Tableaux alternatifs rhomboïdaux, assemblées de permutations et le PASEP

II

(work with Olya Mandelshtam, Berkeley)

LACIM, UQAM, Montréal
29 Juin 2016

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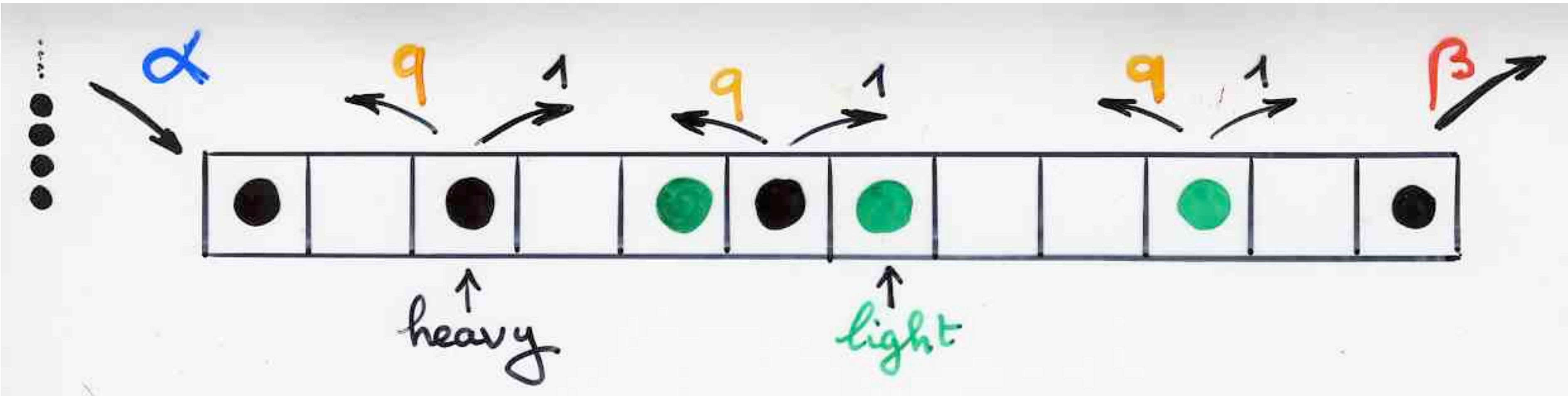
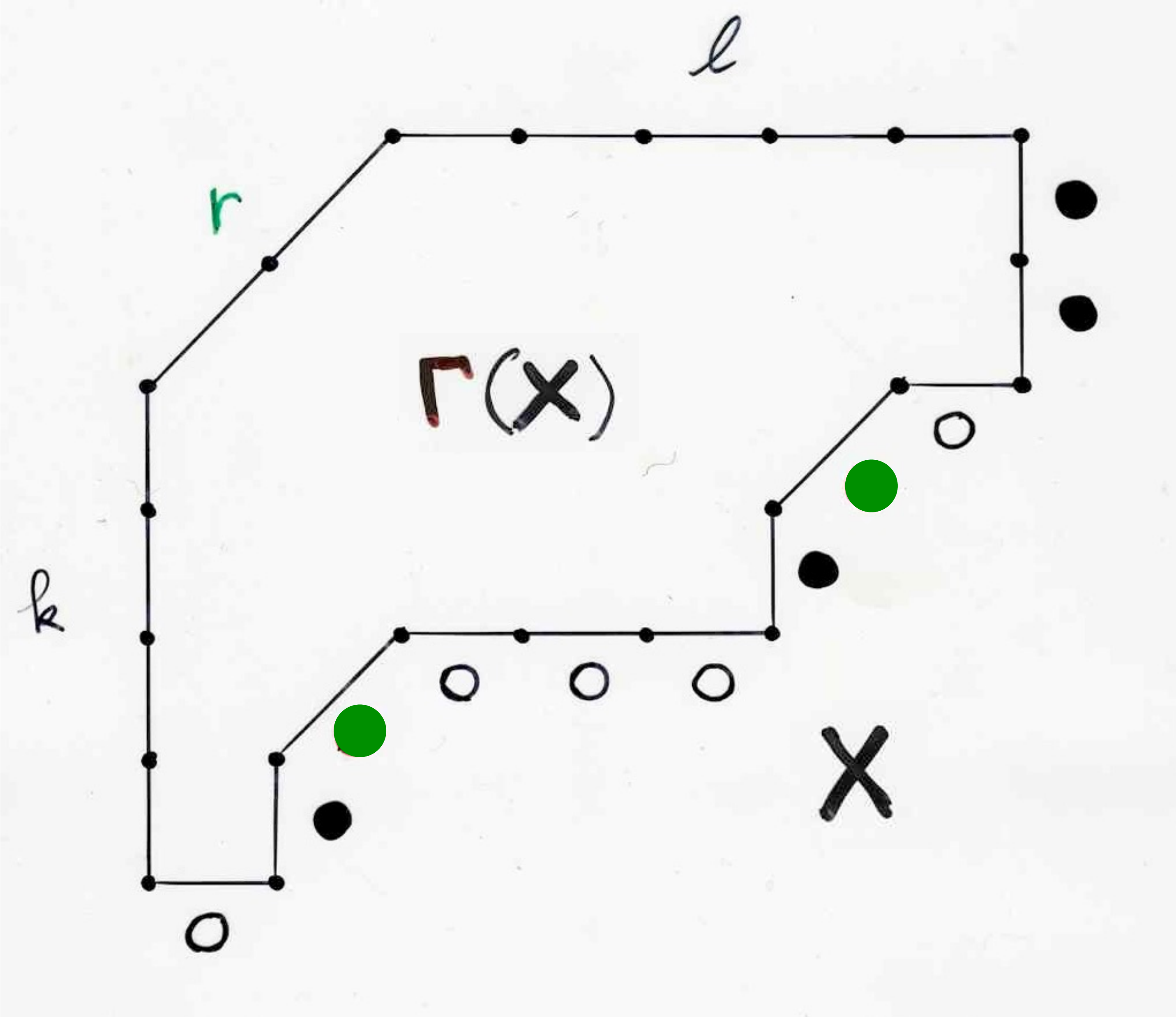
rappel:
résumé de l’exposé précédent

The 2-species PASEP

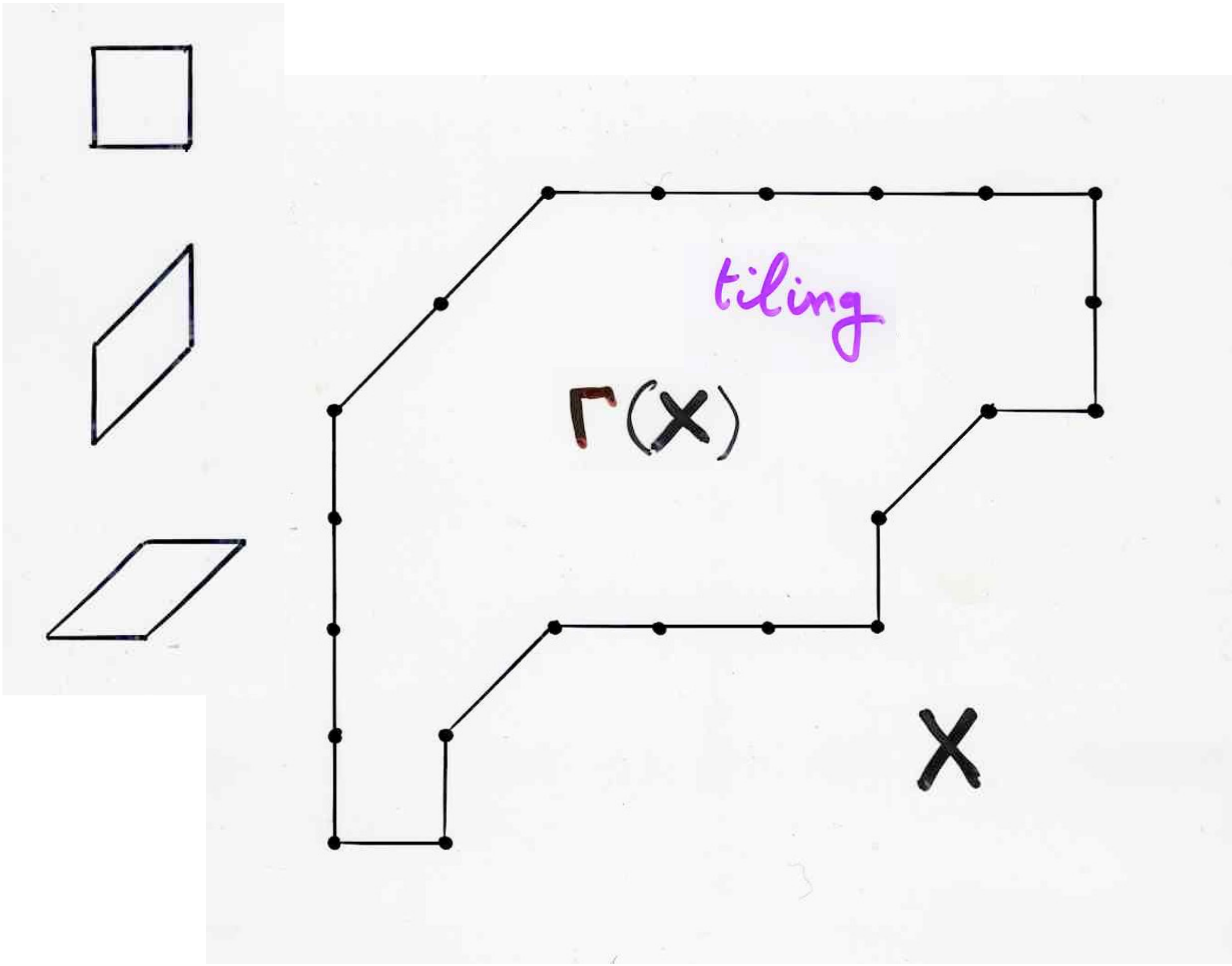


Rhombic alternative tableaux

(tableaux alternatifs rhomboïdaux)

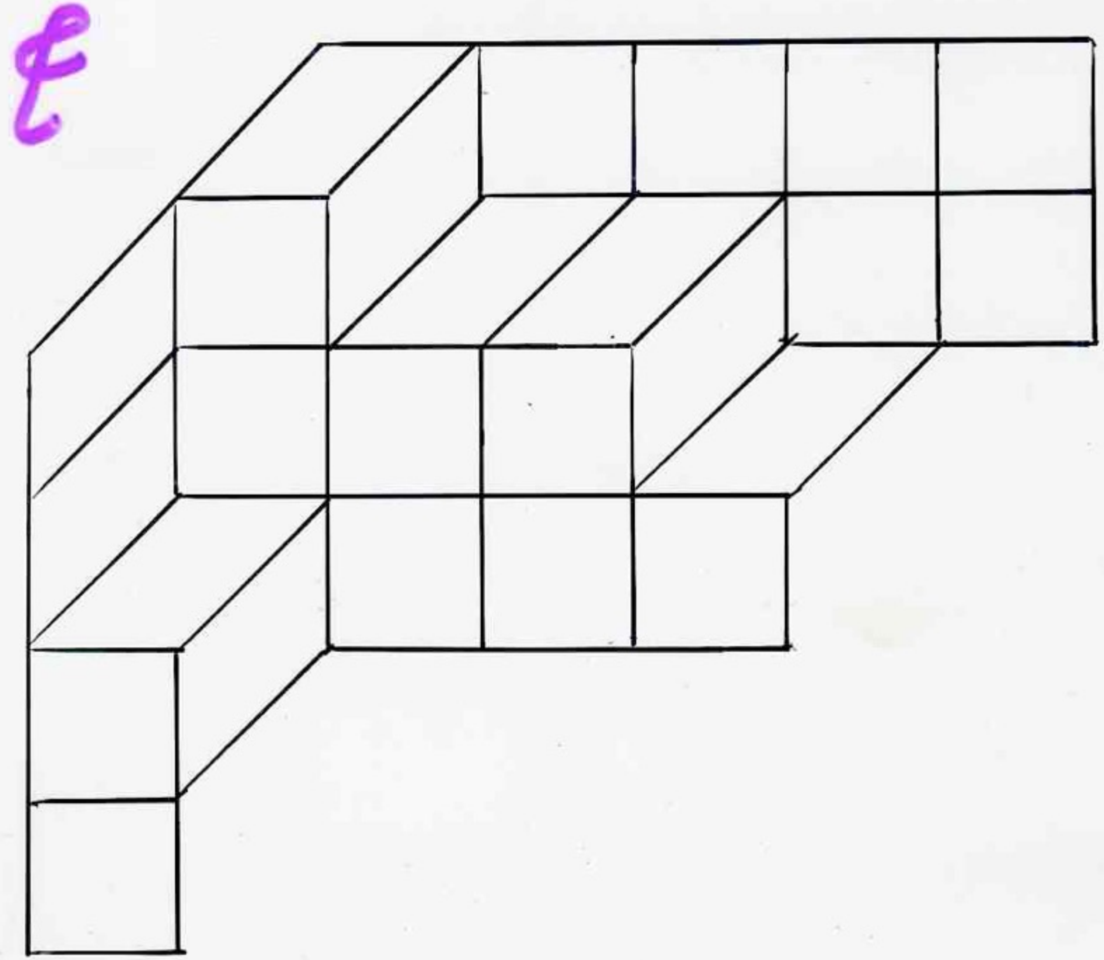


ℓ
r
Γ(X)
k
X



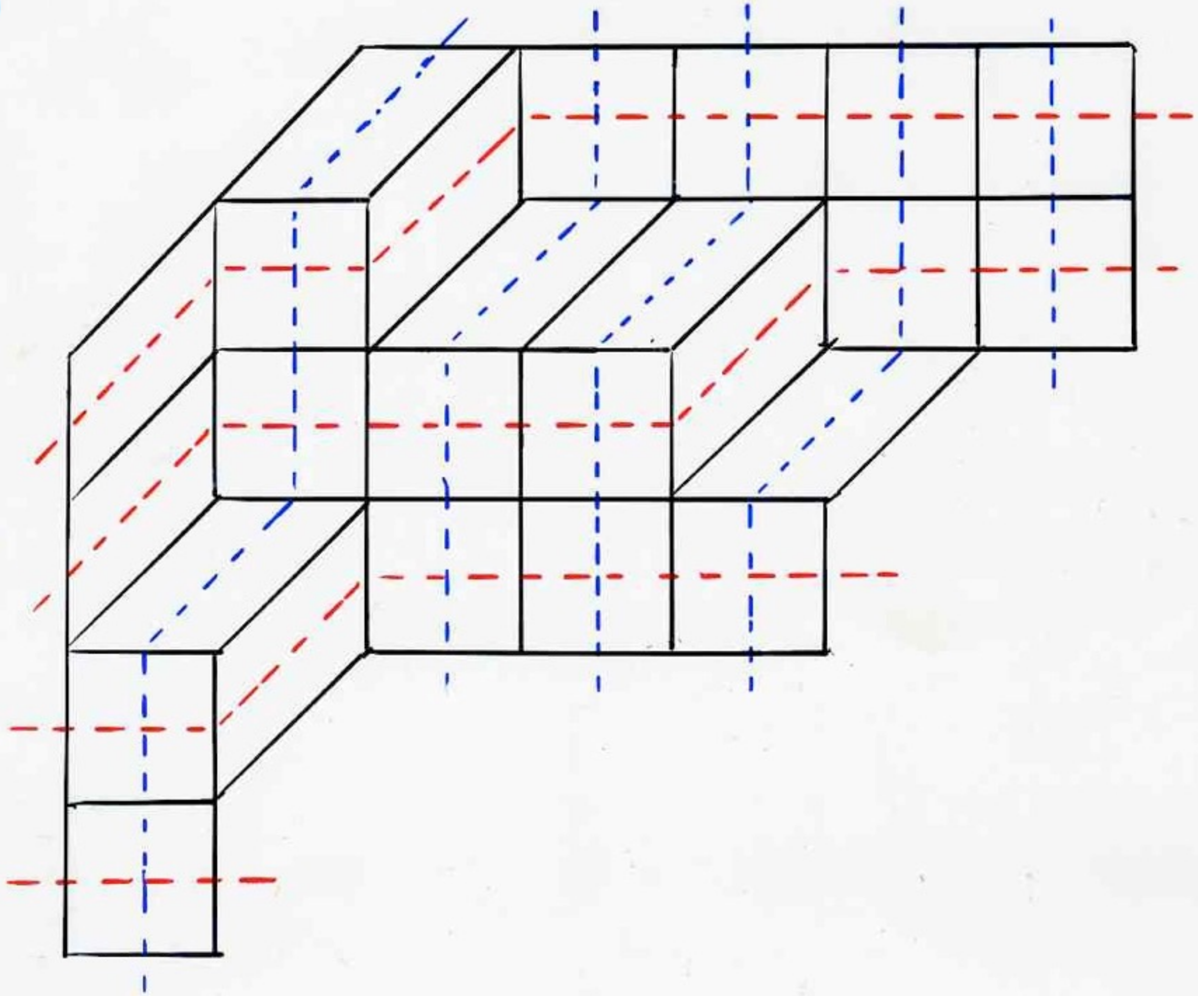
tiling
Γ(X)
X

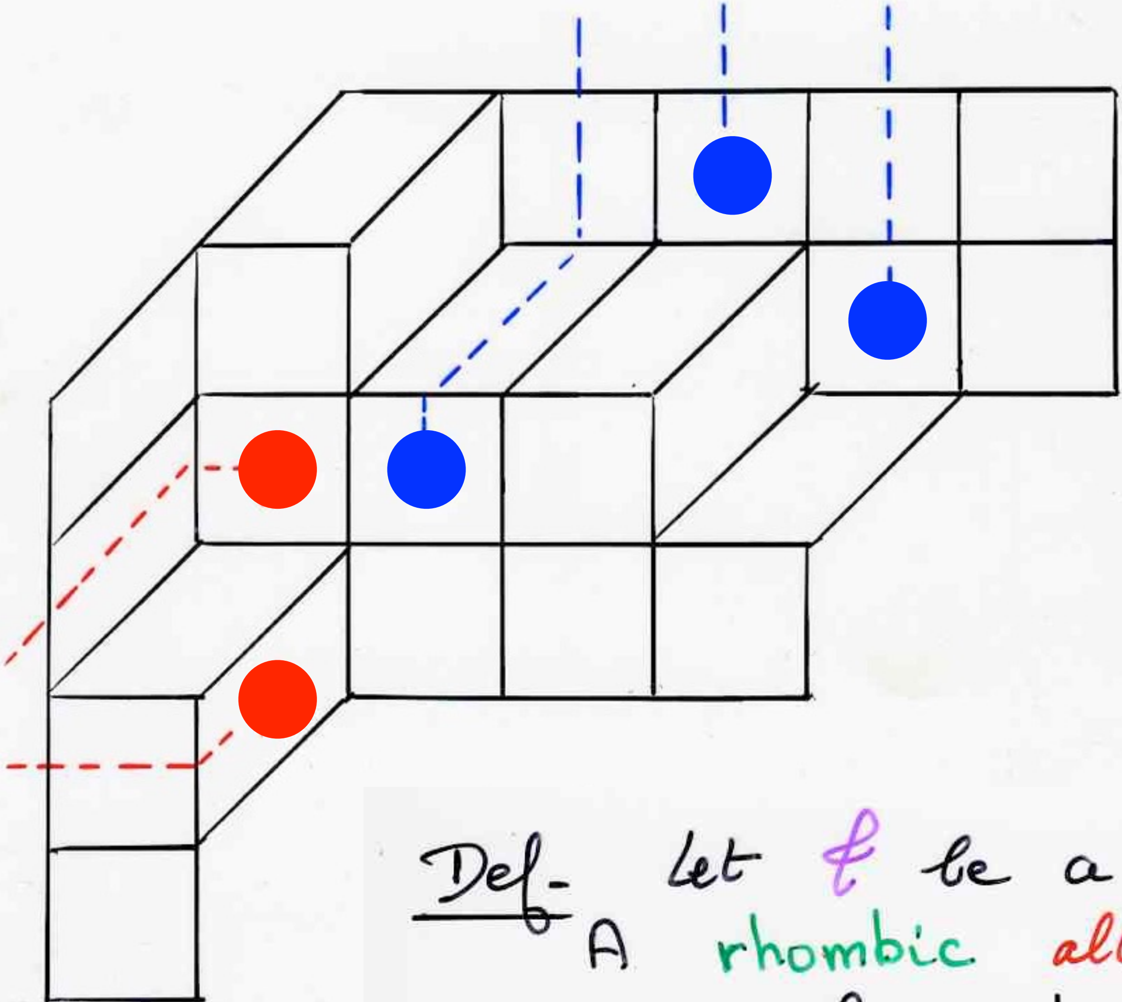
tiling



west- strips

north-strips



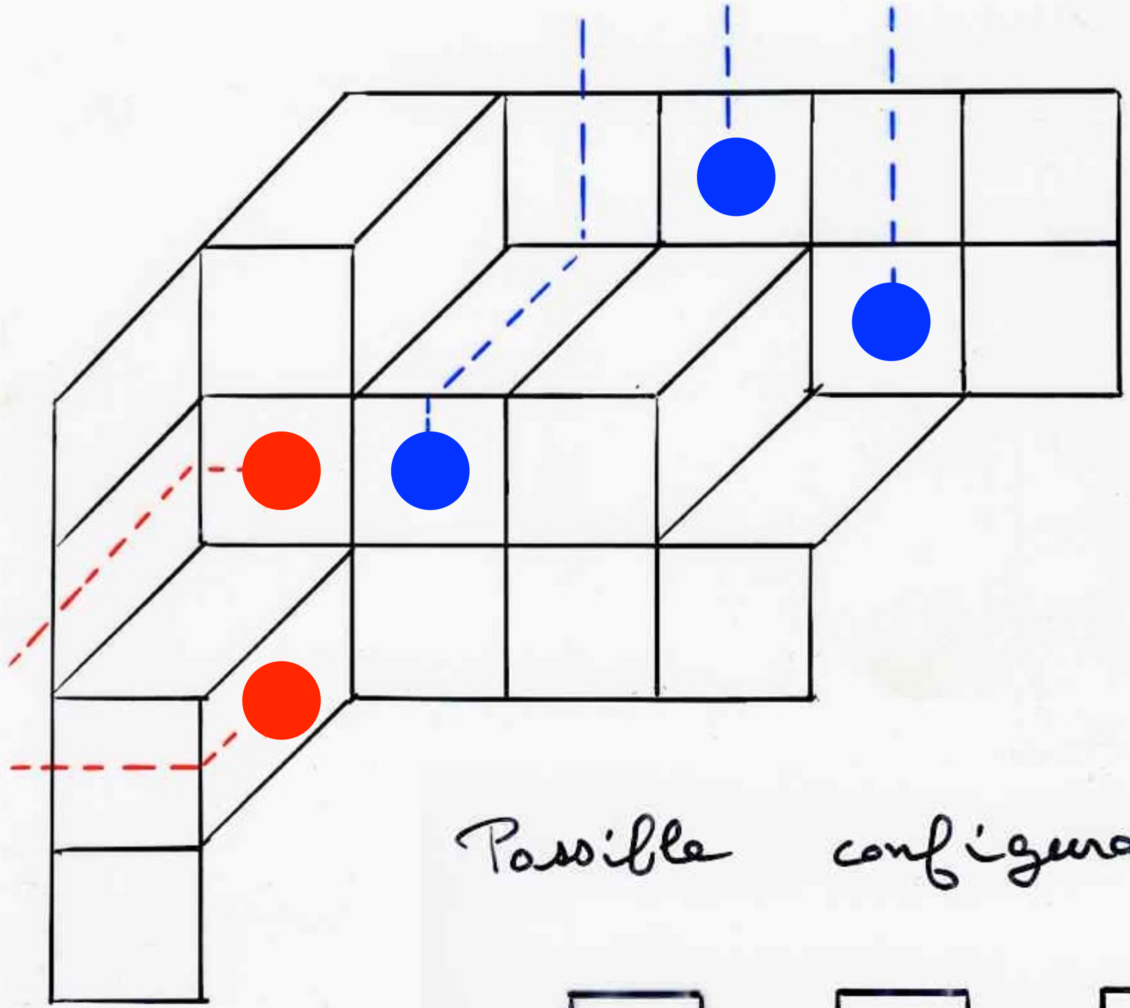


<u>Def-</u> Let $\mathcal{T}$ be a tiling of $\Gamma(X)$

A rhombic alternative tableau T

is a placement of $\color{blue}{\bullet}$, $\color{red}{\bullet}$ in the tiles such that:

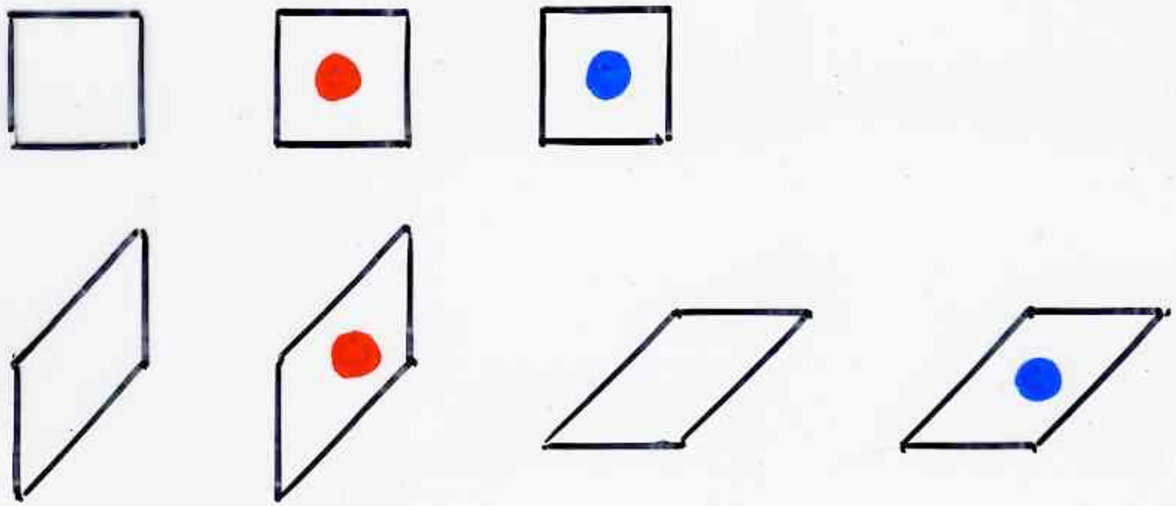
- a $\color{red}{\bullet}$ is on a west-strip and any tile left of this $\color{red}{\bullet}$ is empty
- a $\color{blue}{\bullet}$ is on a north-strip and any tile north of this $\color{blue}{\bullet}$ is empty



west-strips

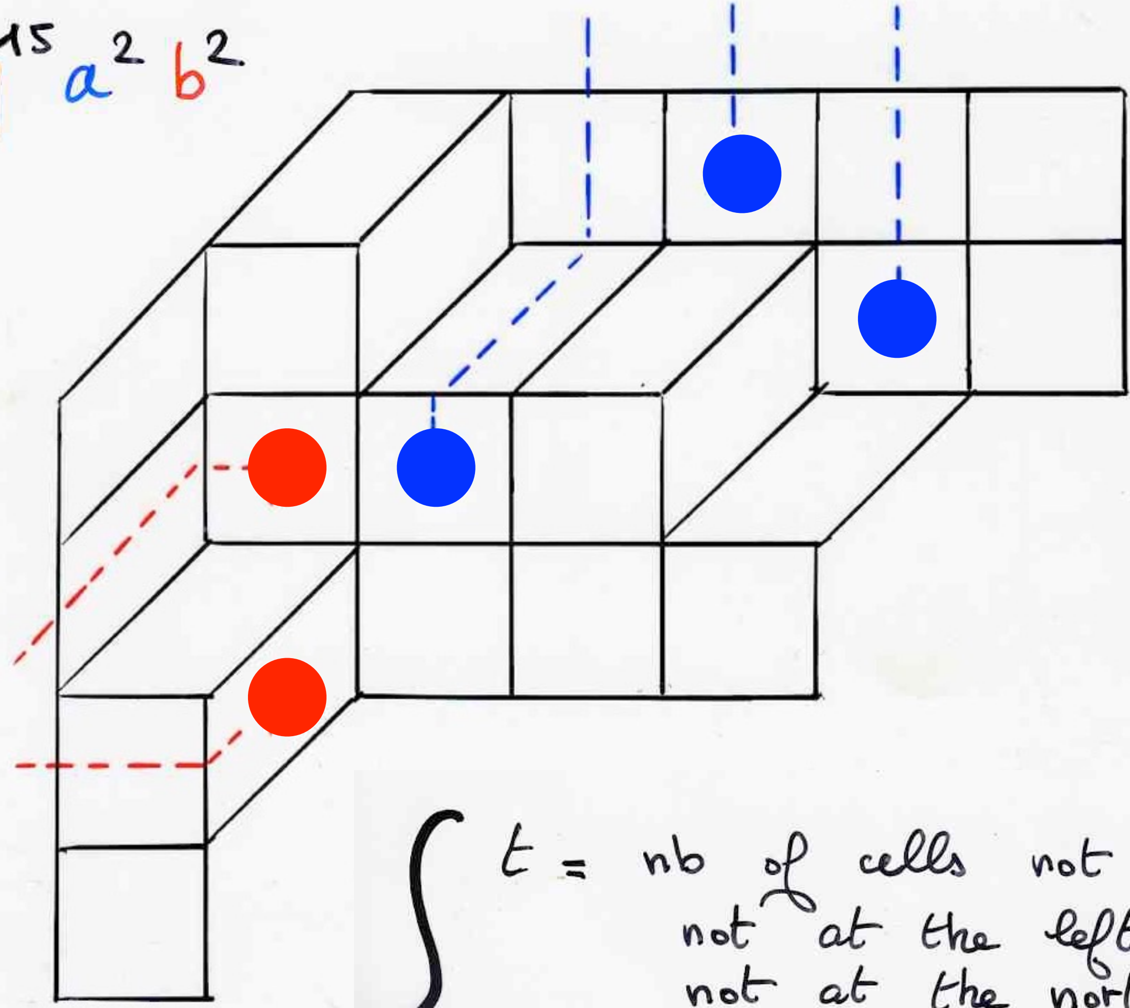
north-strips

Possible configurations for a tile :



weight $wt(T) = q^{t} a^{i} b^{j}$

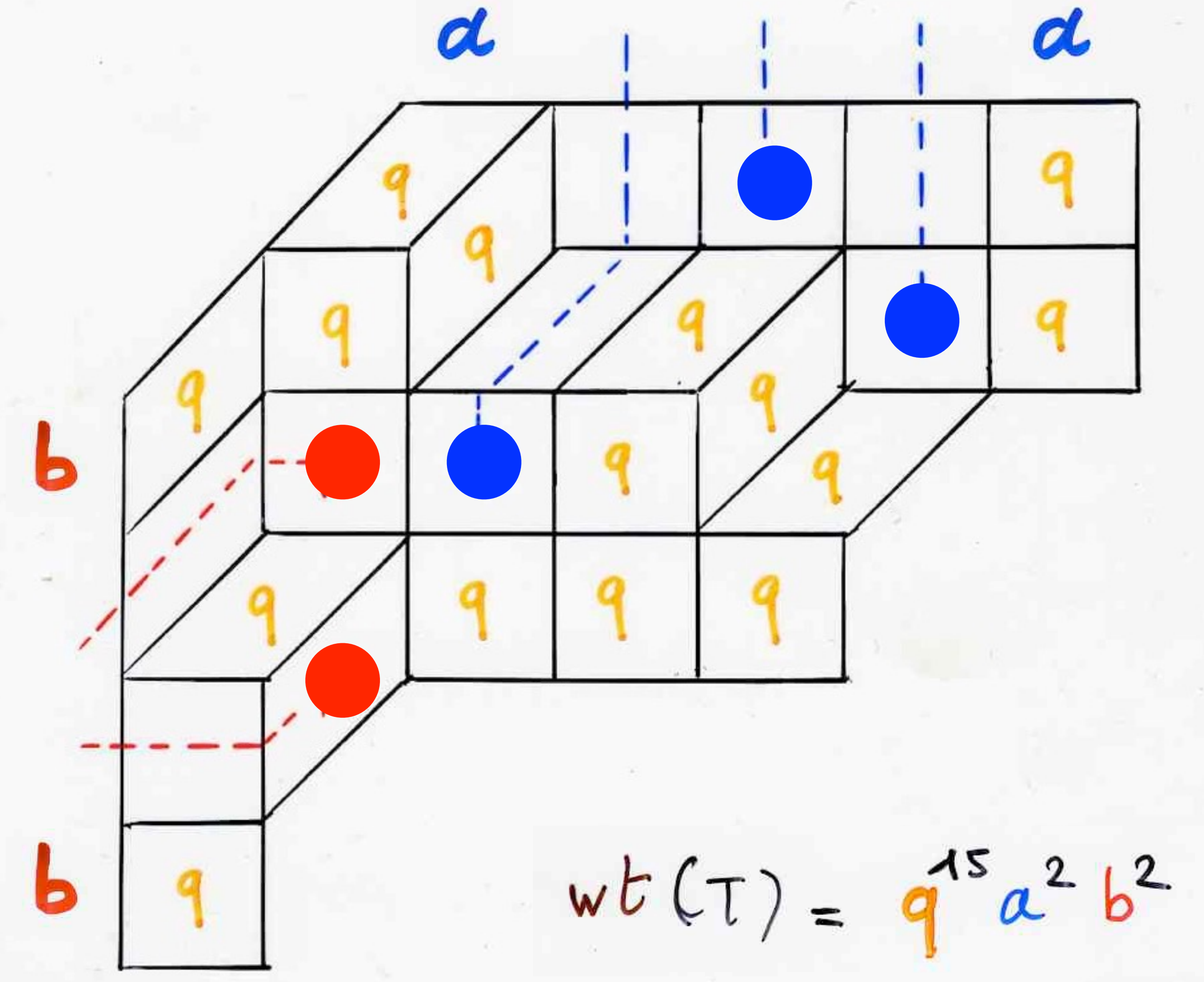
$wt(T) = q^{15} a^{2} b^{2}$



t = nb of cells not ●, not ●, not at the left of a ●, not at the north of a ●

i = nb of north-strips without a ●

j = nb of west-strips without a ●



a
a
b
b
q
wt(T) = q^15 a^2 b^2

$$\mathrm{Prob}(X) = \frac{1}{Z^*_{n,r}} \sum_{T \in R(X,\tau_X)} \underbrace{q^t \left(\frac{1}{\alpha}\right)^i \left(\frac{1}{\beta}\right)^j}_{\mathrm{wt}(T)}$$

$$a = \frac{1}{\alpha}$$

$$b = \frac{1}{\beta}$$

$$Z^*_{n,r} = \sum_{X} \sum_{T \in R(X,\tau_X)} q^t \left(\frac{1}{\alpha}\right)^i \left(\frac{1}{\beta}\right)^j$$

$R(X, \mathfrak{t})$ set of rhombic alternative tableaux related to X, with the tiling $\mathfrak{t}$ of $\Gamma(X)$

Prop X, $\Gamma(X)$ diagram
$\mathfrak{t}$, $\mathfrak{t}'$ tiling of $\Gamma(X)$

$$\sum_{T \in R(X, \mathfrak{t})} wt(T) = \sum_{T \in R(X, \mathfrak{t}')} wt(T)$$

$$Z^{*}_{n,r}(\alpha=\beta=q=1) = \binom{n}{r}\frac{(n+1)!}{(r+1)!}$$

Lah numbers

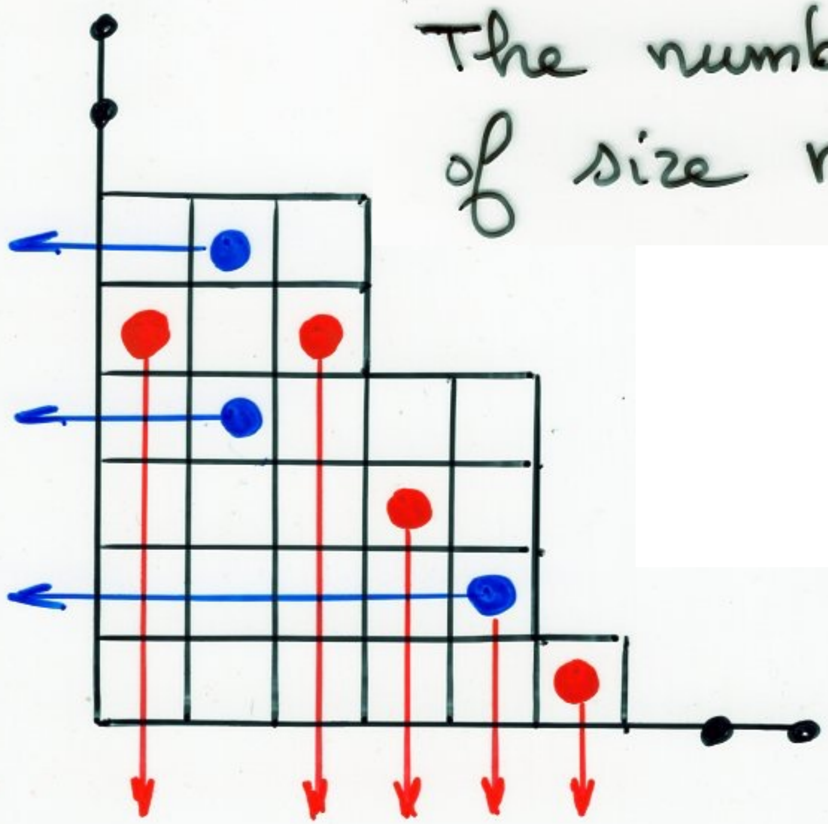
nb of "assemblées" of permutations

$$\left\{ [7,10,5,8] \quad [3,1,4] \quad [9,2,11,6] \right\}$$

$$Z^{*}_{n,r}(\alpha,\beta,q=1) = \binom{n}{r}\prod_{i=r}^{n-1}(\alpha^{-1}+\beta^{-1}+i)$$

number of alternative tableaux

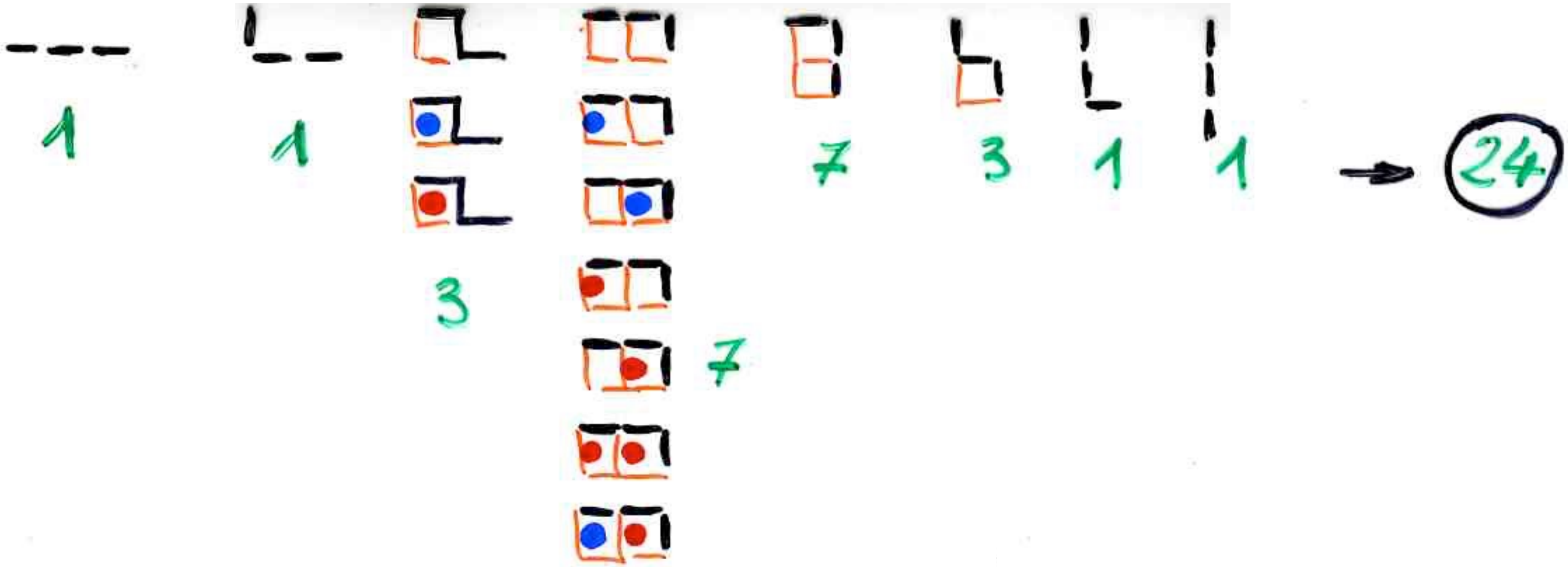
$r = 0$



The number of alternative tableaux of size n is

$$(n+1)!$$

ex: n=2



bijection

permutations --- alternative tableaux

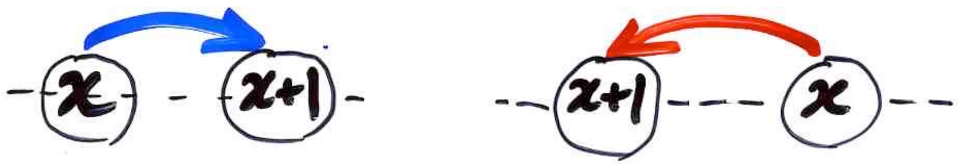
The "exchange-fusion" algorithm

Déf. Permutation $\sigma = \sigma(1)\cdots\sigma(n)$

$x = \sigma(i)$, $1 \leq x < n$

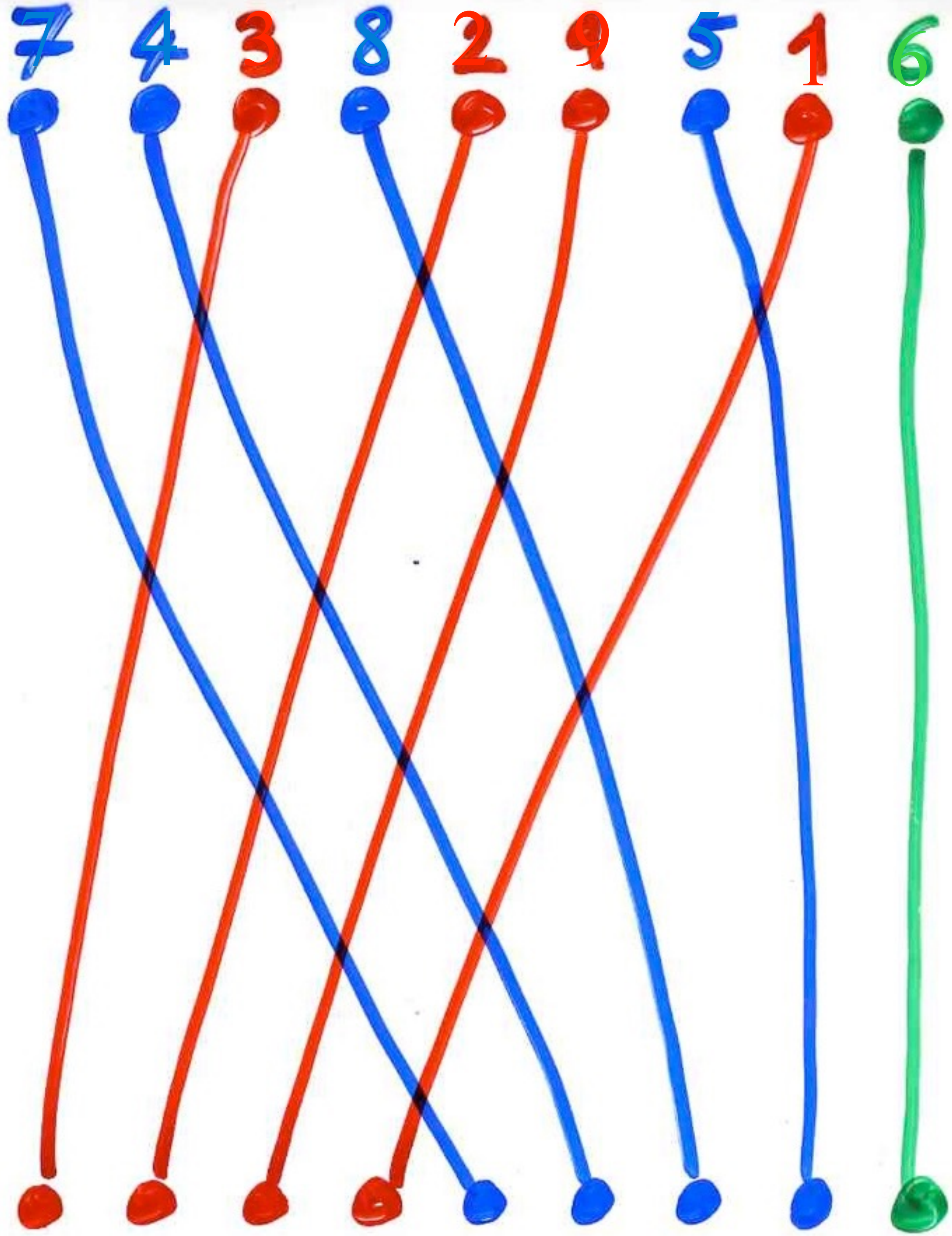
(valeur) x $\begin{cases} \text{avance} \\ \text{recul} \end{cases}$ $x+1 = \sigma(j)$, $\begin{cases} i<j \\ j<i \end{cases}$

- convention $x = n$ est un recul



$\sigma = 7\ 4\ 3\ 8\ 2\ 9\ 5\ 1\ 6$

$\begin{cases} \text{increase} & \ldots x \text{ ------ } x+1 \text{ --} \\ \text{decrease} & \ldots x+1 \text{ --- } x \text{ ----} \quad (\text{max}) \end{cases}$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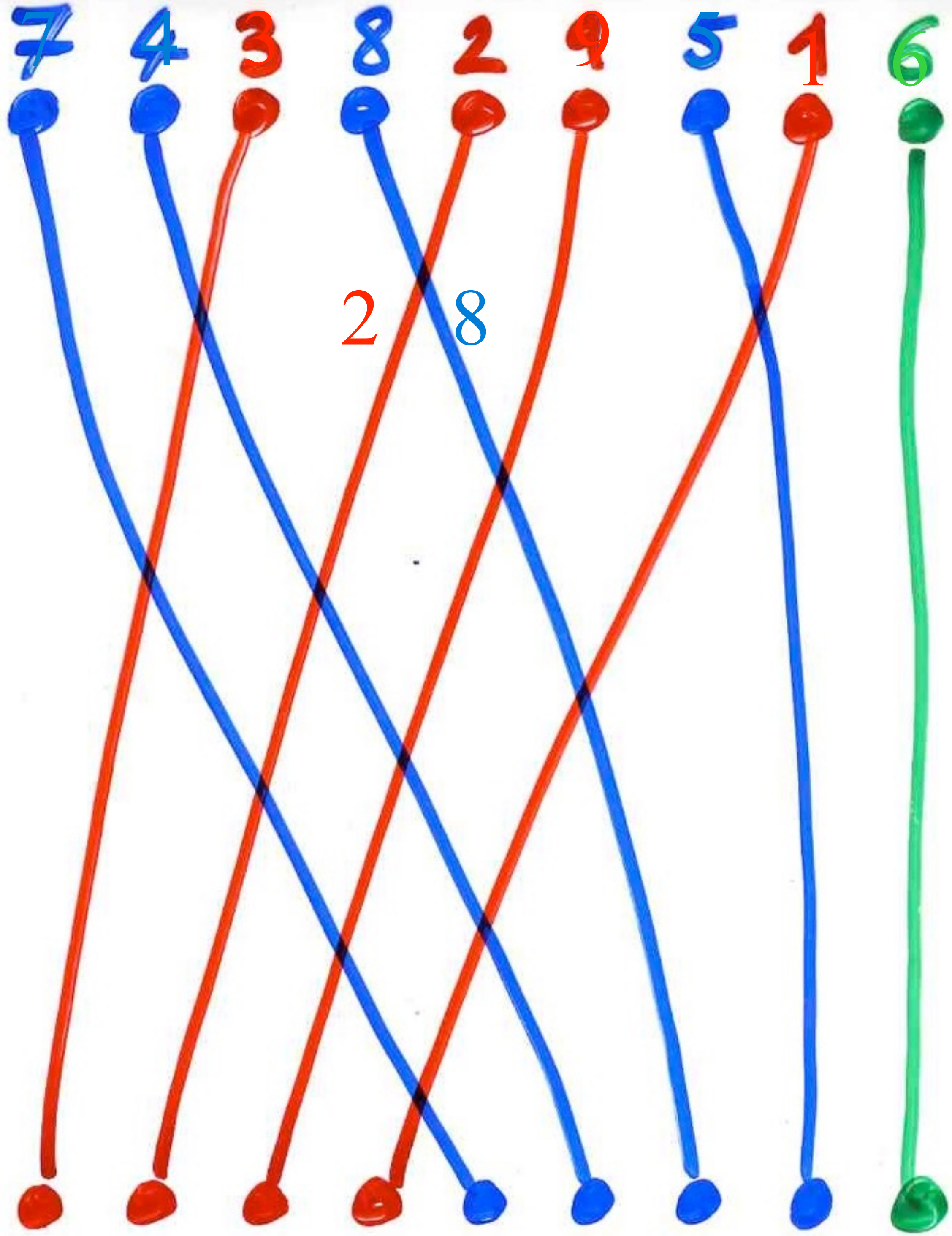
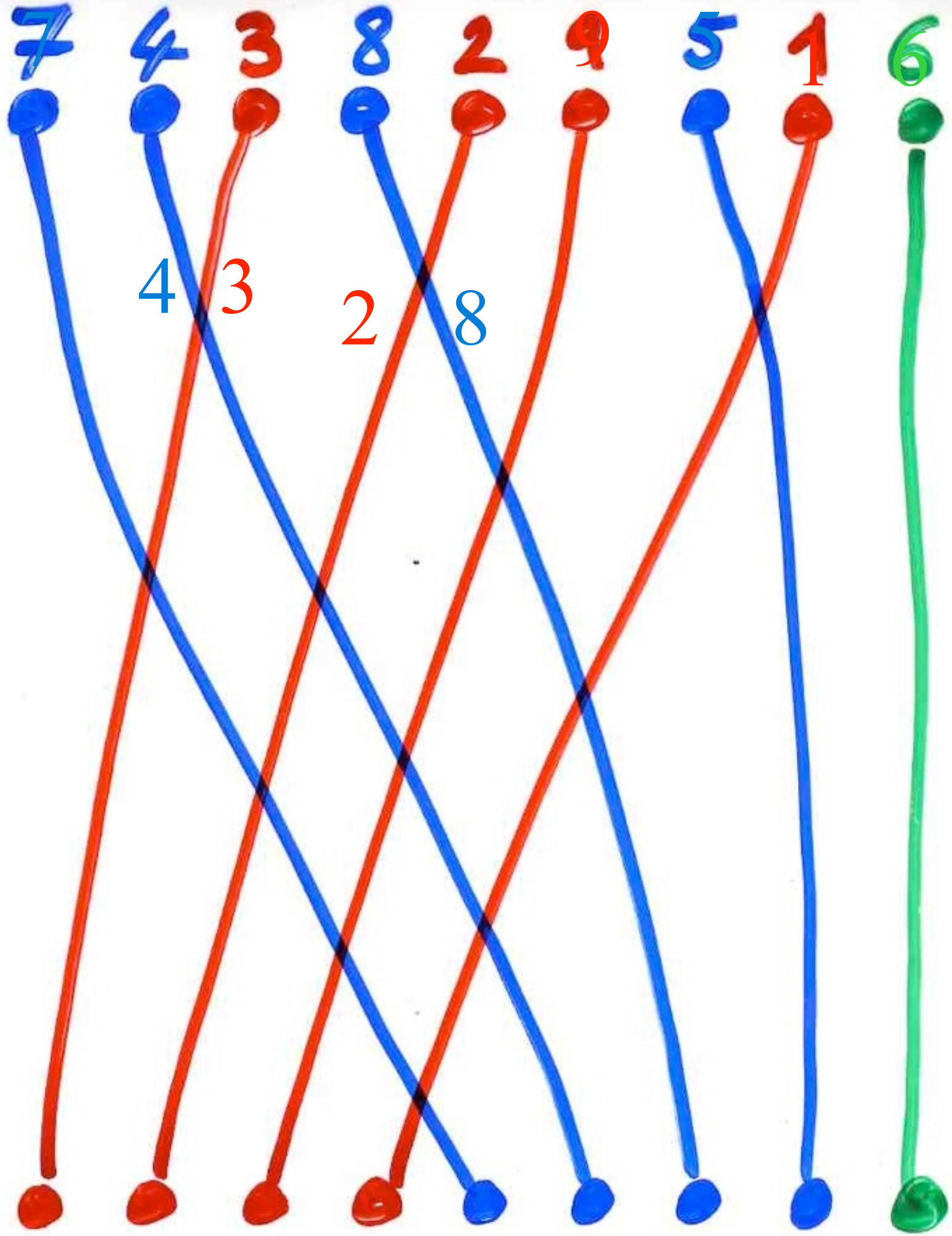
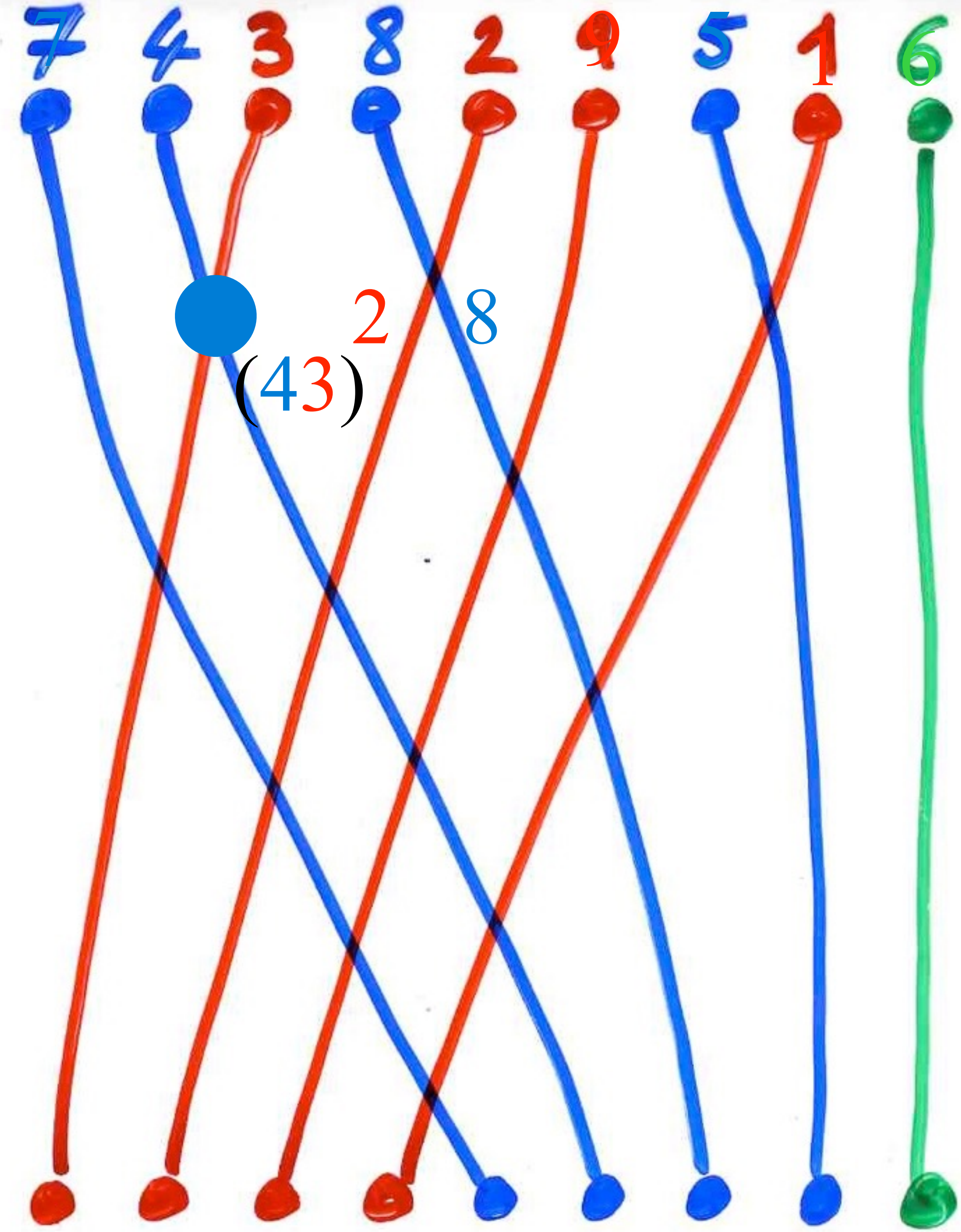
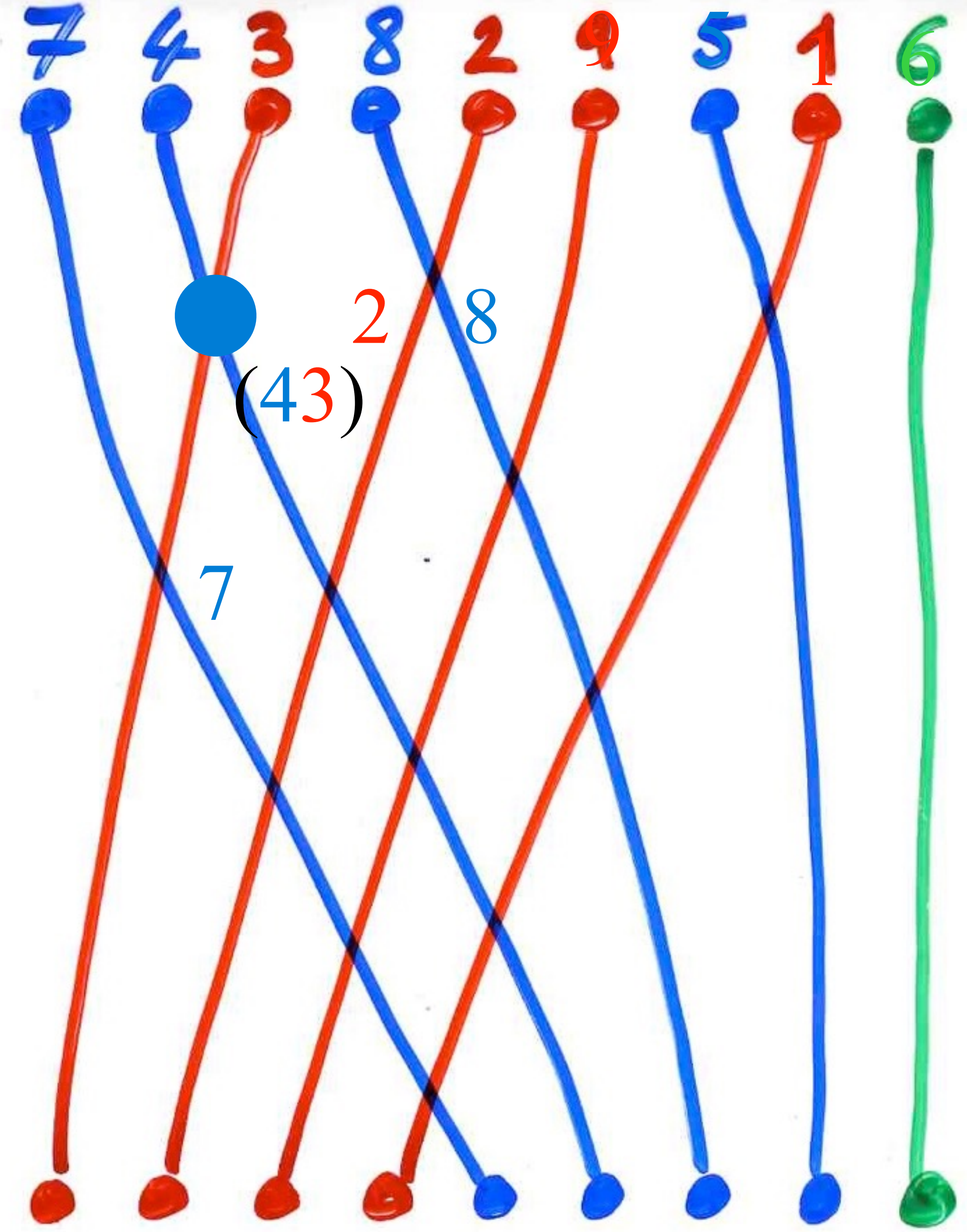

7 4 3 8 2 9 5 1 6
2 8



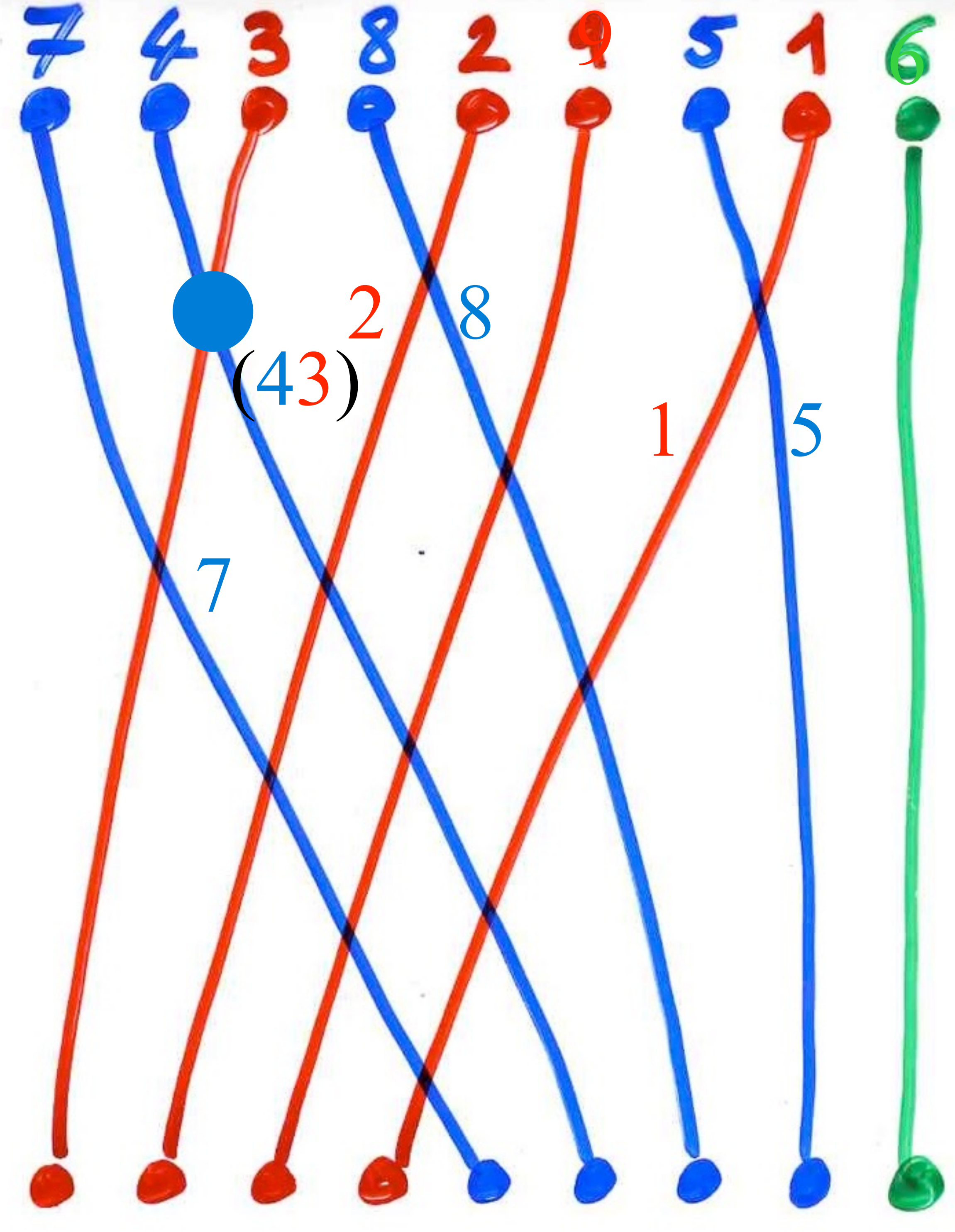
7 4 3 8 2 9 5 1 6
4
3
2
8

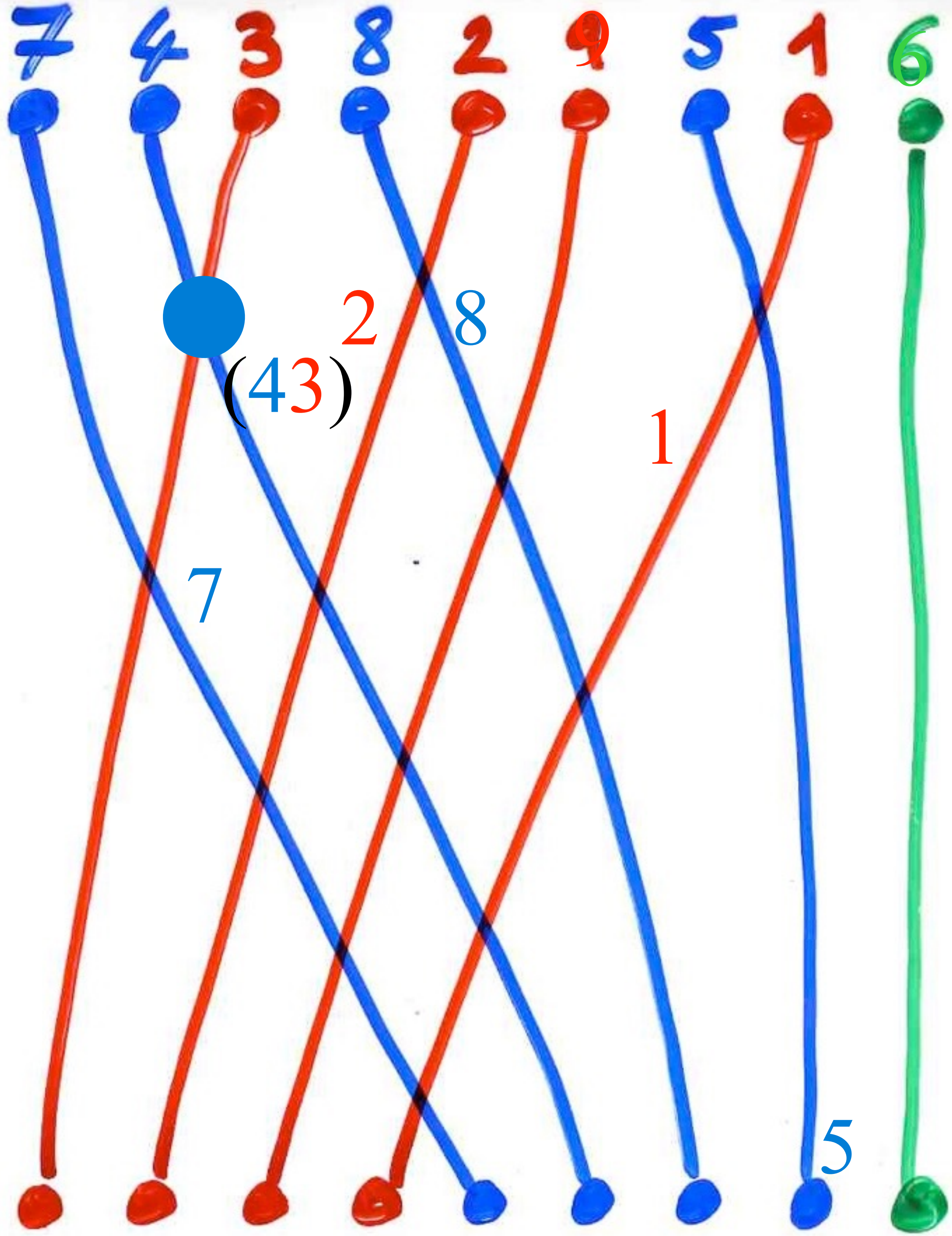


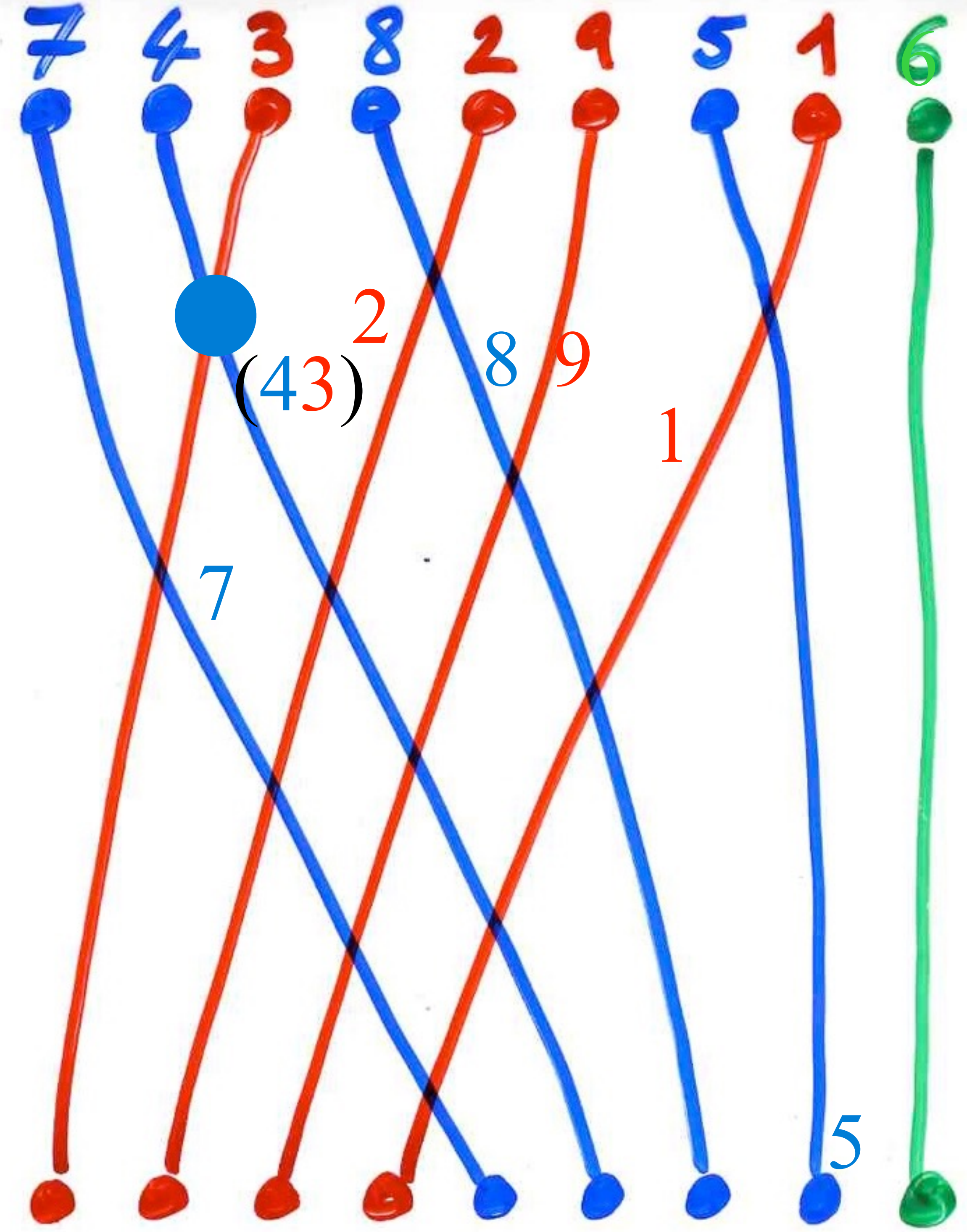
7 4 3 8 2 9 5 1 6
(43)
2
8



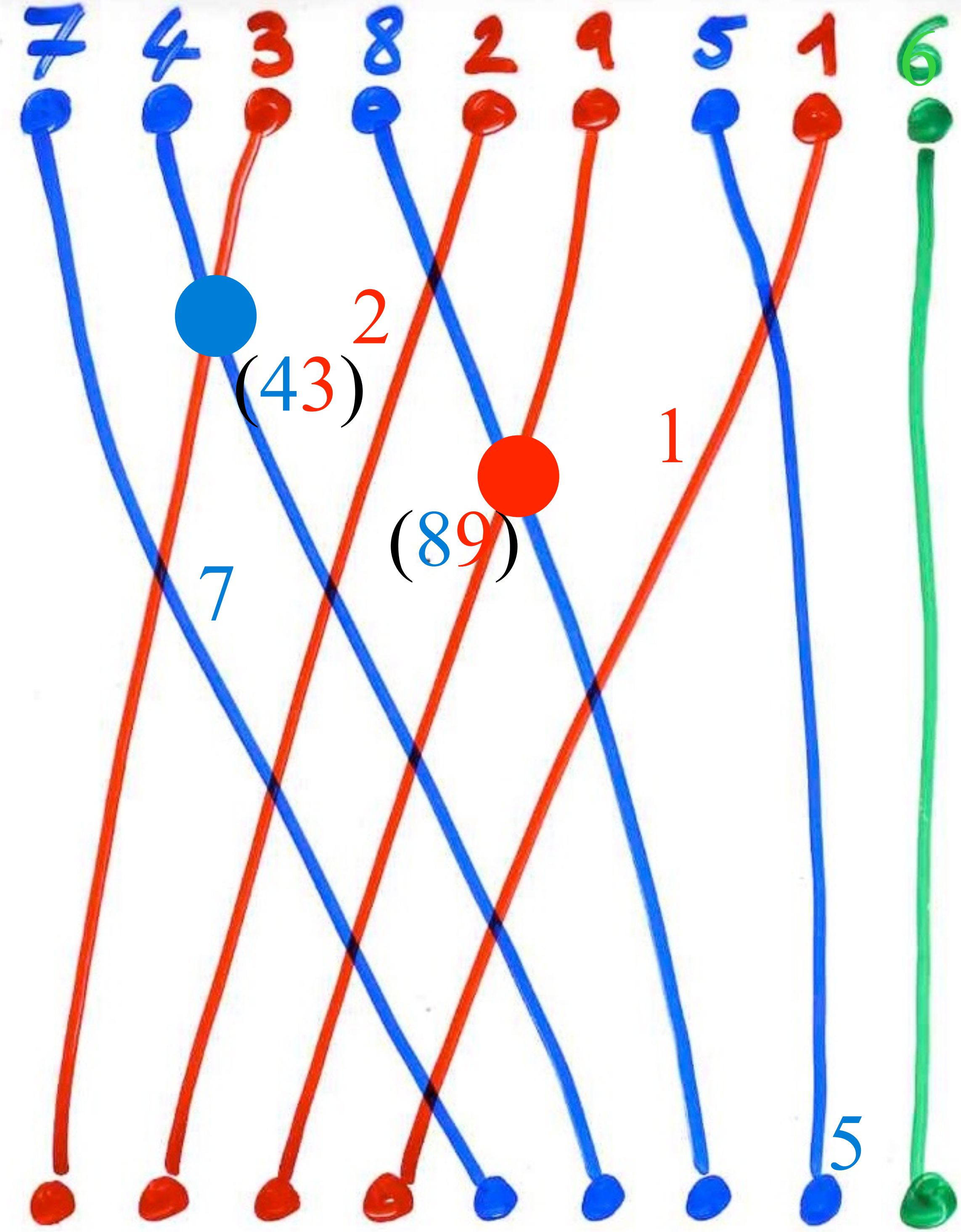
7 4 3 8 2 9 5 1 6
2
8
(43)
7

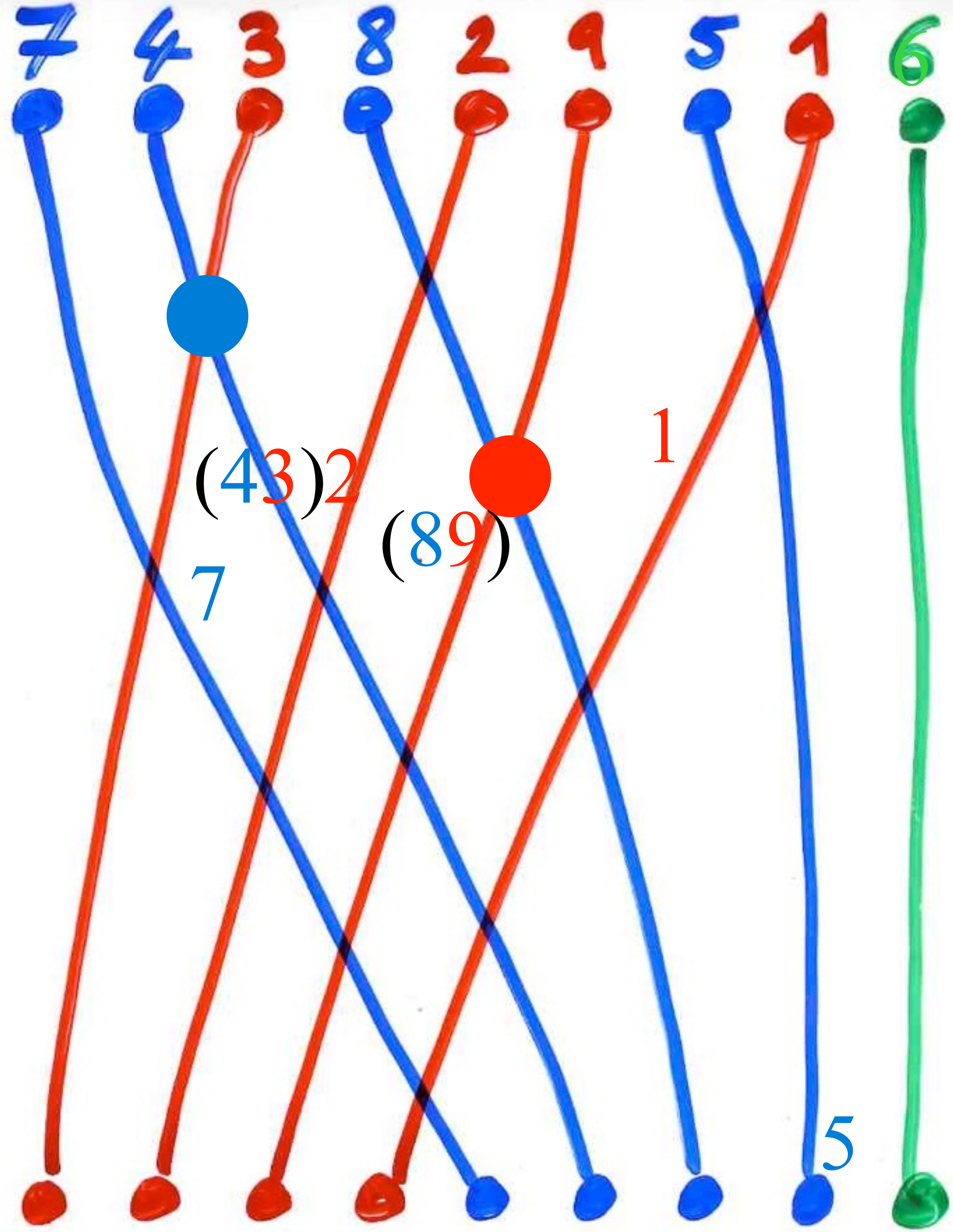

7 4 3 8 2 9 5 1 6
(43)
2
8
1
5
7


7 4 3 8 2 9 5 1 6
(43)
2
8
1
7
5

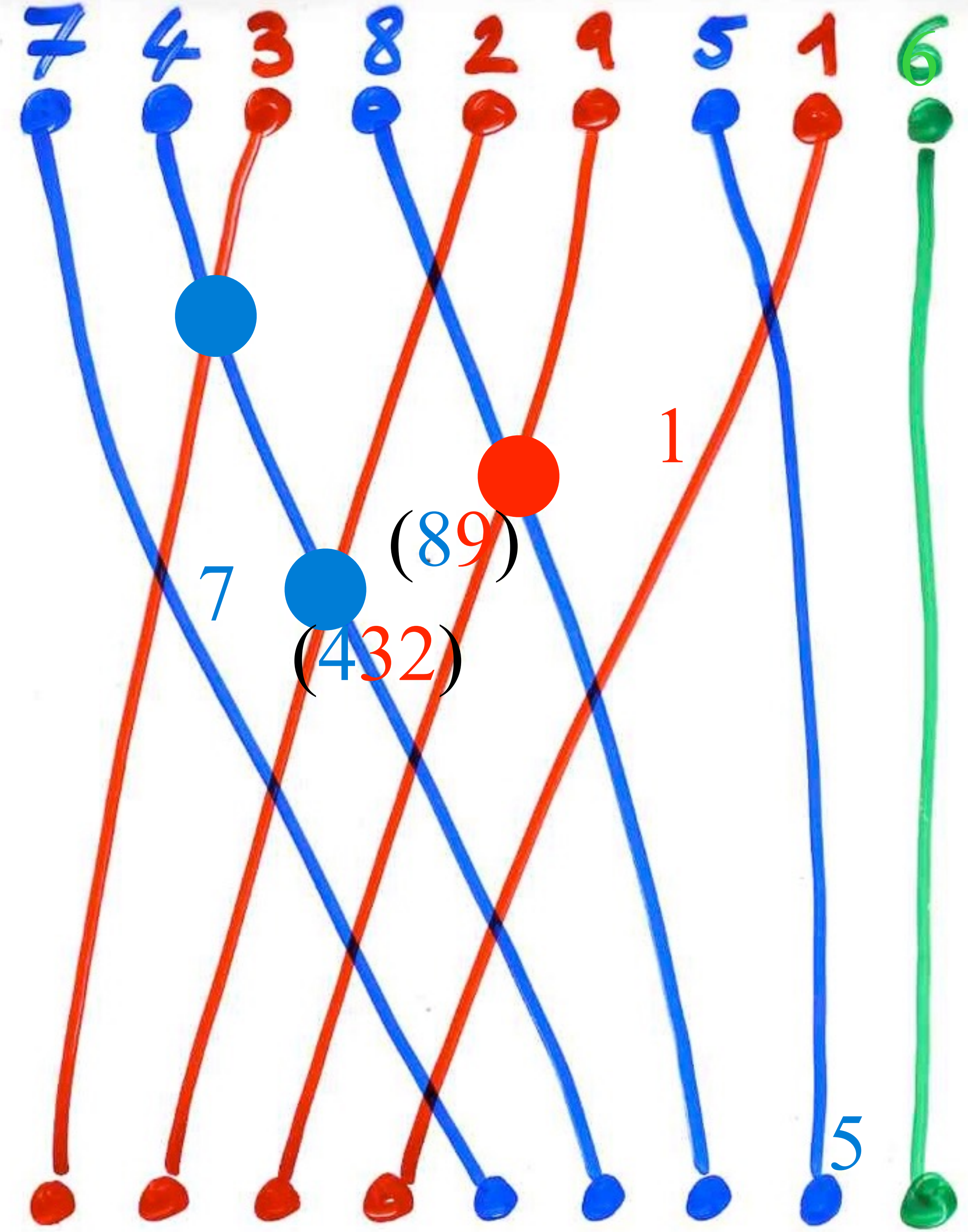


7 4 3 8 2 9 5 1 6
(43)
2
8
9
1
7
5

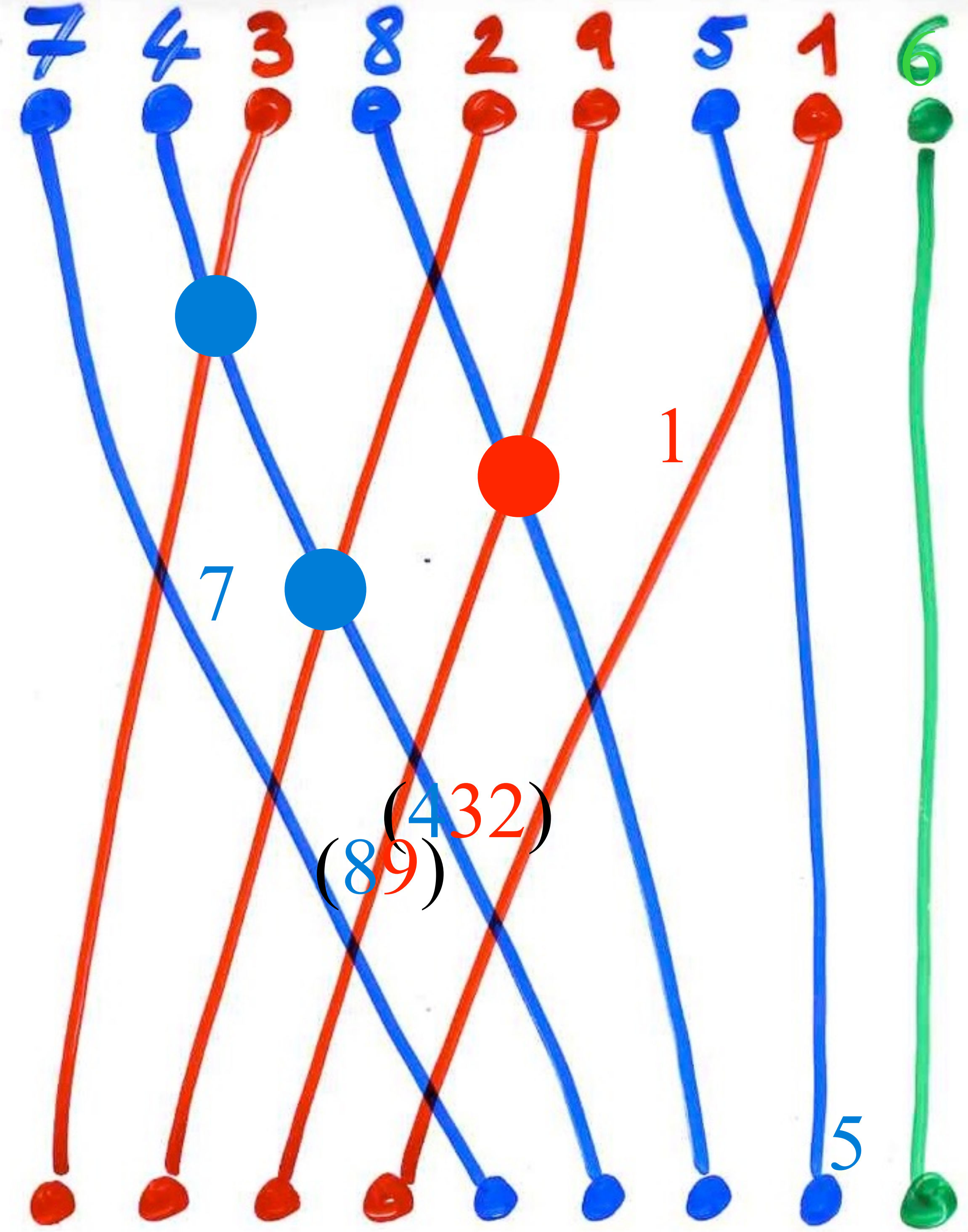

7 4 3 8 2 9 5 1 6
(43)
2
(89)
1
7
5

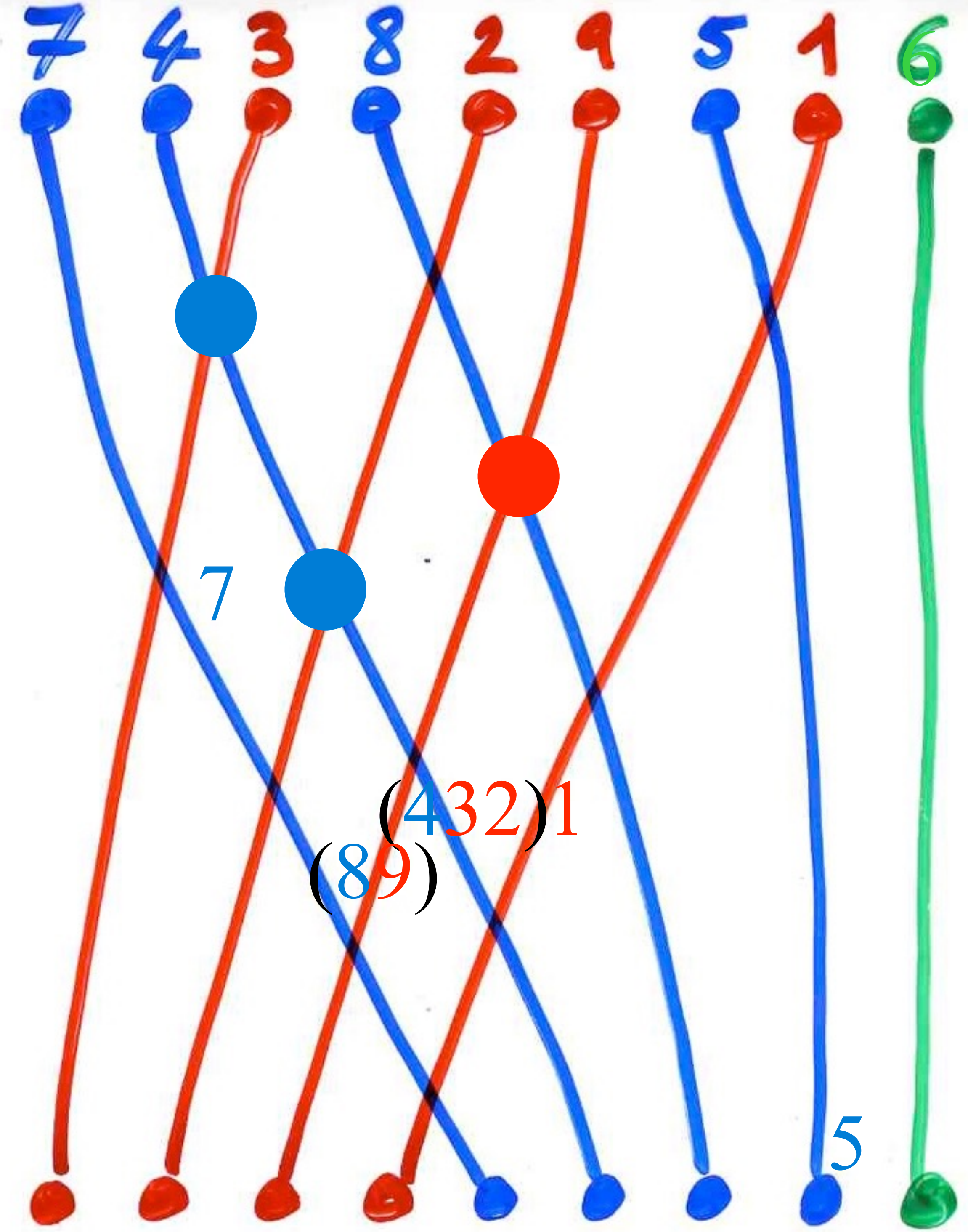


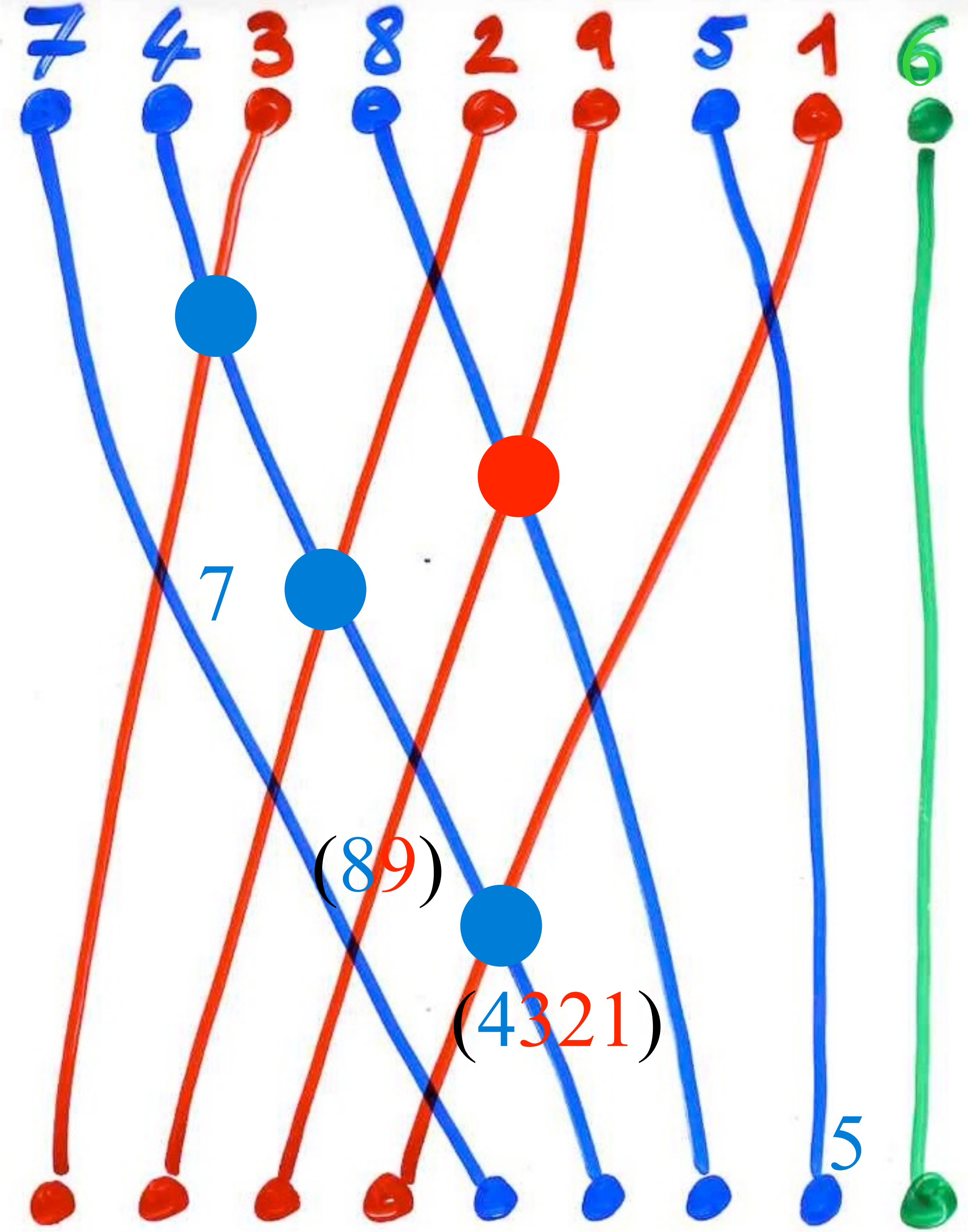
7 4 3 8 2 9 5 1 6
(43)2
(89)
1
7
5



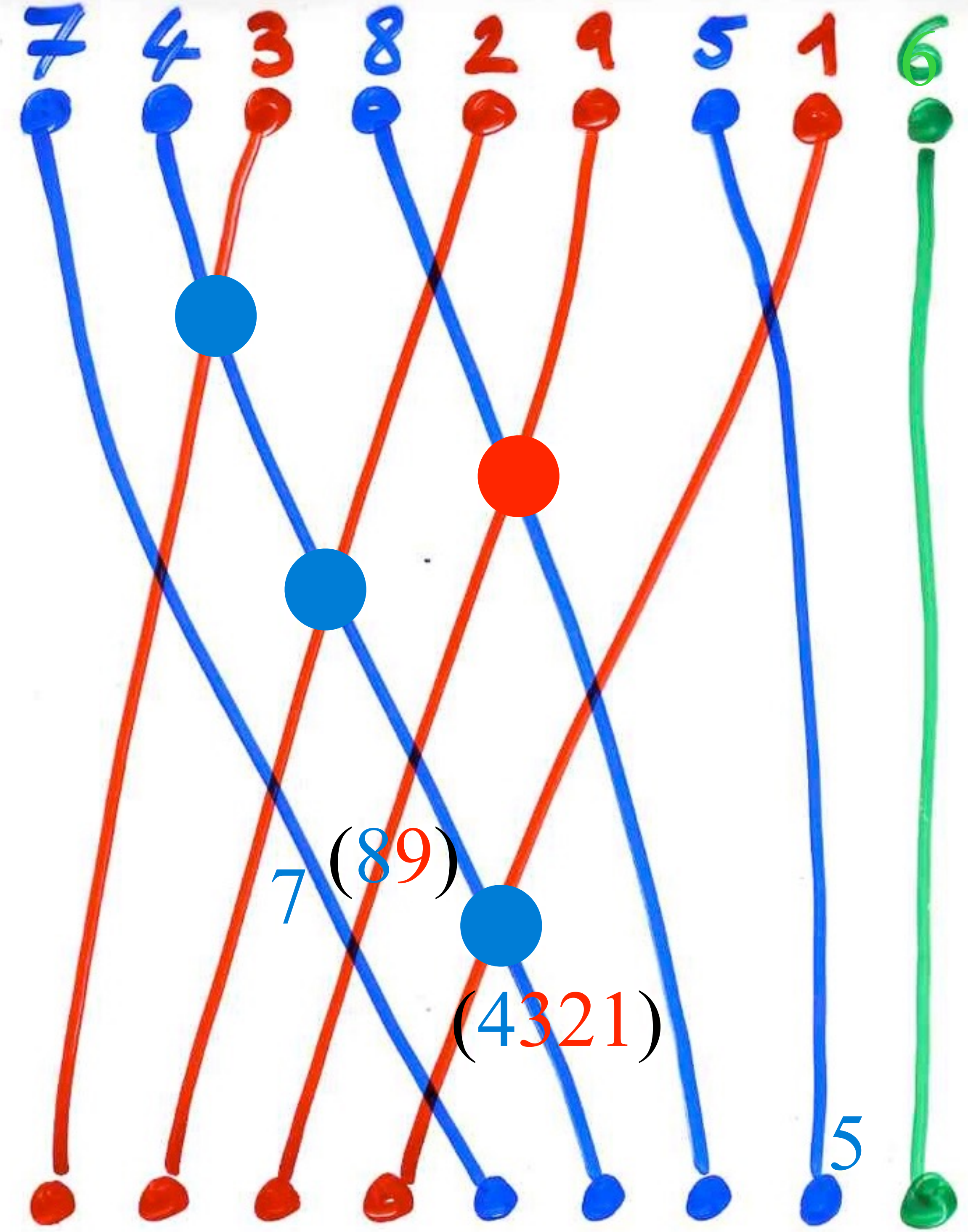
7 4 3 8 2 9 5 1 6
1
(89)
7
(432)
5

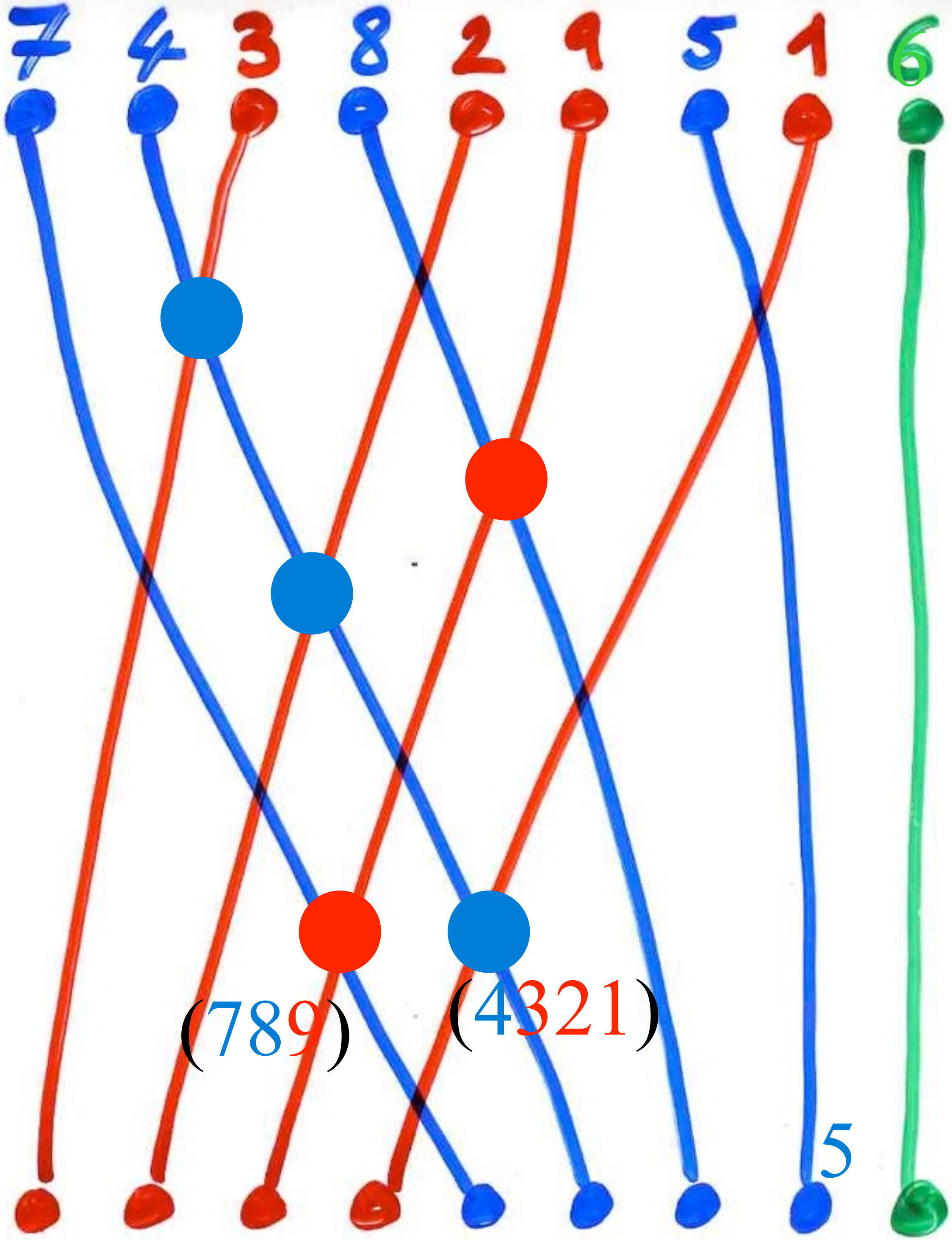

7 4 3 8 2 9 5 1 6
1
7
(432)
(89)
5


7 4 3 8 2 9 5 1 6
7
(432)1
(89)
5



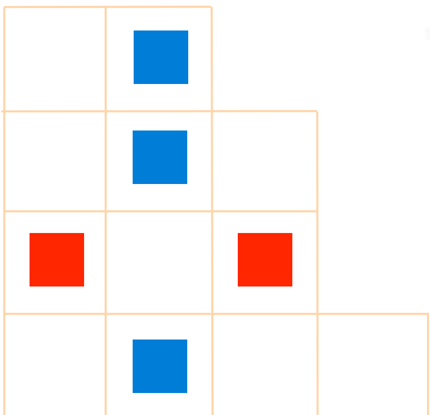
7 4 3 8 2 9 5 1 6
7
(89)
(4321)
5

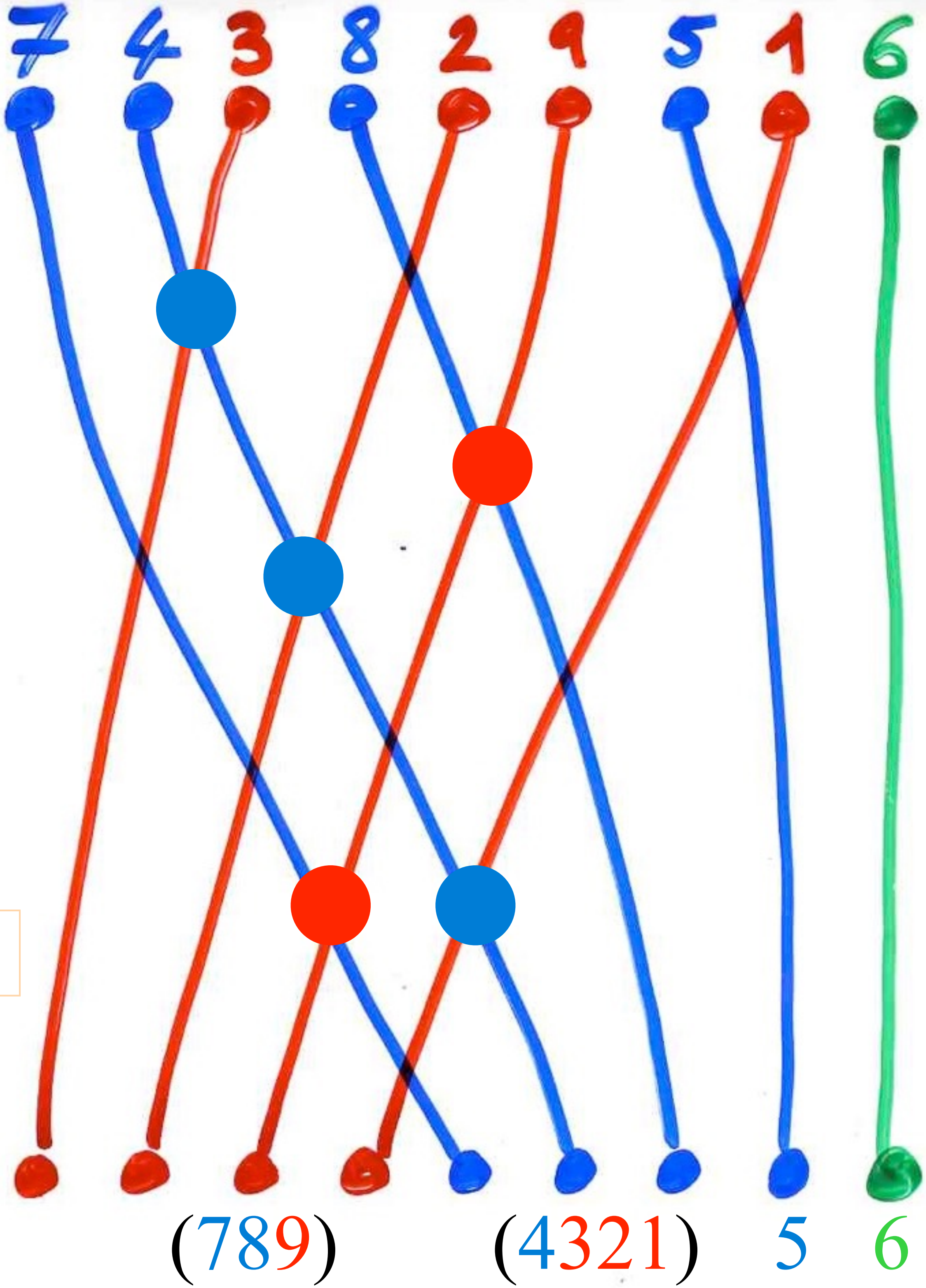

7 4 3 8 2 9 5 1 6
(89)
7
(4321)
5



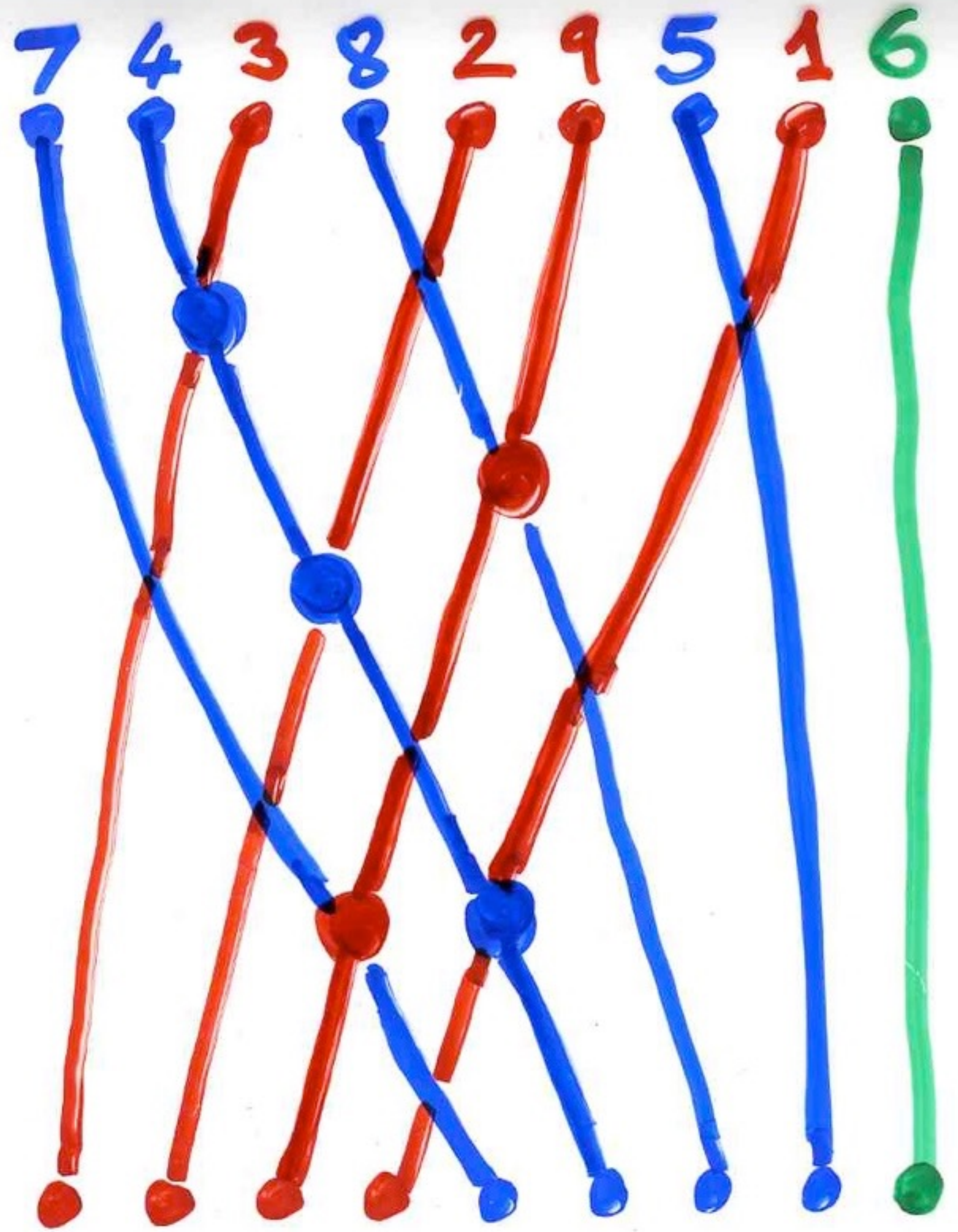
7 4 3 8 2 9 5 1 6
(789)
(4321)
5

"exchange-
fusion"
algorithm





The inverse "exchange-fusion" algorithm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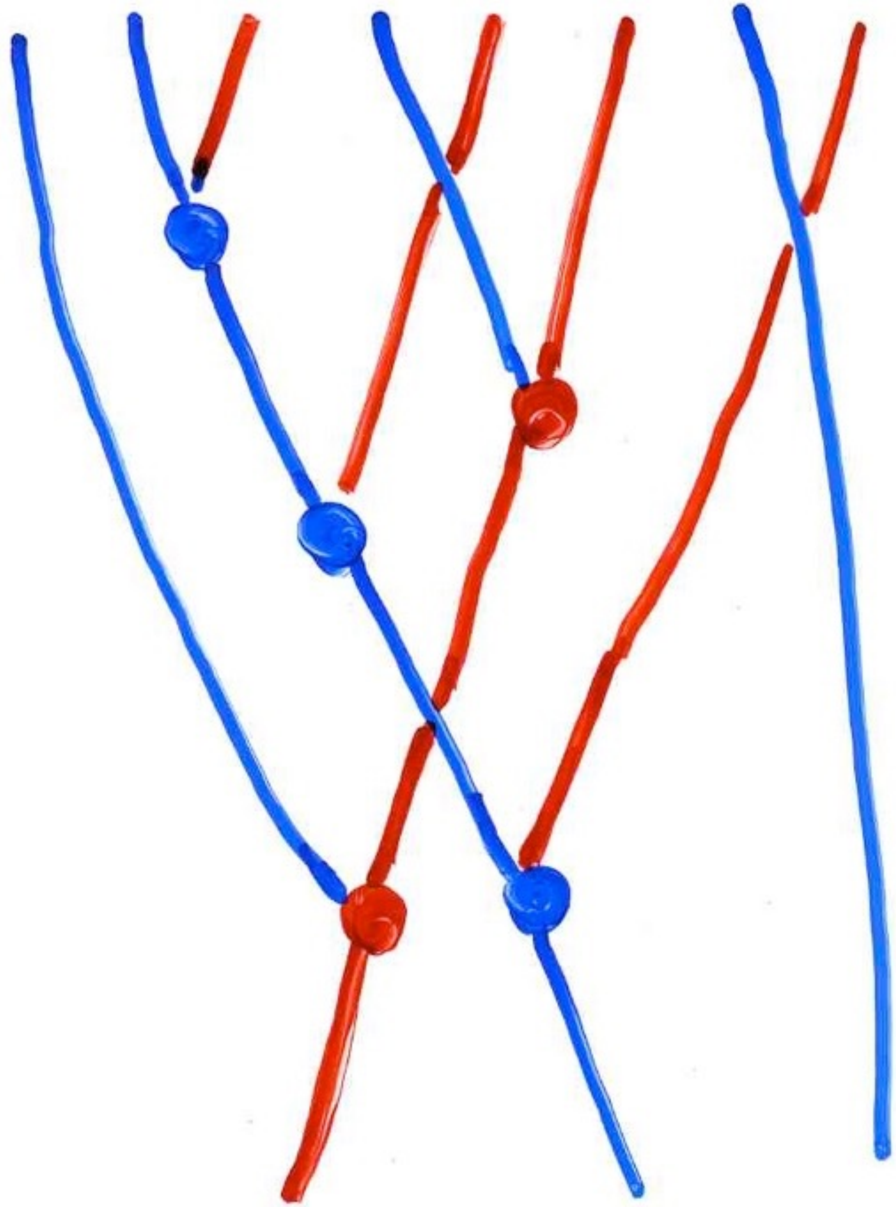
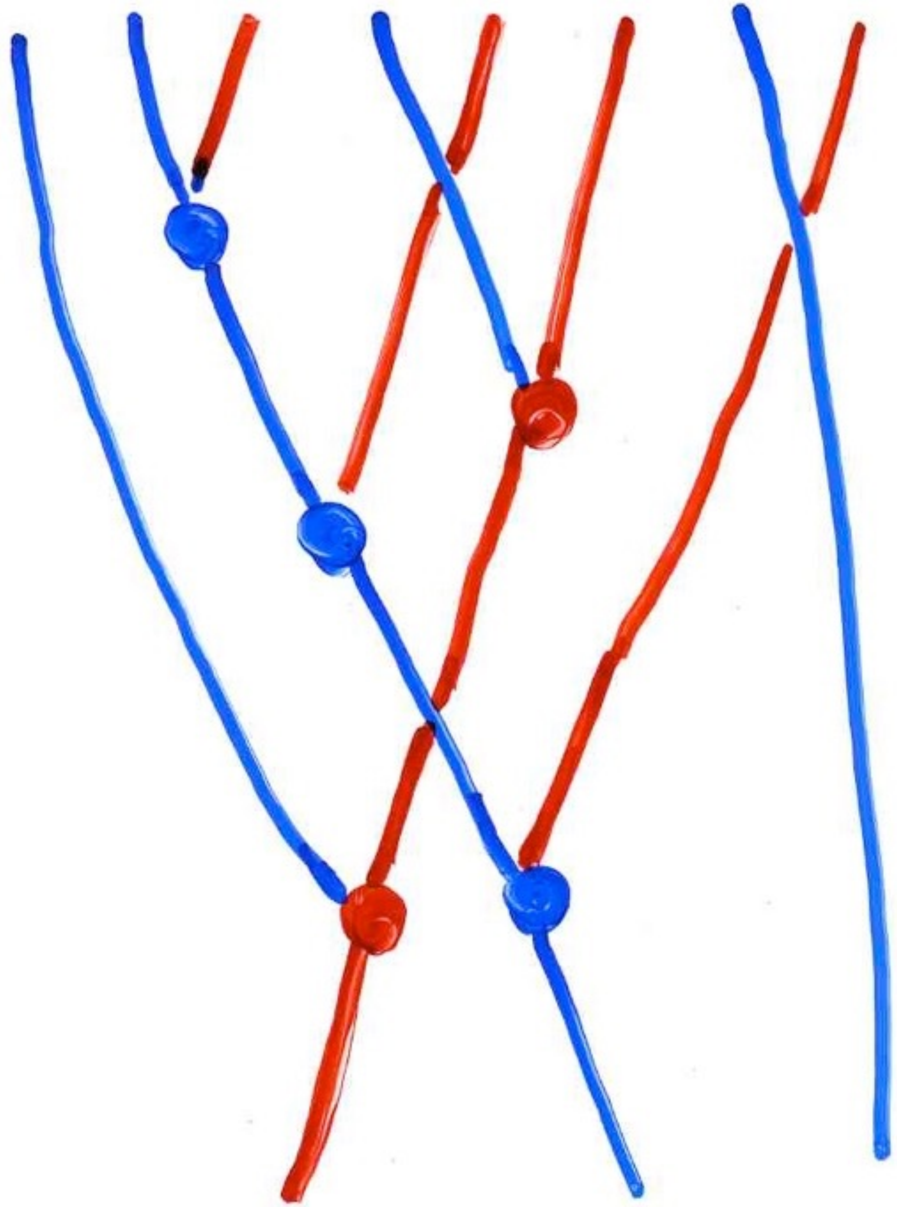
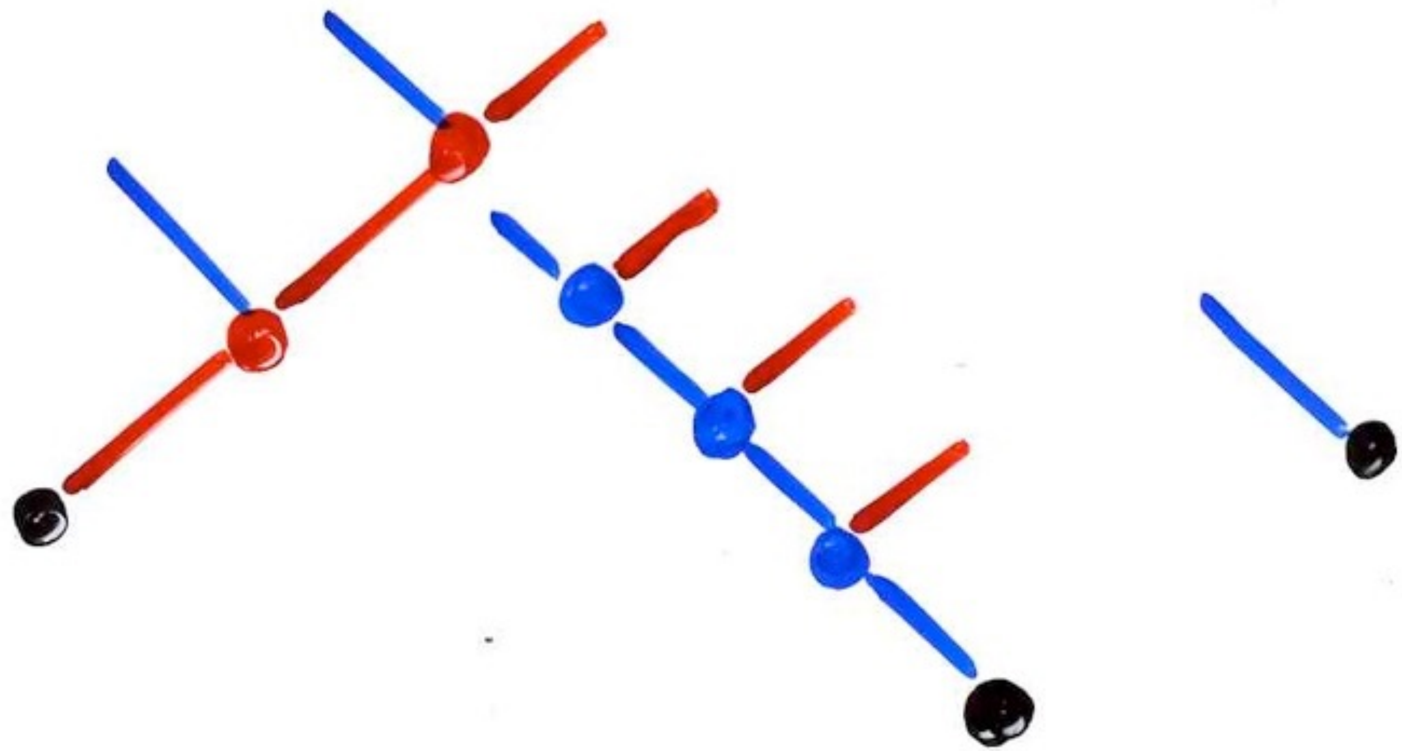
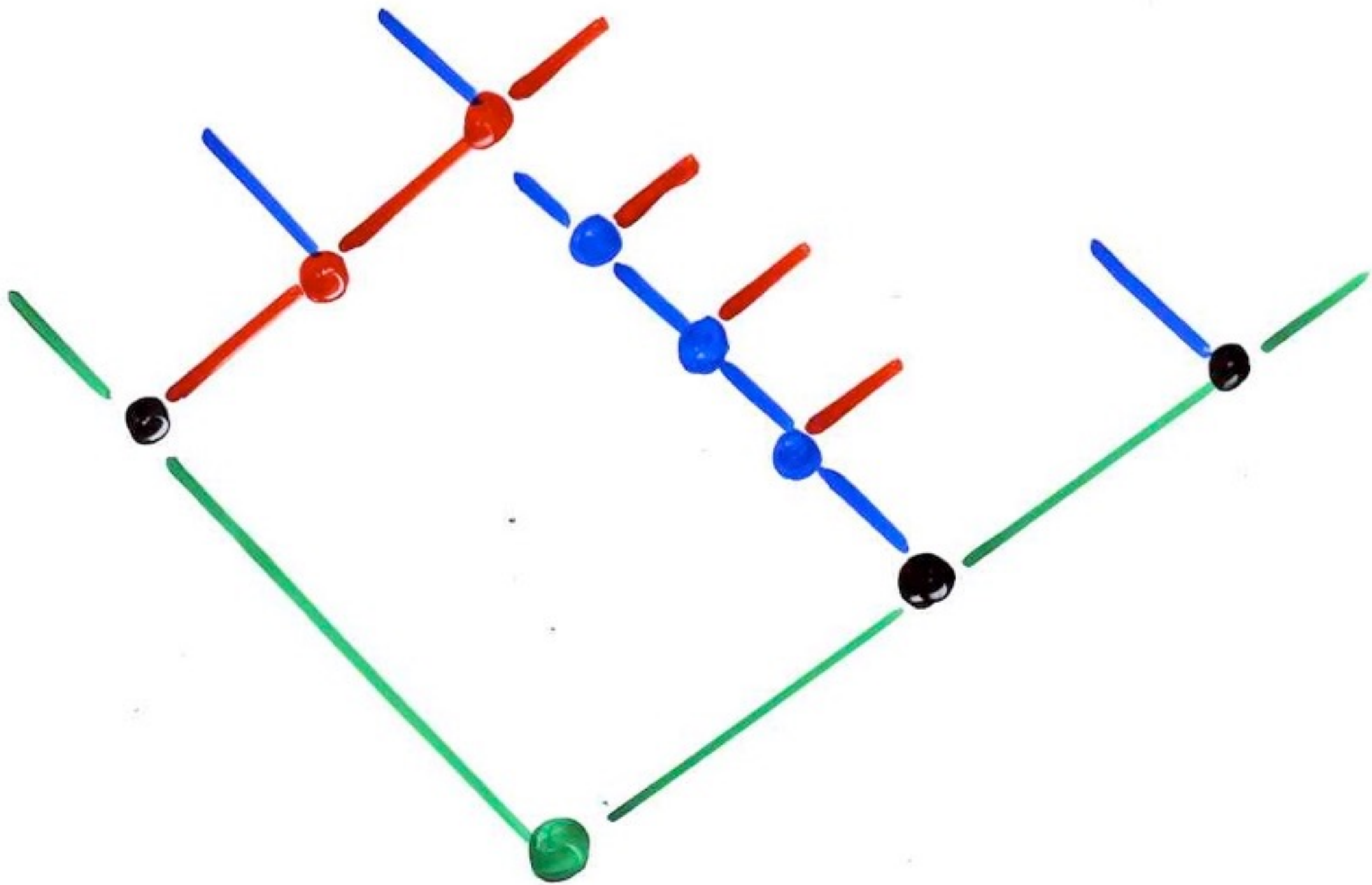
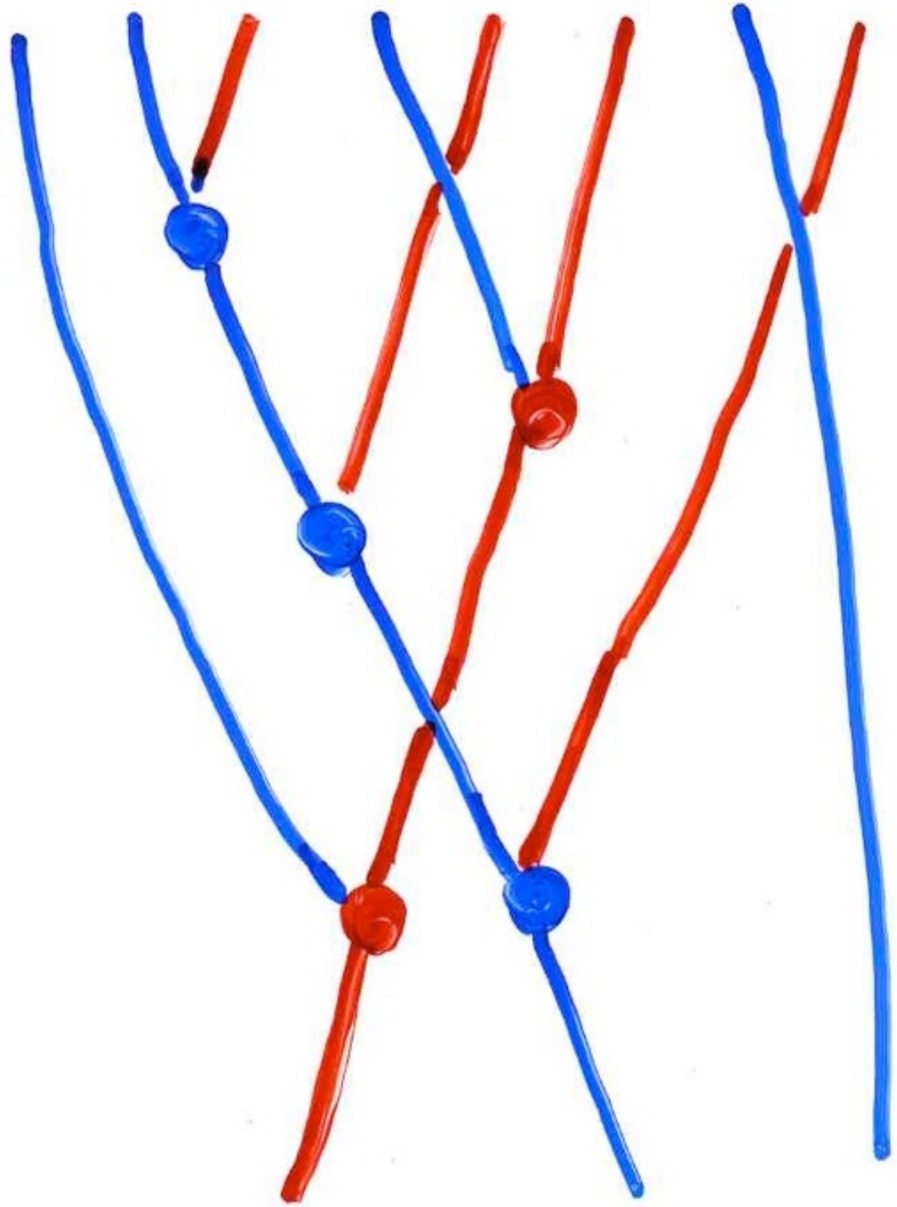
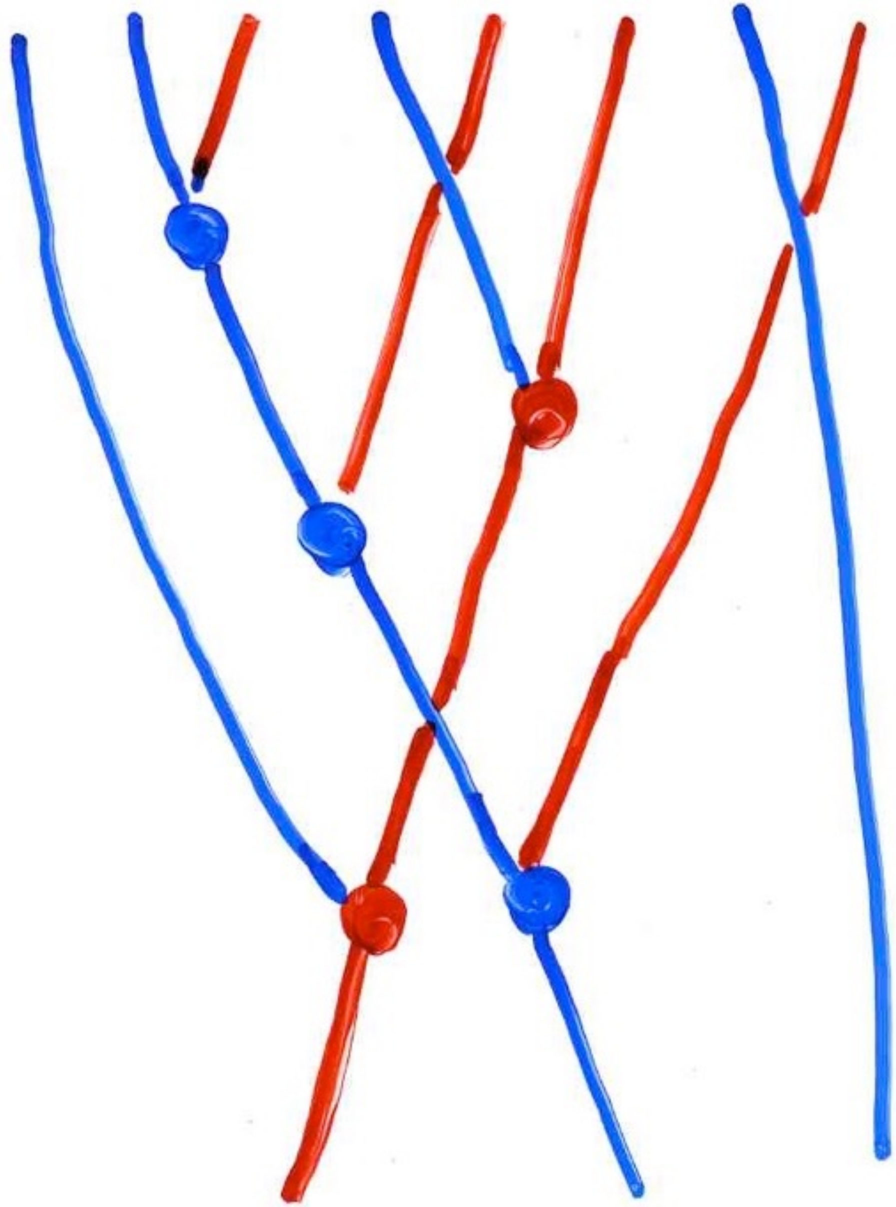




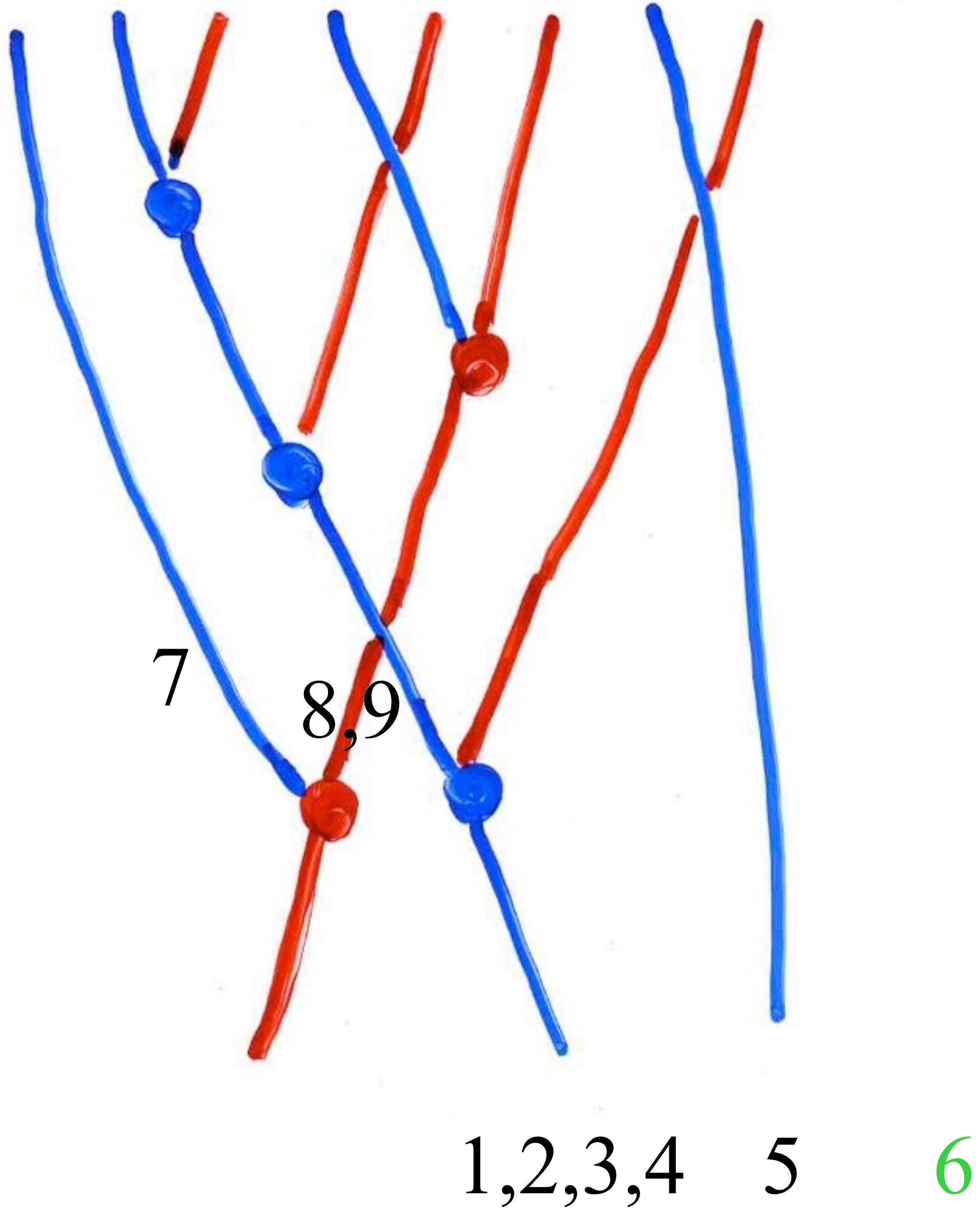




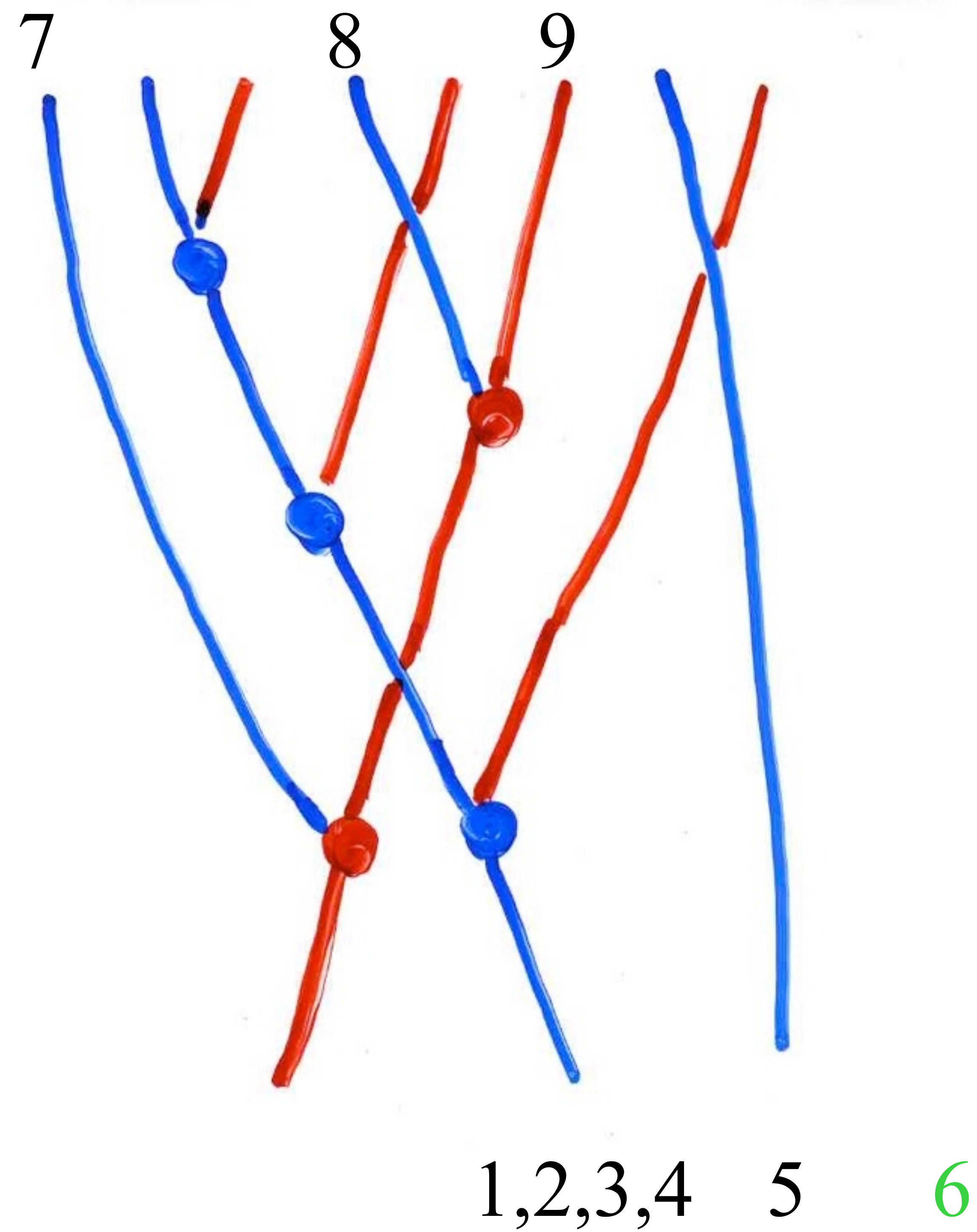




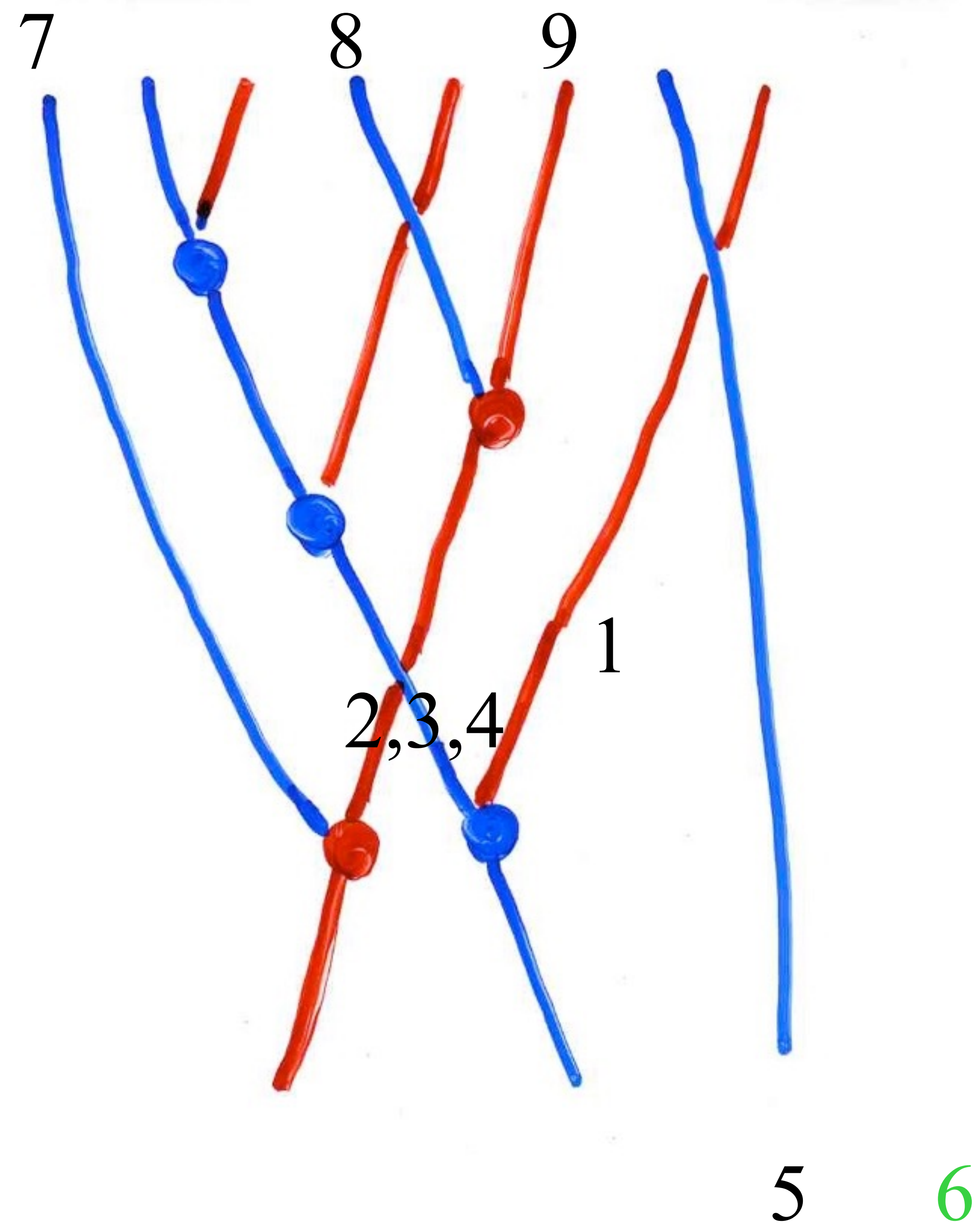
7,8,9 1,2,3,4 5 6



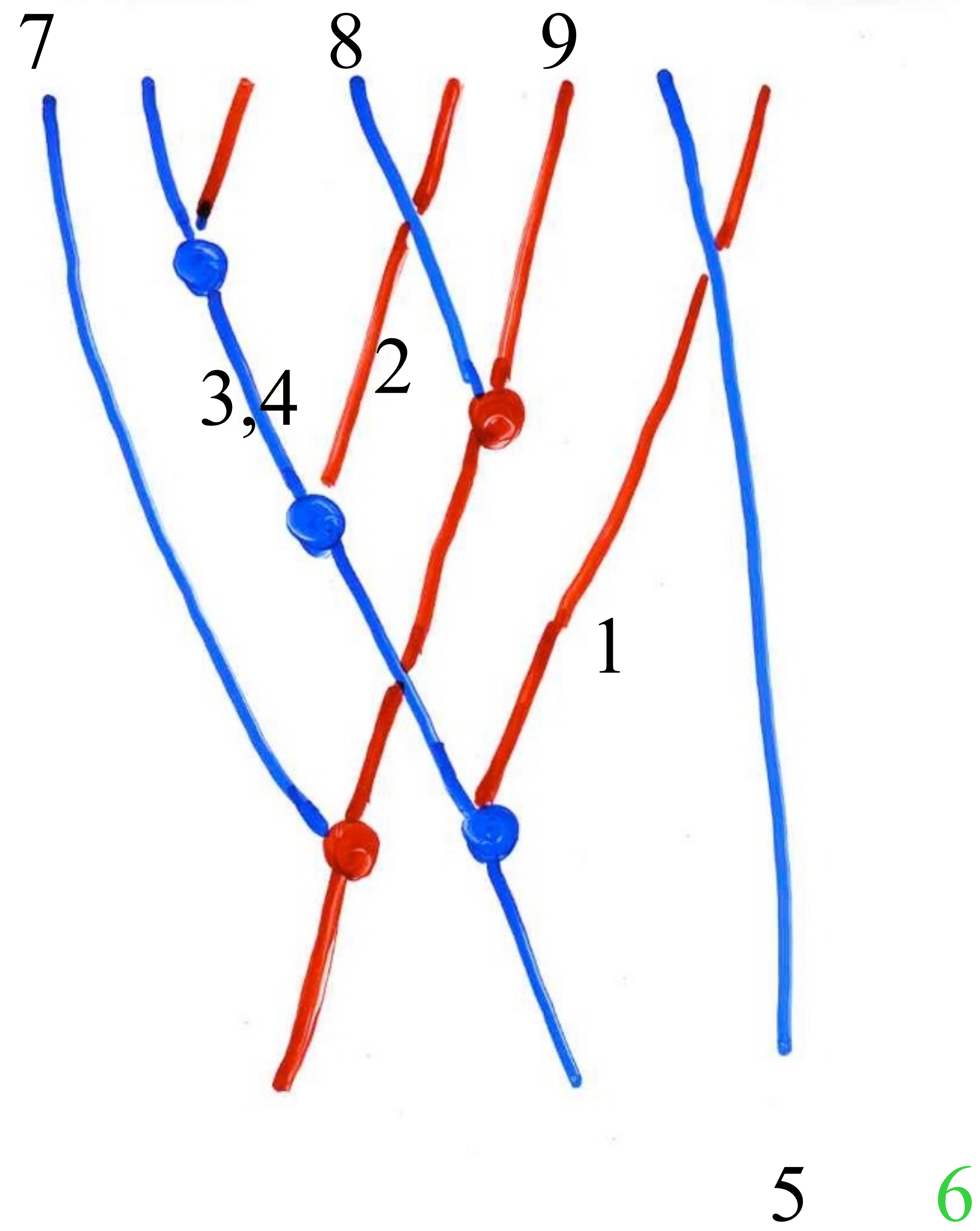
7
8,9
1,2,3,4
5
6



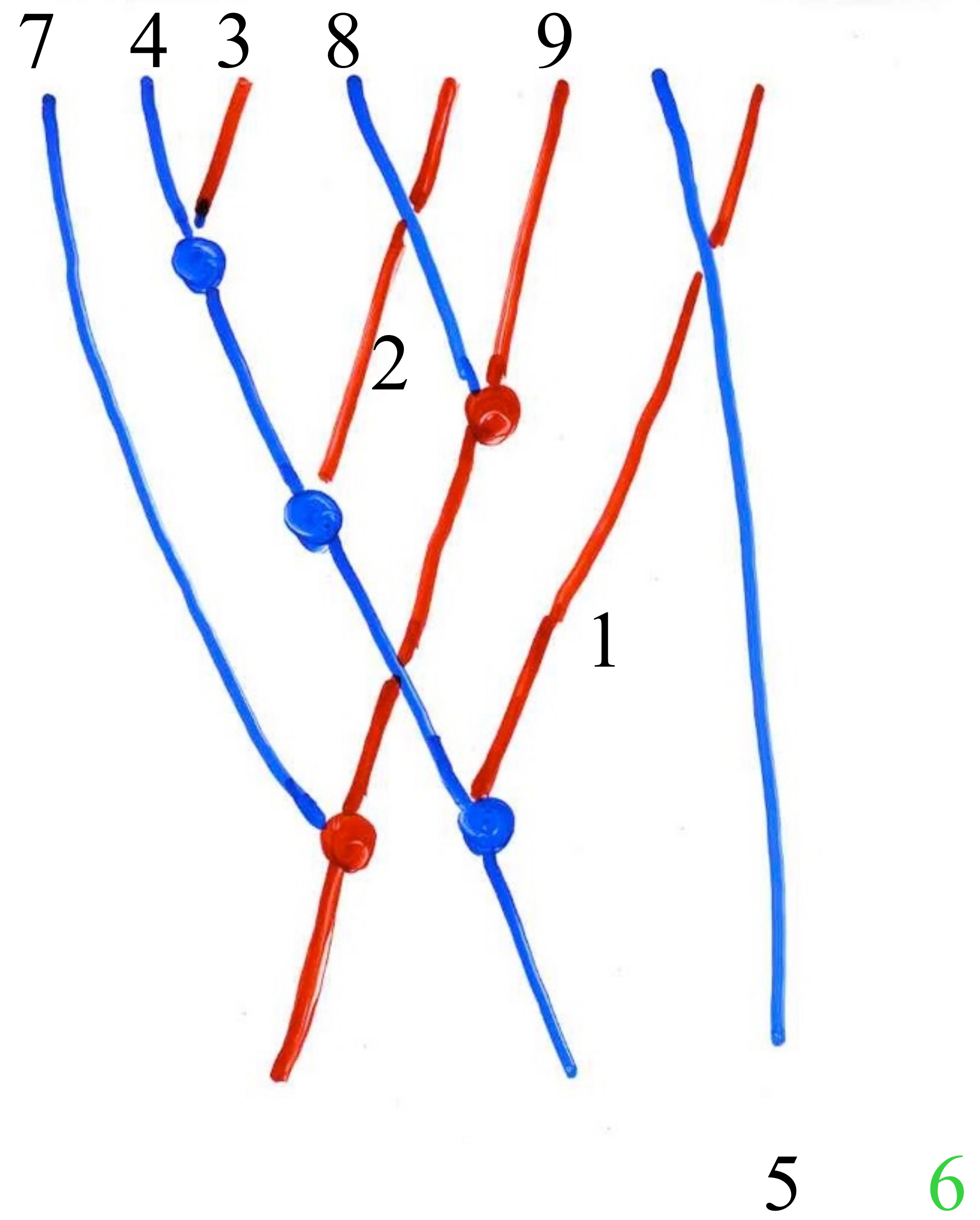
7
8
9
1,2,3,4
5
6



7
8
9
1
2,3,4
5
6



7
8
9
3,4
2
1
5
6


7
4
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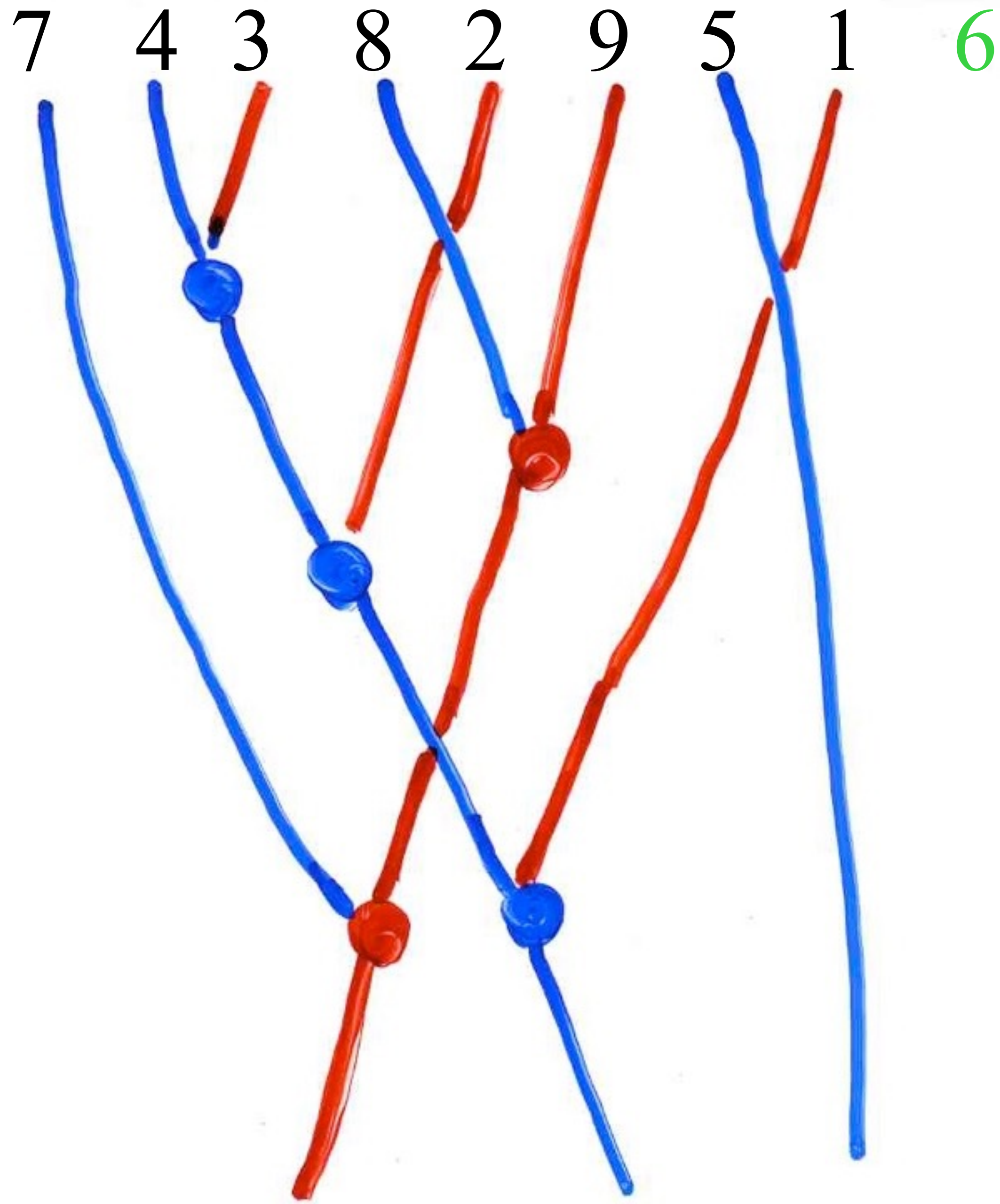
6



7 4 3 8 2 9 5 1 6

«assemblées» and species

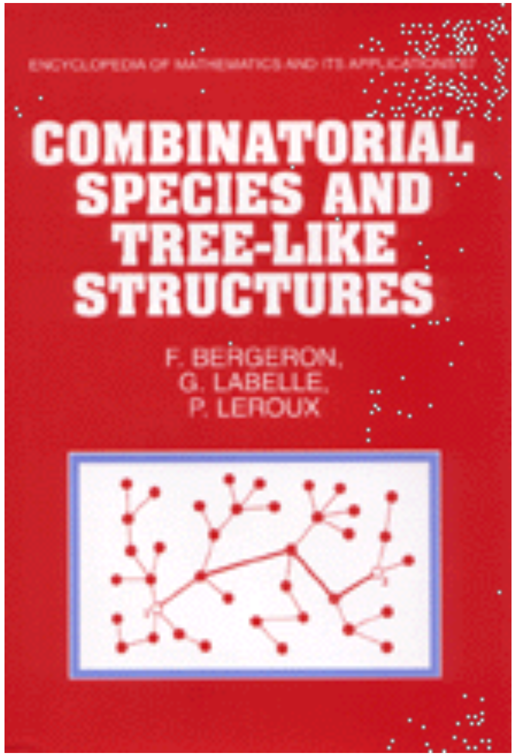
petit rappel

Combinatorial model for exponential generating function

$$f(t) = \sum_{n\geq 0} a_n \frac{t^n}{n!}$$

Species (combinatorial) structures

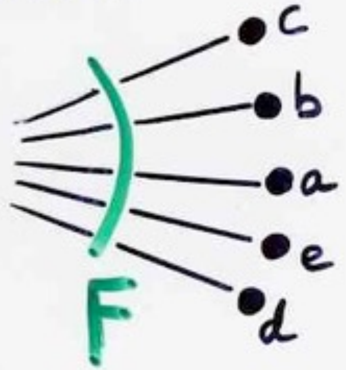
UQAM Montréal Québec

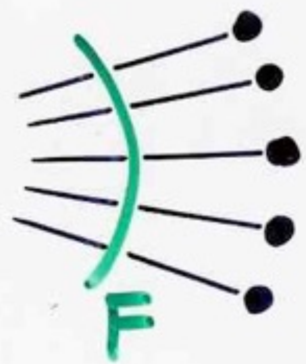


A. Joyal
F. Bergeron
G. Labelle
P. Leroux
- - - - -

Encyclopedia of Maths. and Applications
Cambridge Univ. Press
(1977)

Convention.





enumeration

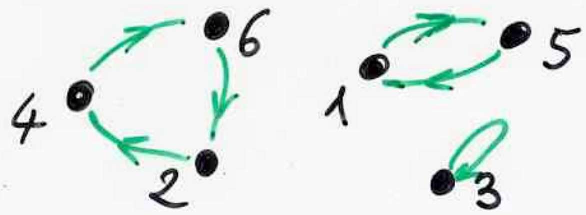
$$a_n = | F\{1,2,\dots,n\} |$$

Def. Generating function of the species F

$$F(t) = \sum_{n \geq 0} a_n \frac{t^n}{n!}$$

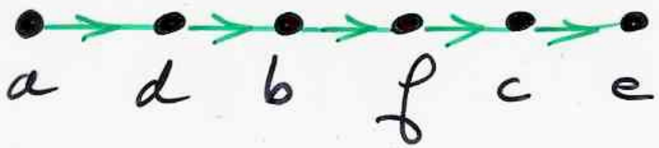
Examples

- **Permutations** S $\quad a_n = n! \quad S(t) = \frac{1}{1-t}$



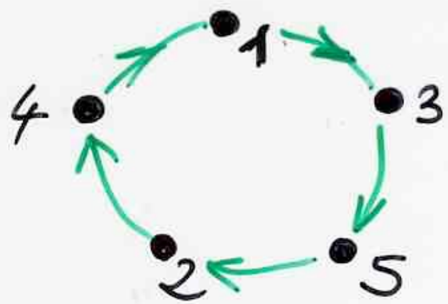
$$\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 5 & 4 & 3 & 6 & 1 & 2 \end{pmatrix}$$

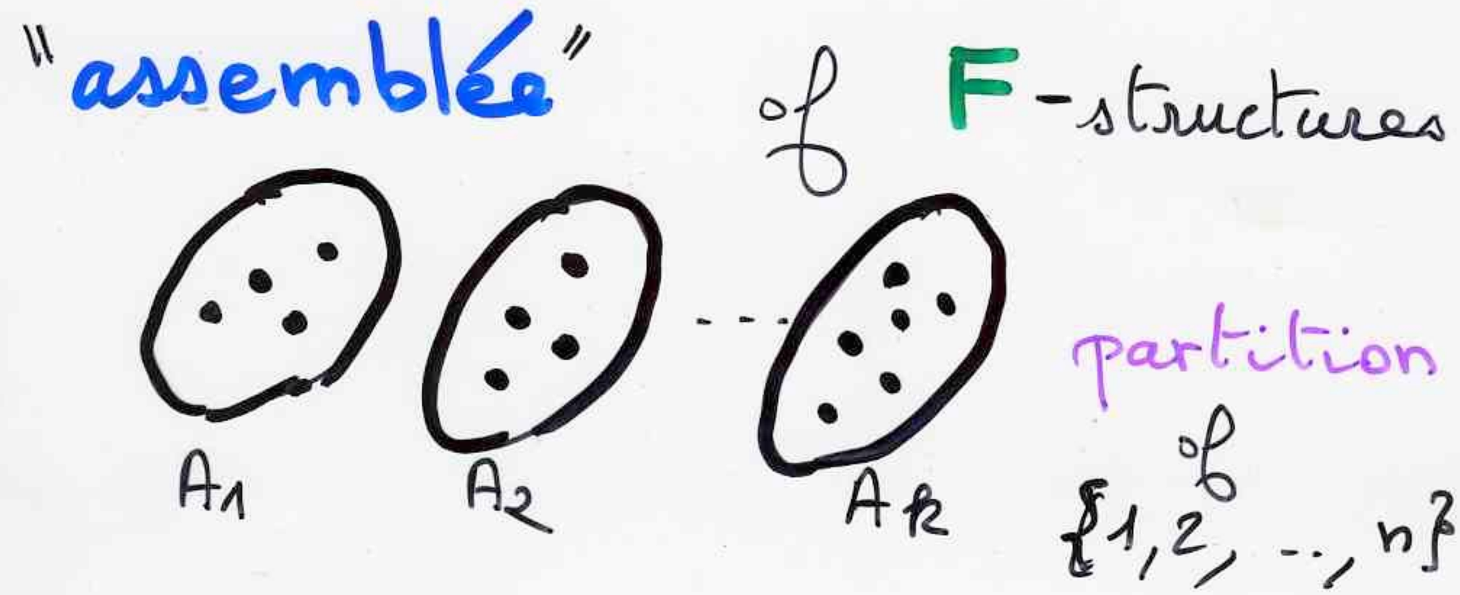
- **Total order** L $\quad a_n = n! \quad L(t) = \frac{1}{1-t}$



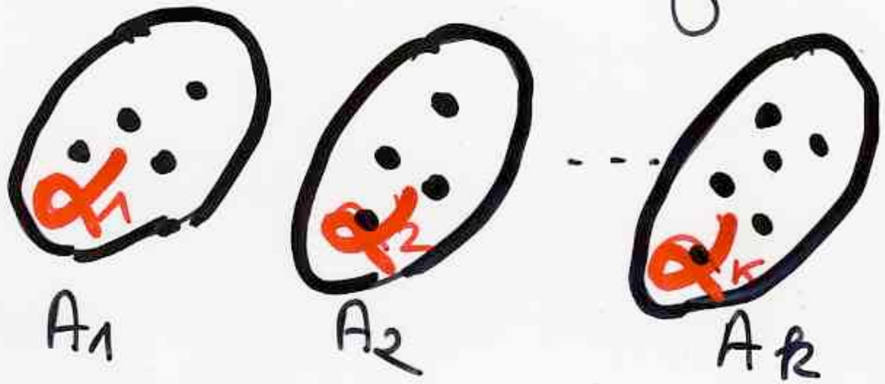
- **Cycle** C $\quad a_n = (n-1)!$

$$C(t) = \sum_{n \geq 1} \frac{t^n}{n} = \log(1-t)^{-1}$$




"assemblée" of F-structures
A1
A2
Ak
partition of {1, 2, ..., n}

"assemblée" of F-structures



partition of $\{1, 2, \ldots, n\}$

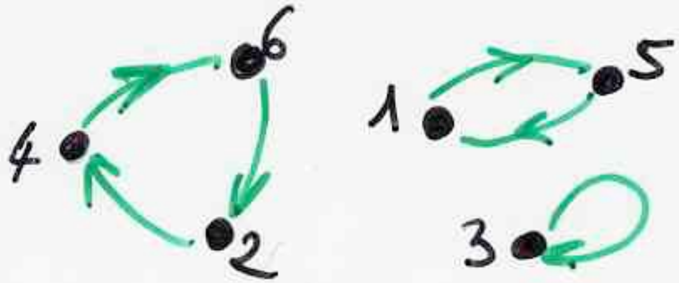
α_i F-structure on A_i

$\mathbf{H} = \exp \mathbf{F}$

$$h(t) = \exp(f(t))$$

$$f(t) = \log(h(t))$$

Permutations $S = \exp C$ cycle



$$\sum_{n\geq 0} n!\,\frac{t^n}{n!} = \frac{1}{1-t}$$

$$\sum_{n\geq 1} (n-1)!\,\frac{t^n}{n!} = \sum_{n\geq 1} \frac{t^n}{n} = \log \frac{1}{1-t}$$

$$Z^{*}_{n,r}(\alpha=\beta=q=1) = \binom{n}{r}\frac{(n+1)!}{(r+1)!}$$

Lah numbers

nb of "assembleés" of permutations

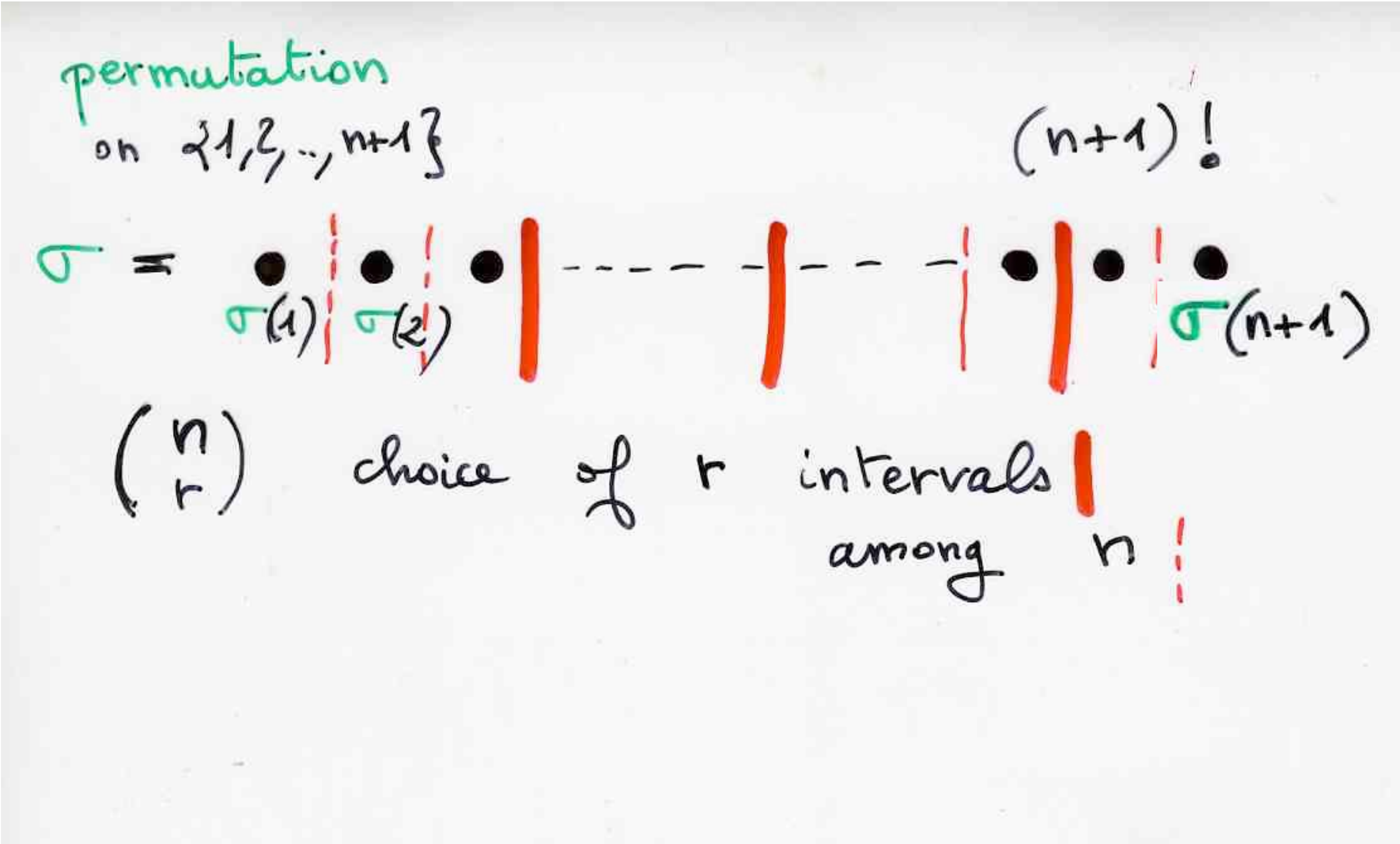
$$\left\{[7,10,5,8]\ [3,1,4]\ [9,2,11,6]\right\}$$

$$\exp\left(\frac{xt}{1-t}\right)$$

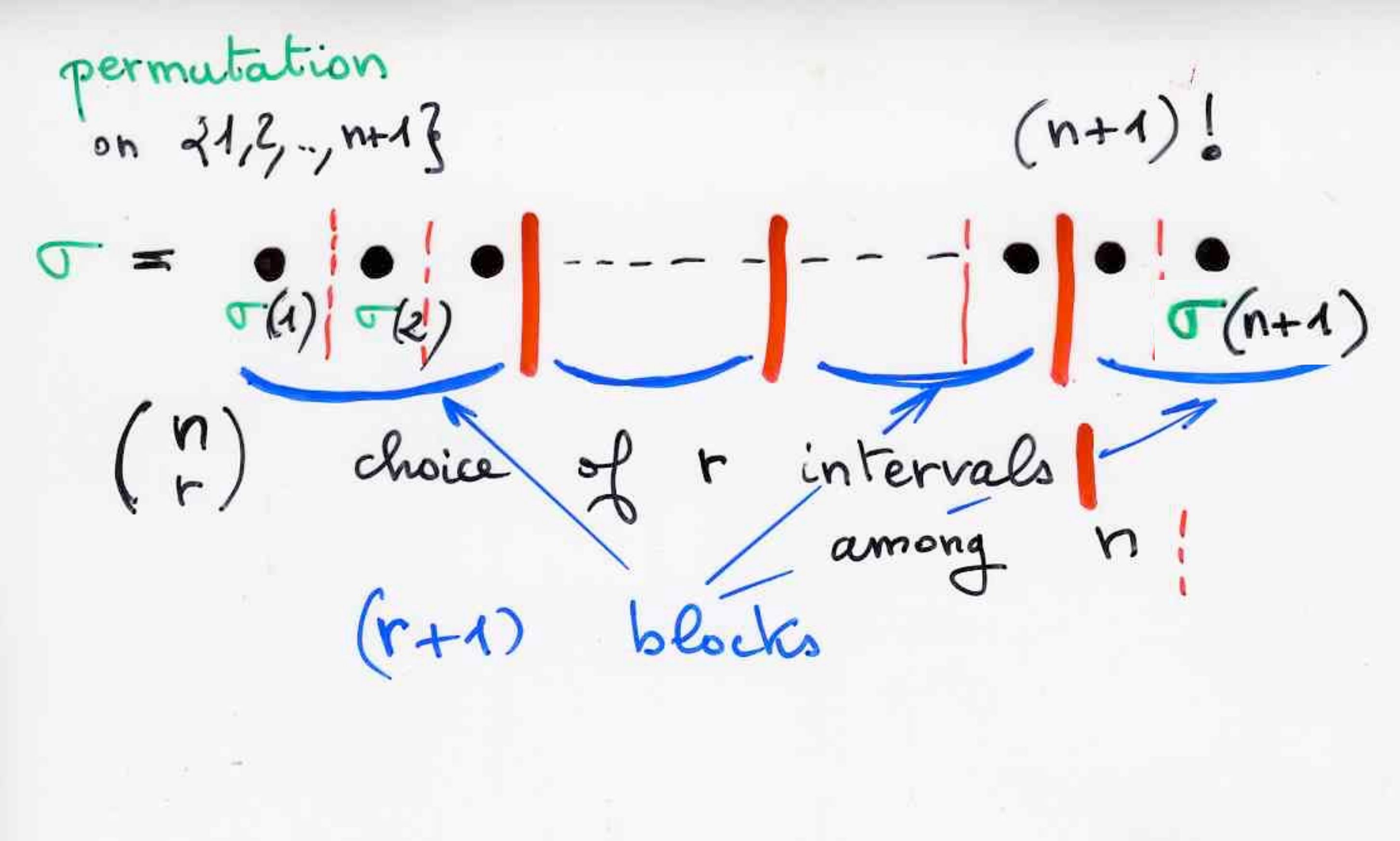
permutation

on $\{1, 2, \ldots, n+1\}$

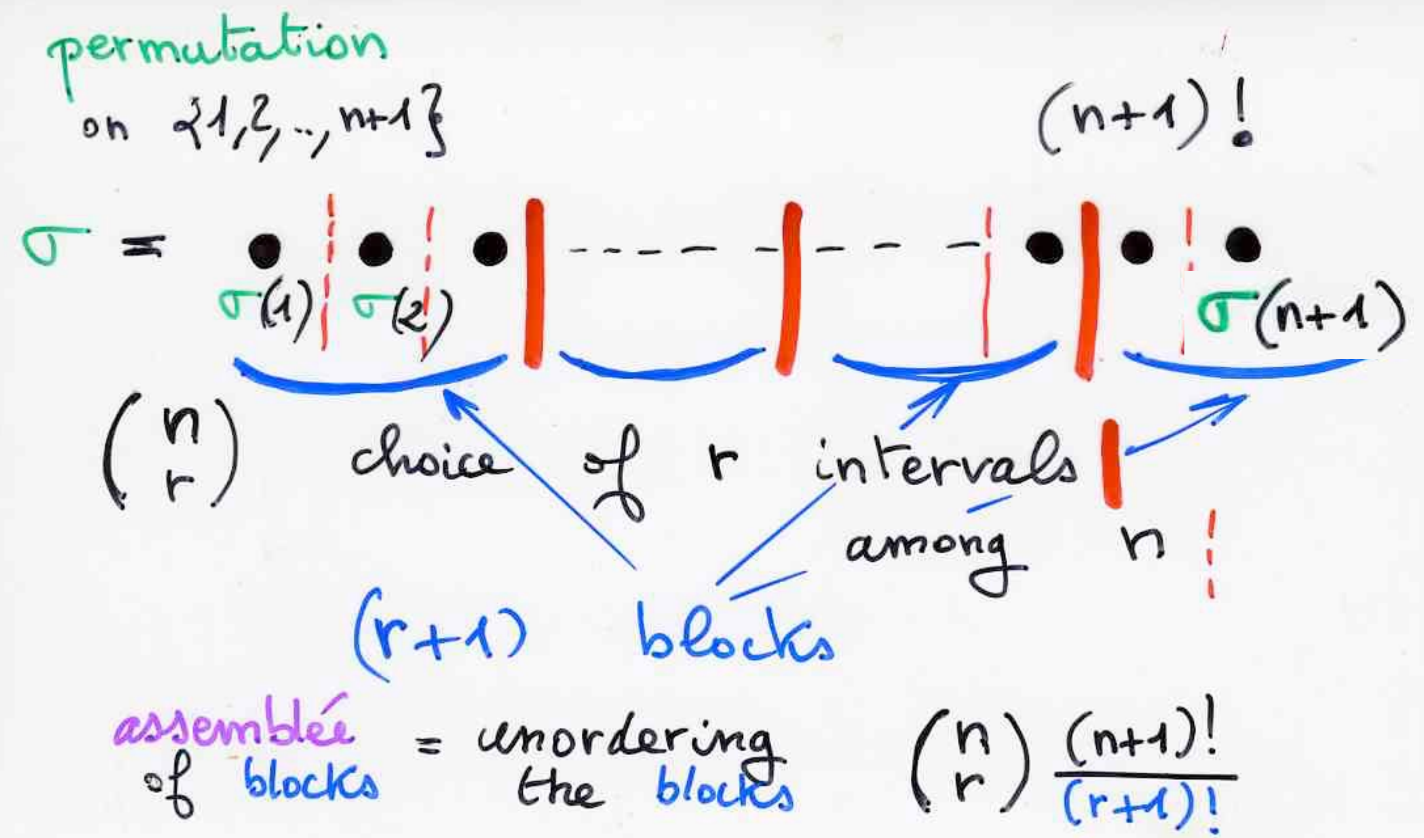
$(n+1)!$



permutation
on {1,2,..,n+1}
(n+1)!
σ =
σ(1)
σ(2)
σ(n+1)
(n r) choice of r intervals
among n



permutation
on {1,2,..,n+1}
(n+1)!
σ =
σ(1)
σ(2)
σ(n+1)
$\binom{n}{r}$
choice of r intervals among n
(r+1) blocks



$$\binom{n}{r}$$

$$\binom{n}{r}\frac{(n+1)!}{(r+1)!}$$

from «assemblées» of permutations
to
rhombic alternative tableaux

$$\left\{ [2, 10, 12, 7] \quad [3, 11, 4] \quad [5, 9, 1, 8, 6] \right\}$$

"assemblée" of permutations

from an "assemblée" of permutations

→ σ permutation

concatenation of the blocks

such that their last elements

go decreasing

2 10 12 7 5 9 1 8 6 3 11 4

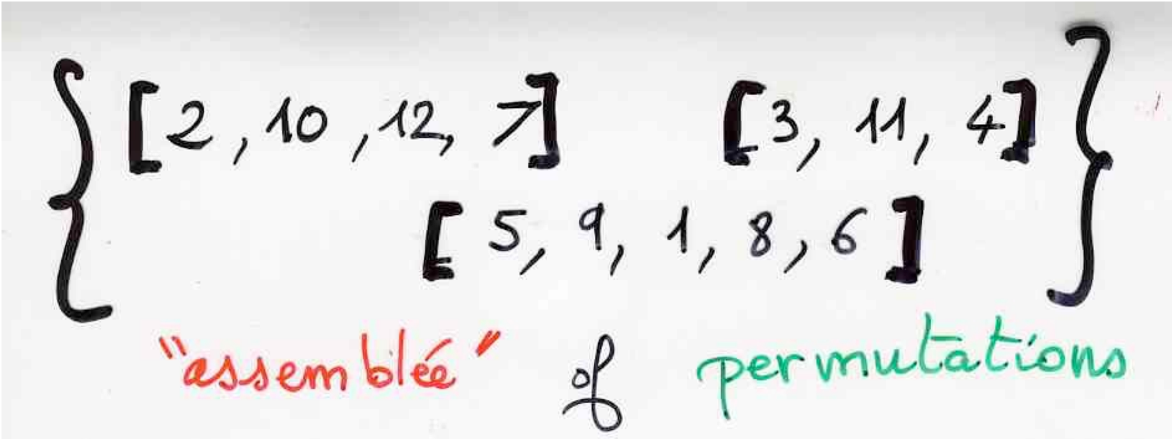
$$\left\{ [2, 10, 12, 7] \quad [3, 11, 4] \quad [5, 9, 1, 8, 6] \right\}$$

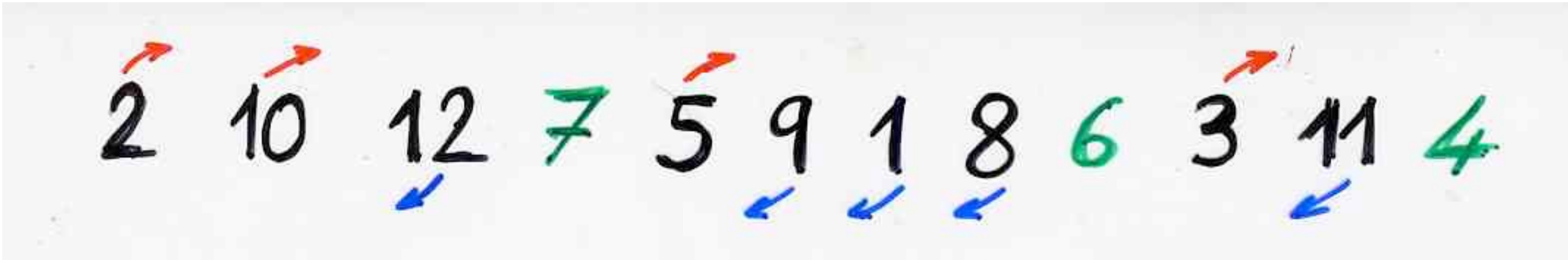
"assemblée" of permutations

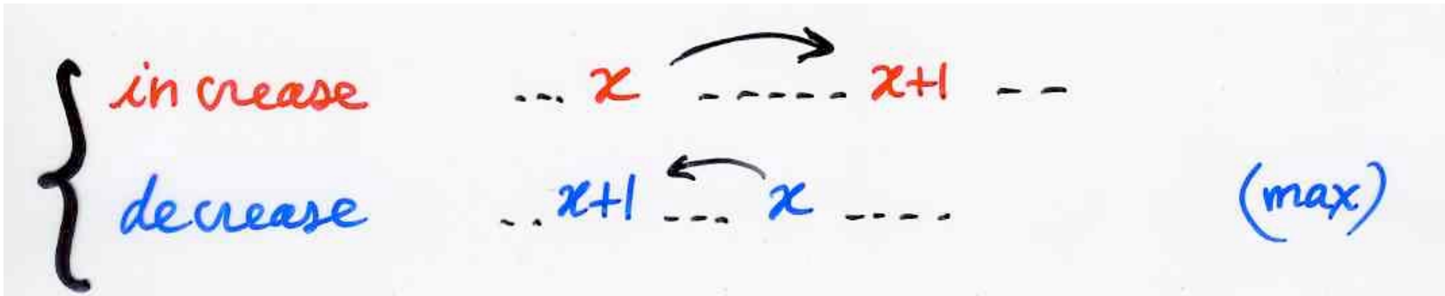
2 10 12 7 5 9 1 8 6 3 11 4

increase ... x ------ $x+1$ --

decrease .. $x+1$ --- x .... (max)


[2, 10, 12, 7]   [3, 11, 4]
[5, 9, 1, 8, 6]
"assemblée" of permutations


2 10 12 7 5 9 1 8 6 3 11 4


increase ... x ------ x+1 --
decrease ..x+1 --- x ----
(max)

$$\left\{ [2, 10, 12, 7] \quad [3, 11, 4] \quad [5, 9, 1, 8, 6] \right\}$$

"assemblée" of permutations

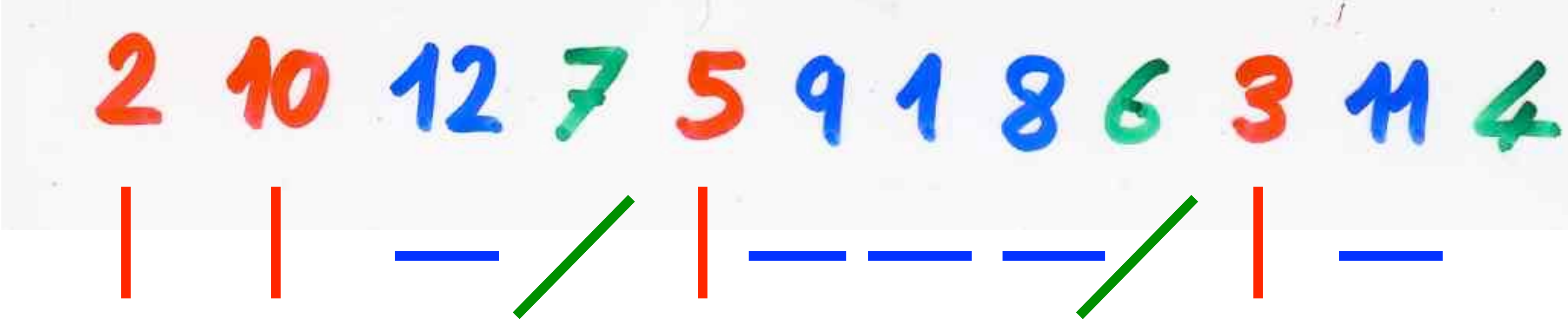
2 10 12 7 5 9 1 8 6 3 11 4

increase ... x ------ $x+1$ --

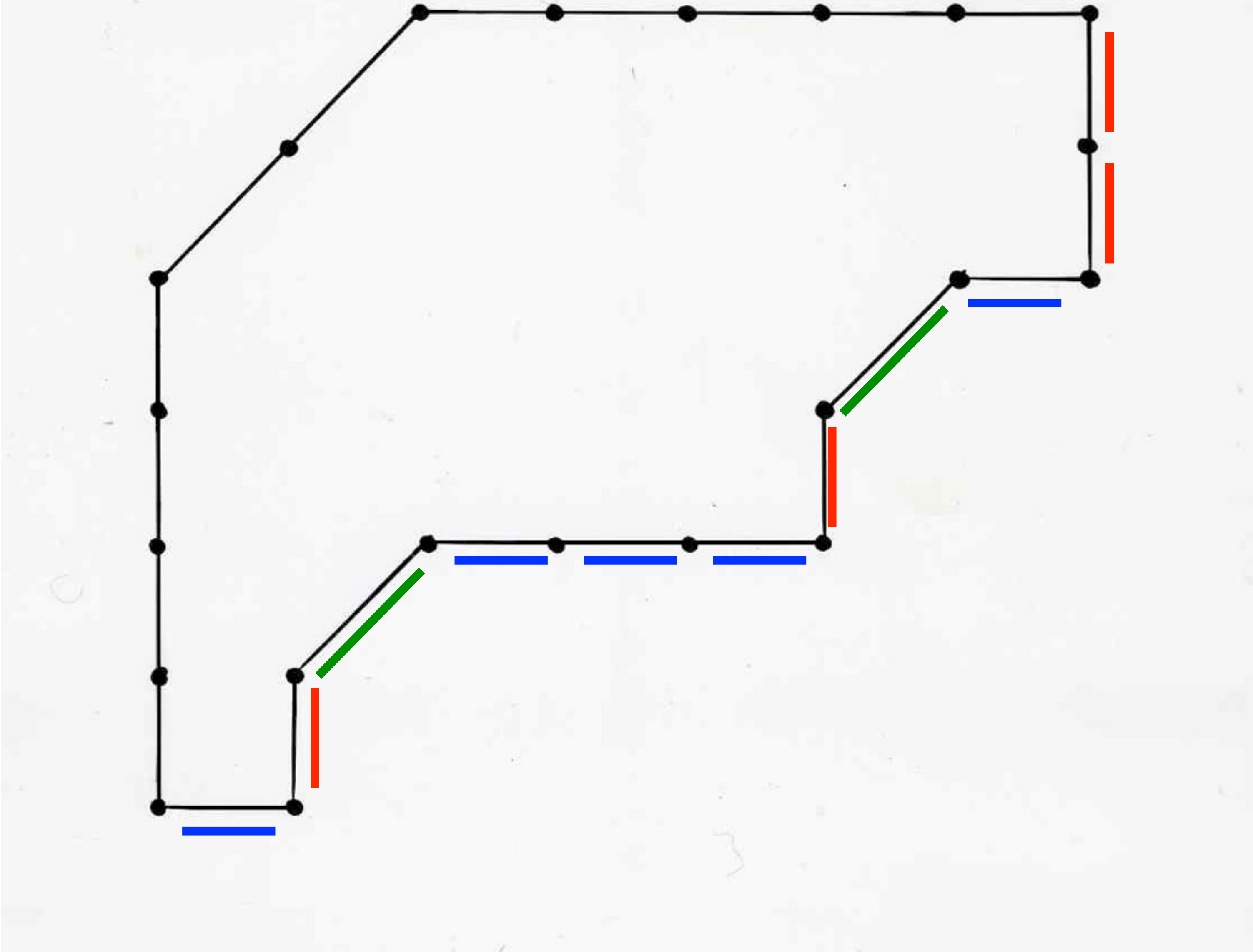
decrease .. $x+1$ --- x ---- (max)

for an assembleé $\alpha \rightarrow$ permutation σ

$\alpha \rightarrow \sigma \rightarrow X$ word $\{\bullet, \bullet, \bullet\} \rightarrow \Gamma(X)$ diagram



2 10 12 7 5 9 1 8 6 3 11 4



exchange-fusion algorithm

Def- A **label** (or *segment*)
is a list (possibly empty) of consecutive integers

for two disjoint nonempty labels A and B,
B covers A (denoted $B \succ A$)
if for the smallest $i \in B$ and the largest
$j \in A$, we have $i = j + 1$

ex: $B = (6, 7, 8)$ $A = (4, 5)$, with $i = 6$ and $j = 5$

then $A \cup B$ is also a label

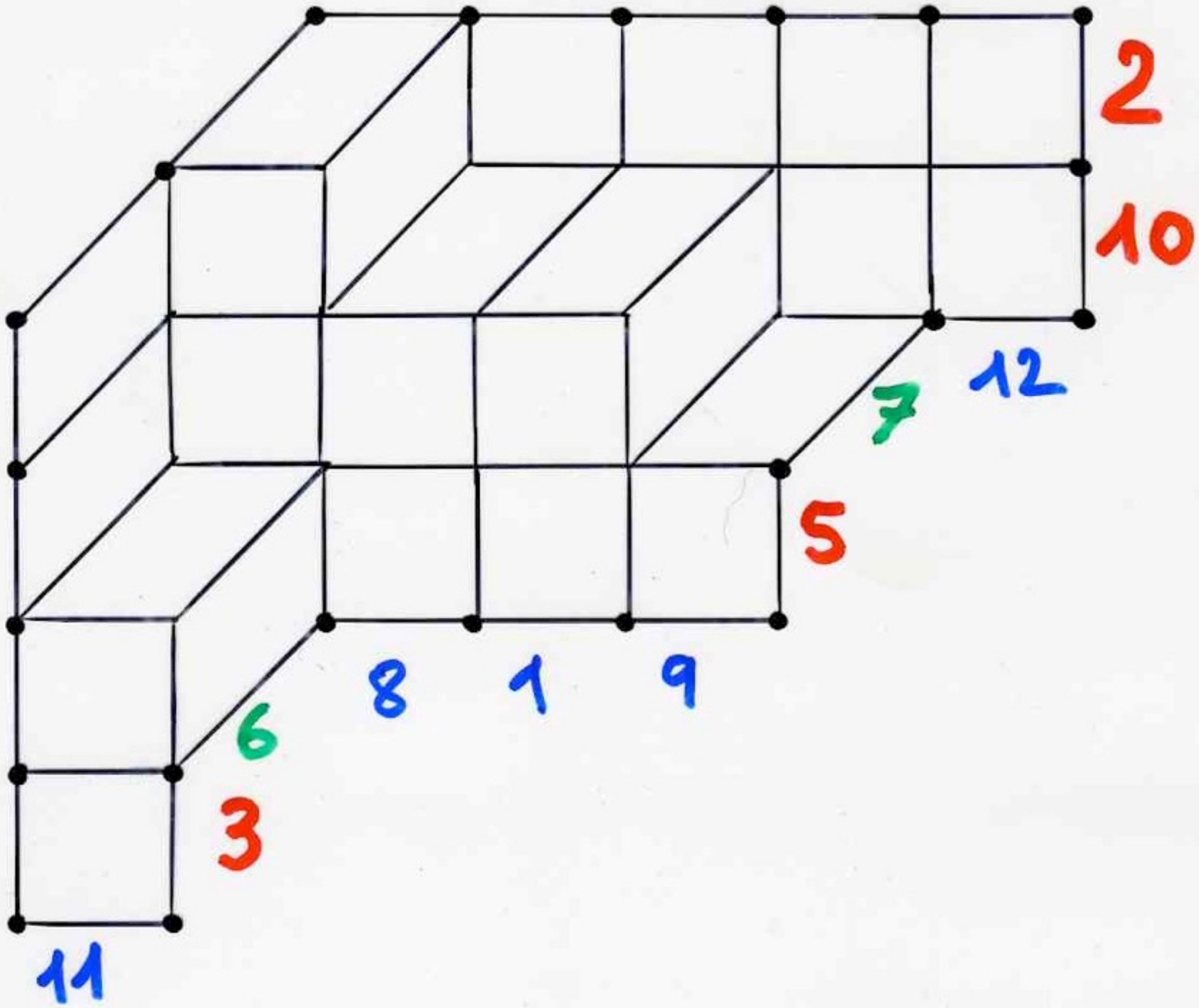
exchange-fusion algorithm

Initiation

every SW edge of $\Gamma(X)$ receive a label reduced to the corresponding element of σ

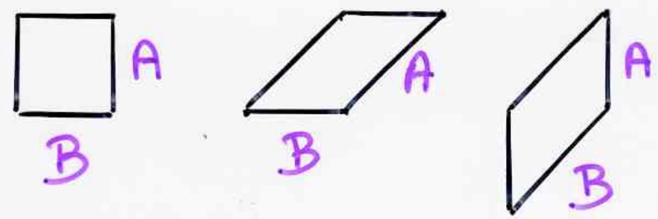
choice of a tiling $\mathcal{E}$ of $\Gamma(X)$

2 10 12 7 5 9 1 8 6 3 11 4

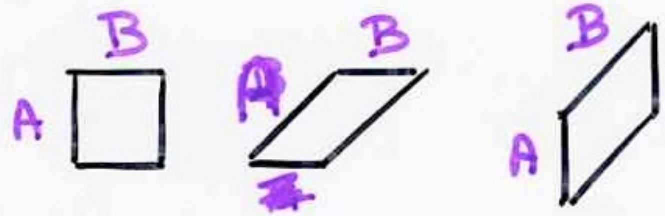


(4)

Let a tile with labels A and B

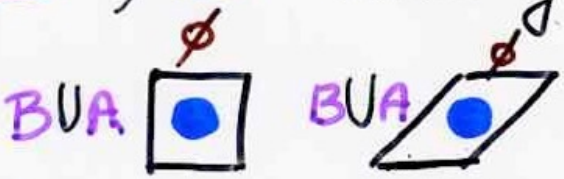


- if $A \not\succ B$ and $B \not\succ A$ then the labels "cross"



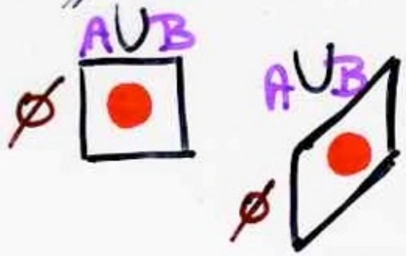
- if $A \succ B$, B labeling a horizontal edge

then



- if $B \succ A$, A labeling a vertical edge

then



- if $A \succ B$, B labeling a diagonal edge then the labels "cross"

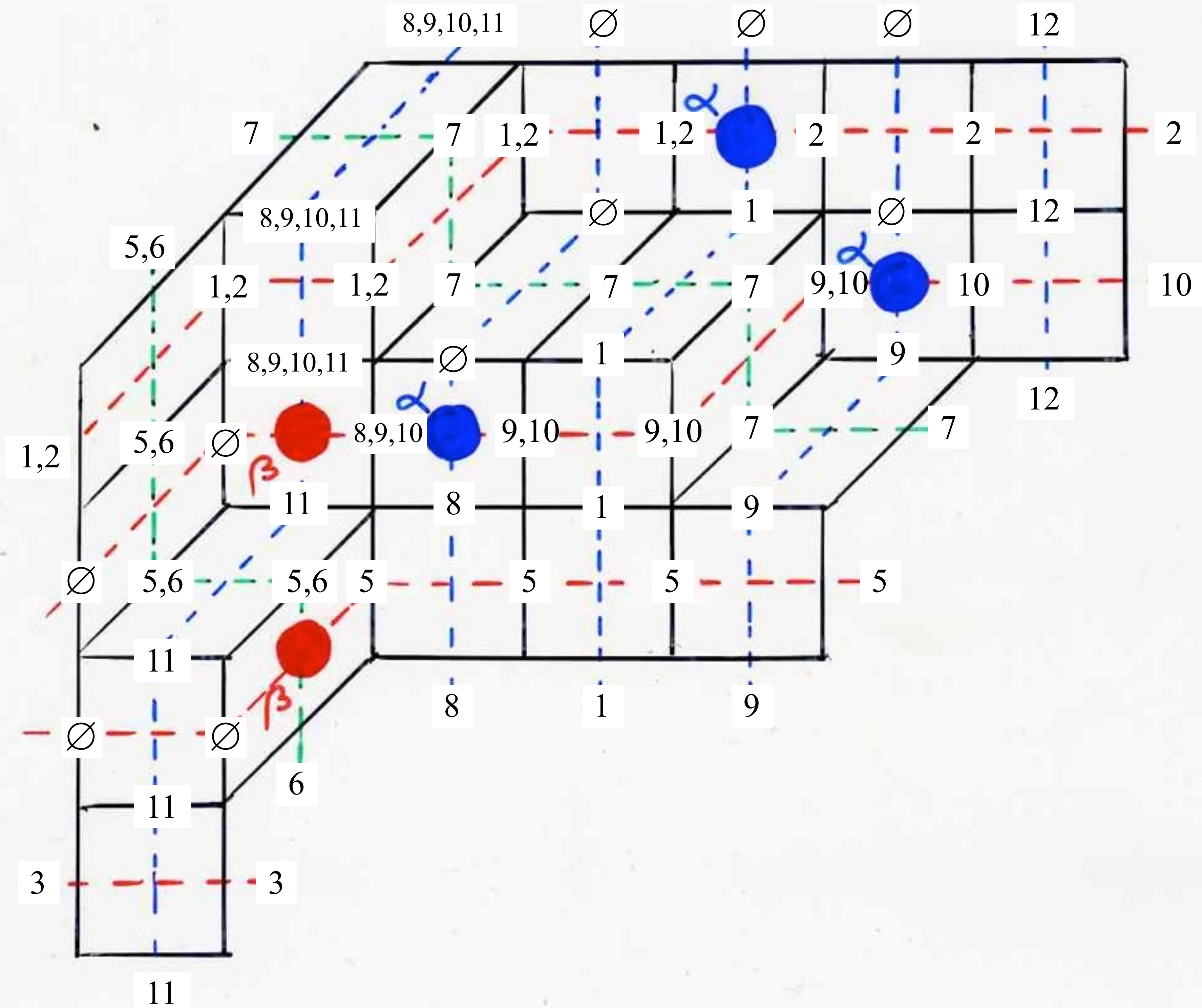
Equivalent formulation:

(i) • if $A \not> B$ and $B \not> A$, the labels "cross"

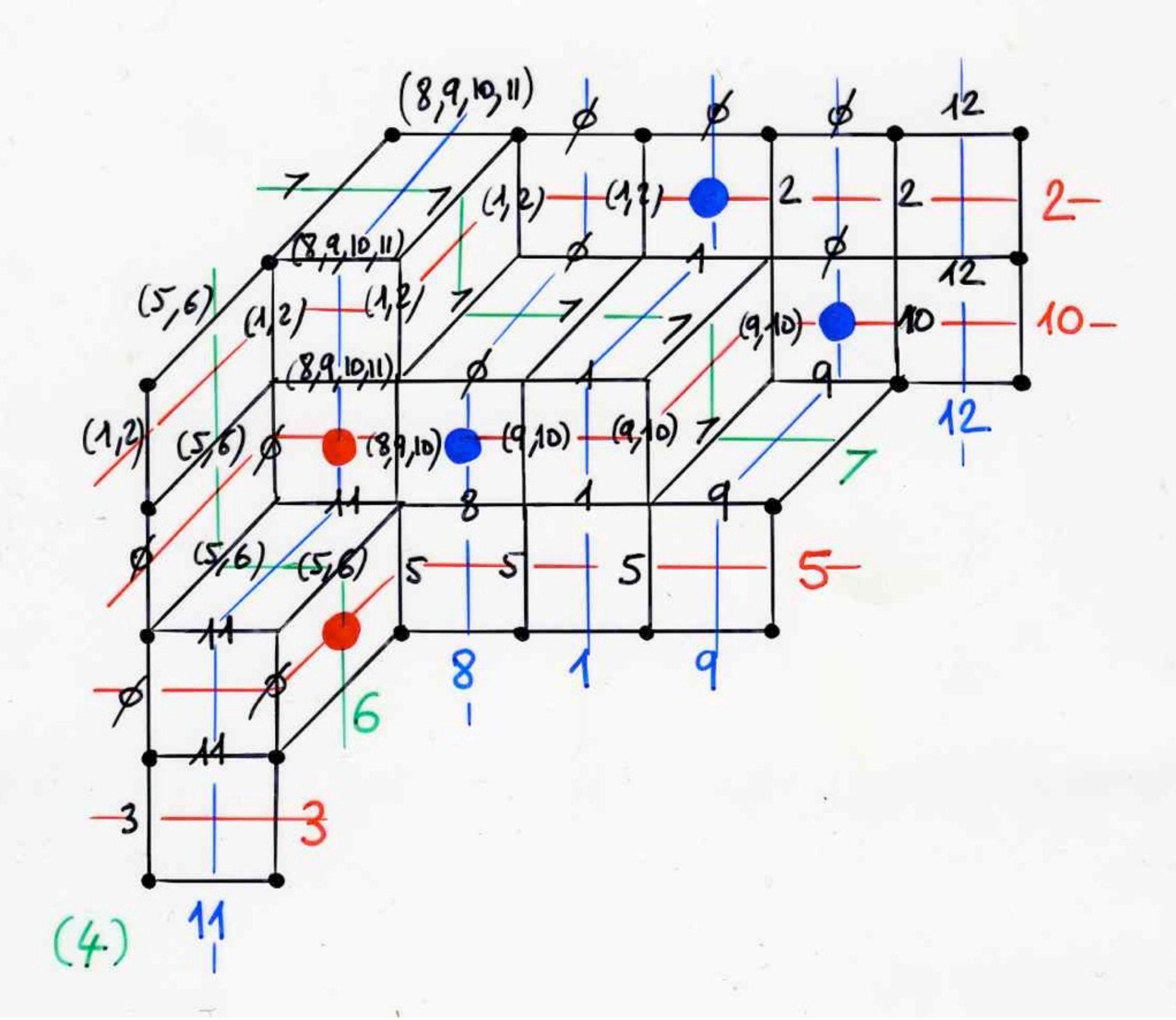
(ii) • if $A > B$ or $B > A$, $A \cup B$ is a label which follows the line of biggest label A or B

- except if the smallest label is on a green line, in that case the two labels "cross"

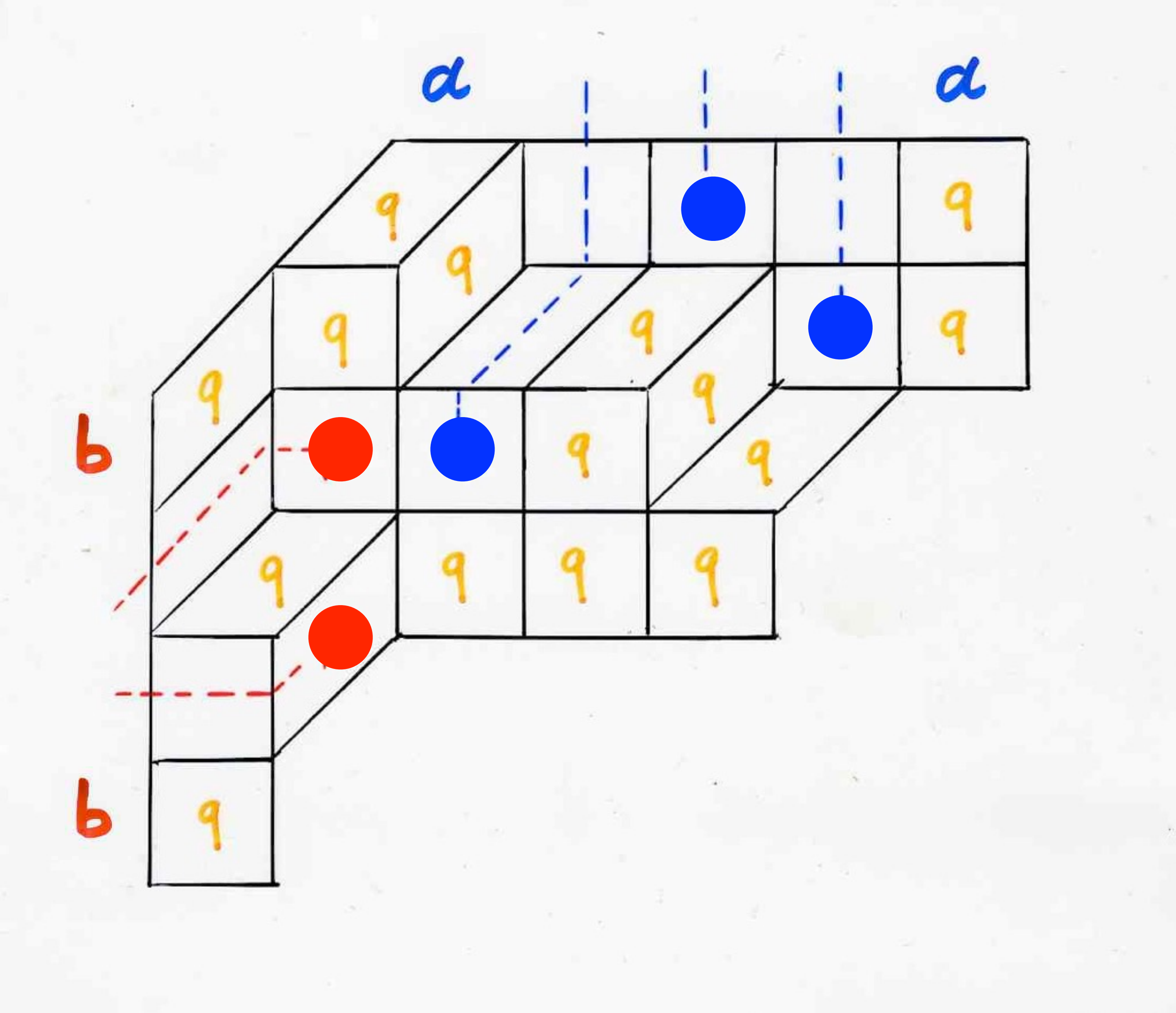
- when $A \cup B$ is a label, the other edge receives the label $\emptyset$


8,9,10,11
∅
∅
∅
12
7
7
1,2
1,2
α
2
2
2
1
∅
12
8,9,10,11
5,6
α
1,2
1,2
7
7
7
9,10
10
10
8,9,10,11
∅
1
9
1,2
5,6
∅
8,9,10
α
9,10
9,10
7
7
12
β
11
8
1
9
∅
5,6
5,6
5
5
5
5
11
β
8
1
9
∅
∅
6
11
3
3
11

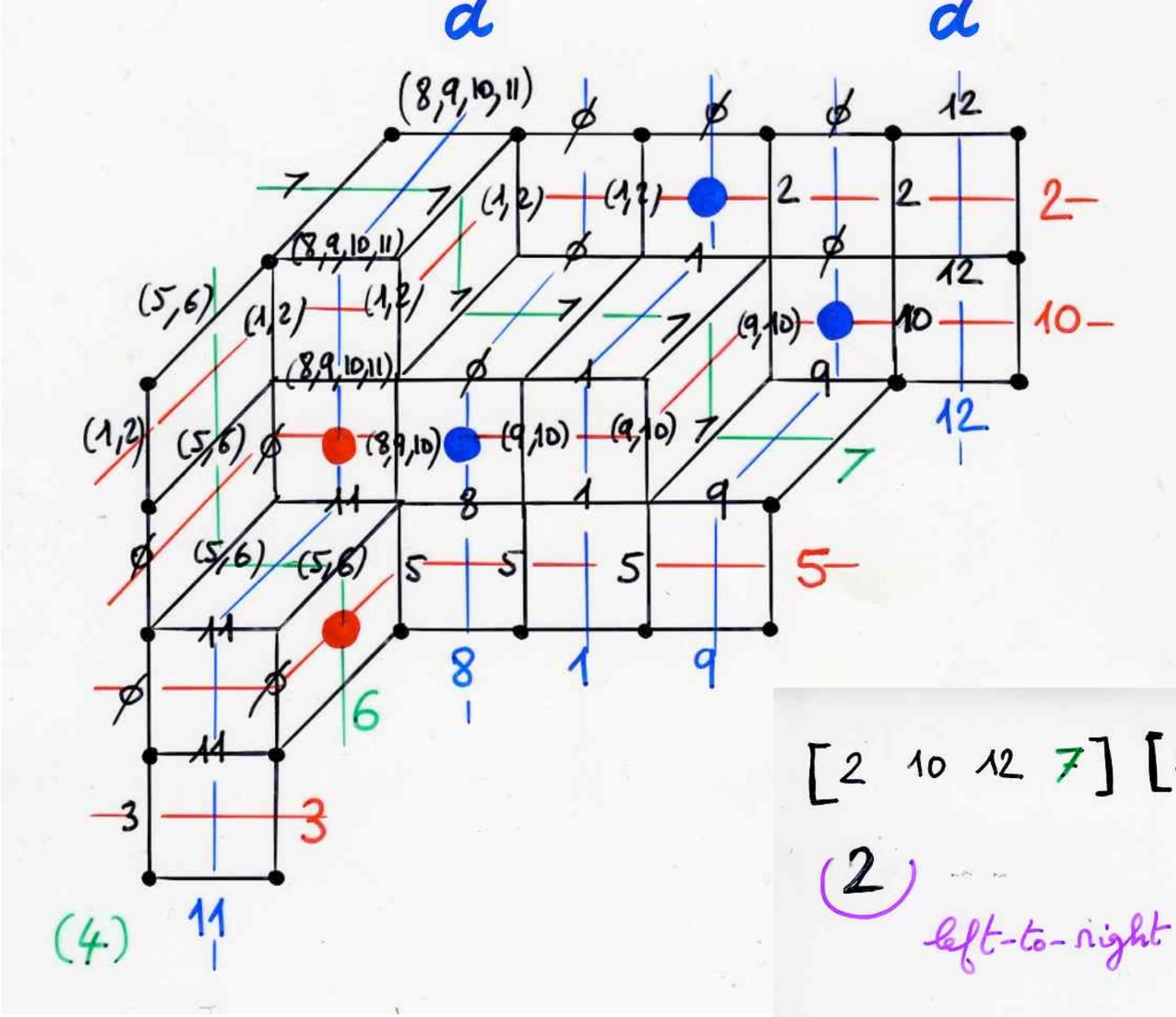
(4)



$$wt(T) = q^{15} a^{2} b^{2}$$



$wt(T) = q^{15} a^2 b^2$



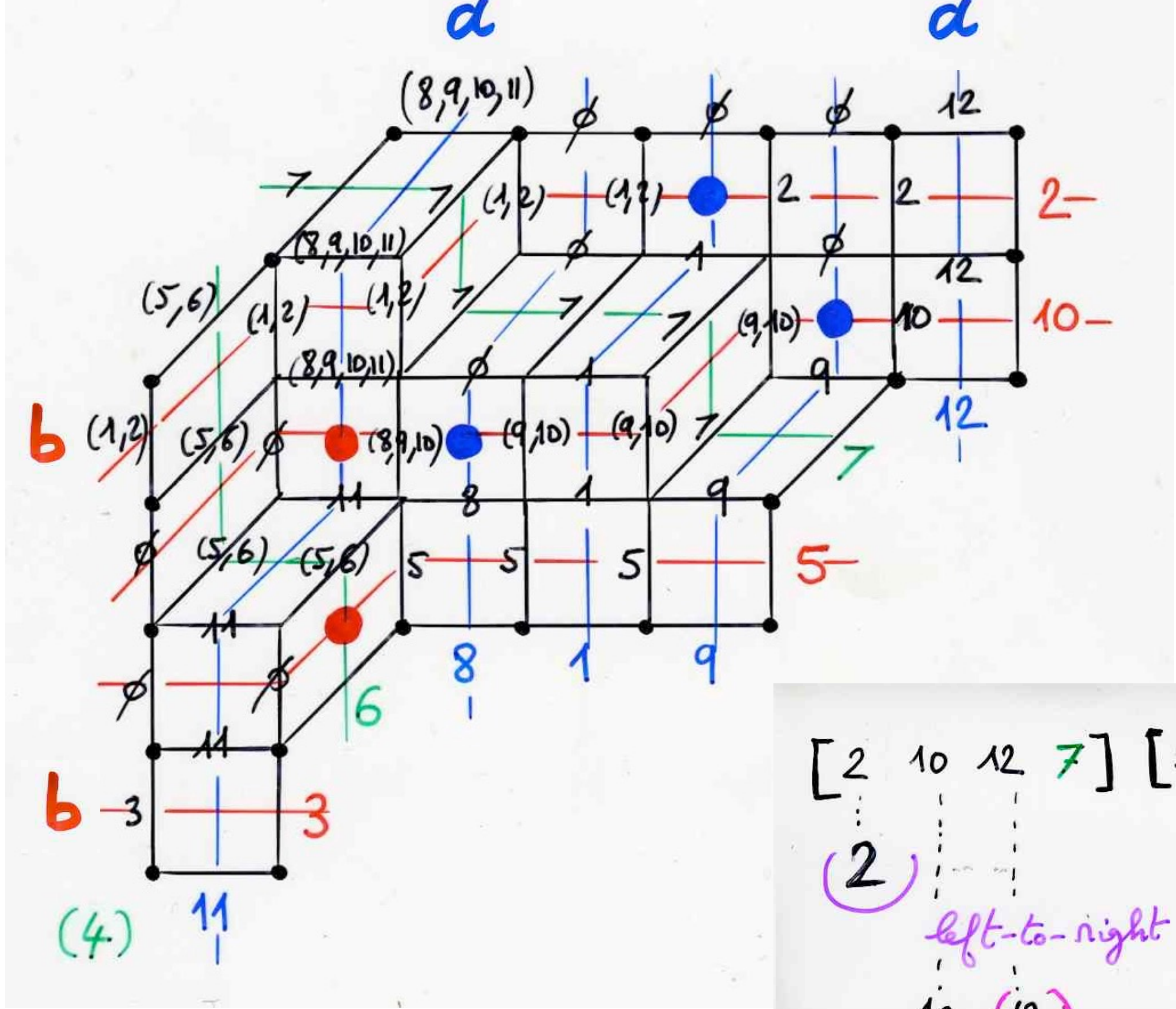
[2 10 12 7] [5 9 1 8 6] [3 11 4]

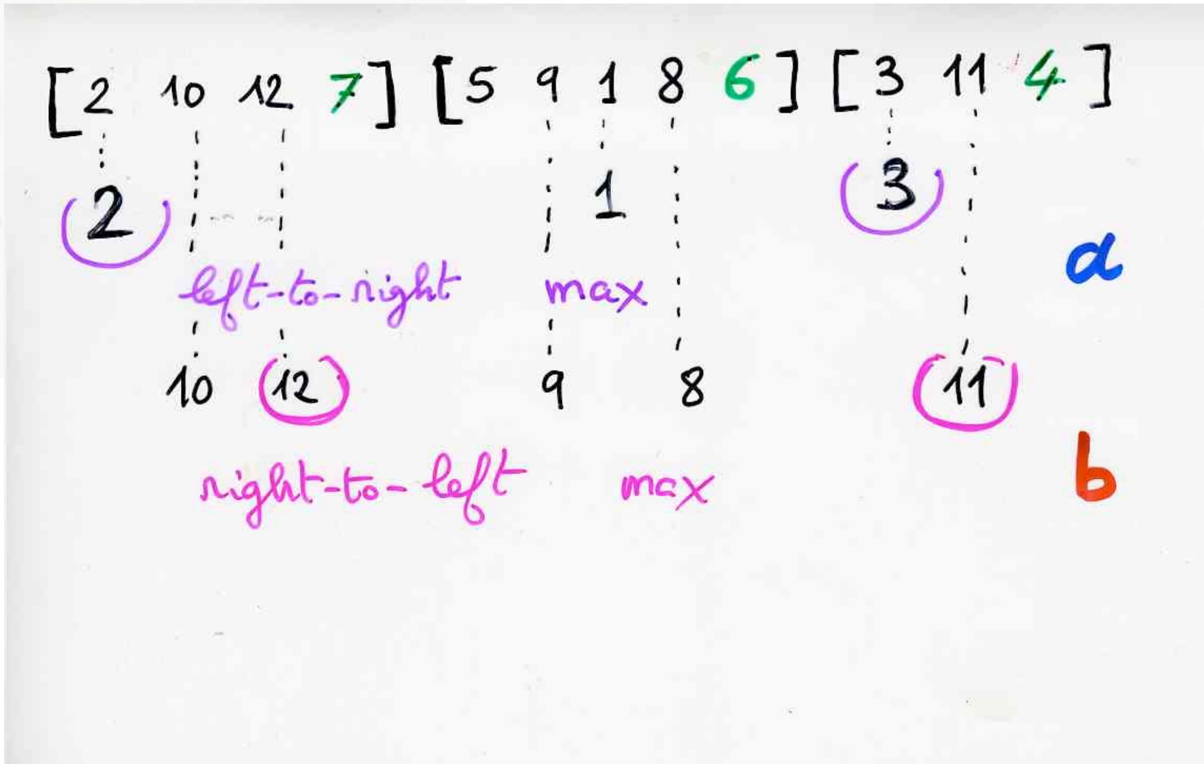
(2) 1 (3)

left-to-right max

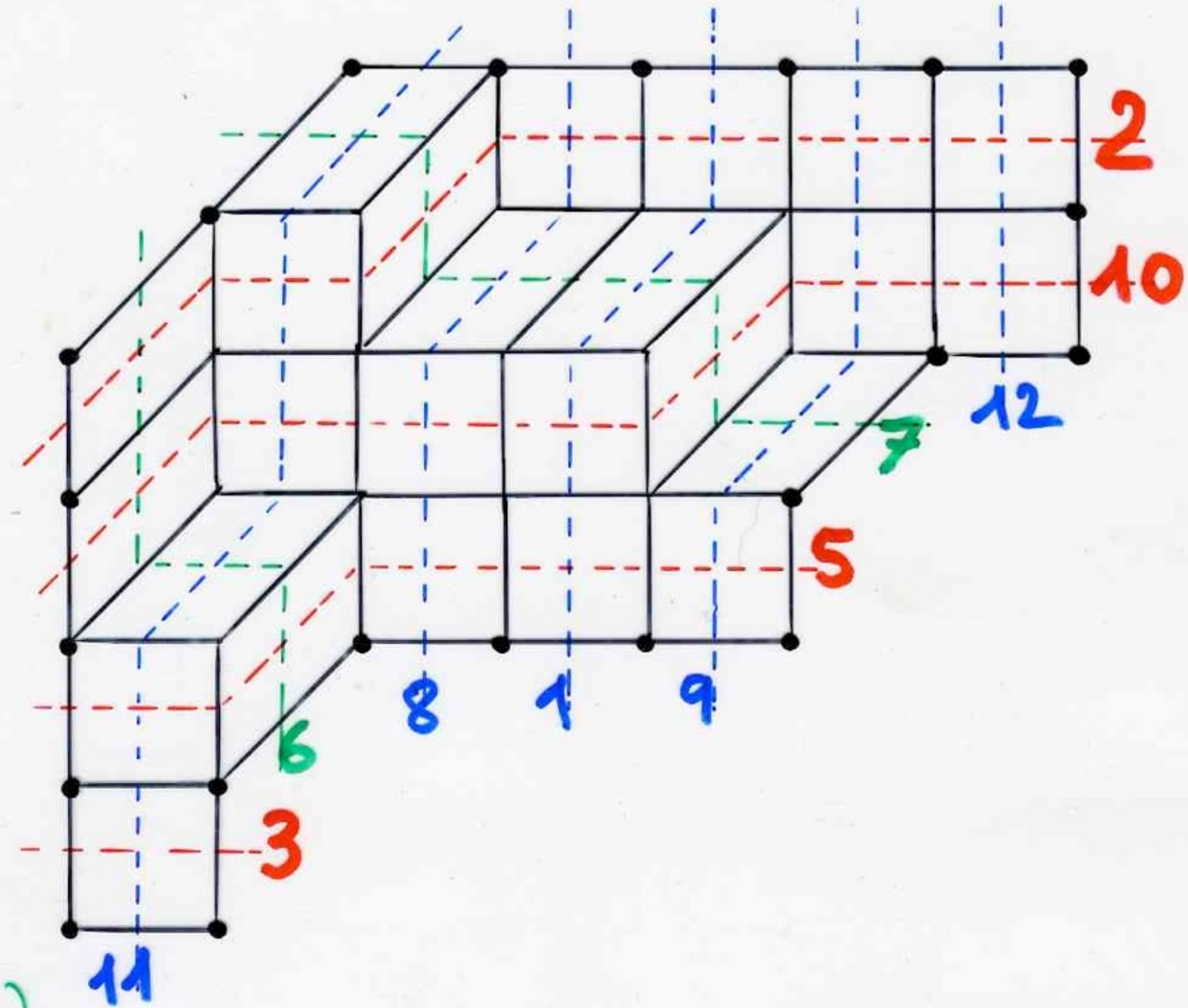
d

$$wt(T) = q^{15} a^{2} b^{2}$$

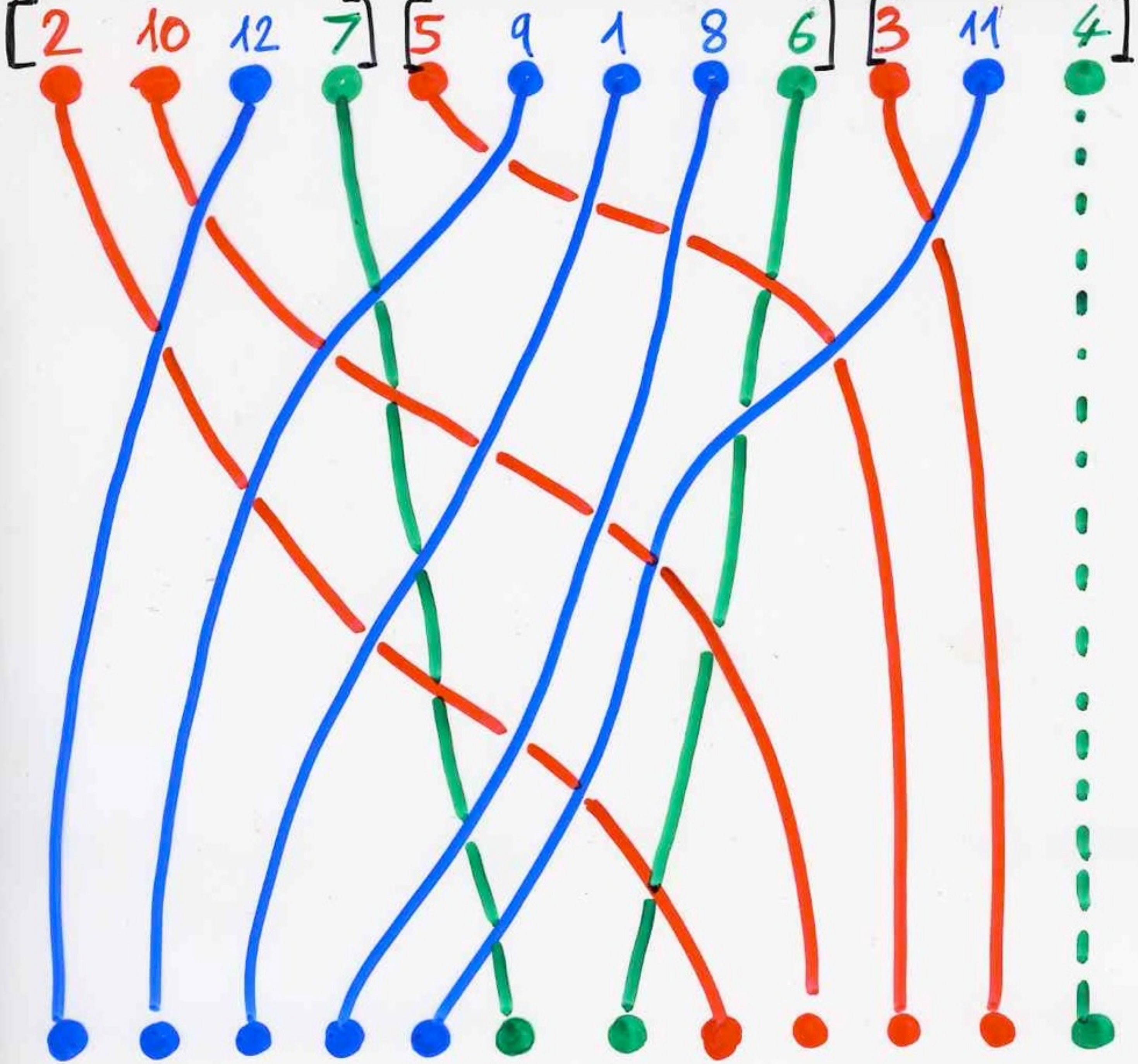

a
a
(8,9,10,11)
12
2
10
12
b
5
8
1
9
7
6
b
3
3
(4)
11

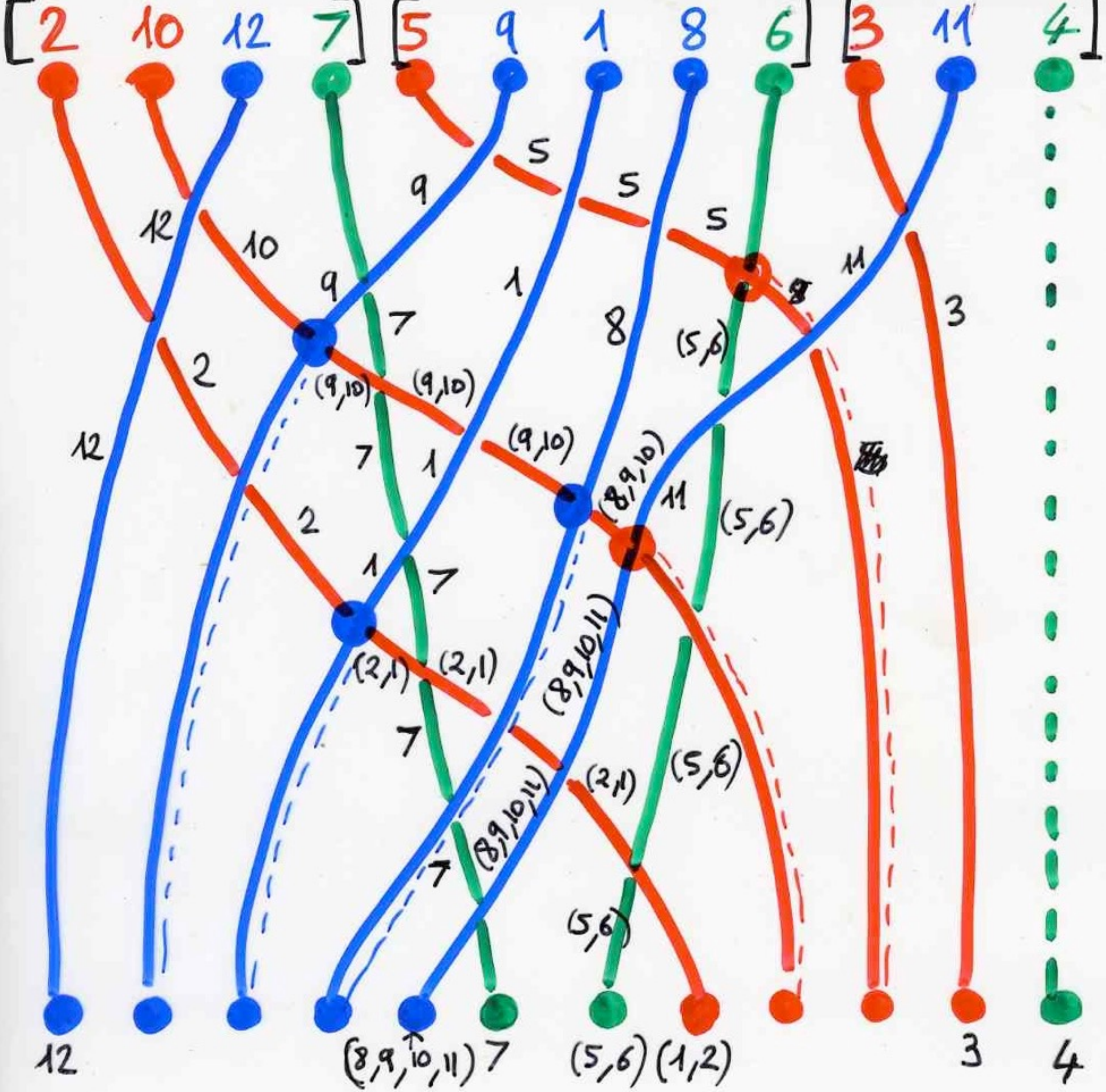

[2 10 12 7] [5 9 1 8 6] [3 11 4]
2
1
3
a
left-to-right max
10 12 9 8 11
b
right-to-left max

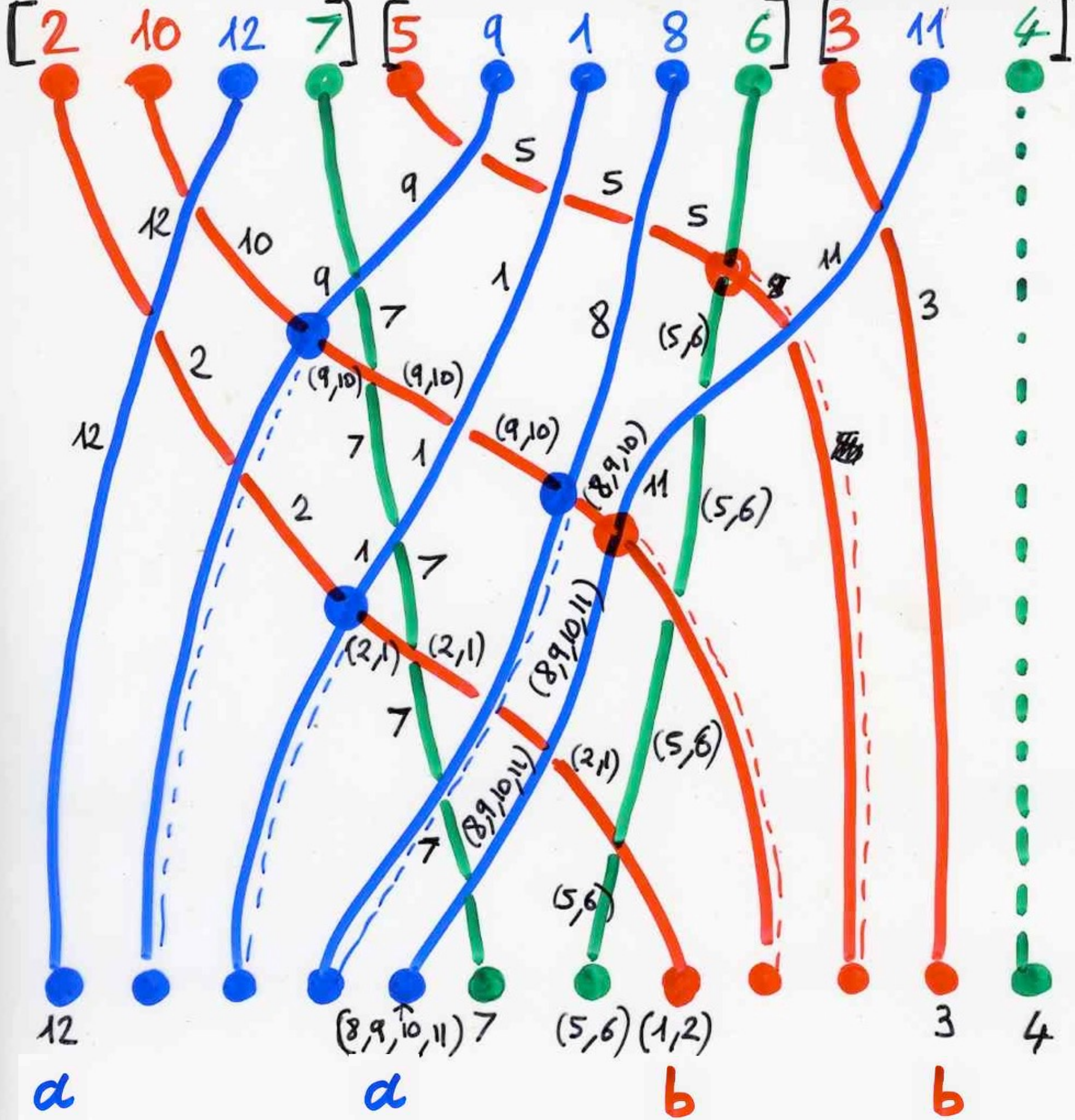
$$wt(T) = q^{15} a^{2} b^{2}$$



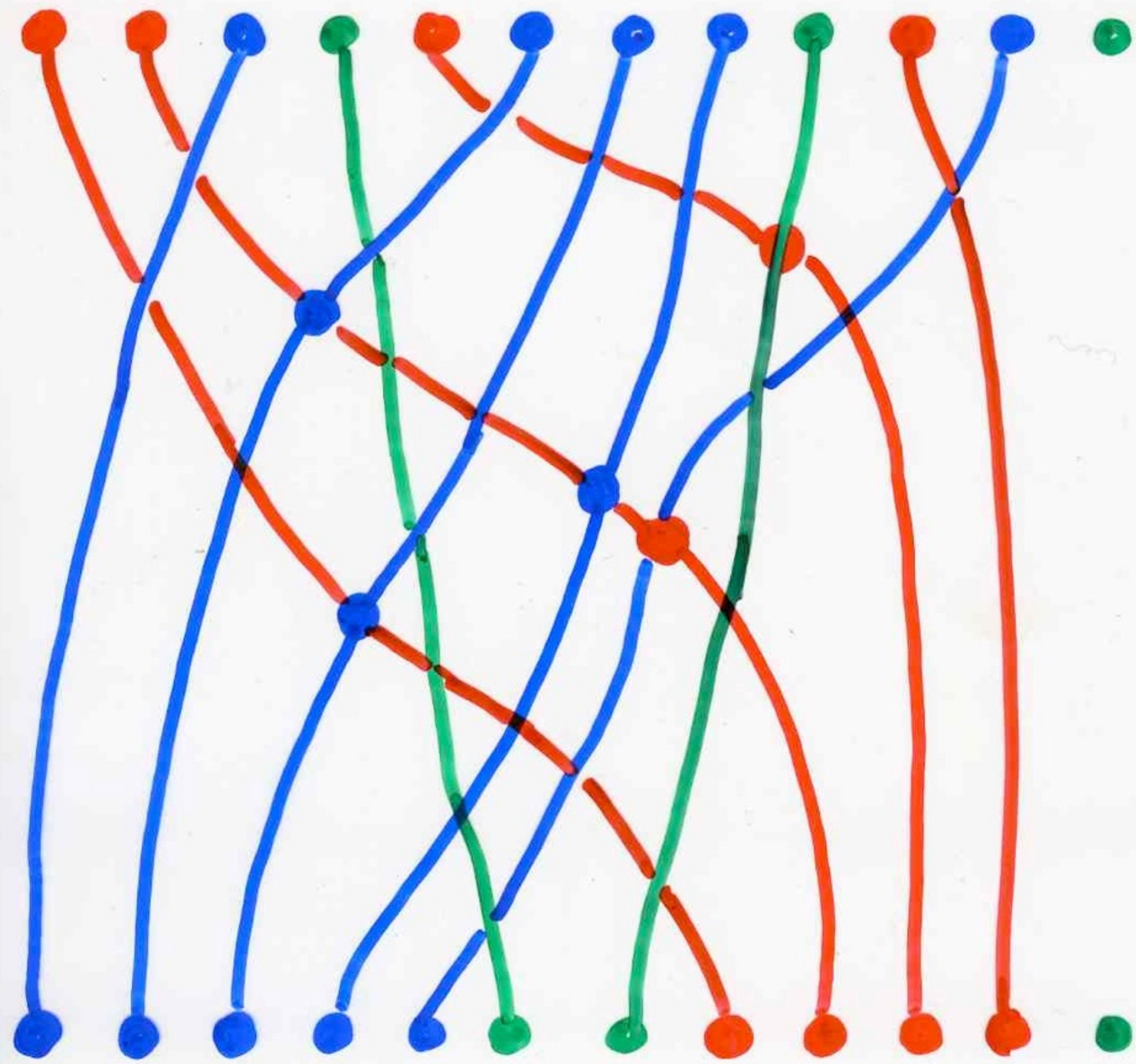
(4)

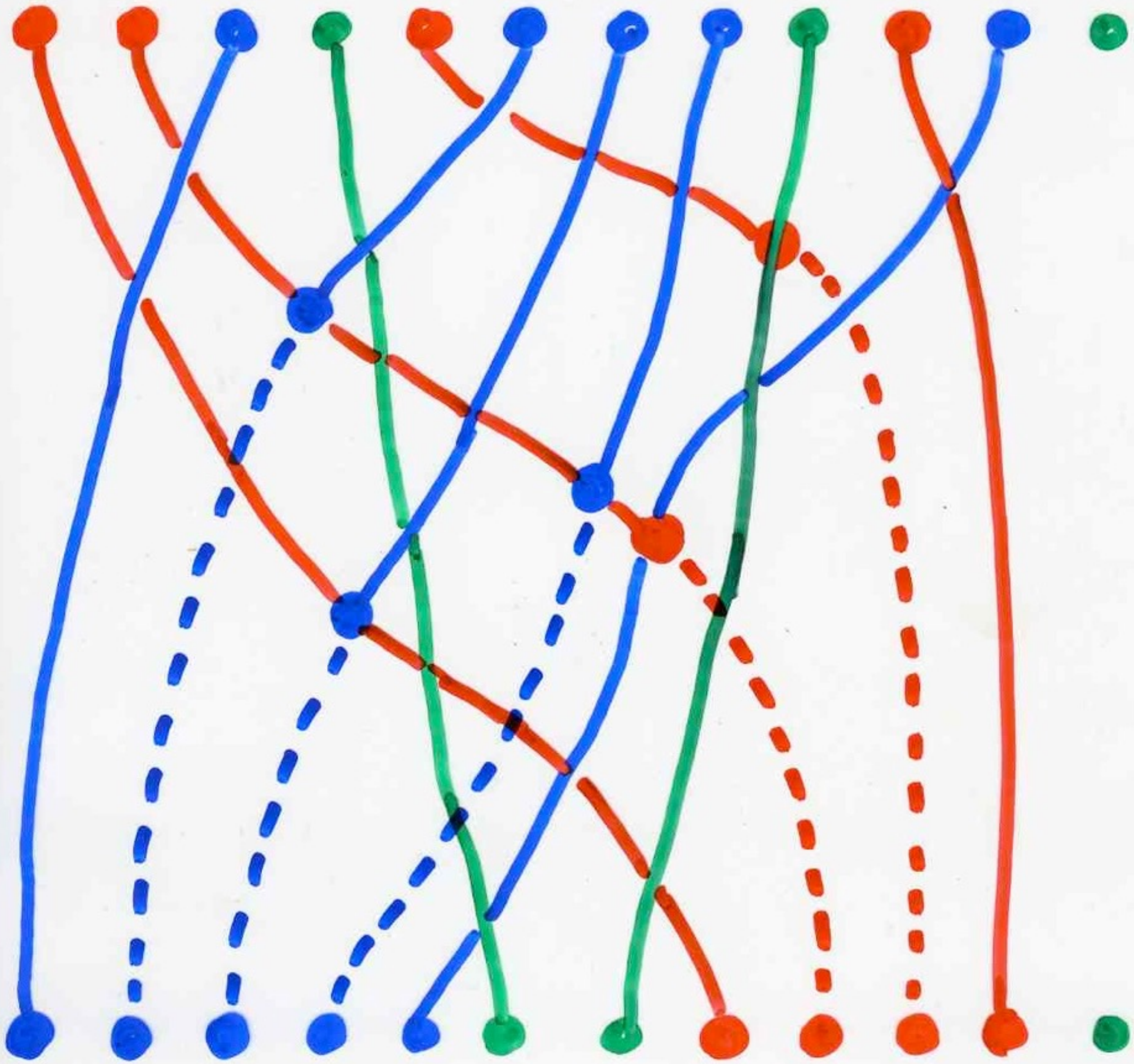

[2 10 12 7] [5 9 1 8 6] [3 11 4]

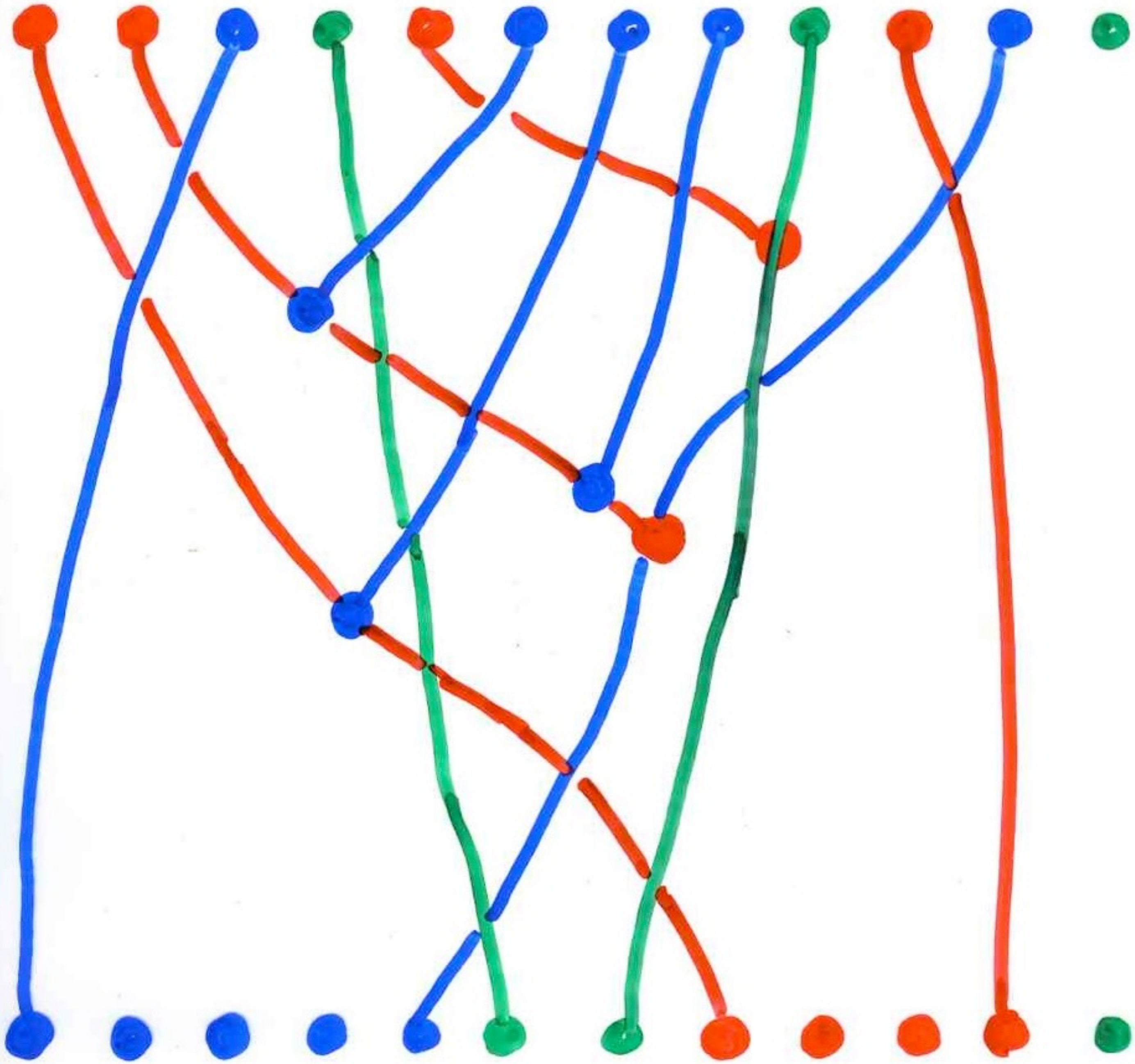

[2 10 12 7] [5 9 1 8 6] [3 11 4]
12 (8,9,10,11) 7 (5,6) (1,2) 3 4

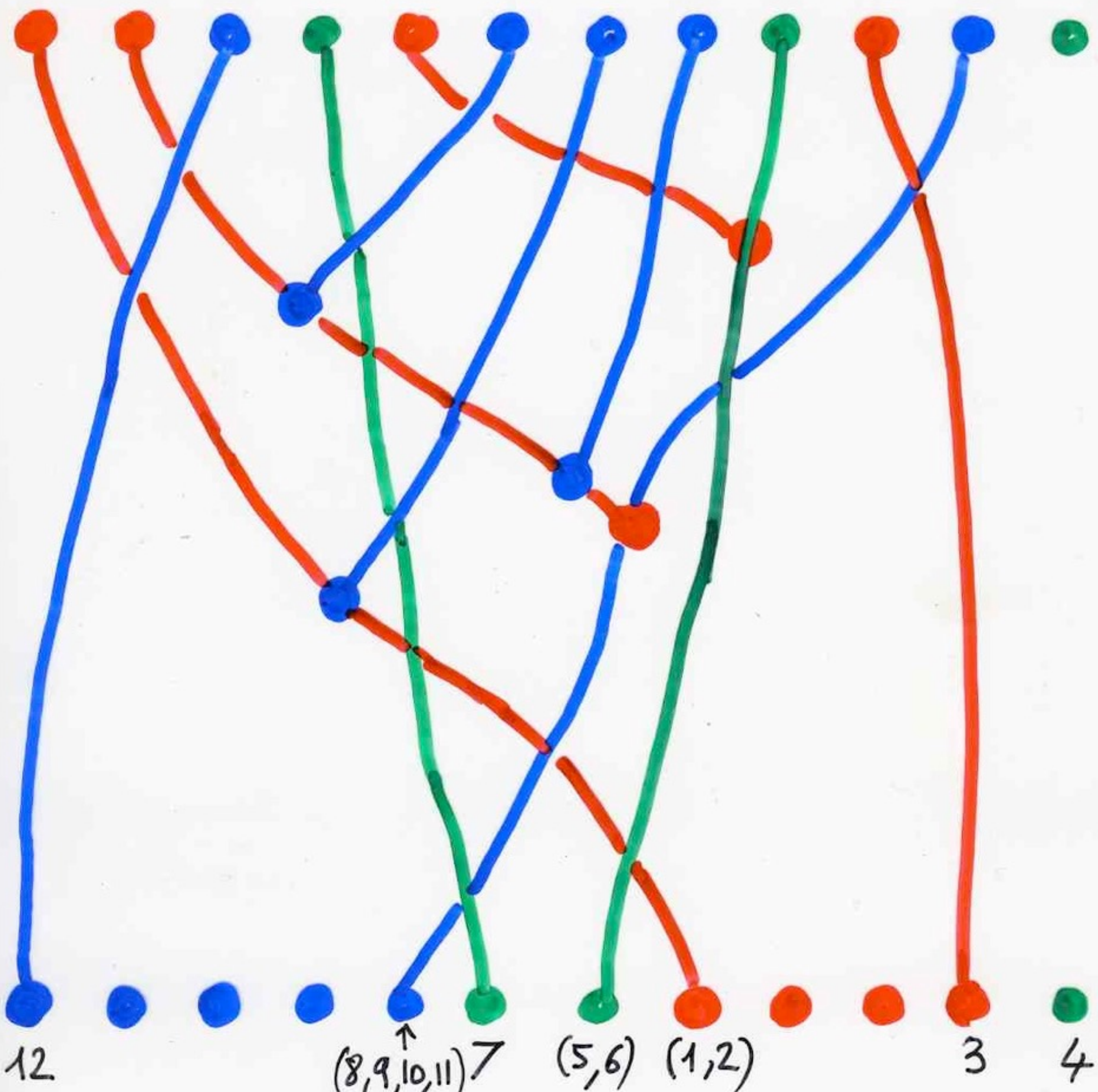

[2 10 12 7] [5 9 1 8 6] [3 11 4]
12
(8,9,10,11) 7
(5,6) (1,2)
3
4
a
a
b
b

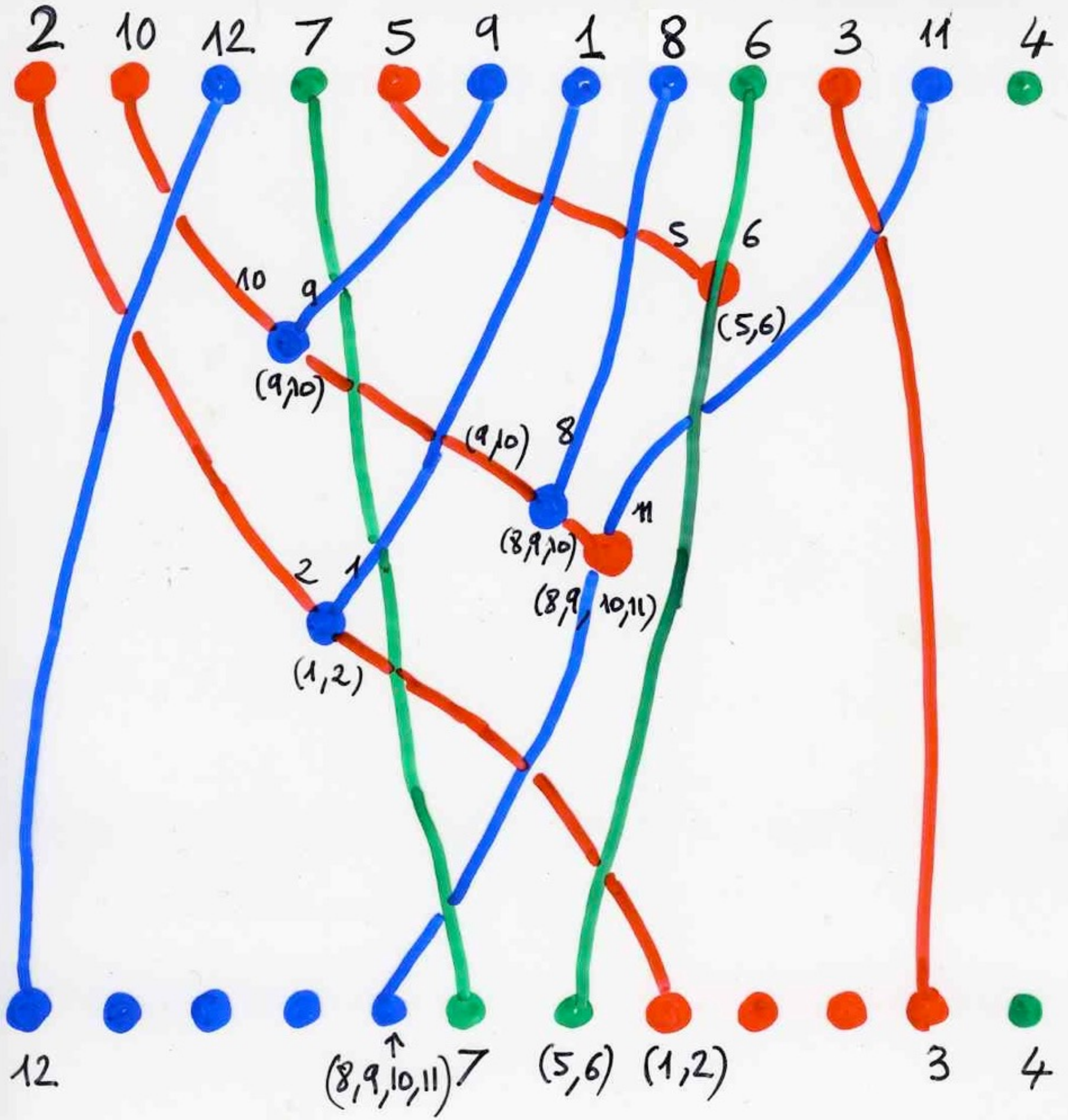
from rhombic alternative tableaux
to
assemblée of permutations



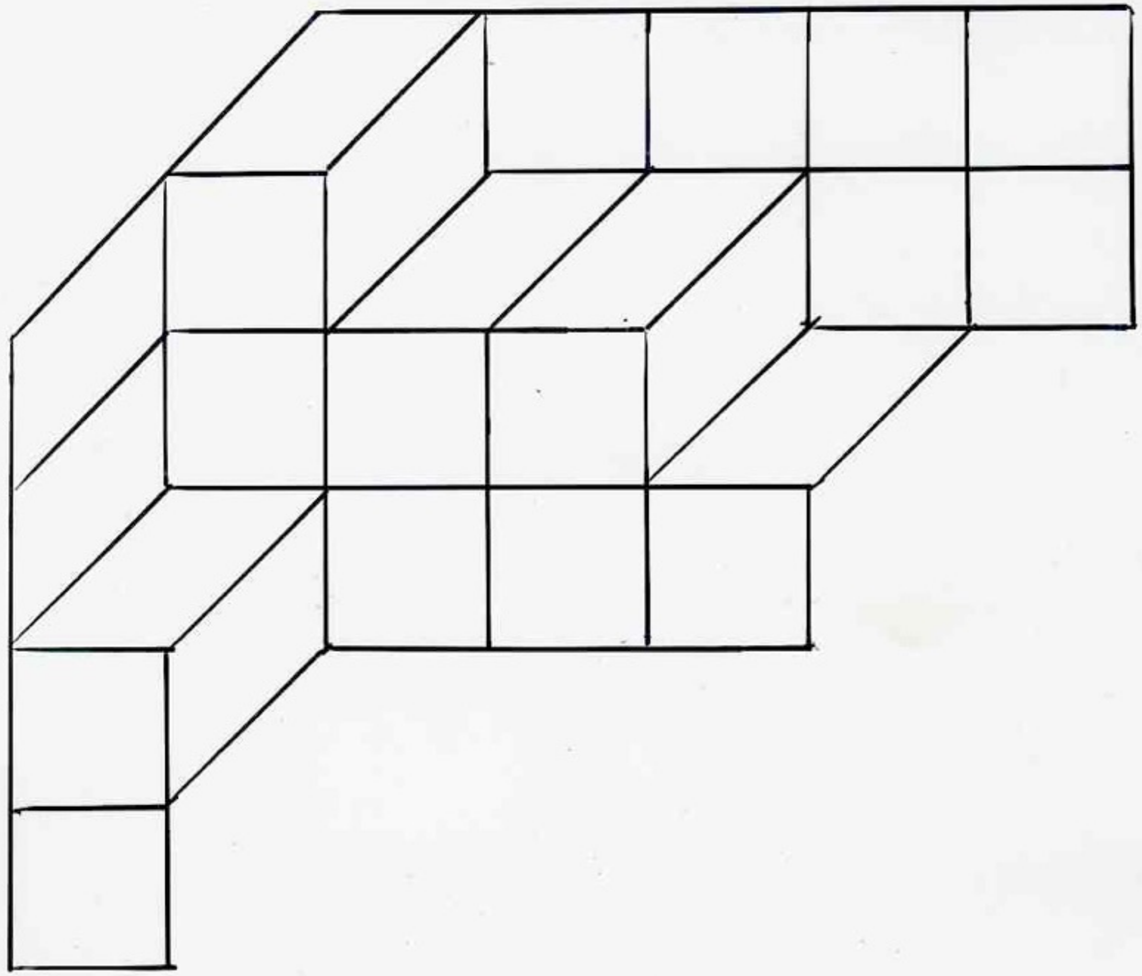


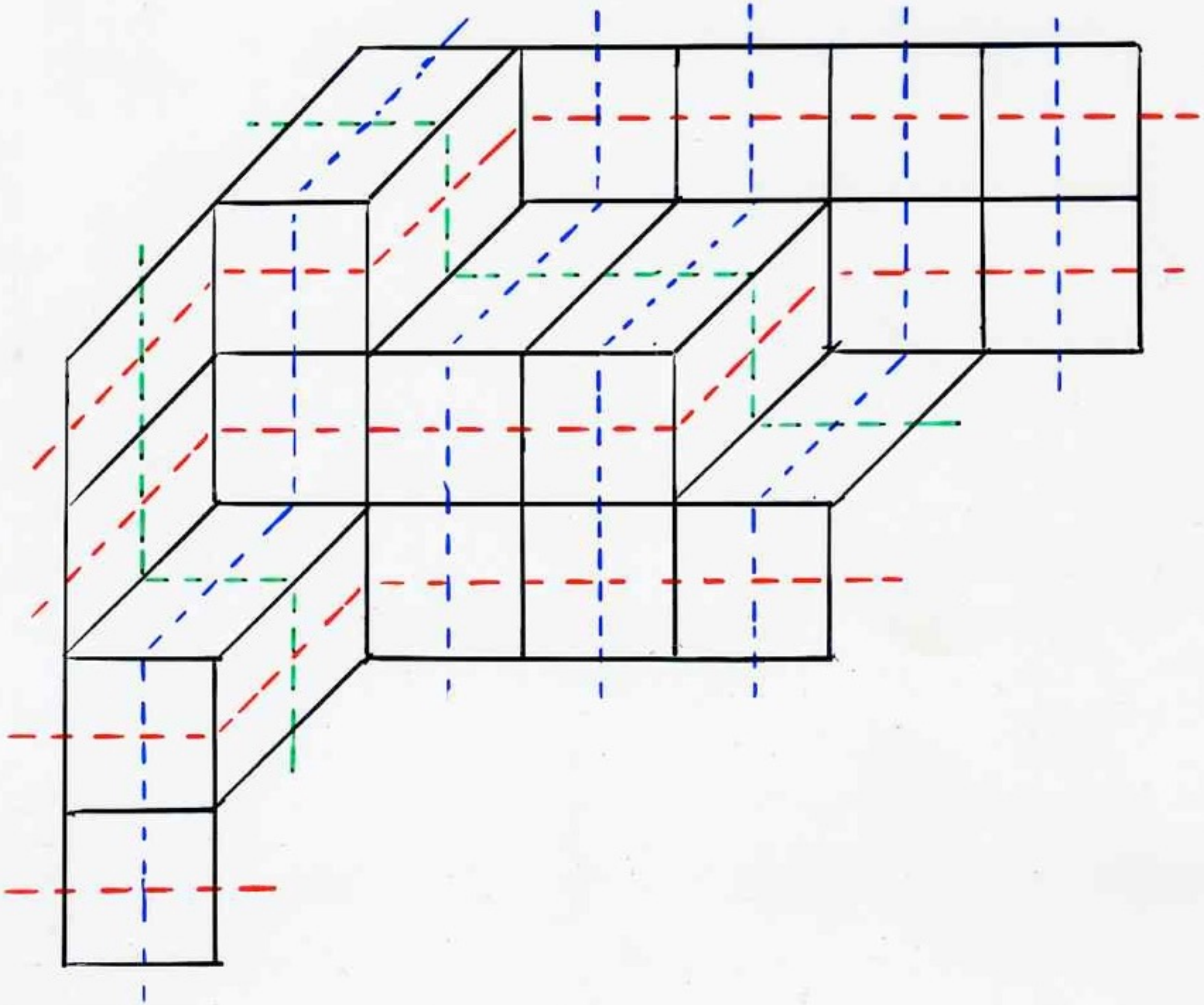


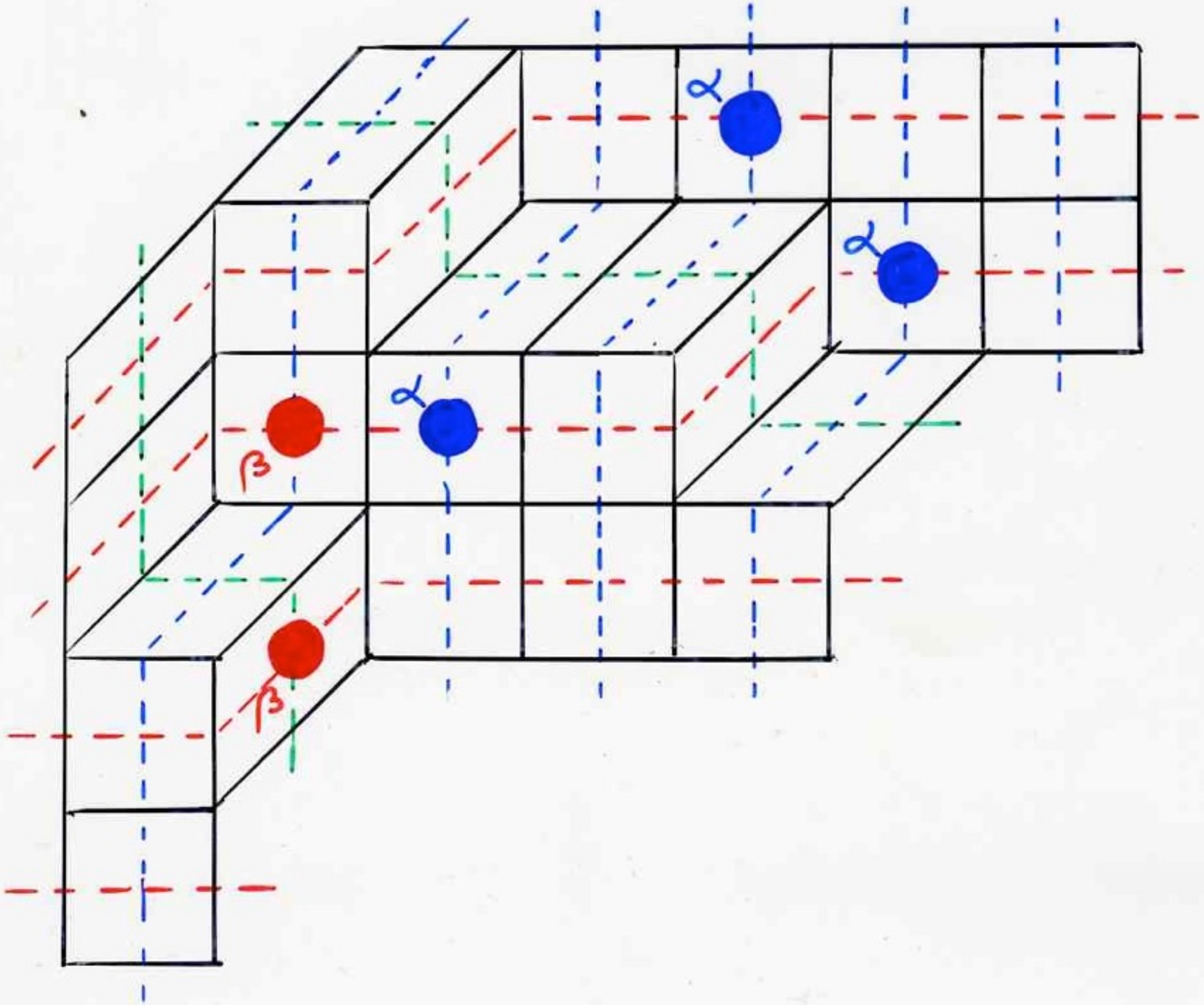

12
(8,9,10,11)
7
(5,6)
(1,2)
3
4

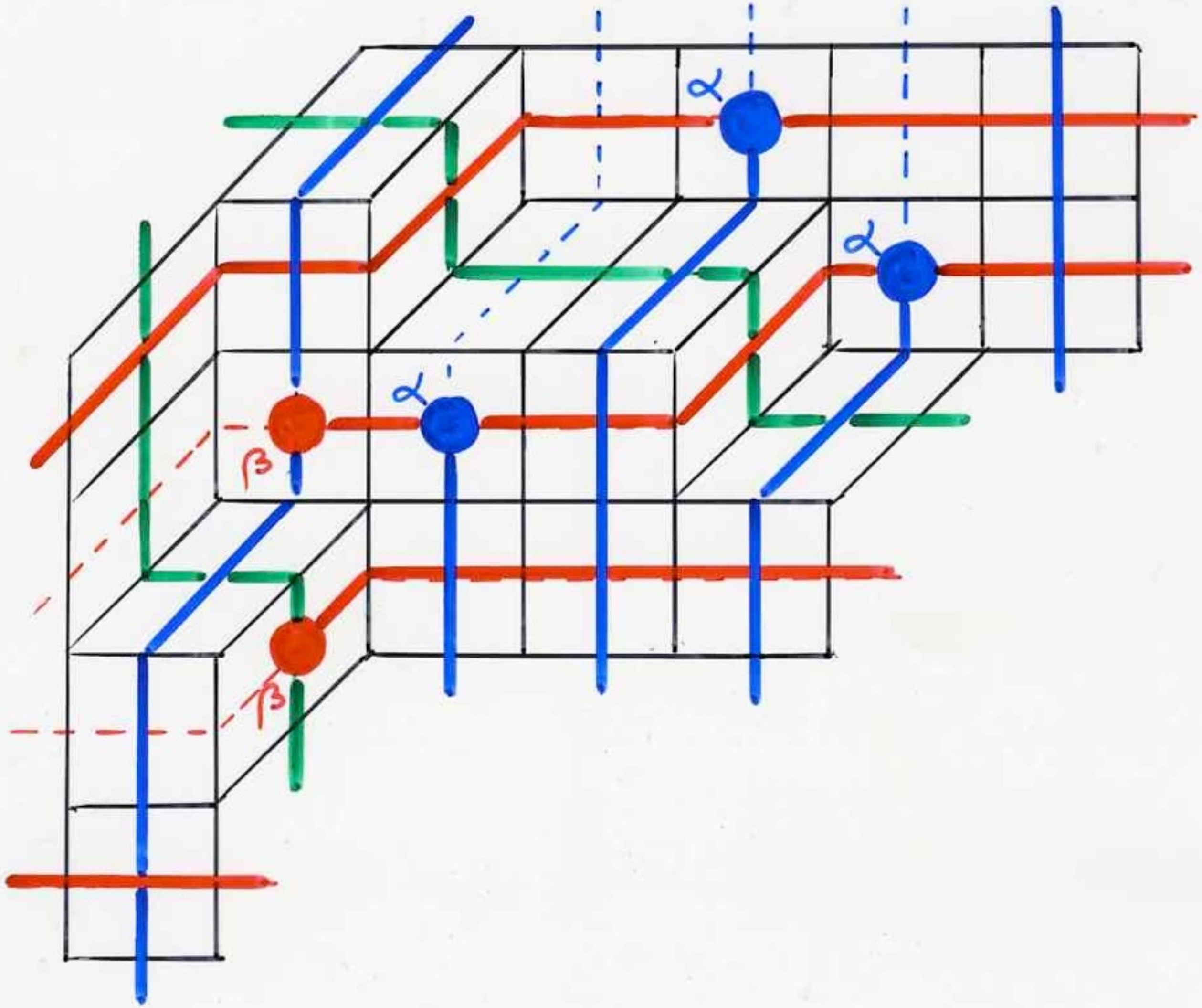


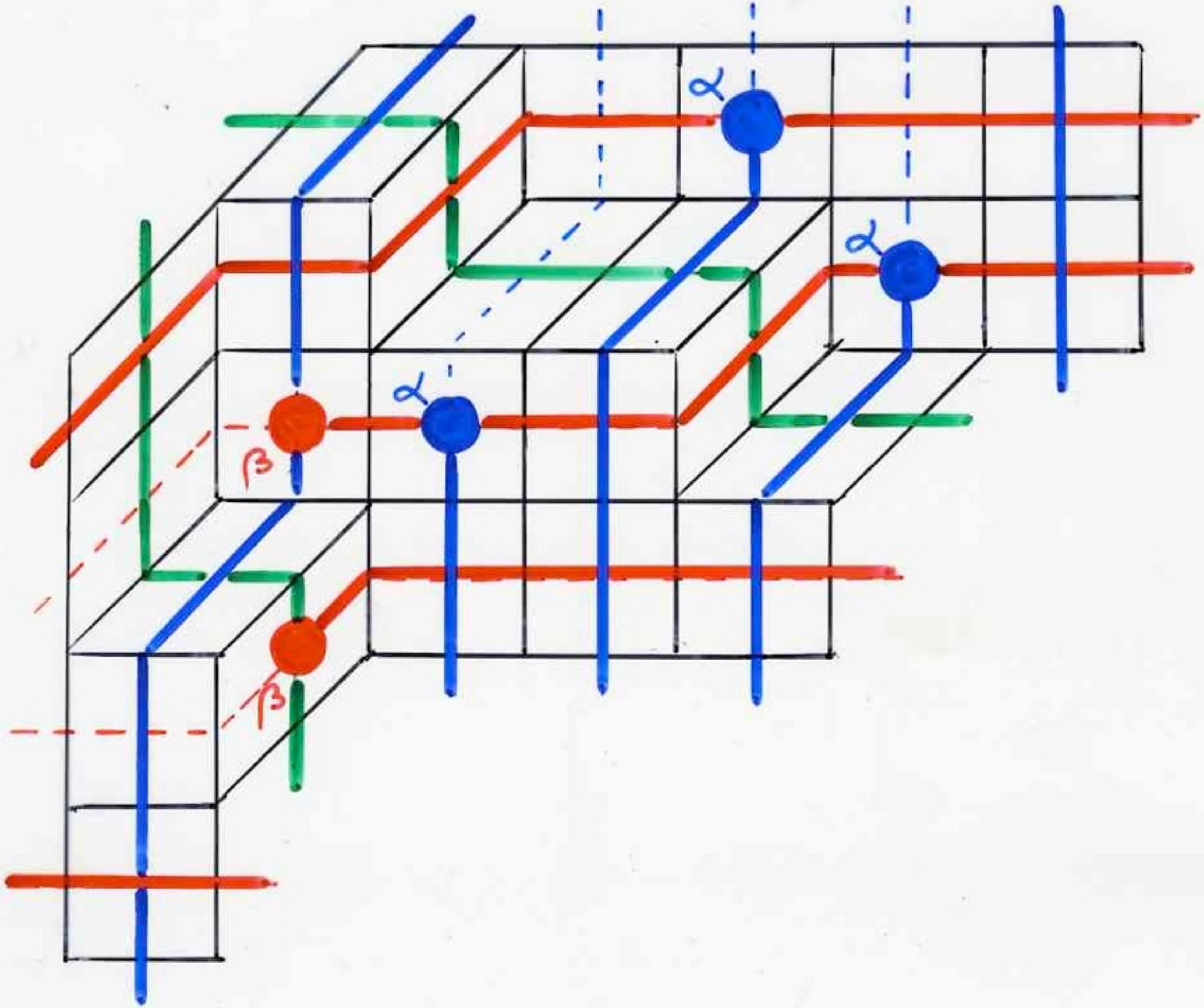
2 10 12 7 5 9 1 8 6 3 11 4
10
9
(9,10)
5
6
(5,6)
(9,10)
8
(8,9,10)
11
(8,9,10,11)
2
1
(1,2)
12
(8,9,10,11)
7
(5,6)
(1,2)
3
4



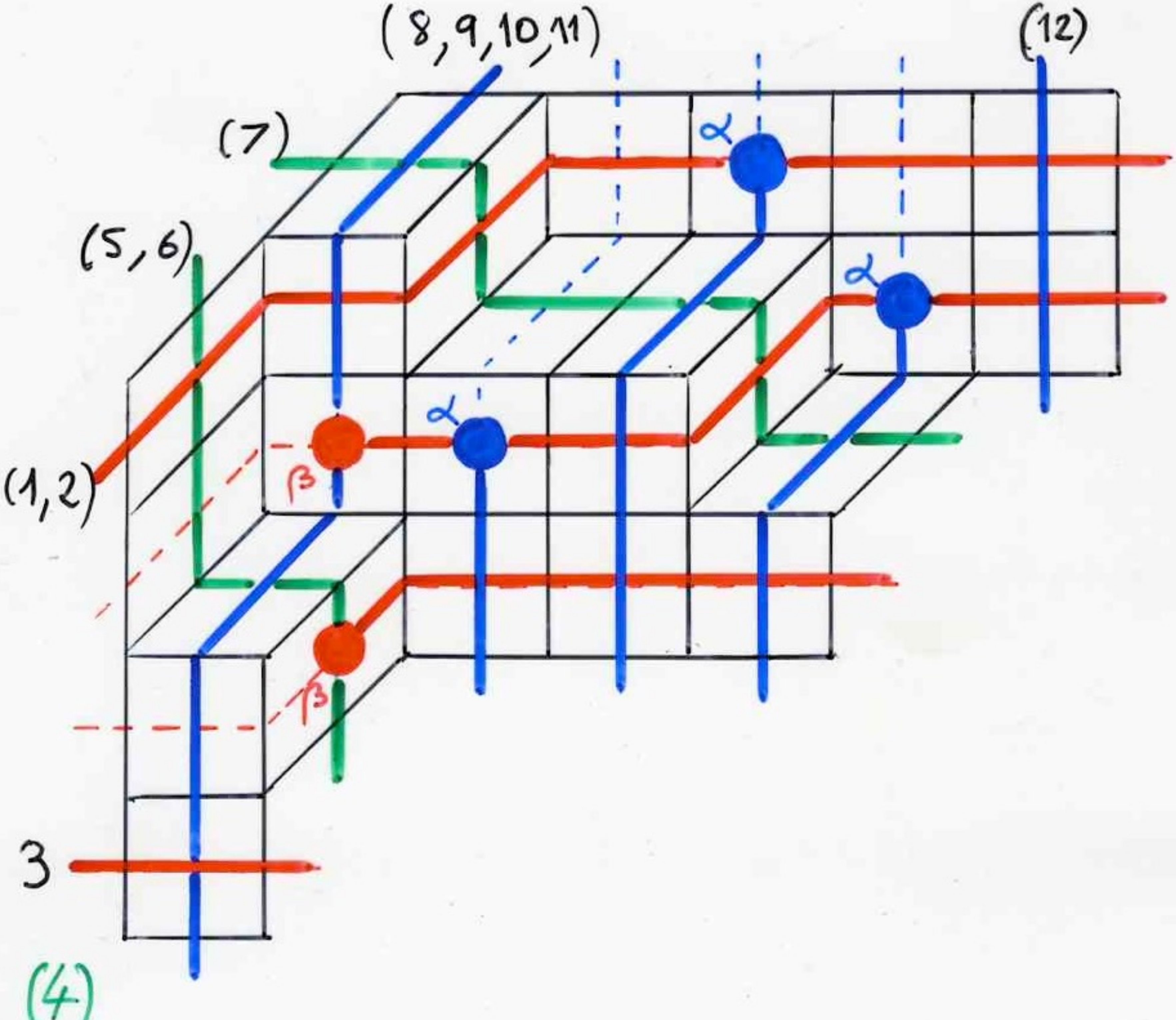


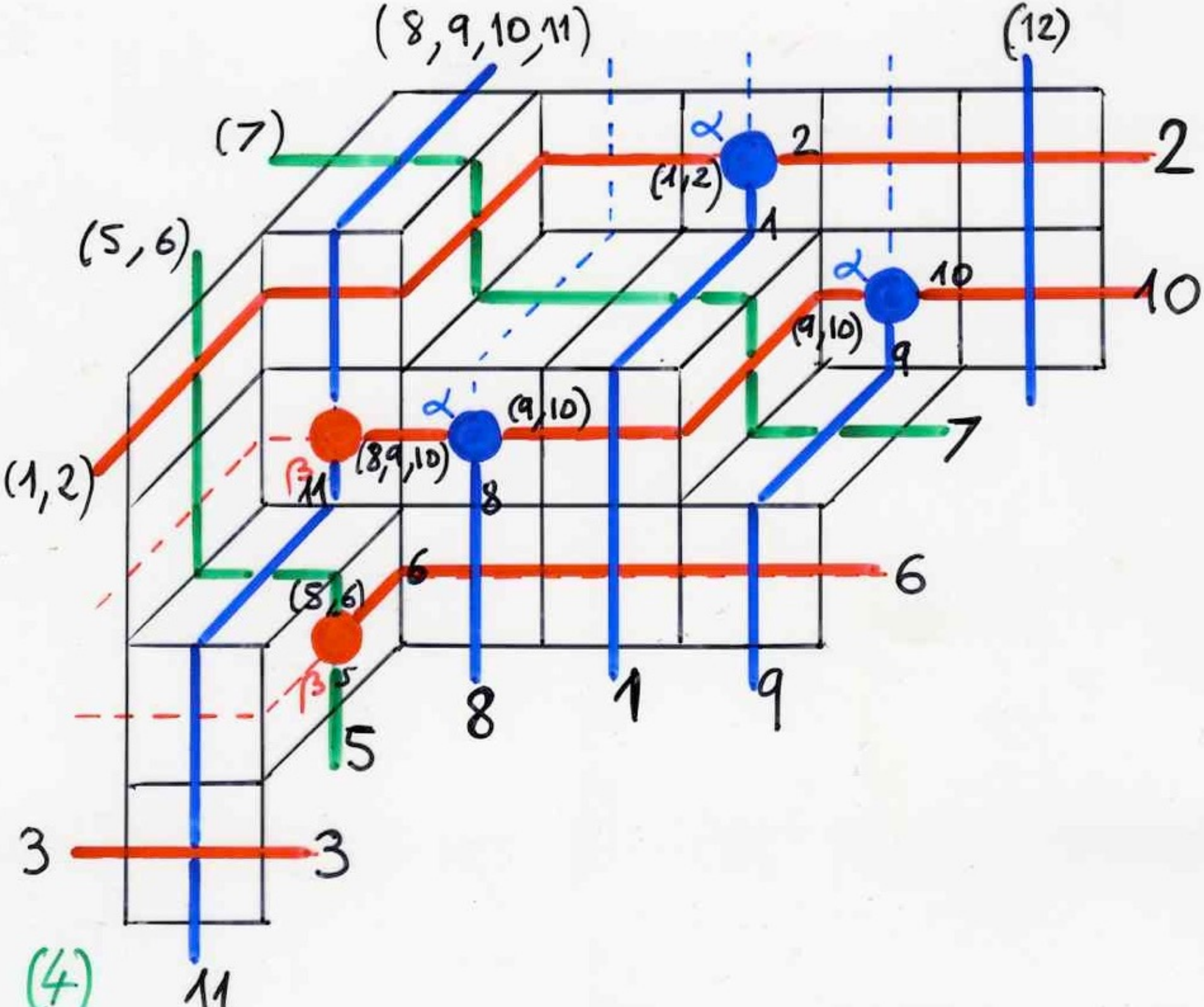

α
α
α
β
β


α
α
α
β
β



α
α
α
β
β


(8, 9, 10, 11)
(12)
(7)
(5, 6)
(1, 2)
3
(4)
α
α
α
β
β



(8,9,10,11)
(12)
(7)
α
2
(1,2)
1
2
(5,6)
α
10
10
(9,10)
9
α
(9,10)
7
(1,2)
β
(8,9,10)
11
8
6
6
(8,6)
β
5
5
8
1
9
3
3
(4)
11

bijection

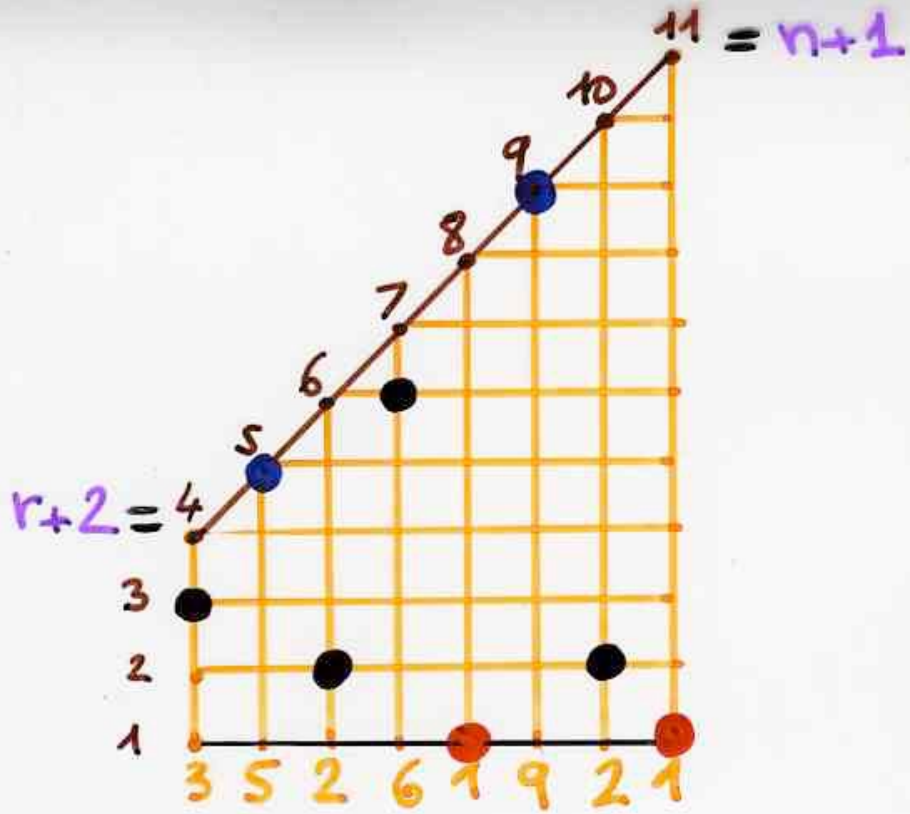
assemblée of permutations

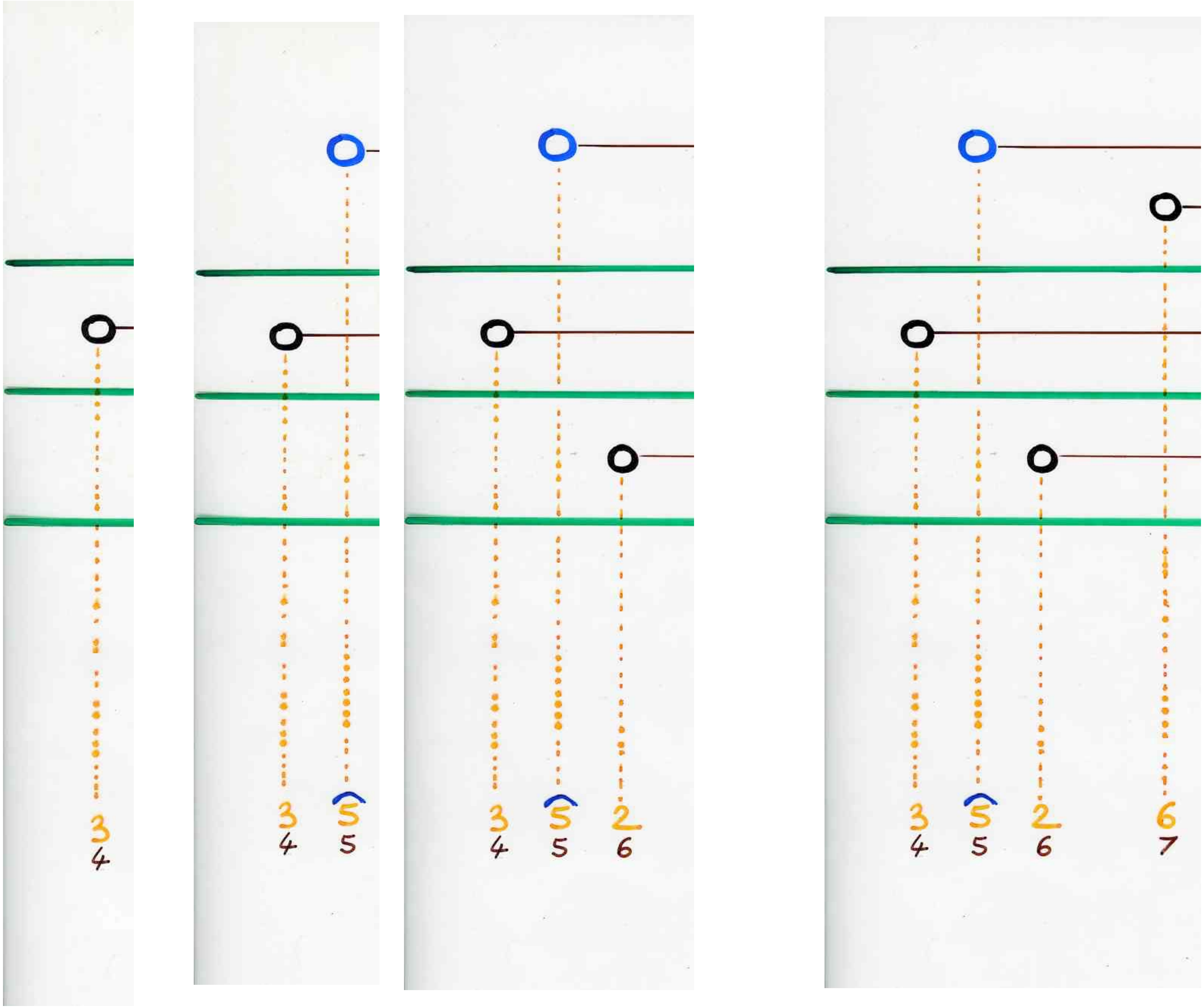
$\updownarrow$

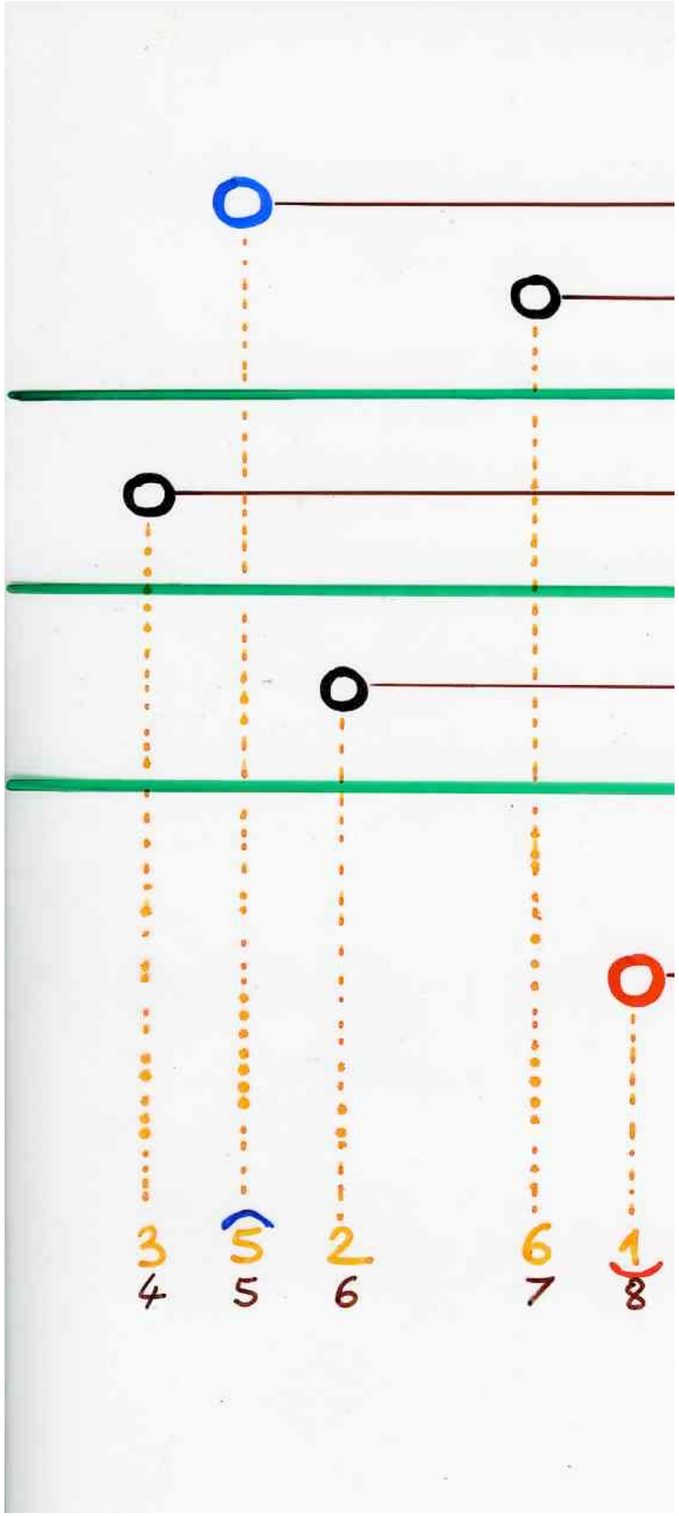
(subset of r elements among n) x
(r-truncated subexceedant functions)

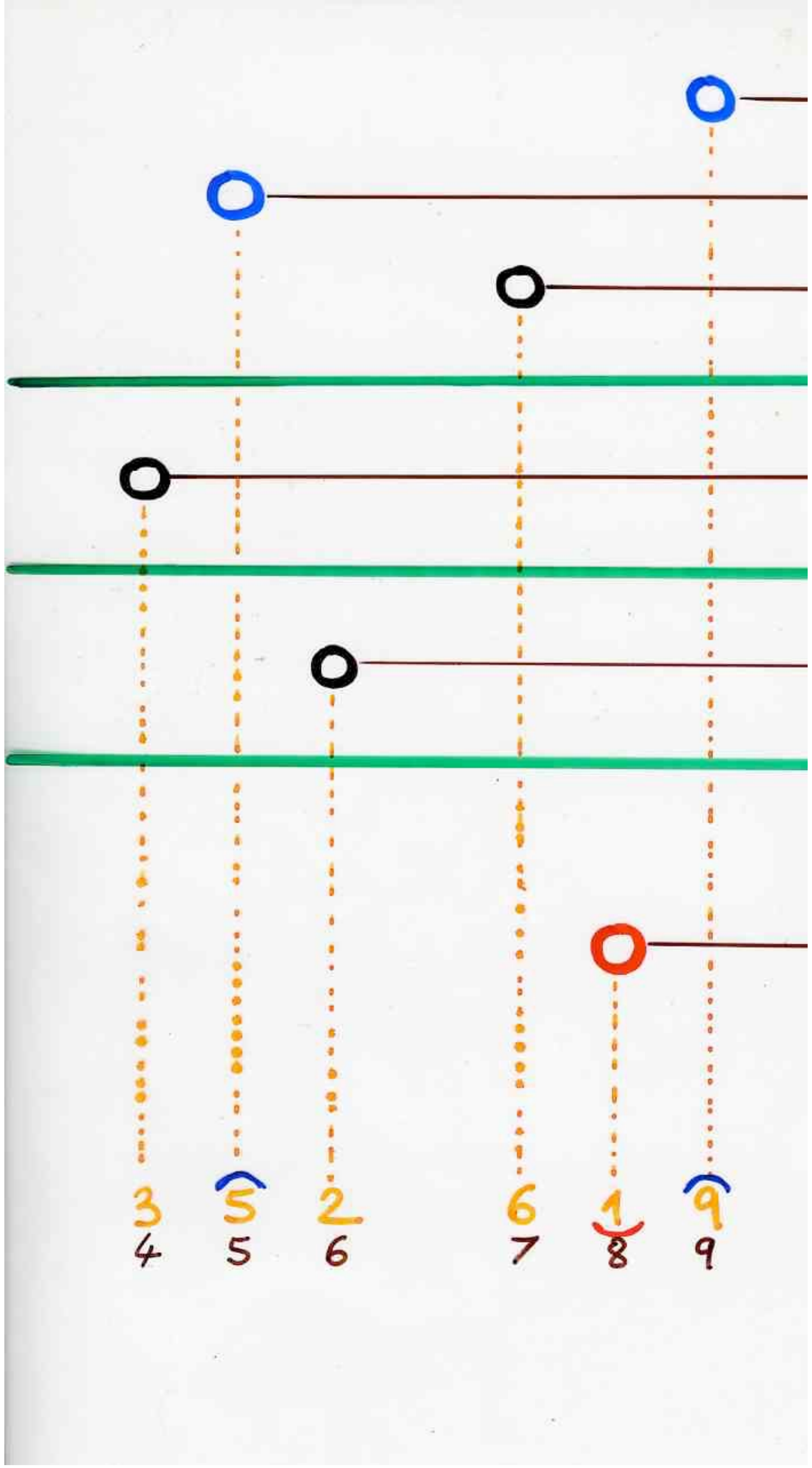
$$Z^{*}_{n,r}(\alpha, \beta, q=1) = \binom{n}{r} \prod_{i=r}^{n-1} \left(\alpha^{-1} + \beta^{-1} + i\right)$$

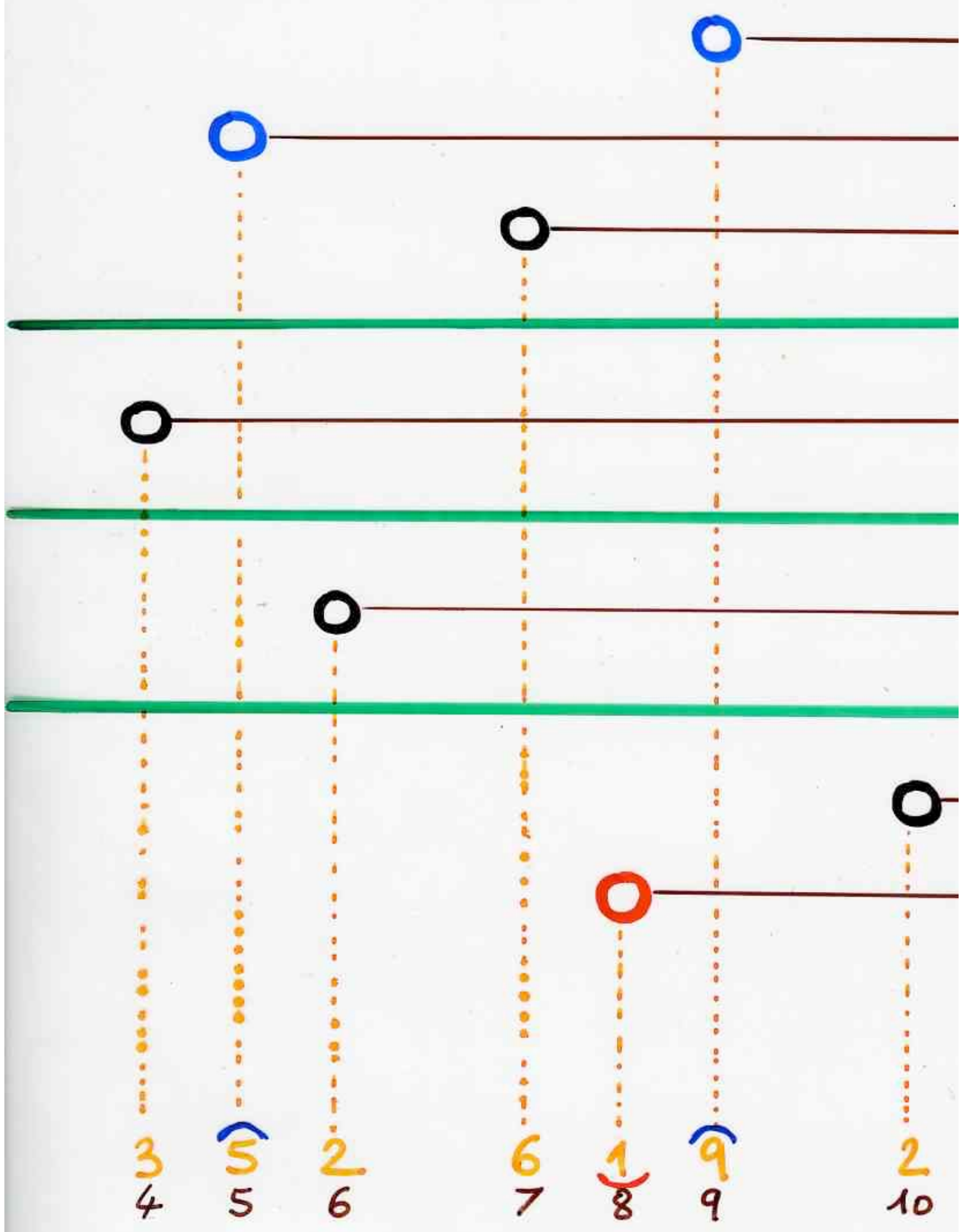
$$\binom{n}{r}$$



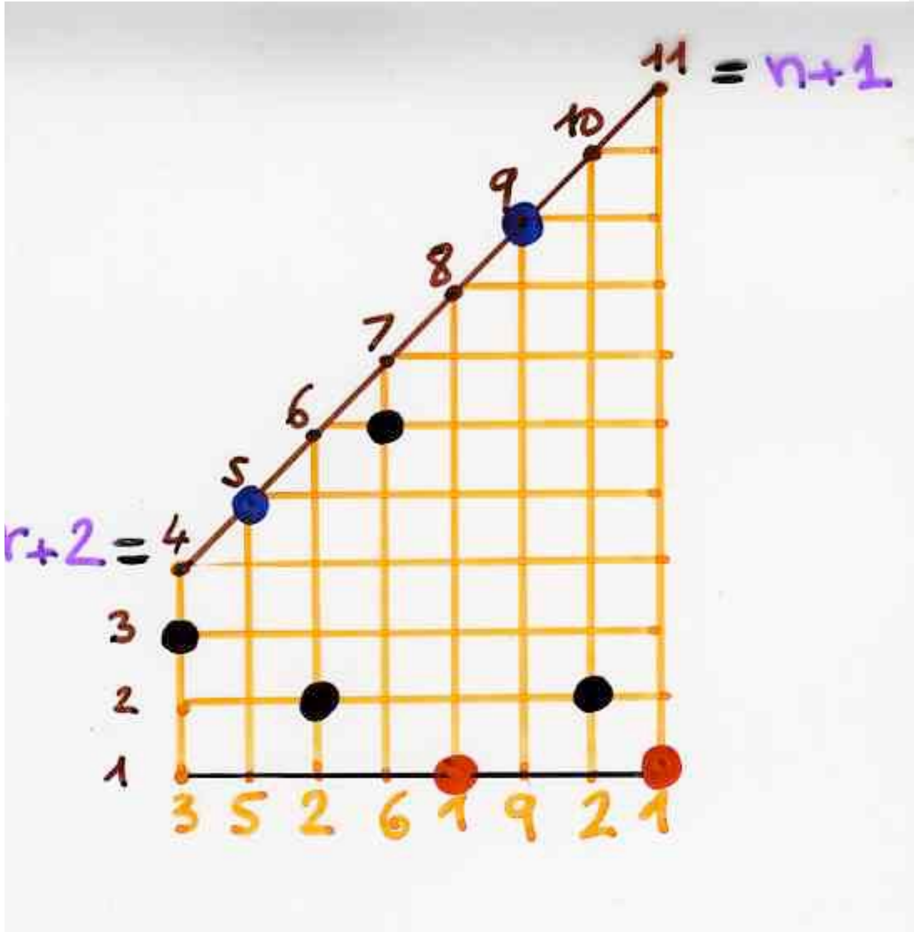

3
4
3 5
4 5
3 5 2
4 5 6
3 5 2 6
4 5 6 7

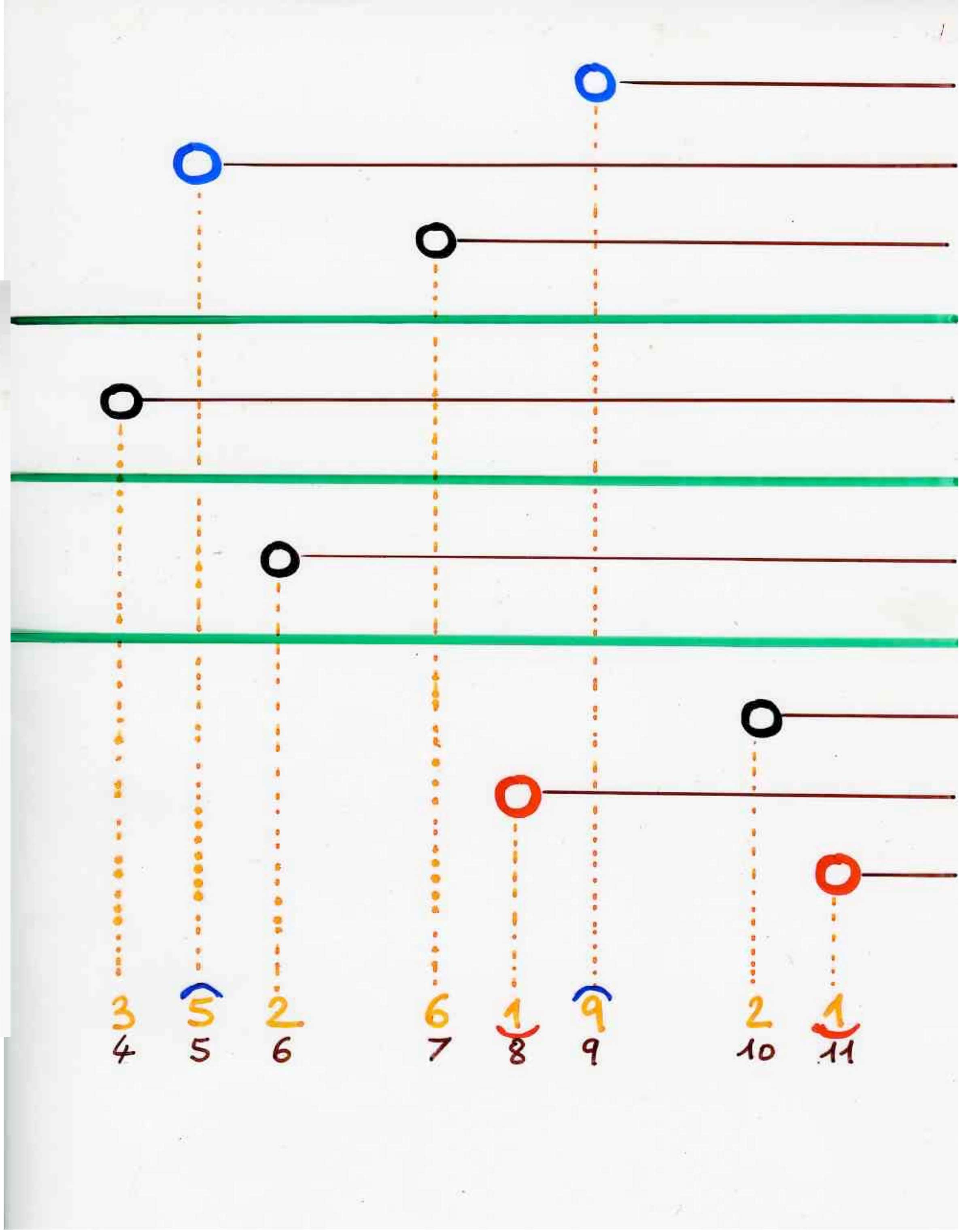

3 5 2 6 1
4 5 6 7 8


3 5 2 6 1 9
4 5 6 7 8 9

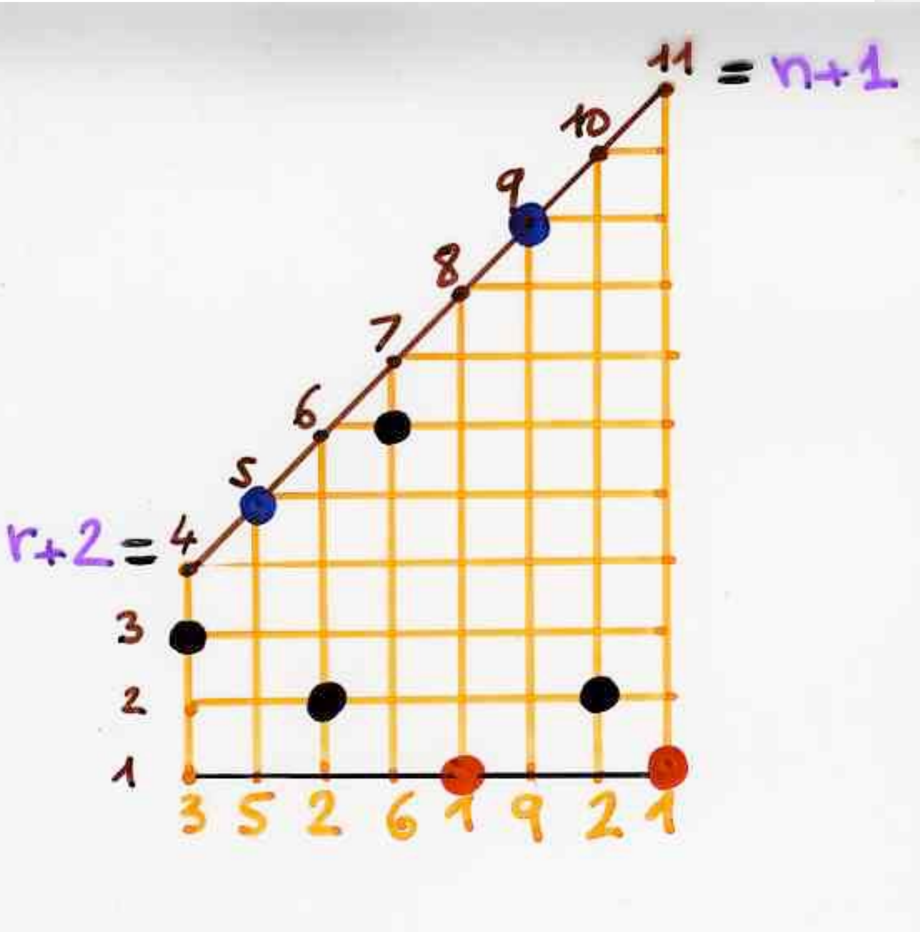


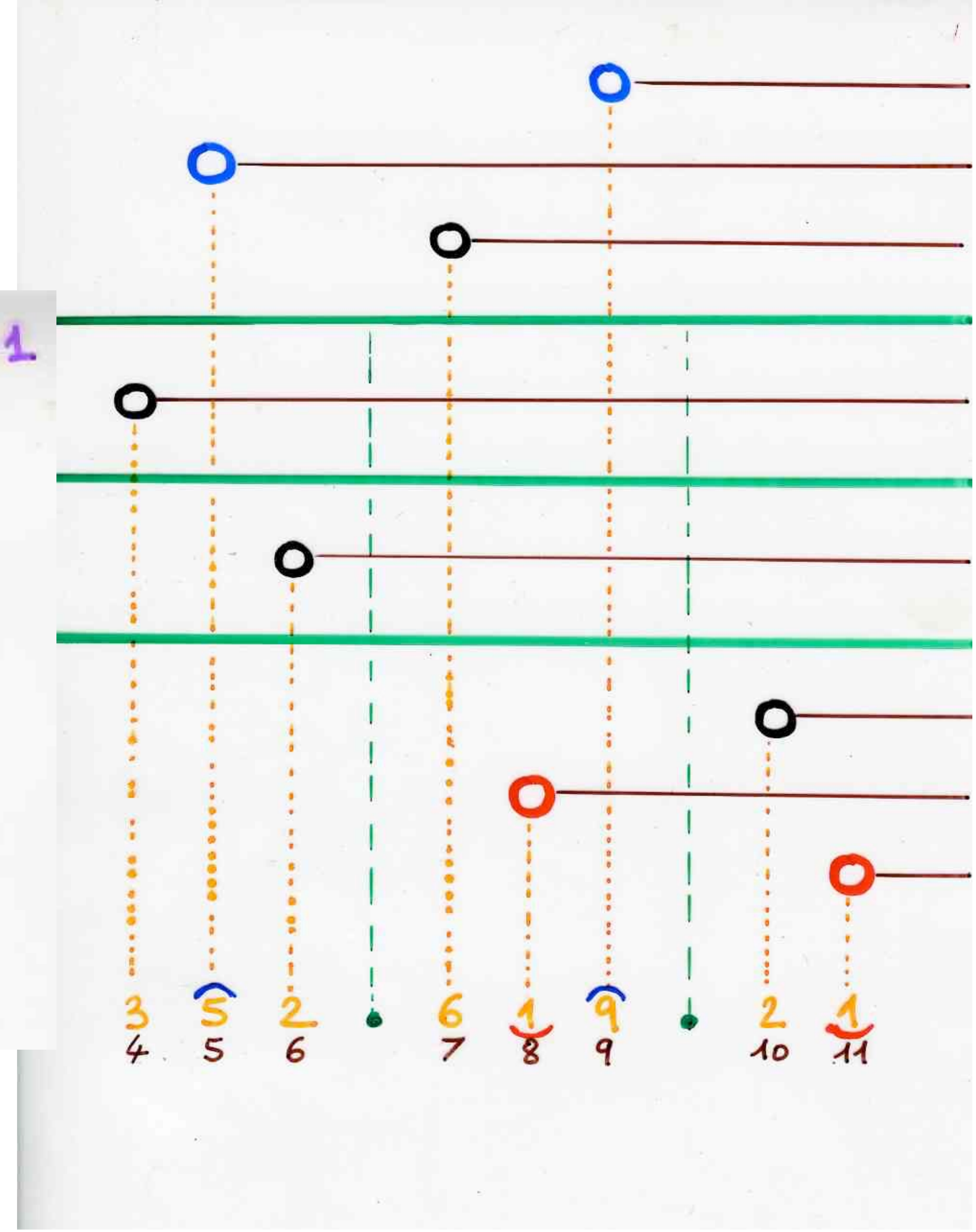
3 5 2 6 1 9 2
4 5 6 7 8 9 10

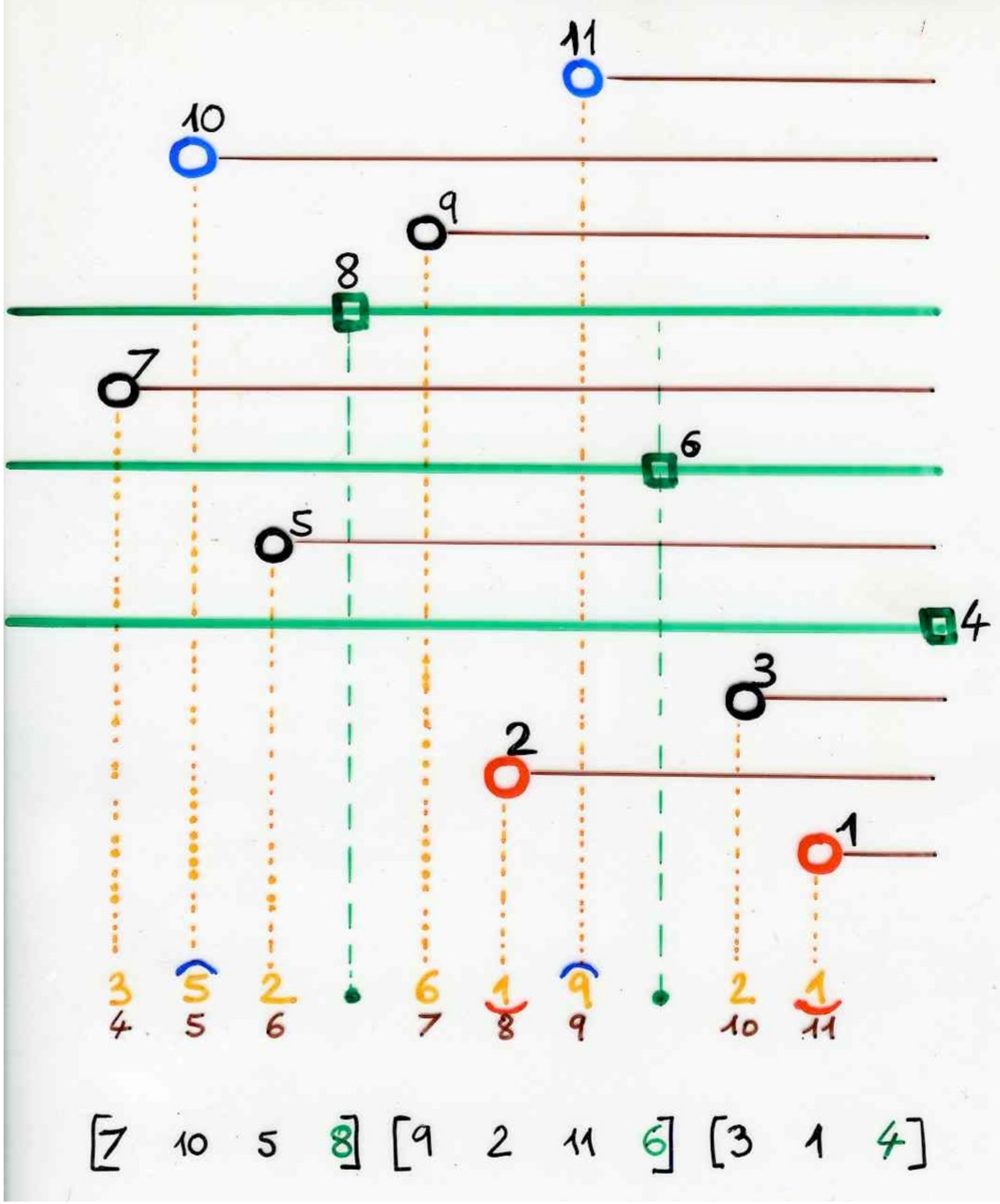

11 = n+1
10
9
8
7
6
5
r+2 = 4
3
2
1
3 5 2 6 1 9 2 1

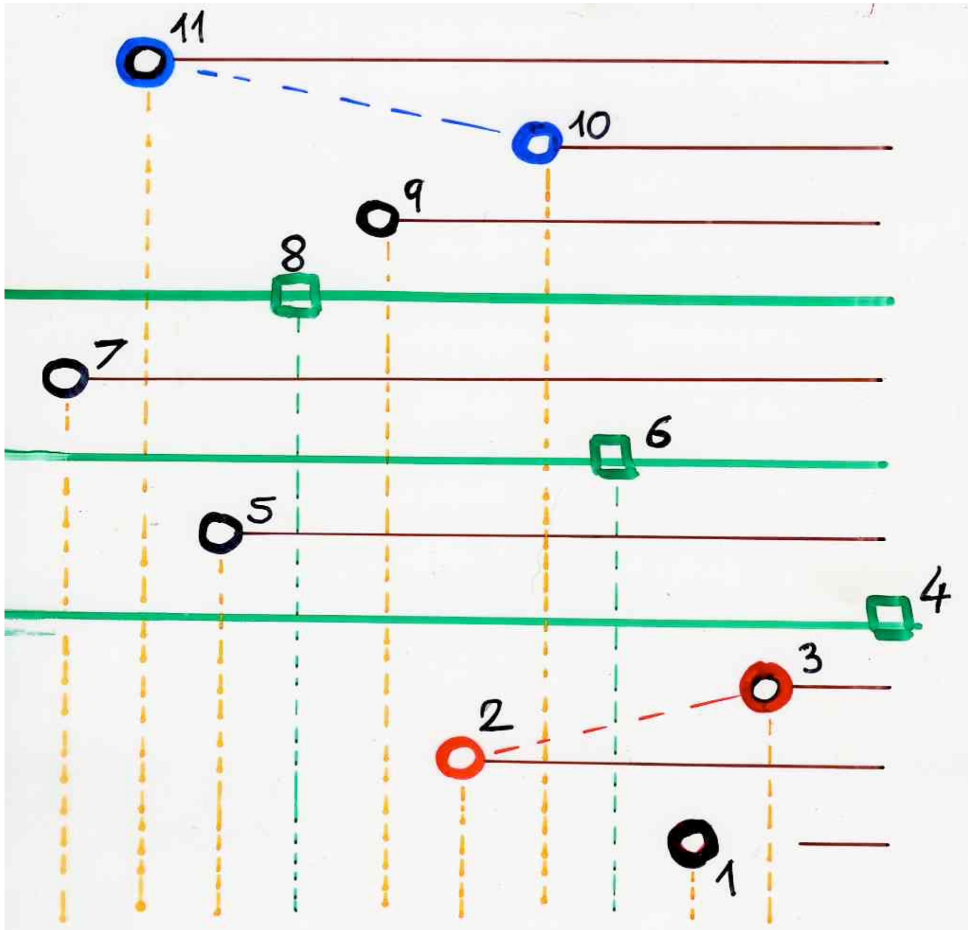

3 5 2 6 1 9 2 1
4 5 6 7 8 9 10 11

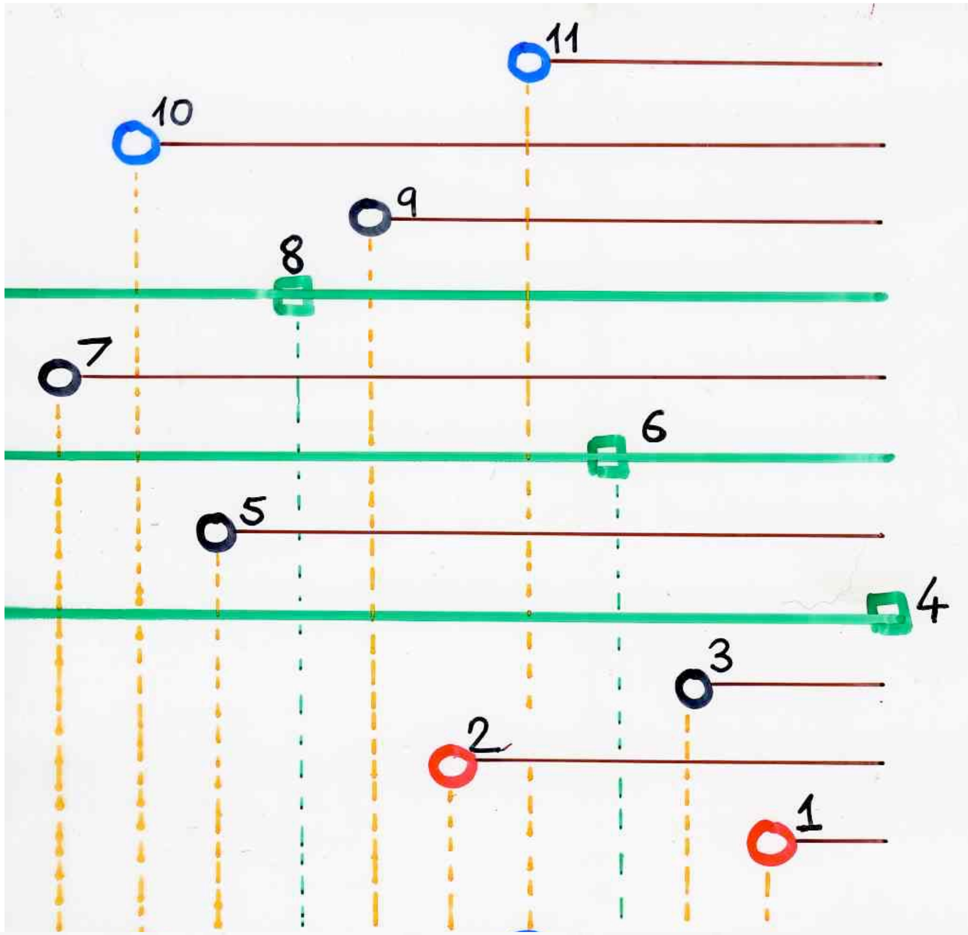
$$\binom{n}{r}\prod_{i=r}^{n-1}\left(\alpha^{-1}+\beta^{-1}+i\right)$$






11
10
9
8
7
6
5
4
3
2
1
3 5 2 6 1 9 2 1
4 5 6 7 8 9 10 11
[7 10 5 8] [9 2 11 6] [3 1 4]





[7 10 5 8] [9 2 11 6] [3 1 4]

[7 10 5 8] [9 2 11 6] [3 1 4]

$u = \underline{2}\ 3\ \underline{1}$ $\qquad u^c = \bar{2}\ 1\ \bar{3}$

$v = \bar{10}\ 9\ \bar{11}$ $\qquad \tilde{v} = \bar{11}\ 9\ \bar{10}$

$[7\ \bar{11}\ 5\ 8]\quad [9\ \bar{2}\ \bar{10}\ 6]\quad [1\ \bar{3}\ 4]$

$$\binom{n}{r}(r+\bar{\alpha}+\bar{\beta})\cdots(n-1+\bar{\alpha}+\bar{\beta})$$

$$[7\ \ 10\ \ 5\ \ 8]\quad[9\ \ 2\ \ 11\ \ 6]\quad[3\ \ 1\ \ 4]$$

$$[7\ \ 10\ \ 5\ \ 8]\quad[9\ \ 2\ \ 11\ \ 6]\quad[3\ \ 1\ \ 4]$$

$$u = \underline{2}\ \ 3\ \ \underline{1} \qquad u^c = \bar{2}\ \ 1\ \ \bar{3}$$

$$v = \bar{10}\ \ 9\ \ \bar{11} \qquad \tilde{v} = \bar{11}\ \ 9\ \ \bar{10}$$

$$[7\ \ \bar{11}\ \ 5\ \ 8]\quad[9\ \ \bar{2}\ \ \bar{10}\ \ 6]\quad[1\ \ \bar{3}\ \ 4]$$

further enumerative results

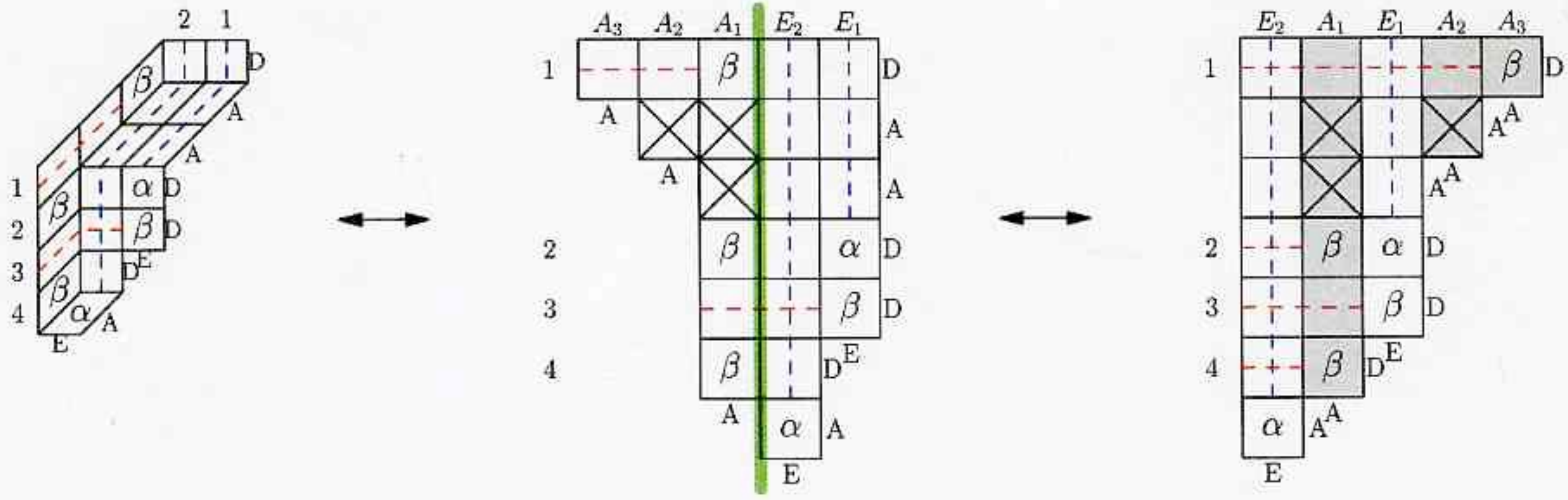
$q = 0$ (Olya Mandelshtam)

$$Z^*_{n,r}(\alpha, \beta, 0) = \sum_{p=1}^{n-r} \frac{2r+p}{2n-p} \binom{2n-p}{n+r} \frac{\bar{\alpha}^{p+1} - \bar{\beta}^{p+1}}{\bar{\alpha} - \bar{\beta}}$$

$$Z^*_{n,r}(1,1,0) = \frac{2(r+1)}{n+r+2} \binom{2n+1}{n-r}$$

nb of RAT size $n = r + k + \ell$, r A's, k D's, ℓ E's

$$\frac{r+1}{n+1} \binom{n+1}{k} \binom{n+1}{\ell}$$



(Olga Mandelshtam)

Multi-Catalan tableaux and the two-species TASEP arXiv: 1502.00948

accepted AIHP-D

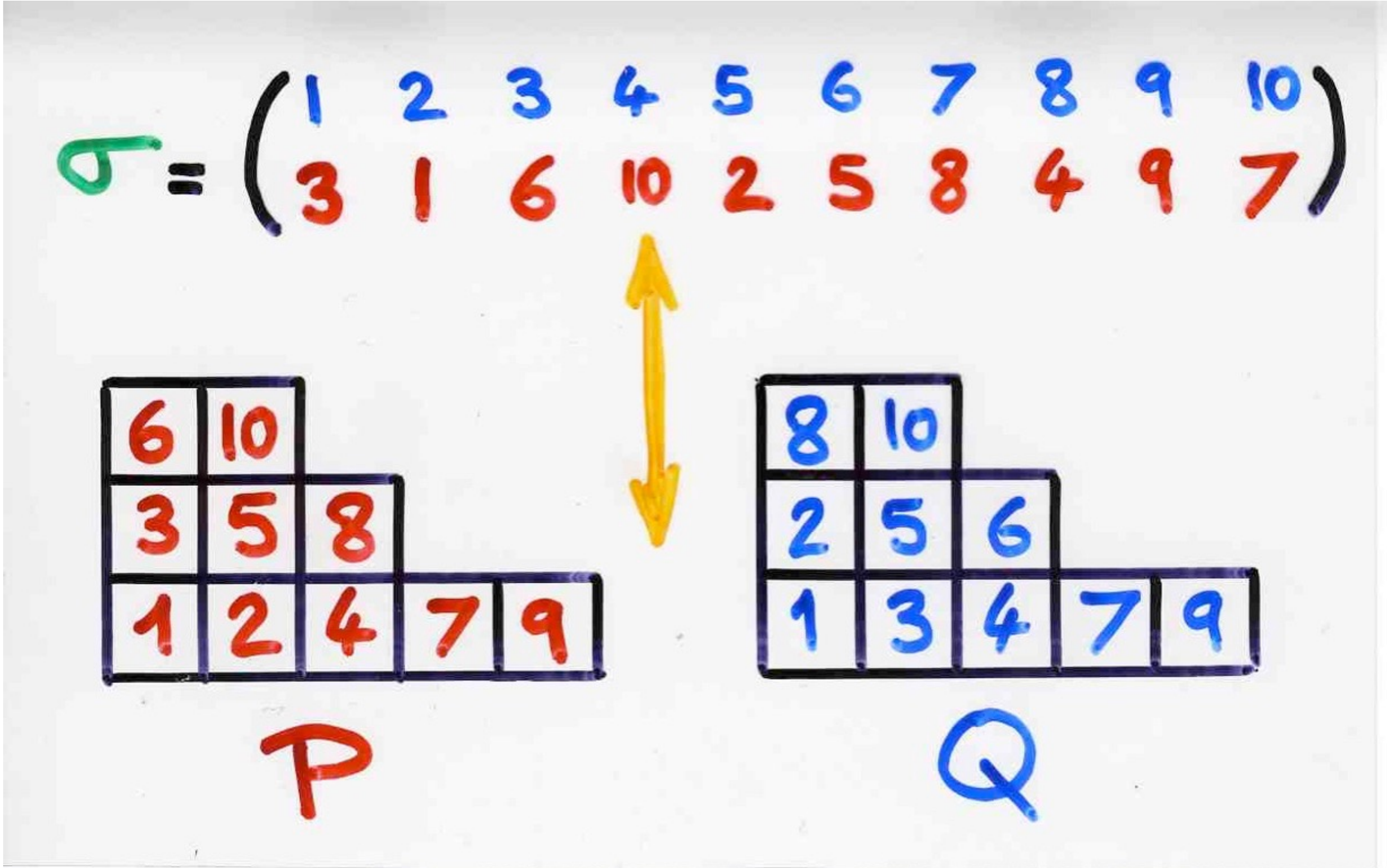
- O.M., Multi-Catalan tableaux and the two-species TASEP, arXiv: 1502.00948 accepted → AIHP-D
- O.M., X.V., Tableaux combinatorics of the two-species PASEP arXiv: 1506.01980
- O.M., X.V. Rhombic alternative tableaux bijection, in preparation

} O.M., X.V. extended abstract accepted FPSAC/2016

- O.M., Matrix ansatz and combinatorics of the k-species PASEP arXiv: 1508.04115

the bijection permutations — alternative tableaux
was constructed
from a combinatorial representation
of the PASEP algebra

and using the methodology of the
«Cellular Ansatz»



The Robinson-Schensted correspondence (RSK) between permutations and pair of (standard) Young tableaux with the same shape

"The cellular Ansatz"

Physics

combinatorial
objects
on a 2d lattice

representation
by operators

bijections

"normal ordering"

$UD = DU + Id$

Weyl-Heisenberg

permutations

RSK

$\longleftrightarrow$

pairs of Tableaux Young

quadratic algebra Q

Q-tableaux

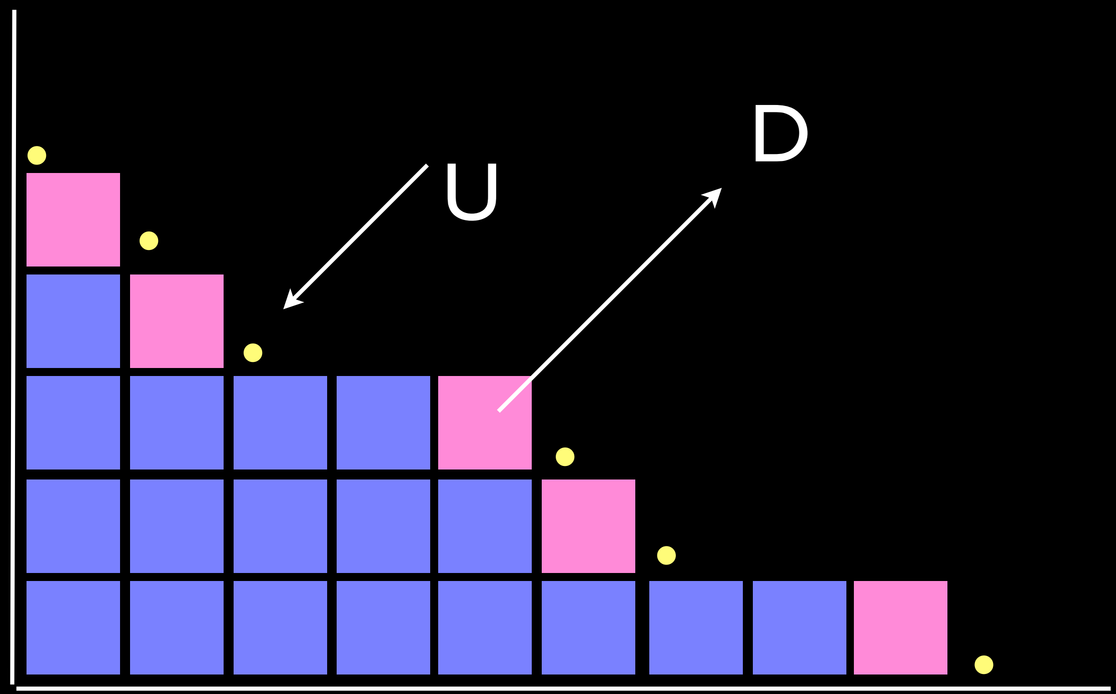
representation of the operators U, D

$UD = DU + I$

Sergey Fomin

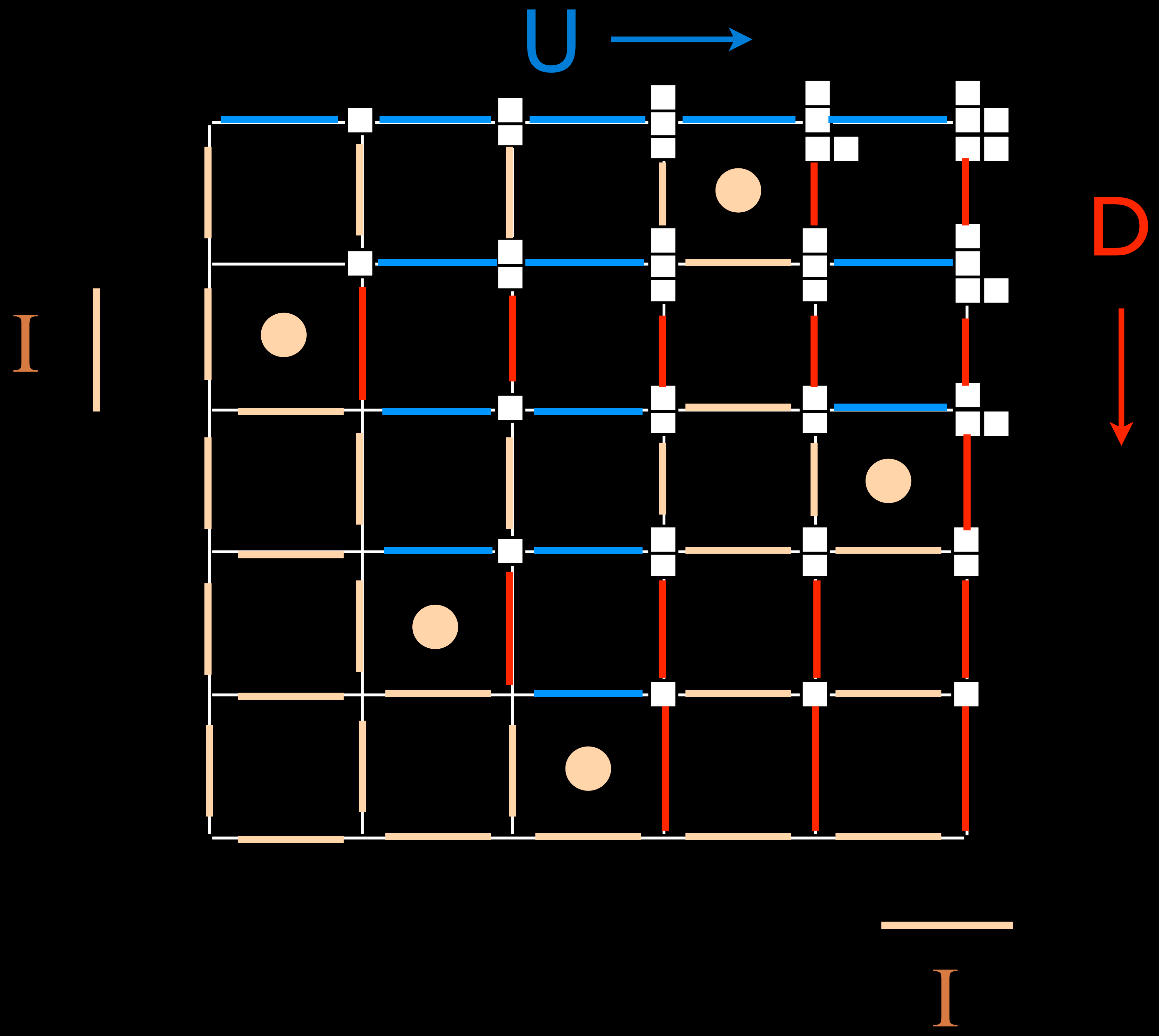
(with C. K.)

Operators U and D



adding
or deleting
a cell in
a Ferrers
diagram

Young lattice


U
D
I
I

The cellular Ansatz

guided construction
of a bijection
(from a representation of the quadratic
algebra Q with "combinatorial operators")

"The cellular Ansatz"

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RSK

permutations ⟷ pairs of Tableaux Young

$DE = qED + E + D$

PASEP

alternative tableaux ⟷ permutations

Laguerre histories

dynamical systems in physics
stationary probabilities

quadratic algebra Q

Q-tableaux

representation
of the
operators
E and D

$$DE = ED + E + D$$

J. Françon 1976
data structure histories

"histoires de fichiers"

24

17

10

8

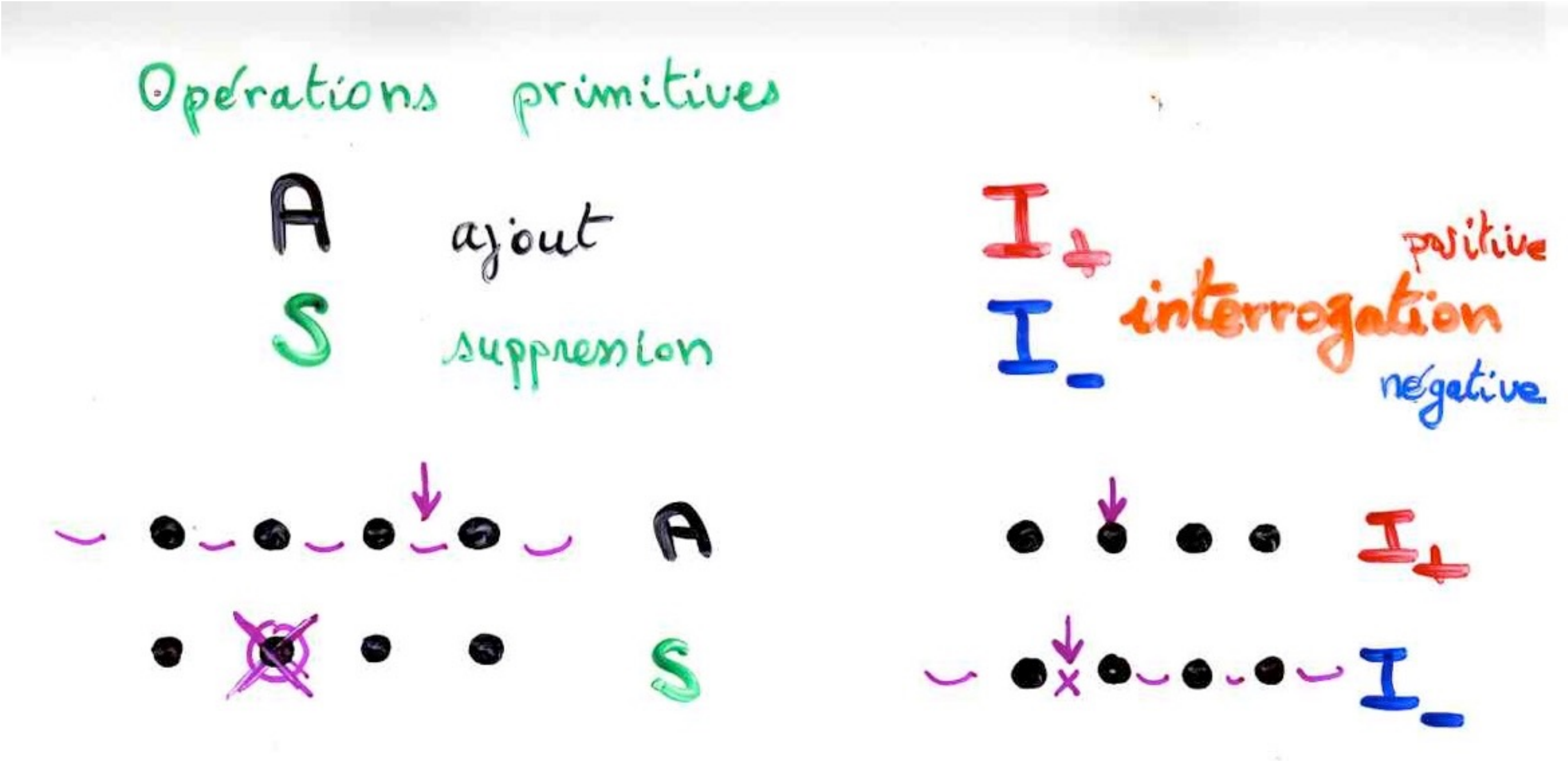
24

17

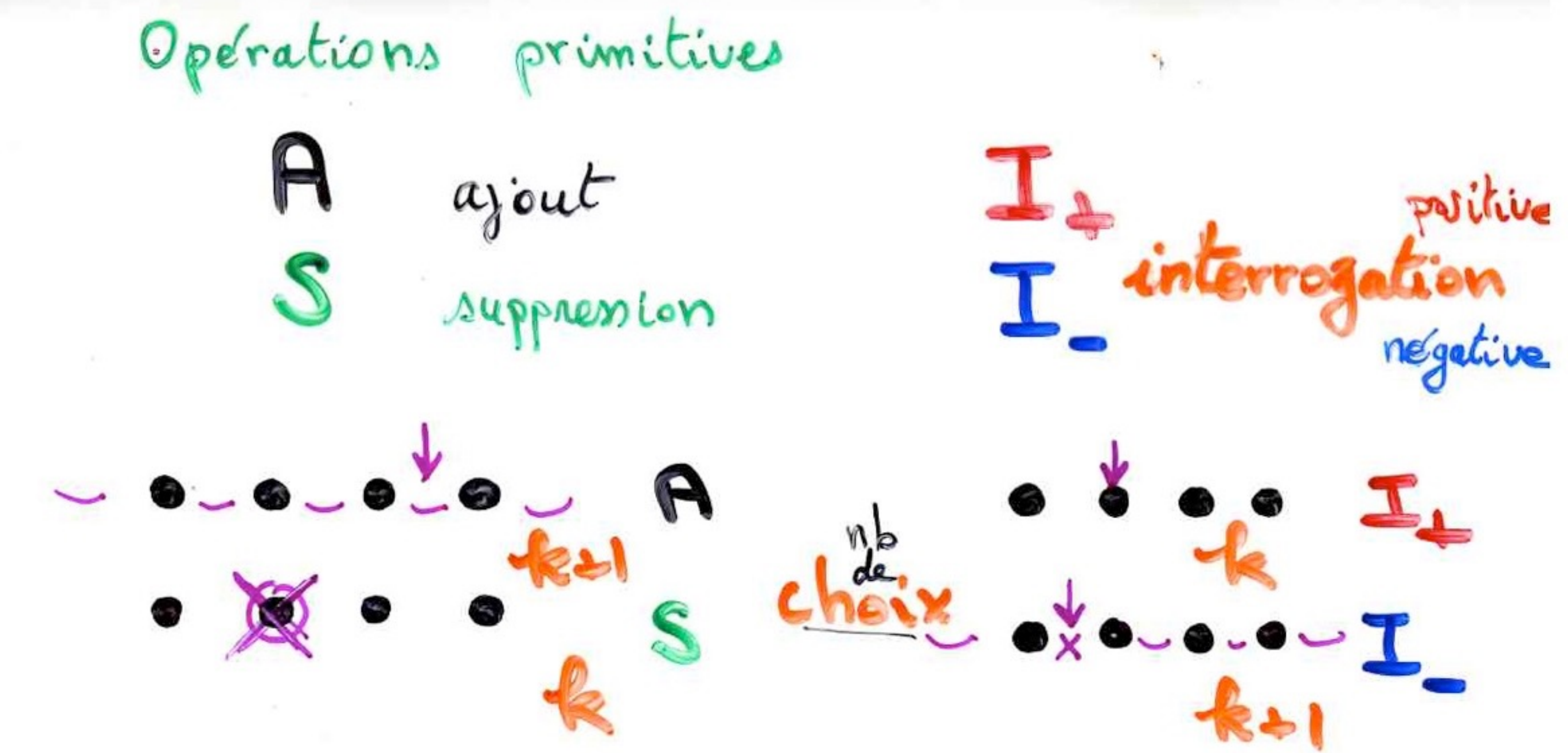
← 12

10

8



Primitive operations
for "dictionnaries" data structure:
add or delete any elements, asking questions (with positive or negative answer)

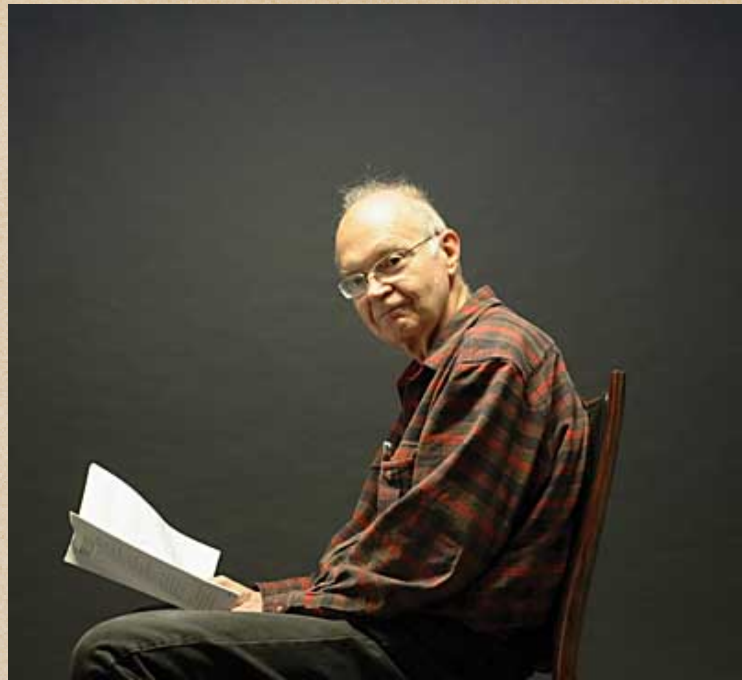


number of choices for each
primitive operations

$$\begin{cases} D = A + I_- \\ E = S + I_+ \end{cases}$$

data structure

integrated cost



D. Knuth

P.Flajolet

representation
of the
operators
E and D

$$DE = ED + E + D$$

V vector space generated by B basis

B alternating words two letters {○, ●}
(no occurrence of ○○ or ●●)

4 operators A, S, J, K

4 operators A, S, J, K, $\quad u \in B$

$\langle u | A = \sum_{\text{letter } \circ \text{ of } u} v$, v obtained by: $\circ \to \circ\bullet\circ$

$\langle u | S = \sum_{\circ \text{ of } u} v$ $\quad v$ obtained by: $\circ \to \bullet$

(and $\bullet\bullet \to \bullet \quad \bullet\bullet\bullet \to \bullet$)

$\langle u | J = \sum_{\circ \text{ of } u} v$ $\quad v$, $\circ \to \bullet\circ$ (and $\bullet\bullet \to \bullet$)

$\langle u | K = \sum_{\circ \text{ of } u} v$ $\quad v$, $\circ \to \circ\bullet$ (and $\bullet\bullet \to \bullet$)

$$\circ\bullet\circ\bullet \,|\, S = \bullet\circ\bullet + \circ\bullet$$

Lemma.

$$AS = SA + J + K$$

$$AK = KA + A$$

$$JS = SJ + S$$

$$JK = KJ$$

$$D = A + J$$

$$E = S + K$$

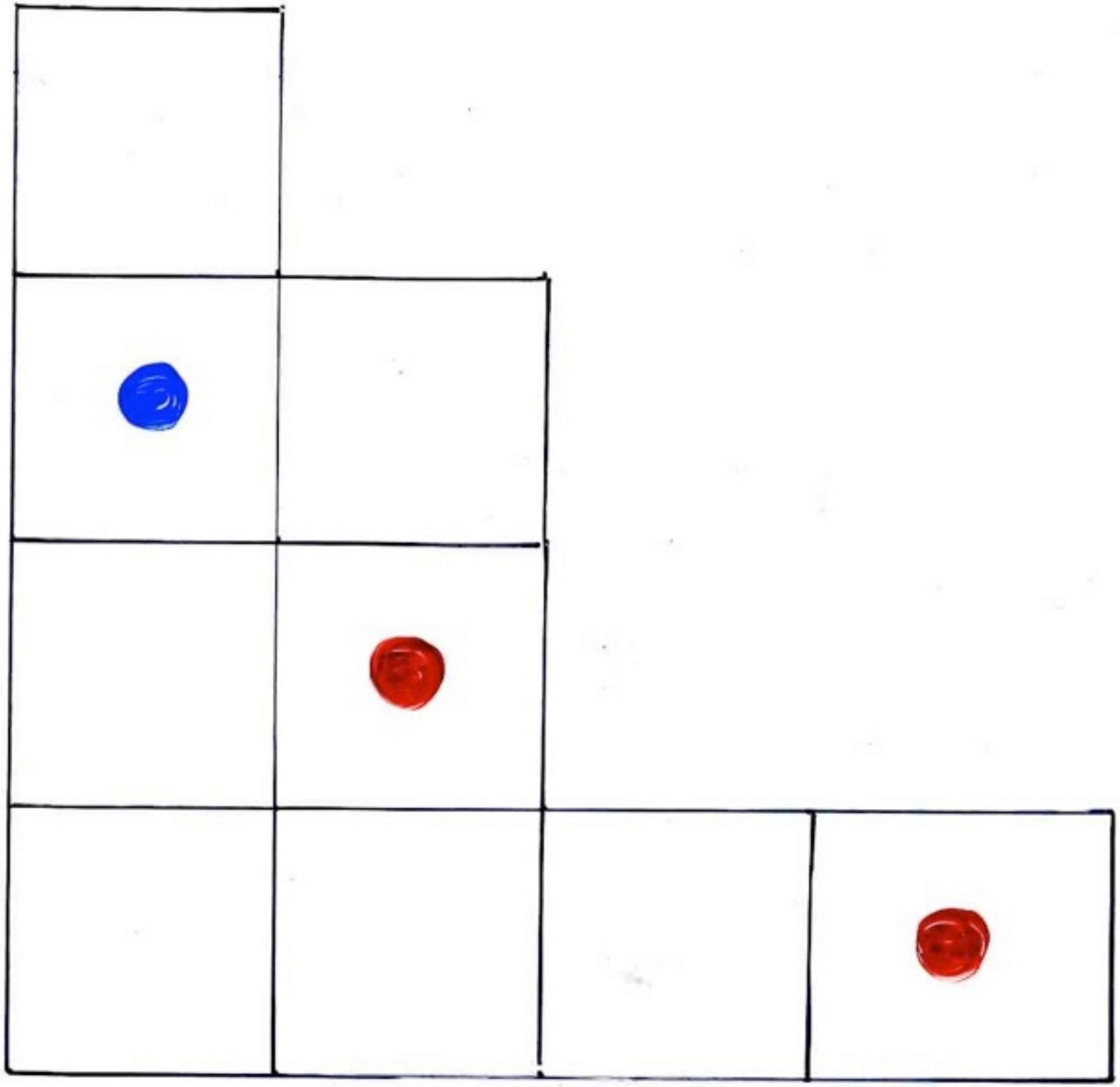
$$D = A + J$$

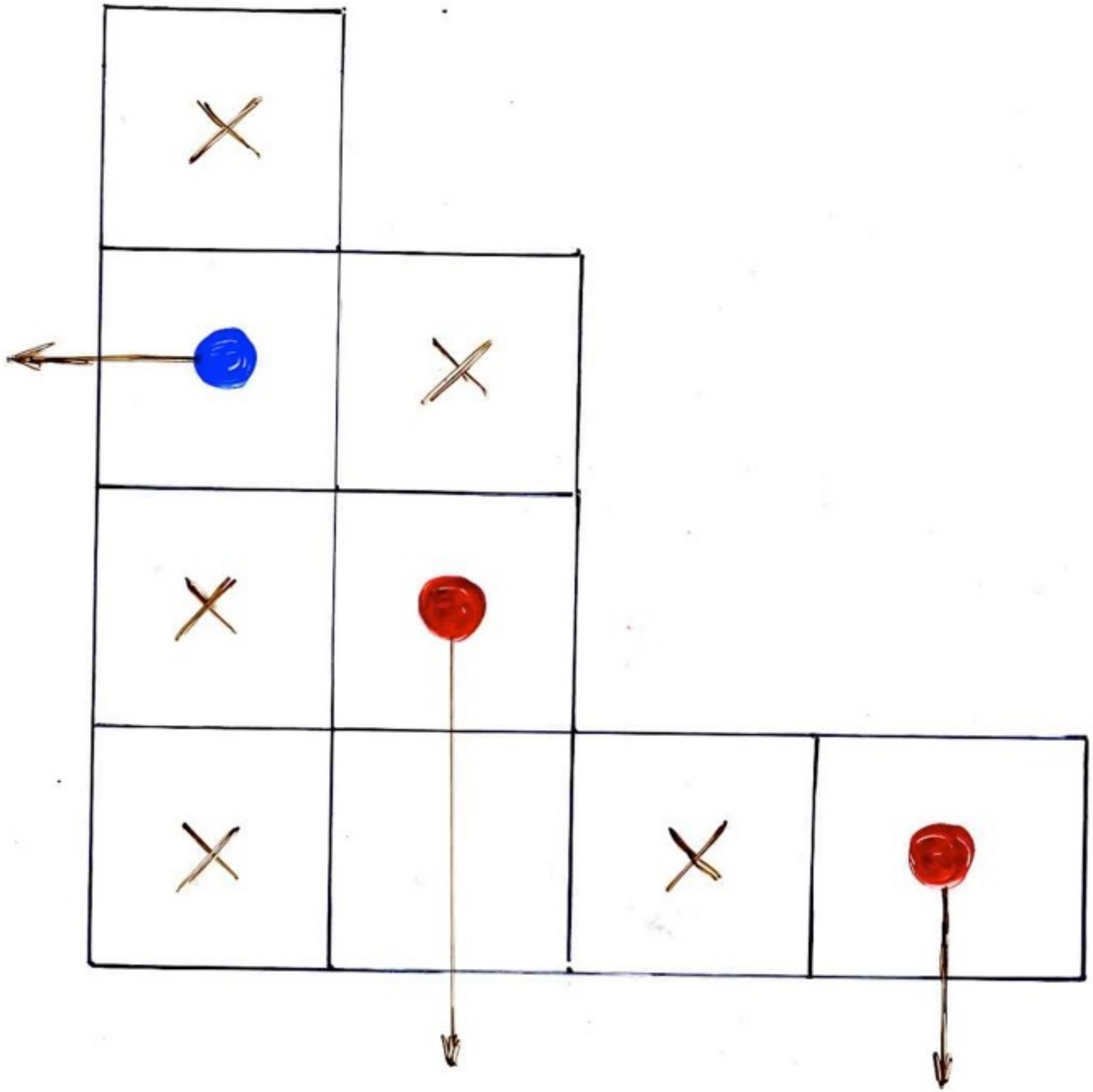
$$E = S + K$$

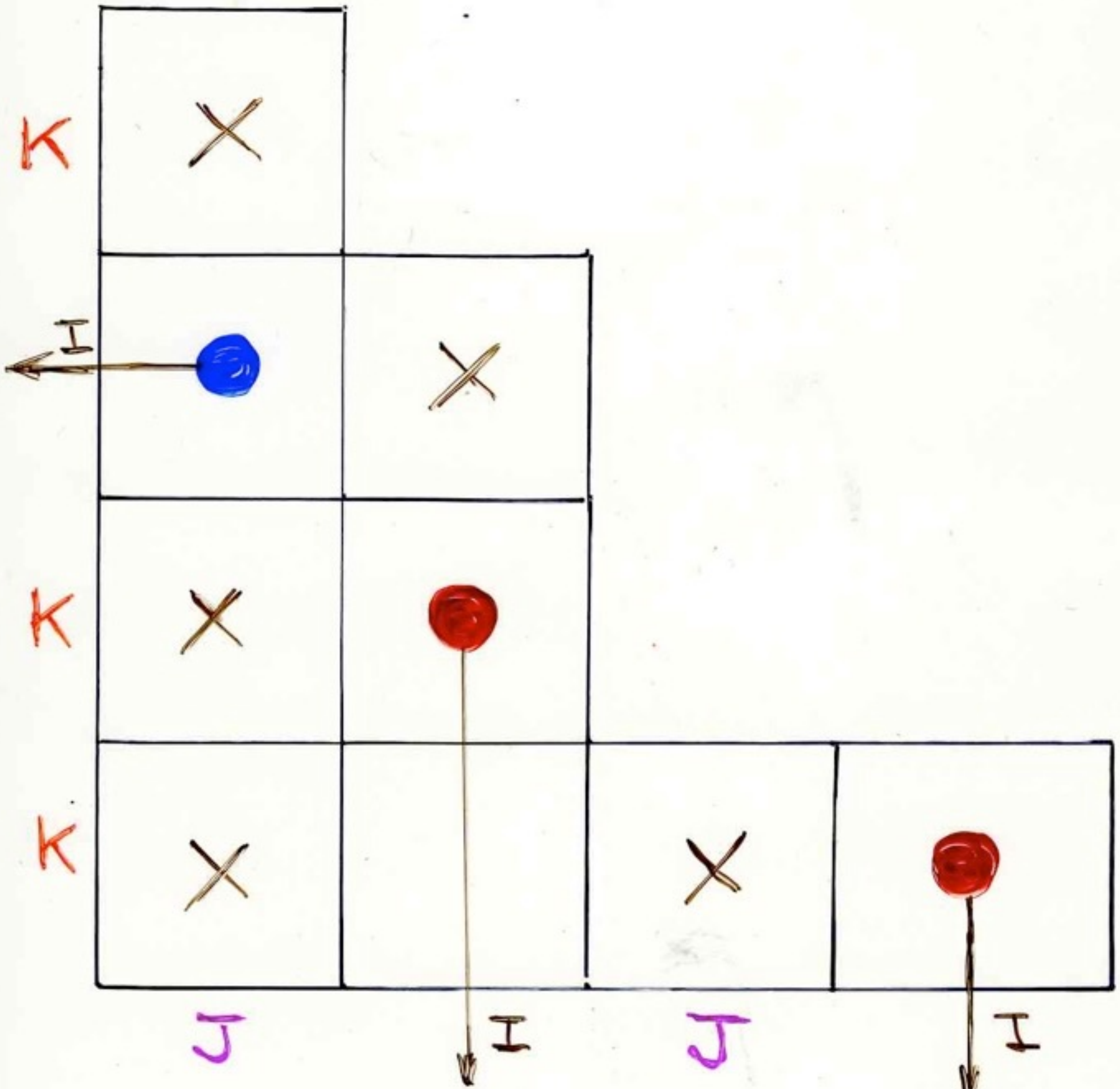
$$DE = (A+J)(S+K)$$

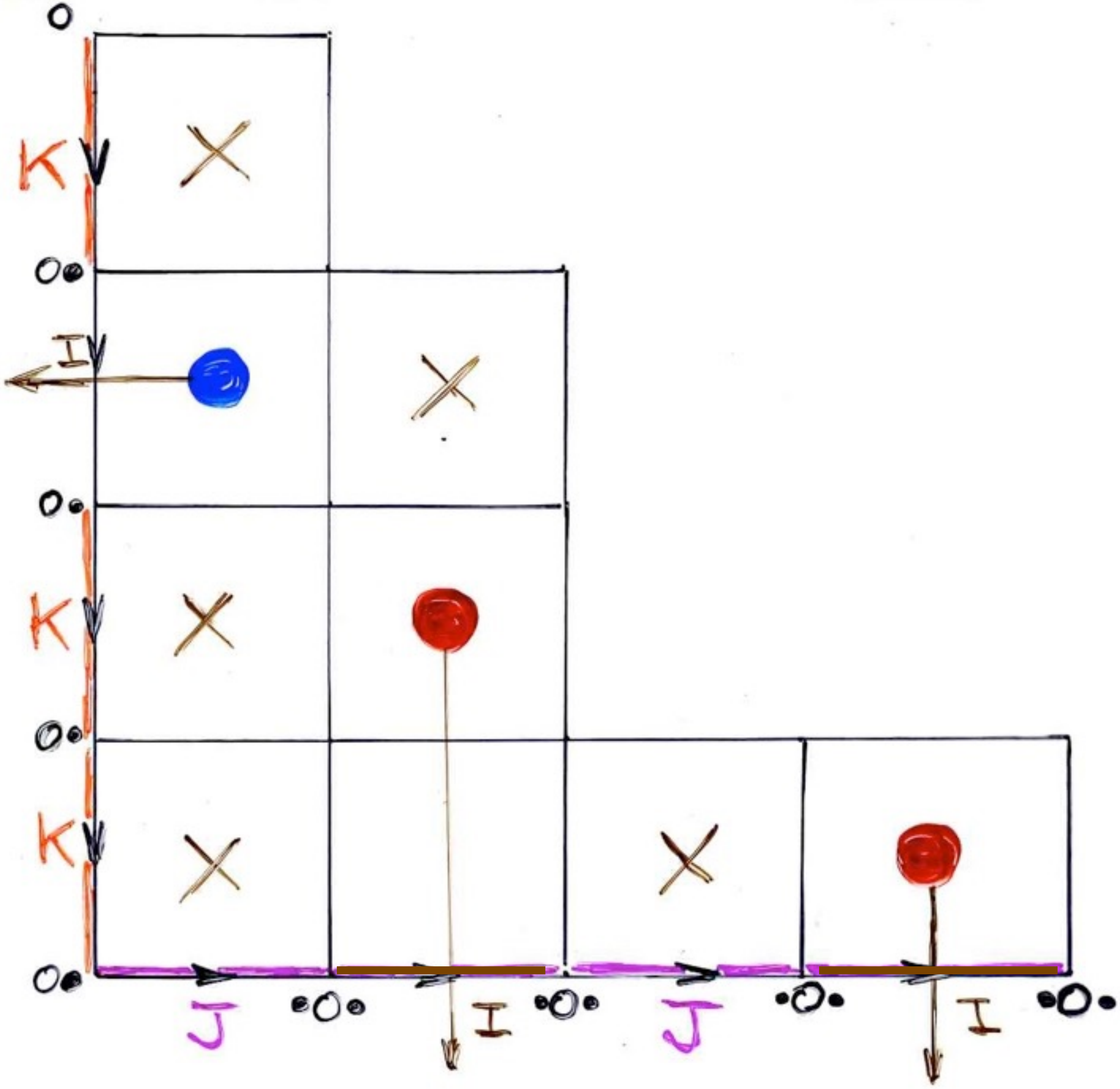
$$= AS + AK + JS + JK$$

$$= \underbrace{\underbrace{(SA + KA + SJ + KJ)}_{(S+K)(A+J)}}_{ED} + \underbrace{J + K + A + S}_{E + D}$$

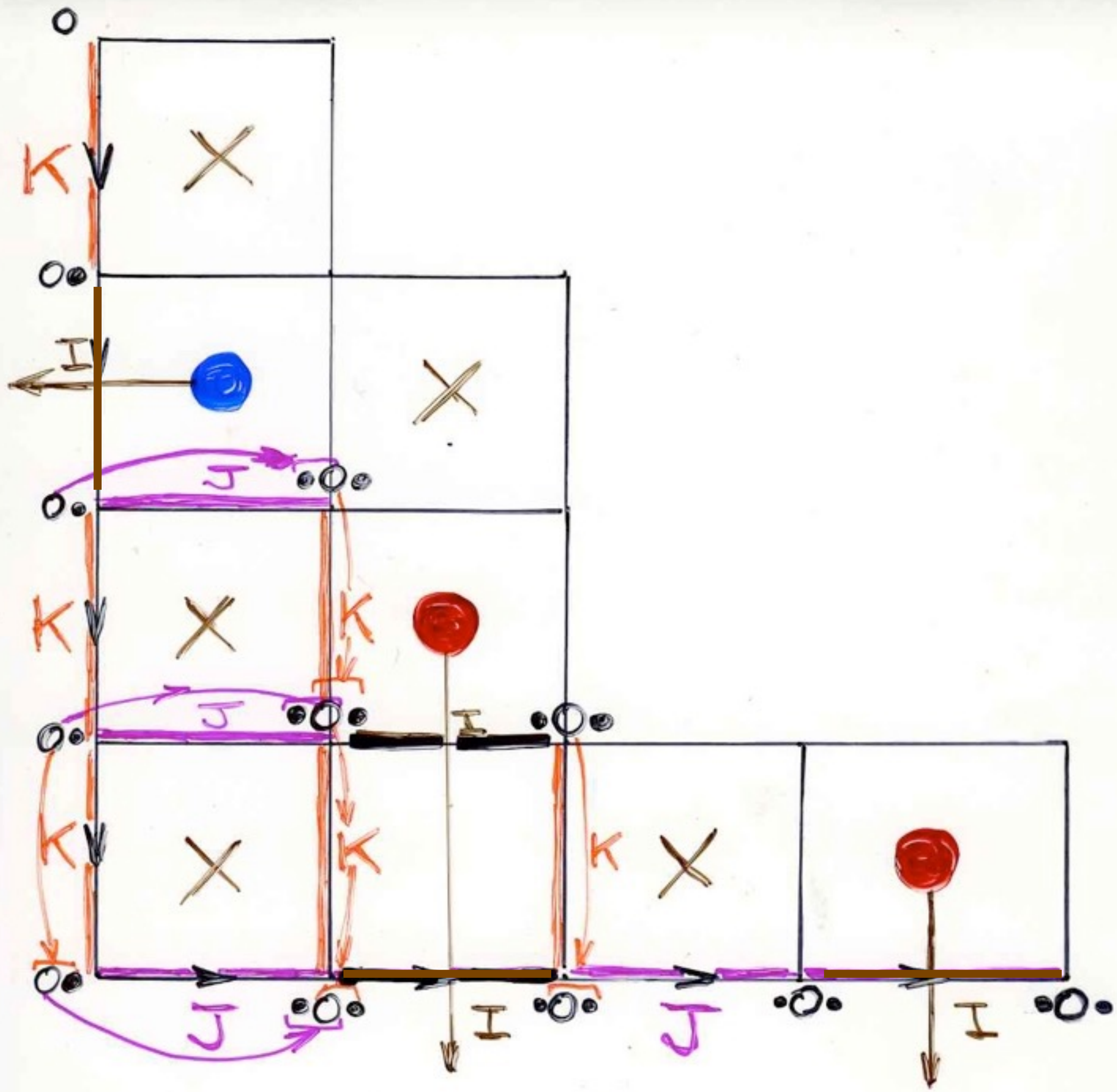





K
I
K
K
J
I
J
I



K
I
K
K
J
I
J
I


K
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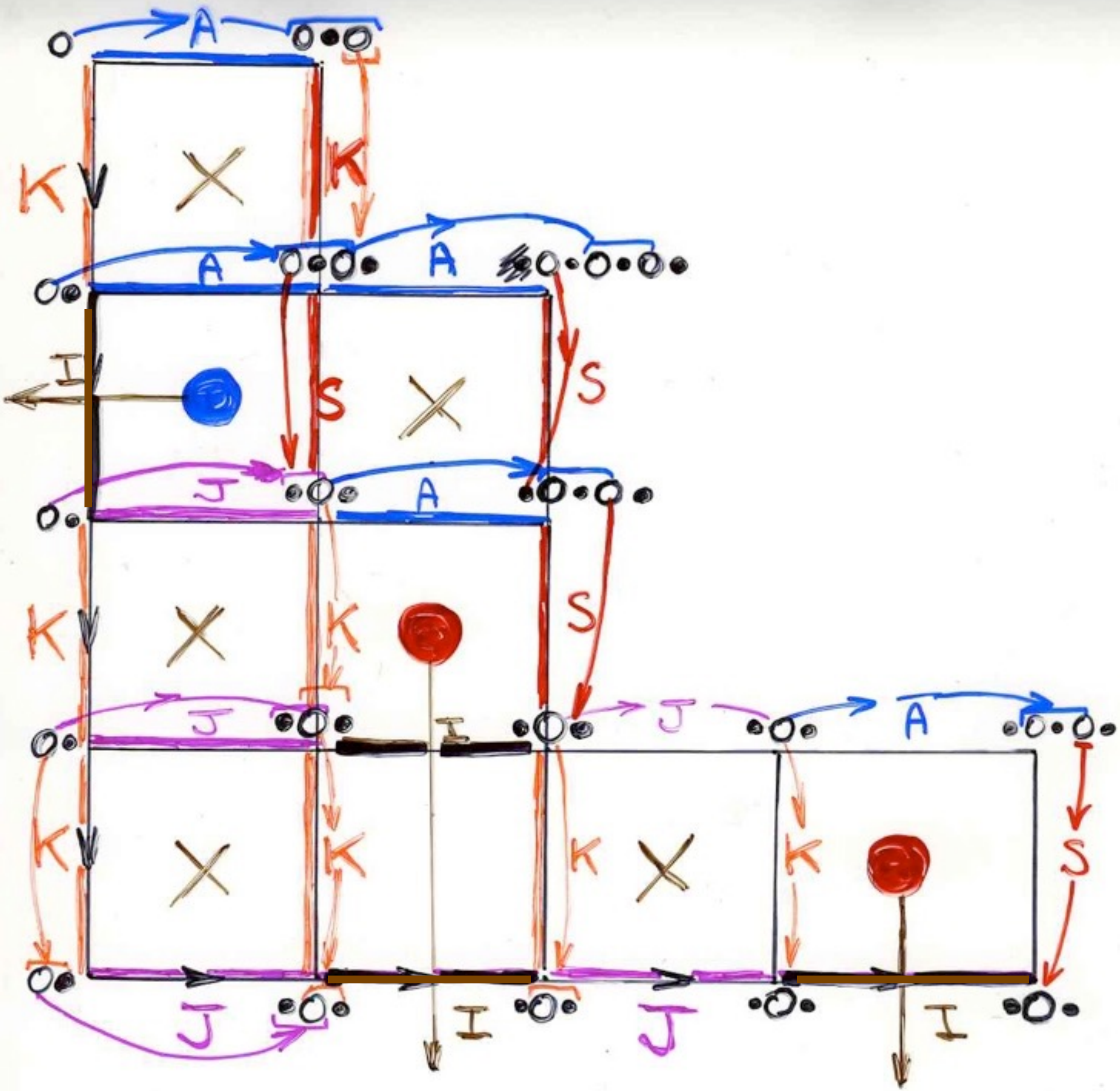
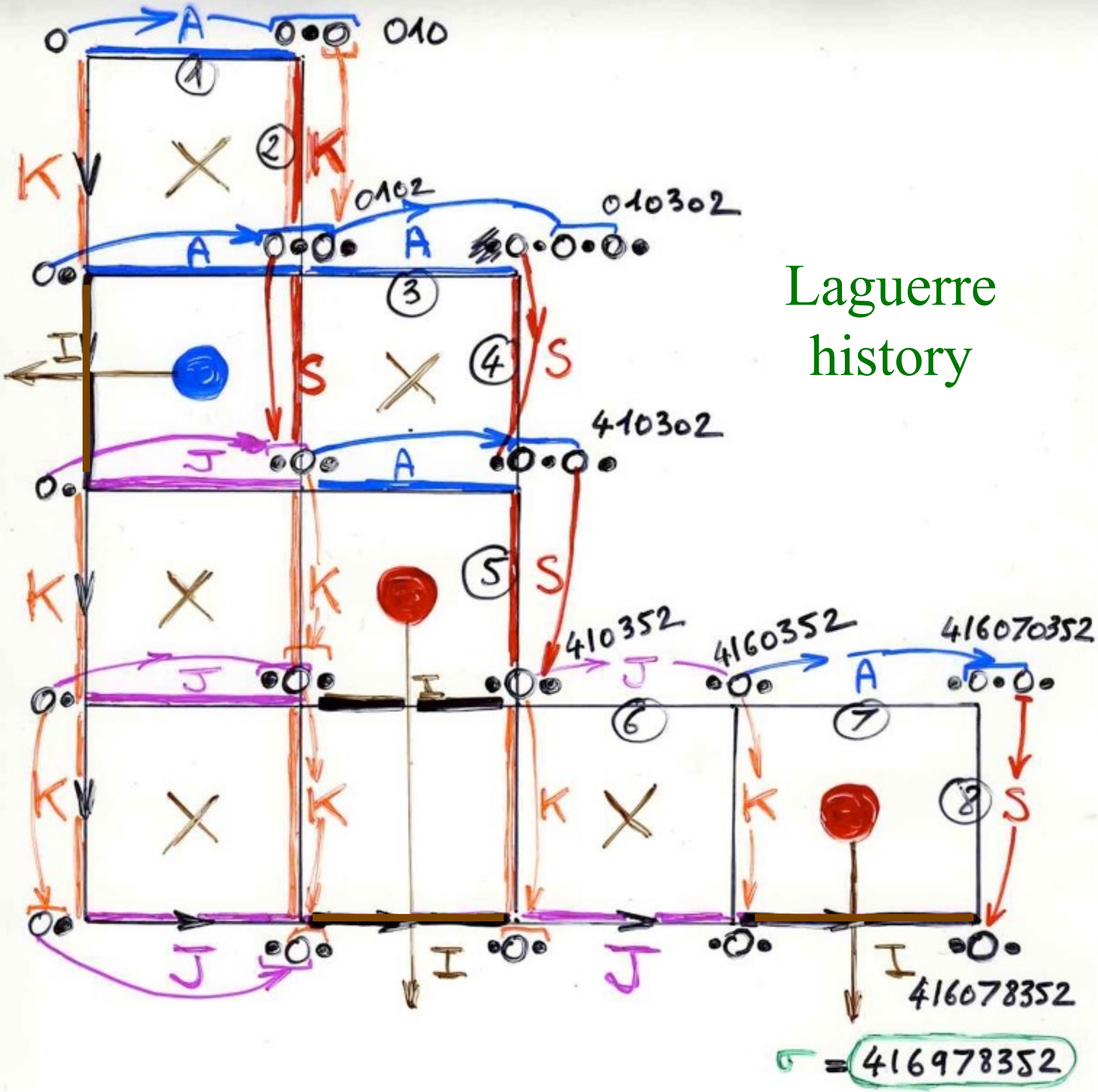
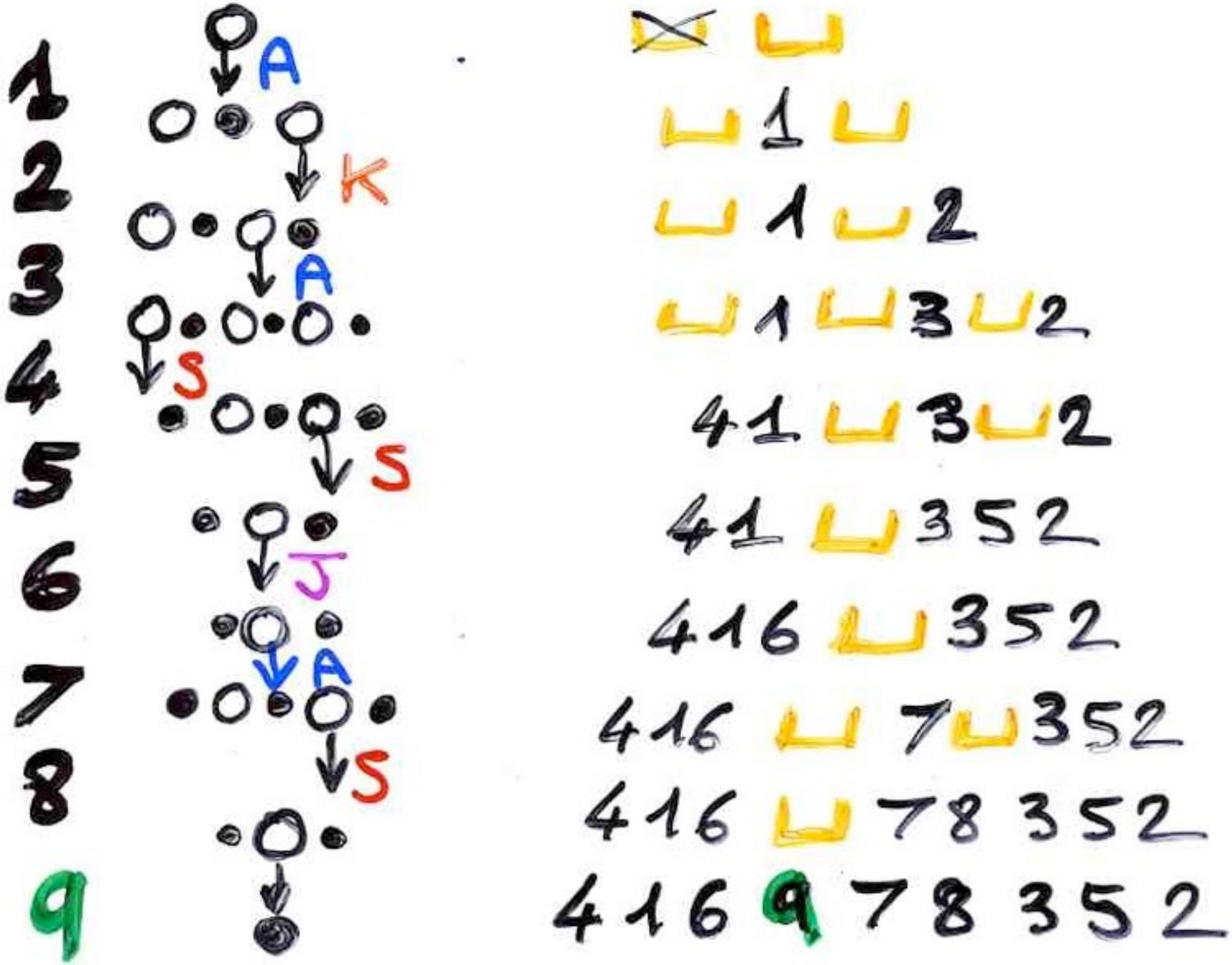
I


A
K
A
A
I
S
S
J
A
K
K
S
J
J
A
K
K
K
K
S
J
I
J
I

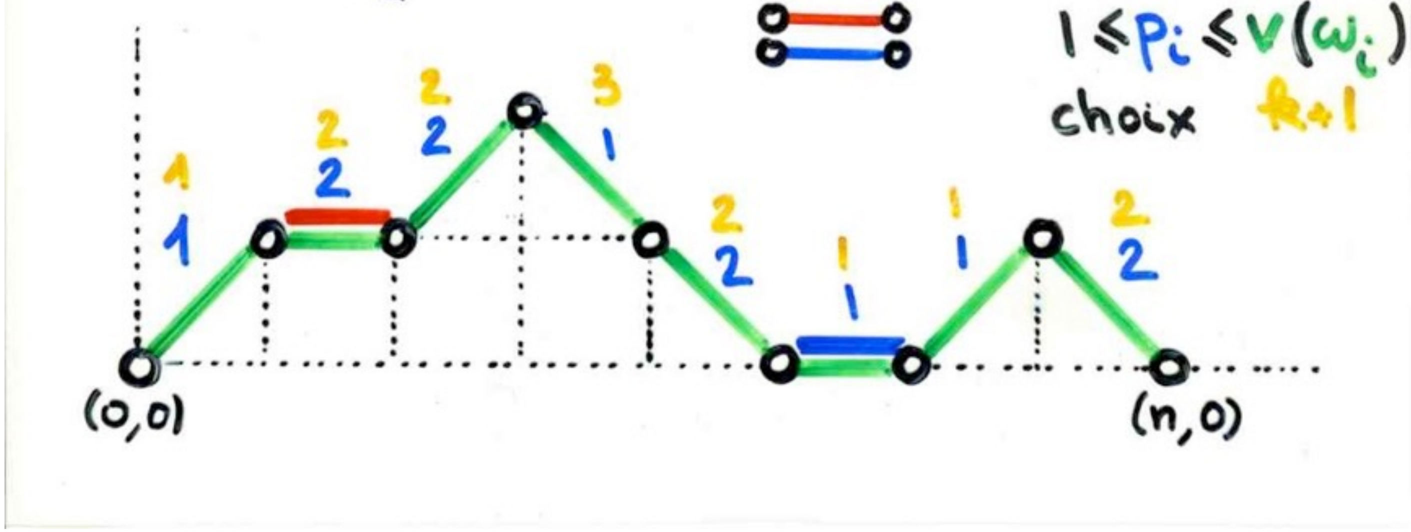
Laguerre history



Laguerre history



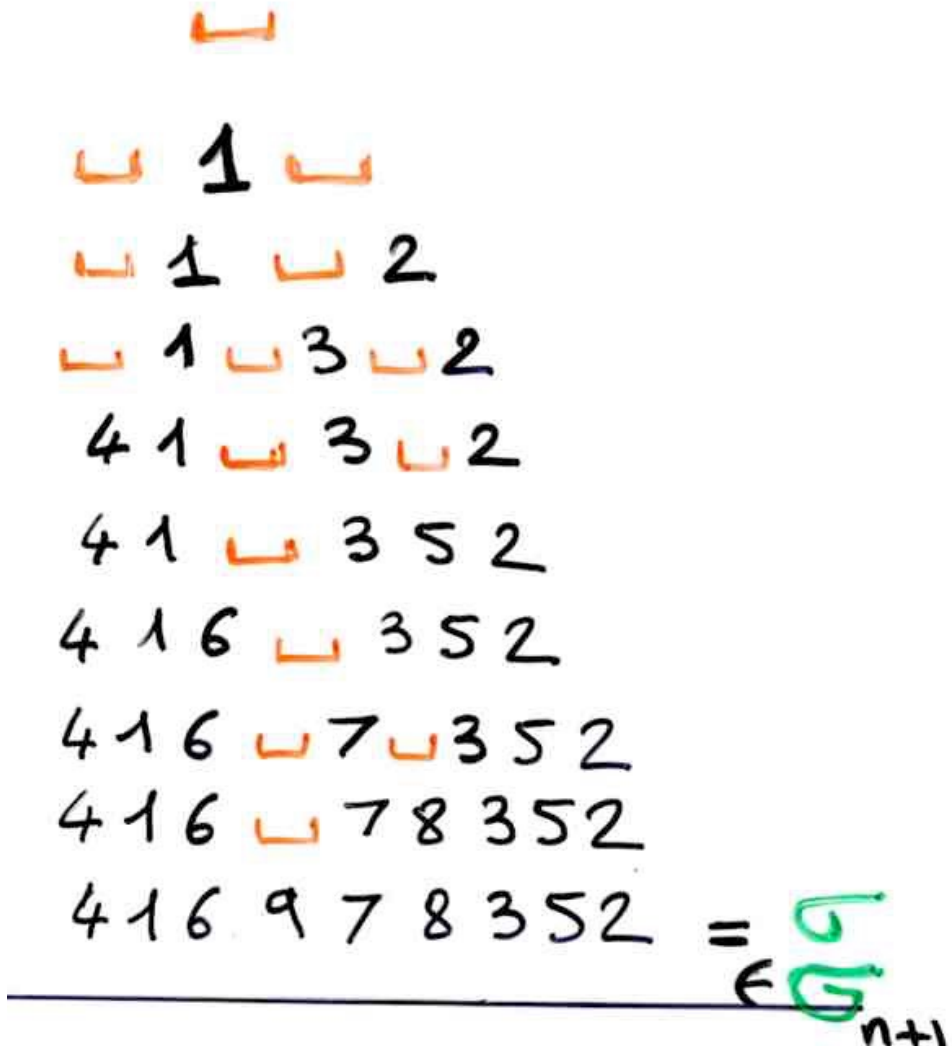
“q-analogue” of Laguerre histories



choice function

1	2	3	4	5	6	7	8
1	2	2	1	2	1	1	2
0	1	1	0	1	0	0	1

q-Laguerre : q^4



q-Laguerre

$$L_n^{I\,\beta}(x;q)$$

$\beta = \alpha + 1$

$$\begin{cases} b_{k,q}^{(\beta)} = [k]_q + [k+1;\beta]_q \\ \lambda_{k,q}^{(\beta)} = [k]_q \cdot [k;\beta]_q \end{cases}$$

$$[k;\beta]_q = \beta + q + q^2 + \dots + q^{k-1}$$

$$\mu_n = \frac{1}{(1-q)^n} \sum_{k=0}^{n} (-1)^k \left(\binom{2n}{n-k} - \binom{2n}{n-k-2} \right) \left(\sum_{i=0}^{k} q^{i(k+1-i)} \right)$$

Corteel, Josuat-Vergès y
Prellberg, Rubey (2008)

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2-PASEP

rhombic alternative tableaux ⟷ assemblée of permutations

quadratic algebra Q

Q-tableaux

commutations rewriting rules

planarisation

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commutations
rewriting rules

ex: ASM,
(alternating sign matrices)
FPL (fully packed loops)
tilings, 8-vertex

?

planar
automata

www.xavierviennot.org

L' ansatz cellulaire:

- petite école 2011/2012
- cours IIT Bombay 203

cours.xavierviennot.org

voir aussi un résumé de la théorie avec l'exposé

(page « exposés » du site principal www.xavierviennot.org)

From automata to RSK correspondence

Journées Christophe Reutenauer, «Words, Codes and Algebraic Combinatorics», Cetraro, Italy, July 2013 slides (pdf, 32,5 Mo)

Pub:

cours IMSc, Channai, Inde, Janvier-Mars 2016

« Introduction à la combinatoire énumérative, algébrique et bijective »

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Merci !