The Strahler analysis of binary trees in computer science and in other sciences

IIT-Bombay
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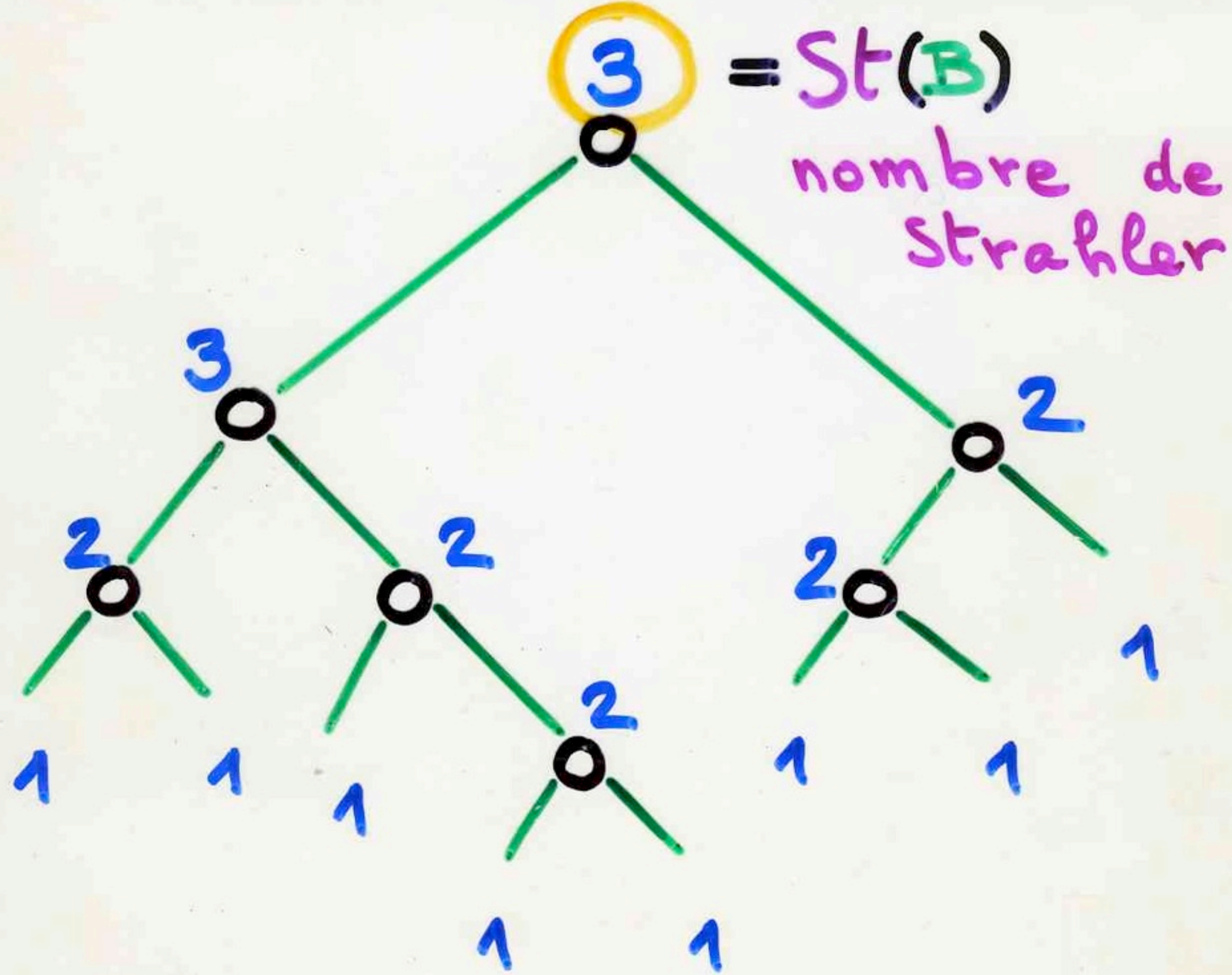
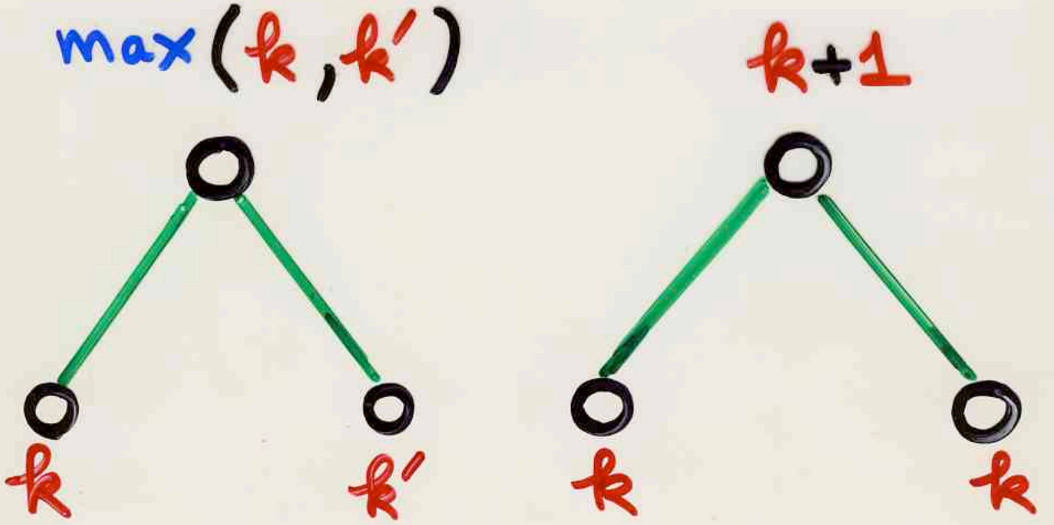
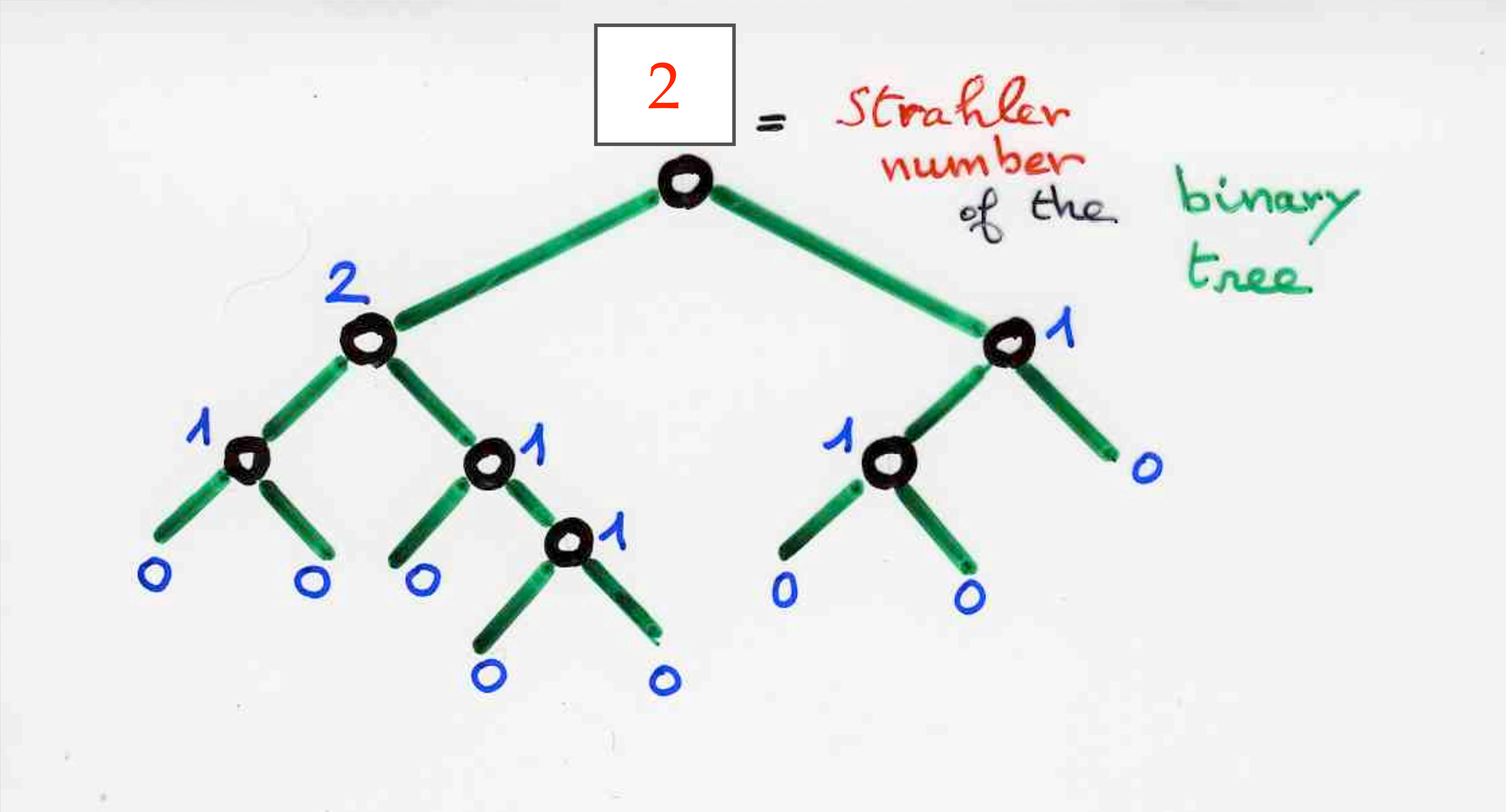
Xavier Viennot
CNRS, LaBRI,
Bordeaux University, France

Strahler number of a binary tree



3
= St(B)
nombre de Strahler
3
2
2
2
2
2
1
1
1
1
1
1
1
1


max(k, k′)
k+1
k
k′
k
k


2
= Strahler number of the binary tree
2
1
1
1
1
1
0
0
0
0
0
0
0
0
0

asymptotic analysis

average Strahler number
over binary trees n vertices

$$St_n = \log_4 n + f(\log_4 n) + o(1)$$

Flajolet, Raoult, Vuillemin (1979) periodic
Kemp

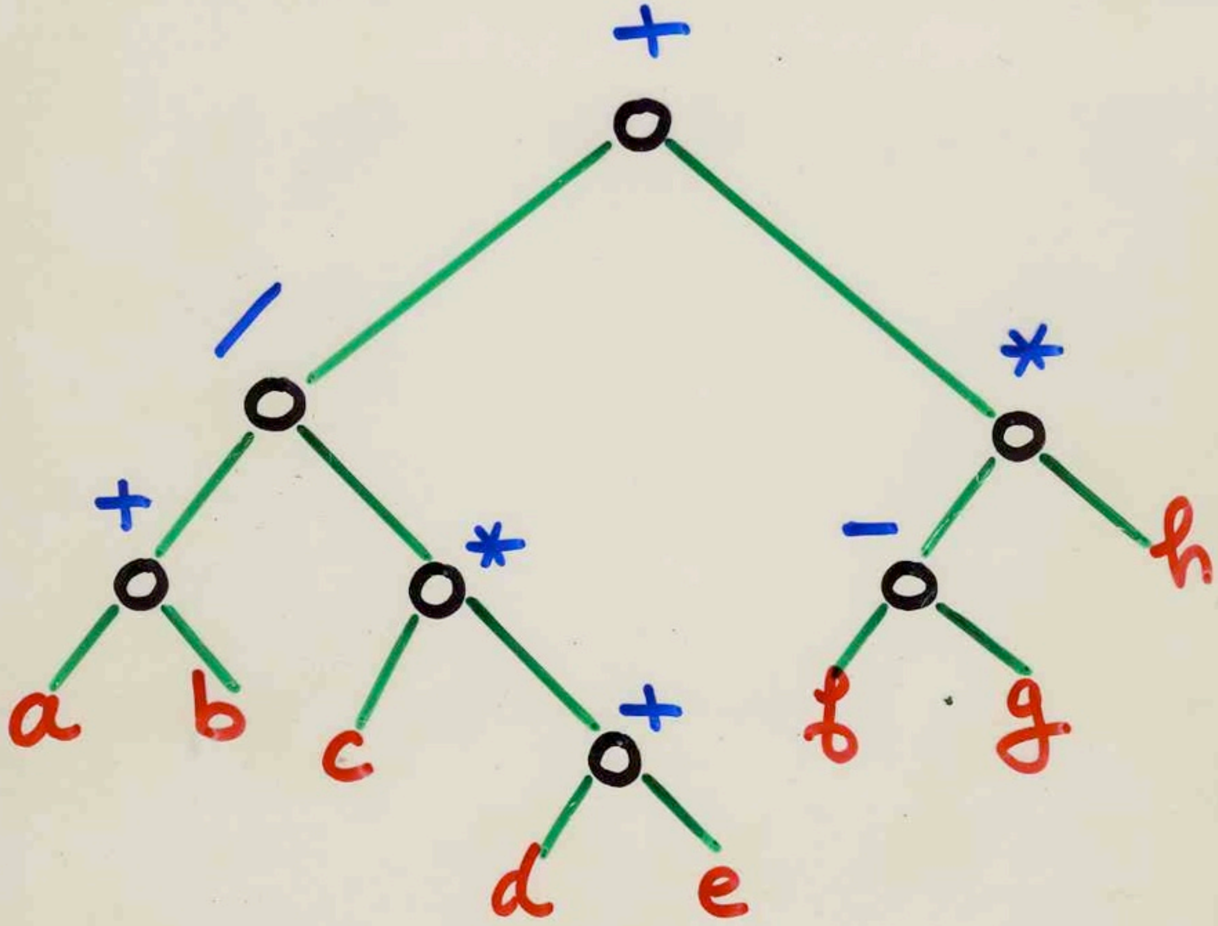
$T(n)$ = number of 1's in the binary expansion of $1, 2, .., (n-1)$

$$T(n) = \frac{1}{2} n \log n + n F(\log n)$$

$$f(t) = 1 - \frac{\gamma}{2 \log 2} - \int_0^\infty t H_4(t) F(\log t + u) e^{-t^2} dt$$

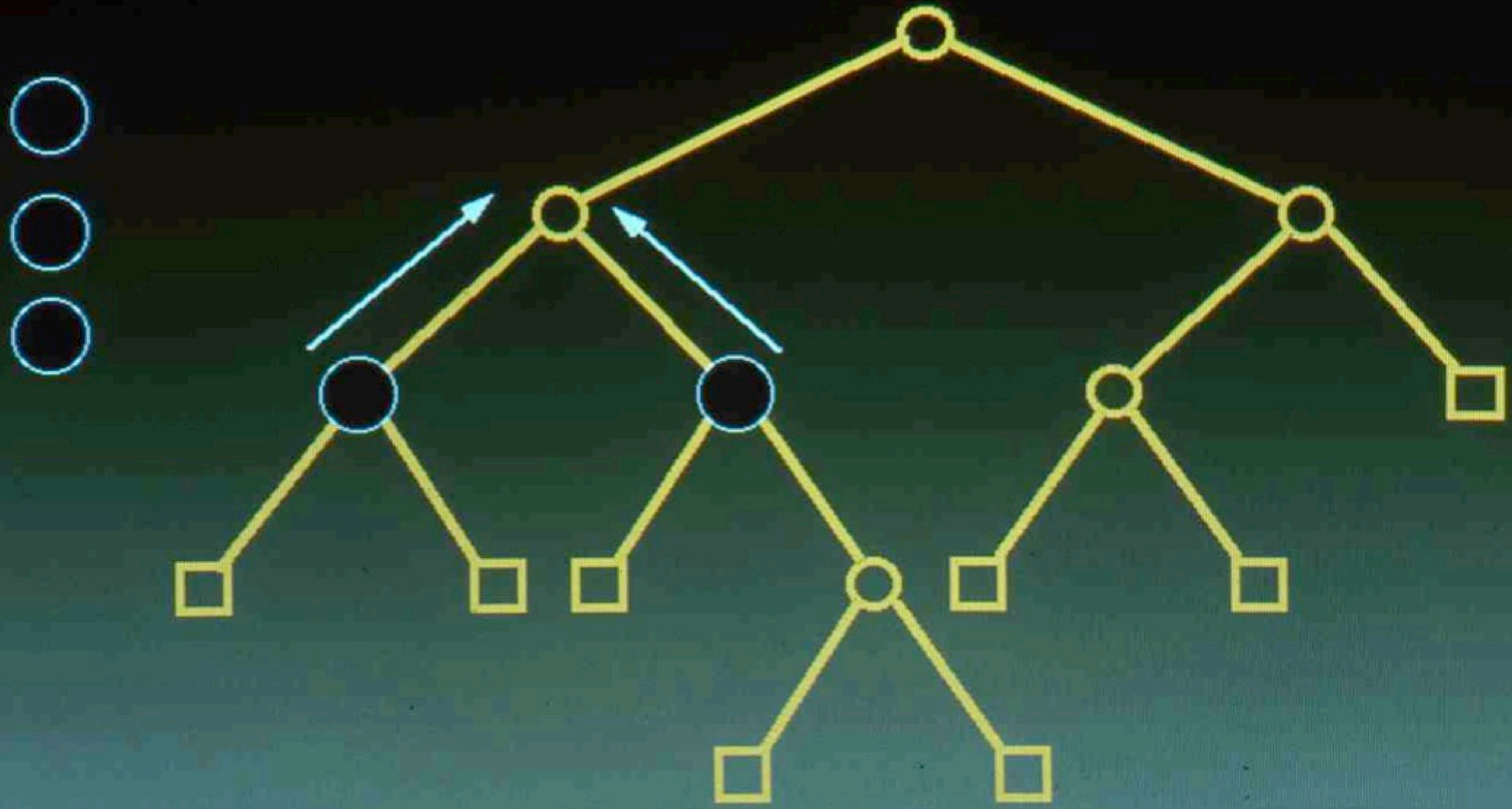
minimum number of registers

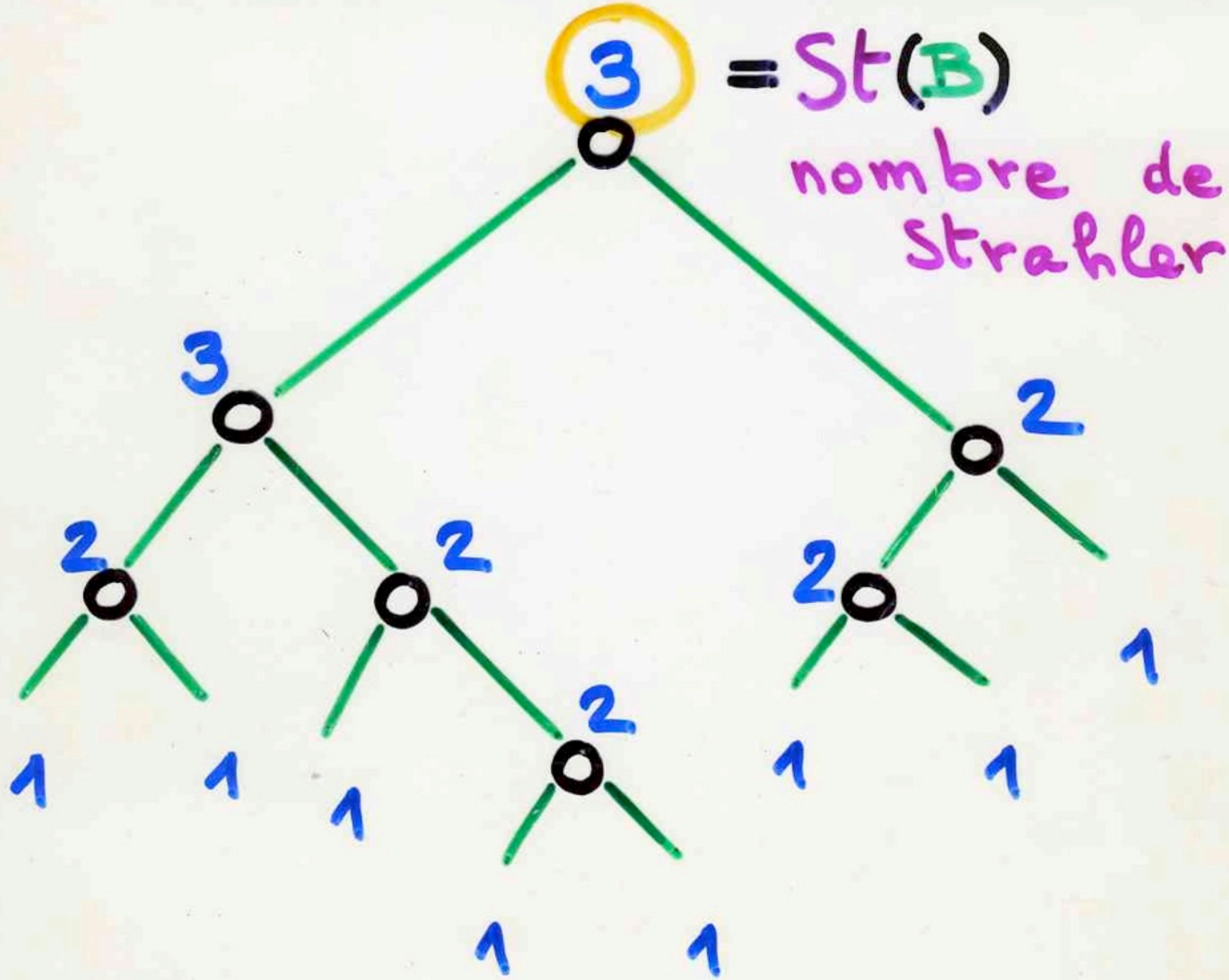
$$\frac{(a+b)}{c(d+e)} + (f-g)^h$$

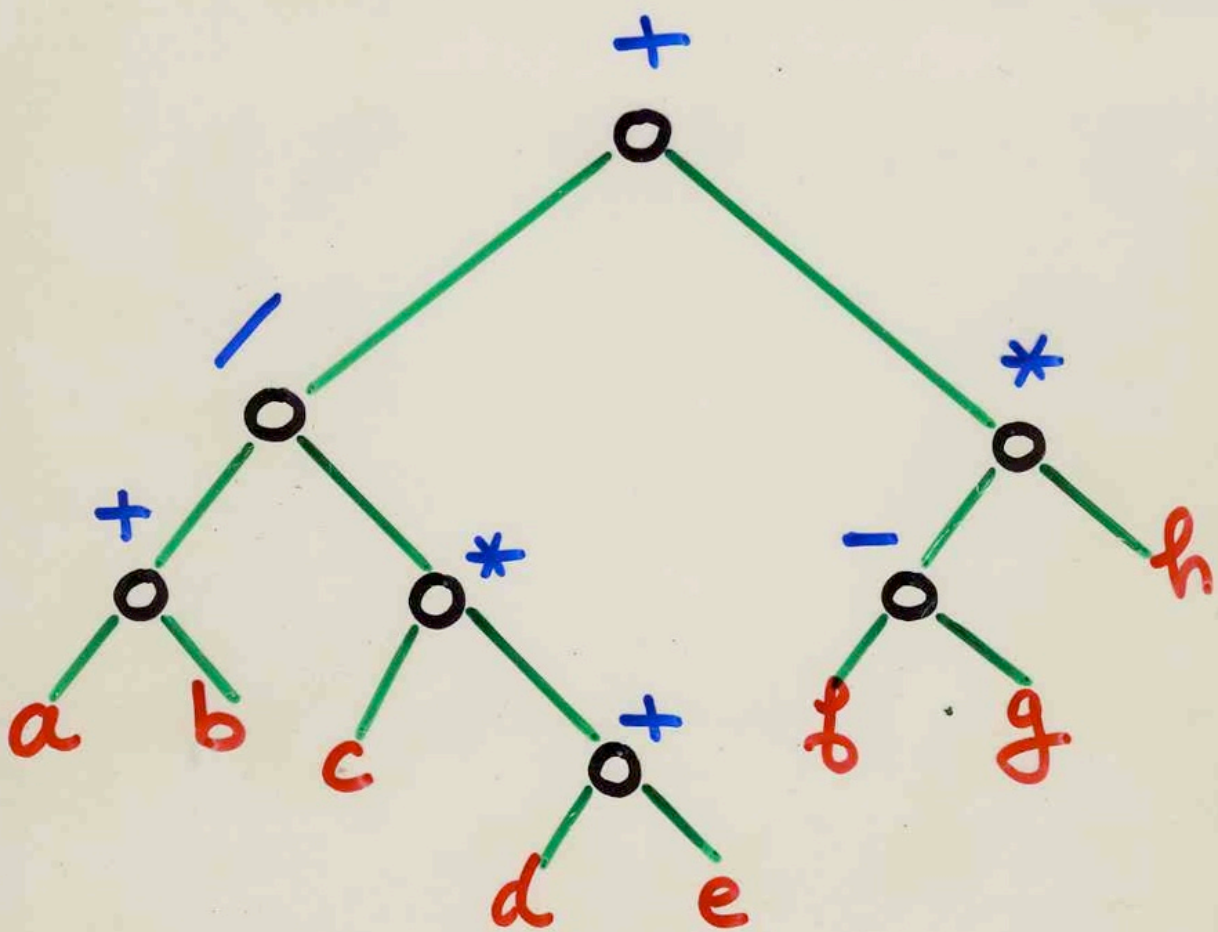


$$\frac{(a+b)}{c\,(d+e)} + (f-g)h$$

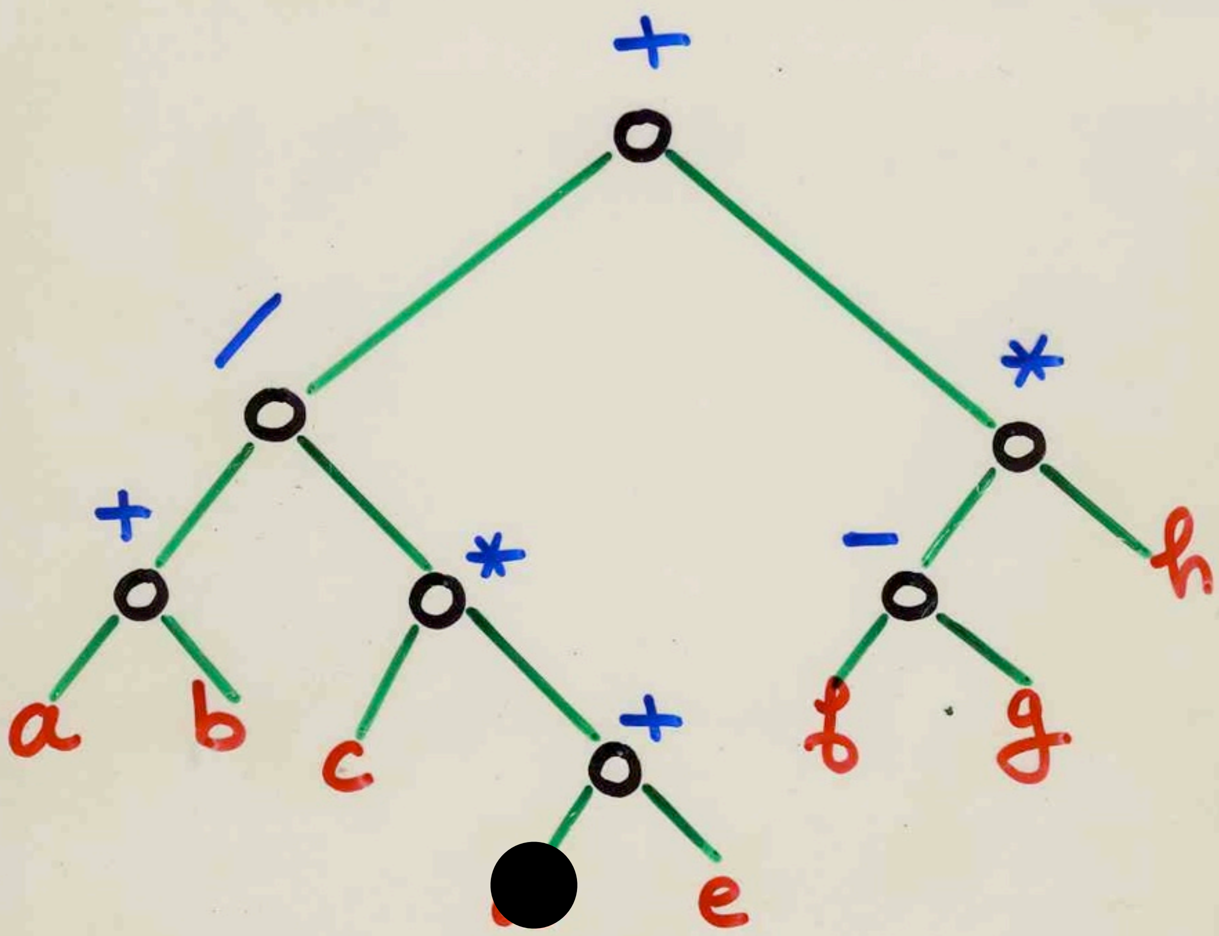
Pebbles problem



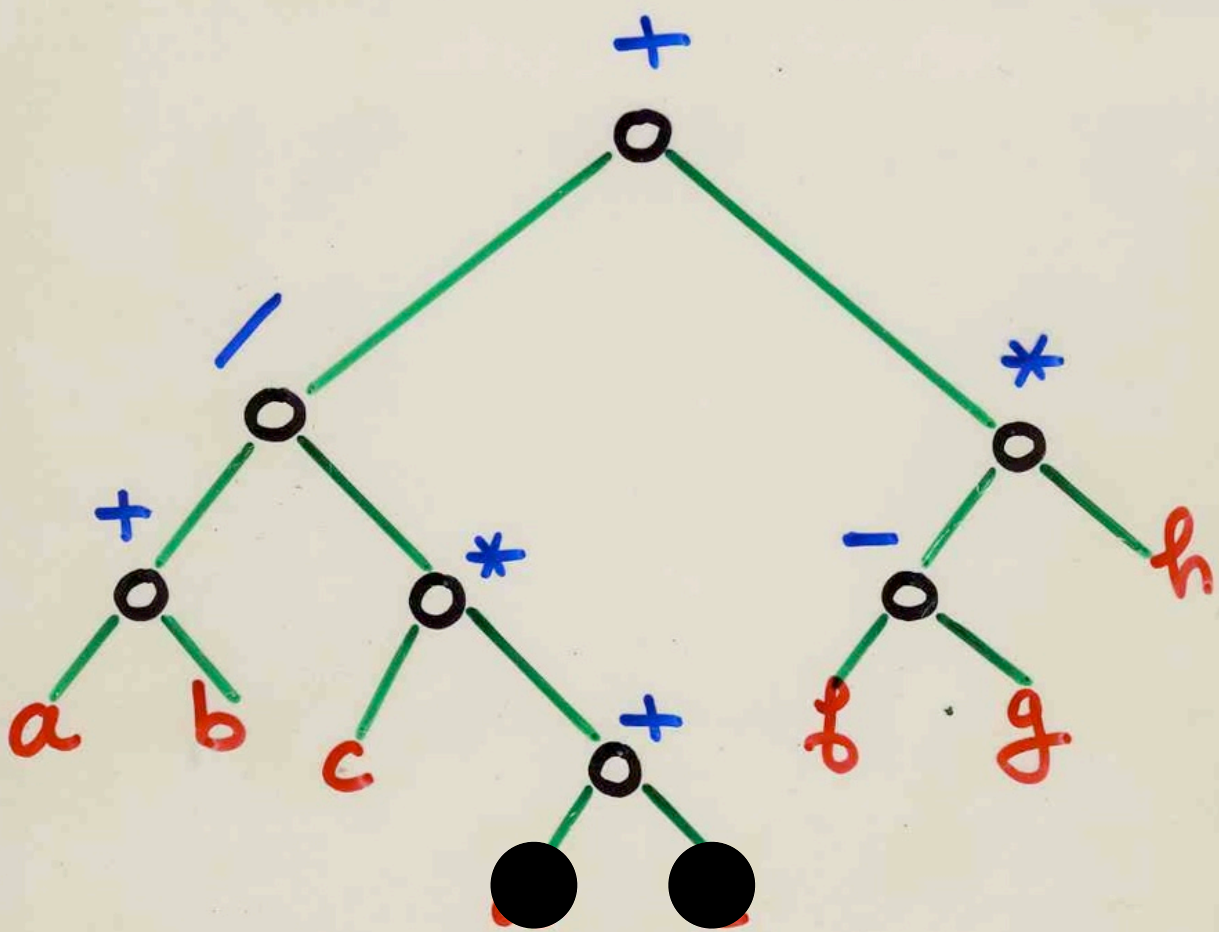

3
= St(B)
nombre de Strahler
3
2
2
2
2
2
1
1
1
1
1
1
1
1
1



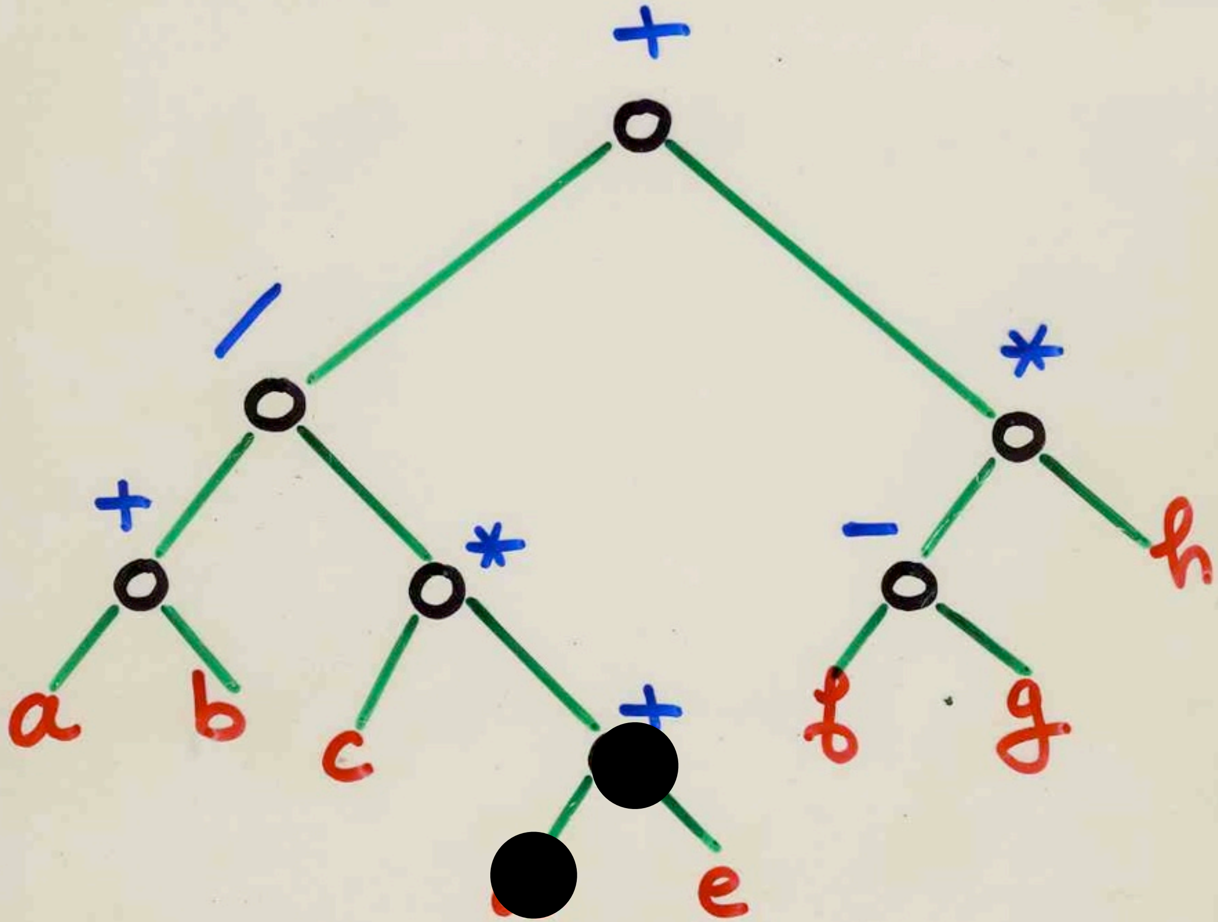
$$\frac{(a+b)}{c\,(d+e)} + (f-g)h$$



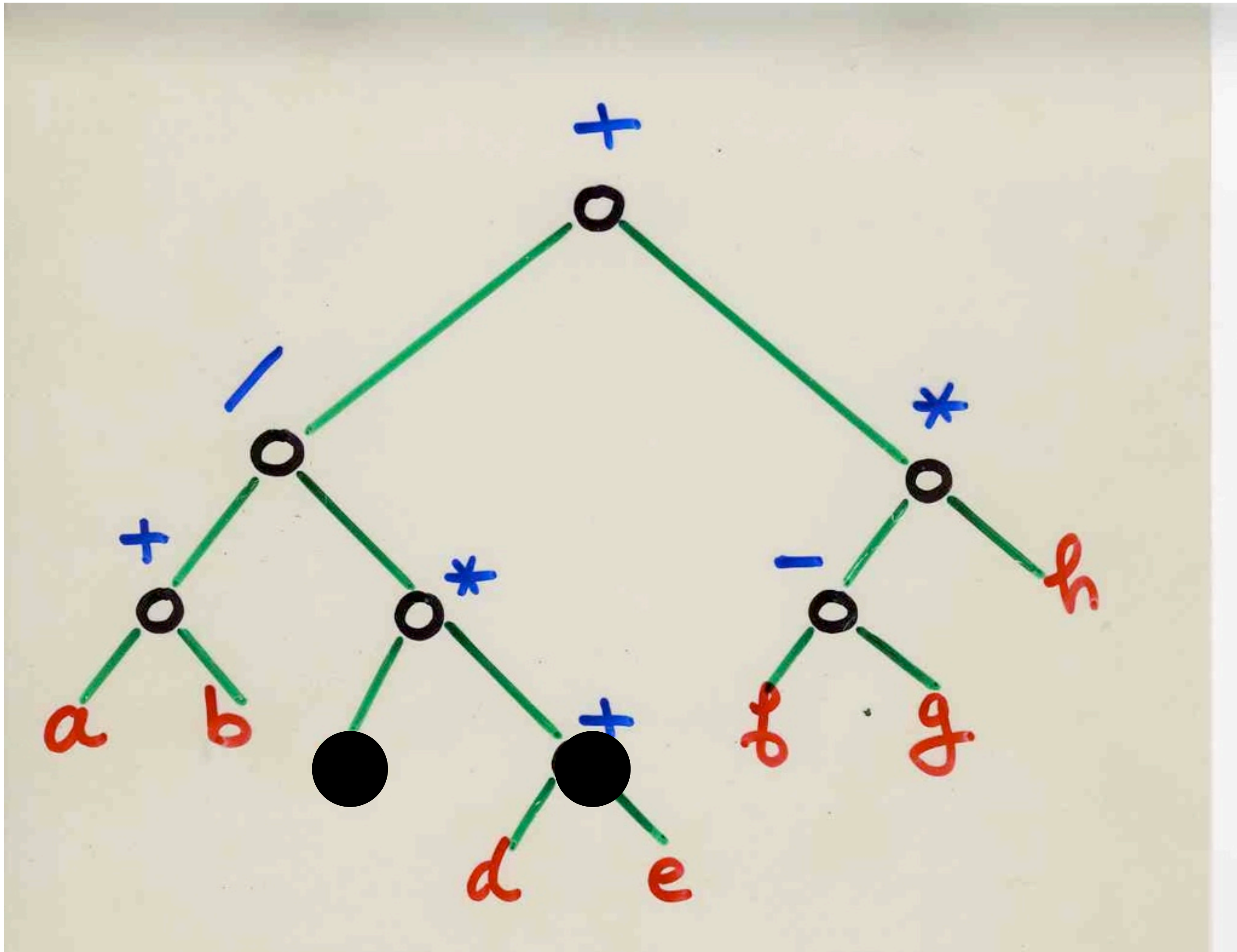
$$\frac{(a+b)}{c\,(d+e)} + (f-g)\,h$$



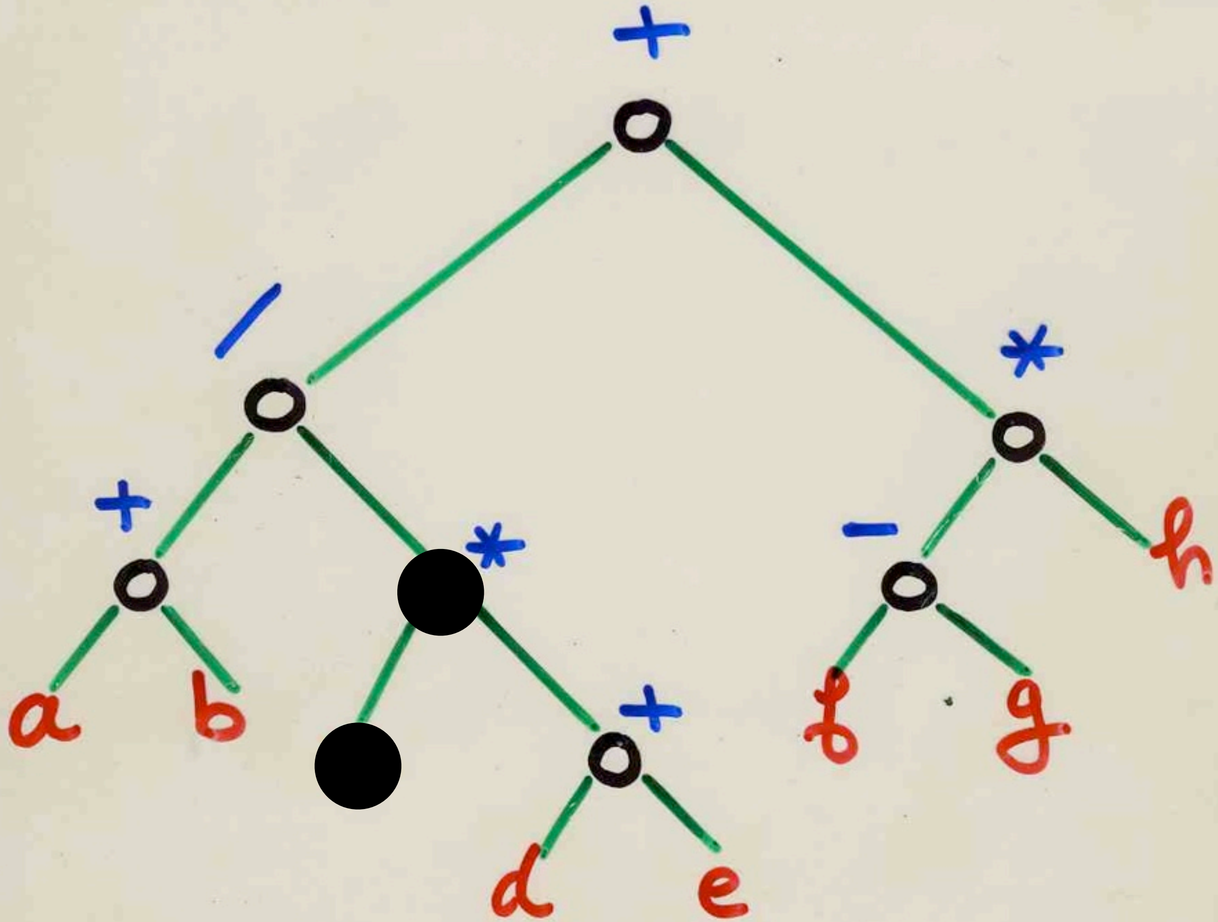
$$\frac{(a+b)}{c\,(d+e)} + (f-g)\,h$$



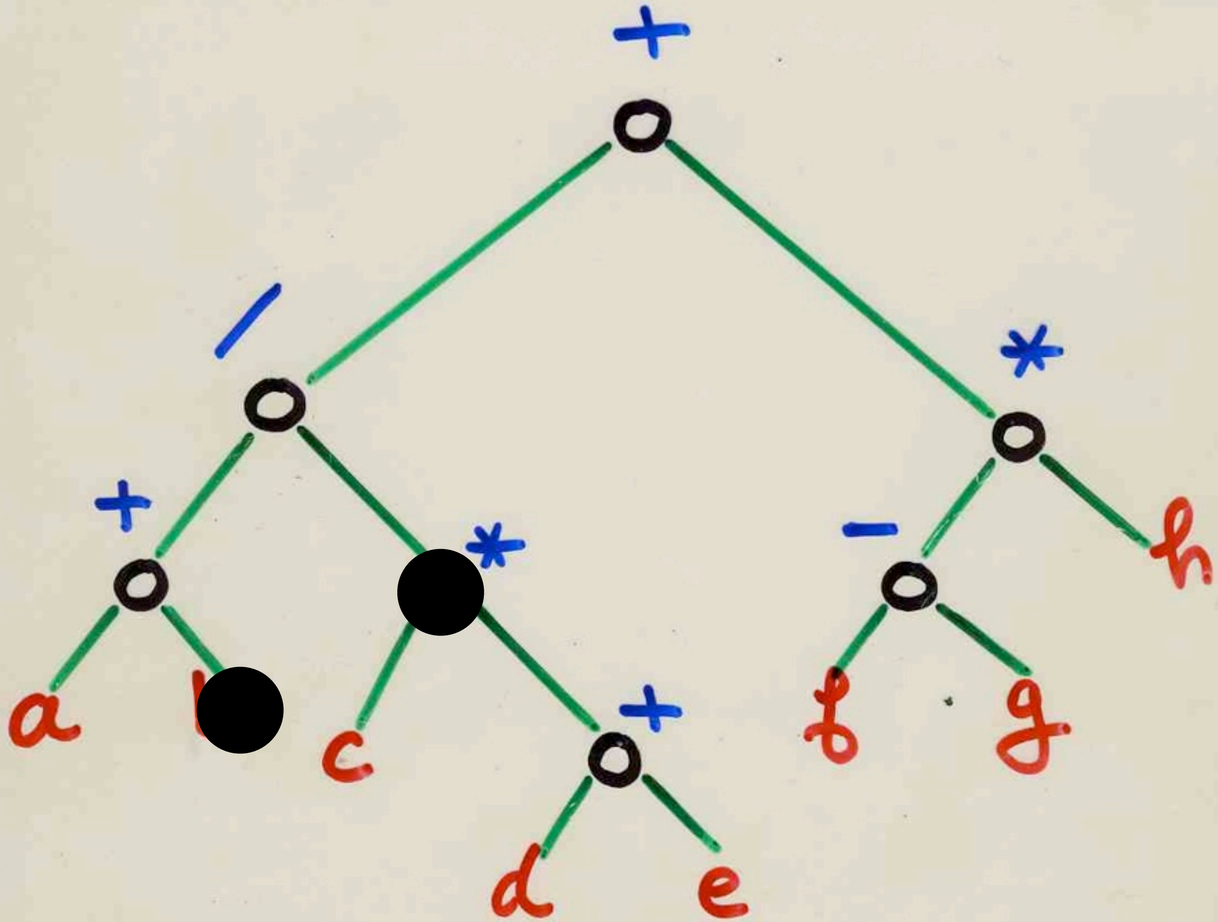
$$\frac{(a+b)}{c\,(d+e)} + (f-g)\,h$$



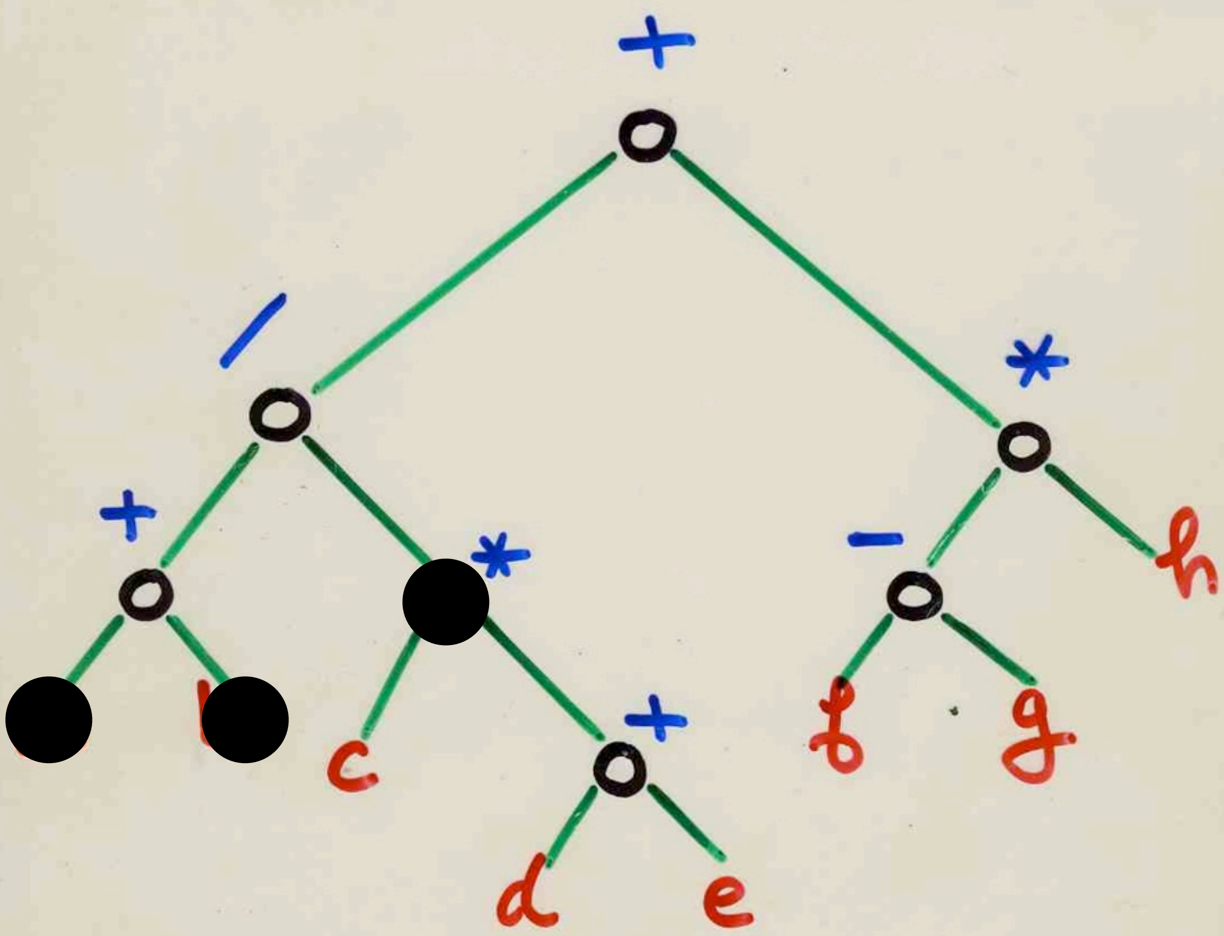
$$\frac{(a+b)}{c\,(d+e)} + (f-g)h$$



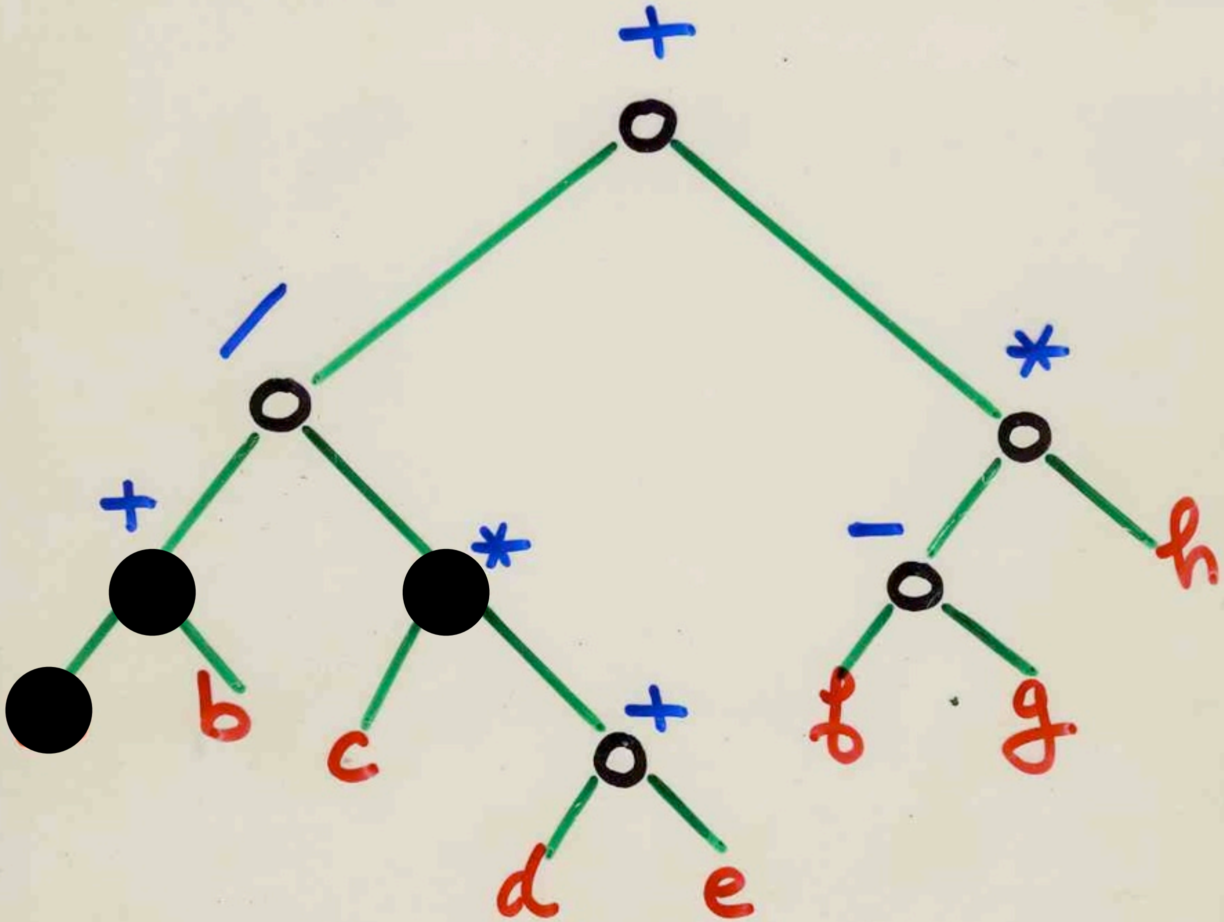
$$\frac{(a+b)}{c\,(d+e)} + (f-g)h$$



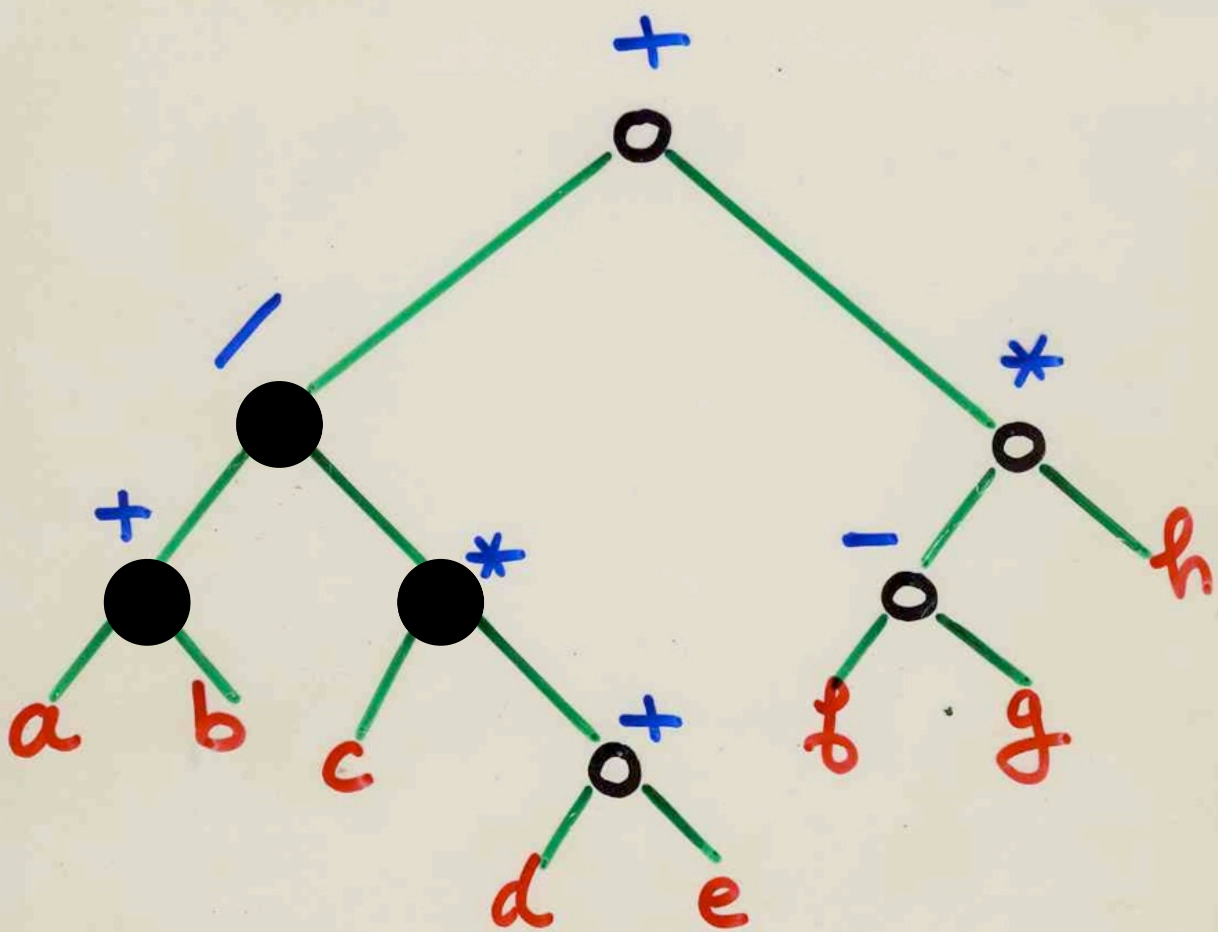
$$\frac{(a+b)}{c\,(d+e)} + (f-g)\,h$$



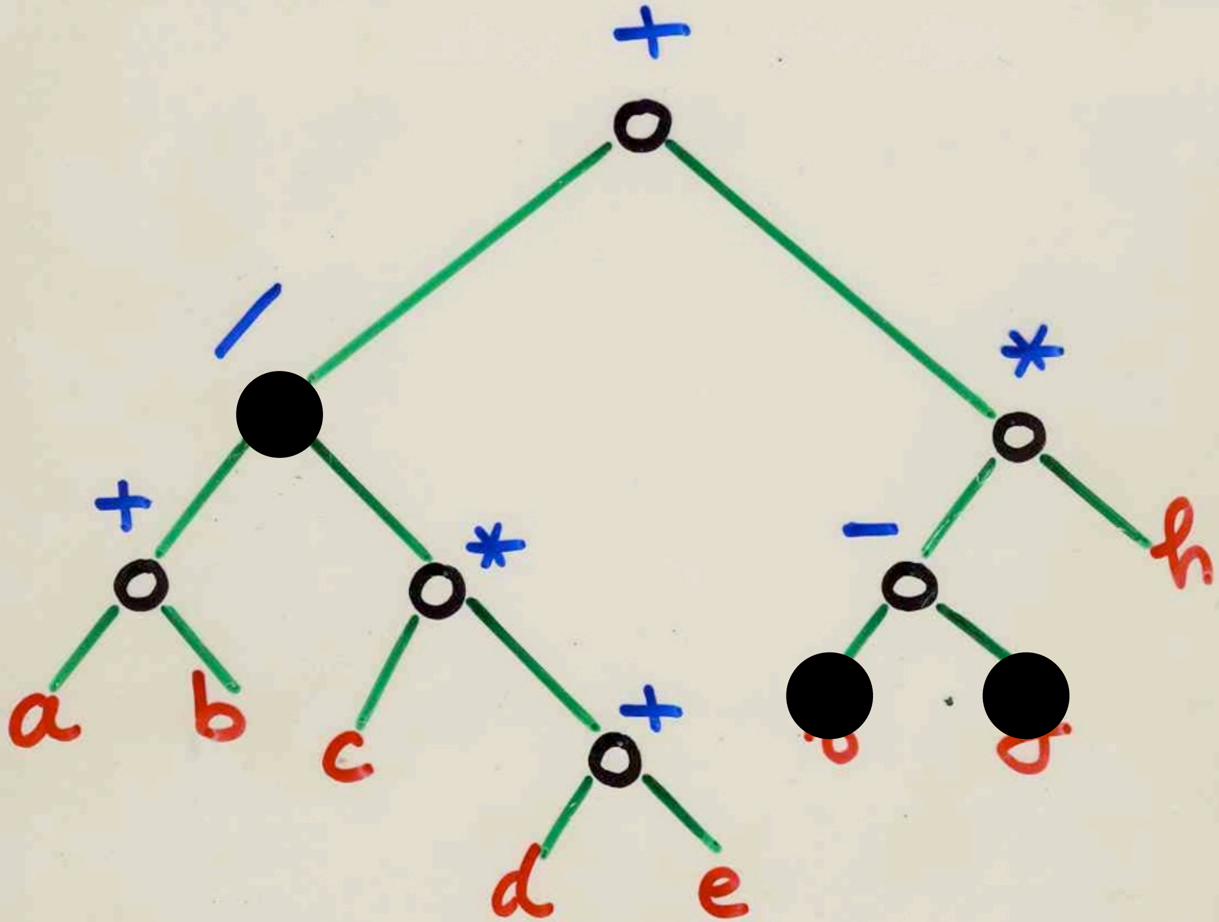
$$\frac{(a+b)}{c\,(d+e)} + (f-g)\,h$$



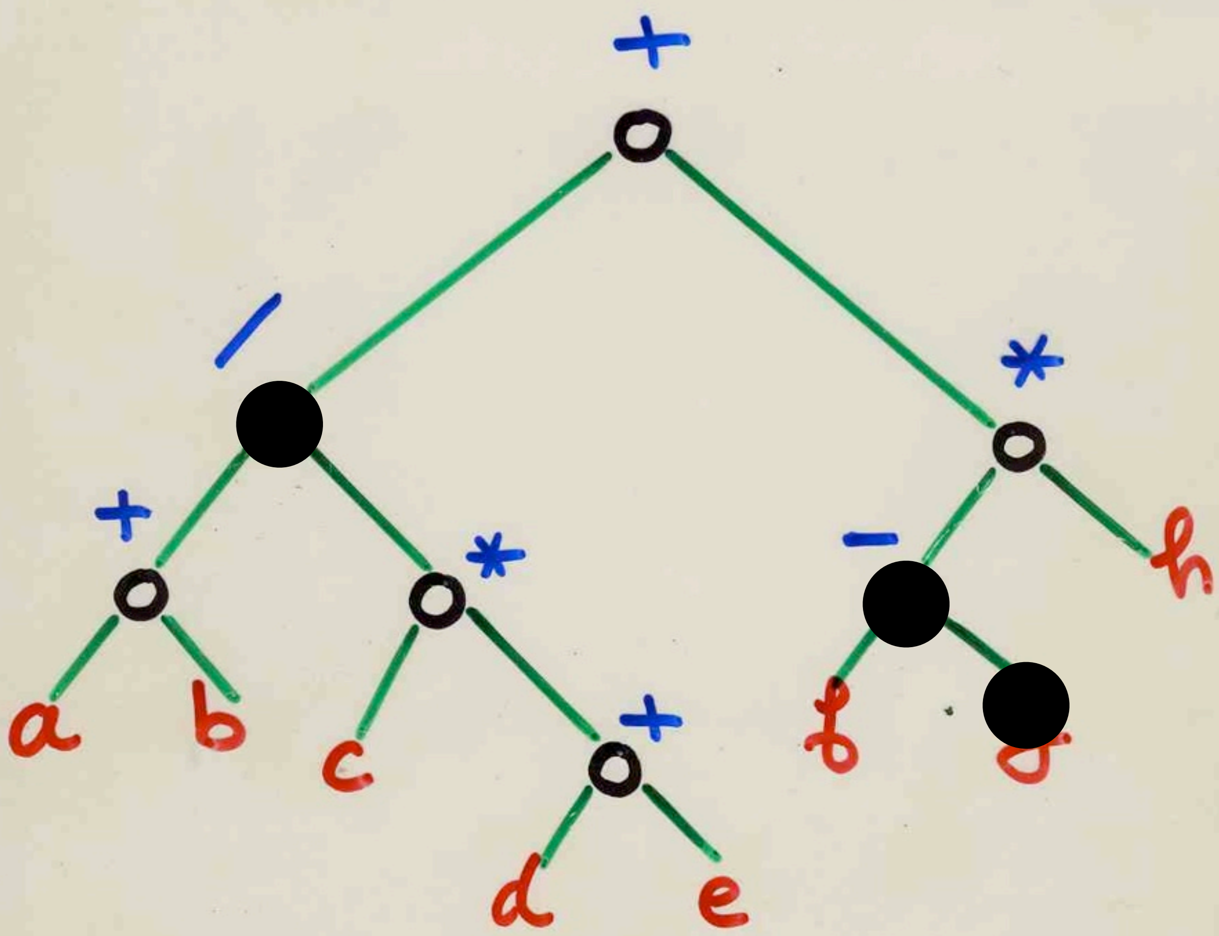
$$\frac{(a+b)}{c\,(d+e)} + (f-g)\,h$$



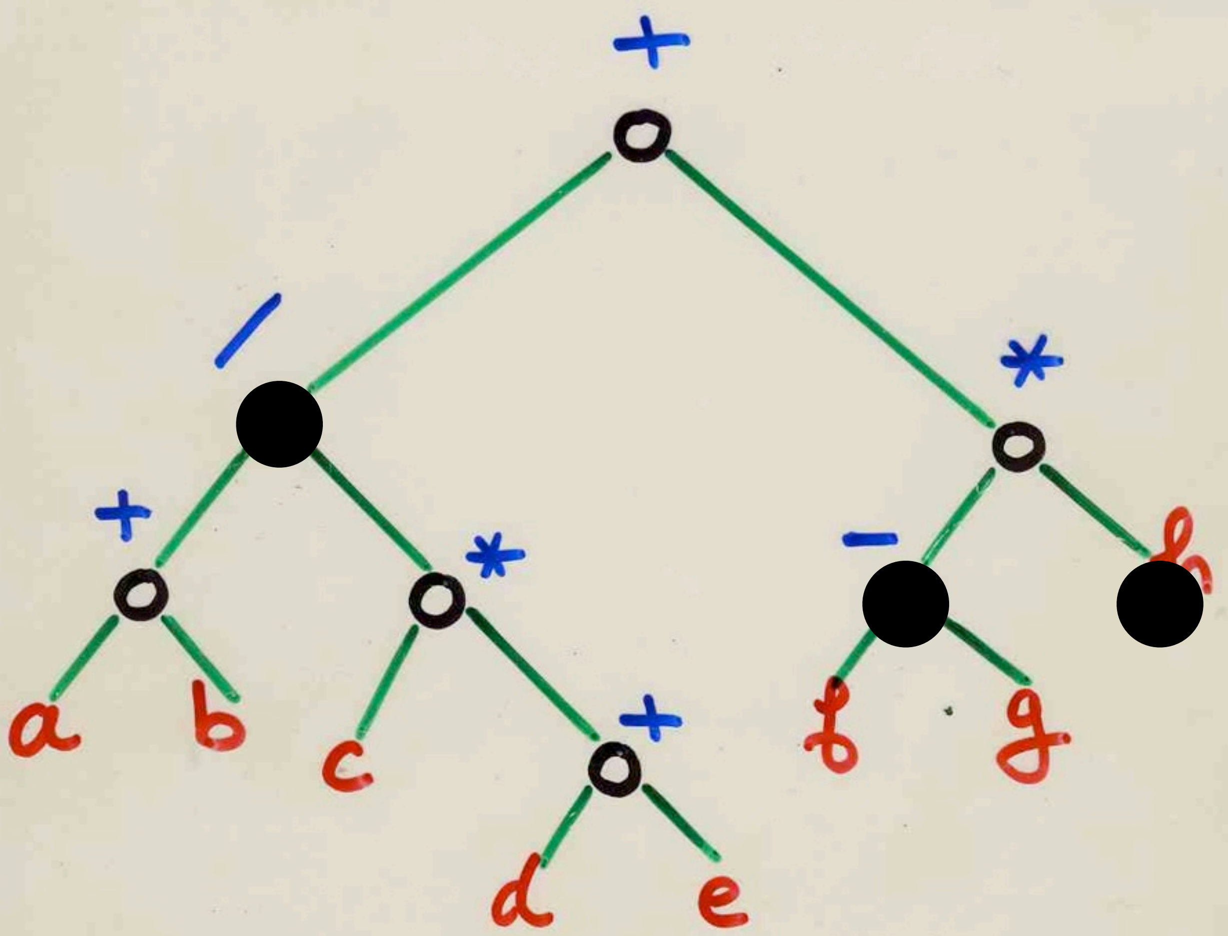
$$\frac{(a+b)}{c\,(d+e)} + (f-g)\,h$$



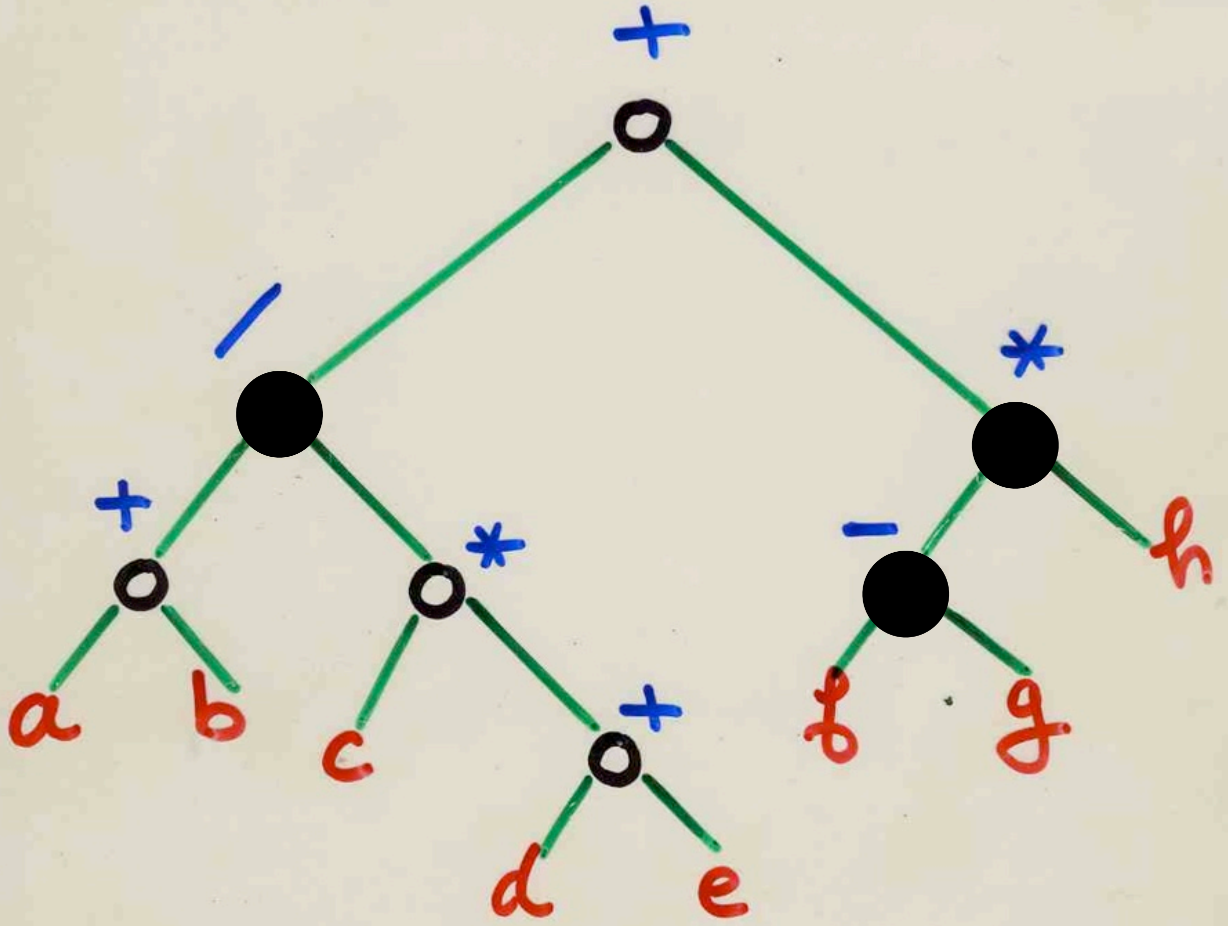
$$\frac{(a+b)}{c\,(d+e)} + (f-g)h$$



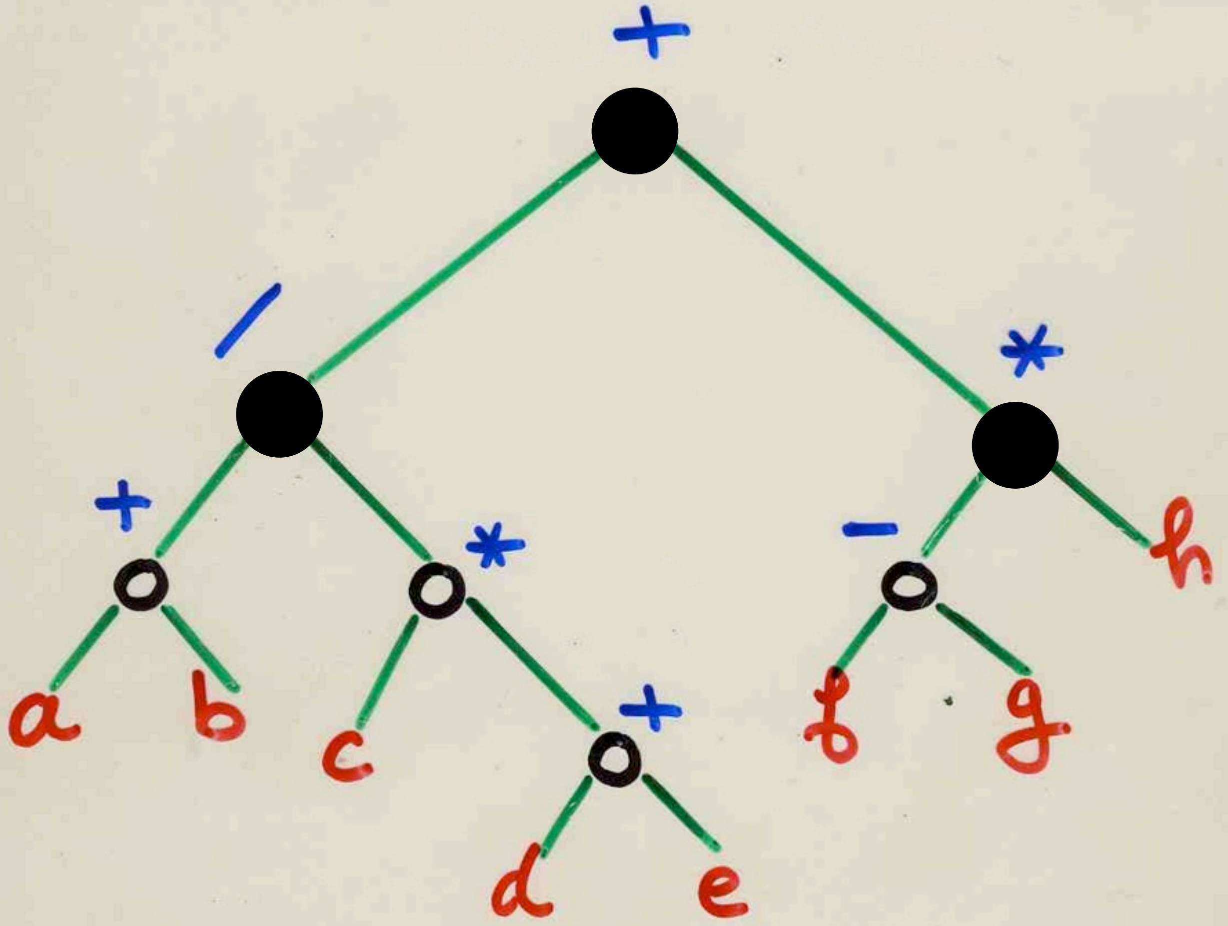
$$\frac{(a+b)}{c\,(d+e)} + (f-g)h$$



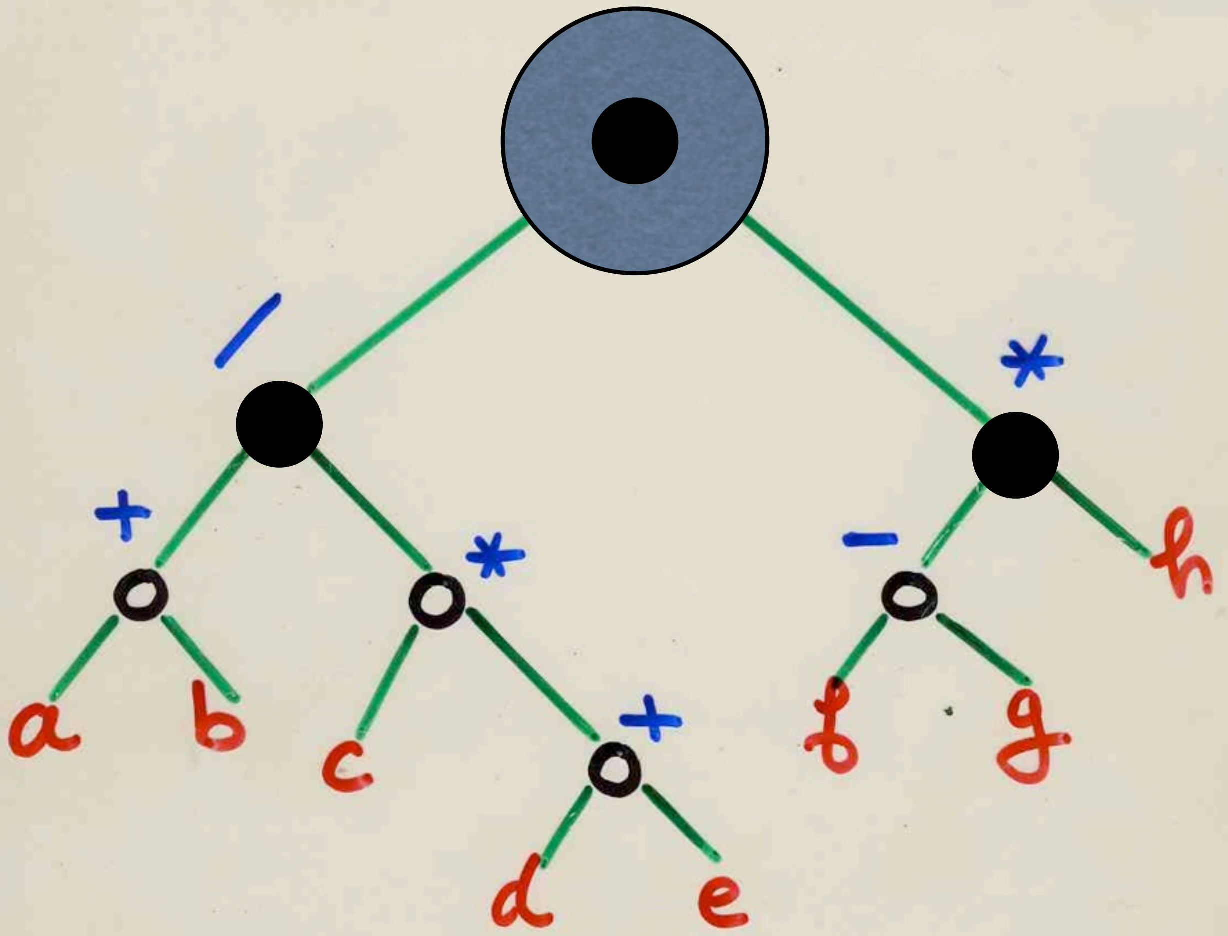
$$\frac{(a+b)}{c\,(d+e)} + (f-g)h$$



$$\frac{(a+b)}{c\,(d+e)} + (f-g)\,h$$



$$\frac{(a+b)}{c\,(d+e)} + (f-g)\,h$$



$$\frac{(a+b)}{c\,(d+e)} + (f-g)\,h$$

Hydrogeology

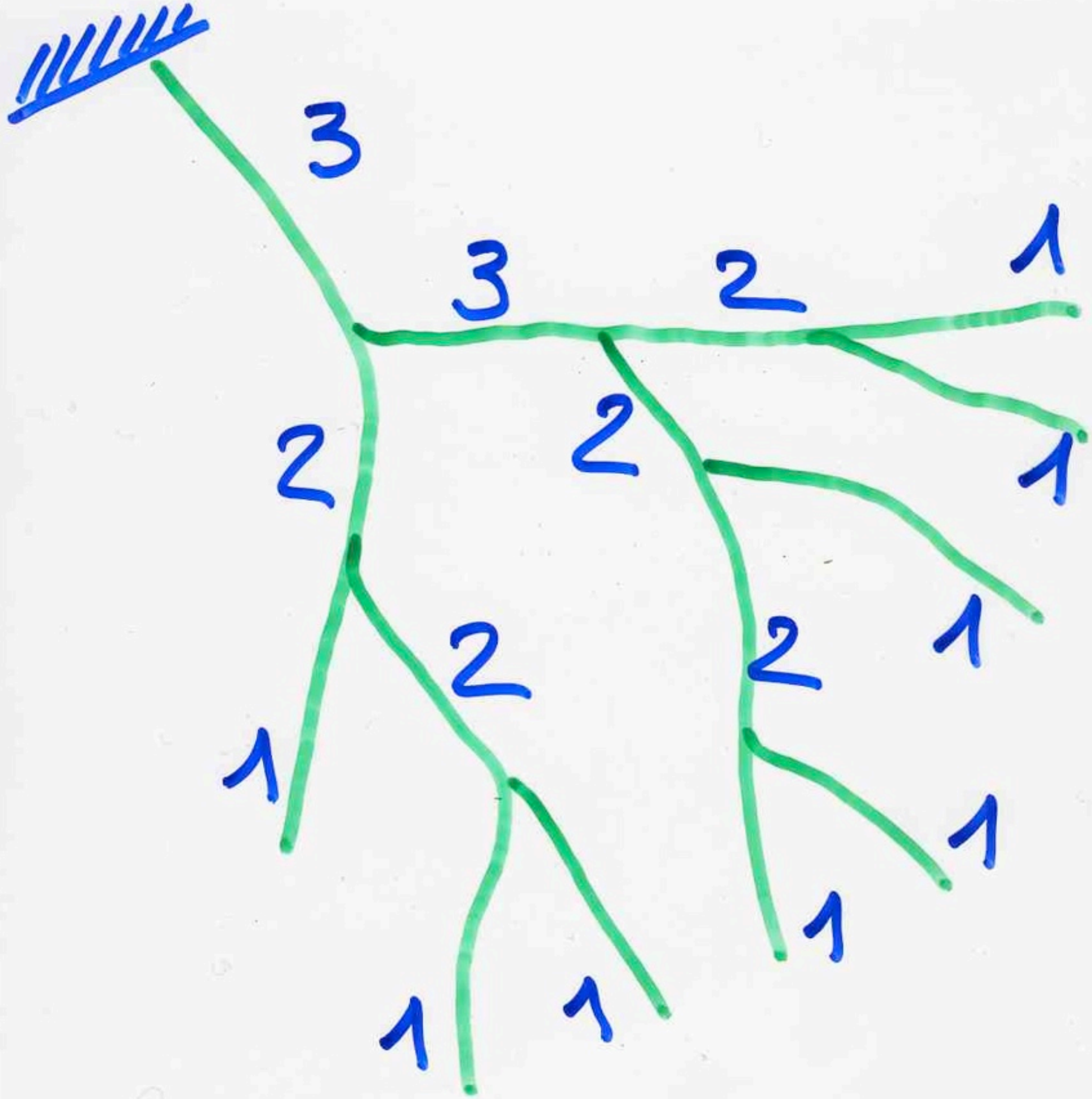
Horton (1945)

Strahler (1952)

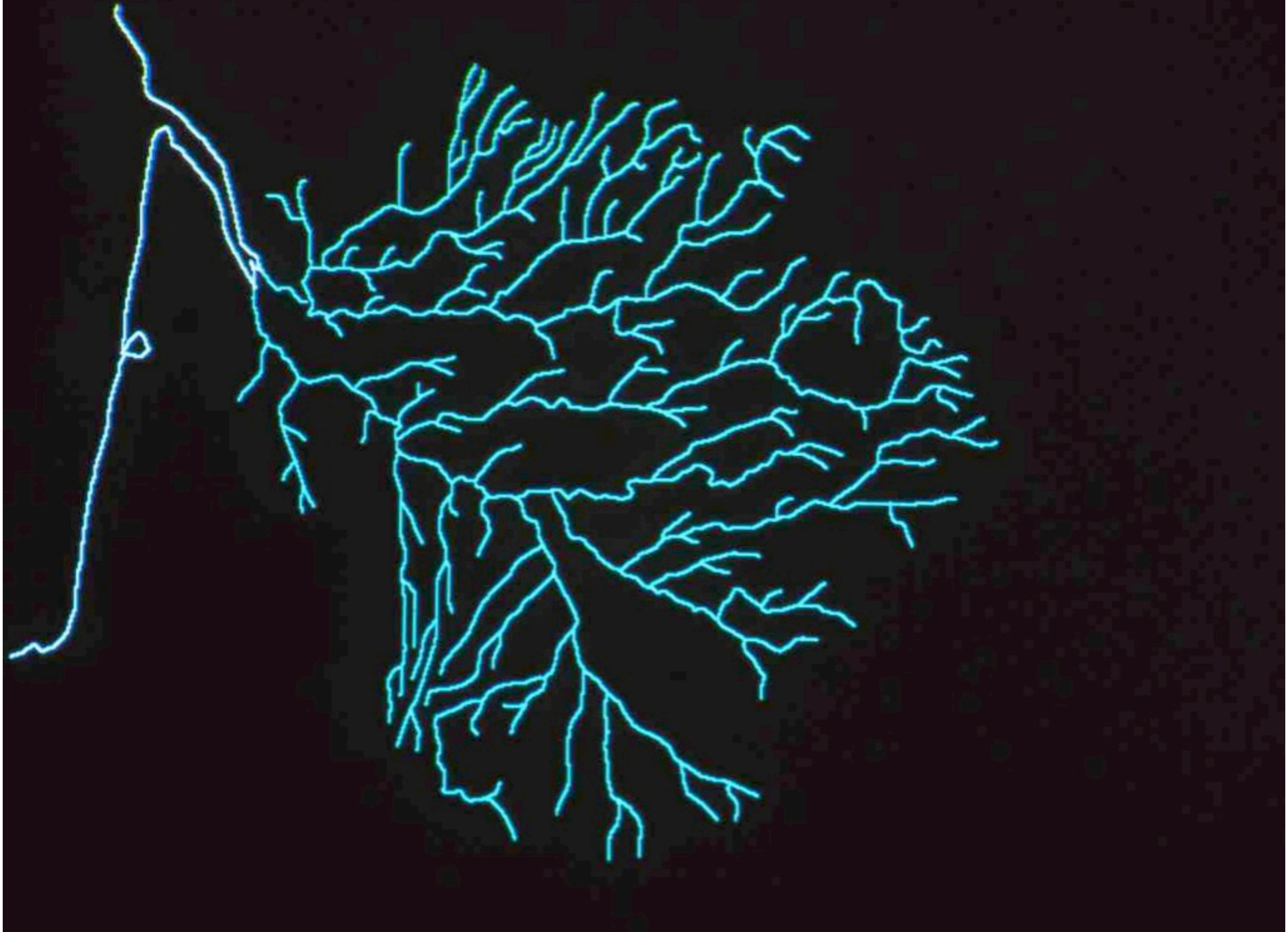
Hydrogeology

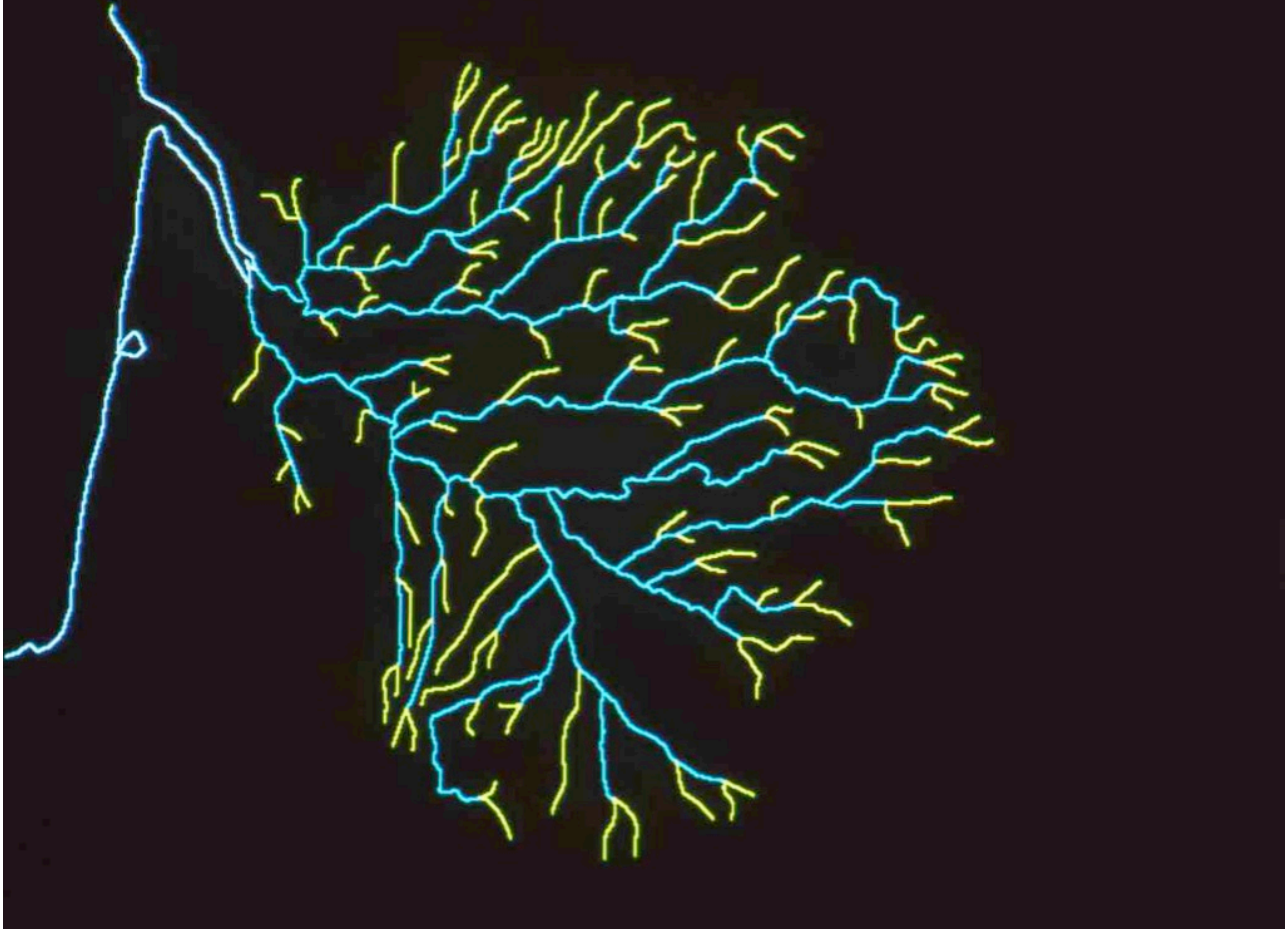
morphology of rivers network

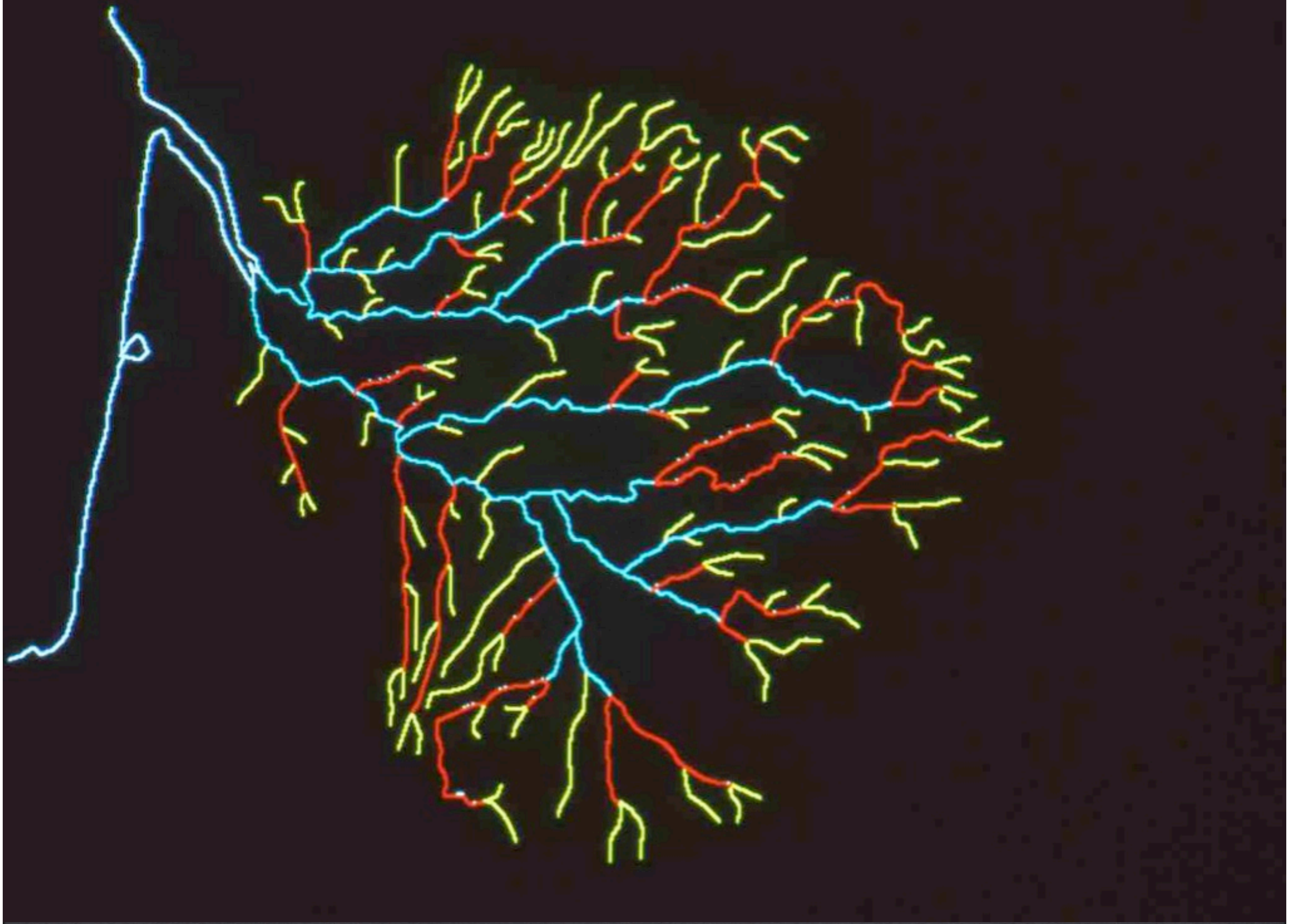
Order of a river

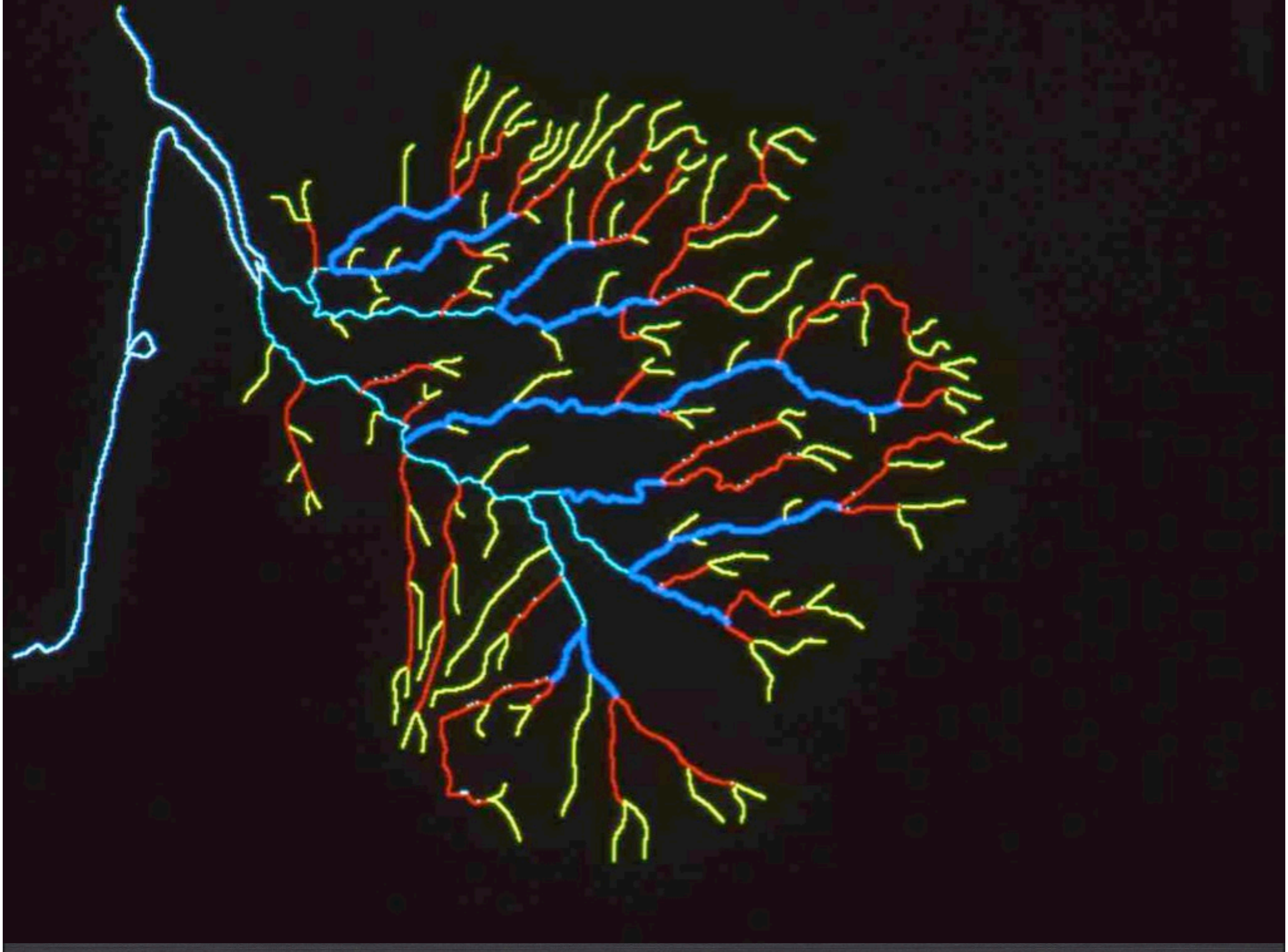


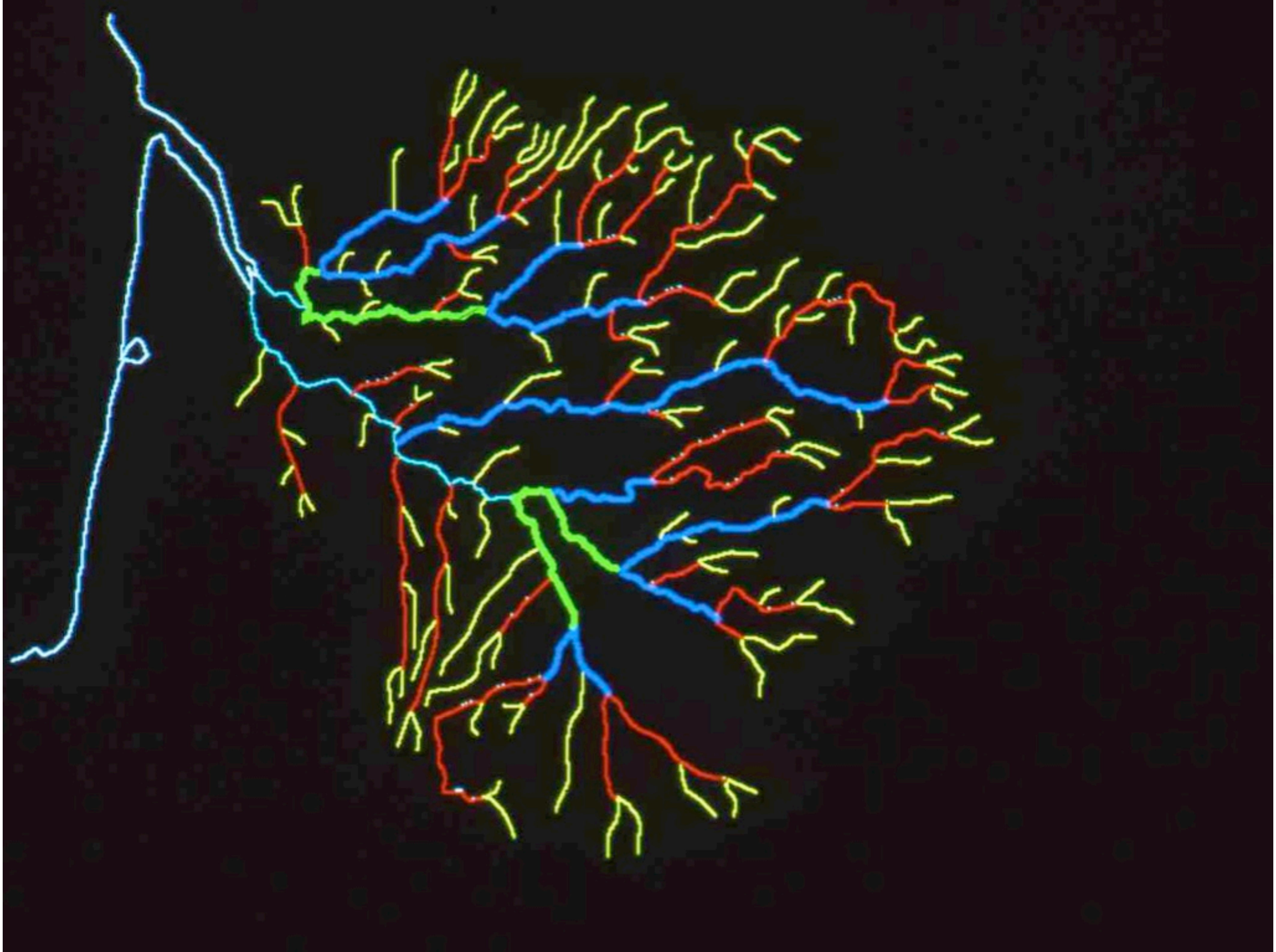
3
3
2
1
2
1
2
2
1
2
1
1
1
1
1

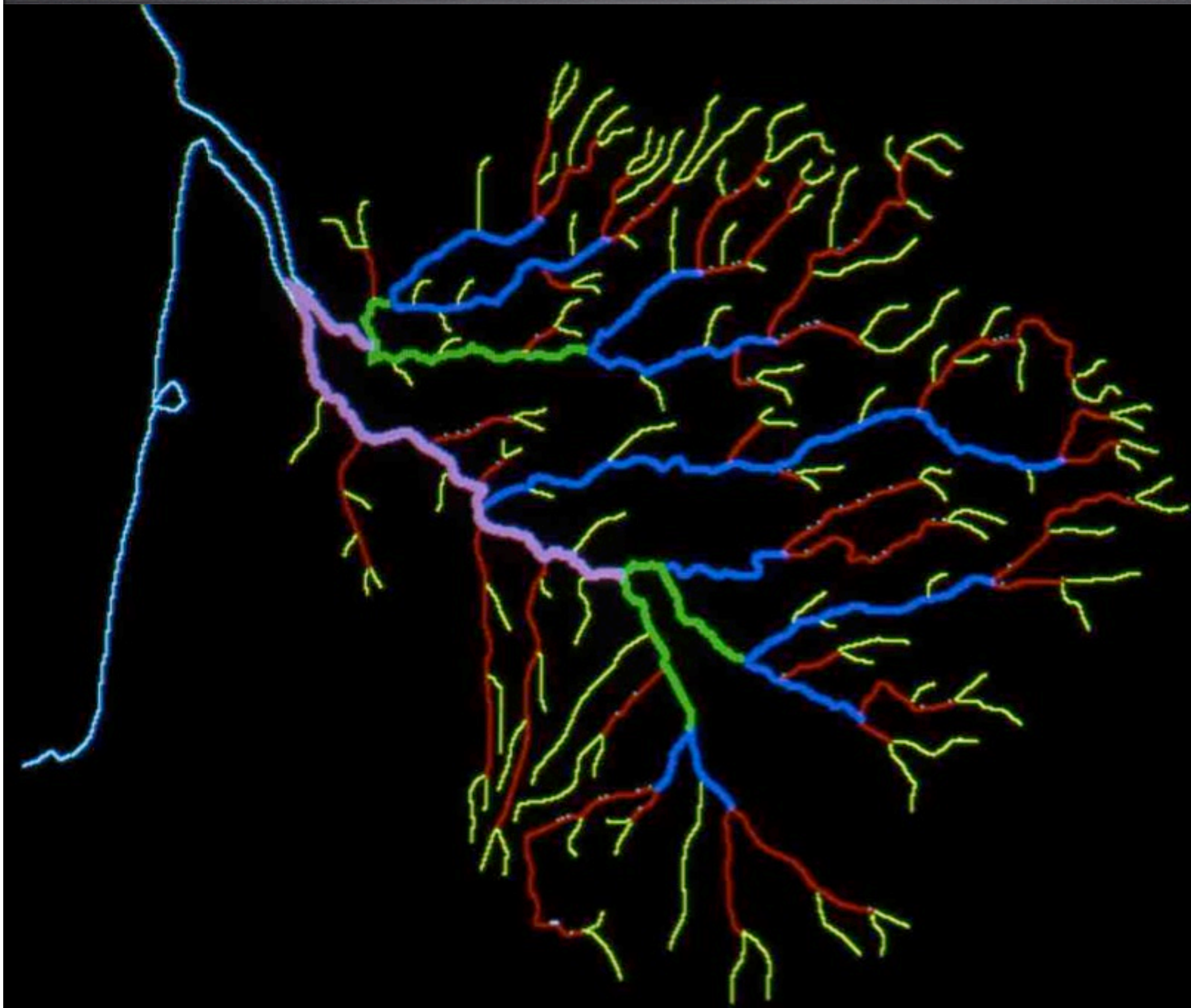


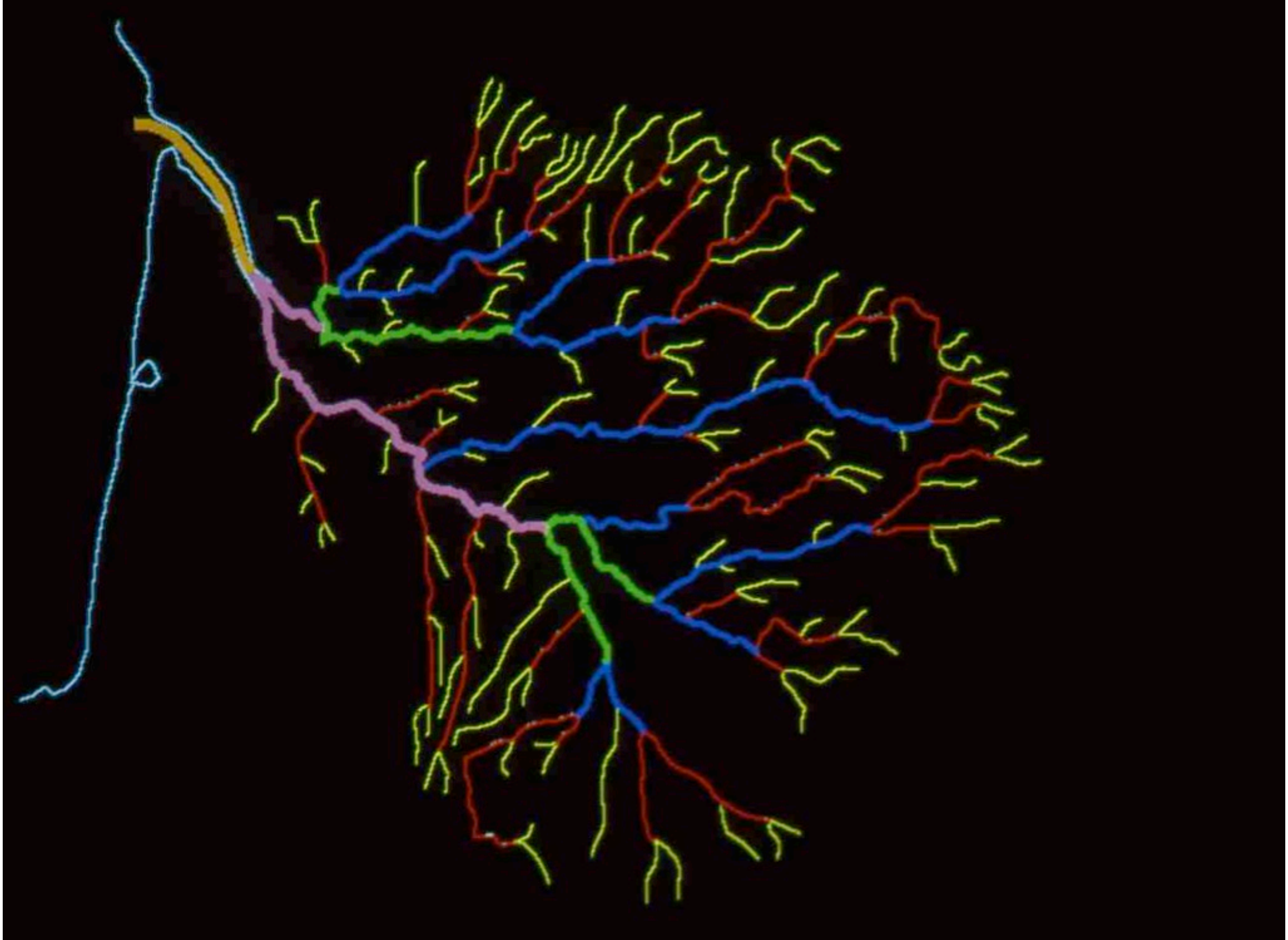




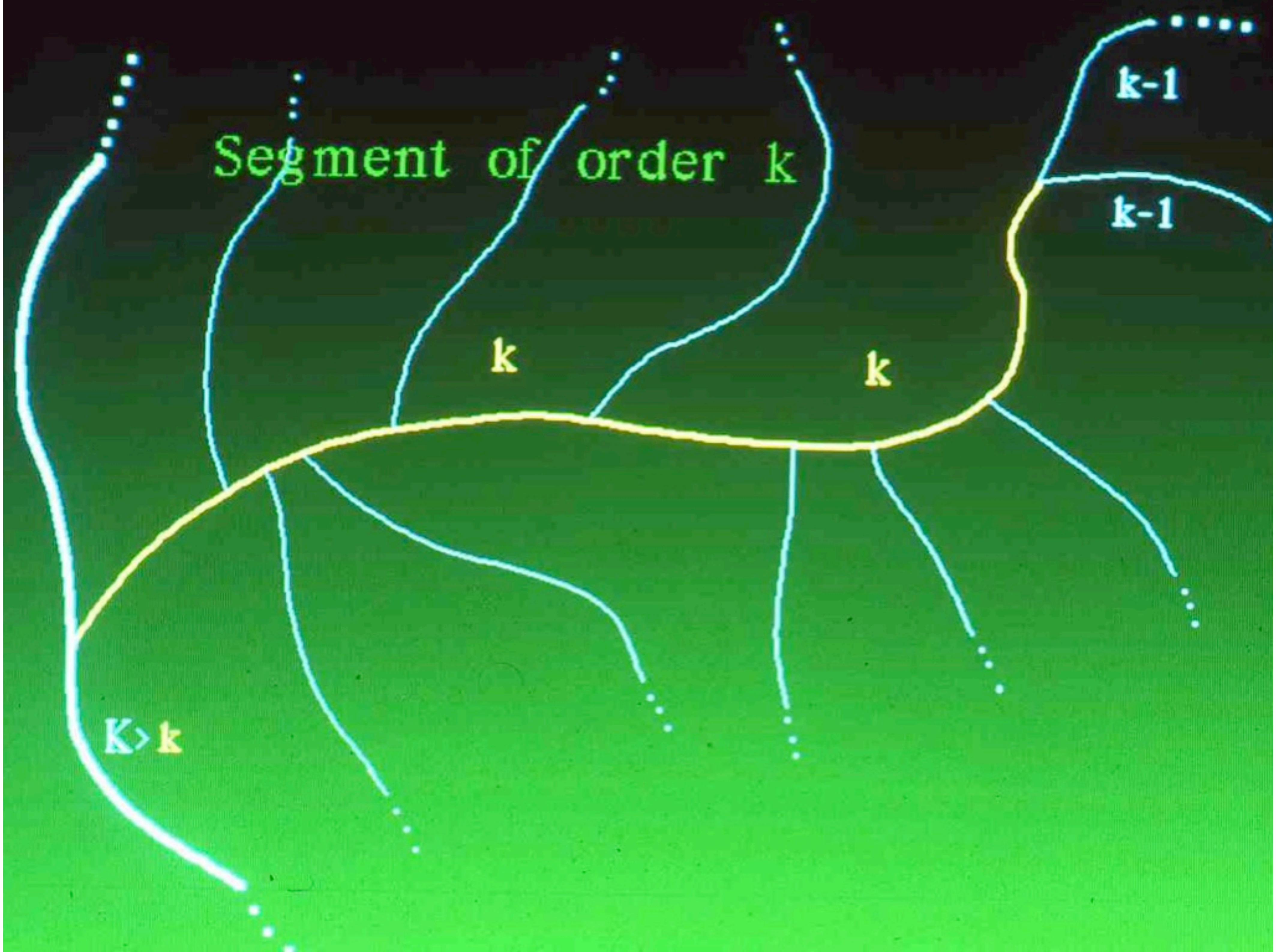








Strahler analysis with bifurcation ratios



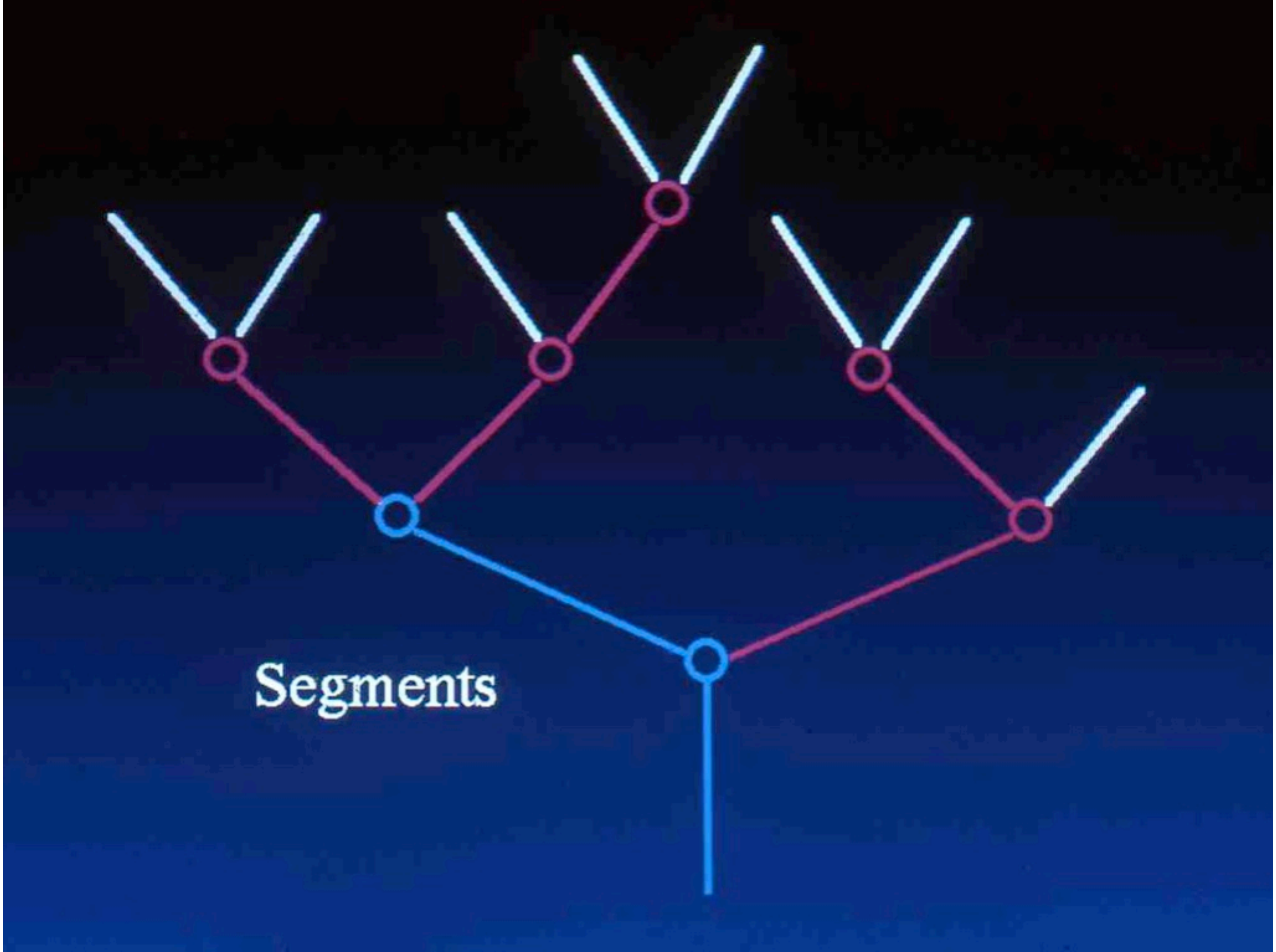
Segment of order k
k-1
k-1
k
k
K>k

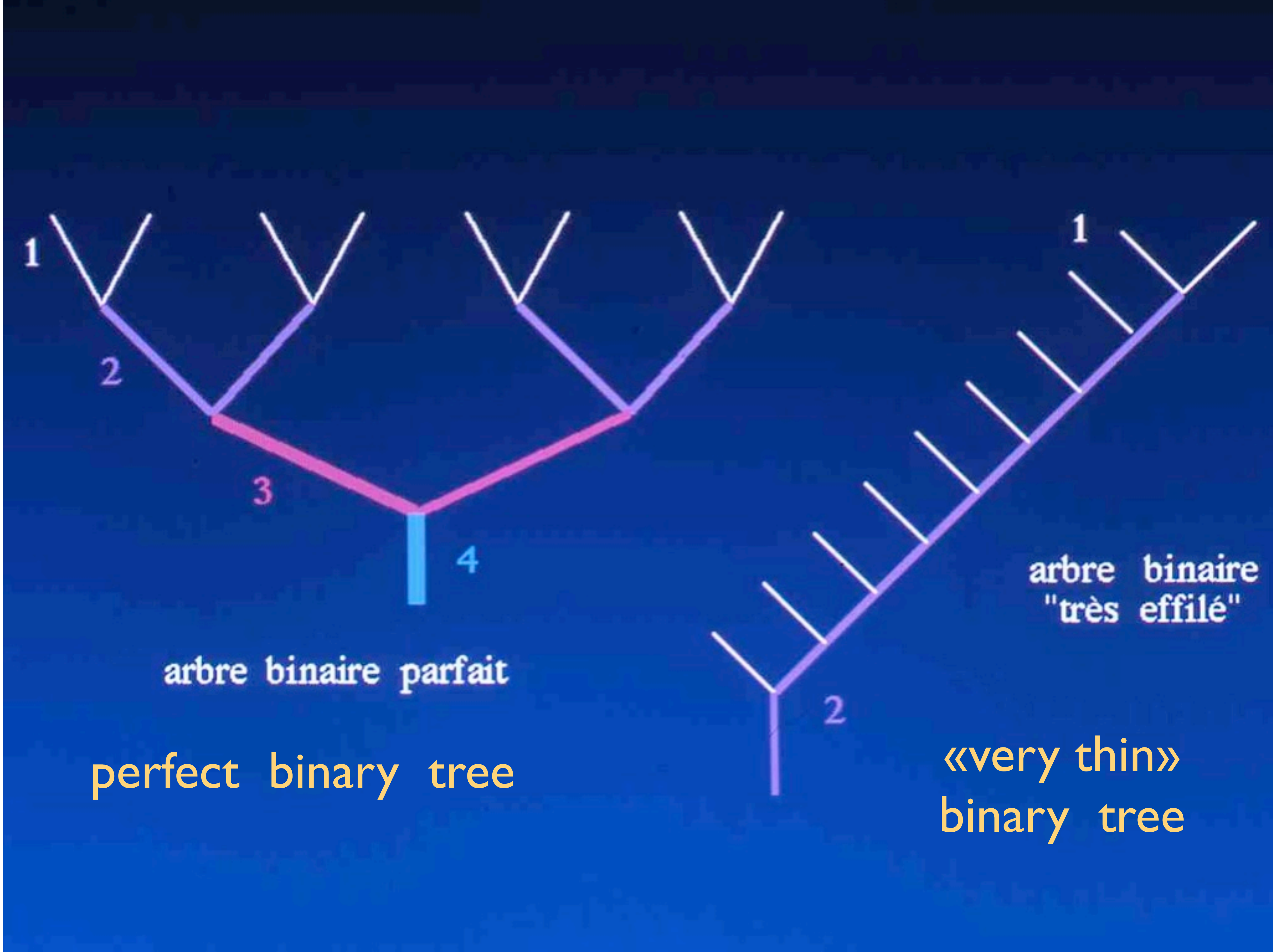
bifurcation ratio

$3 < \beta_k = \beta < 5$

$$\beta_k = \frac{b_k}{b_{k+1}}$$

b_k = number of segments of order k


Segments


1
2
3
4
arbre binaire parfait
perfect binary tree
1
2
arbre binaire
"très effilé"
«very thin»
binary tree

bifurcation ratio

s_k^n = mean numbers of segments of order k (among all binary trees with n internal vertices)

$$\frac{s_k^n}{s_{k+1}^n} = 4 - \frac{4^k}{2n} + O\left(\frac{1}{n^2}\right)$$

Moon 1980

$\beta_k \longrightarrow \beta = 4$

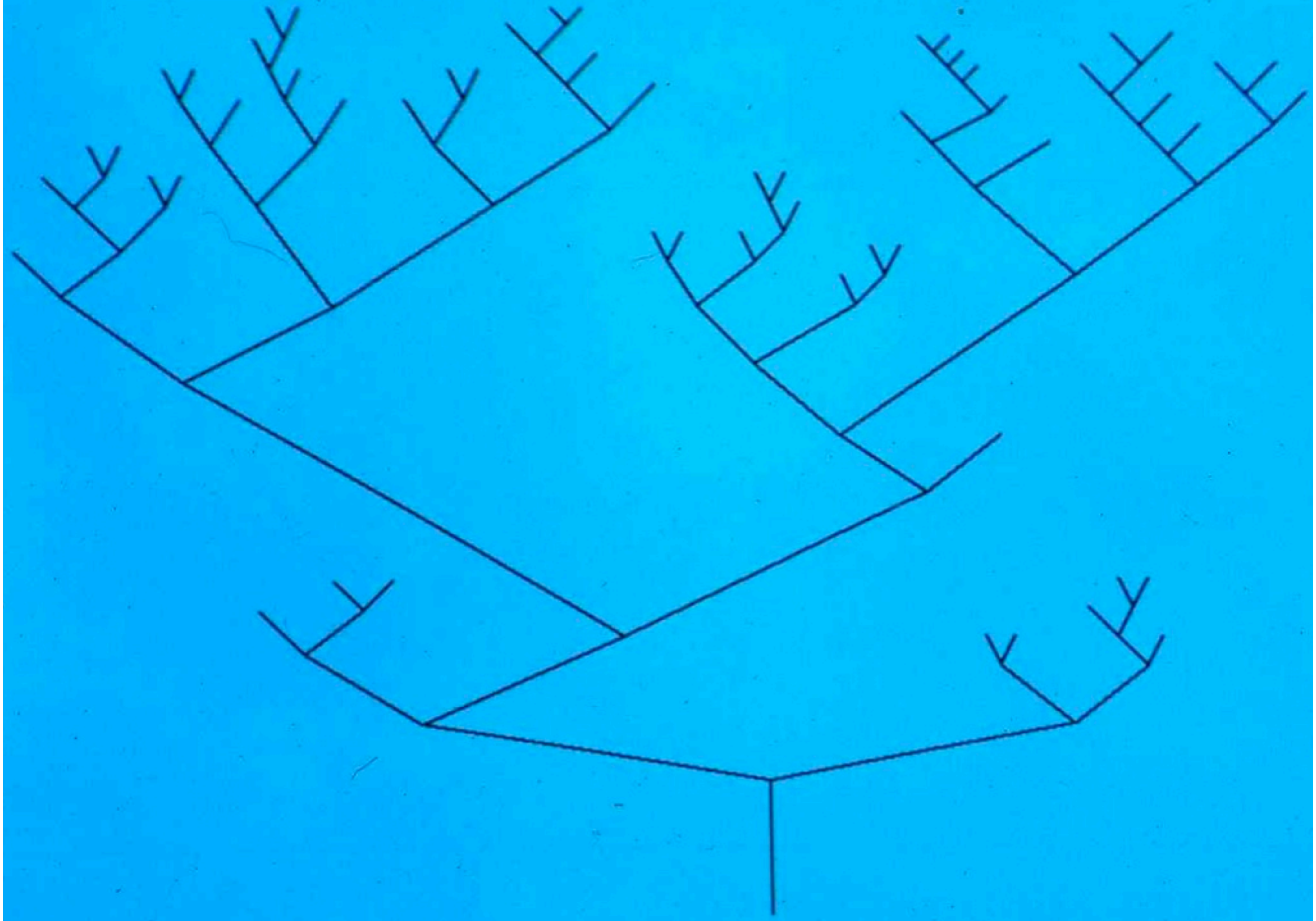
correlation between the «shape» of the river network
and
the structure of the deep underground

Prud'homme, Nadeau, Vigneaux, 1970, 1980

computer graphics

ramification matrix of a binary tree

Arquès, Eyrolles, Janey, X.V.

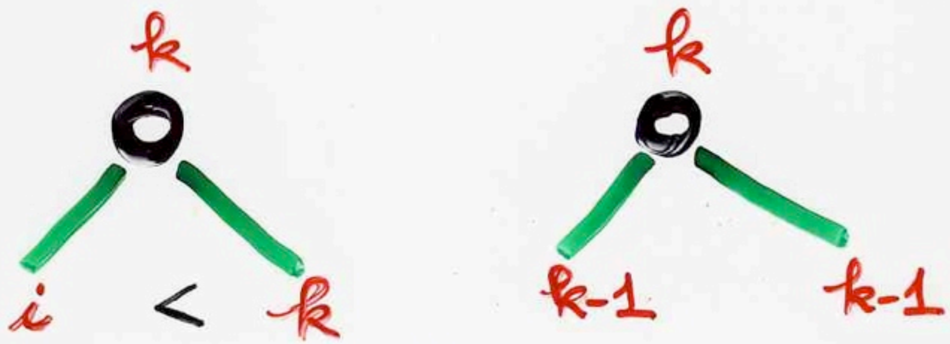


Synthetic images of

trees, leaves, landscapes ...

Arquès, Eyrolles, Janey, X.V.

Ramification matrix

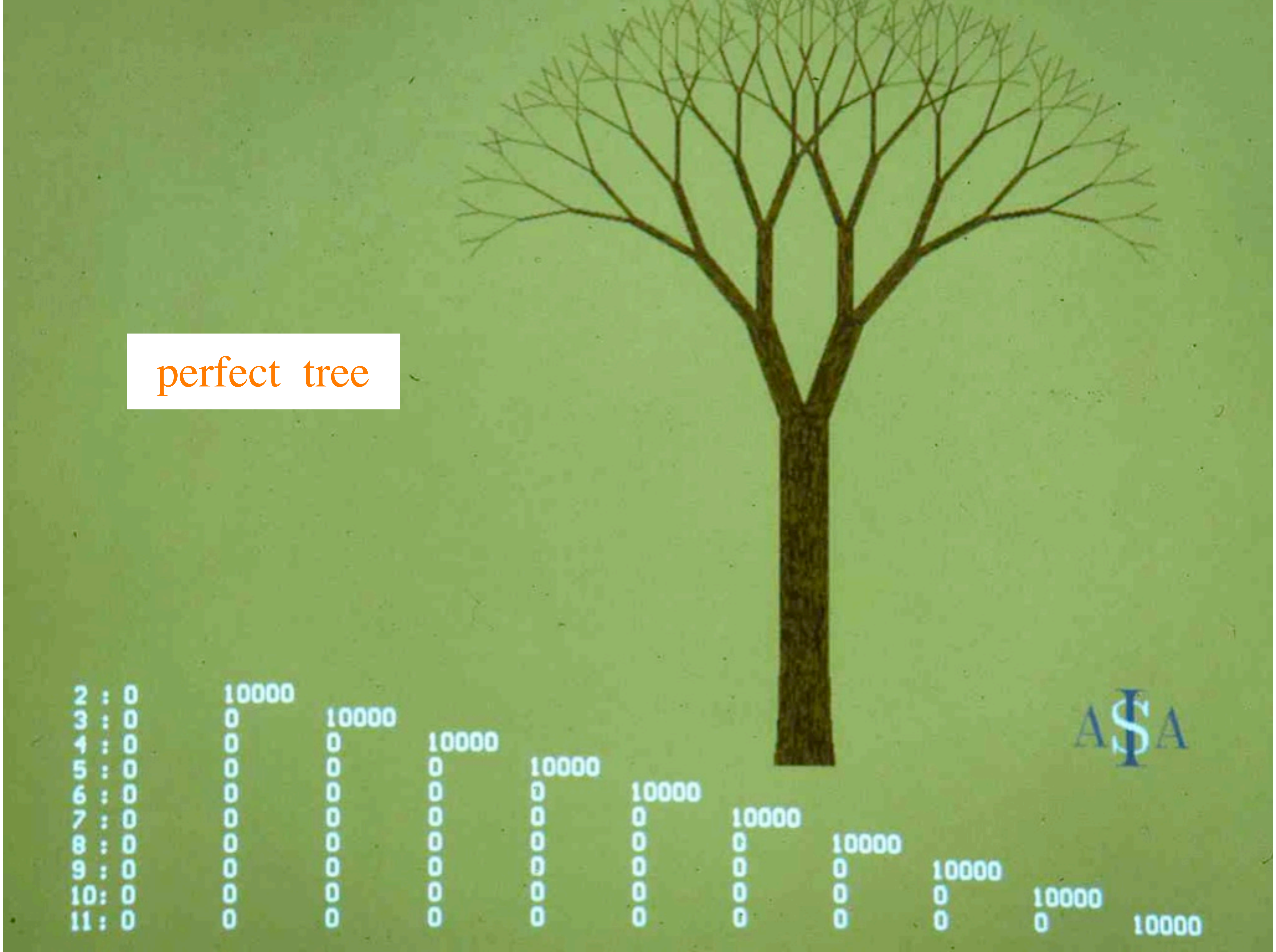


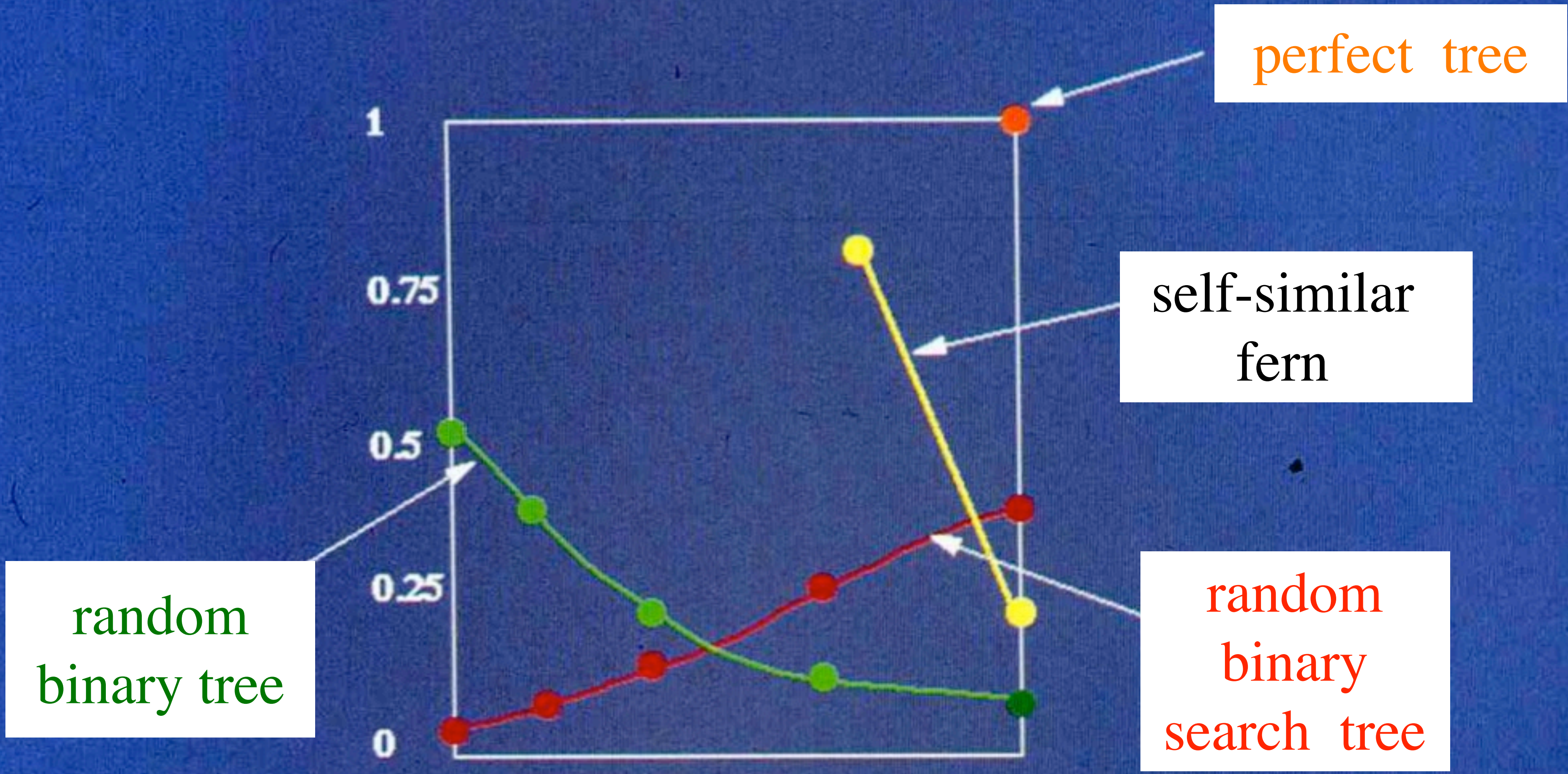
$$P_{k,i} = \frac{b_{k,i}}{a_k}$$

random
binary
search tree

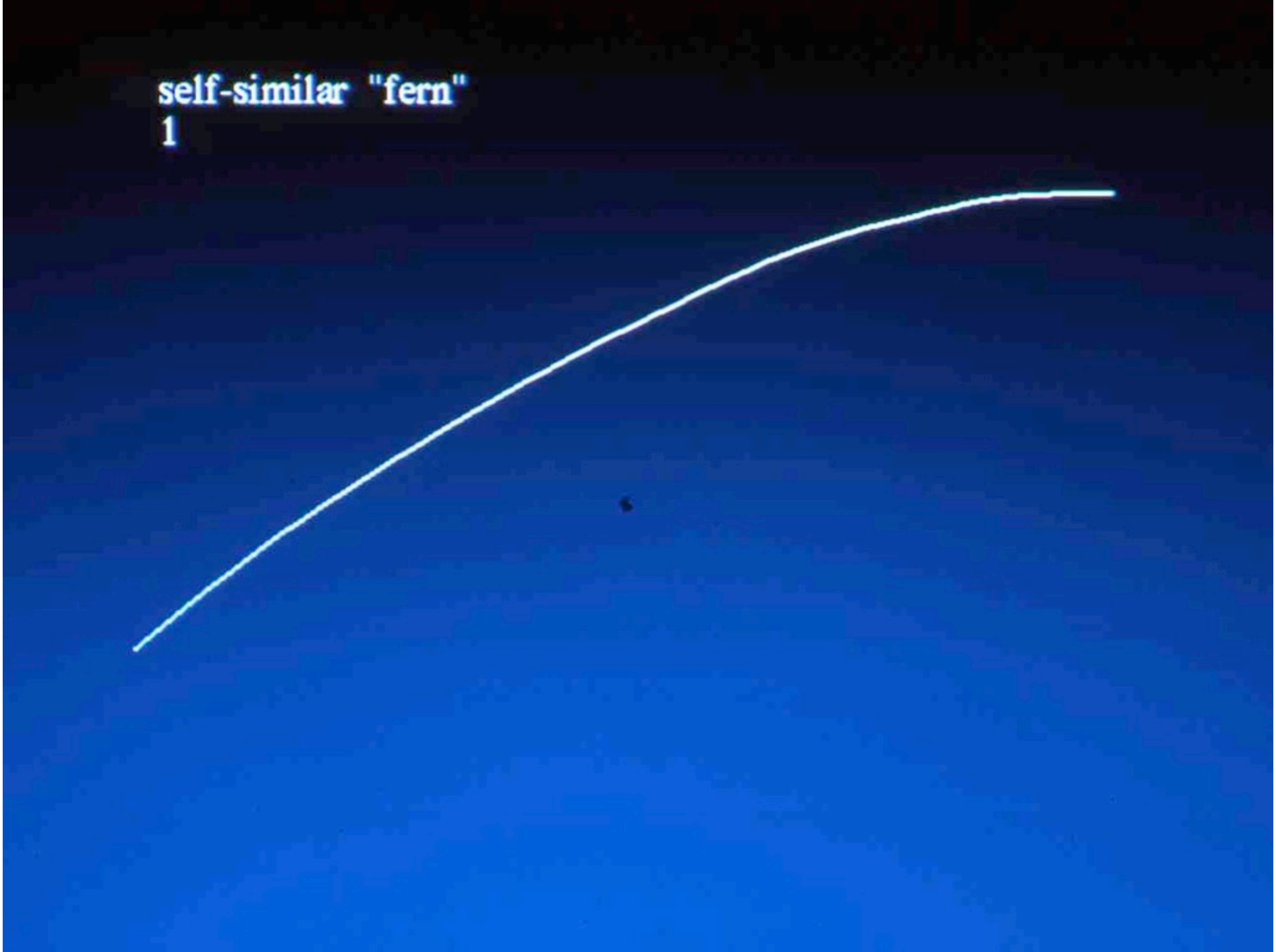
2 :	4000	6000									
3 :	2000	3000	5000								
4 :	1000	2000	3000	4000							
5 :	500	1000	2000	3000	3500						
6 :	250	500	1000	2000	3000	3250					
7 :	125	250	500	1000	2000	3000	3125				
8 :	63	125	250	500	1000	2000	3000	3062			
9 :	31	63	125	250	500	1000	2000	3000	3031		
10:	15	31	63	125	250	500	1000	2000	3000	3016	
11 :	7	15	31	63	125	250	500	1000	2000	3000	3024

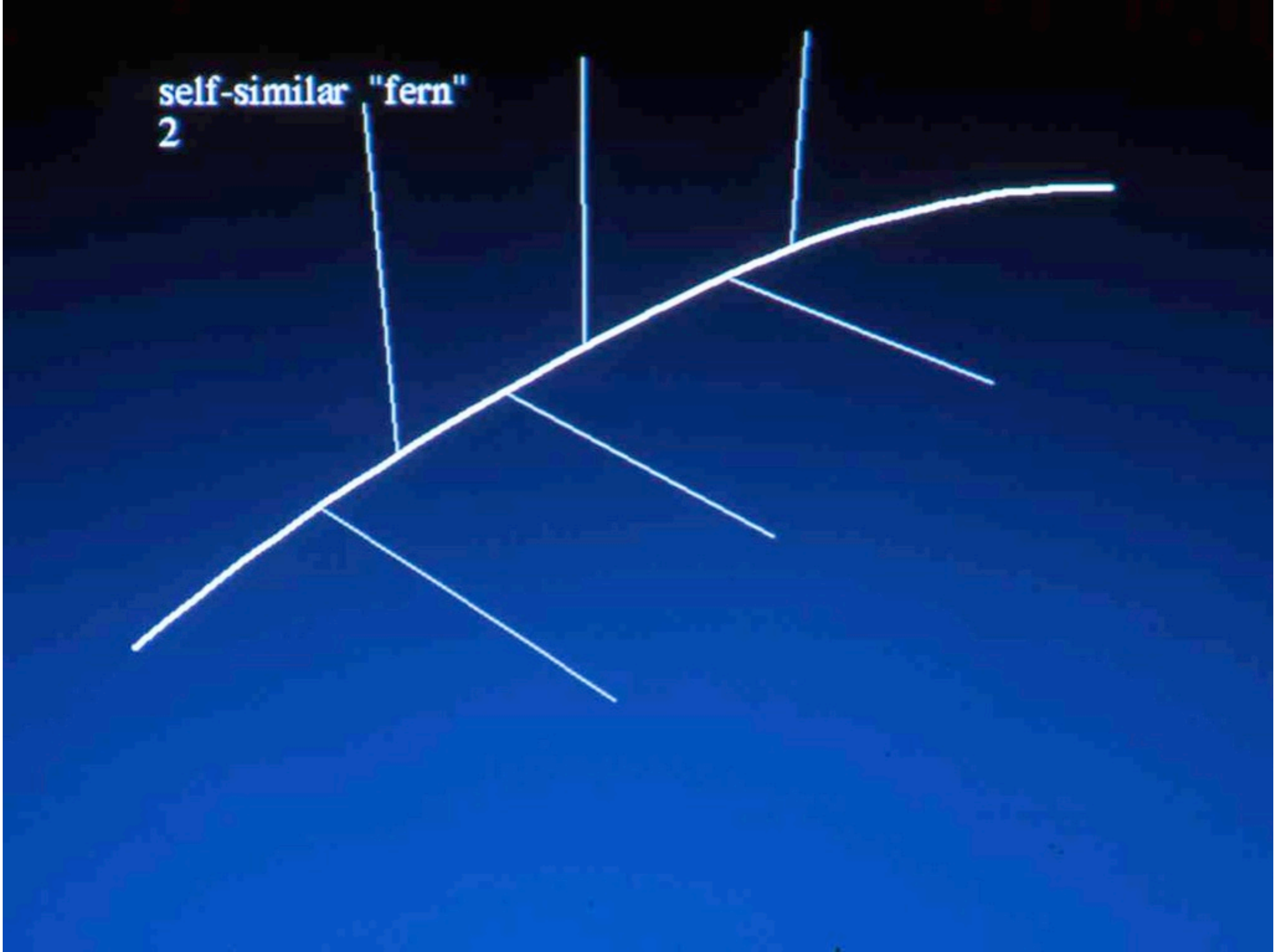

random
binary tree
Arbre binaire aléatoire
en 3D


perfect tree
2 : 0 10000
3 : 0 0 10000
4 : 0 0 0 10000
5 : 0 0 0 0 10000
6 : 0 0 0 0 0 10000
7 : 0 0 0 0 0 0 10000
8 : 0 0 0 0 0 0 0 10000
9 : 0 0 0 0 0 0 0 0 10000
10: 0 0 0 0 0 0 0 0 0 10000
11: 0 0 0 0 0 0 0 0 0 0 10000

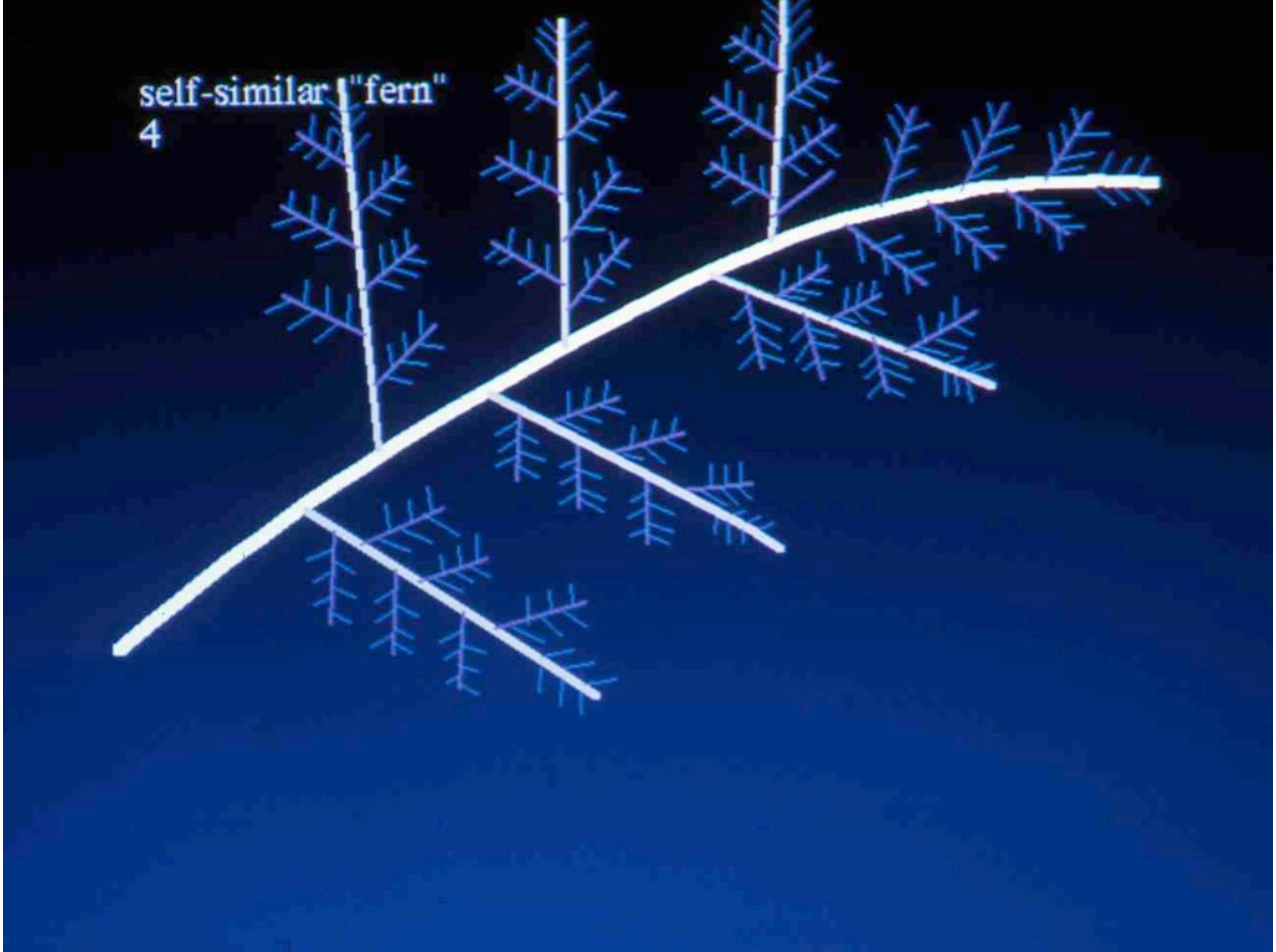


self-similar ramification matrices


self-similar "fern"
1


self-similar "fern"
2

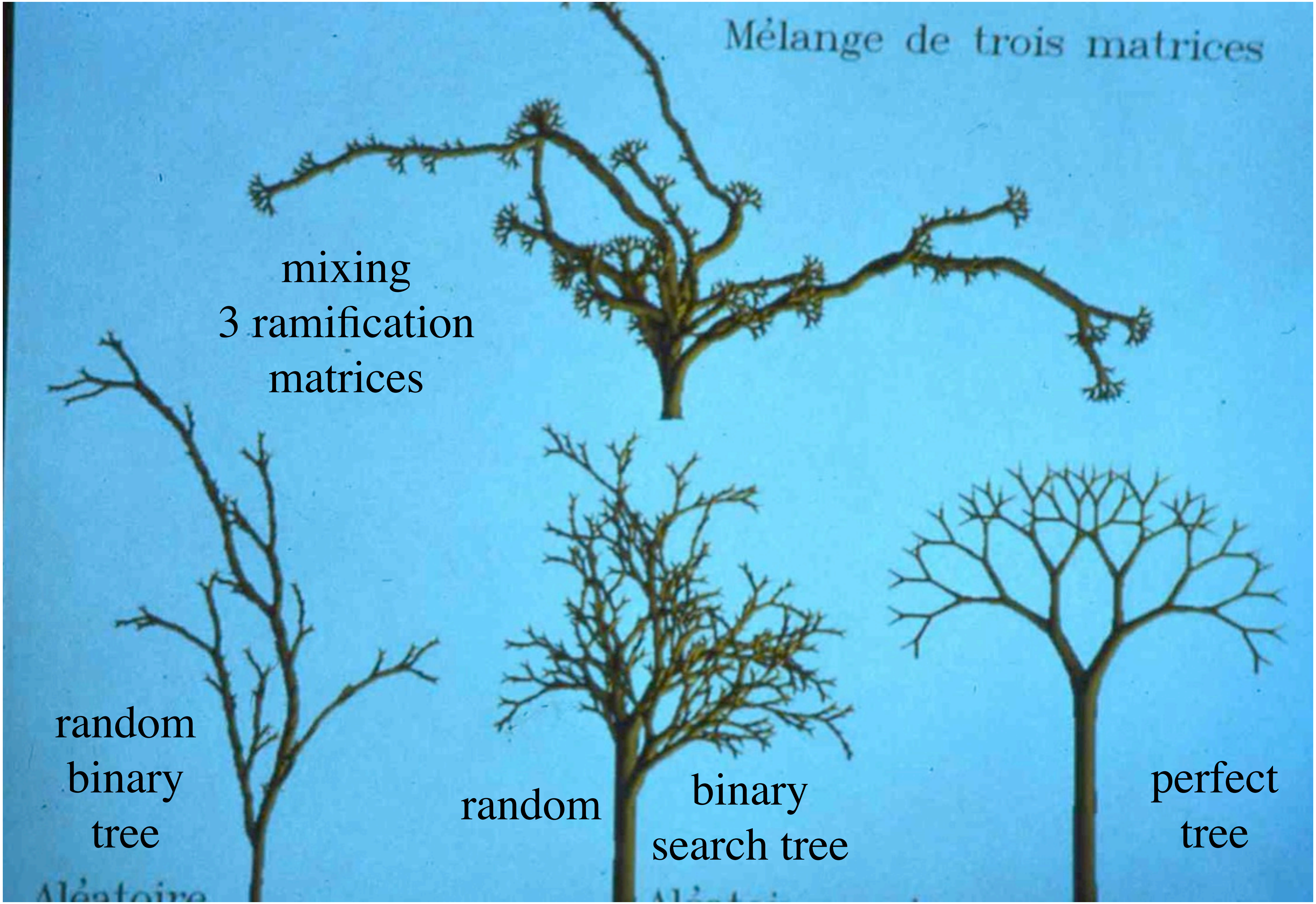

self-similar "fern"
3


self-similar "fern"
4

mixing
3 ramification
matrices

3 «shapes»

2 :	0	10000								
3 :	0	0	10000							
4 :	0	0	0	10000						
5 :	5000	2500	1250	625	625					
6 :	5000	2500	1250	625	313	312				
7 :	125	250	500	1000	2000	3000	3125			
8 :	63	125	250	500	1000	2000	3000	3062		
9 :	31	63	125	250	500	1000	2000	3000	3031	
10:	15	31	63	125	250	500	1000	2000	3000	3016
11 :	7	15	31	63	250	125	500	1000	2000	3000	3009

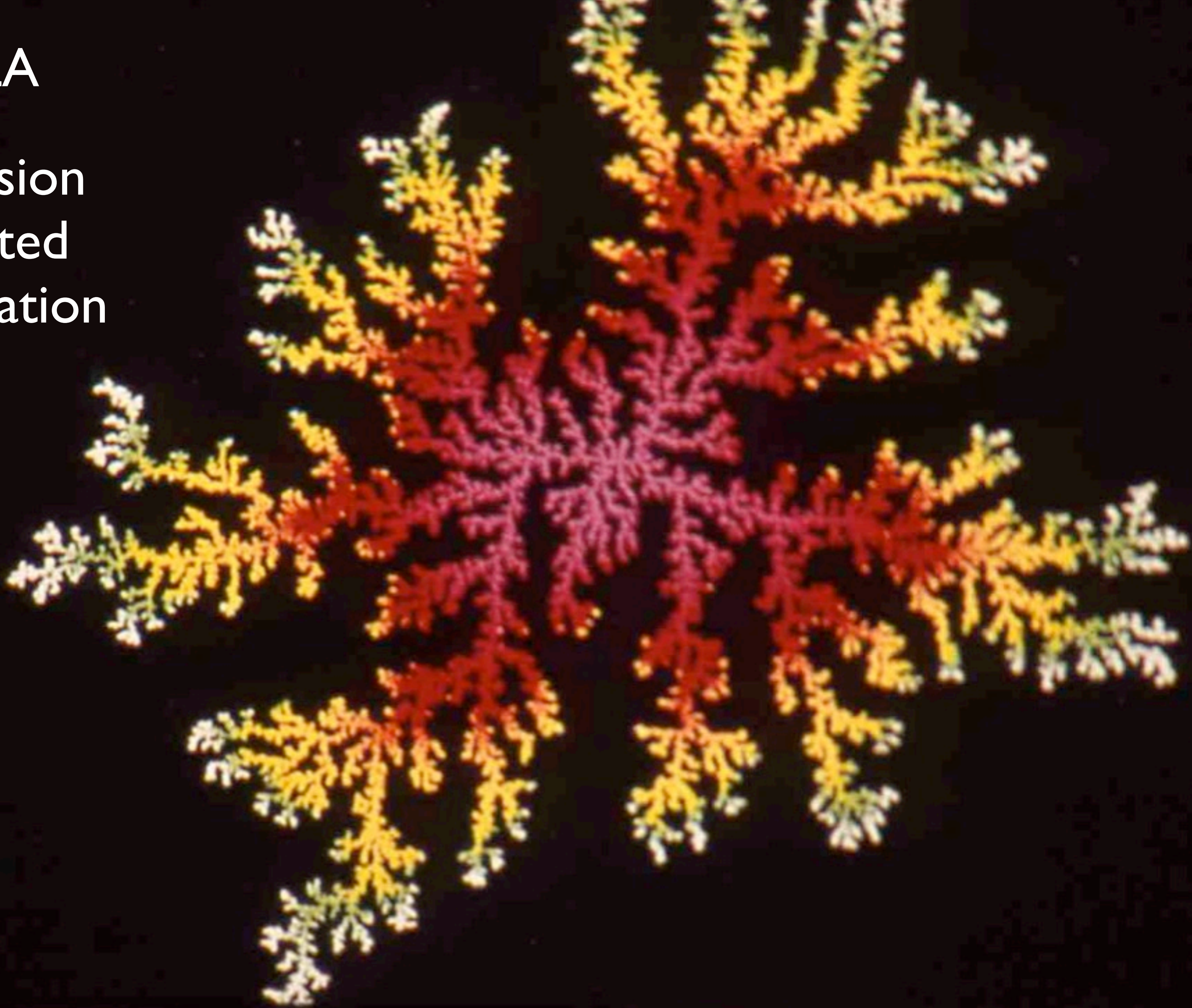

Mélange de trois matrices
mixing
3 ramification
matrices
random
binary
tree
random
binary
search tree
perfect
tree

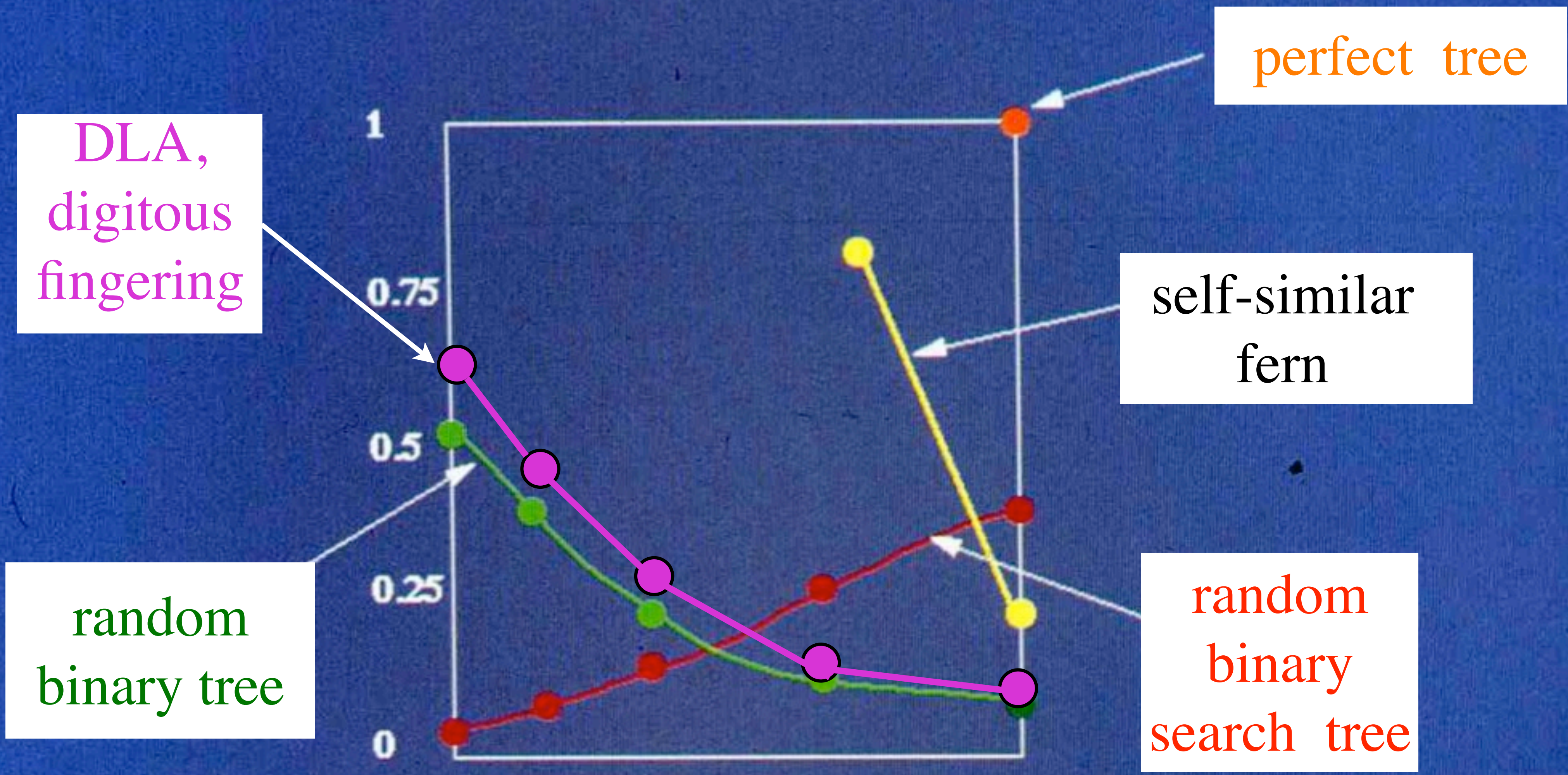
ramification matrix
in physics


digitous
fingering

DLA

Diffusion
Limited
Agregation





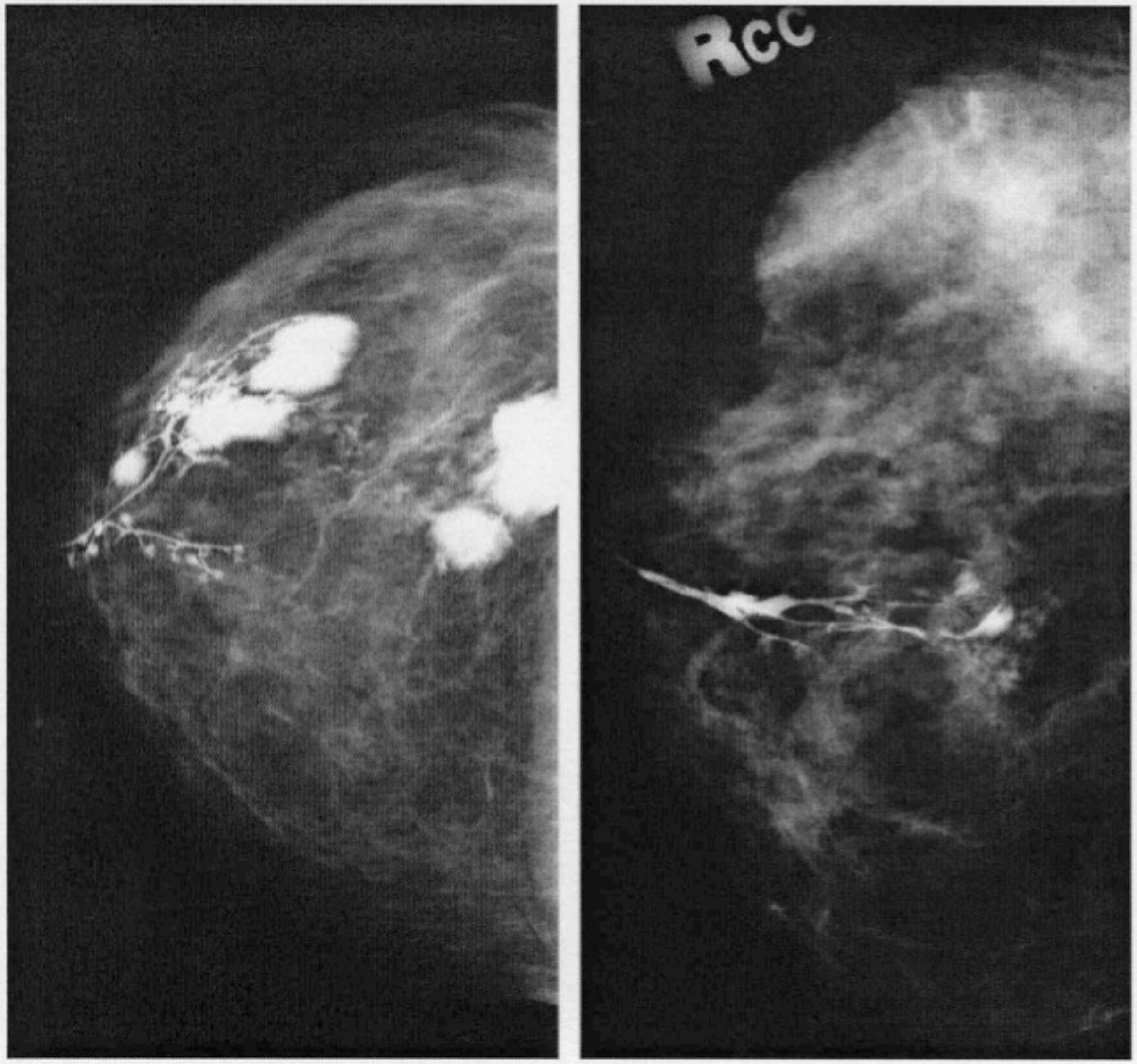
self-similar ramification matrices

- Classification of **Galactograms** with **ramification matrices**

P. Bakic, M. Albert, A. Maidment

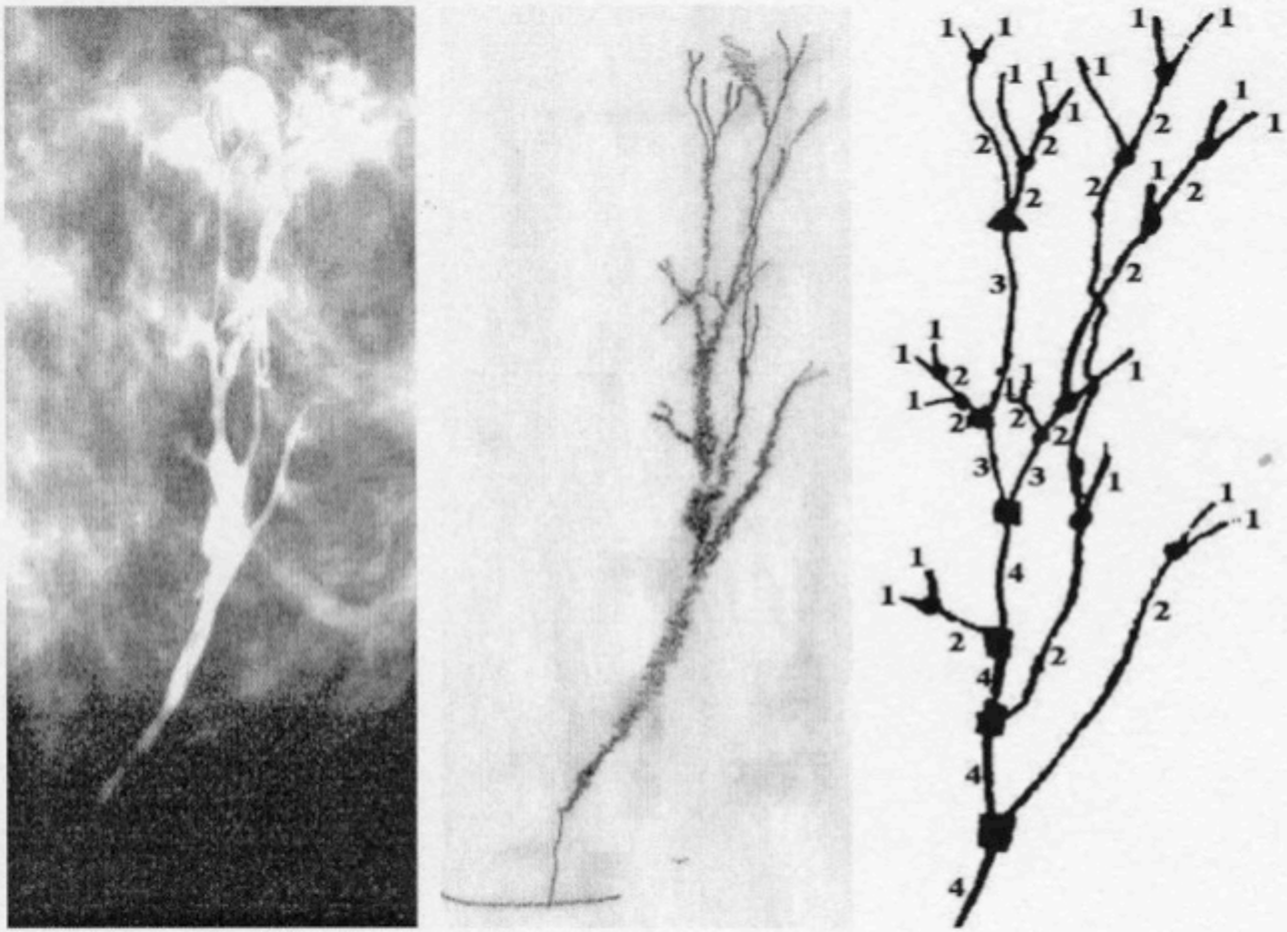
(2003)

Digital mammography



a. b.

Figure 4. Two examples of galactograms that have been correctly classified by means of R matrices. **(a)** Galactogram with no reported findings (patient age, 45 years; right CC view; $r_{3,2} = 0.5$ and $r_{3,3} = 0.19$). (Large bright regions seen in this galactogram are due to extravasation, which did not affect the segmentation of the ductal tree.) **(b)** Galactogram with a reported finding of cysts (patient age, 55 years; right CC view; $r_{3,2} = 0.33$ and $r_{3,3} = 0.67$).



a. b. c.

$$R = \begin{bmatrix} r_{2,1} & r_{2,2} & . & . \\ r_{3,1} & r_{3,2} & r_{3,3} & . \\ r_{4,1} & r_{4,2} & r_{4,3} & r_{4,4} \end{bmatrix} = \begin{bmatrix} 0.43 & 0.57 & . & . \\ 0 & 0.33 & 0.67 & . \\ 0 & 0.75 & 0 & 0.25 \end{bmatrix}$$

d.

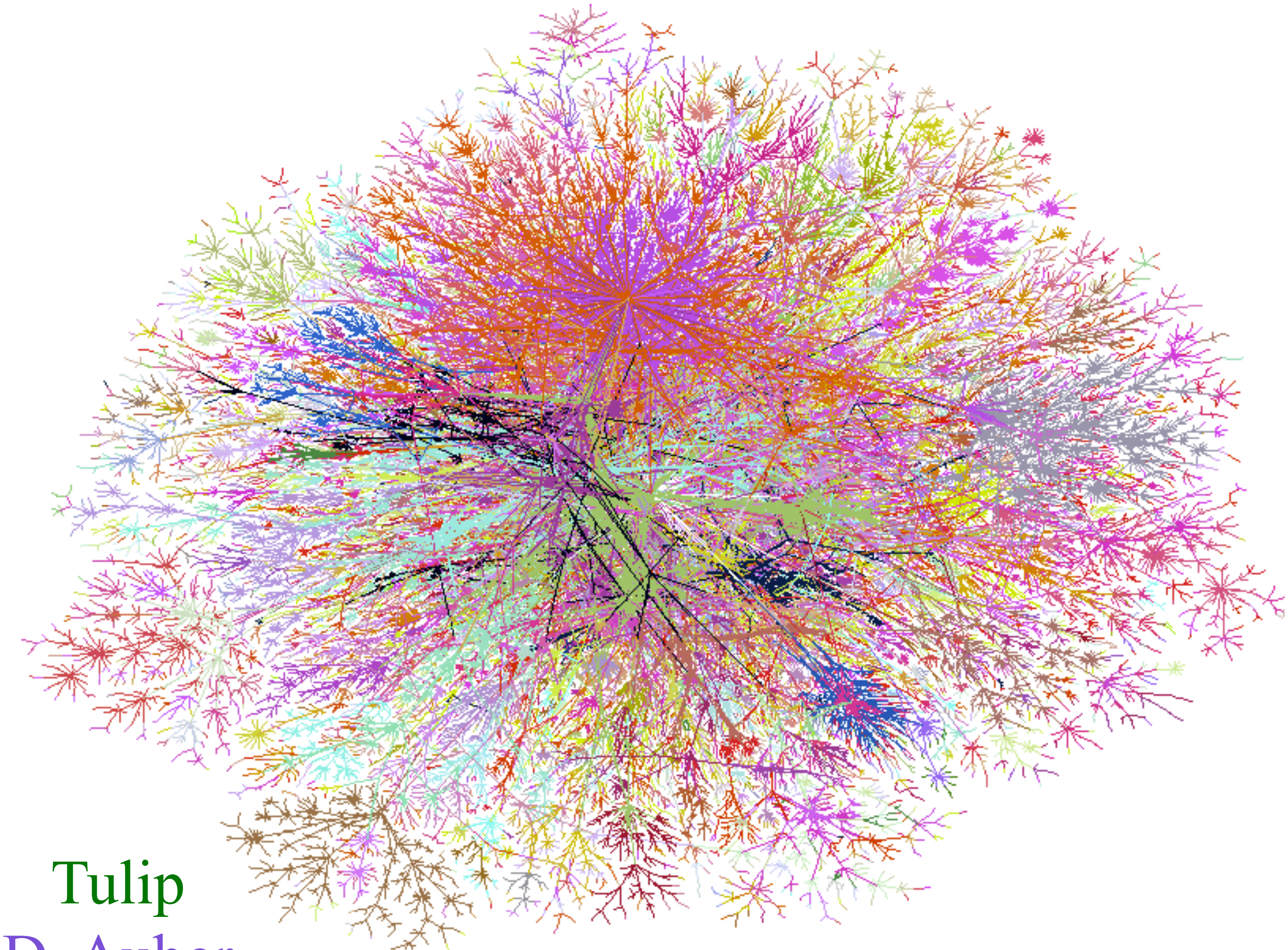
Figure 1. Segmentation of a ductal tree, showing **(a)** part of a galactogram with a contrast–enhanced ductal network, **(b)** the manually traced network of larger ducts from the contrast–enhanced portion of the galactogram, **(c)** numeric labeling of branches in the ductal network, and **(d)** the R matrix computed from the branching pattern. The dots, triangles, and squares denote branching points of different levels of the tree.

visualization of information

Visualization of information for very large graphs

- Visualisation de l'information
très grands graphes
D. Auber, M. Delest
Y. Chiricota, G. Melançon, J.M. Fedou
analyse de Horton-Strahler

extension of Horton-Strahler analysis for graphs



Tulip

D. Auber

Synthetic images of
trees, leaves, landscapes ...

Arquès, Eyrolles, Janey, X.V.

SIGGRAPH'89, IMAGINA' 90

Génération et visualisation des arbres

Génération de l'arbre combinatoire

choix de la matrice de ramification

Génération de l'arbre géométrique

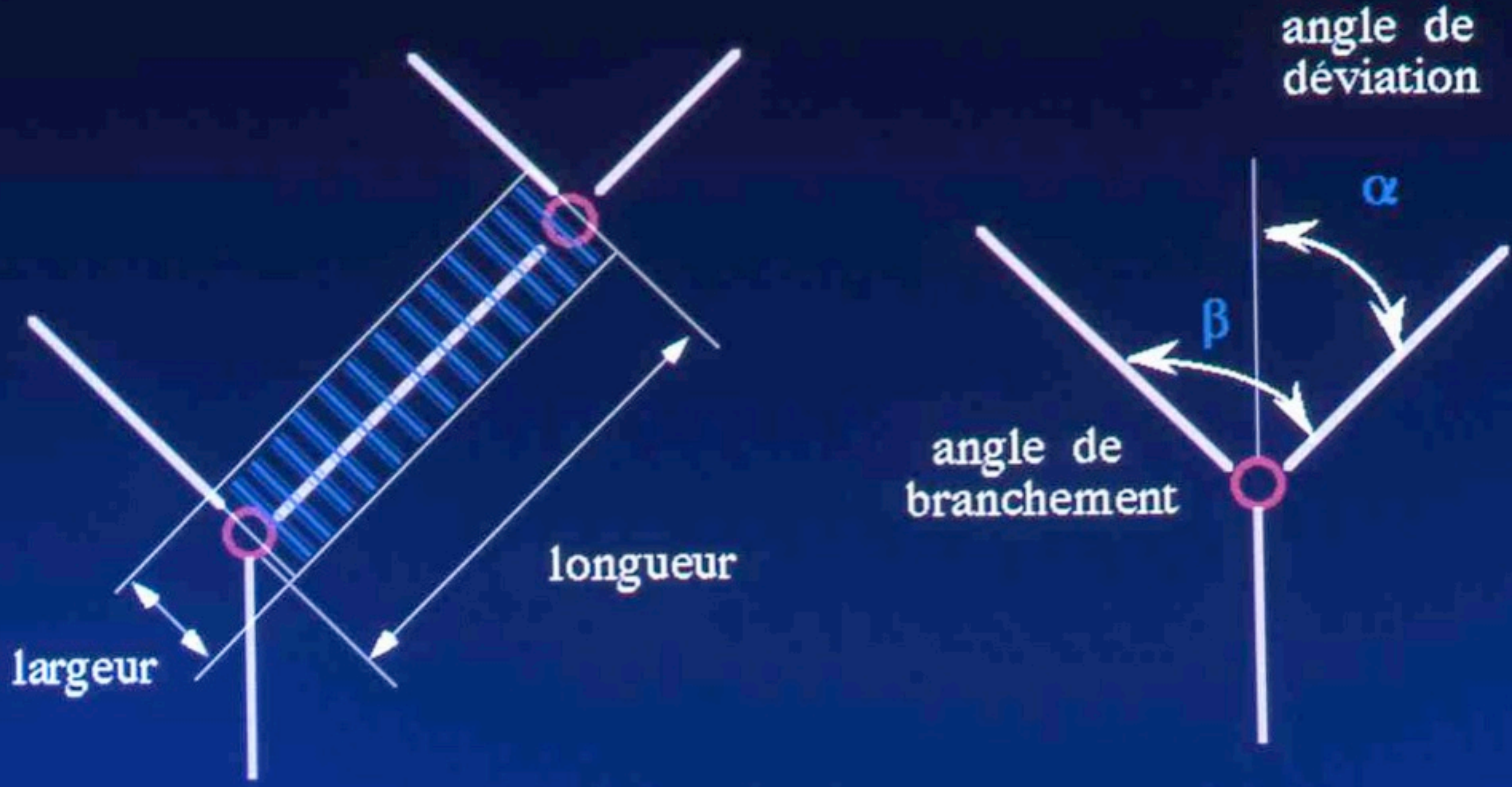
- largeur w(k)
- longueur L(k)

} fonction de l'ordre

- angle de déviation
- angle de branchement

} fonction du biordre

dessin en 2D

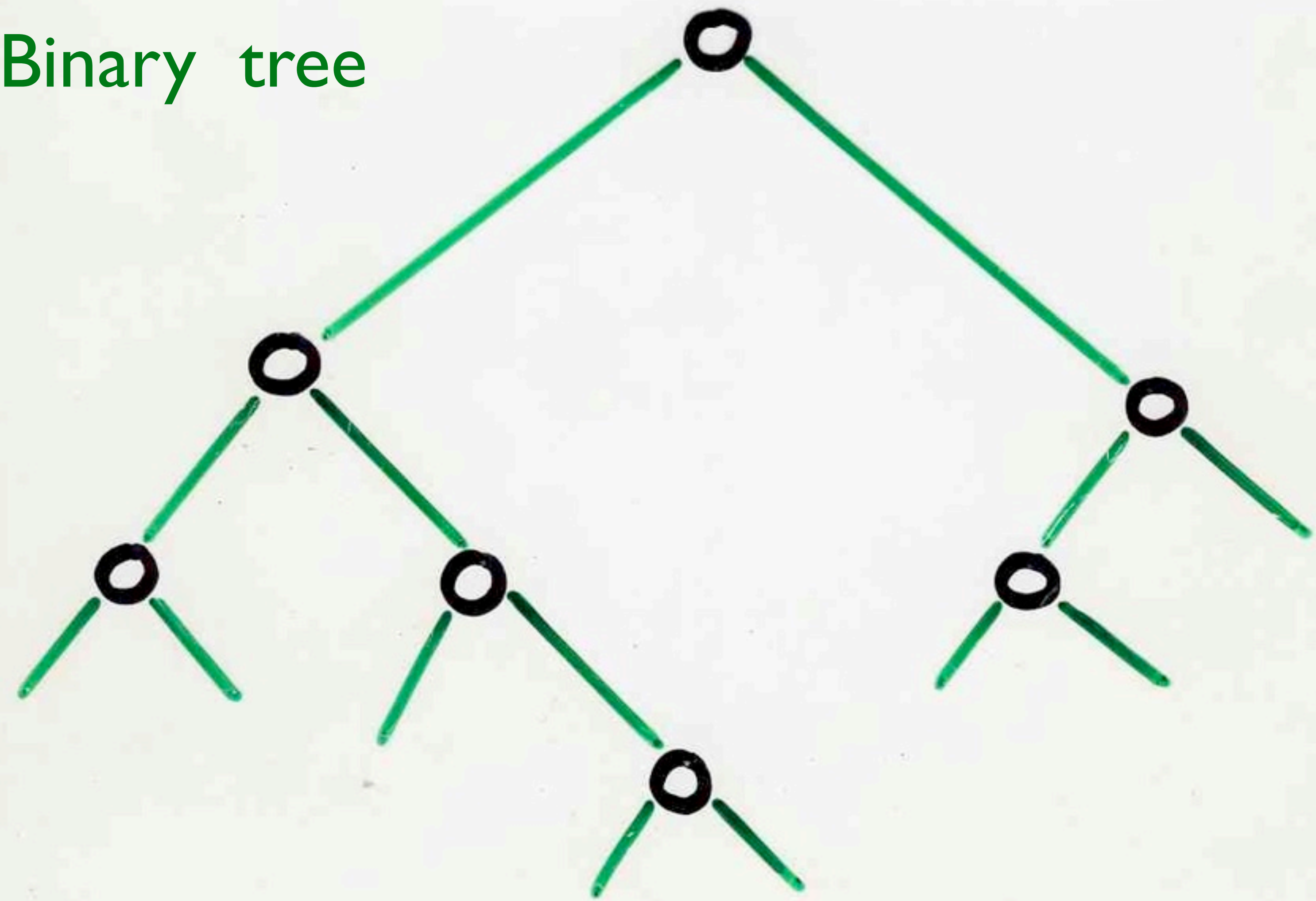


angle de
déviation
α
β
angle de
branchement
longueur
largeur


ASA

If there exist some beauty in these
synthetic images of trees,
it is only the pale reflection of the
extraordinary beauty of the
mathematics hidden behind the
algorithms generating these images

Binary tree



number of binary trees
having n internal vertices
(or n+1) leaves (external vertices)

C_n = number of **binary trees**

with n "internal nodes"

(= n+1 "external nodes")

Catalan number

$$C_n = \frac{1}{n+1}\binom{2n}{n}$$

$$= \frac{(2n)!}{(n+1)!\, n!}$$

$$n! = 1 \times 2 \times \dots \times n$$

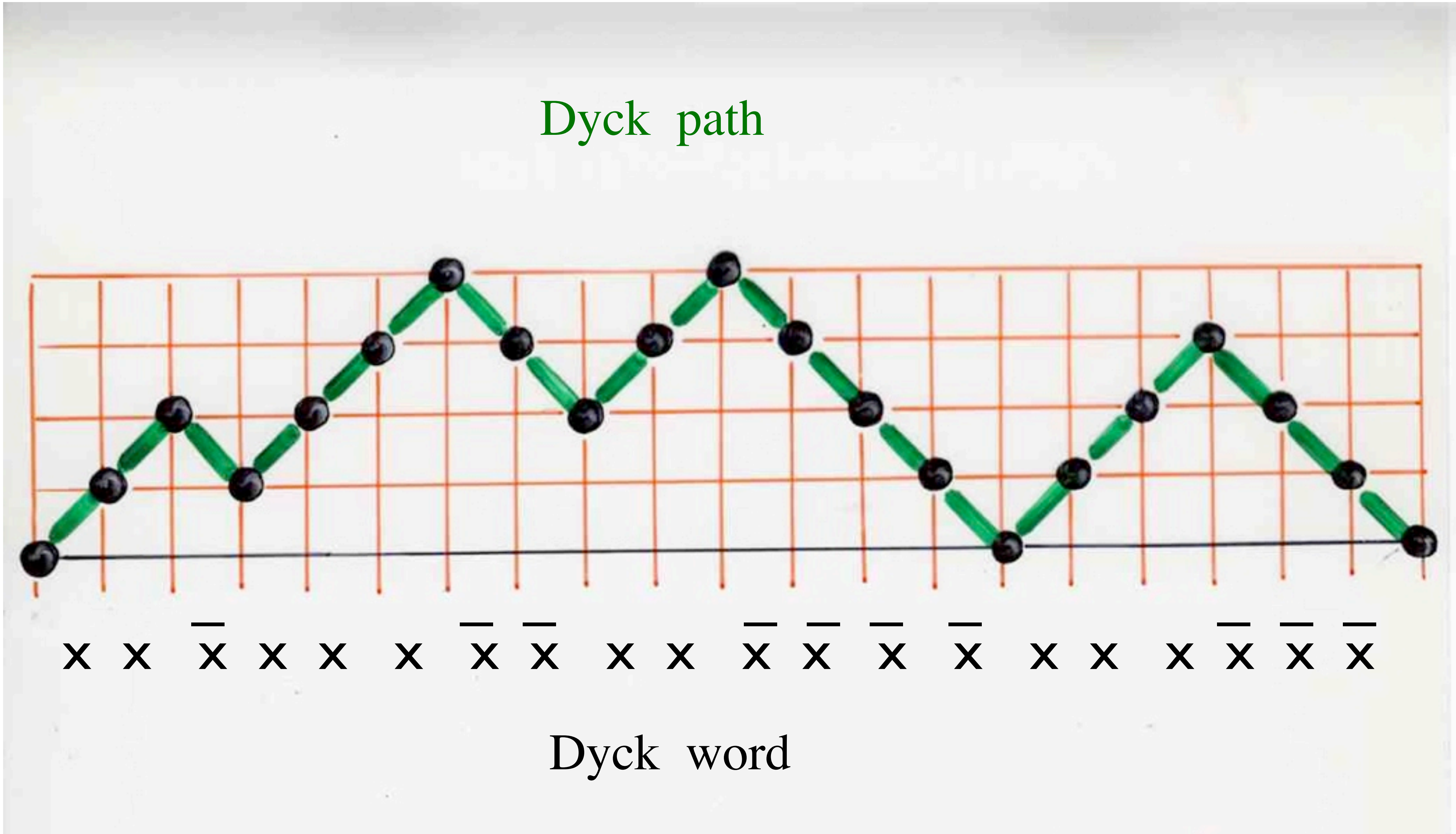
from binary trees

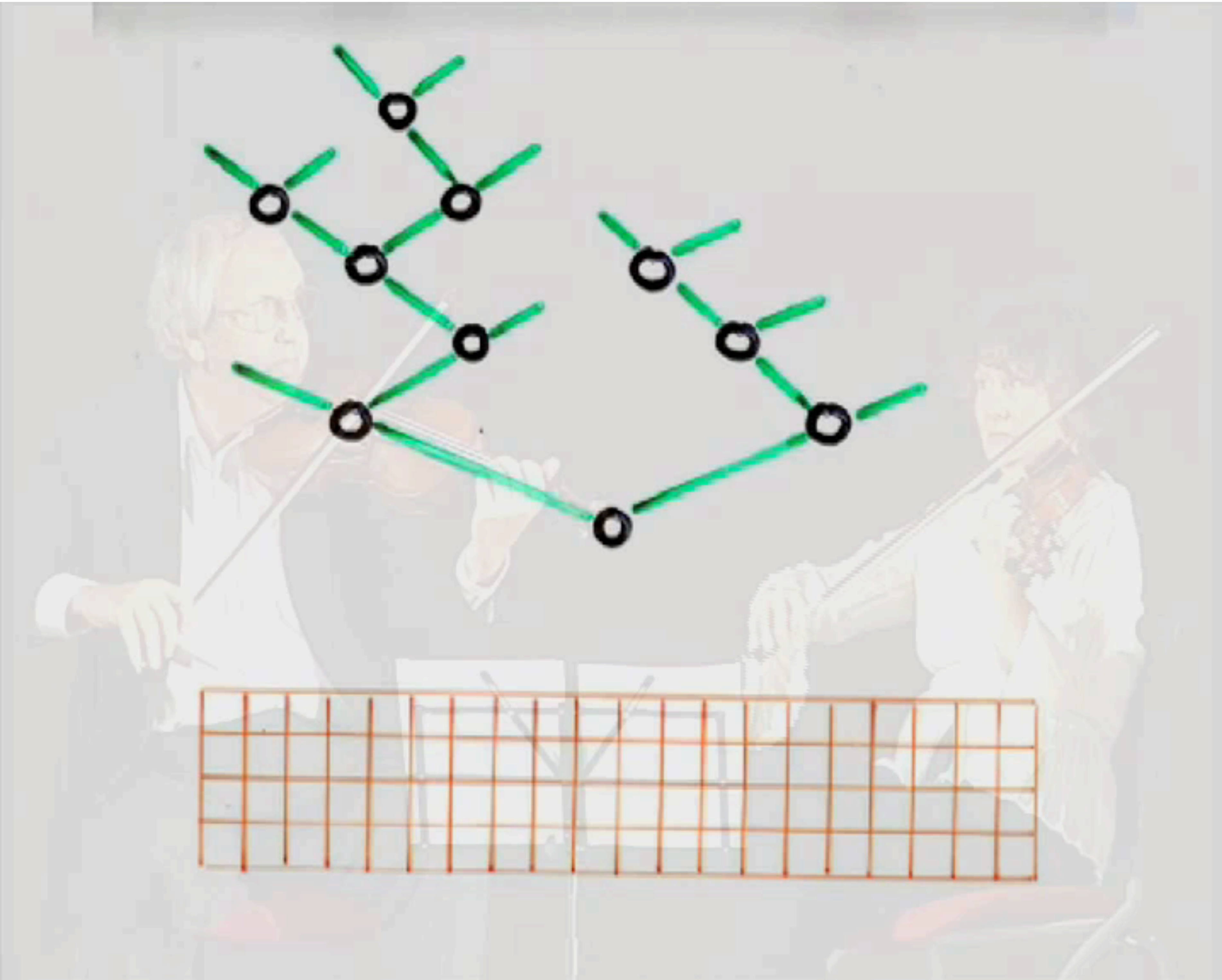
to Dyck paths ....

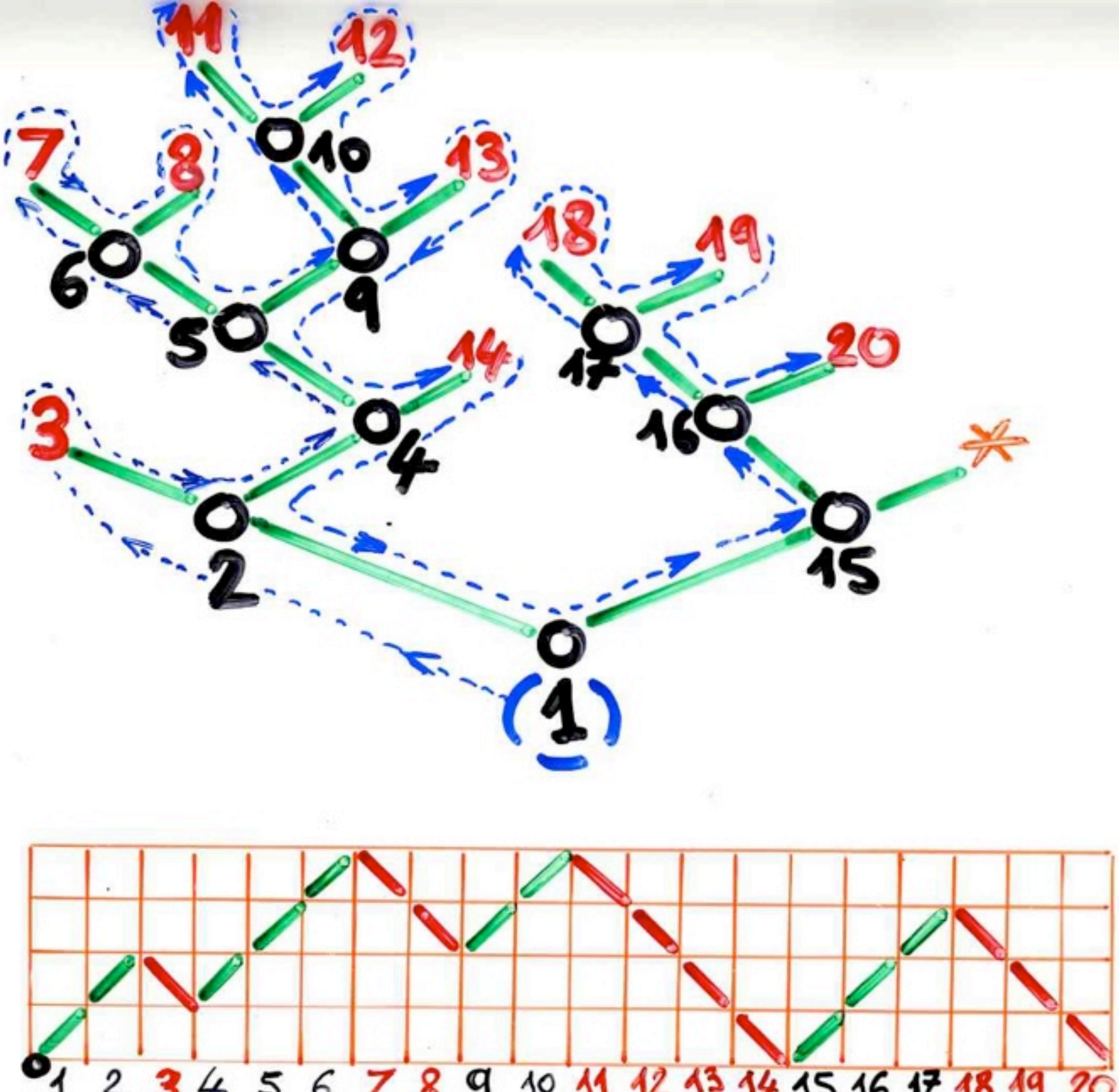
Dyck paths

Dyck words

(nested strings)

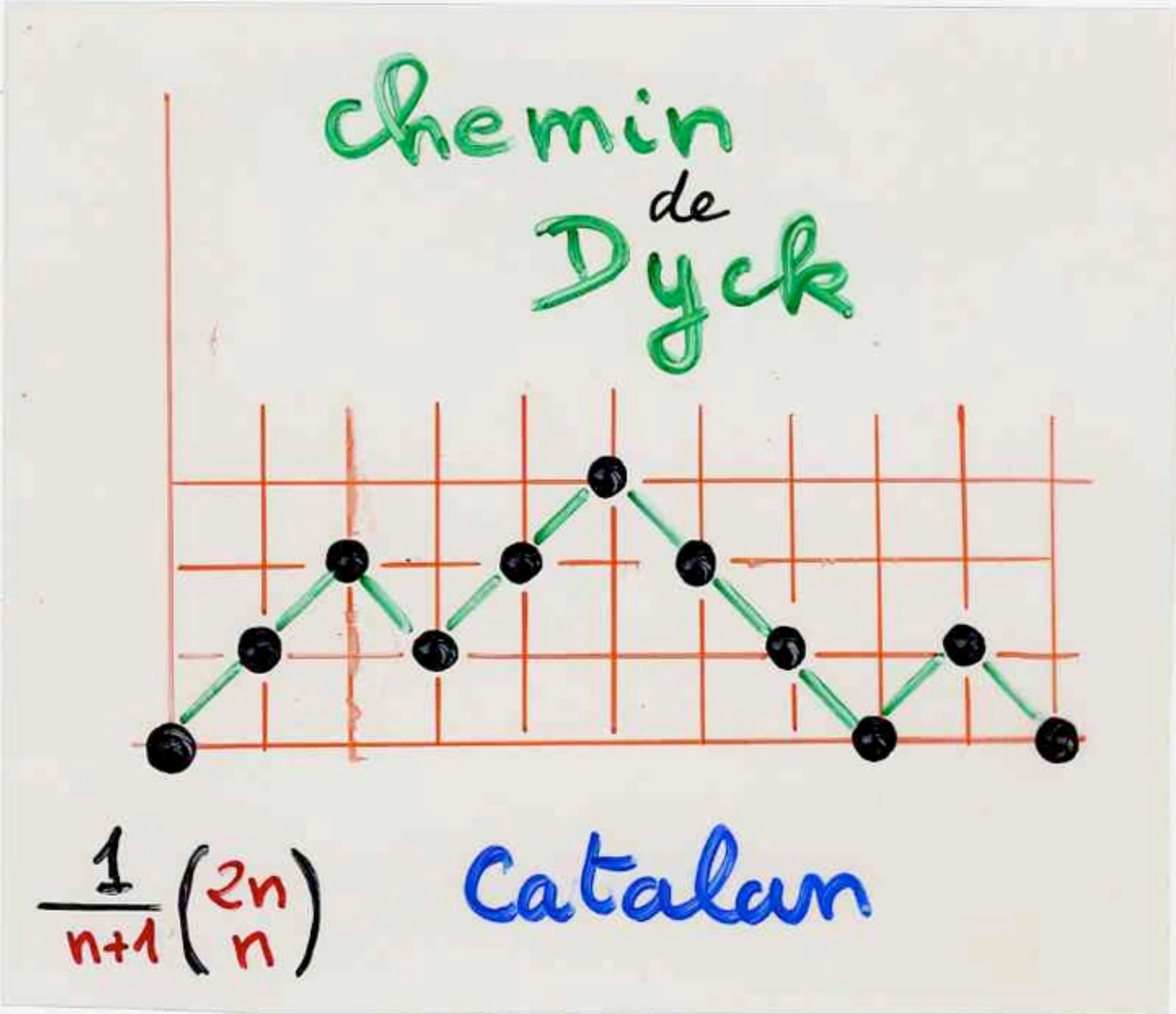

Dyck path
x x $\bar{x}$ x x x $\bar{x}$ $\bar{x}$ x x $\bar{x}$ $\bar{x}$ $\bar{x}$ $\bar{x}$ x x x $\bar{x}$ $\bar{x}$ $\bar{x}$
Dyck word





11
12
7
8
10
13
18
19
6
9
5
14
17
20
3
16
4
*
2
15
1
0
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

logarithmic height



Dyck path

Dyck path w

Height $h(w)$

logarithmic height $lh(w)$

$$= \lfloor \log_2 (1+h(w)) \rfloor$$

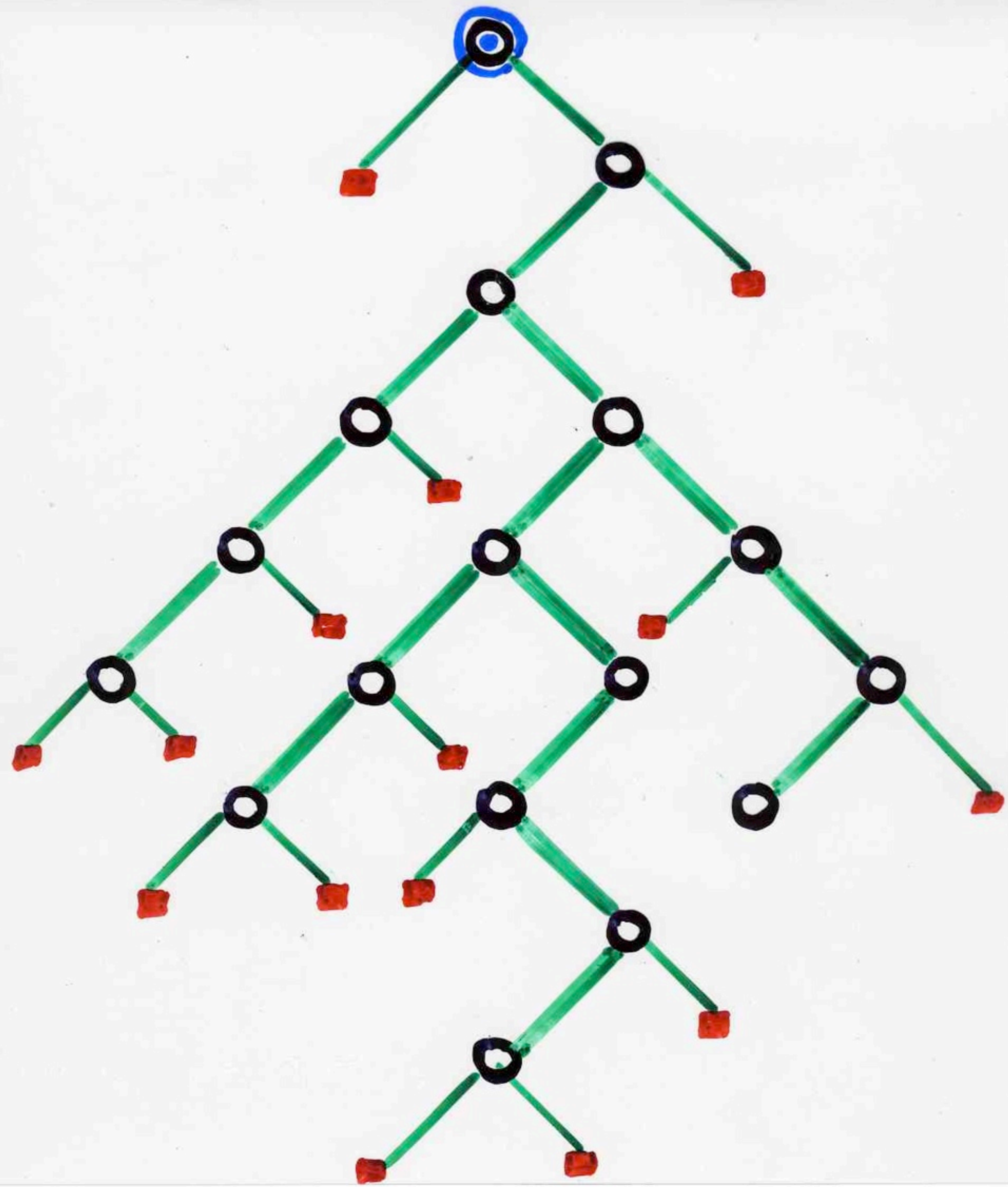
$$lh(w) = k$$

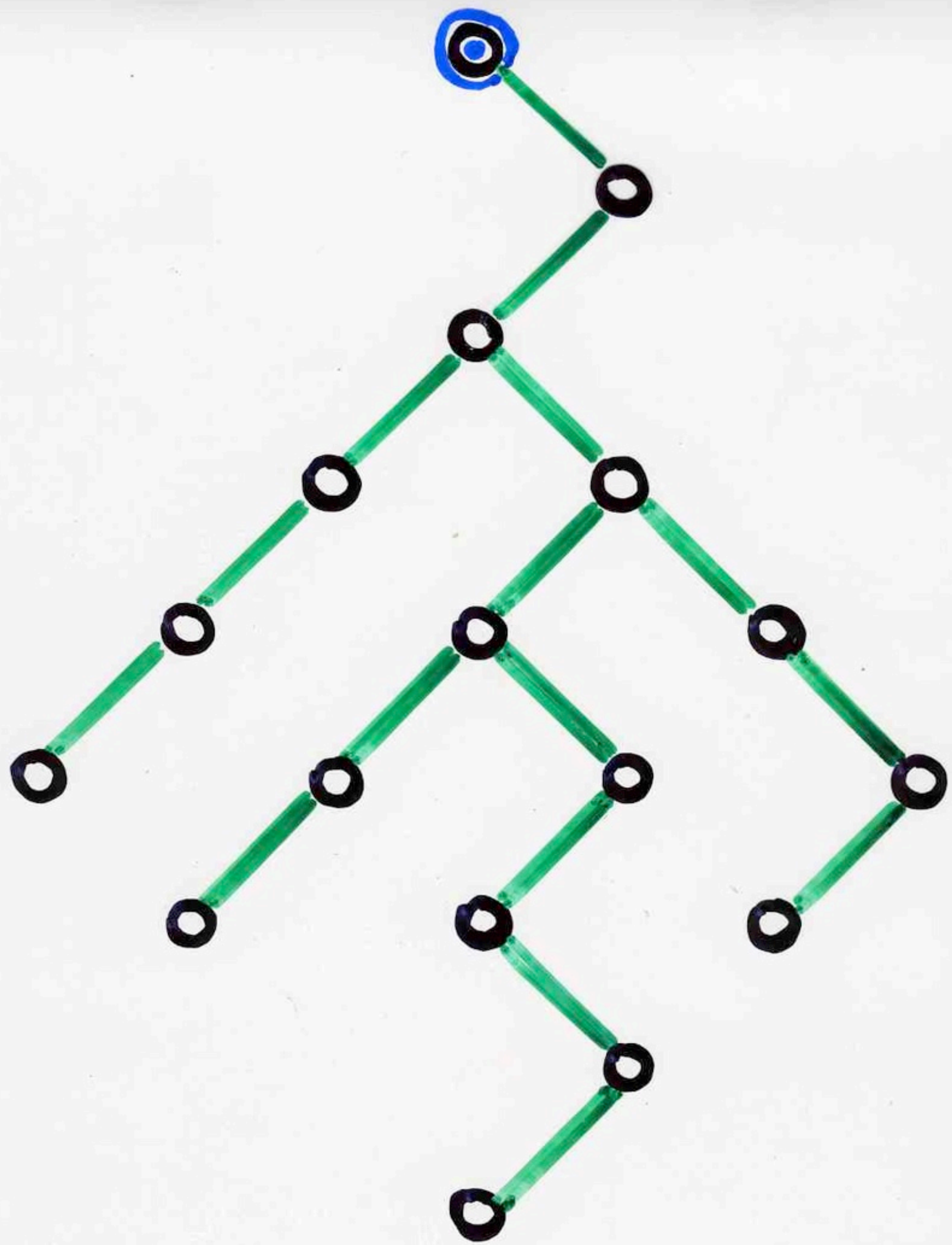
$$\Longleftrightarrow \quad 2^{k}-1 \leq h(w) < 2^{k+1}-1$$

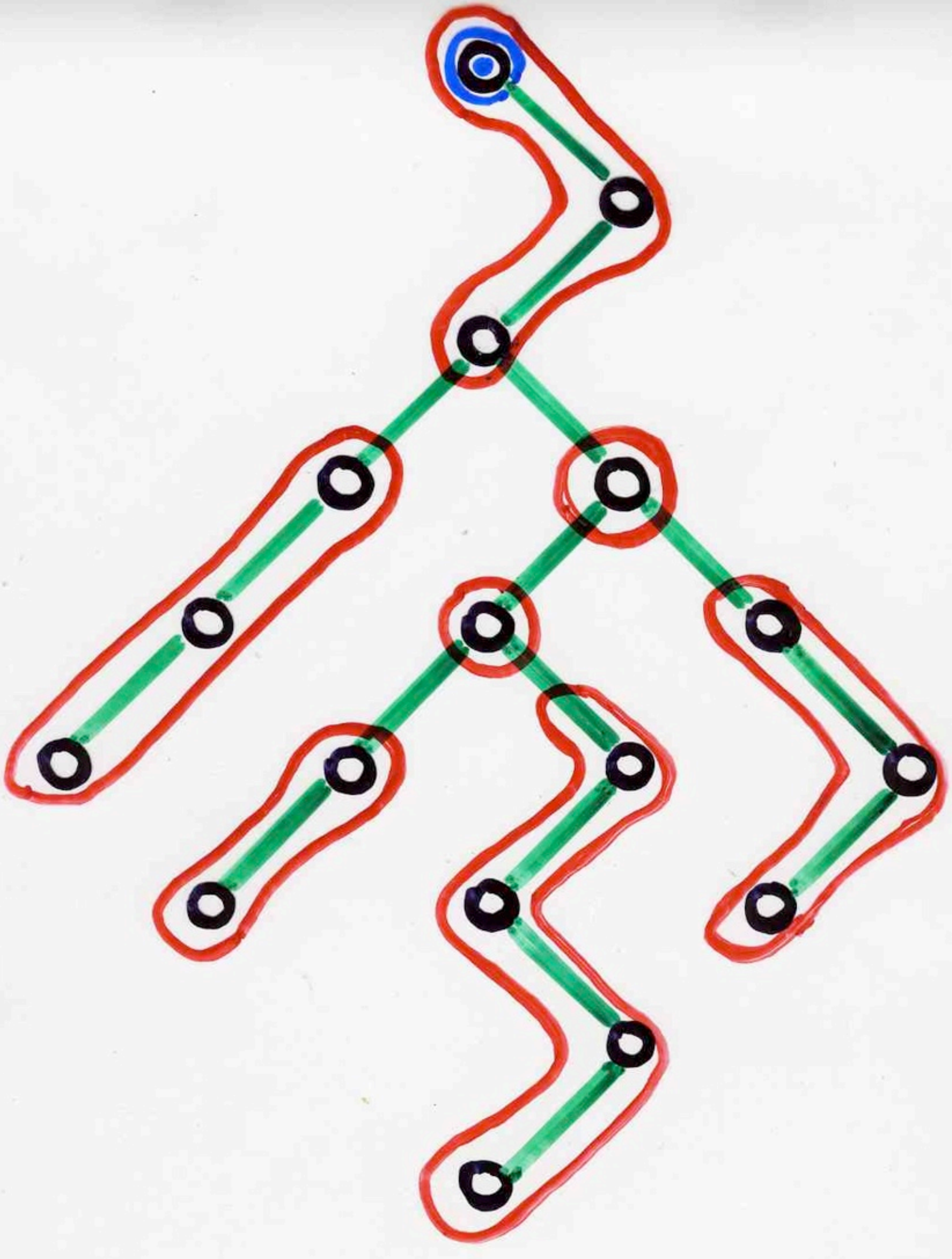
(complete) binary trees ⟷ Françon (1984) ⟷ Dyck paths

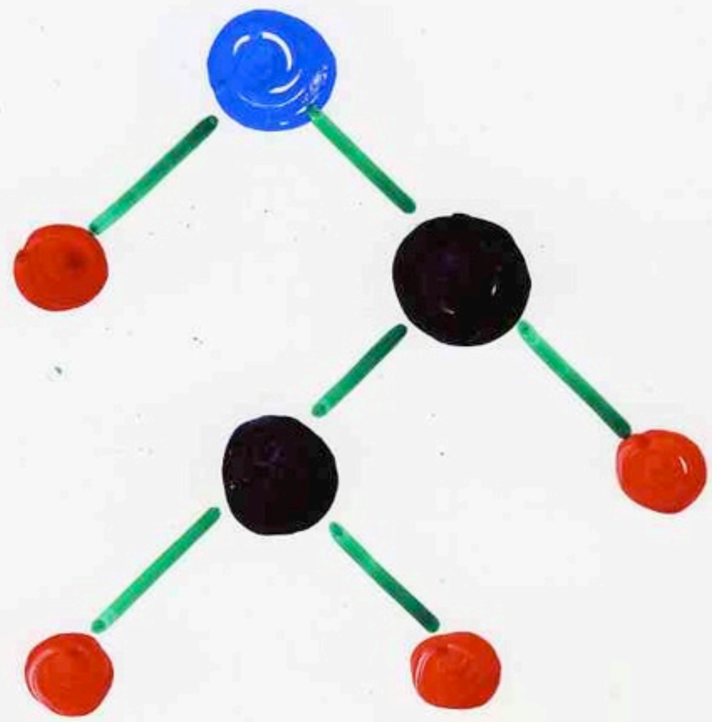
n (internal) vertices — length 2n

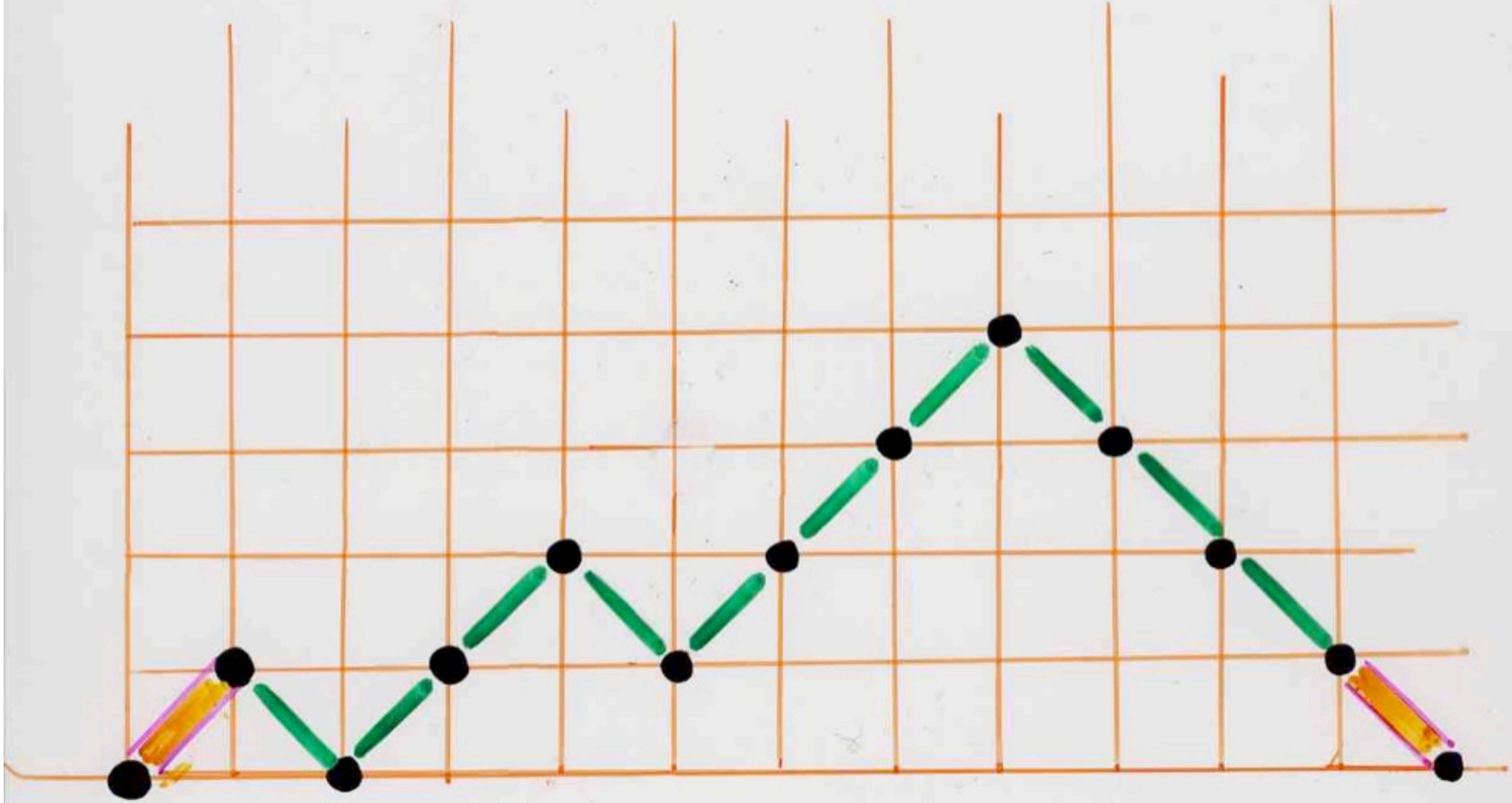
Strahler nb = k — log. height $lh(w) = k$

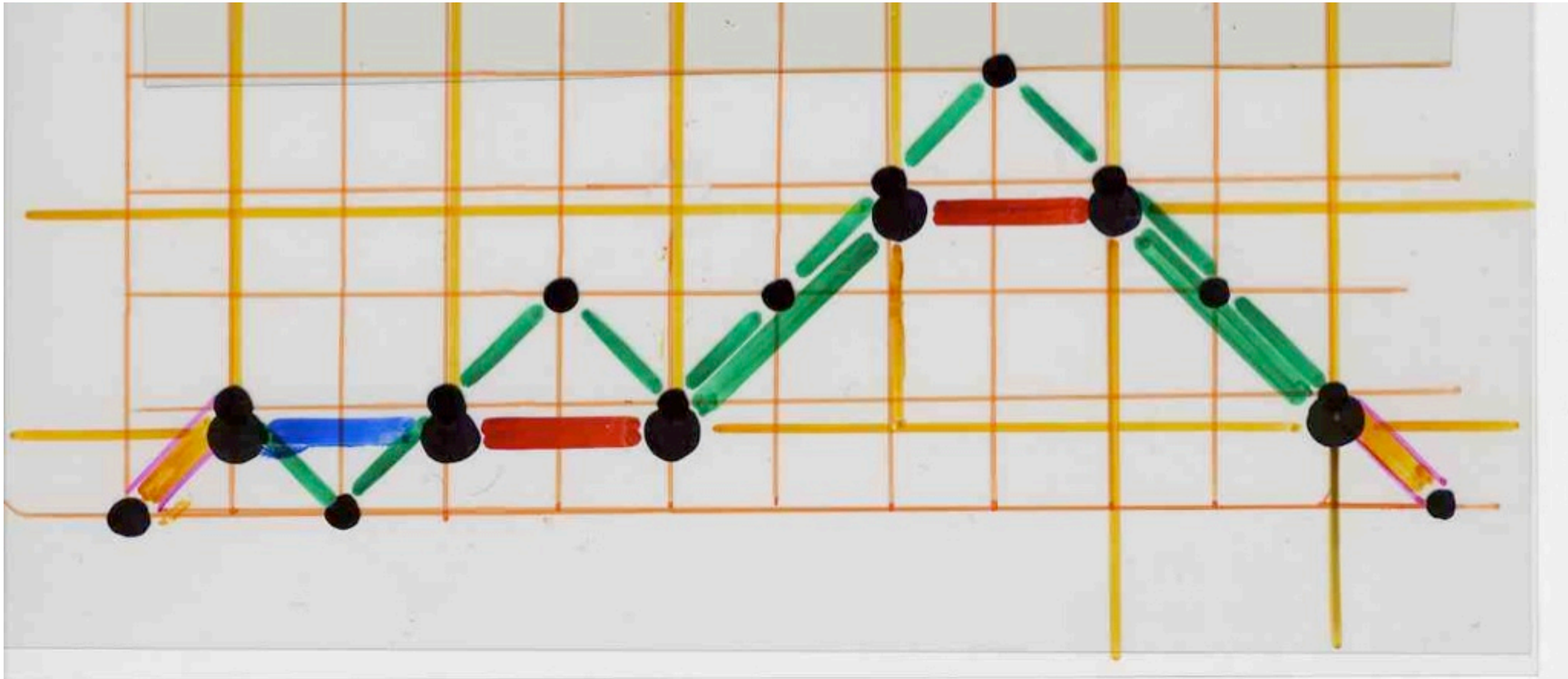


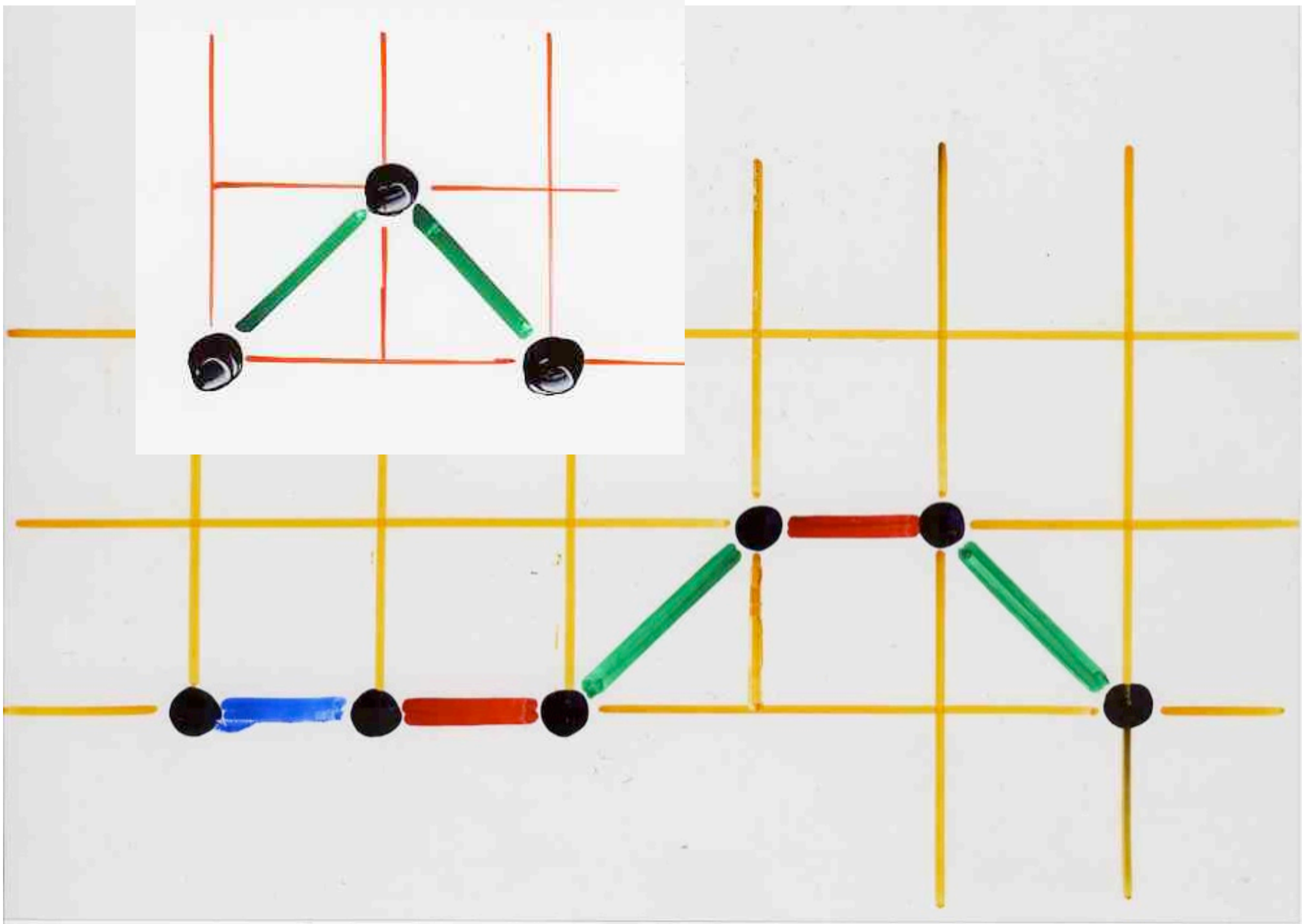


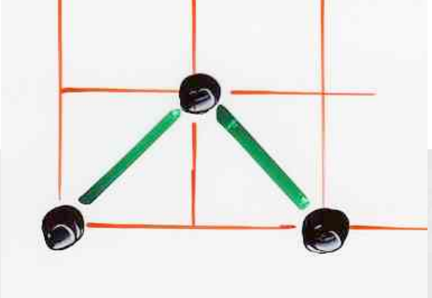
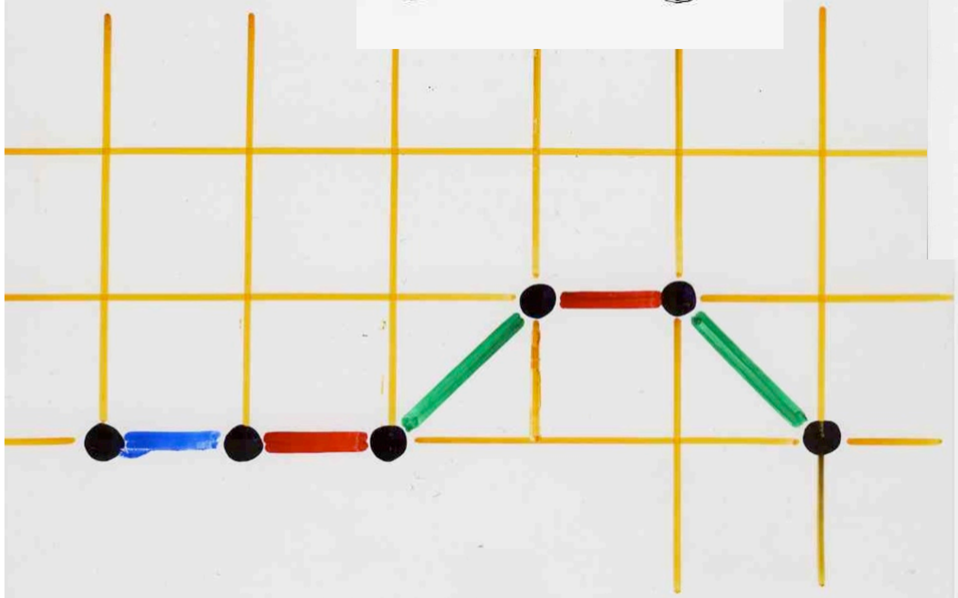
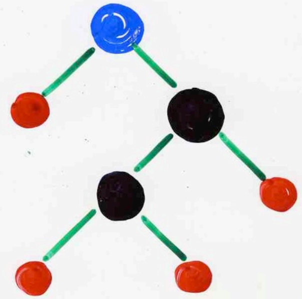
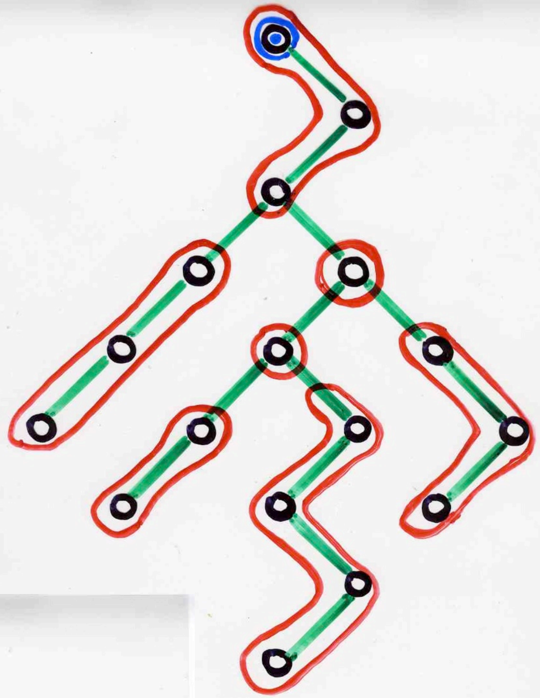
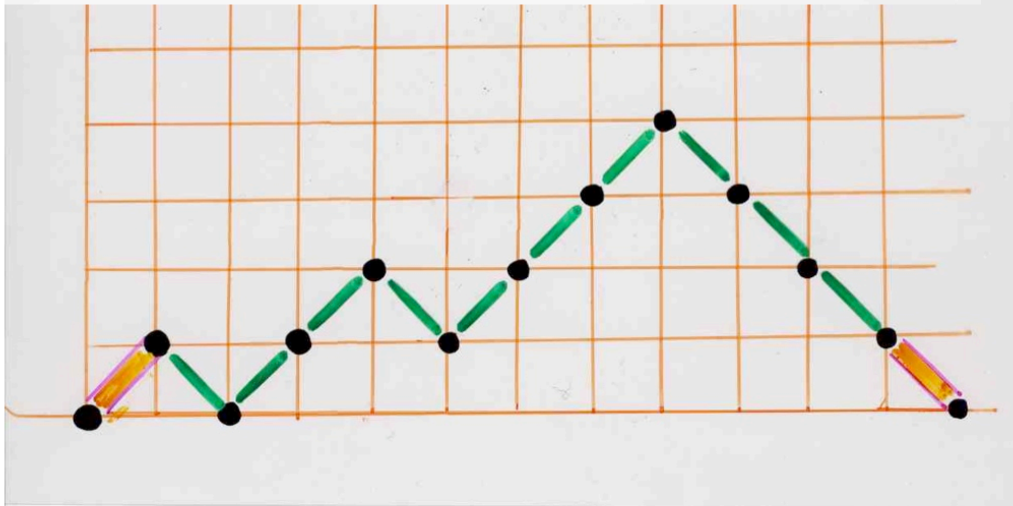
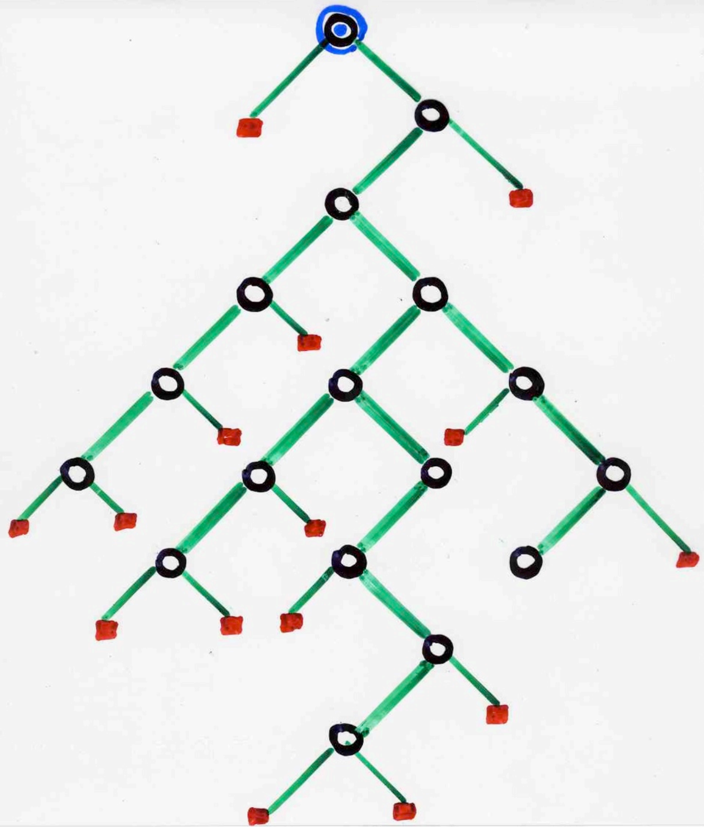










(complete)

binary trees ⟷ Françon (1984) ⟷ Dyck paths

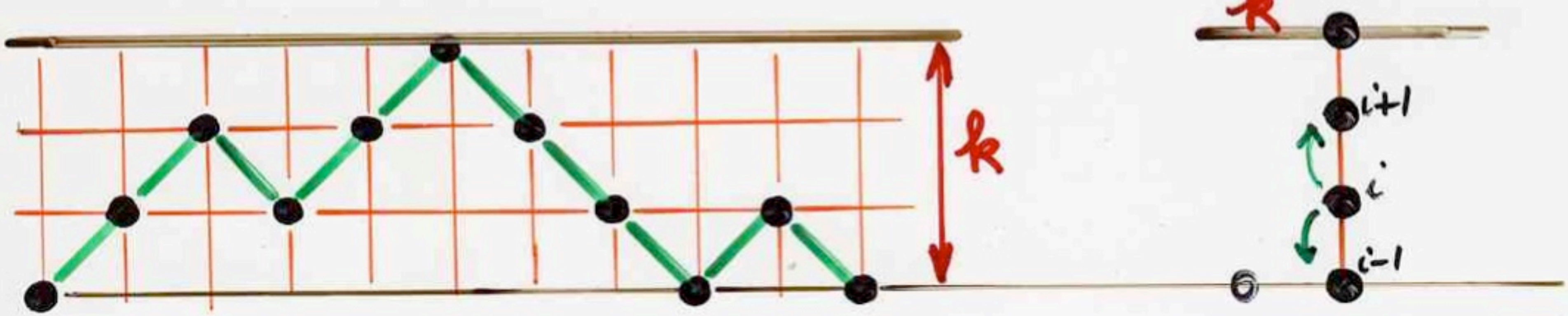
n (internal) vertices — length $2n$

Strahler nb $= k$ — log. height

$lh(w) = k$

ex : Dyck path

bounded at height k



generating function

$S_{n,k}$ = nb of (complete) binary trees B
n (internal) vertices
Strahler nb $St(B) = k$

$$S_k(t) = \sum_{k \geq 0} S_{n,k} t^n$$

$$S_{\leq k}(t) = \sum_{k \geq 0} S_{n,\leq k} t^n$$

$$\sum_{\omega} t^{|\omega|/2} = \frac{F_k(t)}{F_{k+1}(t)}$$

ω Dyck paths bounded k

$$A = (a_{ij}) = \begin{pmatrix} 0 & t & \cdots & \cdots & 0 \\ t & \ddots & \ddots & & \vdots \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ \vdots & & \ddots & \ddots & t \\ 0 & \cdots & \cdots & t & 0 \end{pmatrix}$$

$$\sin(n+1)\theta = (\sin\theta)\, U_n(\cos\theta)$$

$U_n(t/2)$ reciprocal

$F_n(t^2)$

Fibonacci polynomial

$$F_{n+1}(t) = F_n(t) - t\, F_{n-1}(t) \qquad F_0 = F_1 = 1$$

$$S_k(t) = \frac{t^{(2^{k-1}-1)}}{R_{2^k-1}(t)} = S_{\le k}(t) - S_{\le (k-1)}(t)$$

$$S_{\le k}(t) = \frac{R_{2^k-2}(t)}{R_{2^k-1}(t)}$$

average Strahler number
over binary trees n vertices

$$St_n = \log_4 n + f(\log_4 n) + o(1)$$

Flajolet, Raoult, Vuillemin (1979)
Kemp

periodic

$T(n)$ = number of 1's in the binary expansion of $1, 2, \ldots, (n-1)$

$$T(n) = \frac{1}{2} n \log n + n F(\log n)$$

$$f(t) = 1 - \frac{\gamma}{2\log 2} - \int_0^\infty t H_4(t) F(\log t + u) e^{-t^2} dt$$

(complete)

binary trees ⟷ Françon (1984) ⟷ Dyck paths

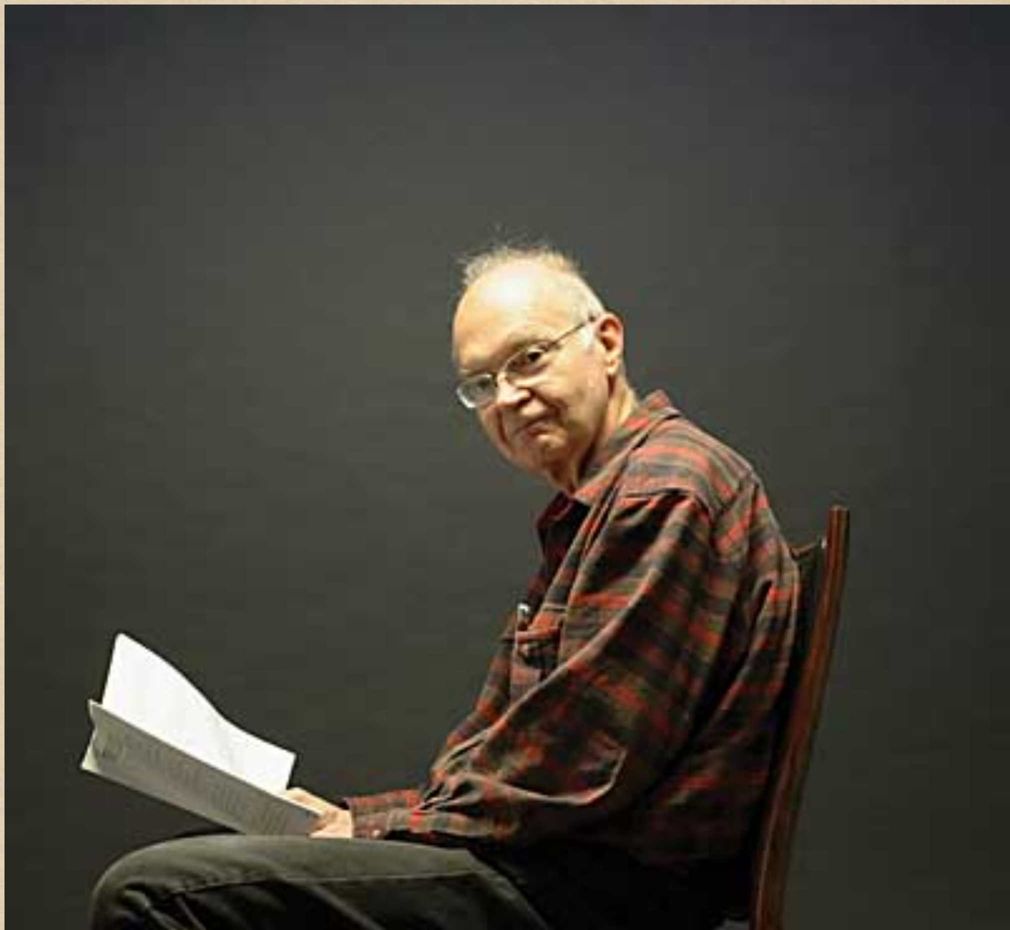
n (internal) vertices — length 2n

Strahler nb $= k$ — log. height

$lh(w) = k$

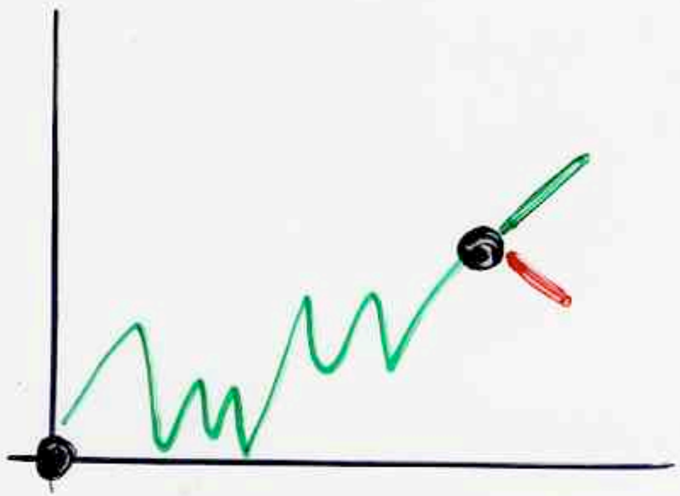
⟷ Knuth (2005) ⟷

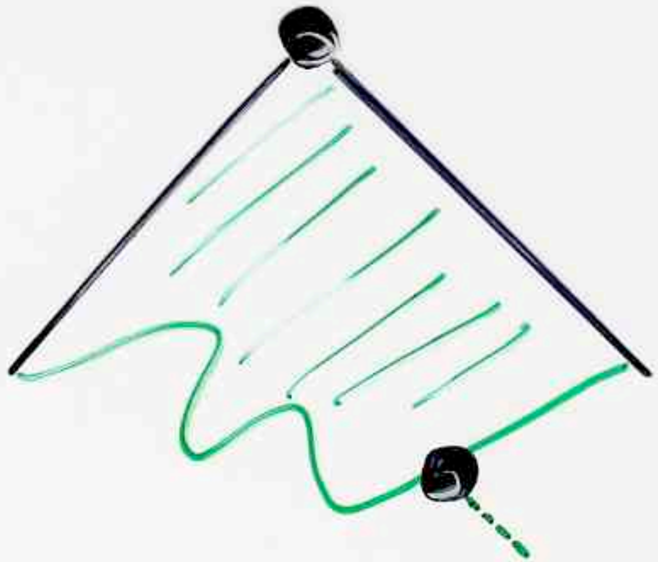
Knuth bijection

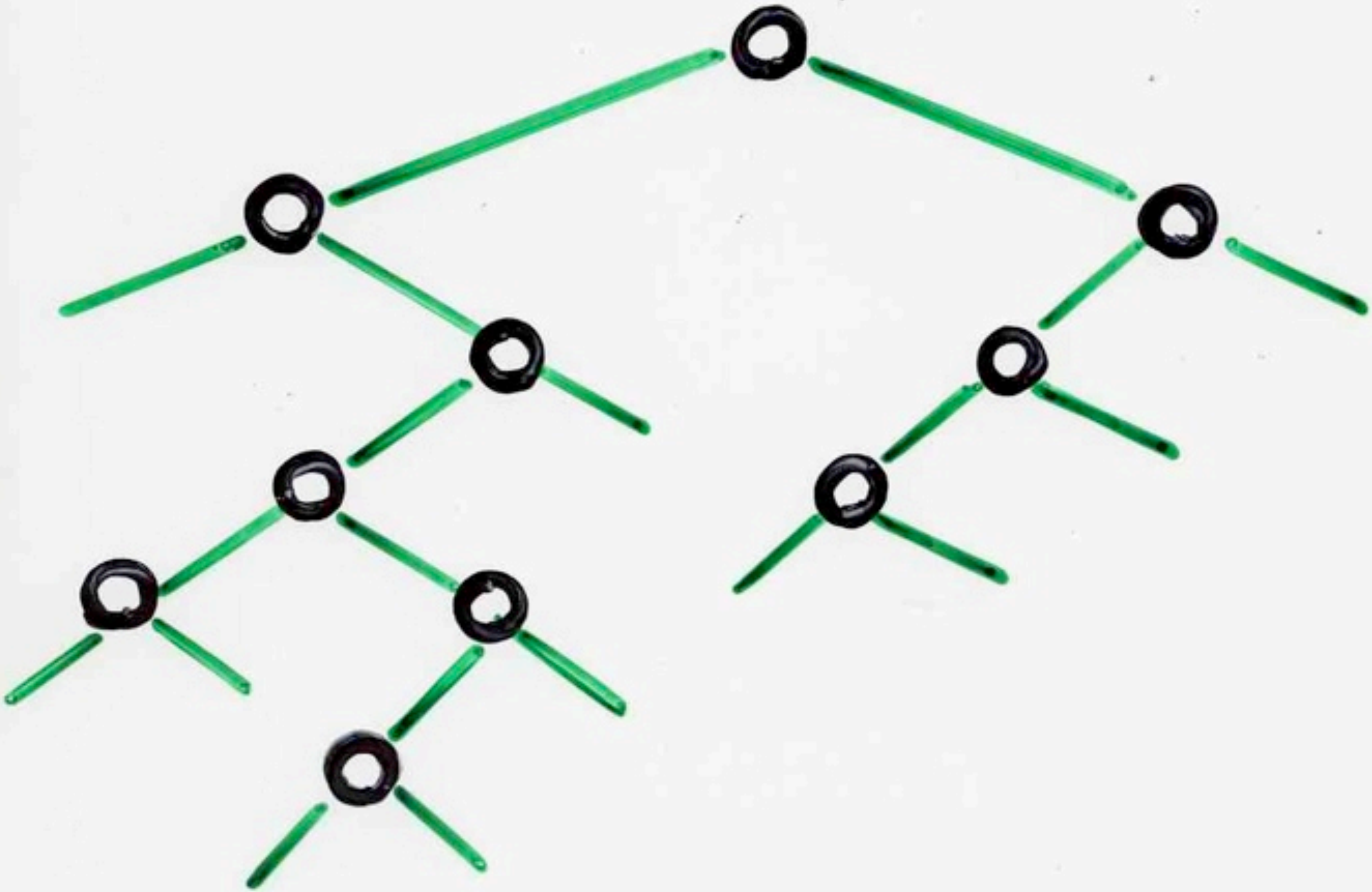


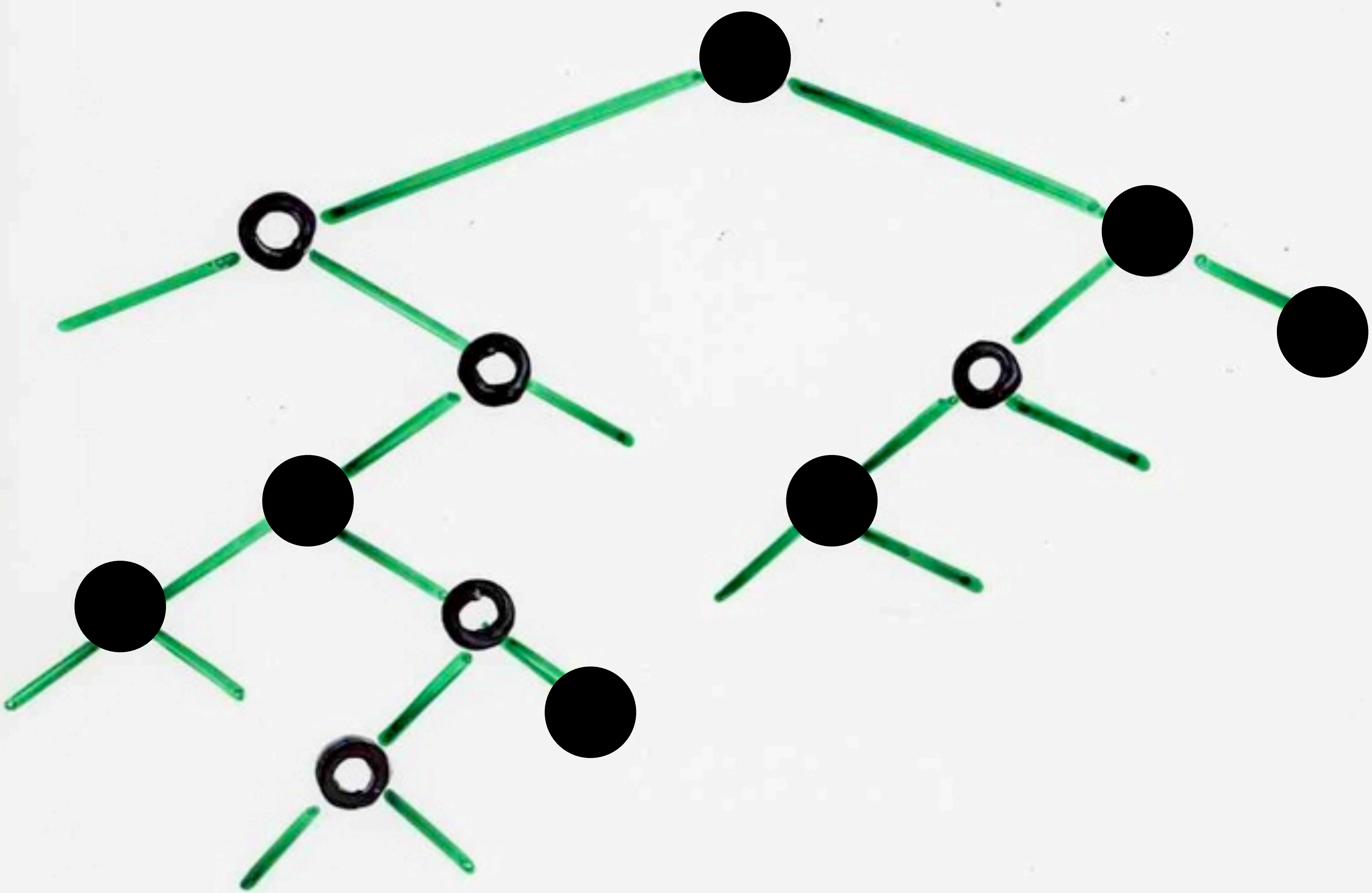
family of bijections

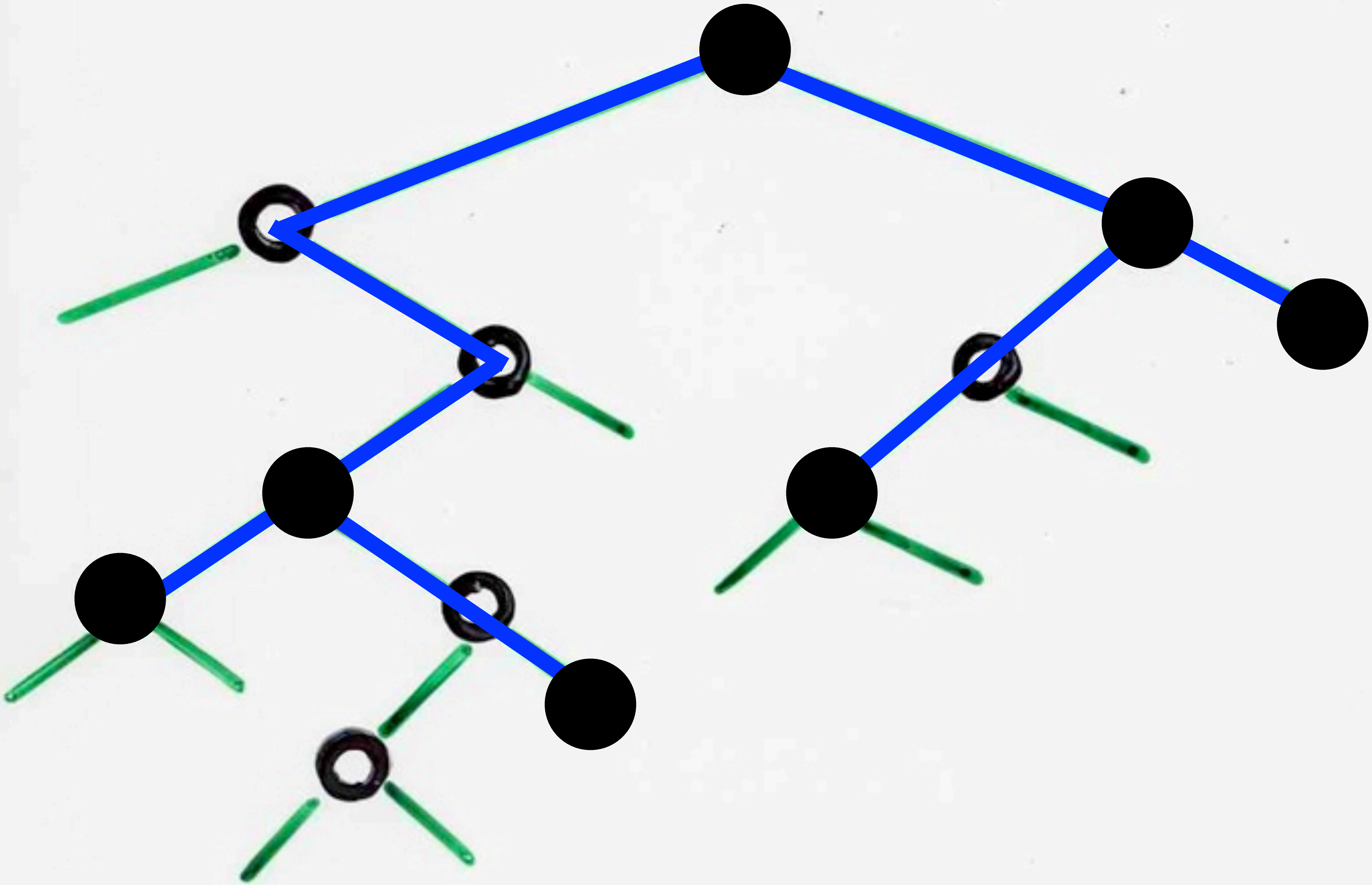
Proskurowski (1980)

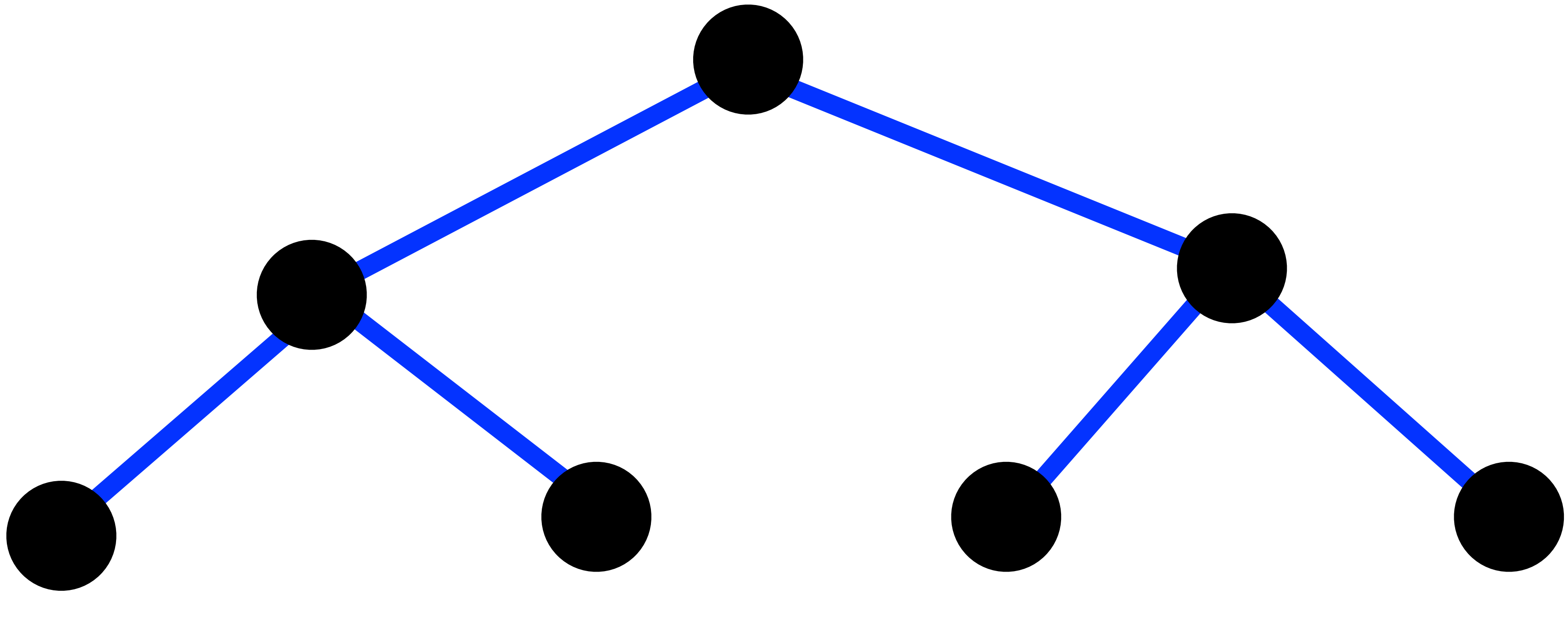


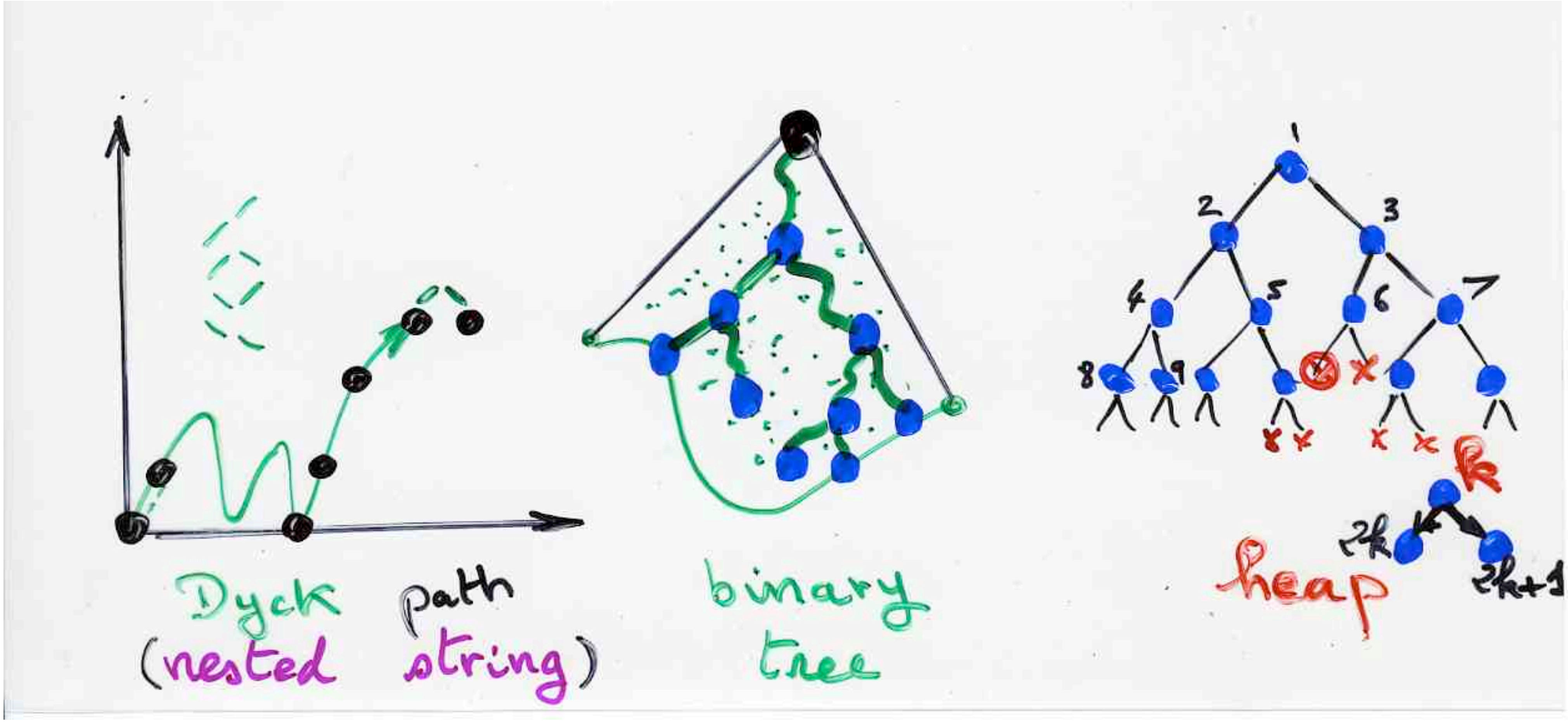











Dyck path
(nested string)
binary tree
heap
1
2
3
4
5
6
7
8
9
k
2k
2k+1

T
A
O
C
P

The
Art
Of
Computer
Programming

T
A
O
C
B

The Art Of Combinatorial Bijections

the trilogy:
binary trees, Dyck paths,
forest of planar tress

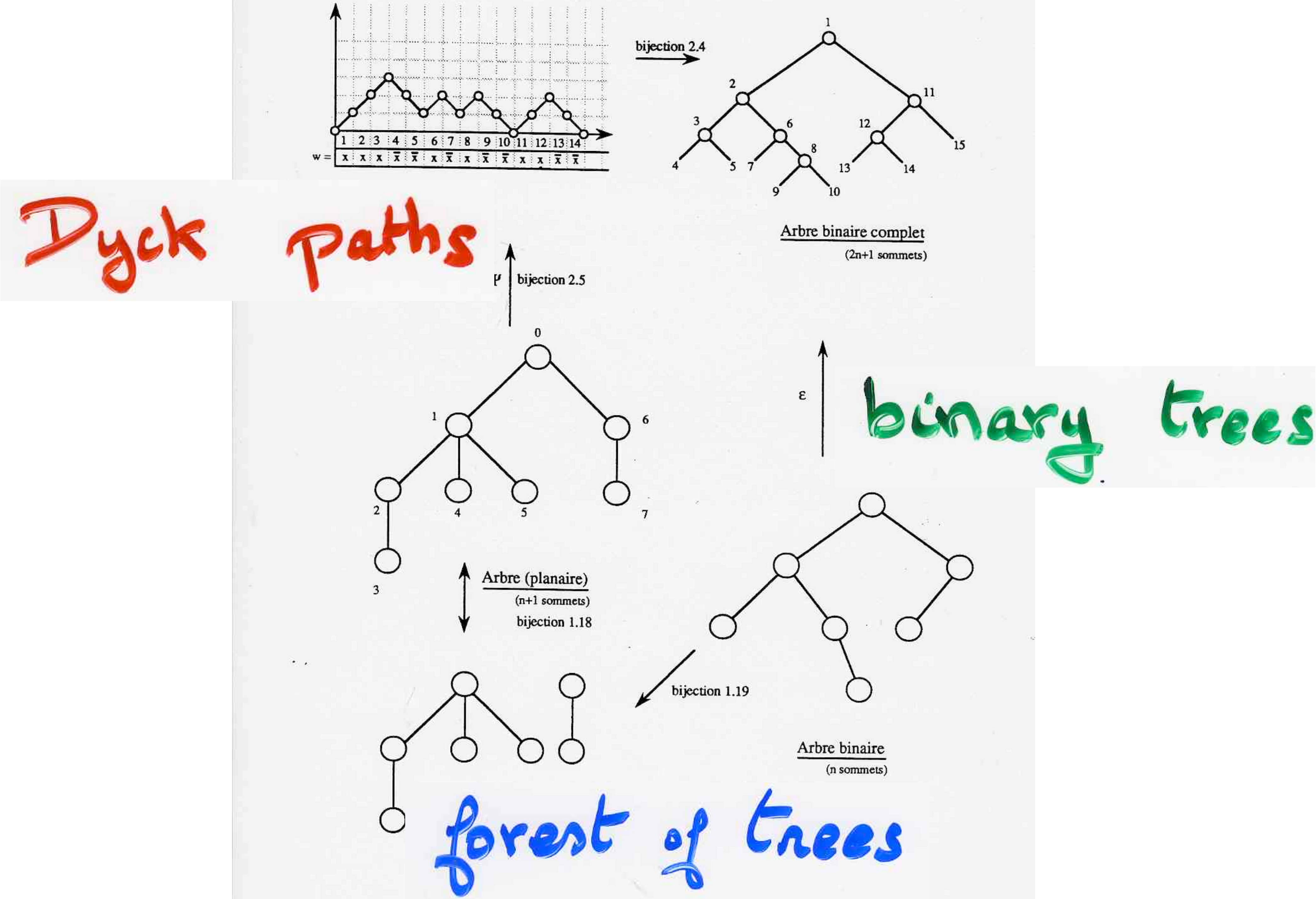
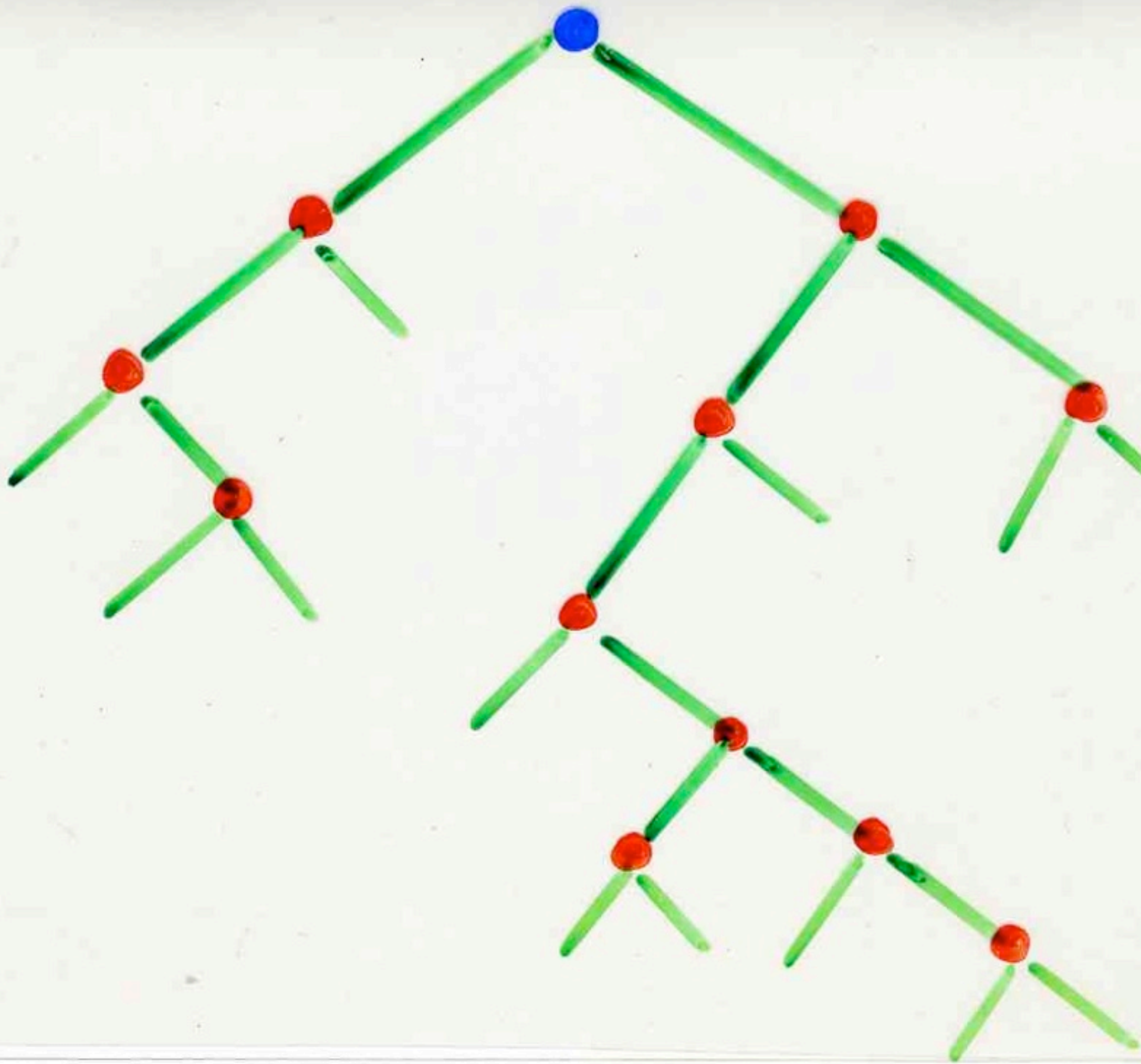
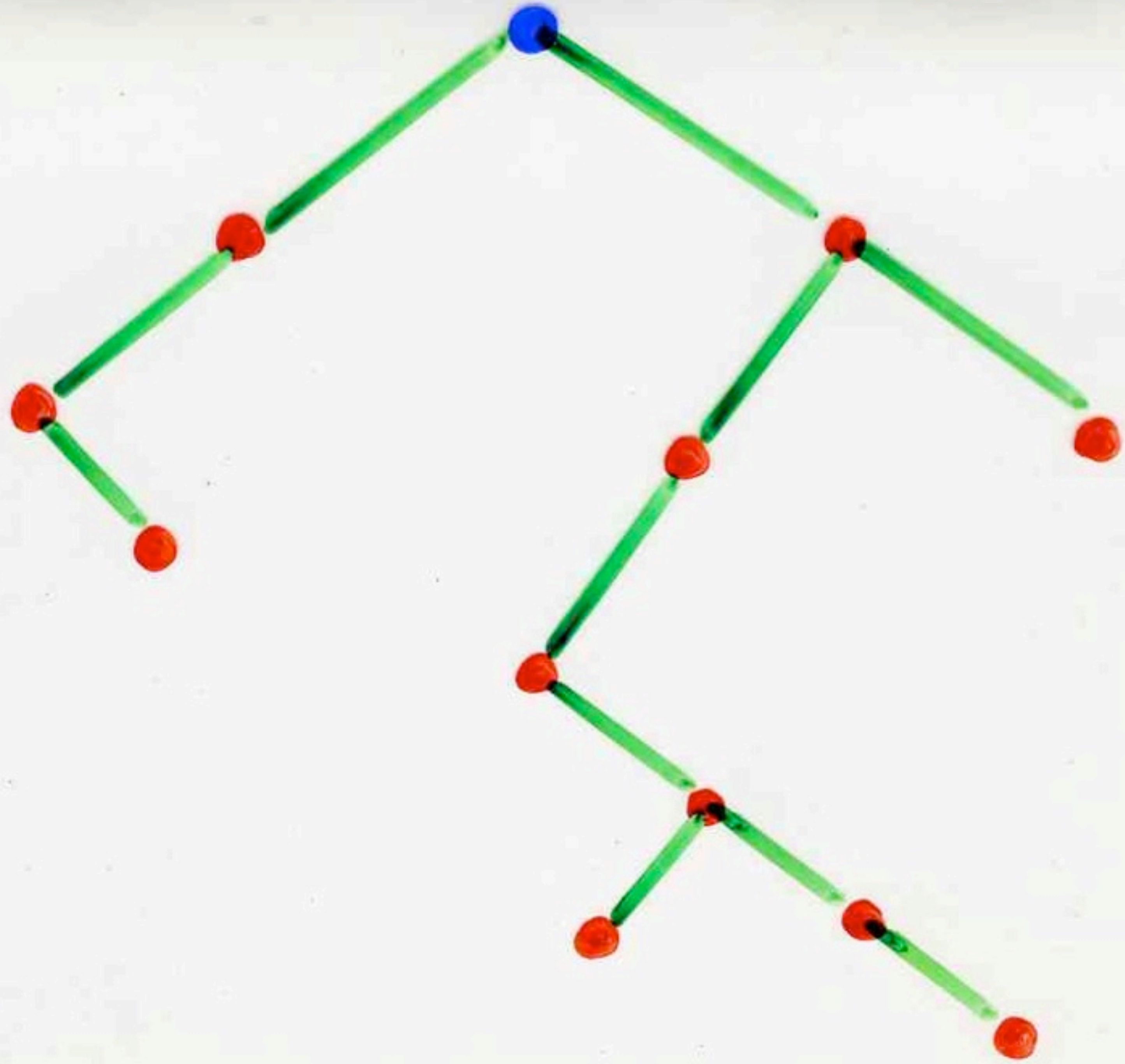


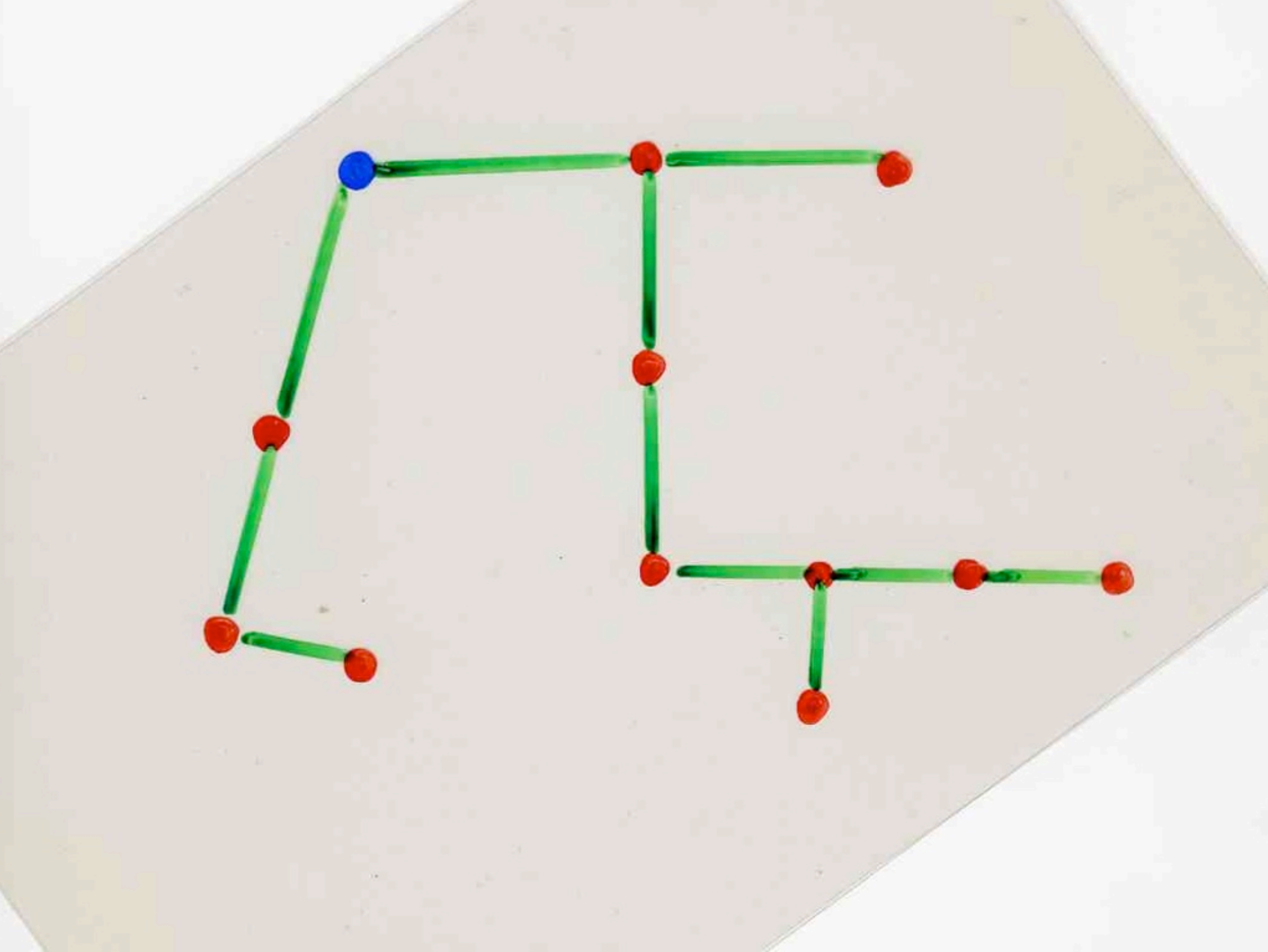
Fig. 10. Diagramme commutatif des bijections φ,Ψ, ε

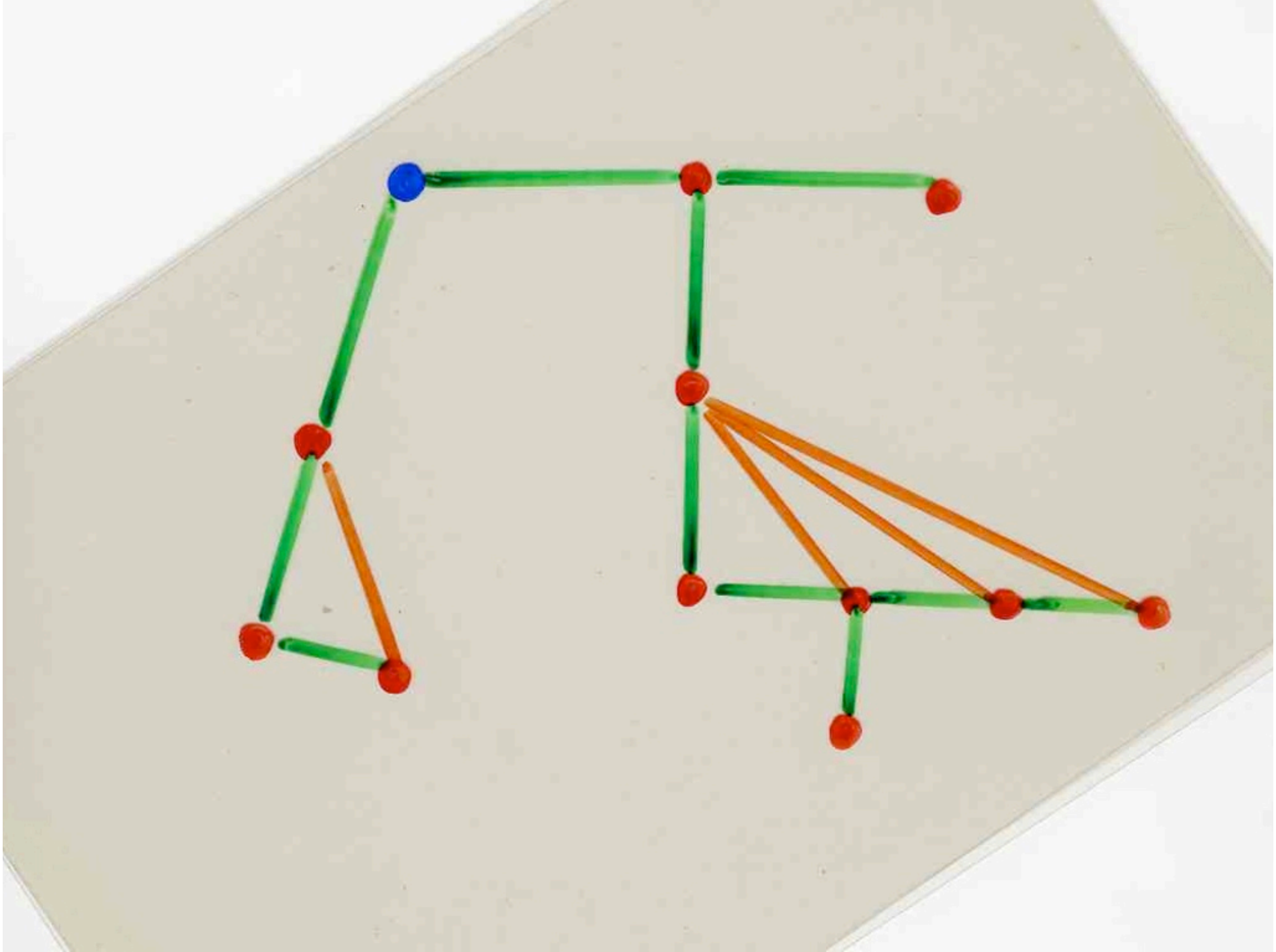
From a binary tree to
an (ordered) forest
of (planar) trees

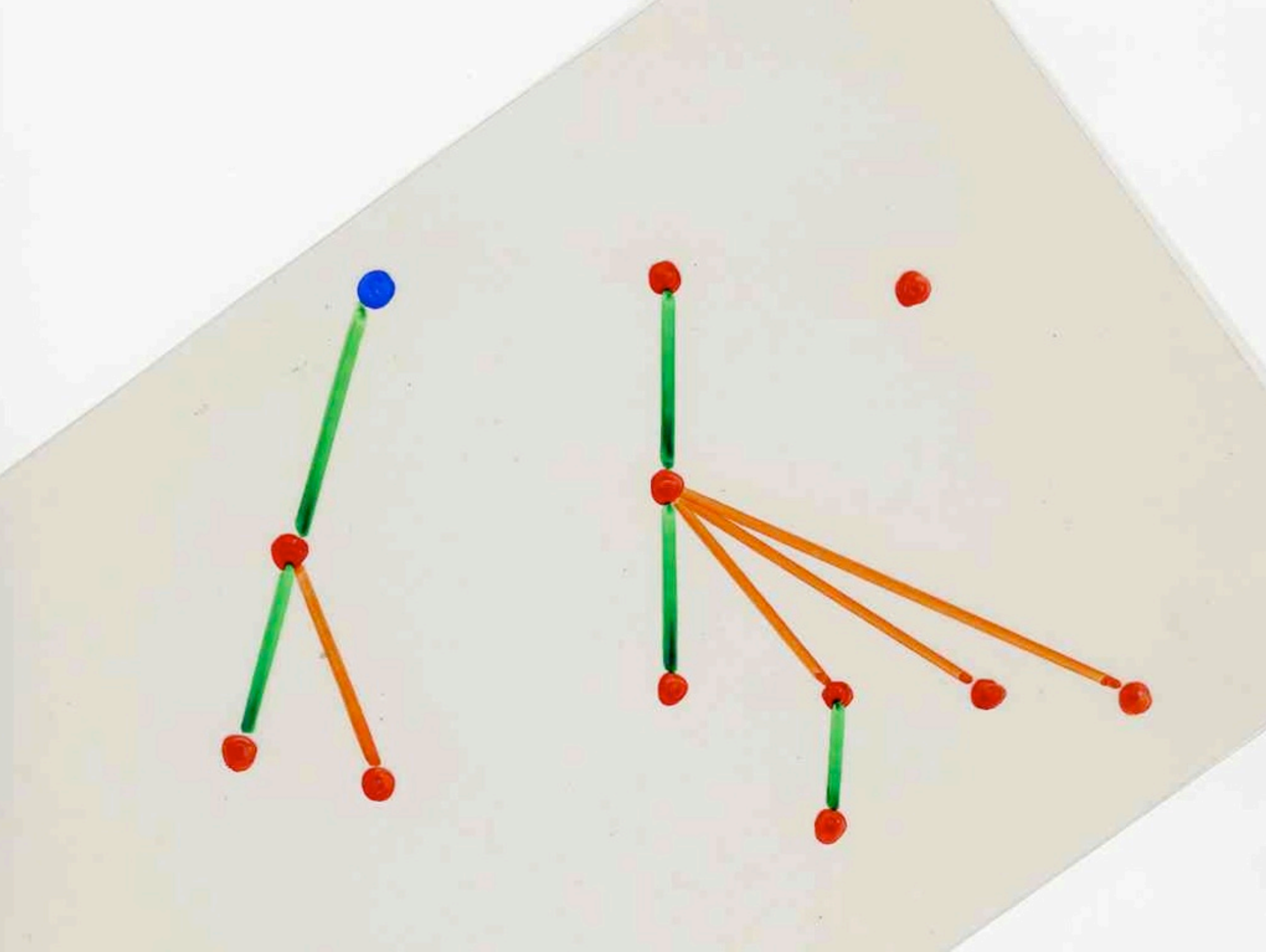
TAOCP
"natural transform"

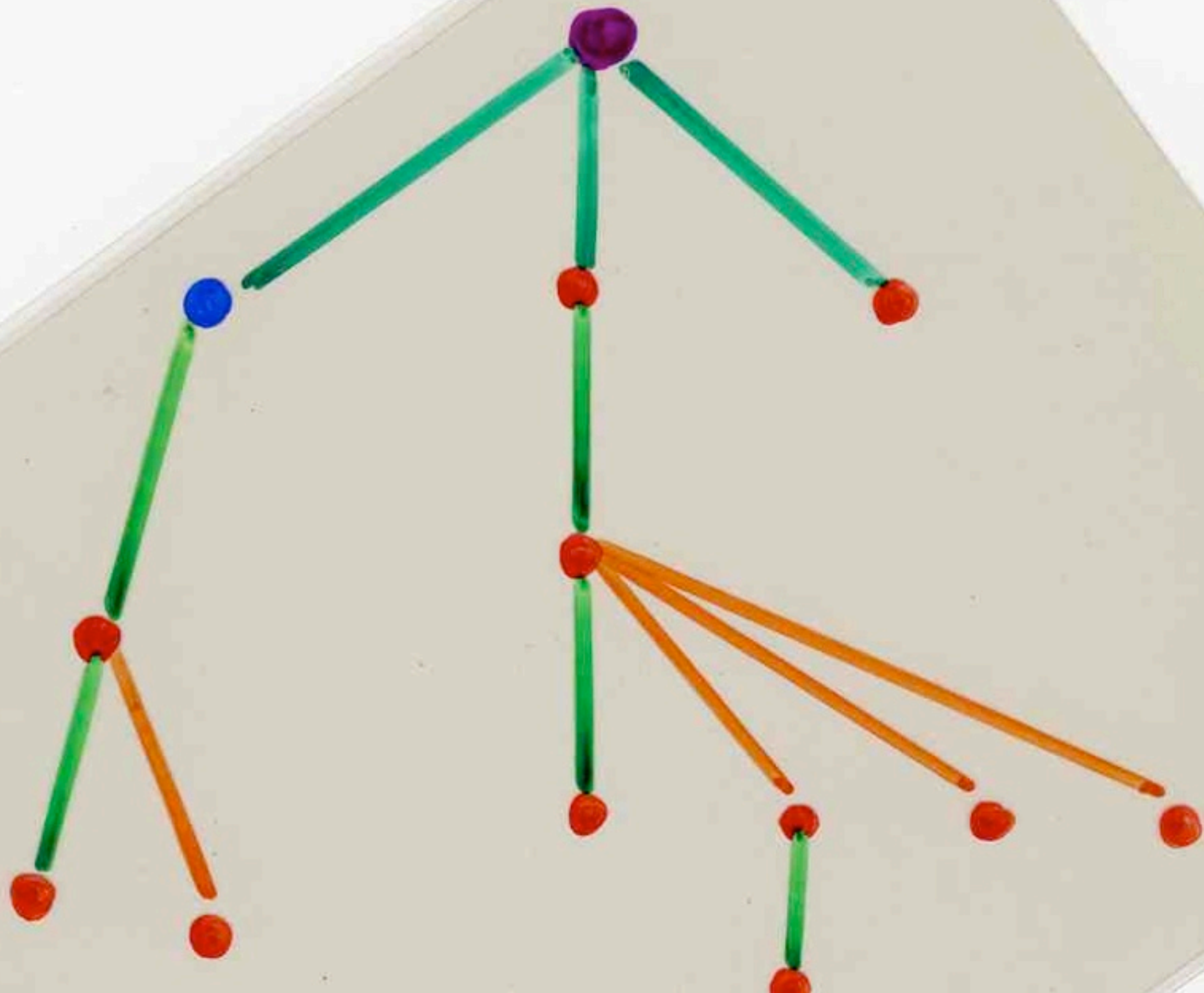


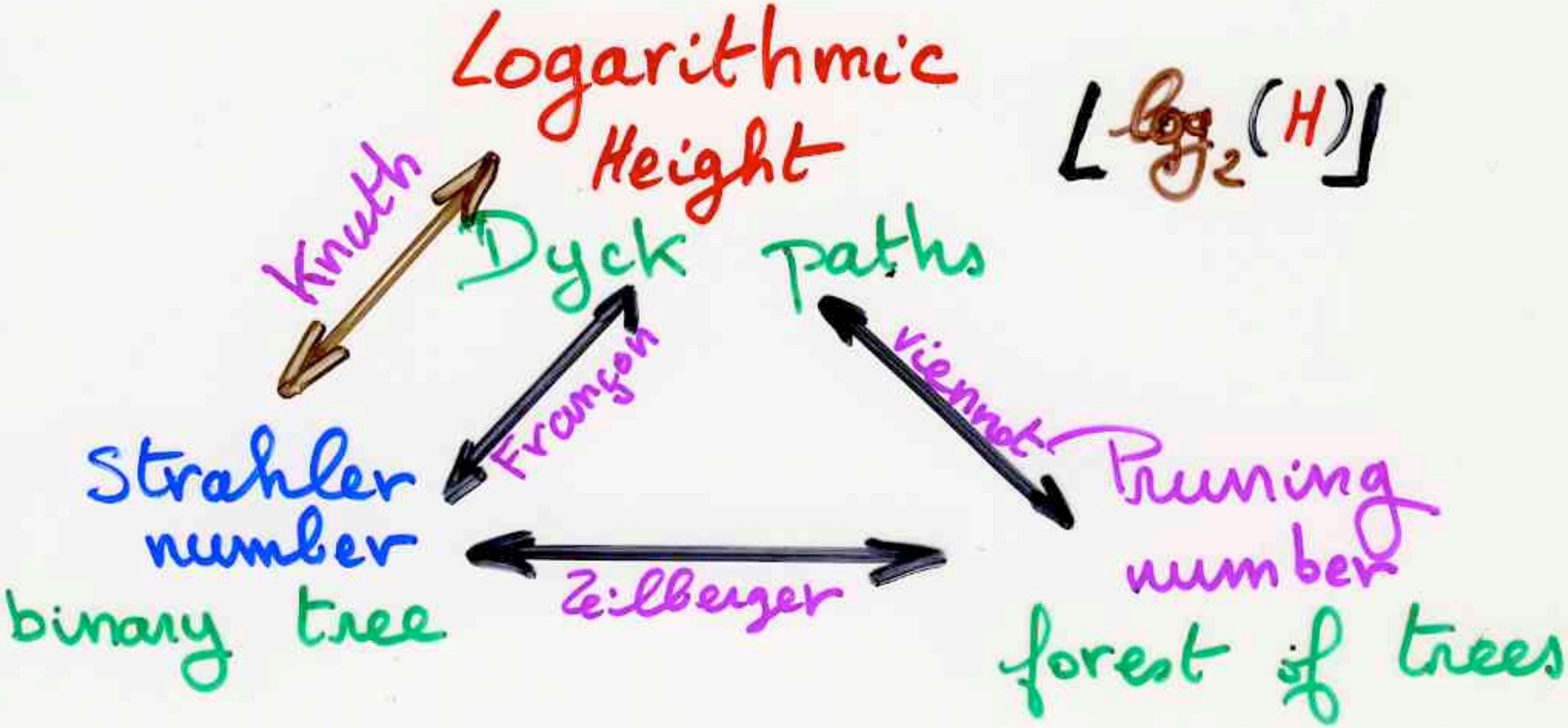






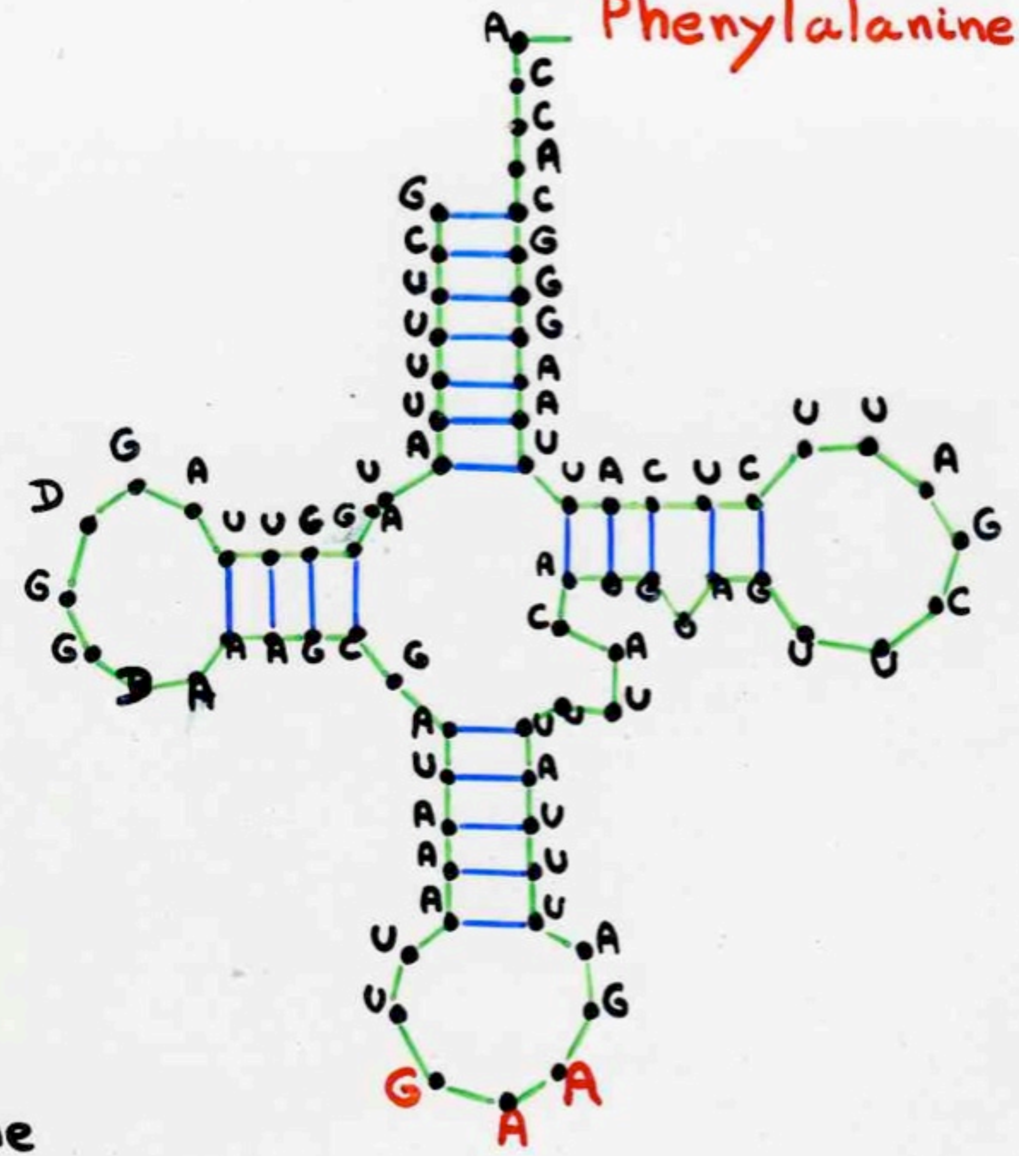






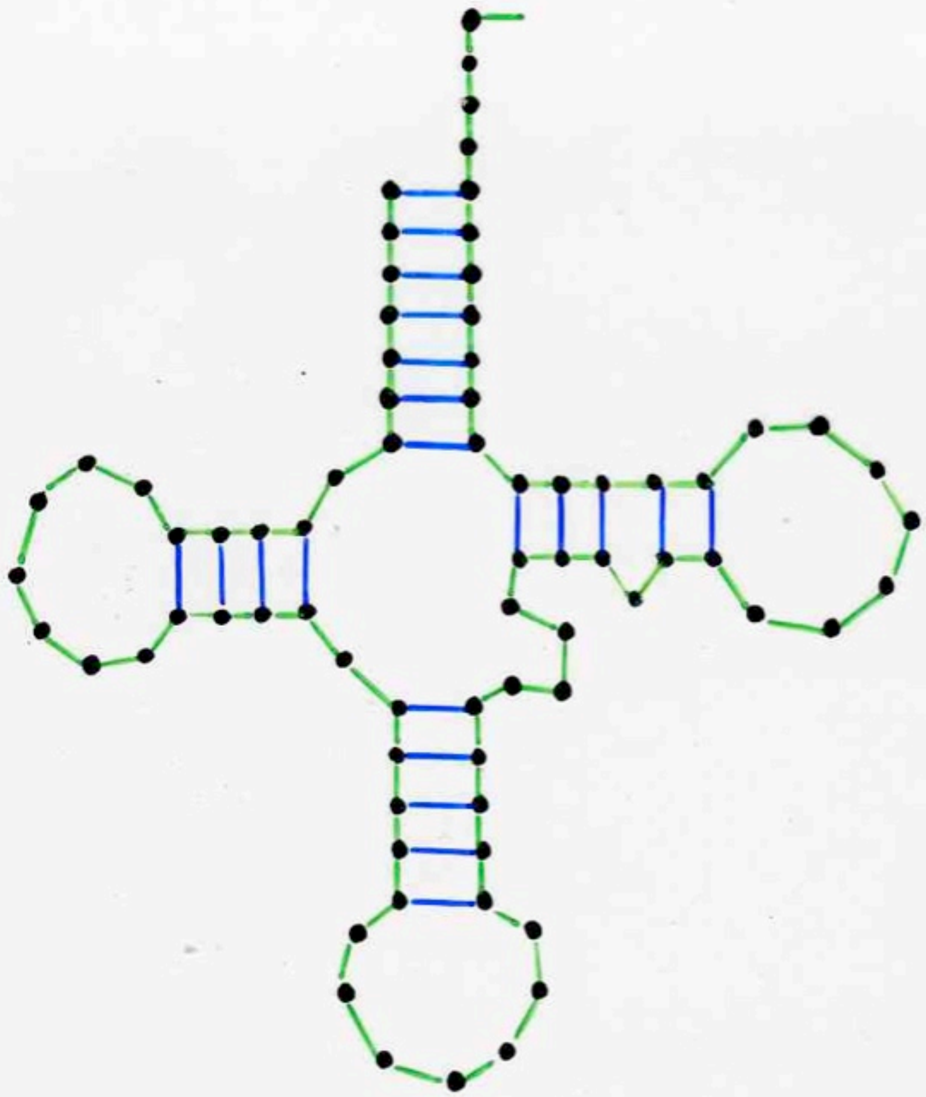
Logarithmic Height
⌊log₂(H)⌋
Dyck paths
Knuth
Françon
Viennot
Strahler number
binary tree
Zeilberger
Pruning number
forest of trees

trees
in secondary structures
of RNA

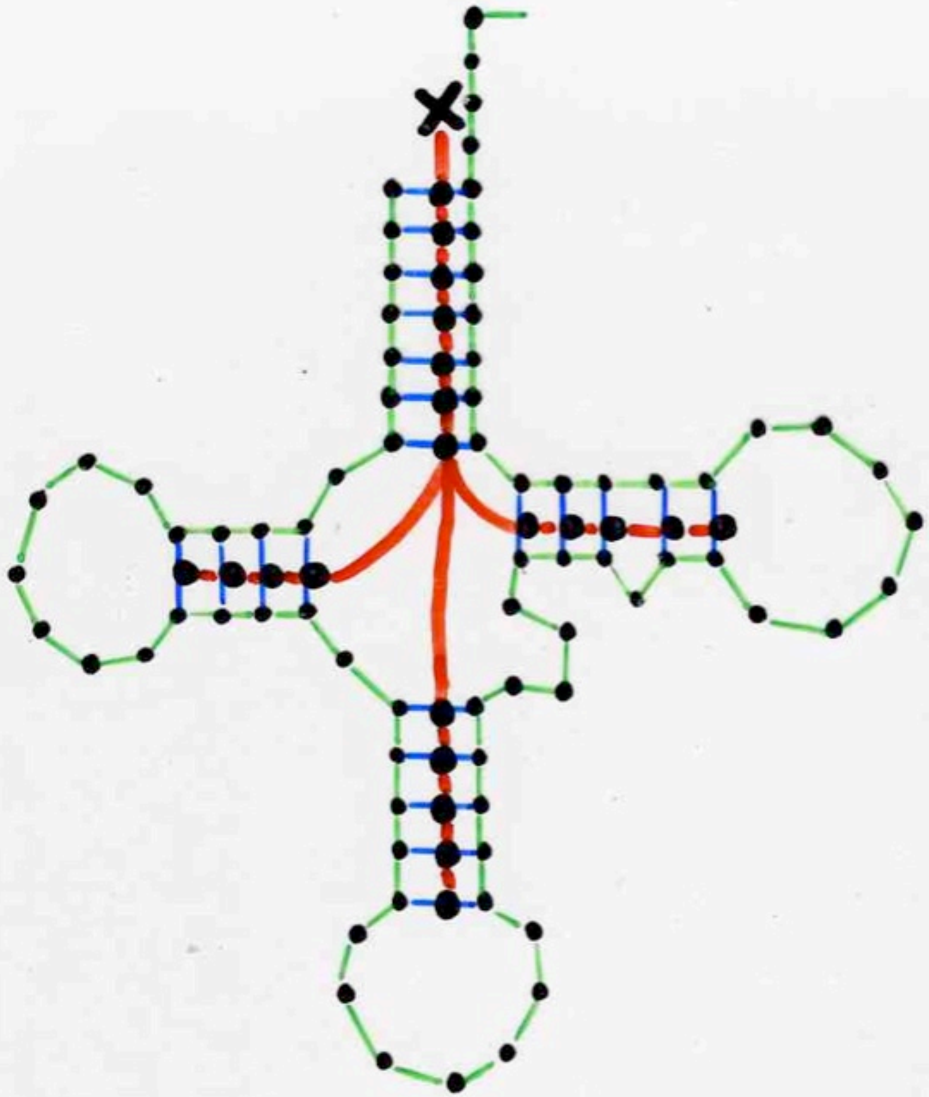


Adenine
Uracyle
Guanine
Cytosine

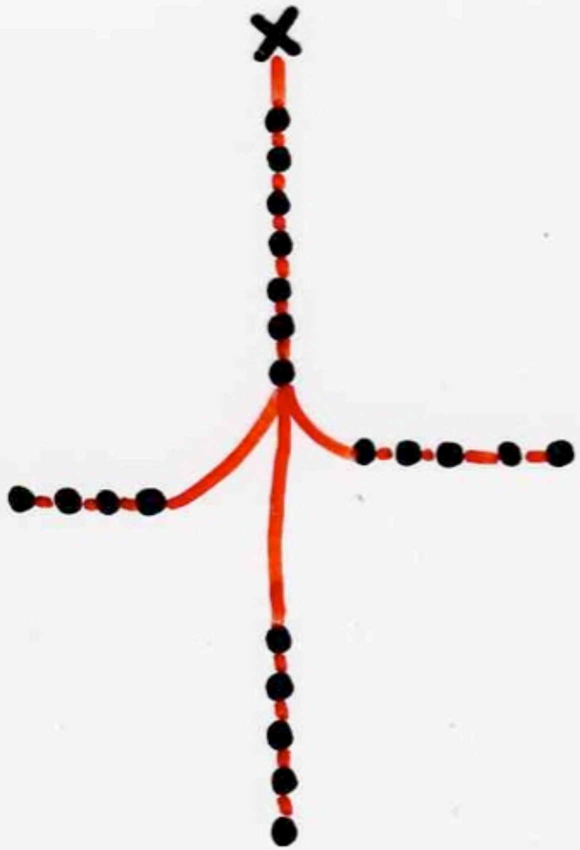
tARN ^{Phe}



tARN ^{Phe}



tARN ^{Phe}



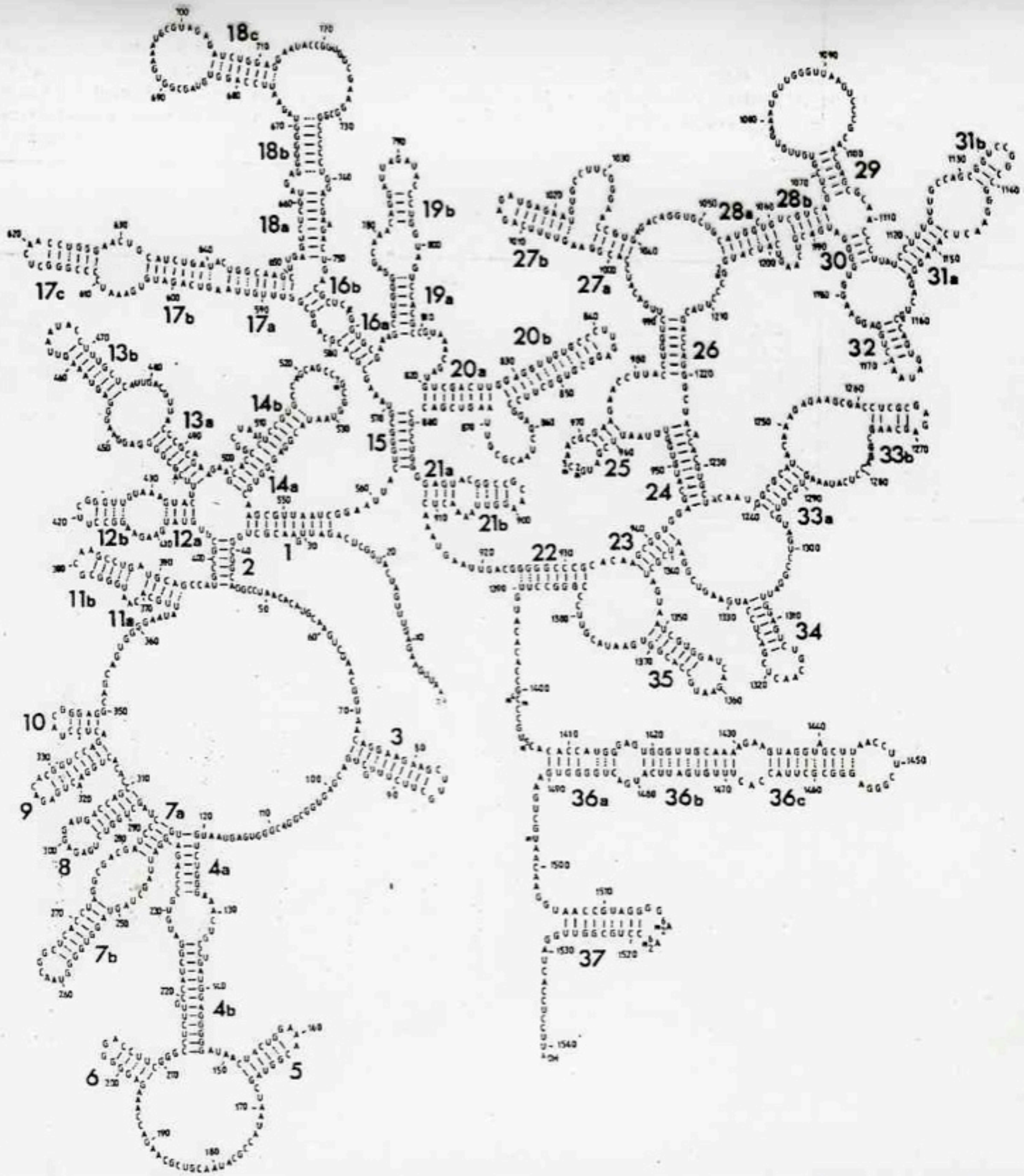


Fig. 1. *Secondary structure model of the 16-S RNA from* E. coli. This model has been fully described elsewhere [18]. The various secondary structure motifs are numbered for reference. Base-pairings 2 and 23 are included in this up-dated scheme and slight modifications have been introduced into helices 18b and 33b

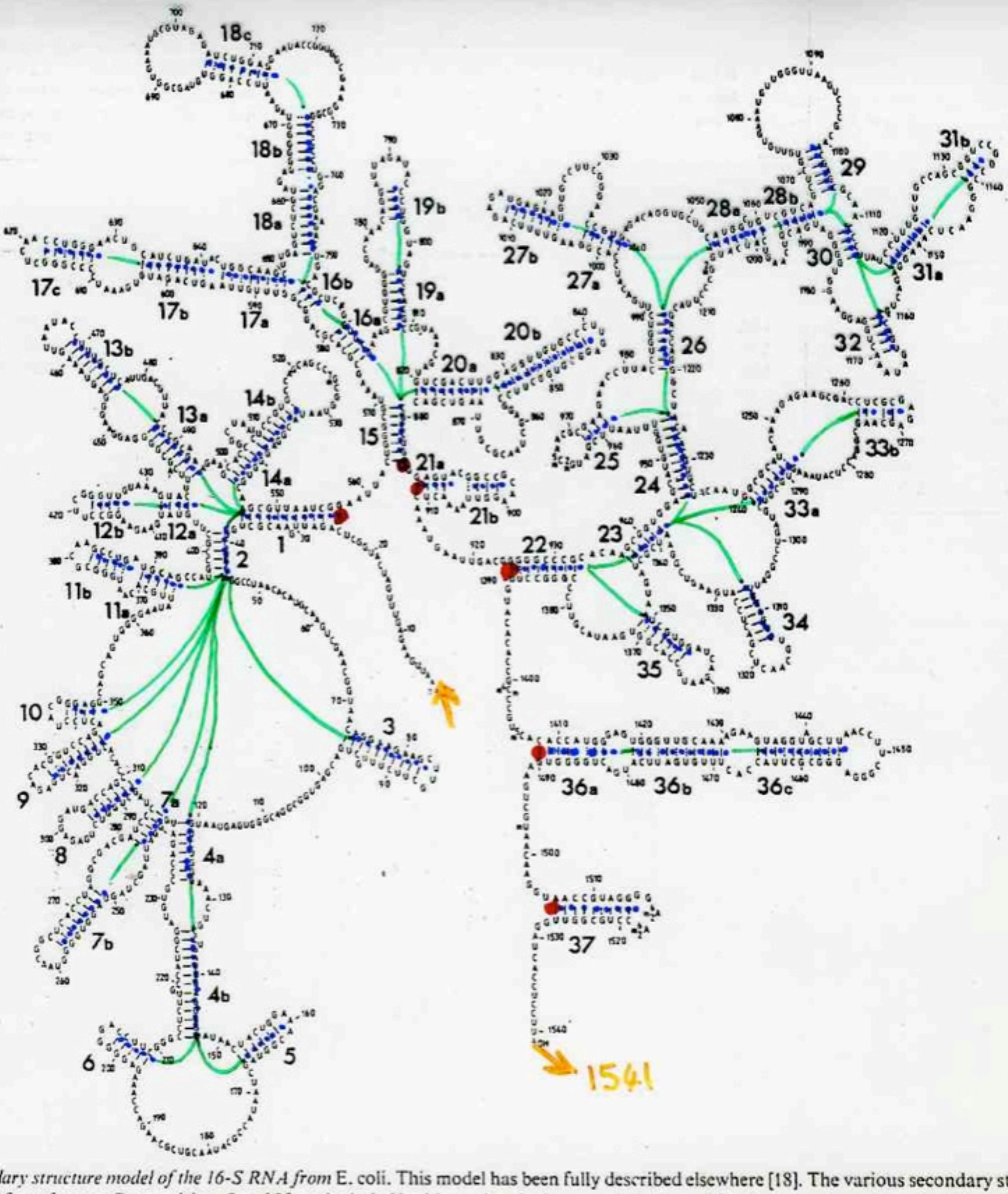


Fig. 1. *Secondary structure model of the 16-S RNA from* E. coli. This model has been fully described elsewhere [18]. The various secondary structure motifs are numbered for reference. Base-pairings 2 and 23 are included in this up-dated scheme and slight modifications have been introduced into helices 18b and 33b

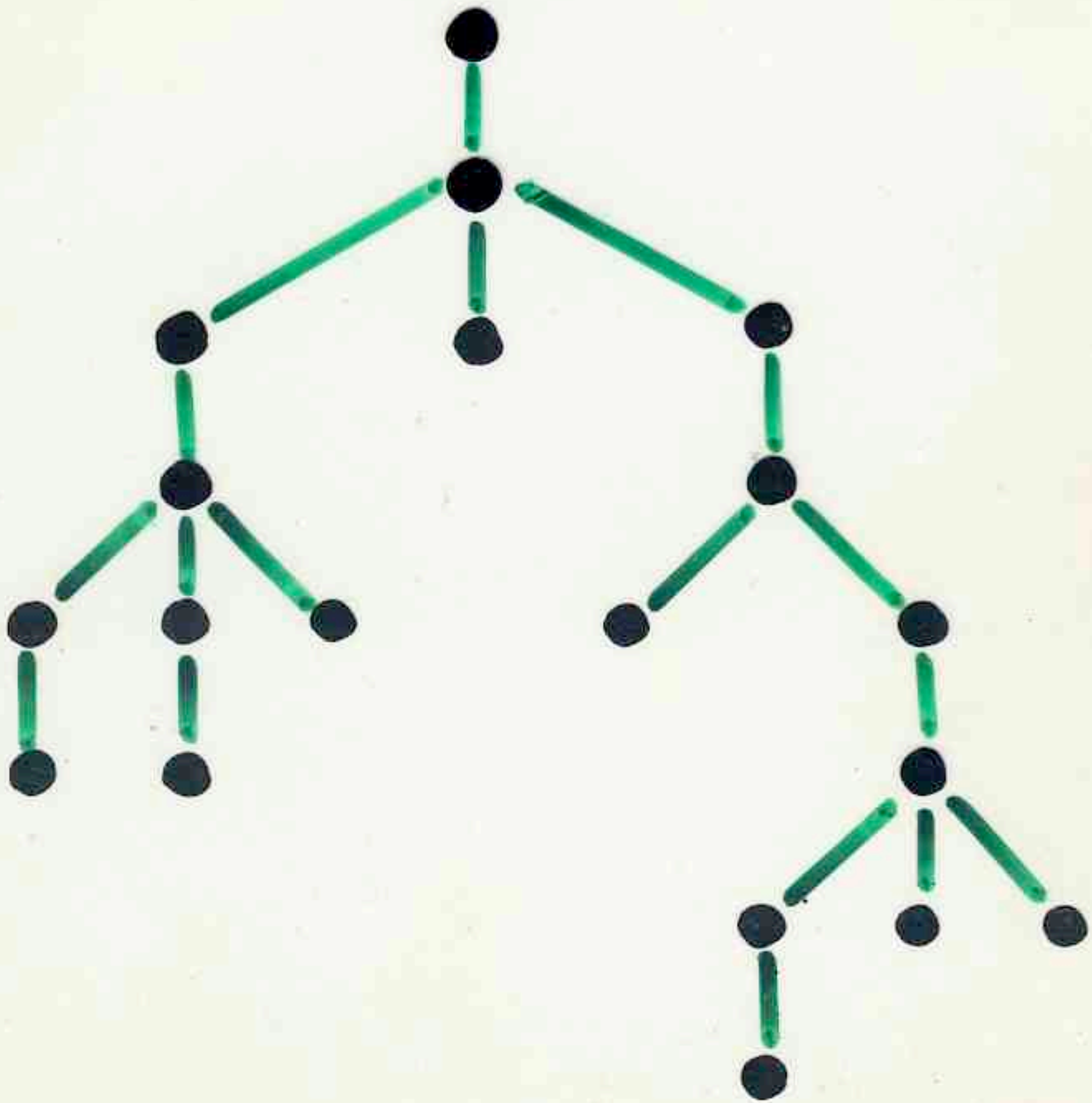
M. Waterman

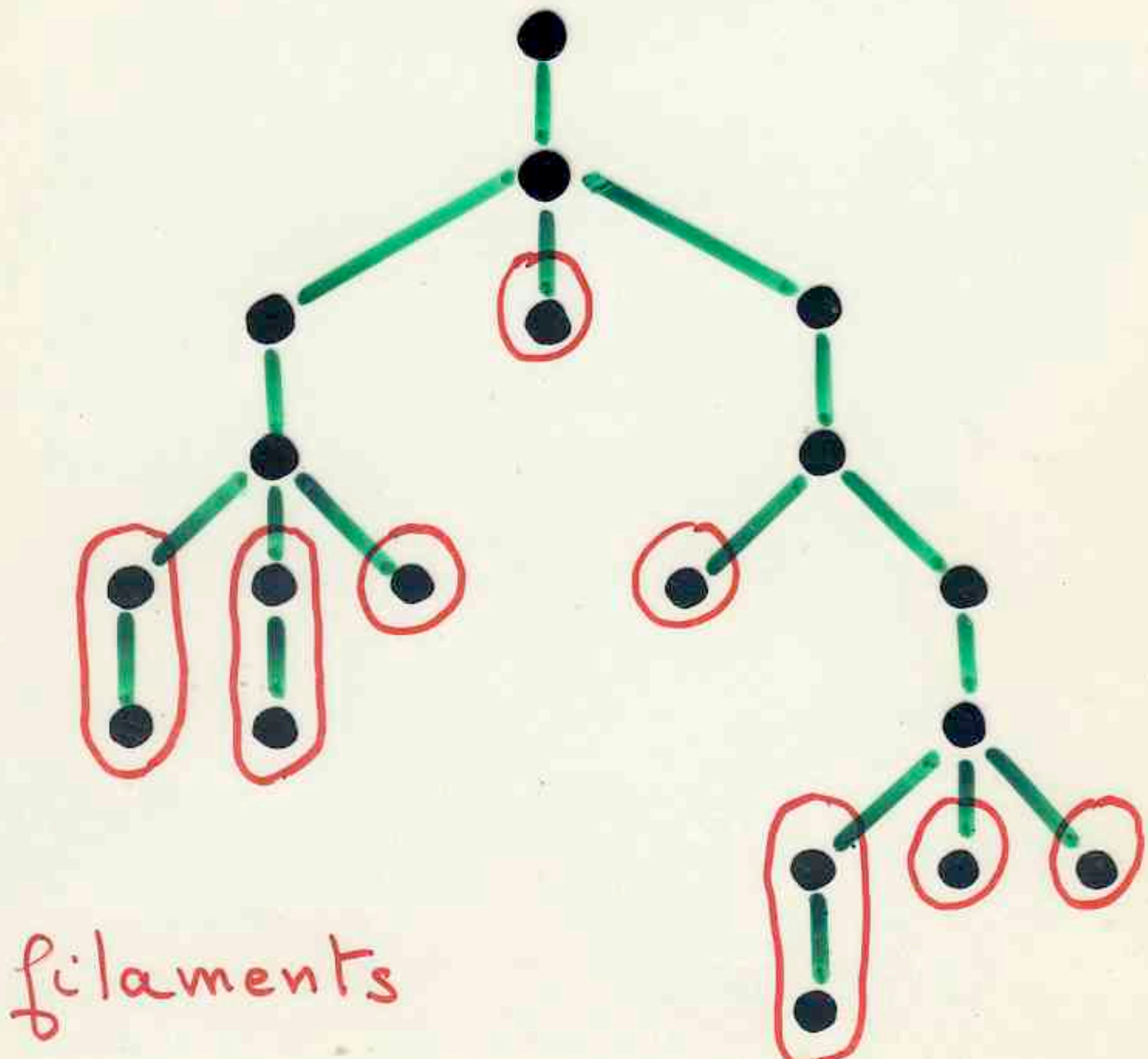
- ordre d'une structure secondaire ARN

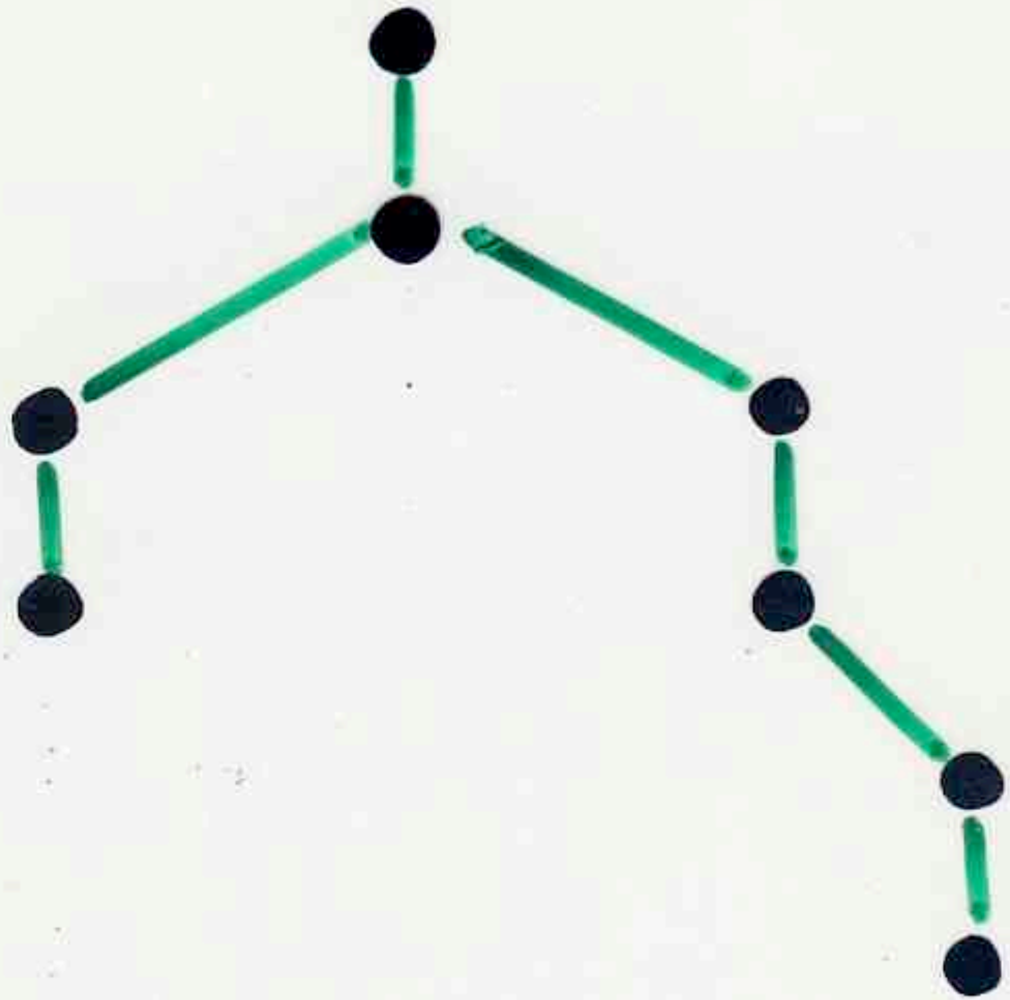
- ordre de la forêt d'arbres sous-jacente

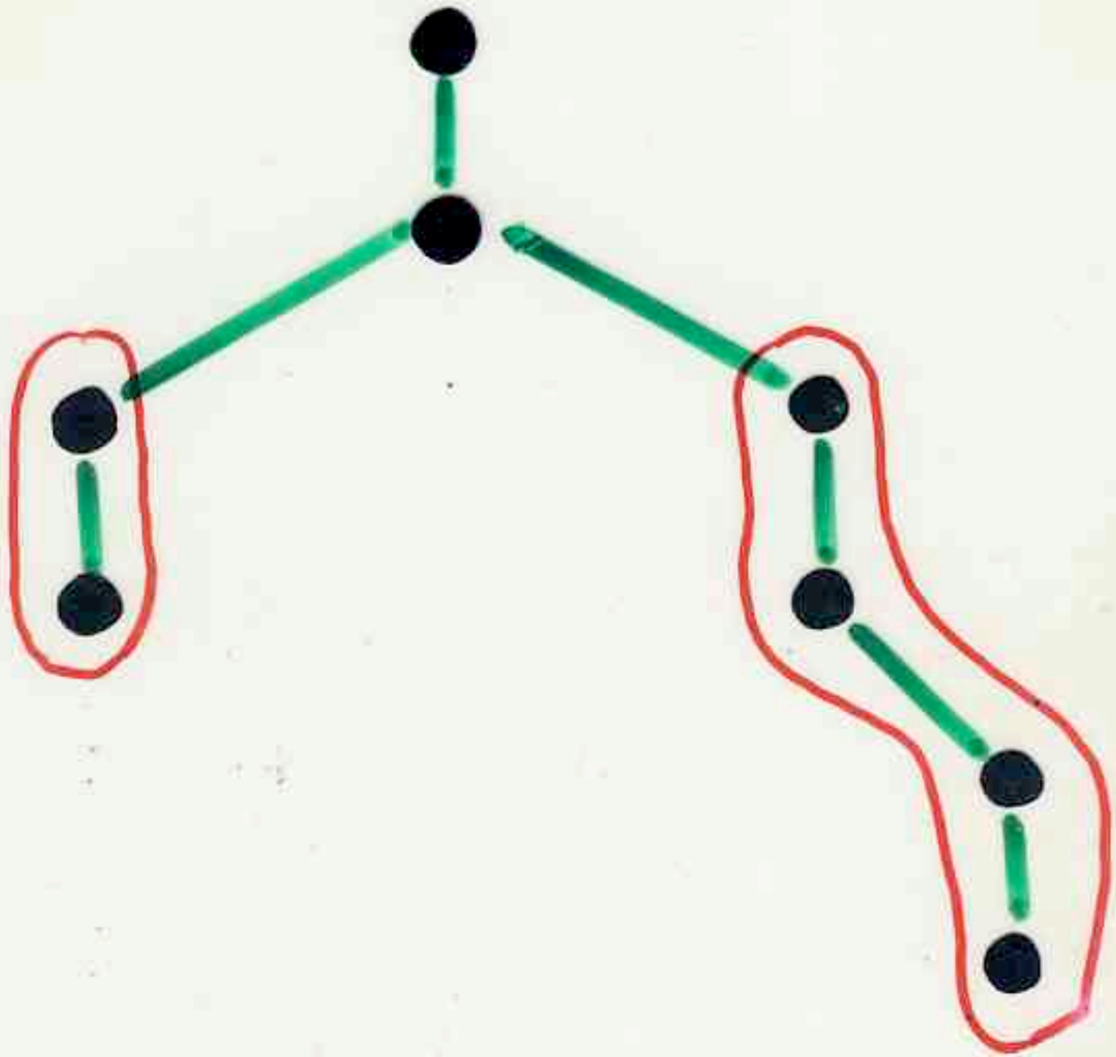
- ordre d'un arbre (planaire)

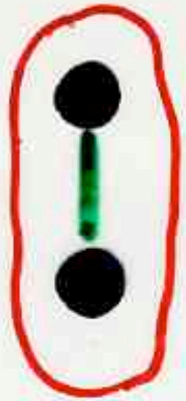
pruning number
of
forests of planar trees




filaments







$F_{n,k}$ = nombre de forêts d'arbres ayant n sommets et d'ordre k

number of forests of planar trees with n vertices and pruning number k

$F_{n,k}$ = nombre de forêts d'arbres ayant n sommets et d'ordre k

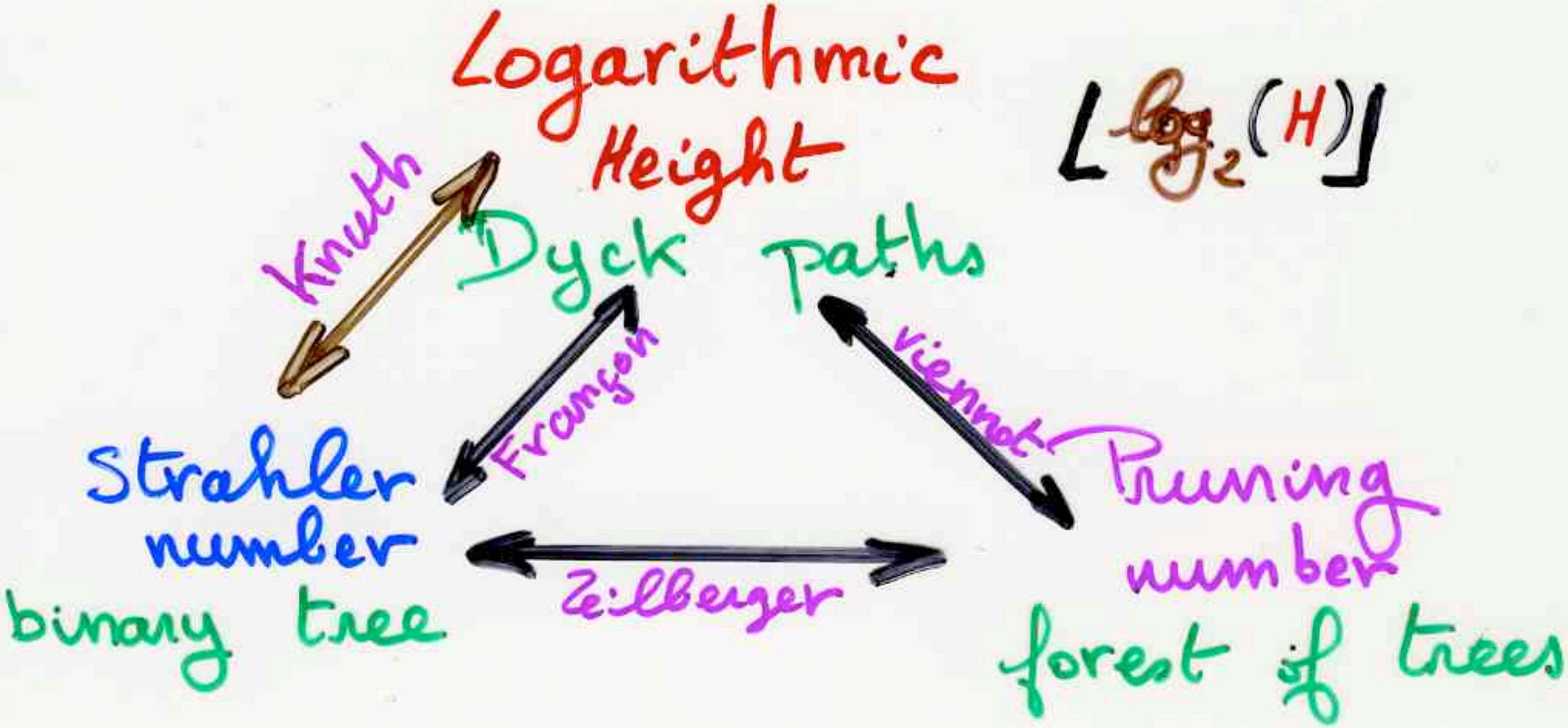
number of forests of planar trees with n vertices and pruning number k

= $S_{n,k}$

Vauchaussade de Chaumont
X. V. (1985) (2001)

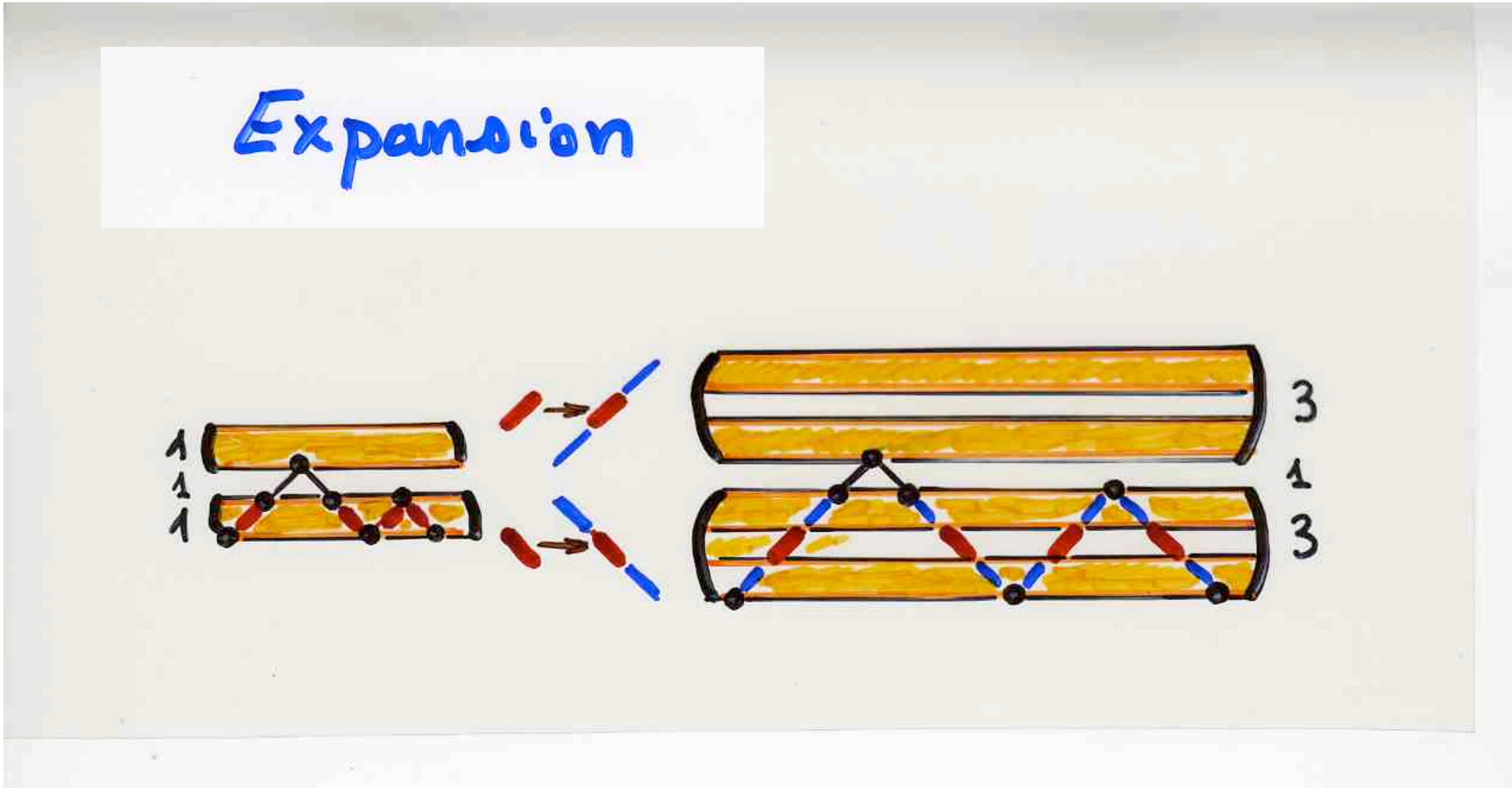
D. Zeilberger (1985)

number of binary trees with n vertices and Strahler number k

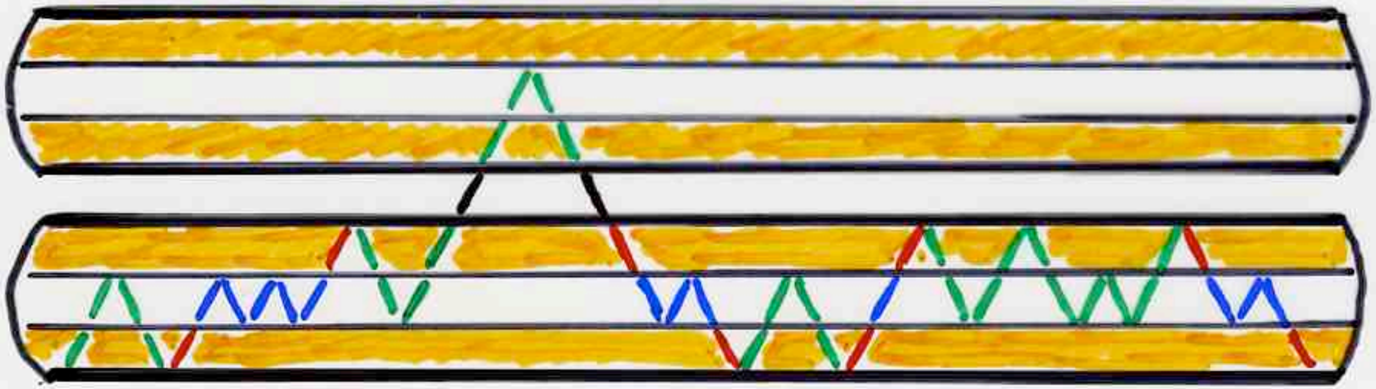


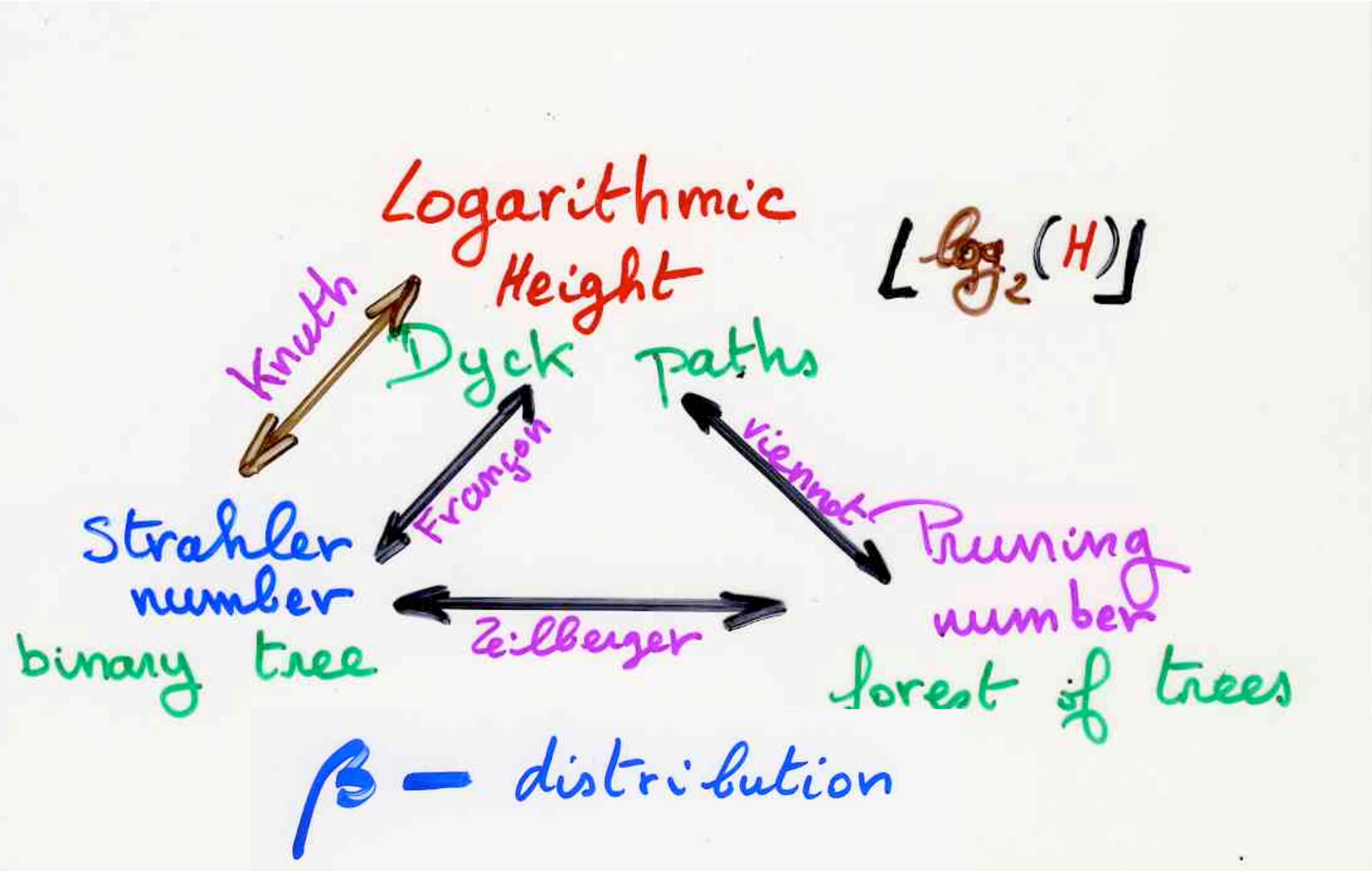
Logarithmic Height
⌊log₂(H)⌋
Knuth
Dyck paths
Françon
Viennot
Strahler number
binary tree
Pruning number
forest of trees
Zeilberger

$$1 + 1 \longrightarrow 2 + 2$$



$1 + 1 \longrightarrow 1+3$





Programs to Read

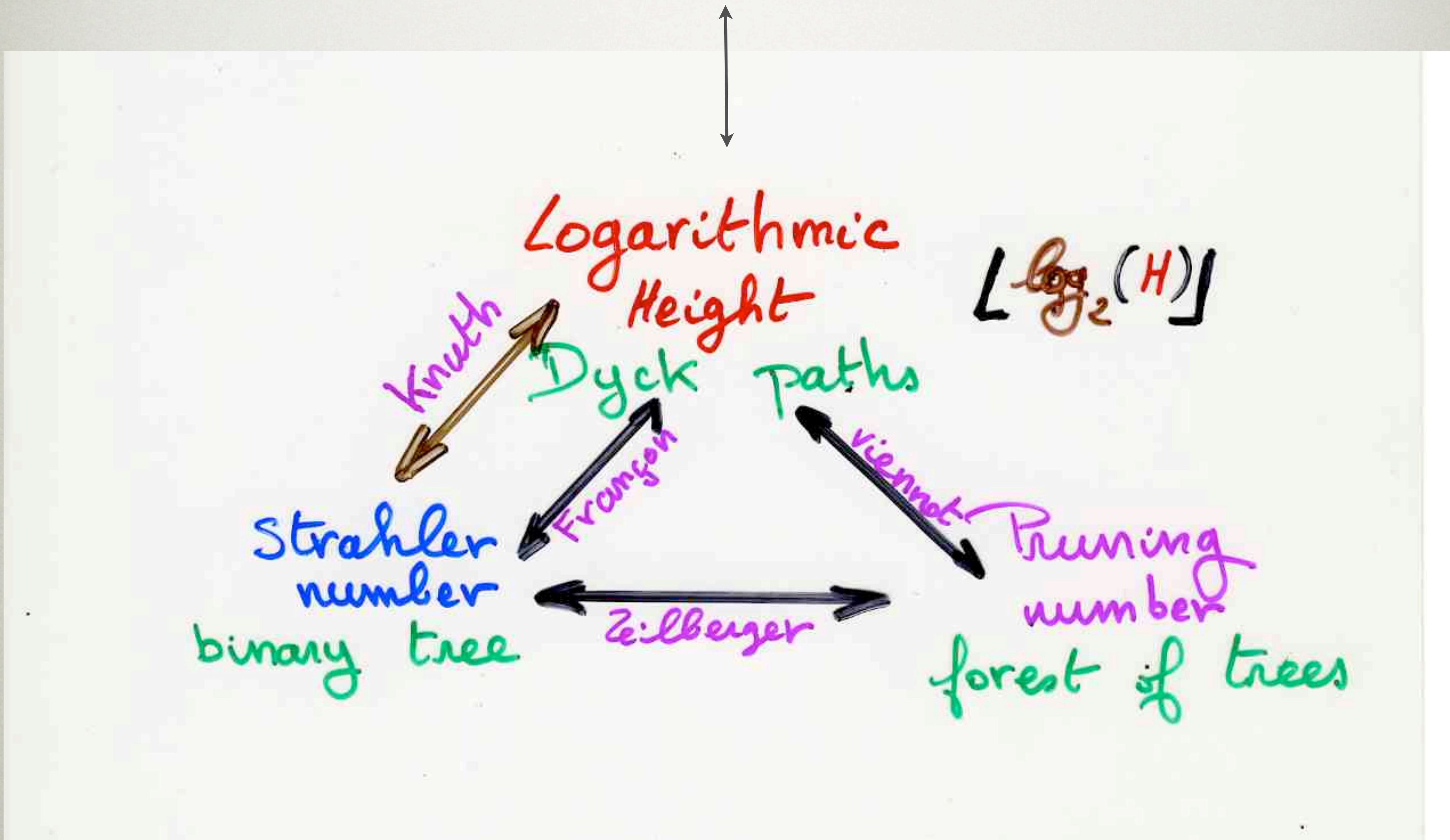
ZEILBERGER, FRANÇON, VIENNOT, an explanatory introduction, and a MetaPost source file for VIENNOT

Three Catalan bijections related to Strahler numbers, pruning orders, and Kepler towers (February 2005)

Kepler towers

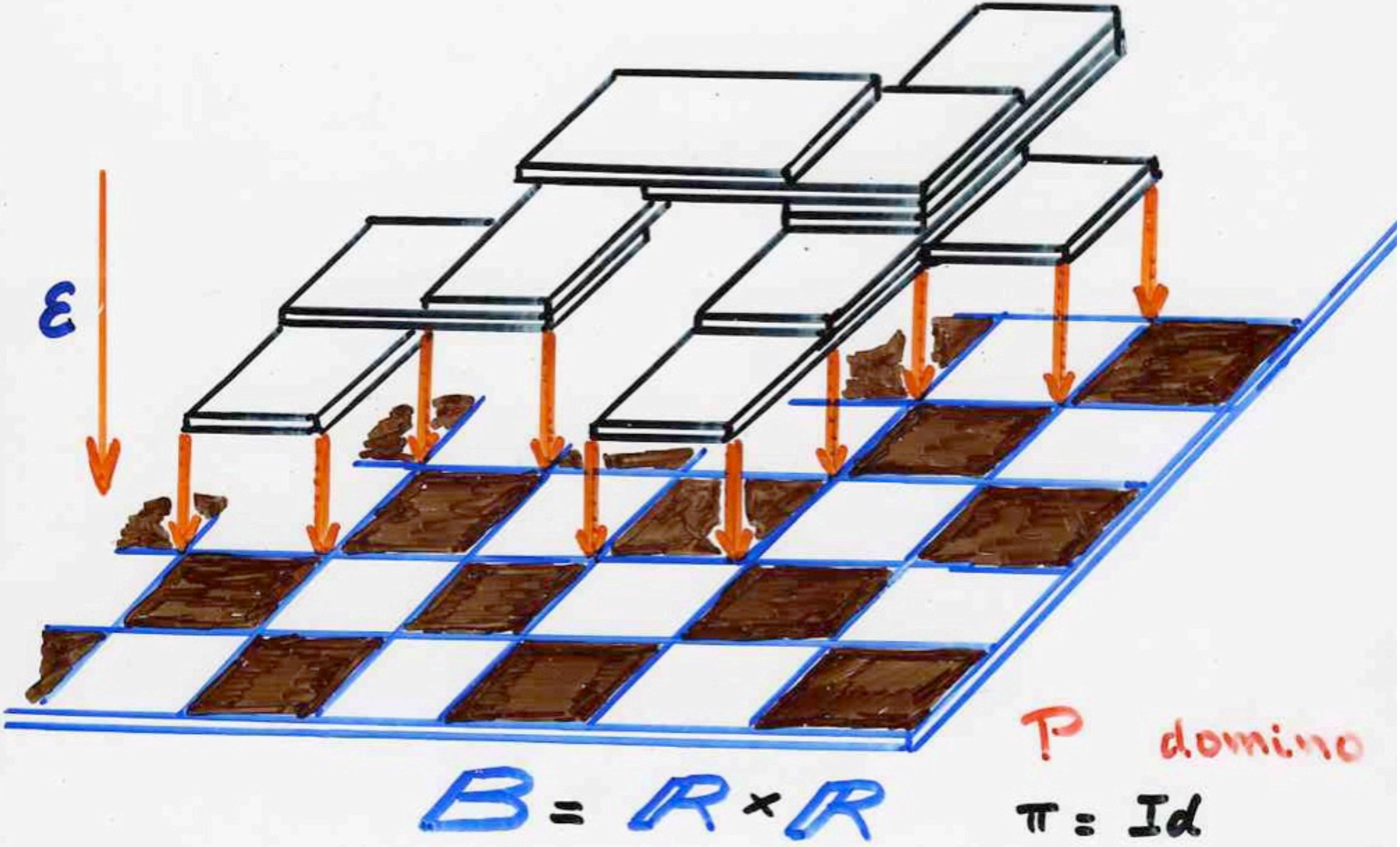
system of Kepler towers

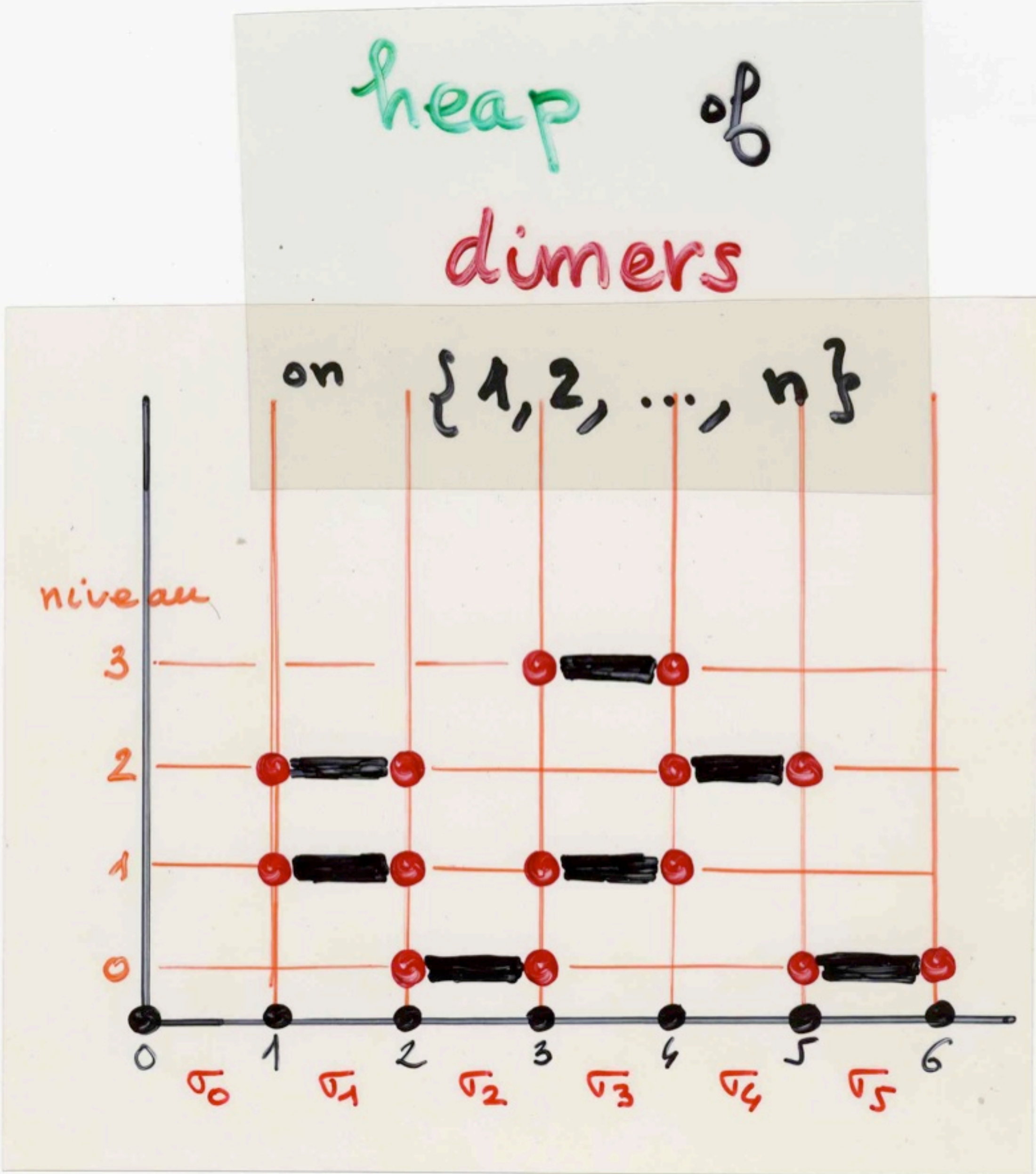
number of towers

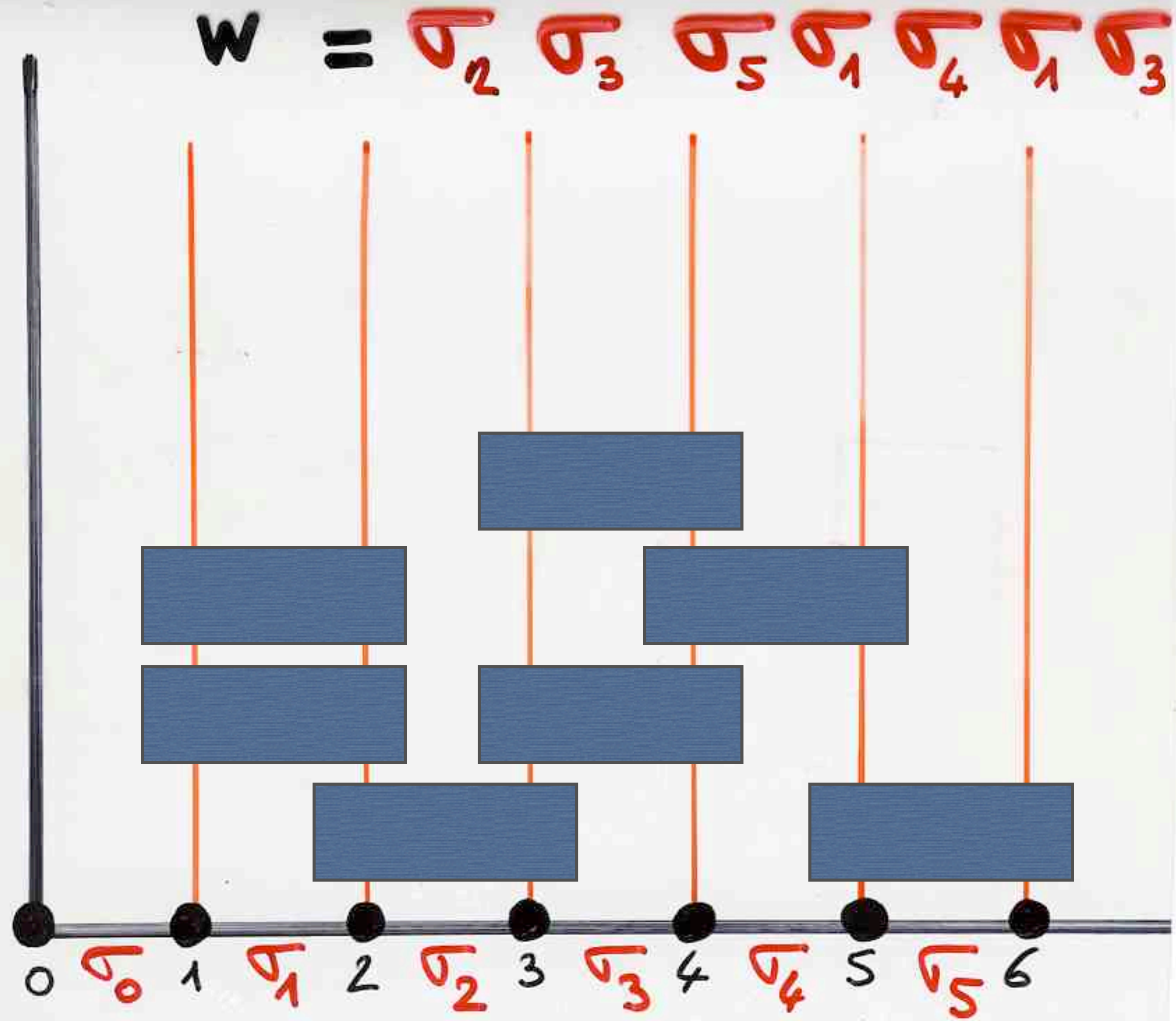


Heaps of pieces

Kepler towers

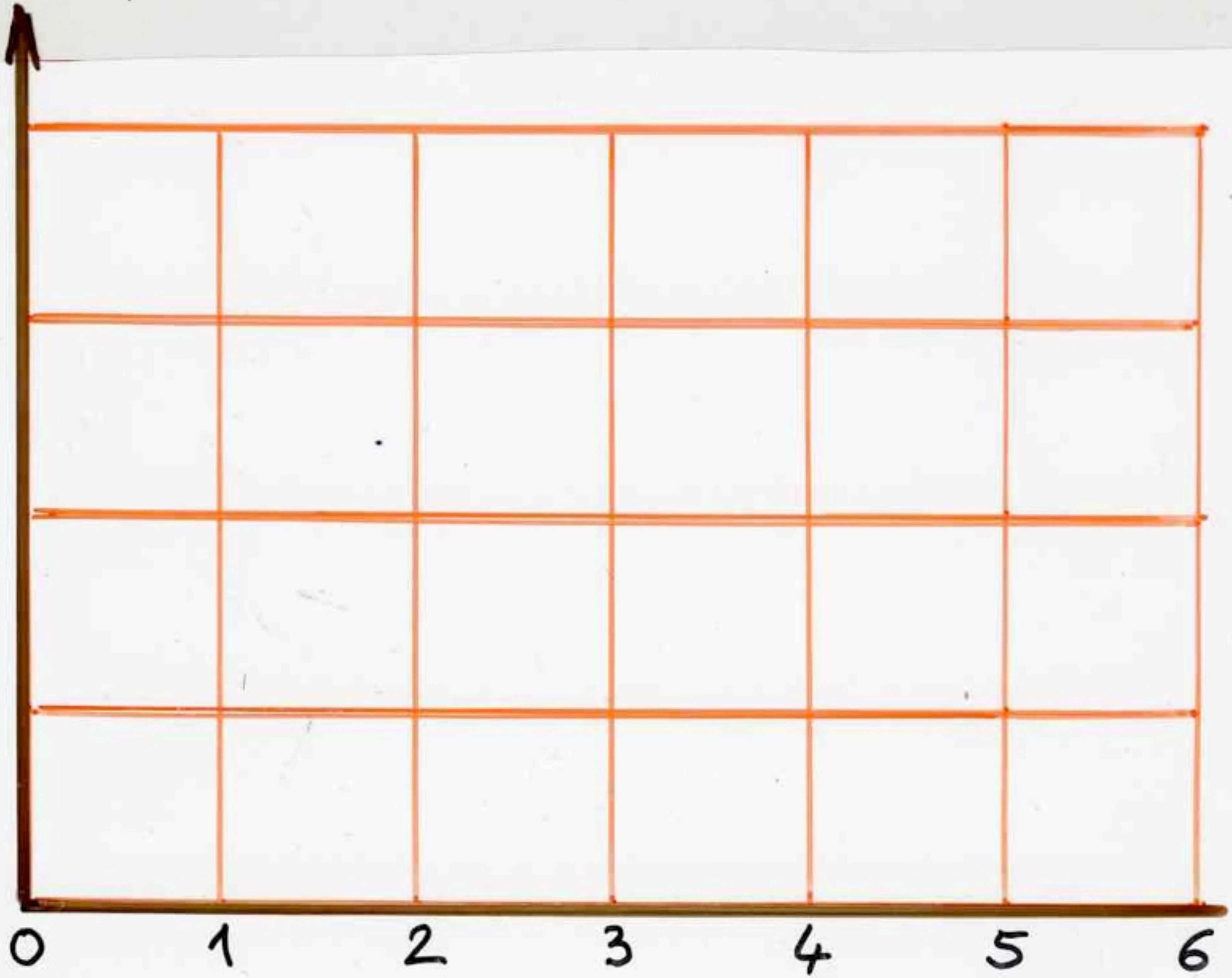

ε
P domino
B = ℝ × ℝ
π = Id


heap of
dimers
on {1, 2, ..., n}
niveau
3
2
1
0
0 1 2 3 4 5 6
σ0 σ1 σ2 σ3 σ4 σ5

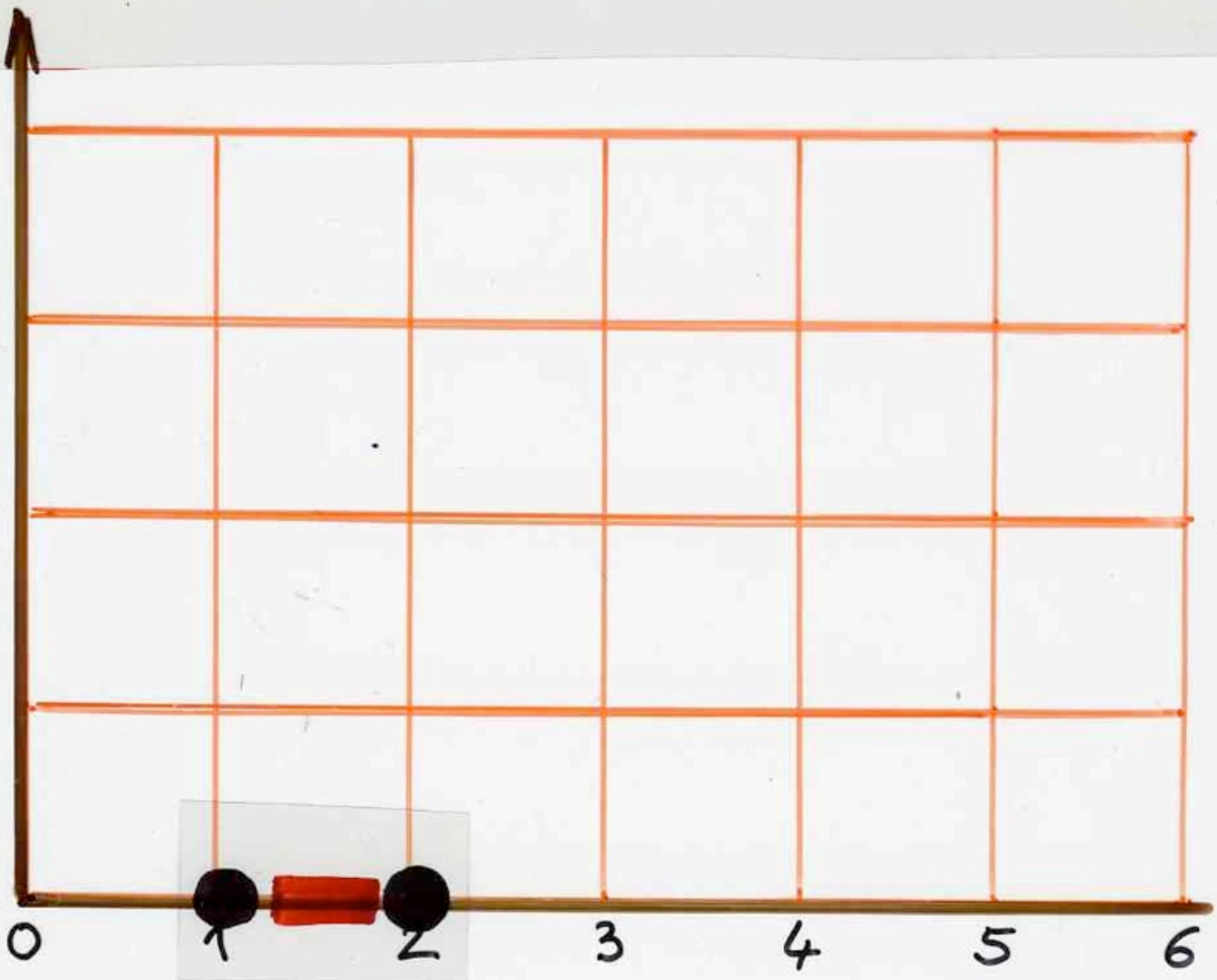


$w = \sigma_2 \sigma_3 \sigma_5 \sigma_1 \sigma_4 \sigma_1 \sigma_3$
0 σ_0 1 σ_1 2 σ_2 3 σ_3 4 σ_4 5 σ_5 6

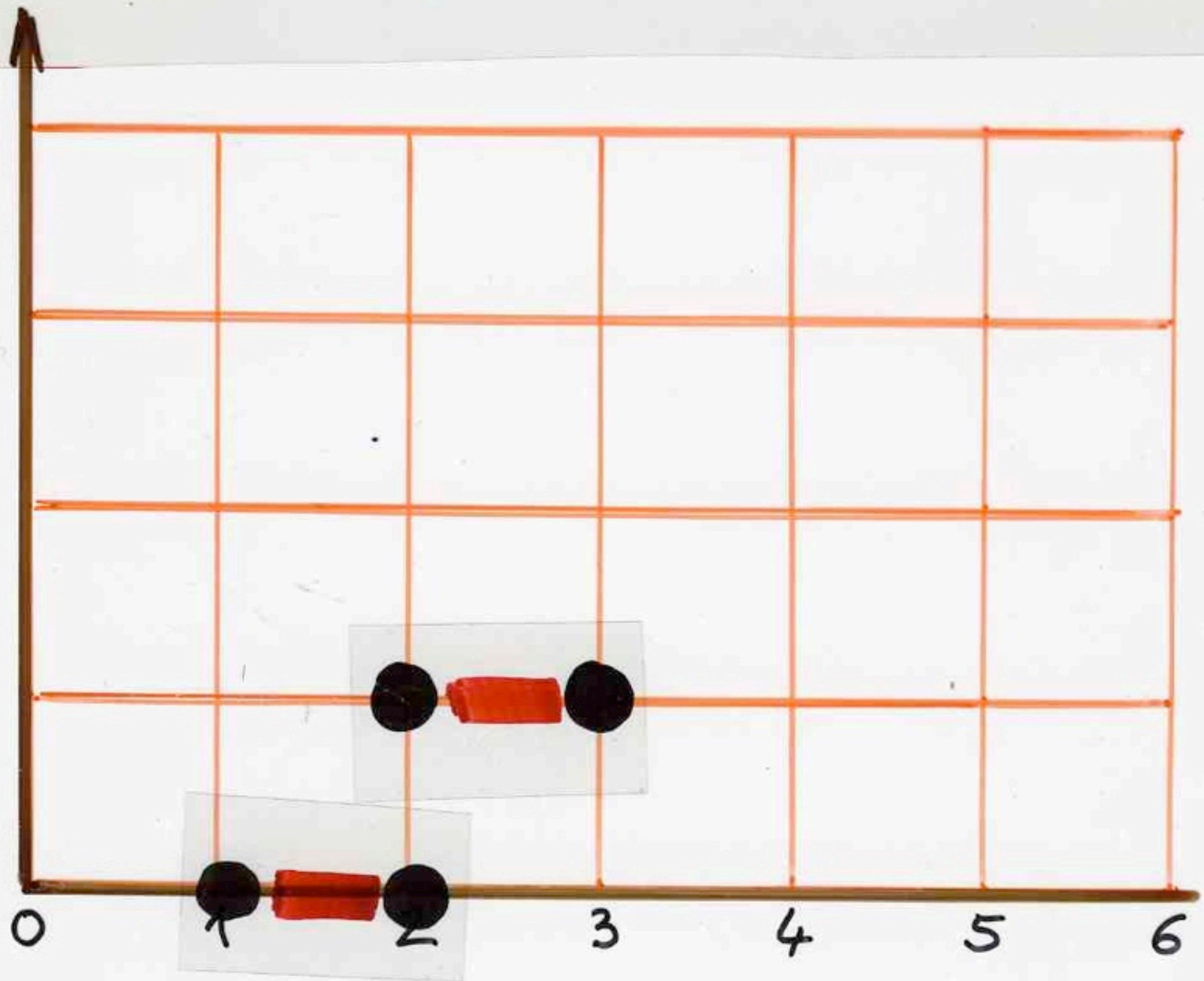
$w = \sigma_1 \sigma_2 \sigma_4 \sigma_1 \sigma_4 \sigma_3 \sigma_0 \sigma_4$



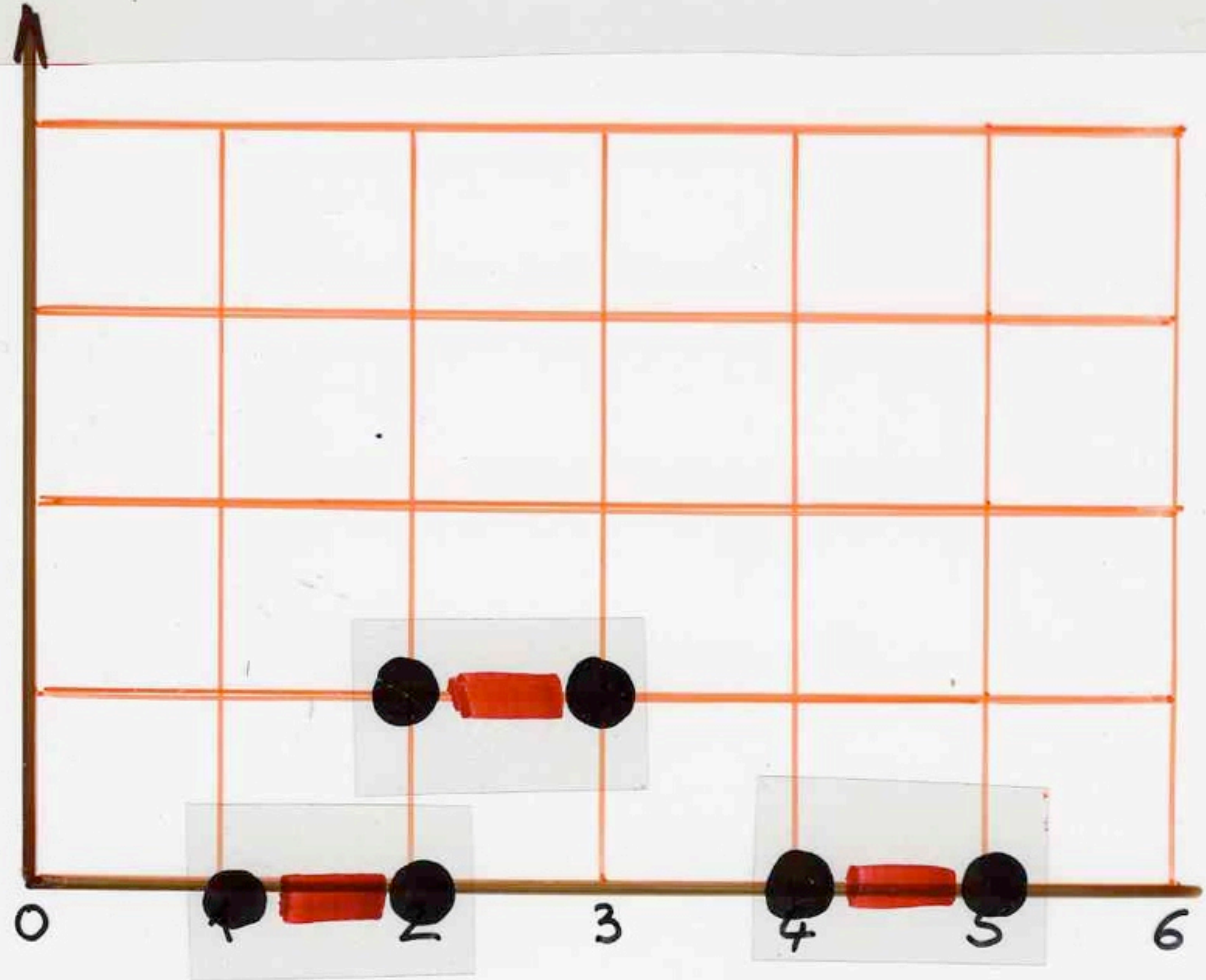
$$w = \sigma_1 \sigma_2 \sigma_4 \sigma_1 \sigma_4 \sigma_3 \sigma_0 \sigma_4$$



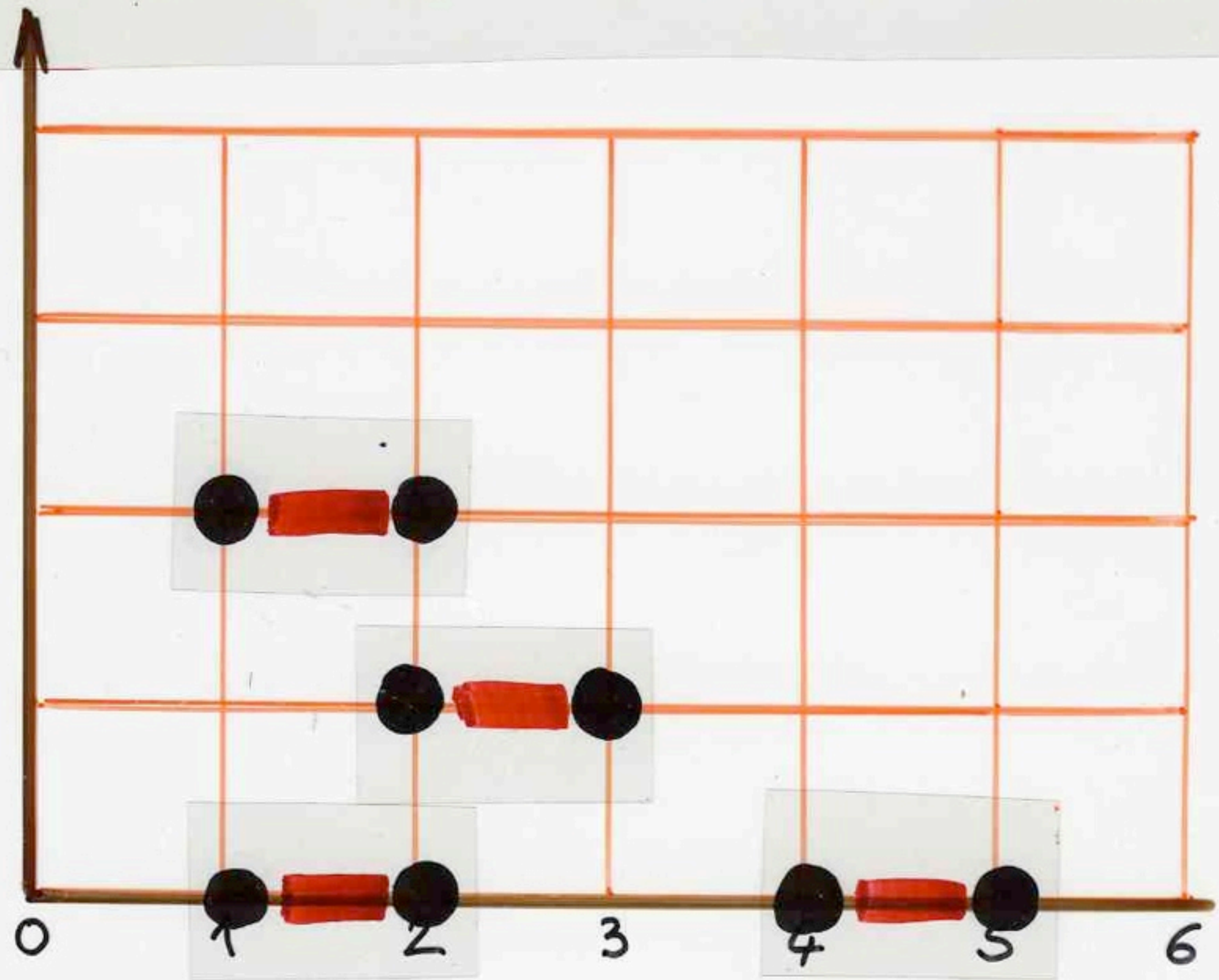
$$w = \sigma_1 \sigma_2 \sigma_4 \sigma_1 \sigma_4 \sigma_3 \sigma_0 \sigma_4$$



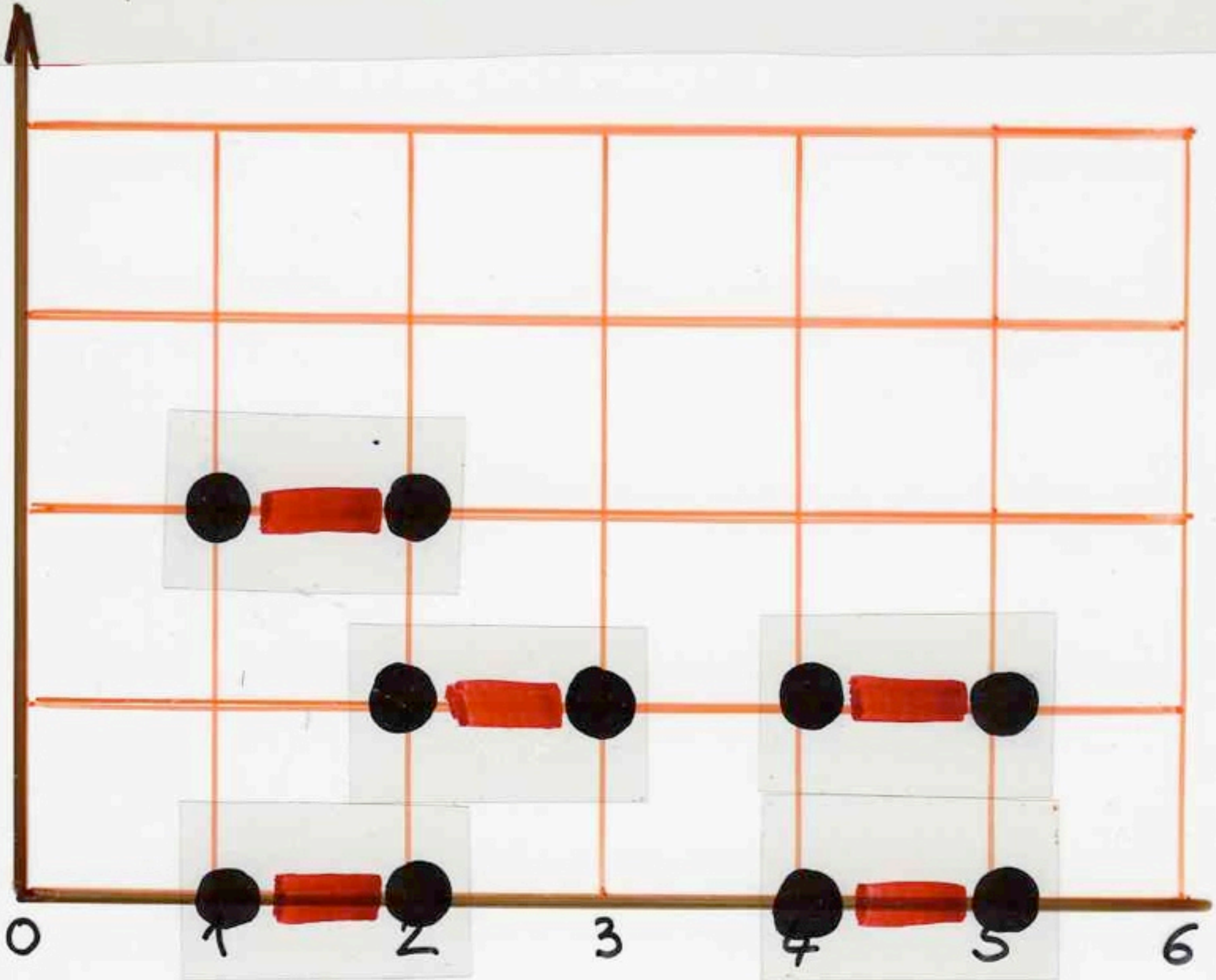
$w = \sigma_1 \sigma_2 \sigma_4 \sigma_1 \sigma_4 \sigma_3 \sigma_0 \sigma_4$



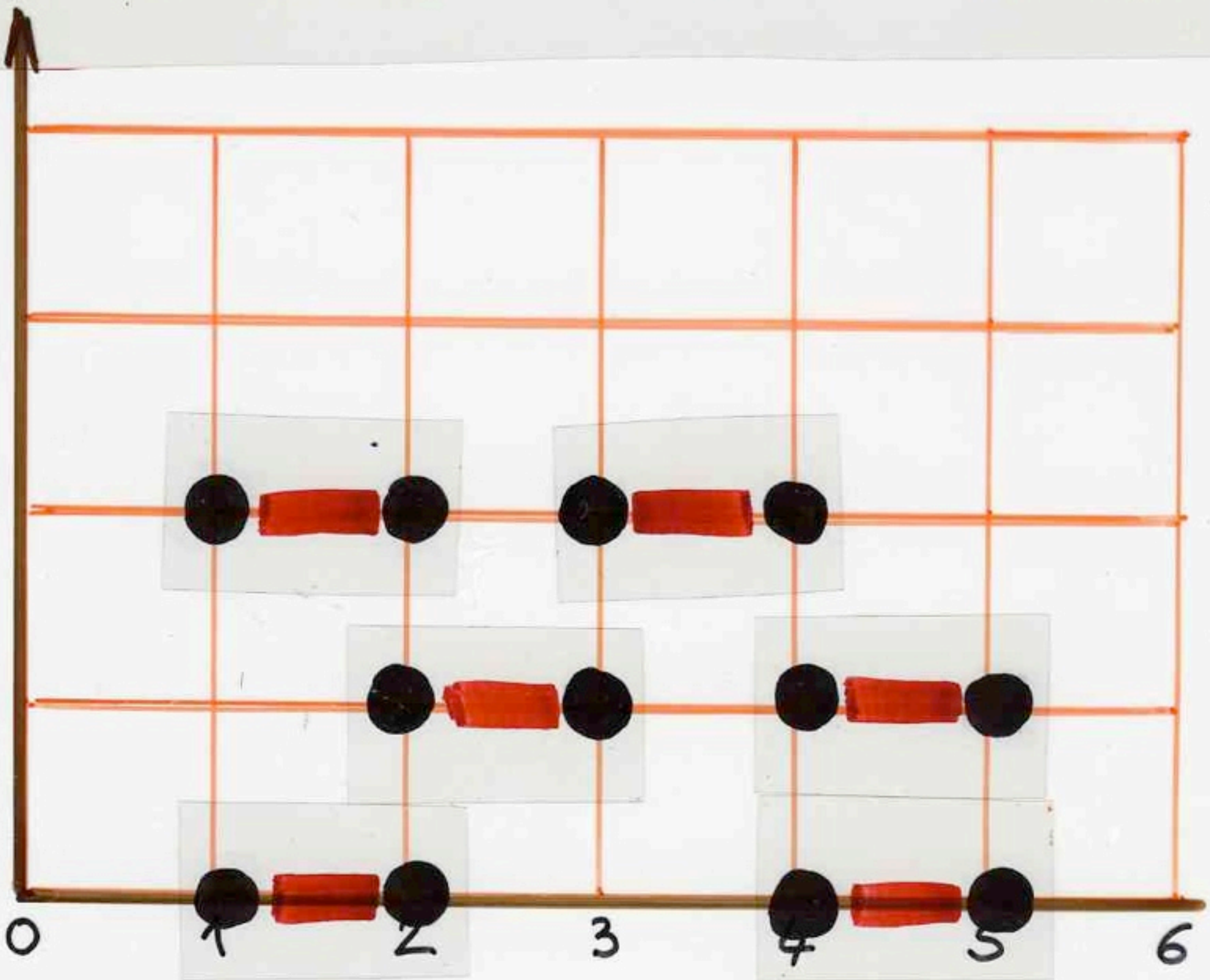
$$w = \sigma_1 \sigma_2 \sigma_4 \sigma_1 \sigma_4 \sigma_3 \sigma_0 \sigma_4$$



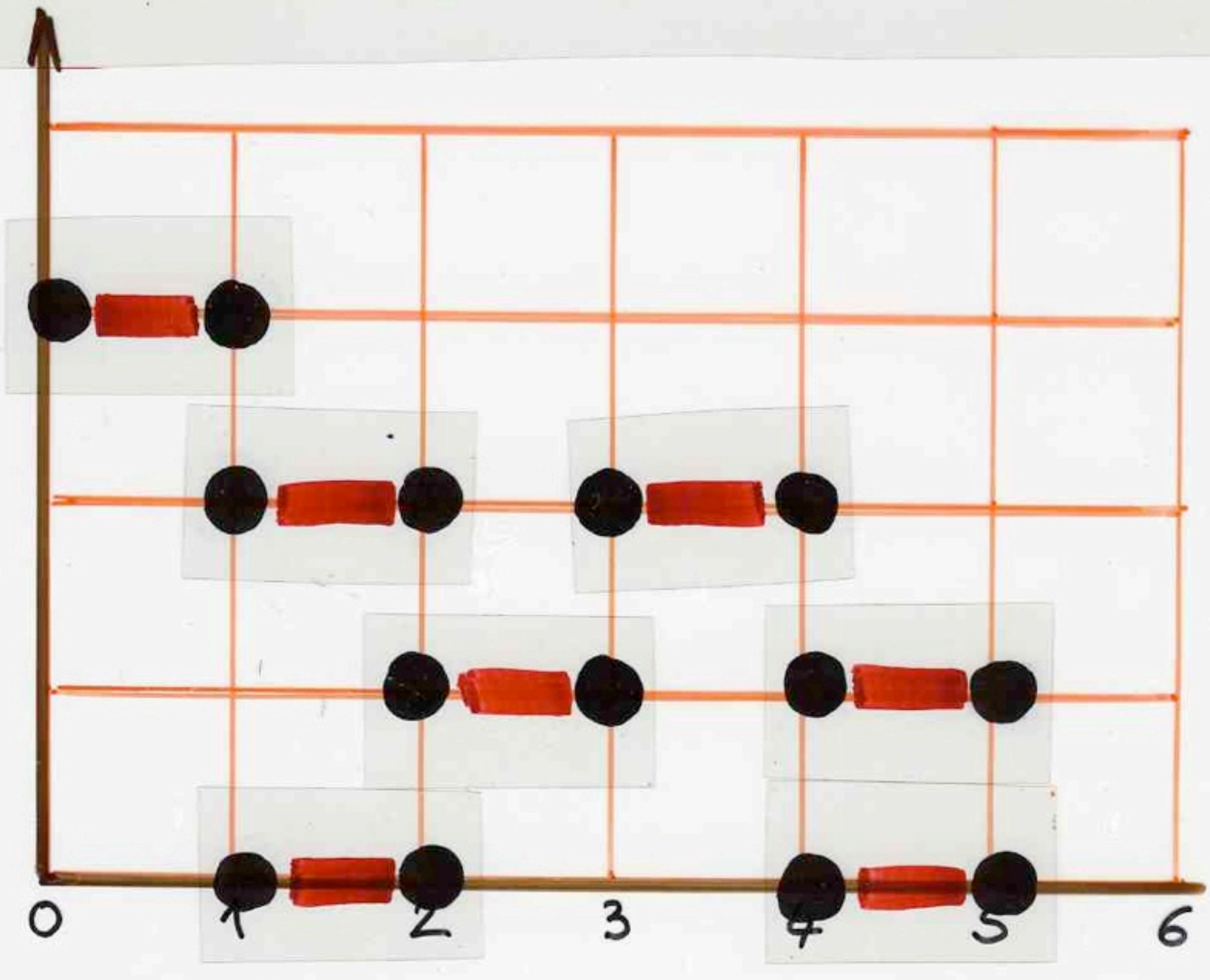
$$w = \sigma_1 \sigma_2 \sigma_4 \sigma_1 \sigma_4 \sigma_3 \sigma_0 \sigma_4$$



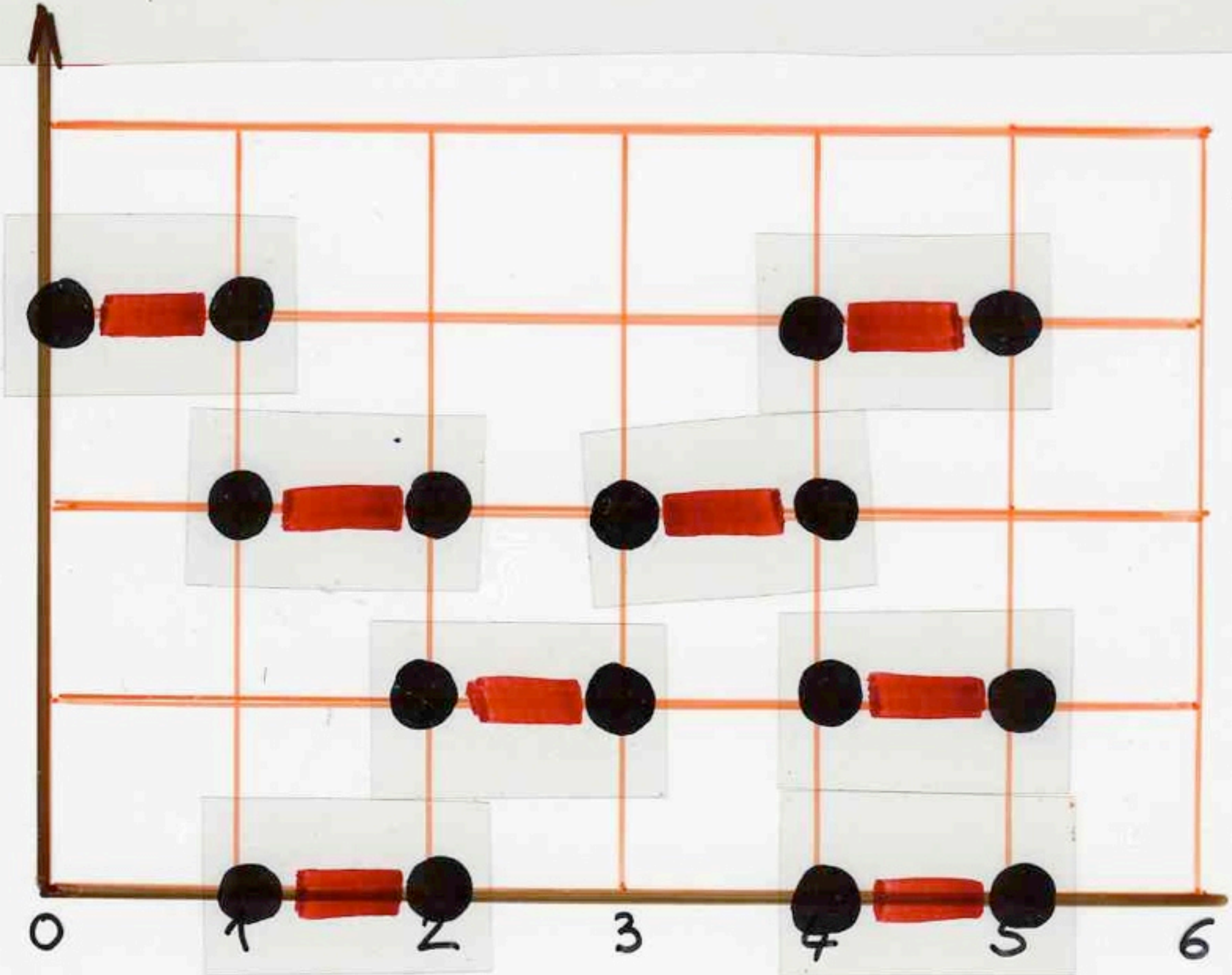
$$w = \sigma_1 \sigma_2 \sigma_4 \sigma_1 \sigma_4 \sigma_3 \sigma_0 \sigma_4$$

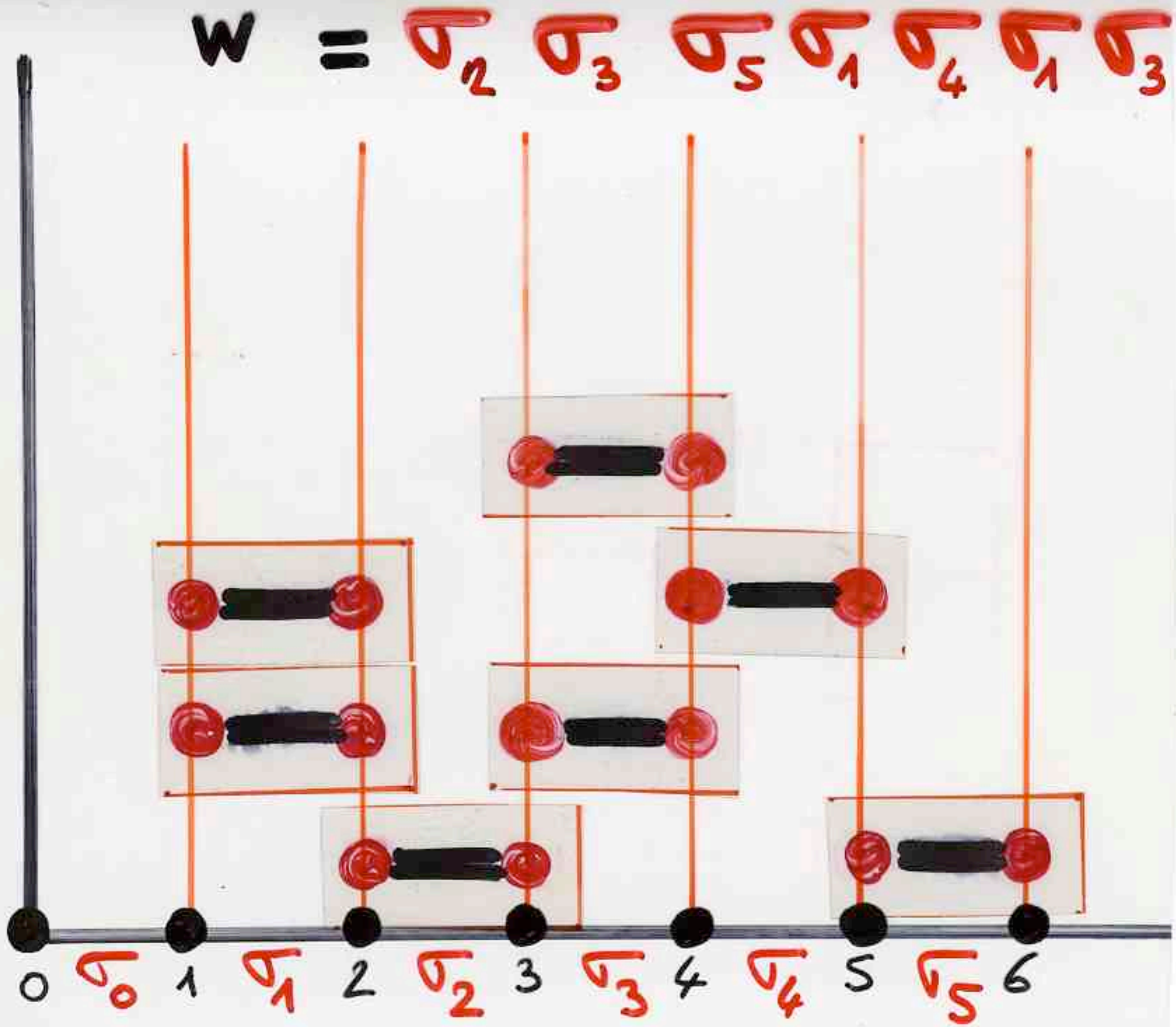


$$w = \sigma_1 \sigma_2 \sigma_4 \sigma_1 \sigma_4 \sigma_3 \sigma_0 \sigma_4$$



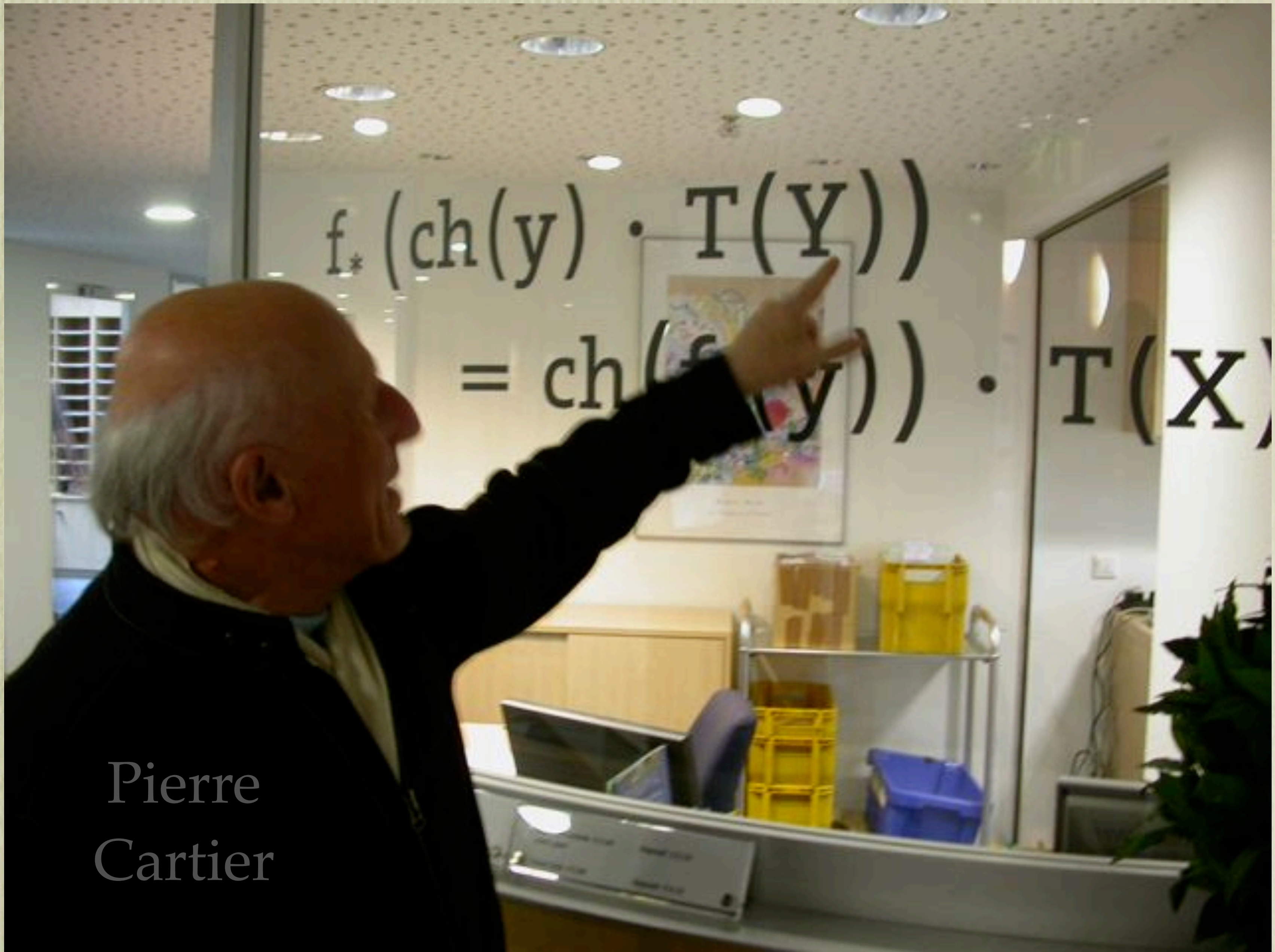
$w = \sigma_1 \sigma_2 \sigma_4 \sigma_1 \sigma_4 \sigma_3 \sigma_0 \sigma_4$




W = σ₂ σ₃ σ₅ σ₁ σ₄ σ₁ σ₃
0 σ₀ 1 σ₁ 2 σ₂ 3 σ₃ 4 σ₄ 5 σ₅ 6

heaps of pieces

commutation
monoids



Pierre Cartier

Dominique
Foata

empilements de pièces

heaps of pieces

commutation monoids

monoïdes de commutation

Trace monoids

Mazurkiewicz

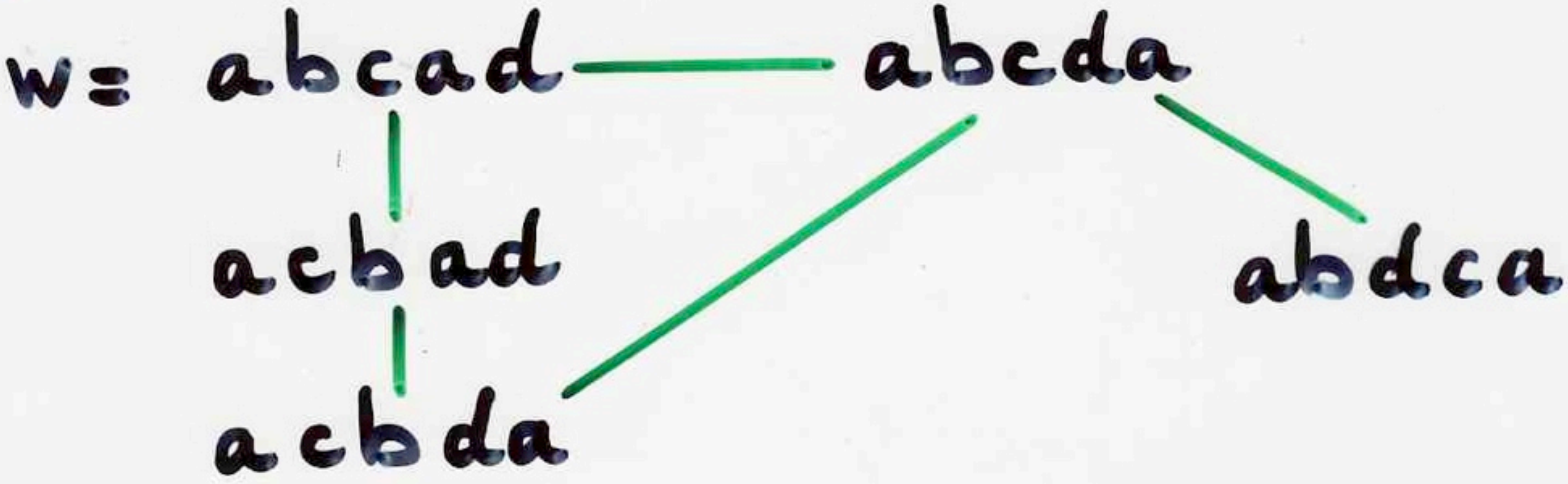
concurrency access to data structures

ex :

$A = \{a, b, c, d\}$

$C \quad \begin{cases} ad = da \\ bc = cb \\ cd = dc \end{cases}$

equivalence class



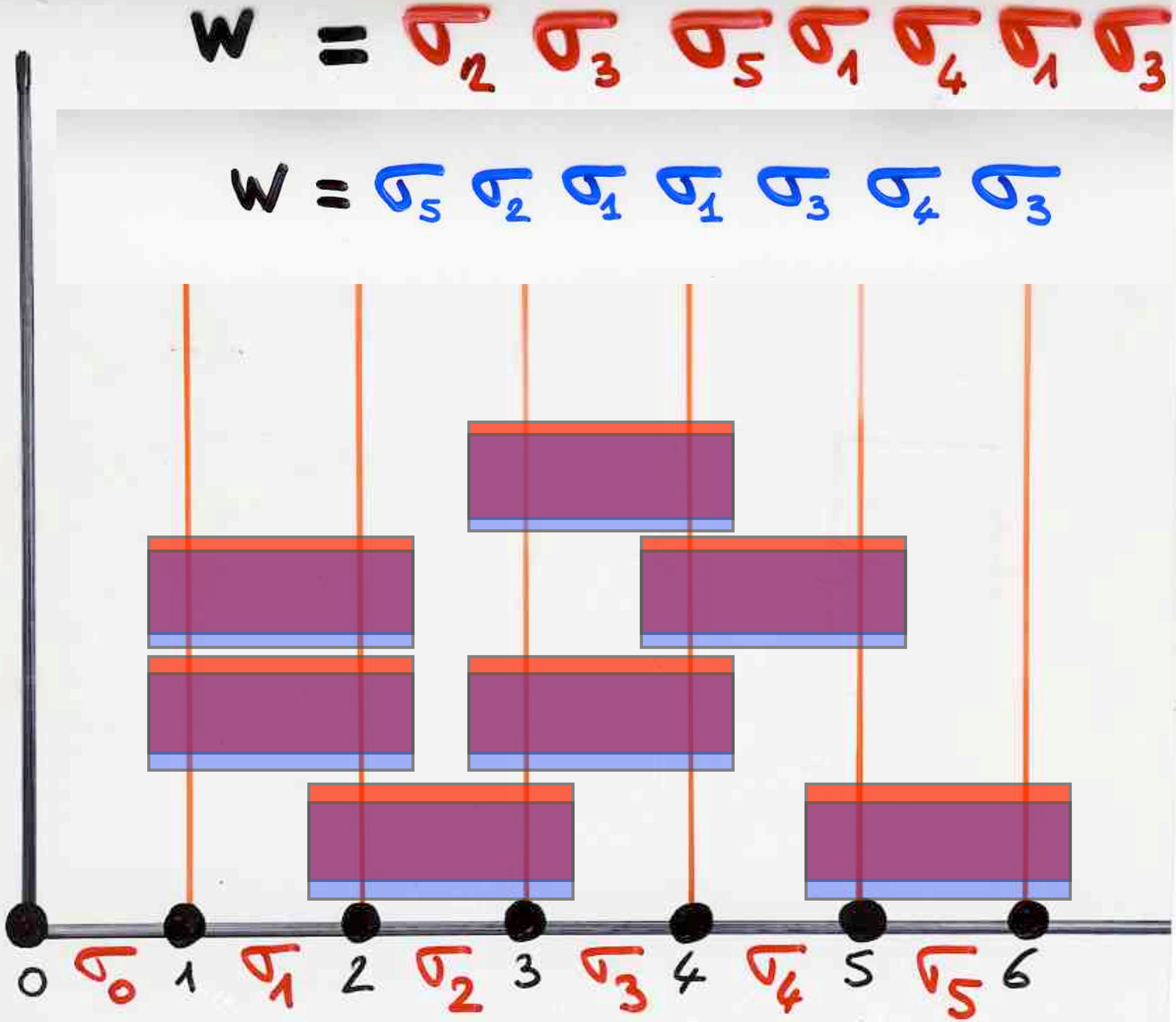
heaps of dimers $(i, i+1)$

on $\{0, 1, \ldots, n-1\}$

generators $\{\sigma_0, \sigma_1, \ldots, \sigma_{n-1}\}$

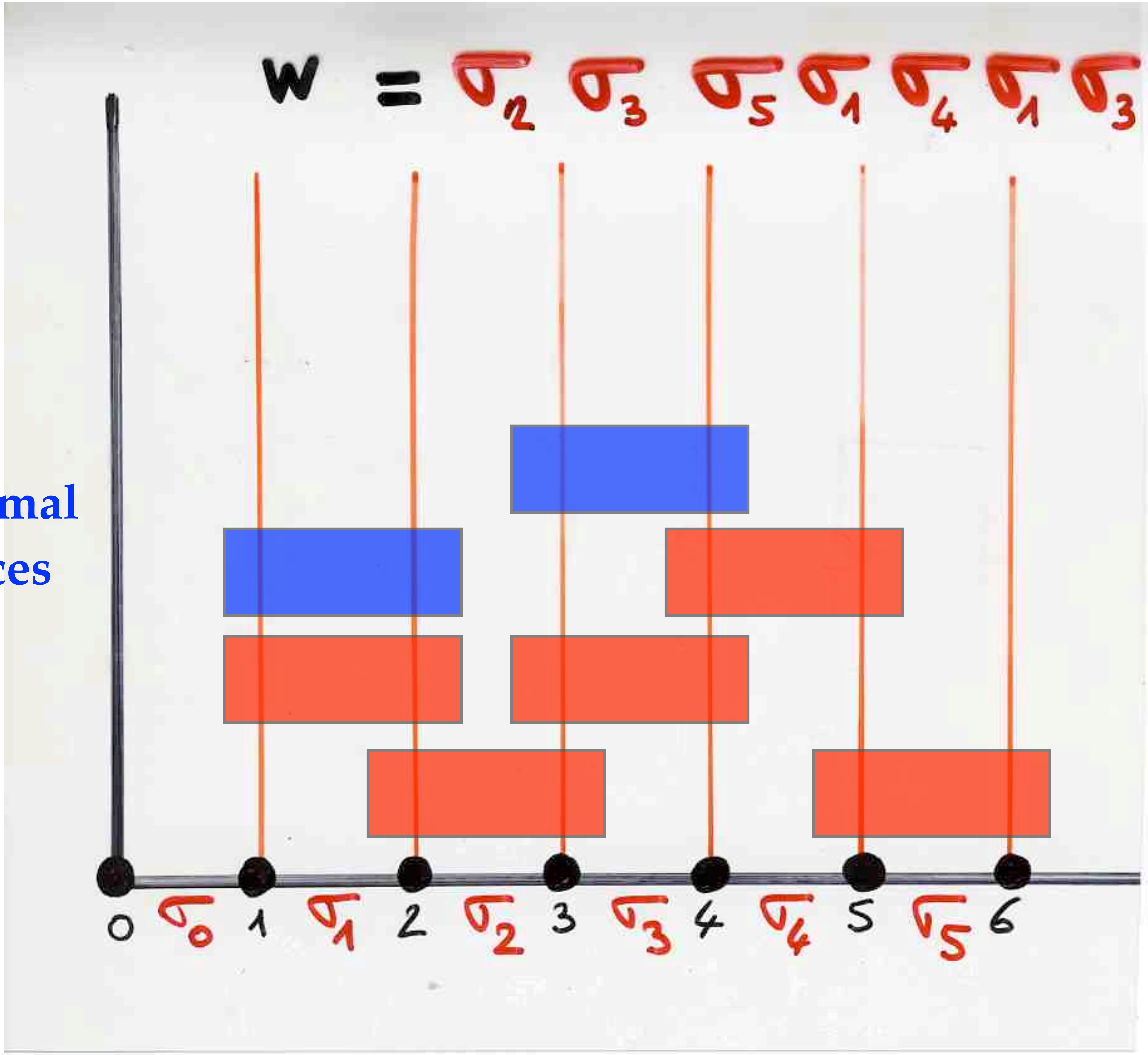
$$\sigma_i \sigma_j = \sigma_j \sigma_i$$

iff $|i-j| \geq 2$



$w = \sigma_2 \sigma_3 \sigma_5 \sigma_1 \sigma_4 \sigma_1 \sigma_3$
$w = \sigma_5 \sigma_2 \sigma_1 \sigma_1 \sigma_3 \sigma_4 \sigma_3$
0 σ_0 1 σ_1 2 σ_2 3 σ_3 4 σ_4 5 σ_5 6

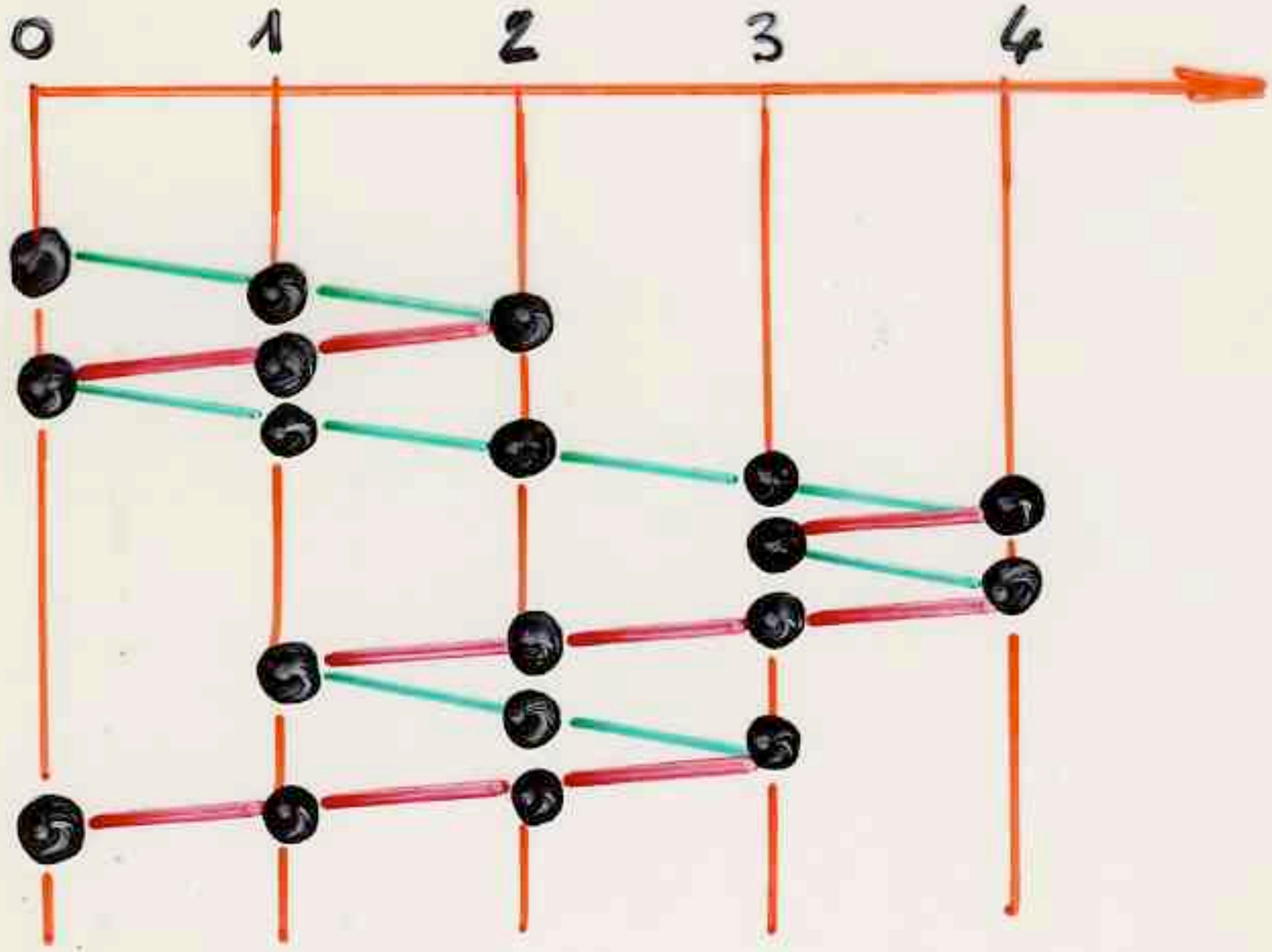
maximal
pieces



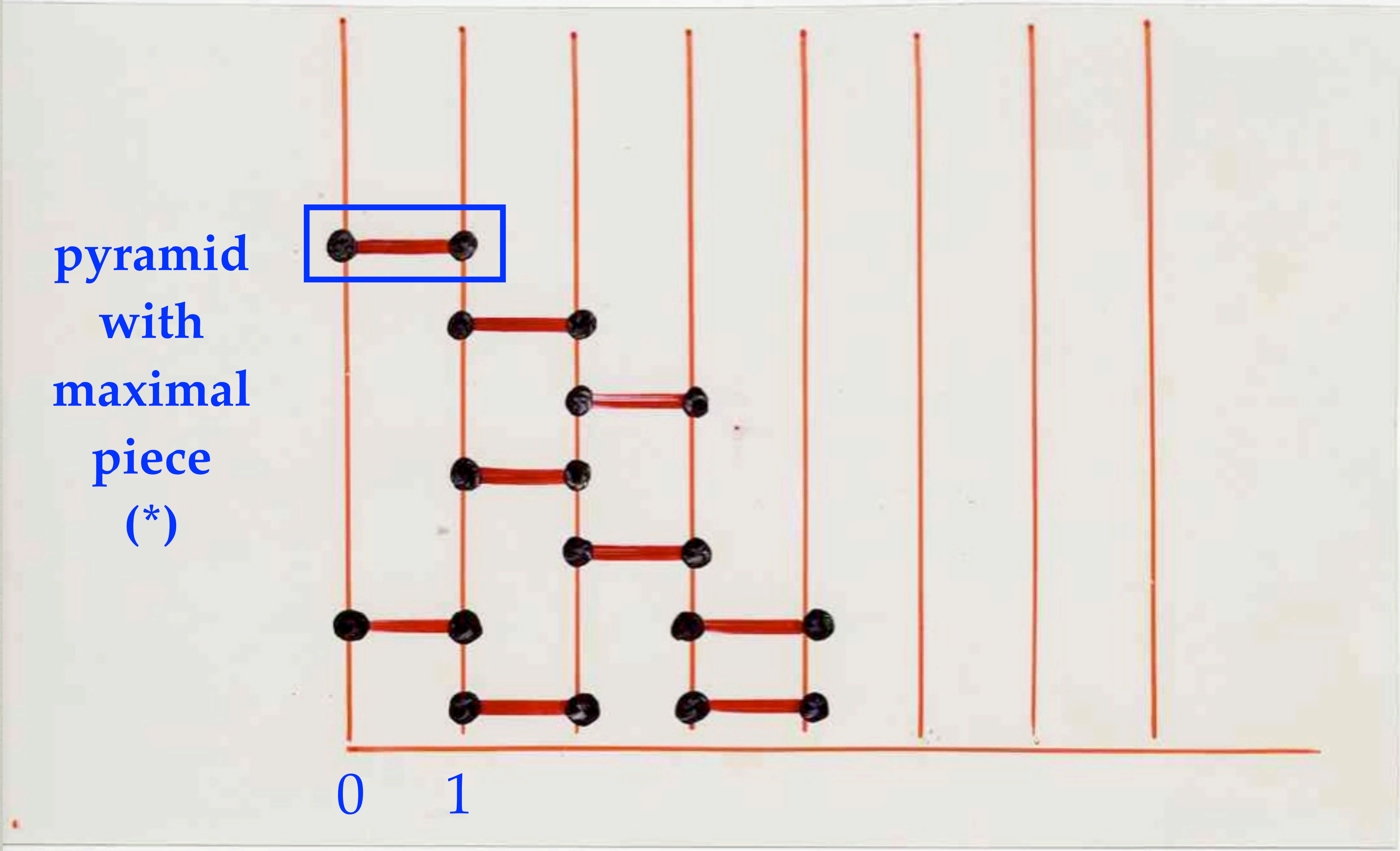
Pyramid

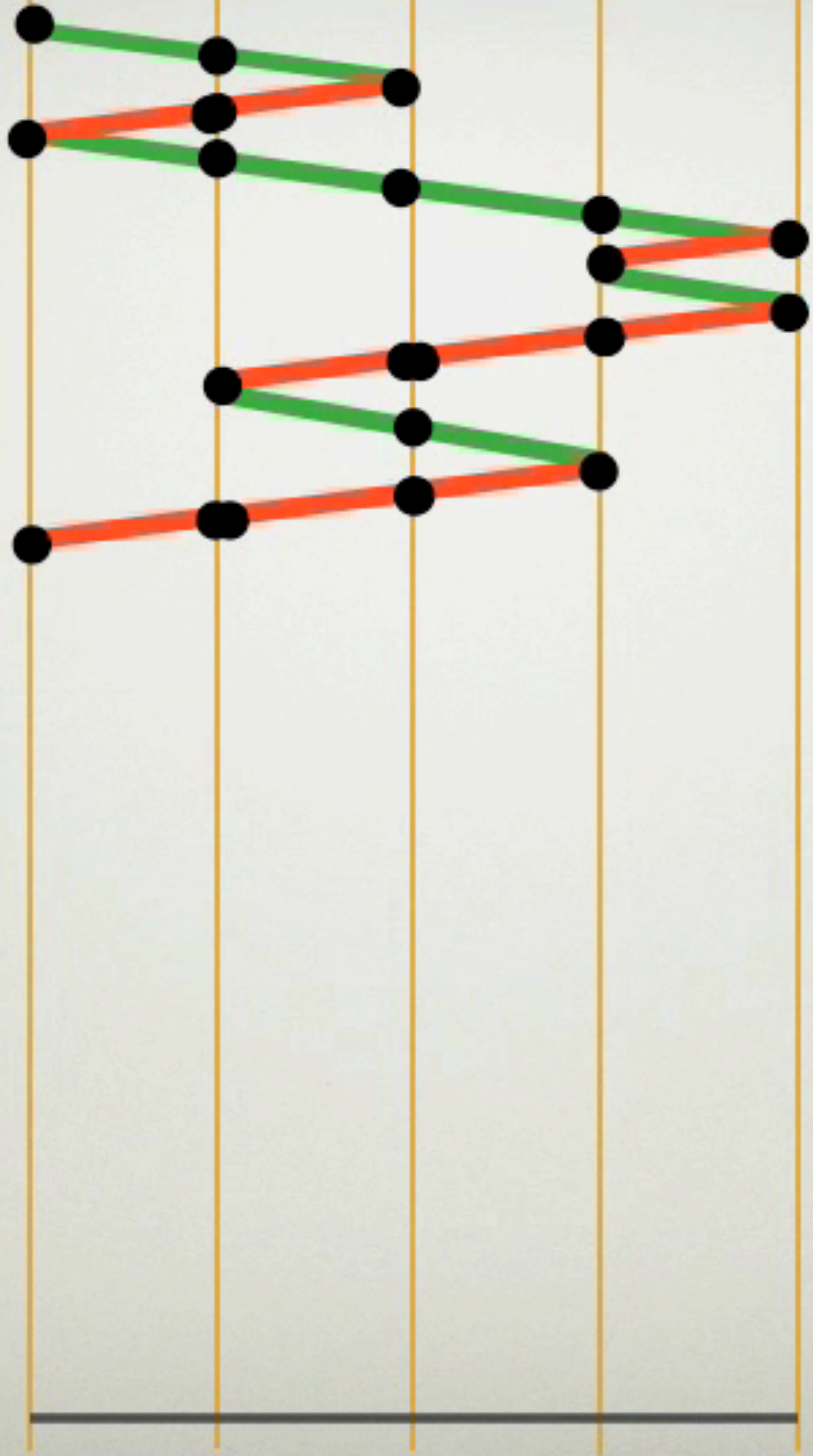
Def- Heap having only one maximal piece

from Dyck paths
to
pyramids of dimers

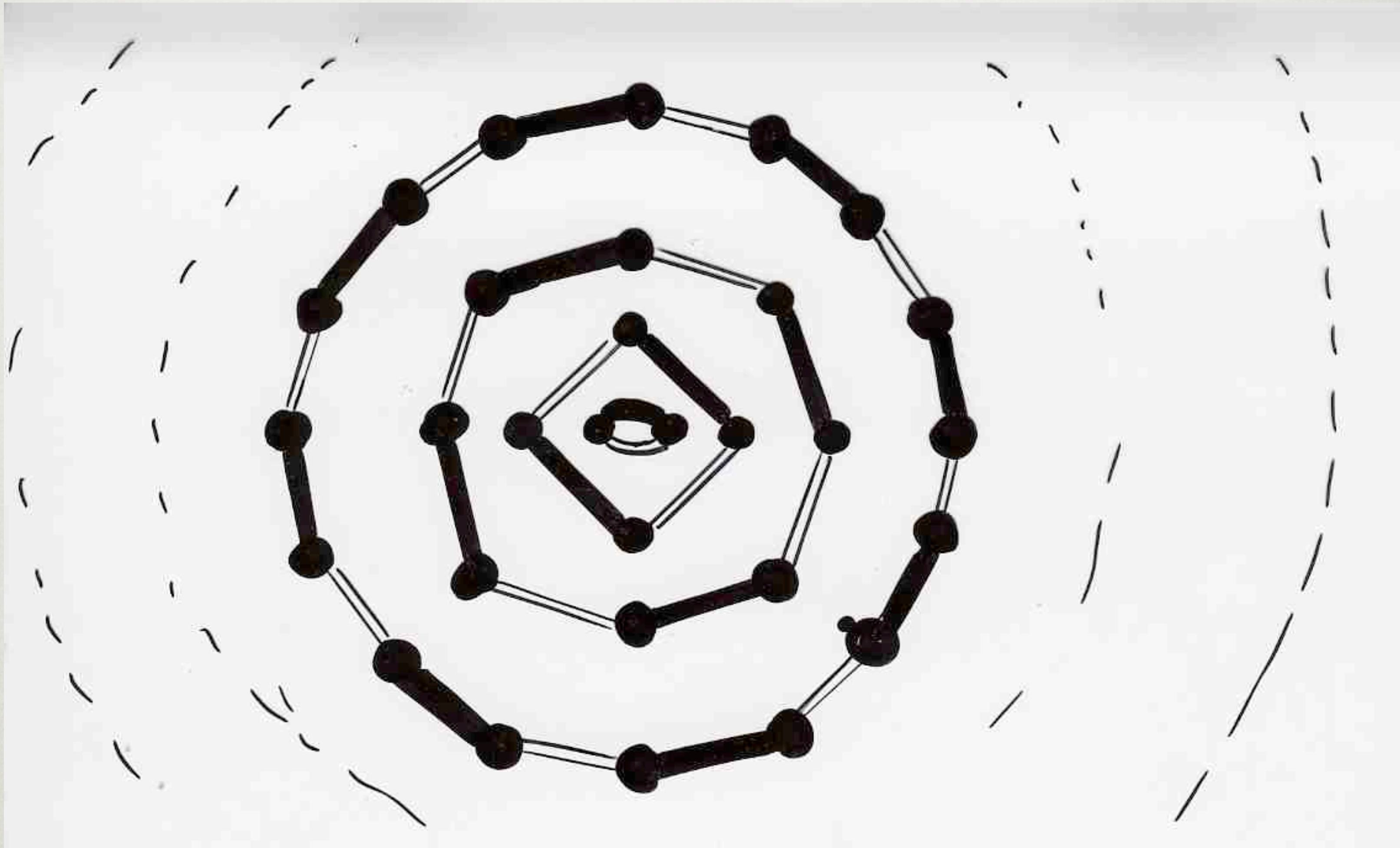


Dyck path


pyramid
with
maximal
piece
(*)
0
1



Kepler towers



System of Kepler towers

- regular polygons $P_2, P_4, P_8, \dots$

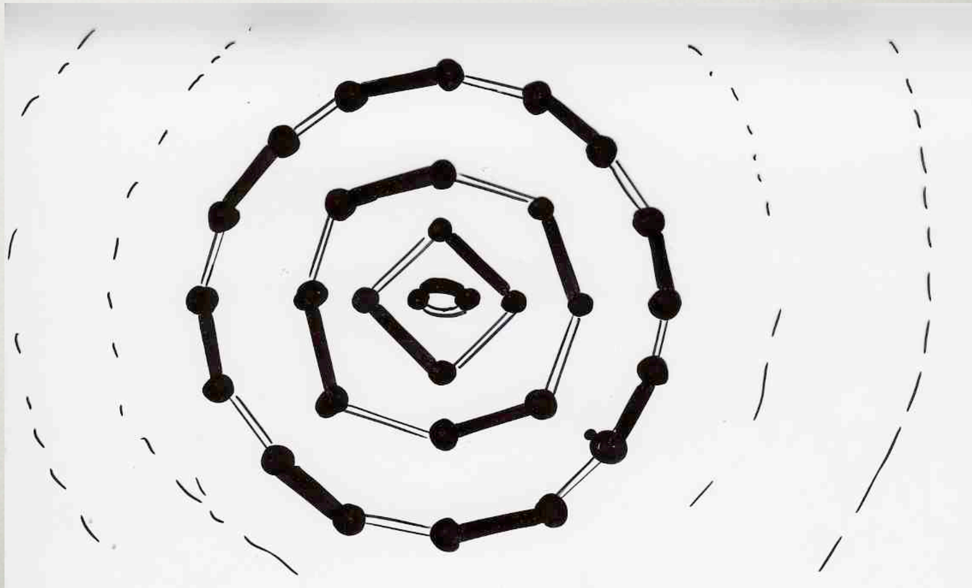
  P_i 2^i edges

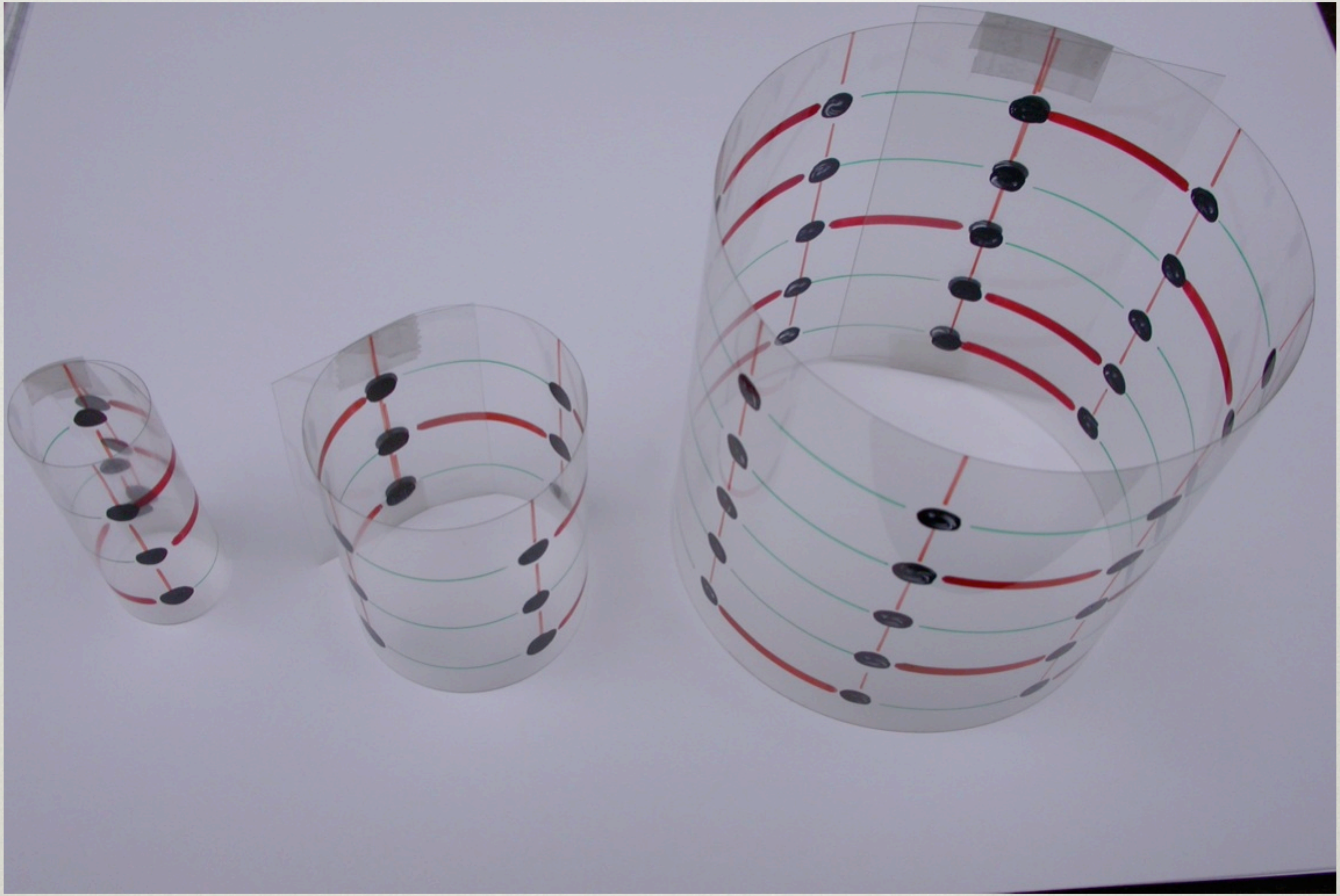
- heaps $H_1, \dots, H_k$

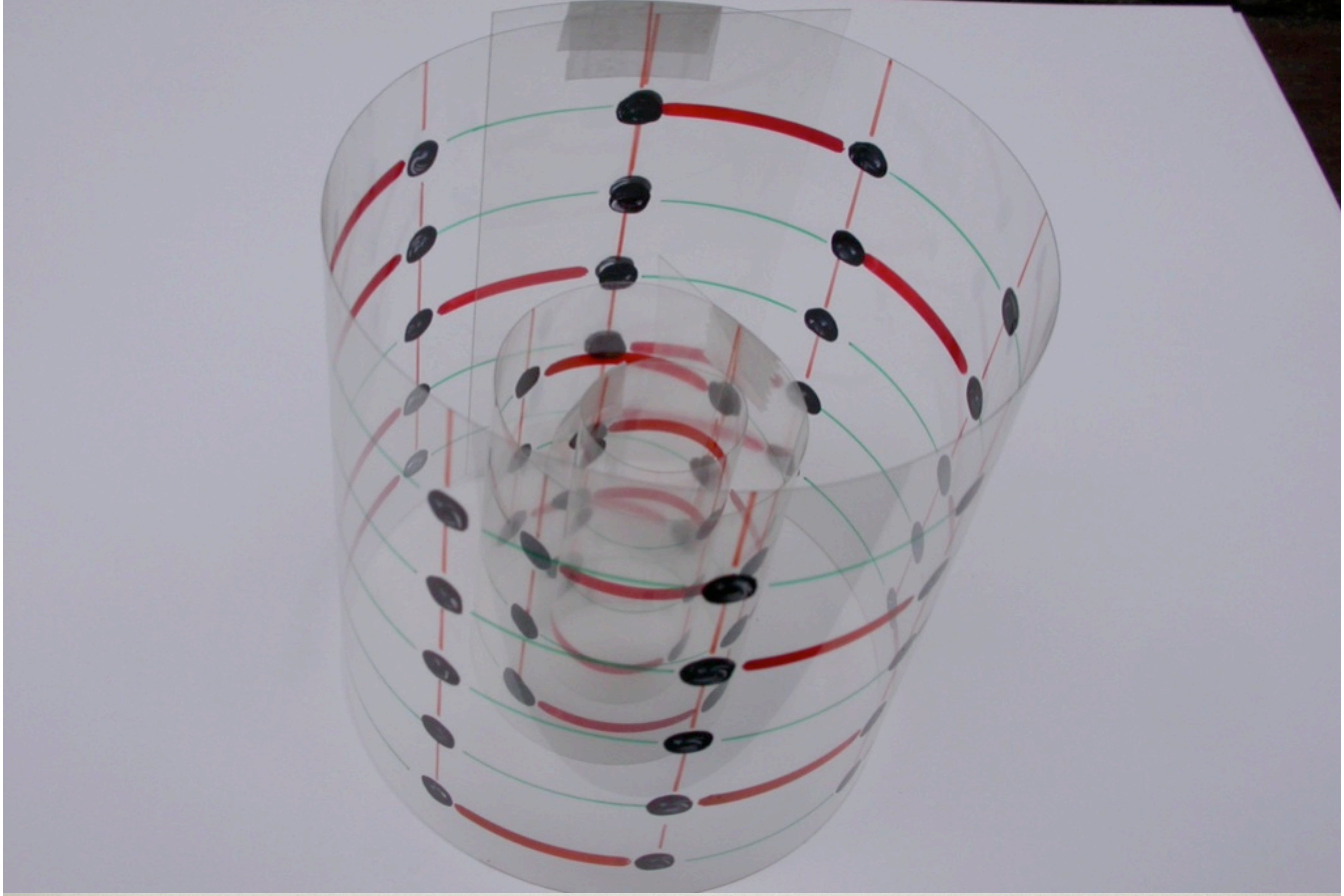
  H_i heap of dimers above P_i (= tower) $1 \le i \le k$

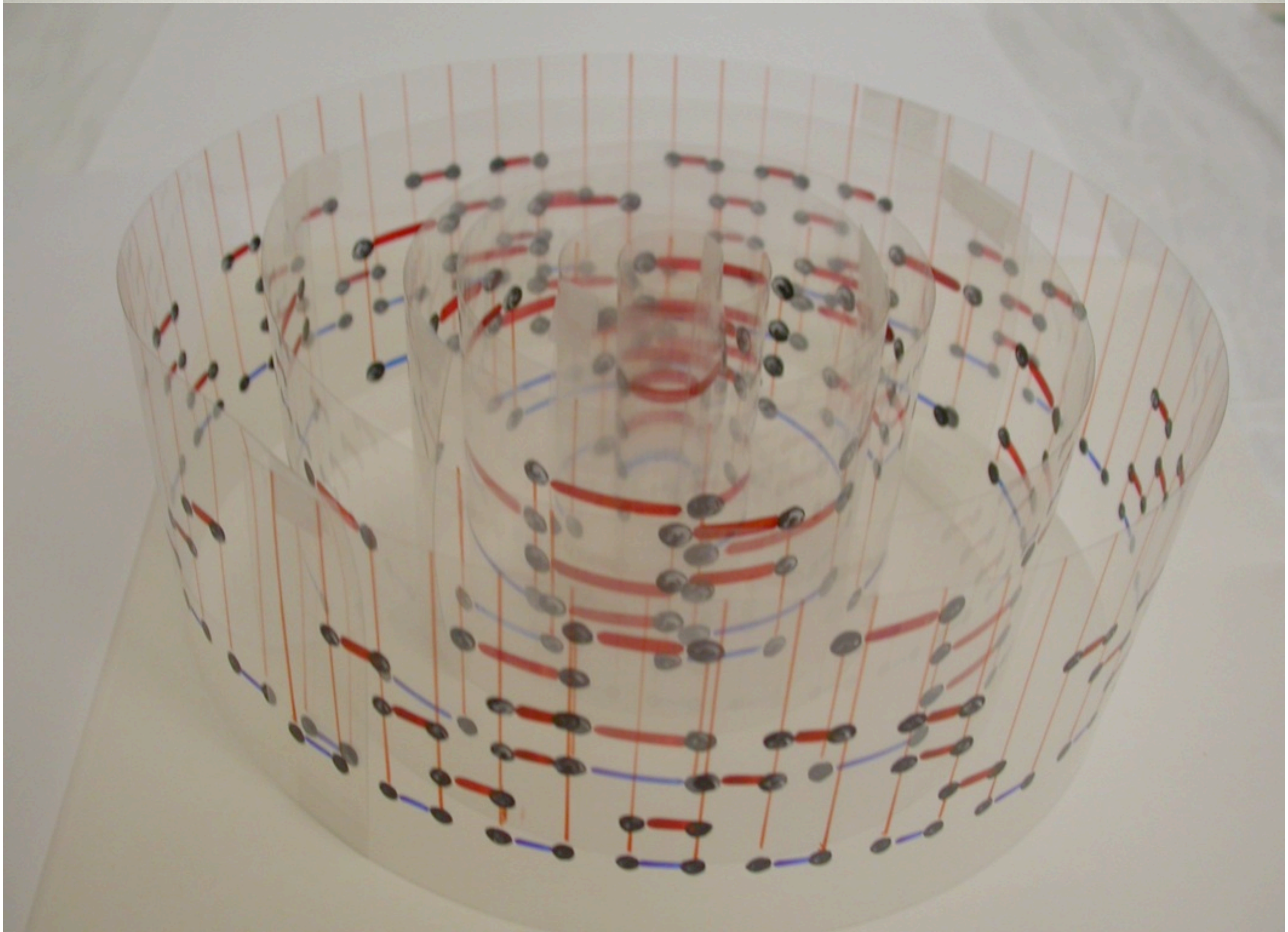
(*) at level 0, H_i contains all 2^{i-1} black edges of P_i

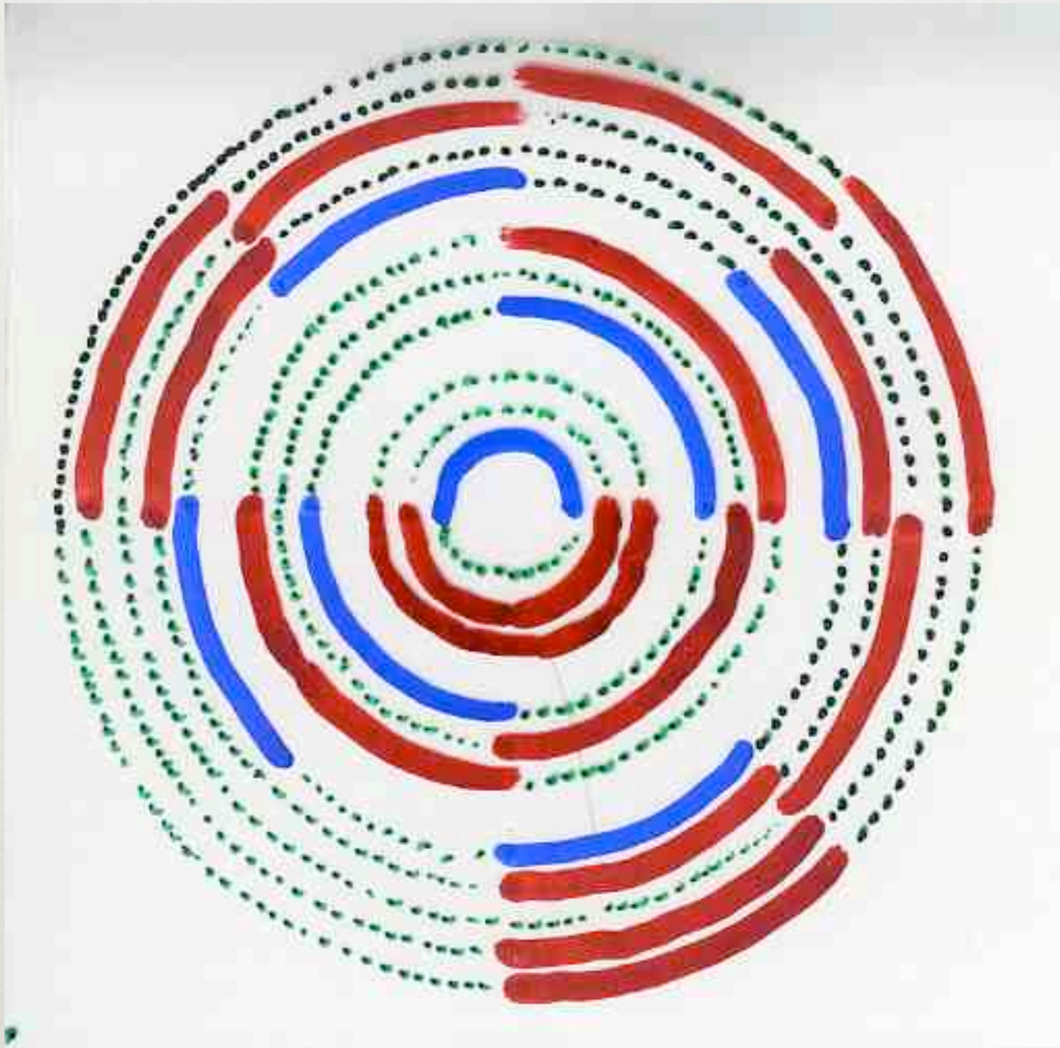
minimal pieces











Prop. The number of system of Kepler towers having n dimers is

Catalan number $C_n = \frac{1}{n+1}\binom{2n}{n}$

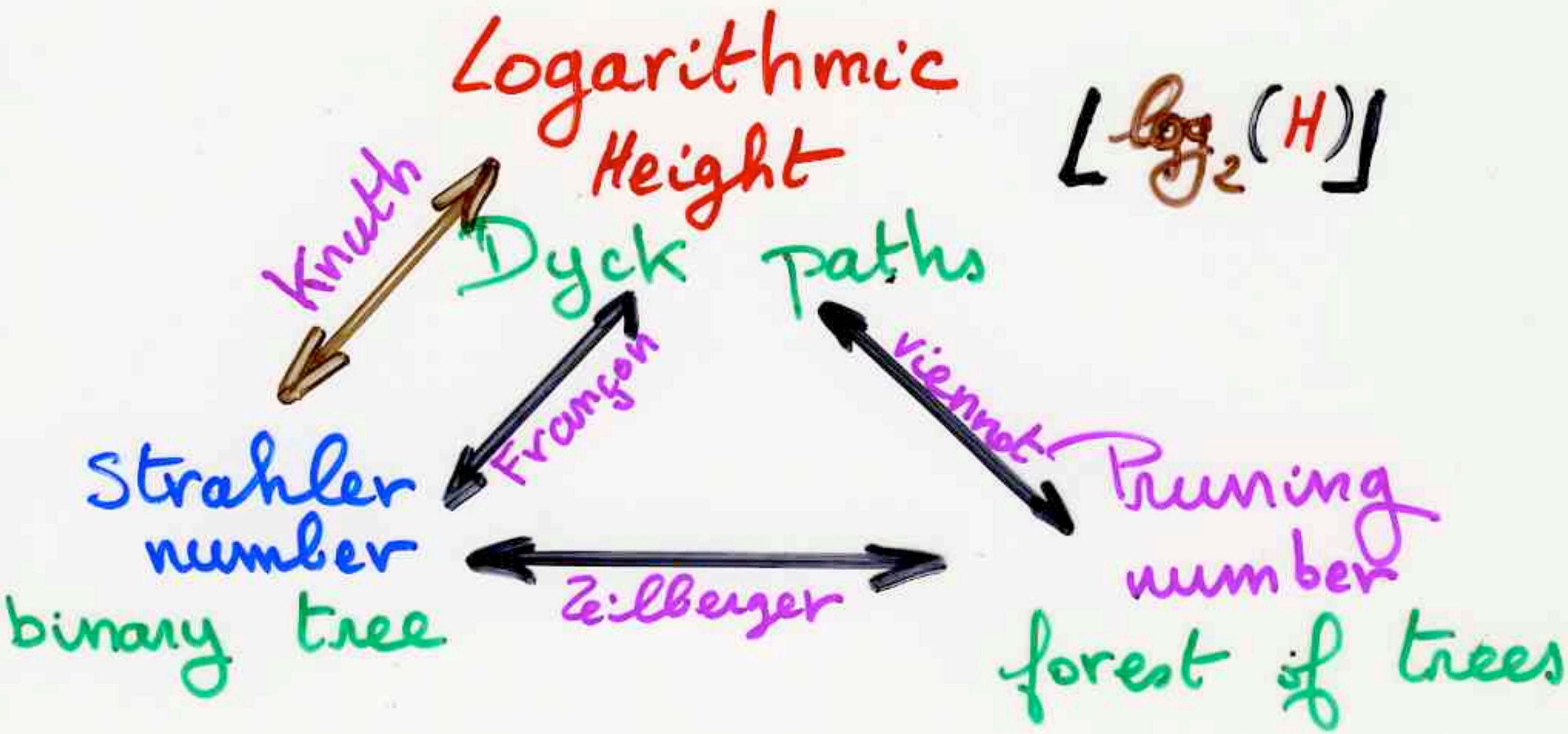
The distribution of system of Kepler towers according to the number of towers is the Strahler distribution system of Kepler towers

number of towers

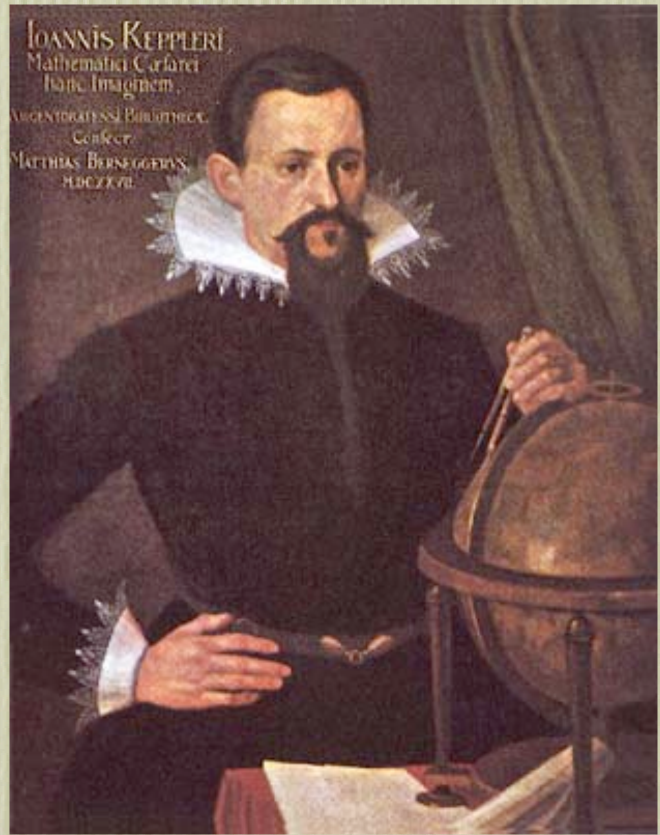
Programs to Read

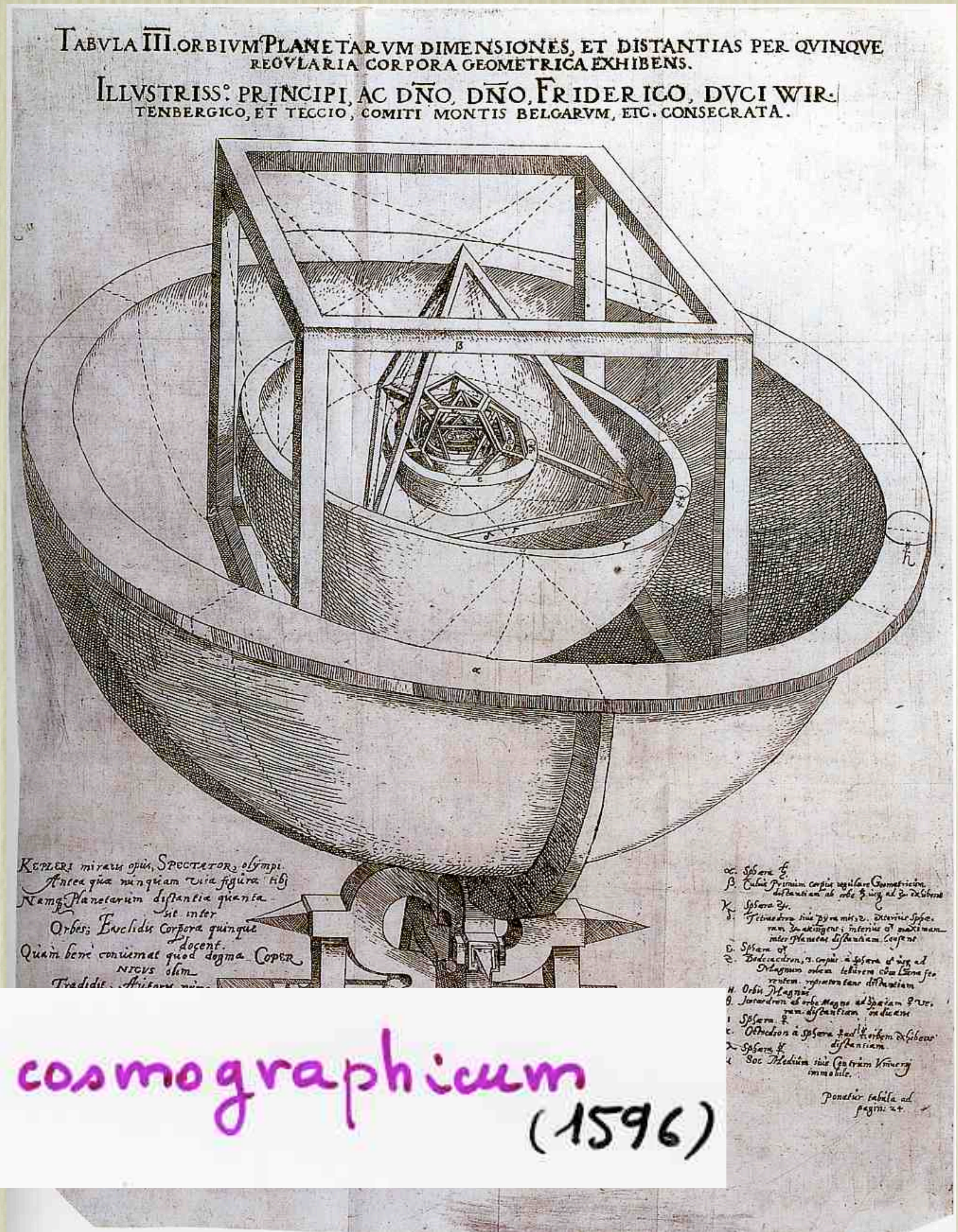
ZEILBERGER, FRANÇON, VIENNOT,
an explanatory introduction,
and a MetaPost source file for VIENNOT

Three Catalan bijections
related to Strahler numbers,
pruning orders,
and Kepler towers (February 2005)

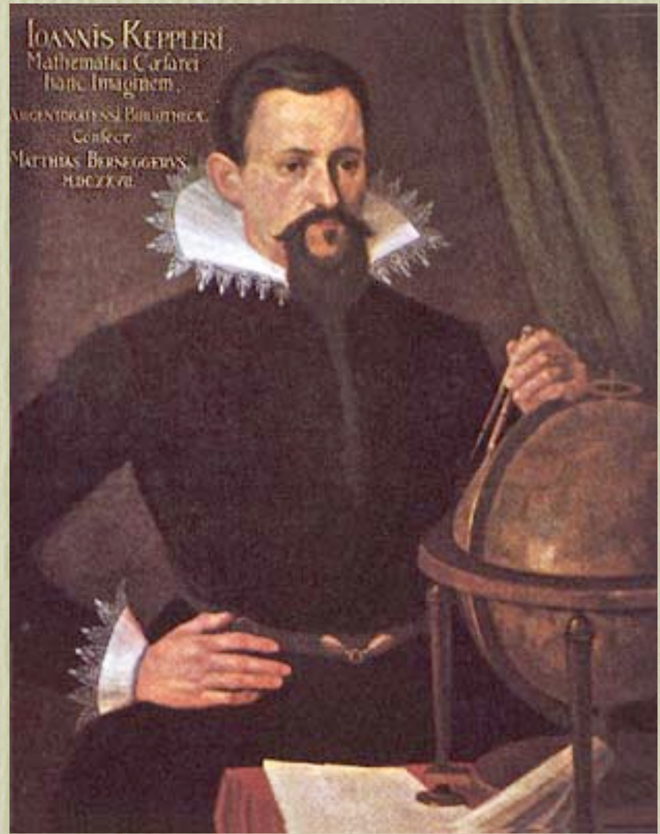


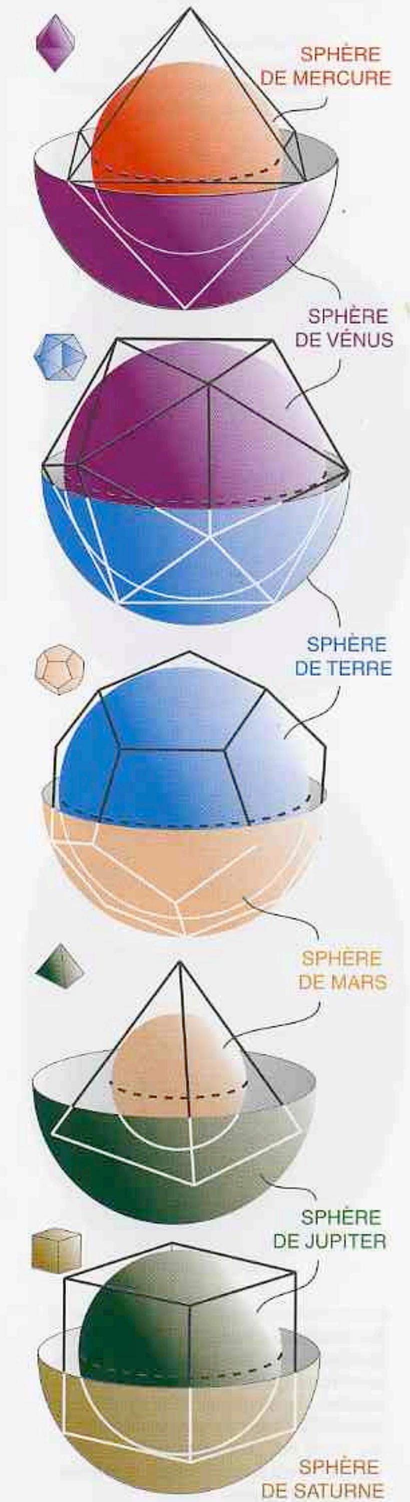
Kepler

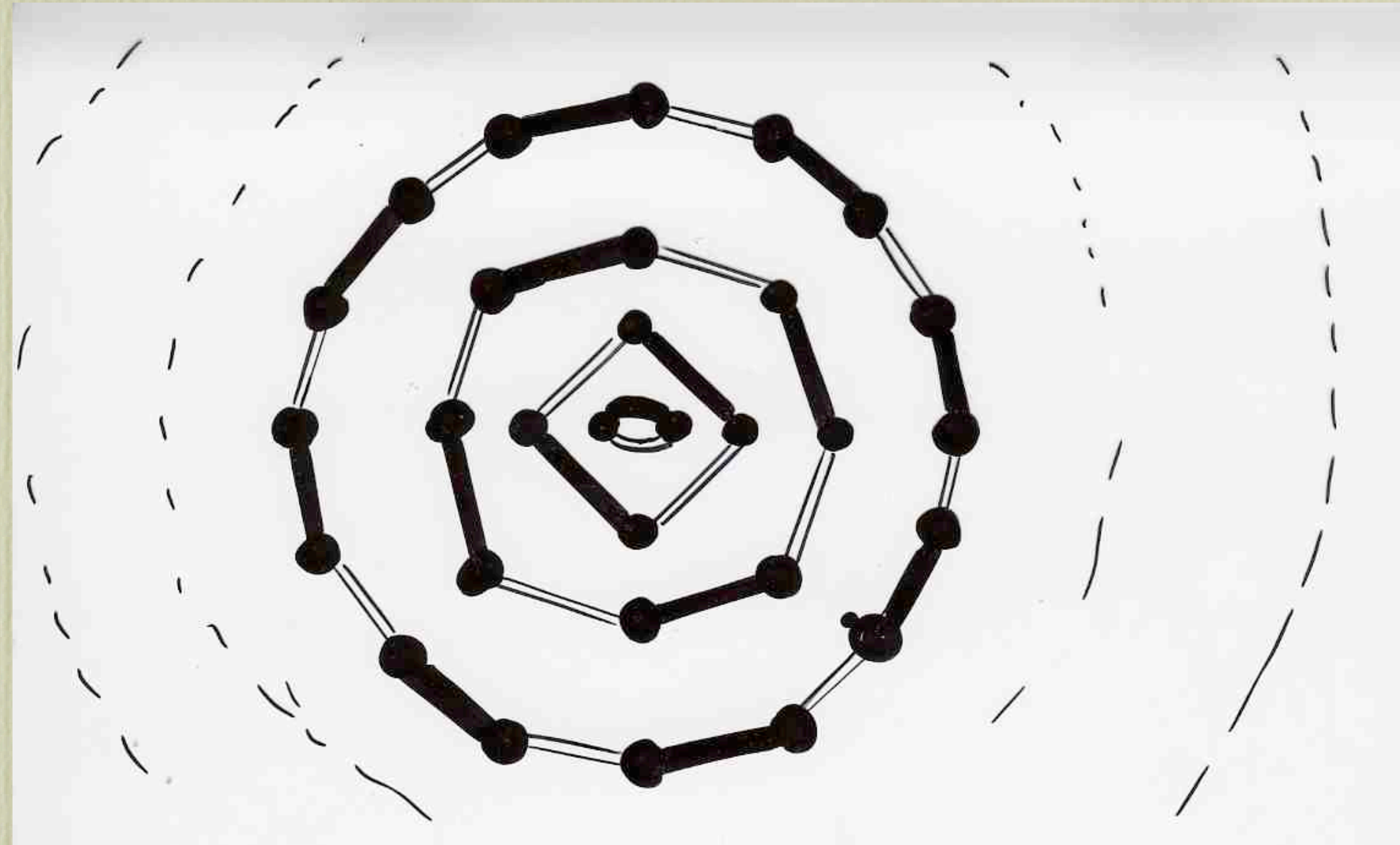




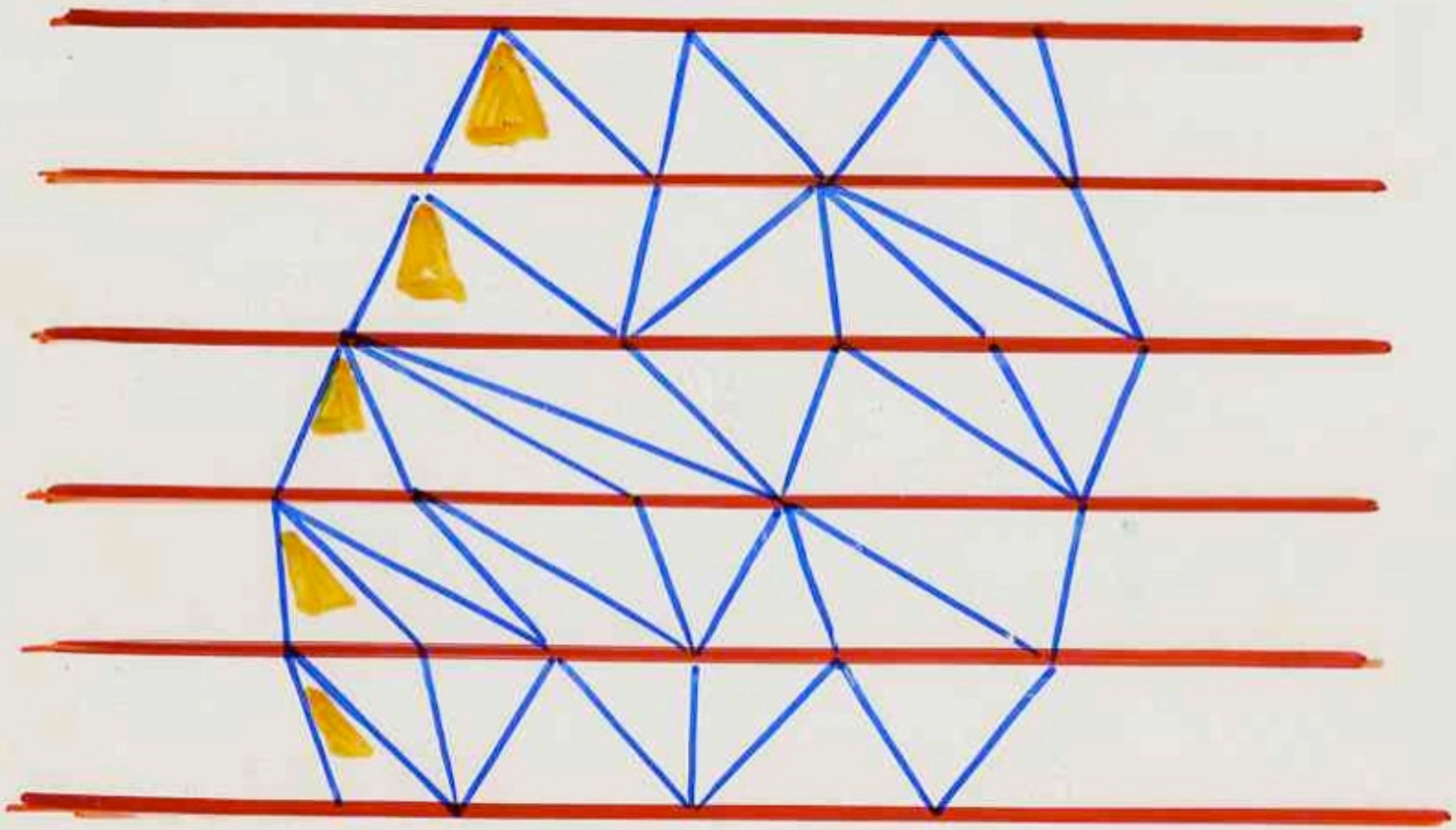
Mysterium cosmographicum (1596)


IOANNIS KEPPLERI
MATTHIAS BERNEGGERUS


SPHÈRE
DE MERCURE
SPHÈRE
DE VÉNUS
SPHÈRE
DE TERRE
SPHÈRE
DE MARS
SPHÈRE
DE JUPITER
SPHÈRE
DE SATURNE



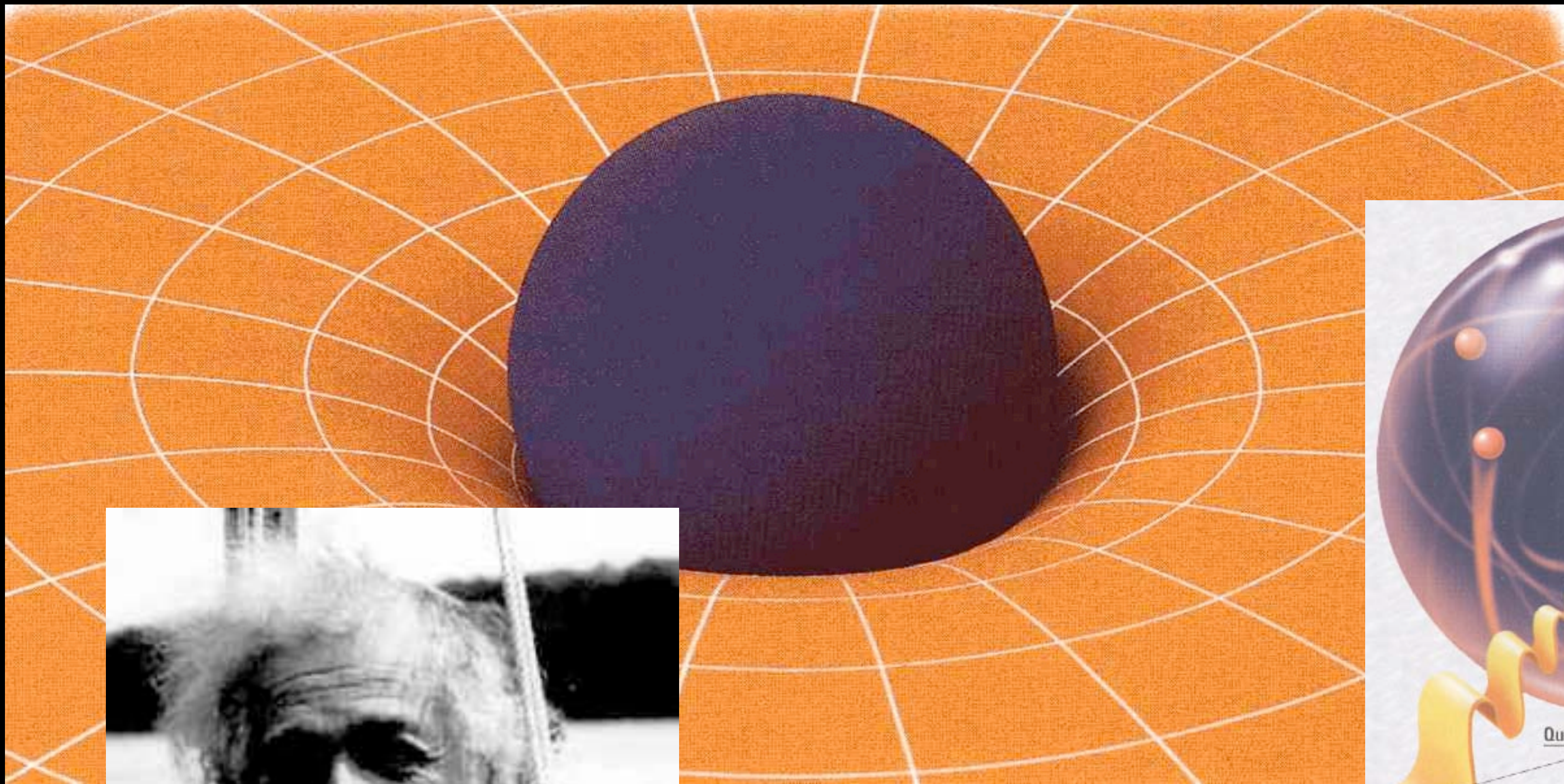
Heaps of dimers

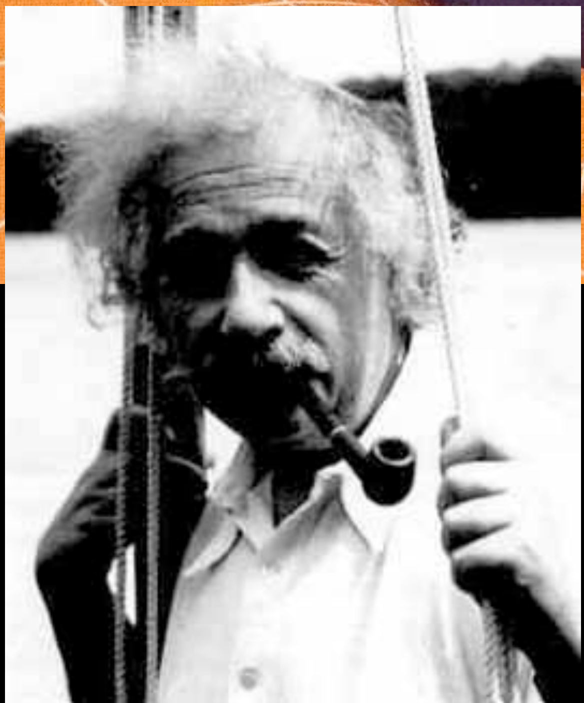


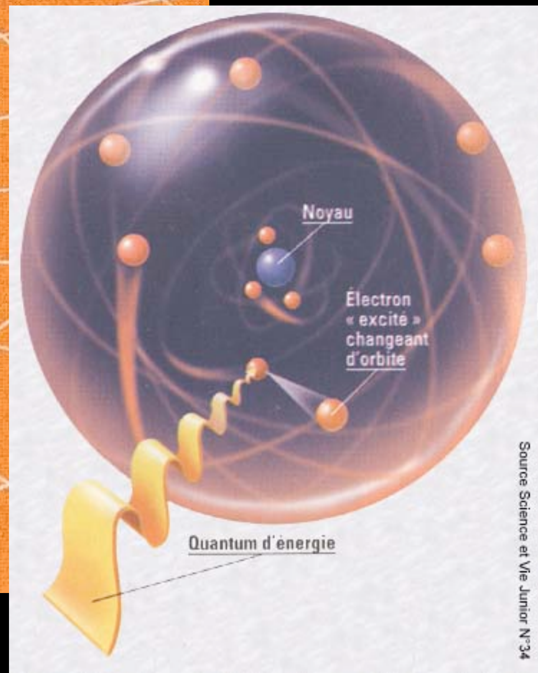
2D Lorentzian causal triangulations

general relativity

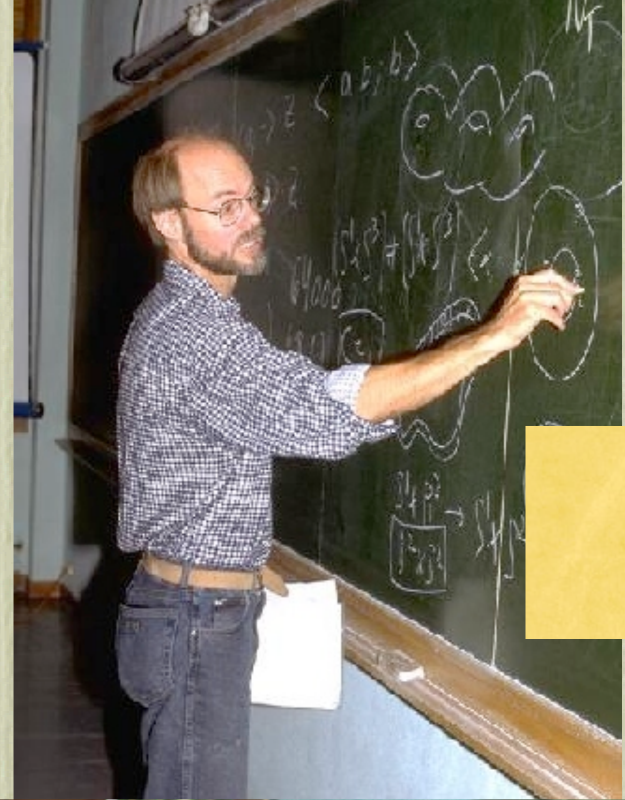
quantum mechanics



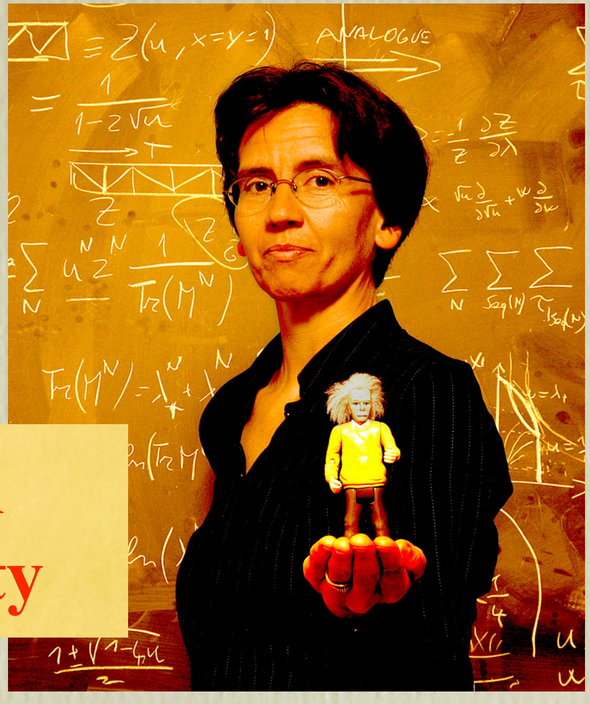




quantum gravity ?



J.Ambjørn



R. Loll

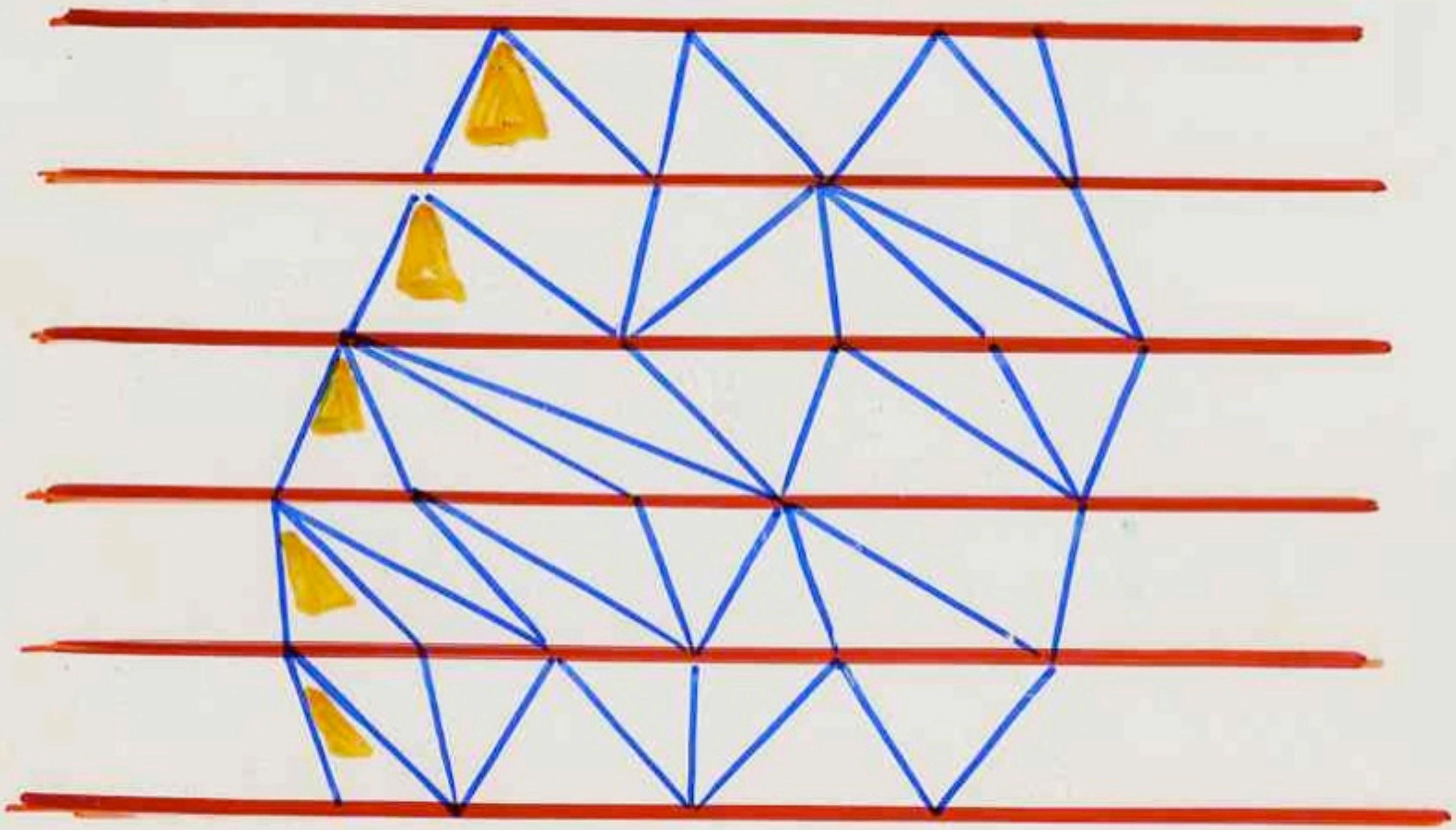
2D Lorentzian quantum gravity

P. Di Francesco

E.Guitter

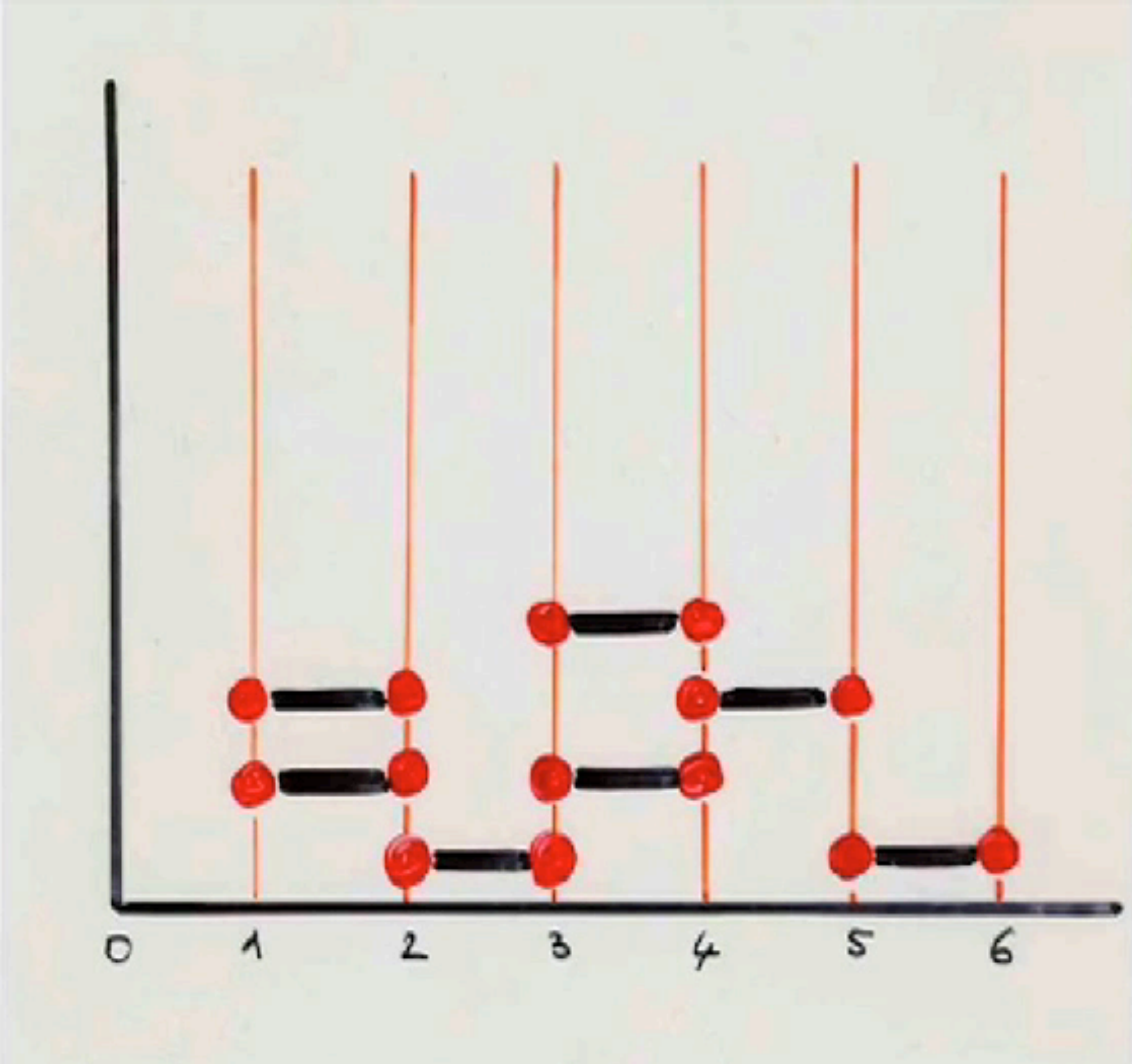
C. Kristjansen

Heaps of dimers



2D Lorentzian causal triangulations

From heaps of dimers
to Lorentzian triangulations



0
1
2
3
4
5
6

metamorphosis:

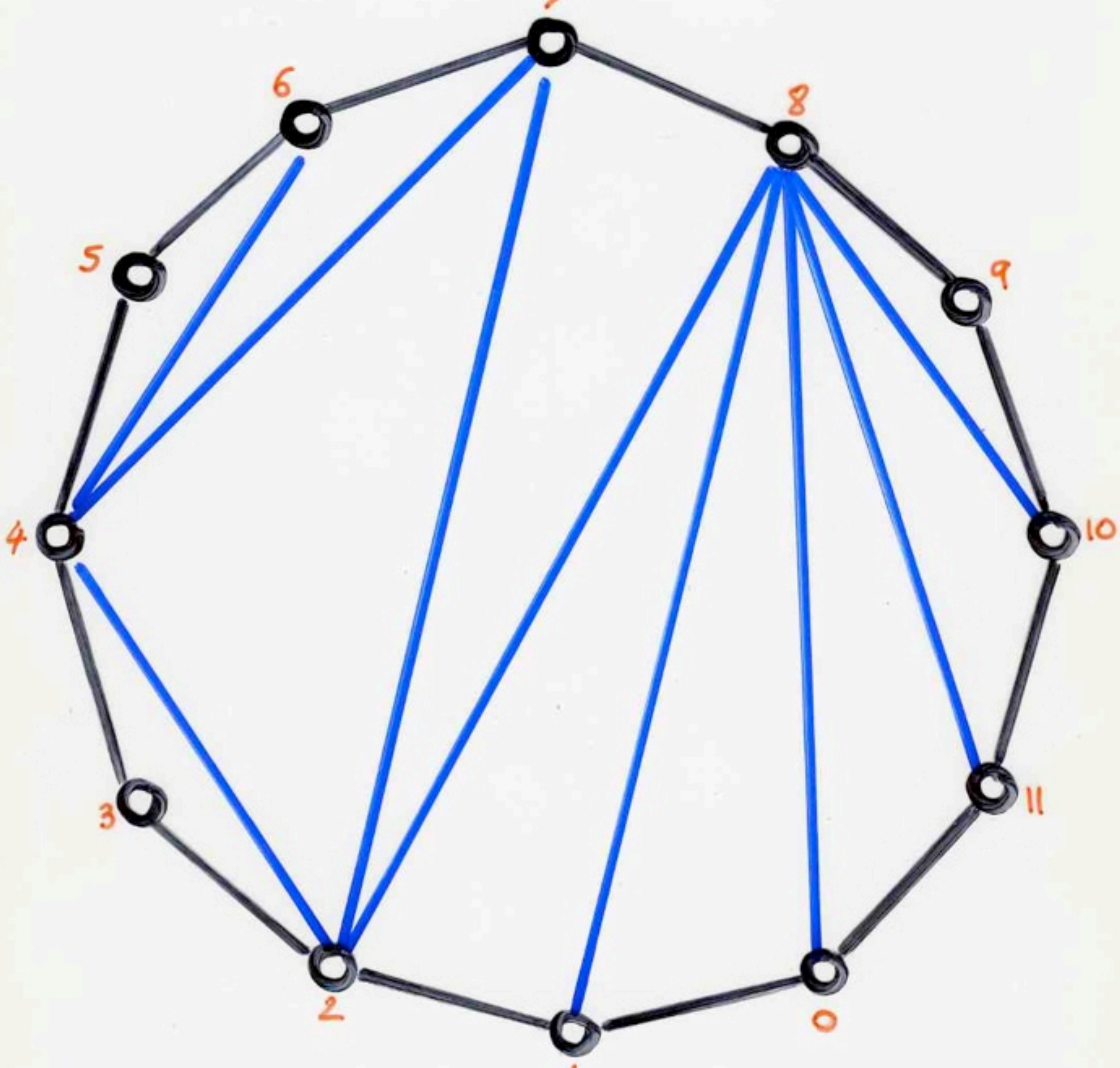
triangulation (Euler)

binary tree

Dyck path

dimer pyramid

Lorentzian triangulation



7
6
8
5
9
4
10
3
11
2
0
1

Trees everywhere !

"Trees sprout up just about everywhere in computer science,
as we've seen in Section 2.3 and in nearly every section of
The Art of Computer Programming"

Don Knuth
Vol 4, Fascicle 4 of TAOCP (2006)
Generating All Trees; History of Combinatorial Generation

Trees everywhere !

Some references from X.V. website: www.xavierviennot.org

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[83] Kepler towers, Catalan numbers and Strahler distribution, FPSAC'05 (Formal Power Series and Algebraic Combinatorics), Session spéciale dédiée à Ardriano Garsia (75 ans), Juin 2005, Taormina, Italie, résumé étendu (10 p.) dans les actes provisoires du colloque (distribué sur CD).

other references:

from D. Knuth website, programs to read:
ZEILBERGER, FRANÇON, VIENNOT, an explanatory introduction,
and a MetaPost source file for VIENNOT
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D. Auber et al. “New Strahler numbers for rooted plane trees”. In: Mathematics and Computer Science III. Basel: Birkhäuser, 2004, pp. 203– 215.

G. Louchard and H. Prodinger. “The register function for lattice paths”. In: Discrete Mathematics and Theoretical Computer Science proc. AG (2008), pp. 139–152.

bijections with violins:

Mariette Freudentheil

Gérard H.E. Duchamp

Marcia Pig Lagos

Xavier Viennot

Association Cont'Science

vocal: Bombay S. Jayashri