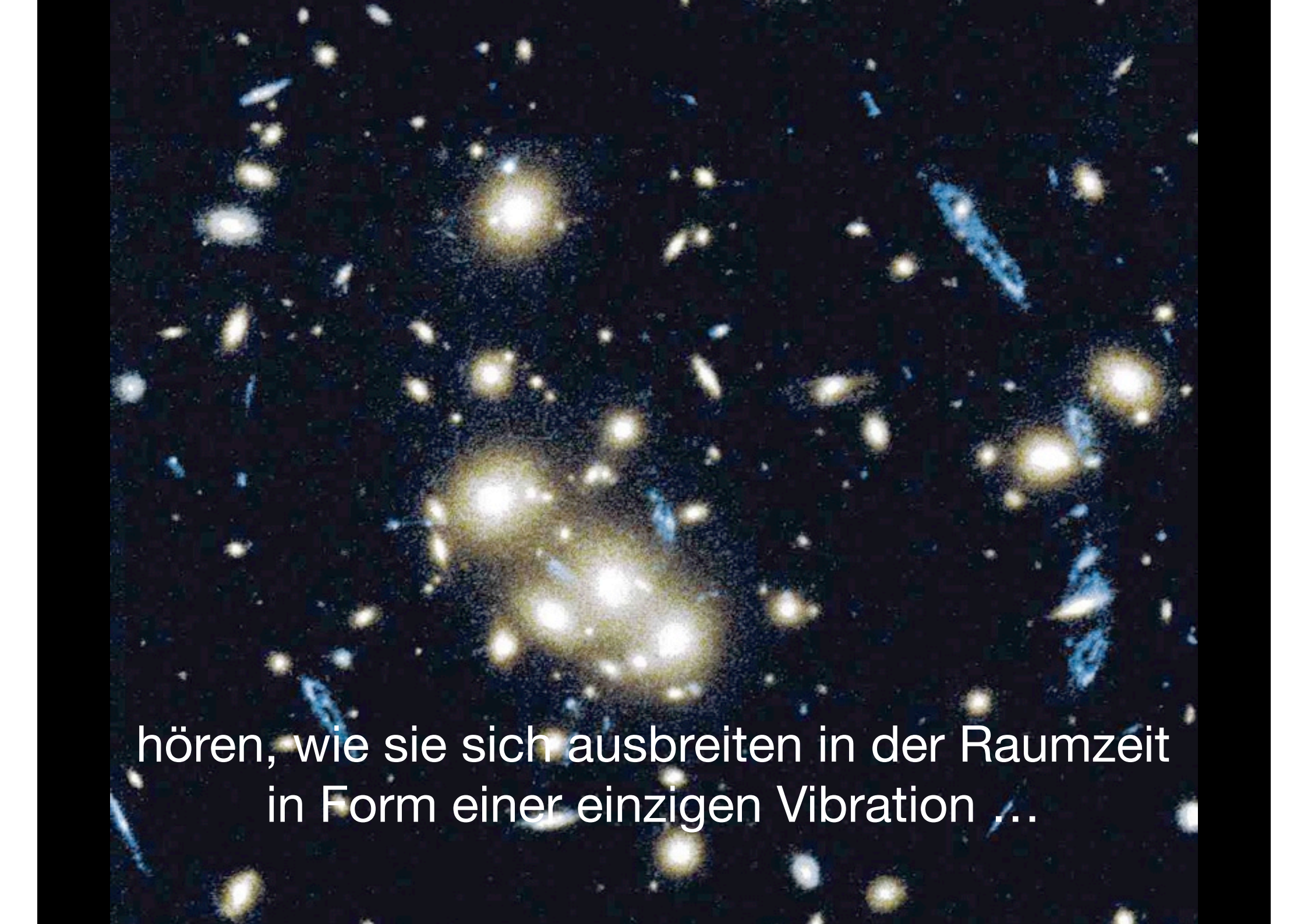
Bei der großartigen Geburt
der Baby-Universen dabei sein.

A deep space photograph showing a vast field of galaxies. The galaxies are primarily yellow and white, indicating older stellar populations, with some blue-tinted galaxies scattered throughout. They are distributed across a black background, representing the cosmic microwave background. The galaxies vary in size, shape, and brightness, some appearing as bright, diffuse clouds and others as more compact, distant points of light.

Den Klang des Universums zu hören.



Diese harmonischen Wellen, erzeugt auf diesen
so feinen, so mächtigen Saiten...



hören, wie sie sich ausbreiten in der Raumzeit
in Form einer einzigen Vibration ...



Der Ur-Ton...





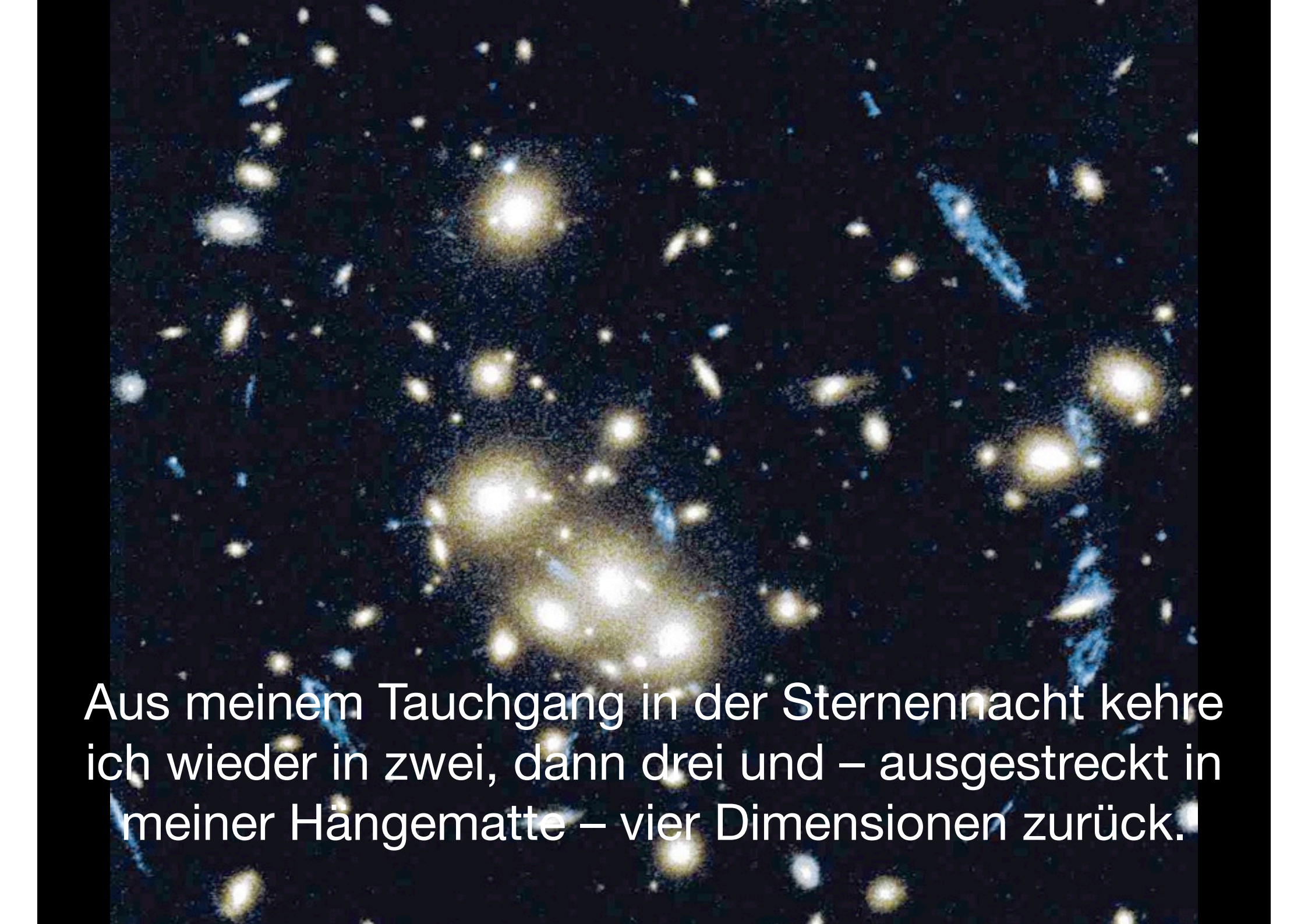
Auf welcher geheimnisvollen
Gleichung gründet sein Wirken?



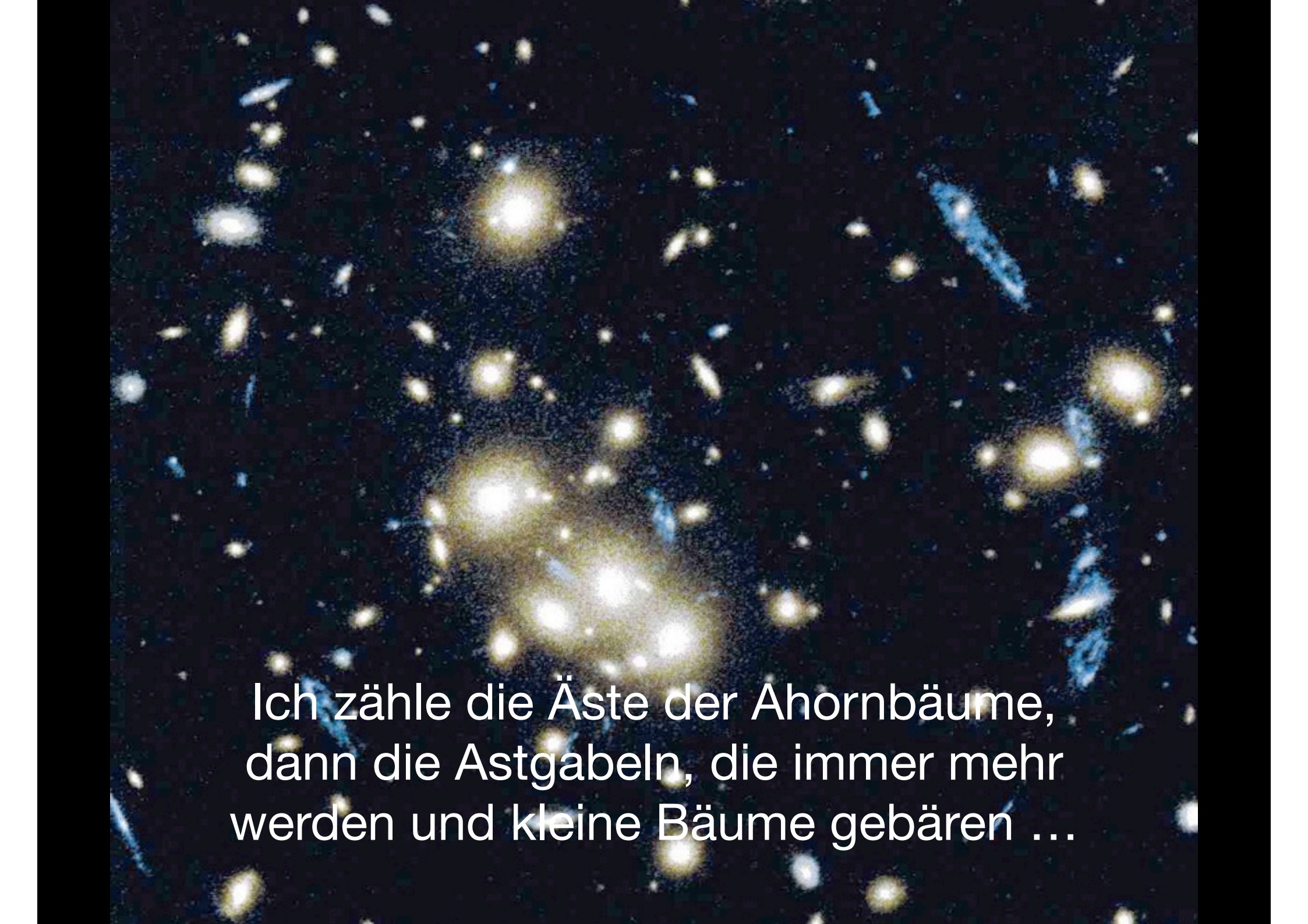
Mit welcher schöner Geometrie
zeichnet er die Schöpfung?



Wie genau verbinden sich die Elemente?



Aus meinem Tauchgang in der Sternennacht kehre
ich wieder in zwei, dann drei und – ausgestreckt in
meiner Hängematte – vier Dimensionen zurück.



Ich zähle die Äste der Ahornbäume,
dann die Astgabeln, die immer mehr
werden und kleine Bäume gebären ...



Mein Blick fängt Himmel und Sterne ein



Es ist Sommer, die Nacht ist klar und ruhig...



Epilogue













NATIONAL GEOGRAPHIC

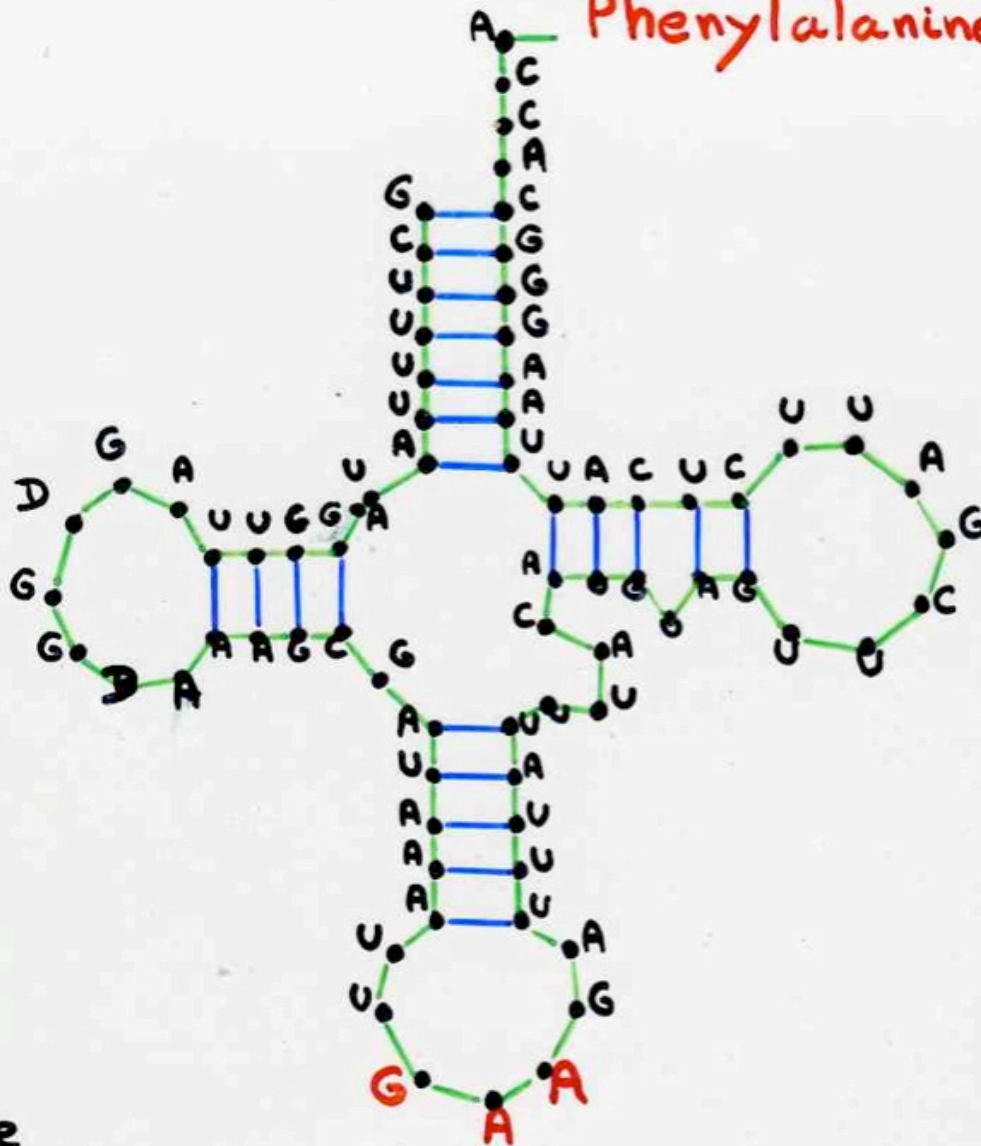




BÄUME,
BAUMSTRUKTUREN
ÜBERALL ...

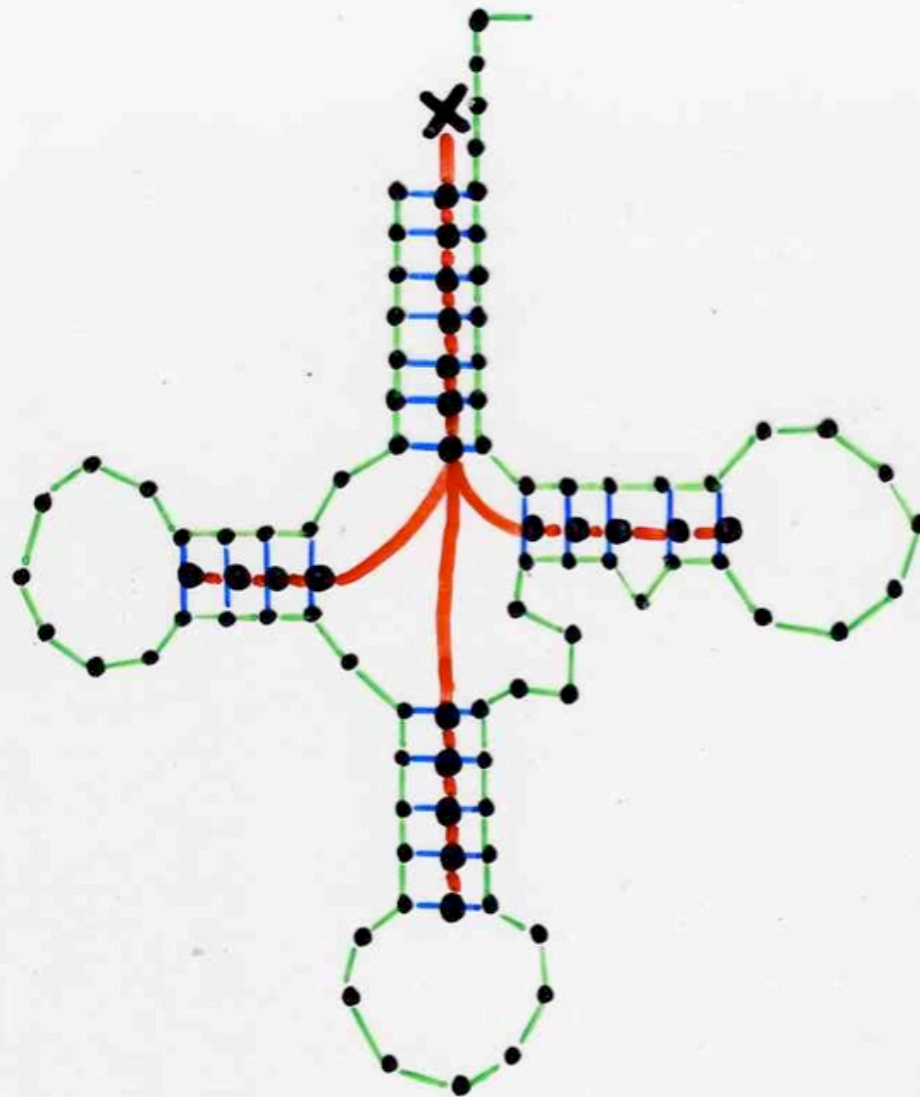
DES ARBRES
DES ARBORESCENCES PARTOUT

Phenylalanine

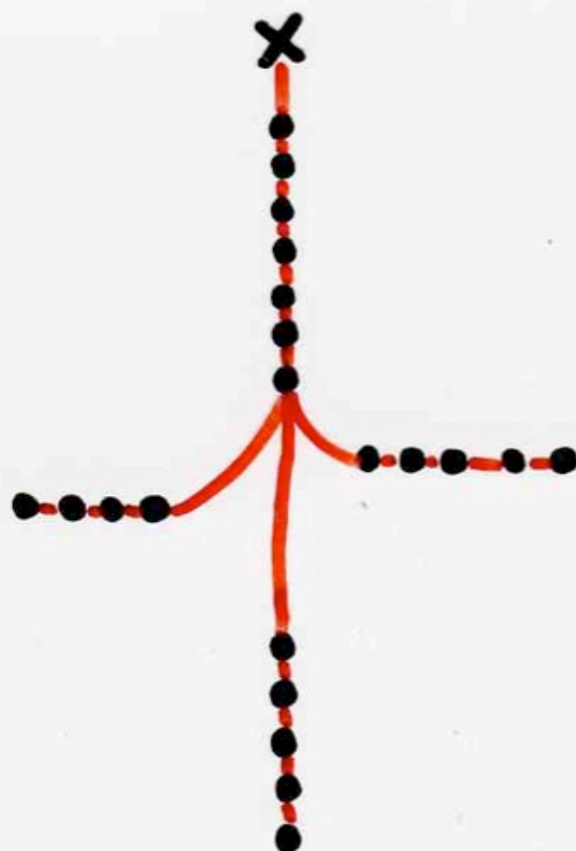


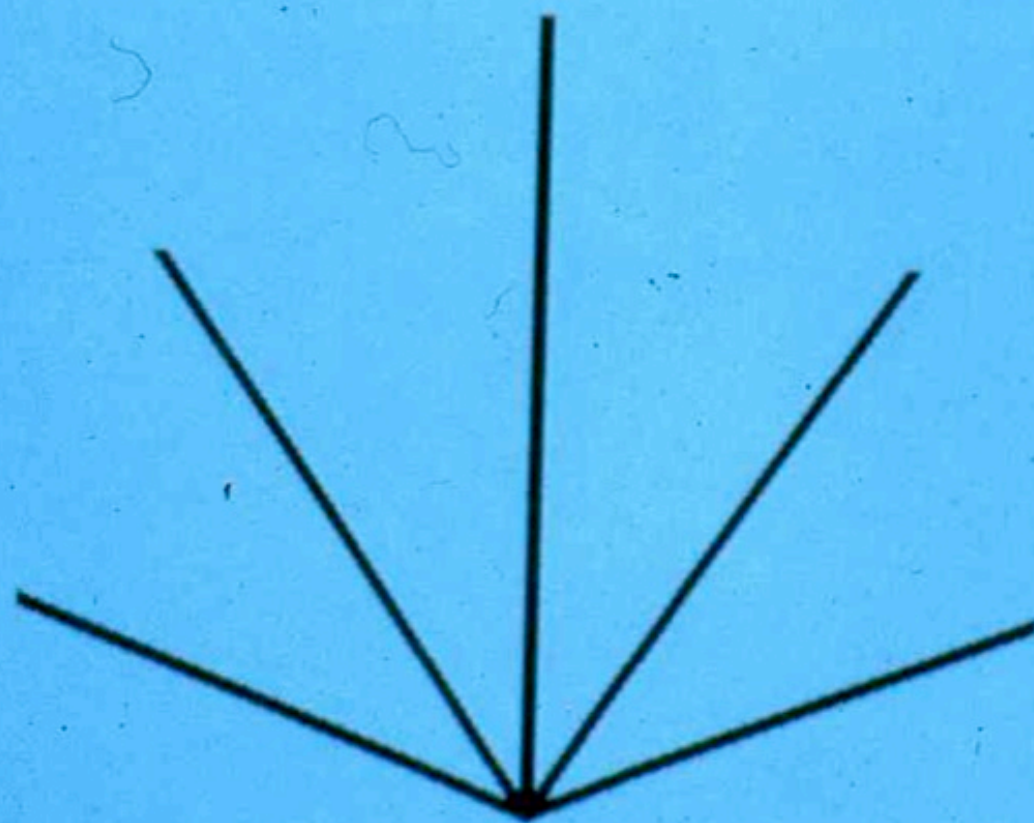
A denine
U racyle
G uanine
C ytosine

tARN^{Phe}



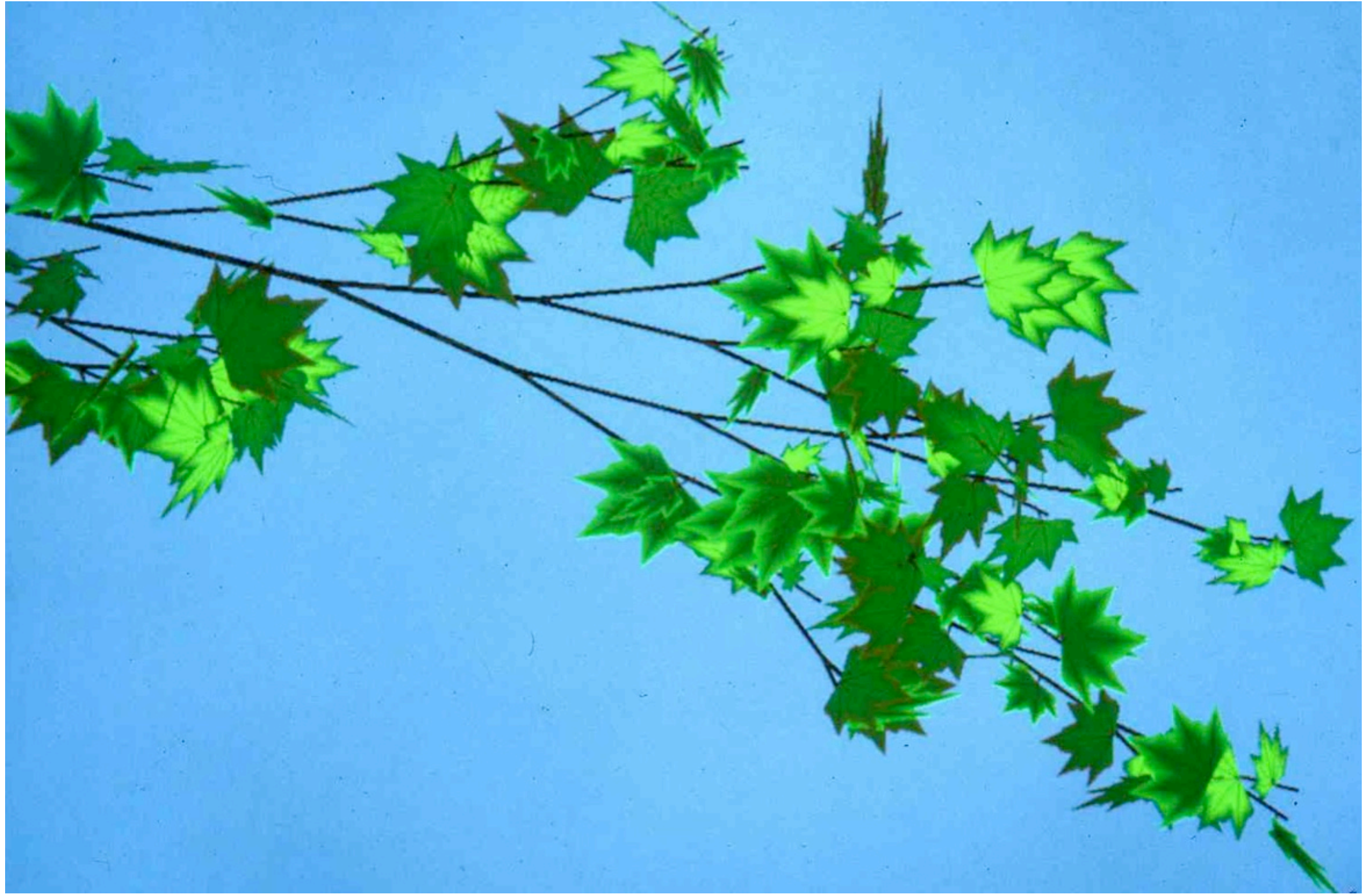
tARN^{Phe}















Il y a des arbres dans les étoiles,
des arbres dans les grains de lumière.

Es gibt Bäume in den Sternen,
Bäume in den Lichtpartikel

Les théories mathématiques s'interpellent,
s'entrecroisent, renaissent, se fondent entre elles.

Mathematische Theorien prallen aufeinander,
überschneiden sich, entstehen neu,
verschmelzen miteinander.

Les grands Maîtres se parlent à travers les siècles
dans le jardin merveilleux des Mathématiques.

Im Wundergarten der Mathematik sprechen
die großen Geister miteinander über die
Jahrhunderte hinweg

A low-angle shot of a gnarled tree against a dark blue sky. The tree's branches are silhouetted against the sky, which is filled with many small, bright white particles that appear to be falling or floating. The foreground shows a rocky, uneven surface. The overall mood is serene and contemplative.

Merci beaucoup

Danke schön

textes:

Le majestueux noyer

L'espace-temps

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Photo Gravitational Lens Galaxy cluster 0024+1654

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