The beauty of mathematics

part III: probabilities and physics, the TASEP model

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§6 bijections and Catalan numbers

probabilities and physics

The TASEP model

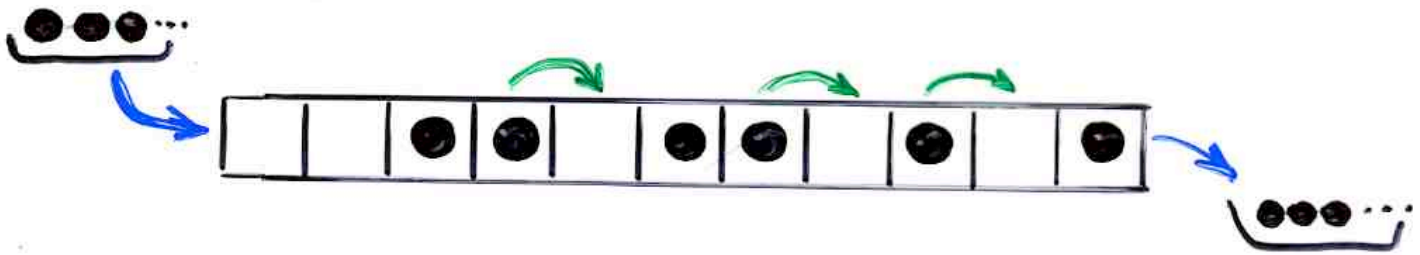
totally asymmetric exclusion process

TASEP

"totally asymmetric exclusion process"

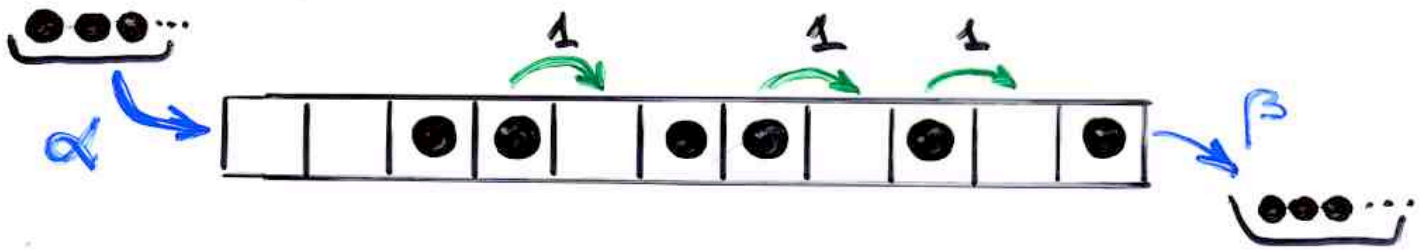
TASEP

"totally asymmetric exclusion process"



TASEP

"totally asymmetric exclusion process"



modèle discret $t = 0, 1, 2, \ldots, i, \ldots$

modèle continu dt

modèle de gaz avec exclusion

physique statistique

systèmes hors de l'équilibre

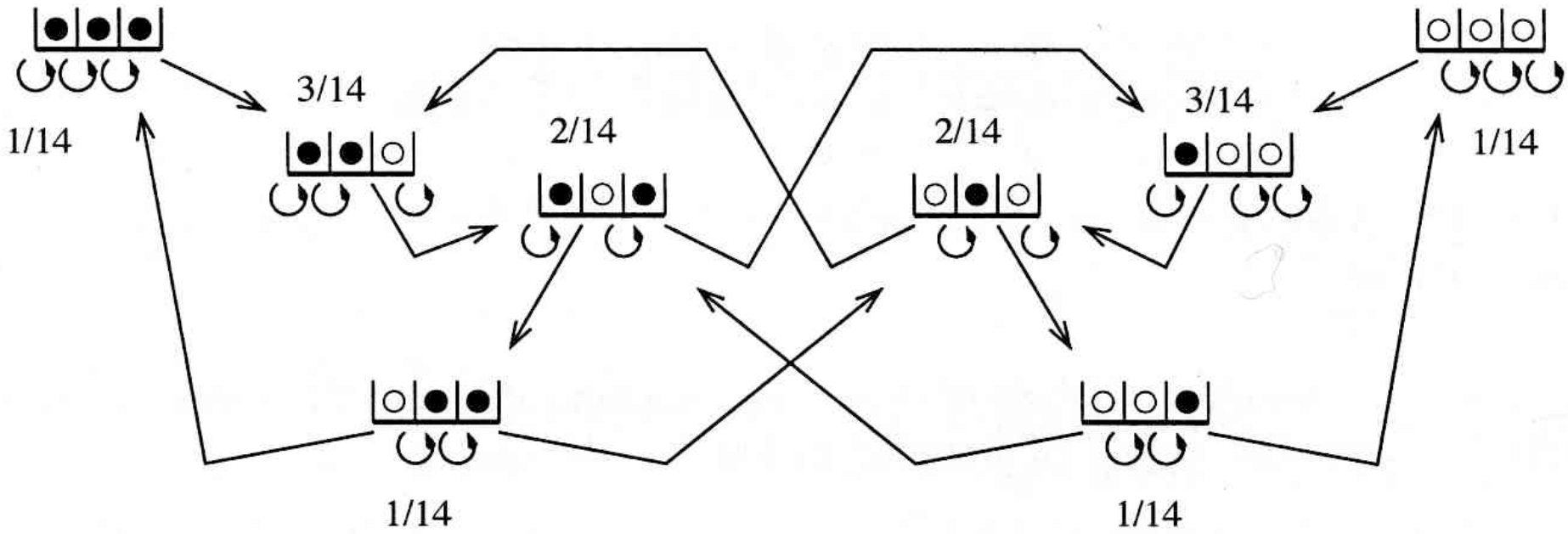
modélisation

- biologie moléculaire
  diffusion enzymes
- flots trafic ...
- formation de décharges

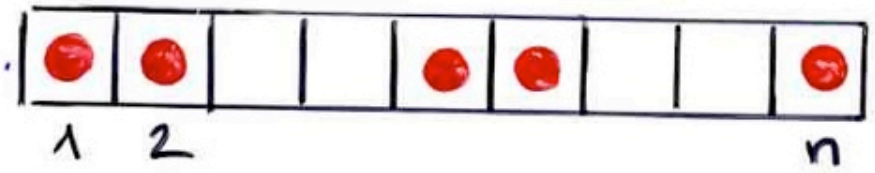
- physique statistique
  transition de phases
  $\alpha = \frac{1}{2}$ $\beta = \frac{1}{2}$
- " résoluble "

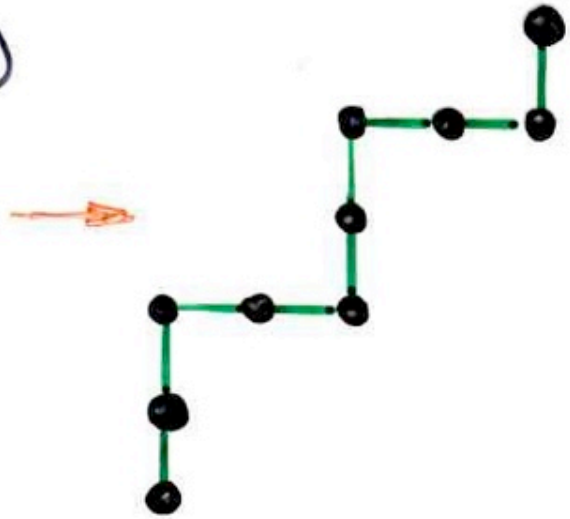
- Combinatoire très riche

stationnary probabilities

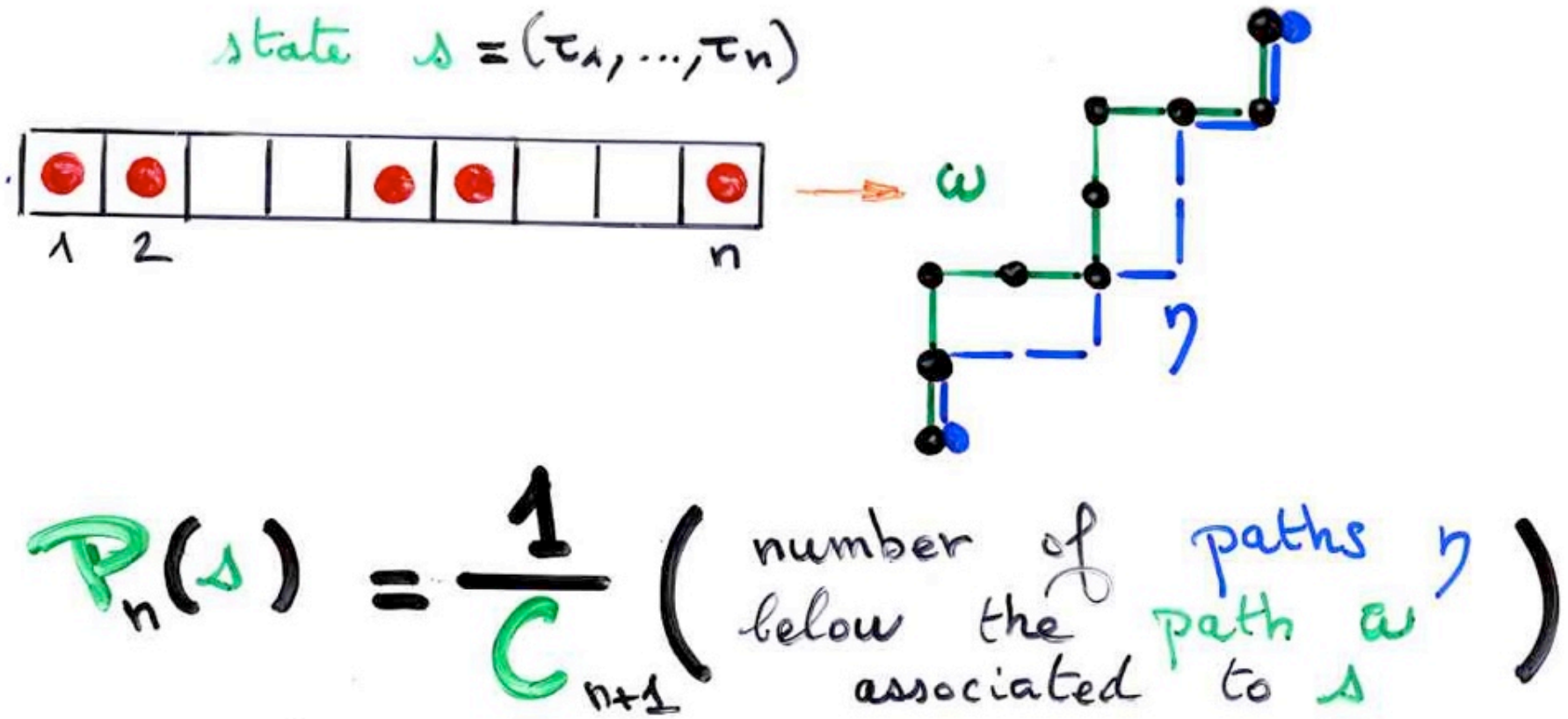

1/14
3/14
2/14
2/14
3/14
1/14
1/14
1/14

state $s = (\tau_1, \ldots, \tau_n)$





$P_n(s) =$

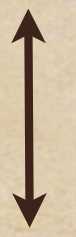


The number of such pairs (ϖ, η) of paths of length n is the Catalan number

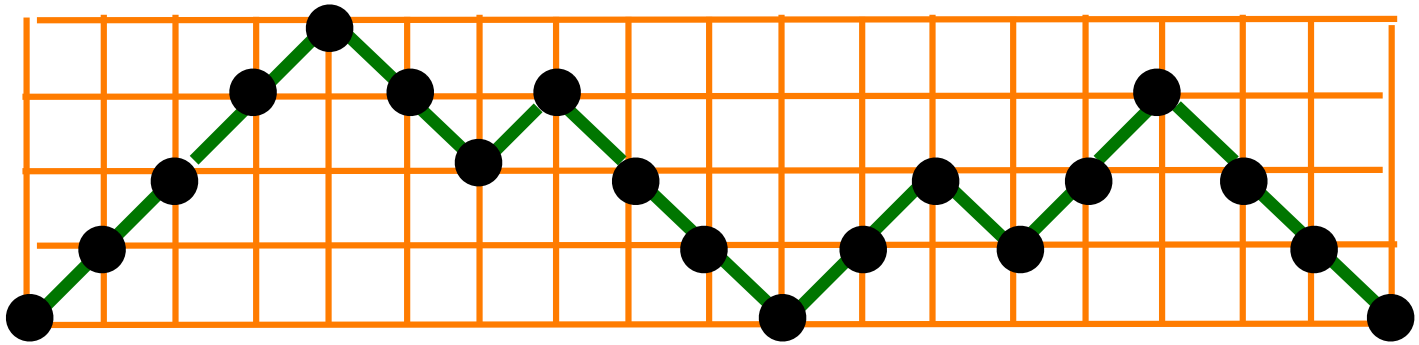
$$C_{n+1}$$

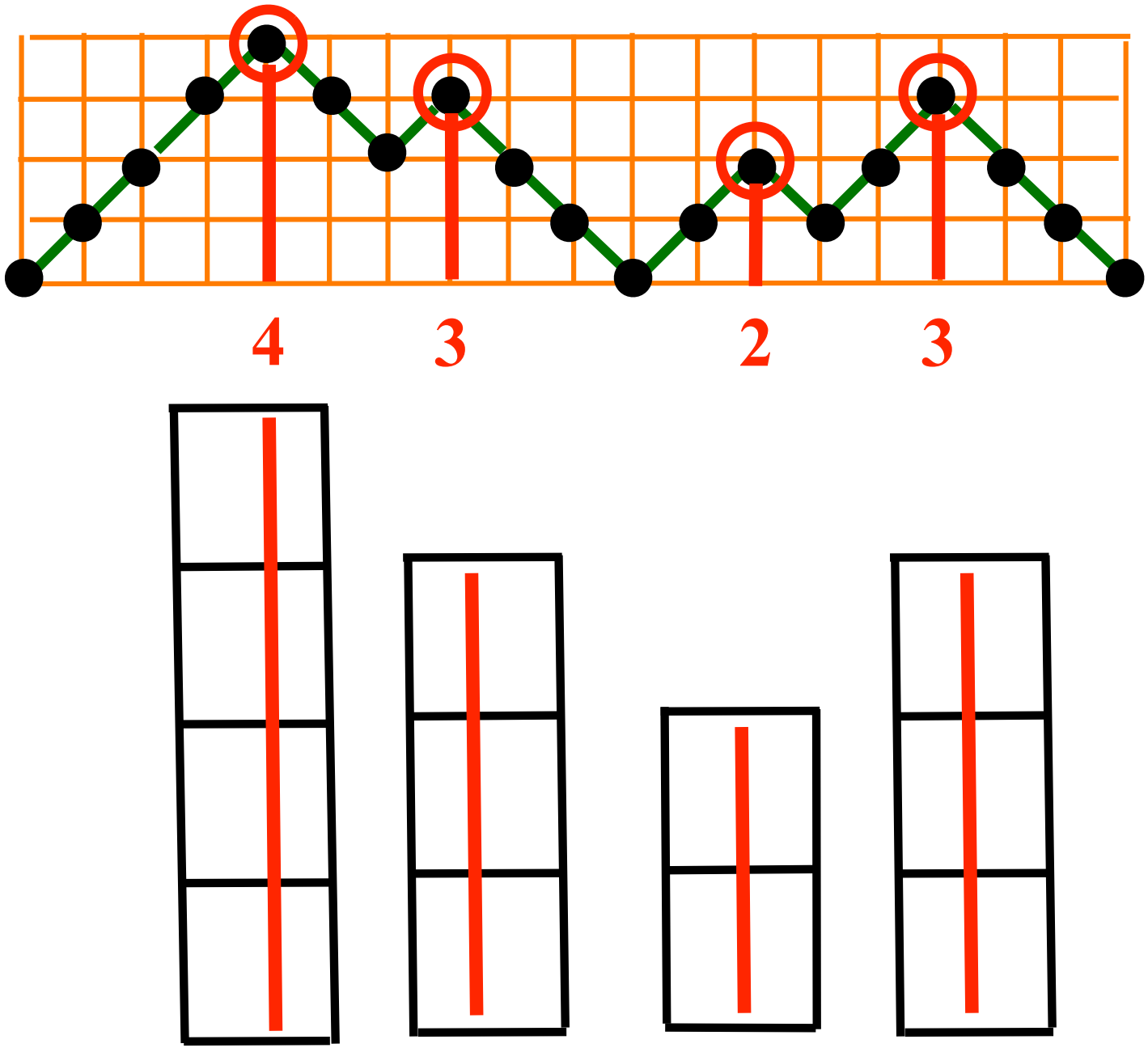
bijection

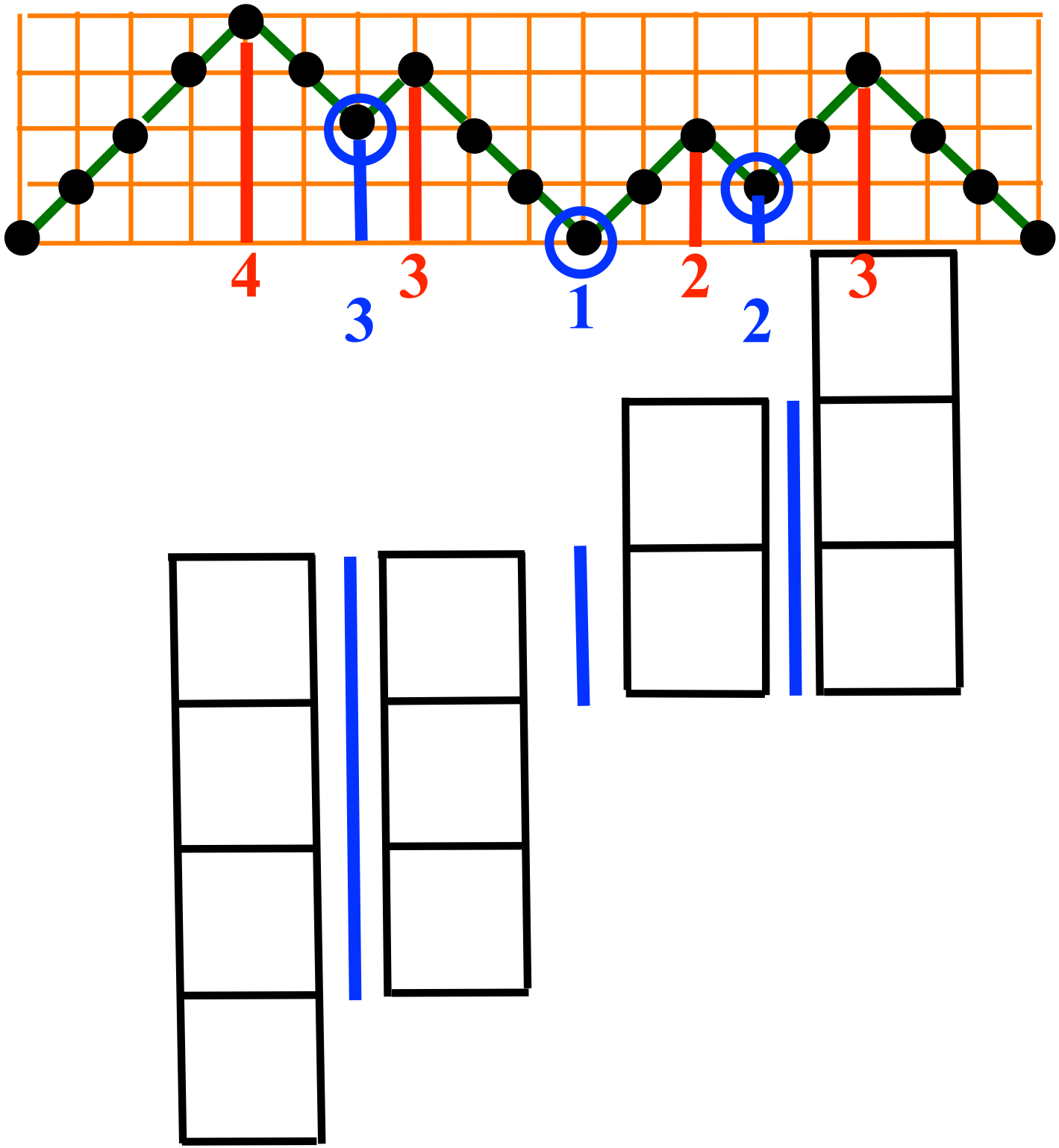
Dyck paths

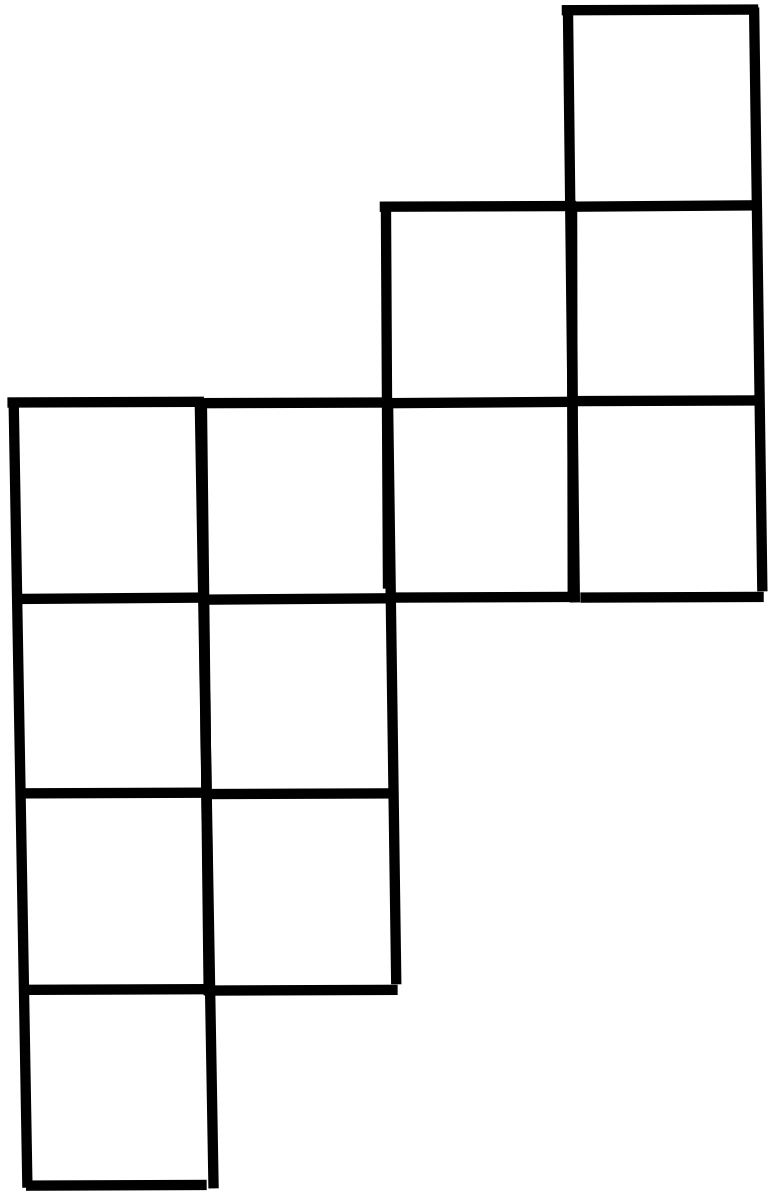


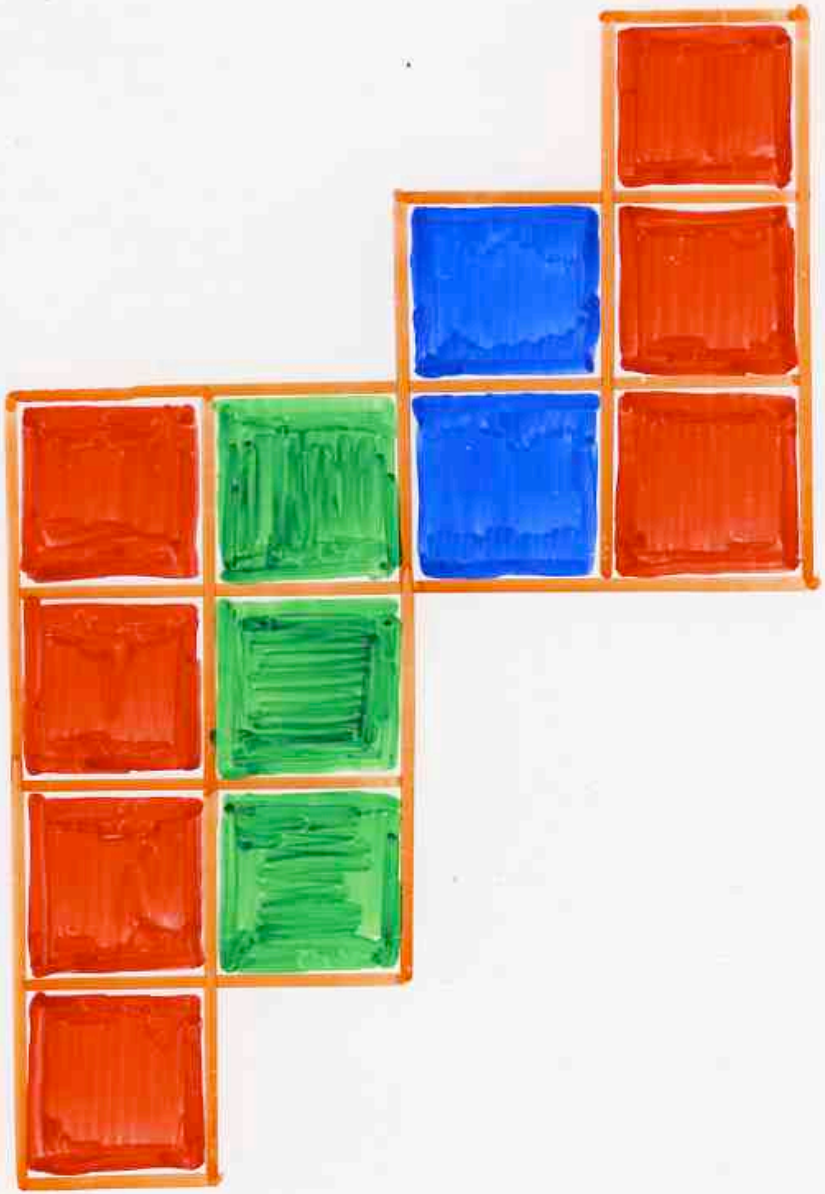
pairs of non-crossing paths

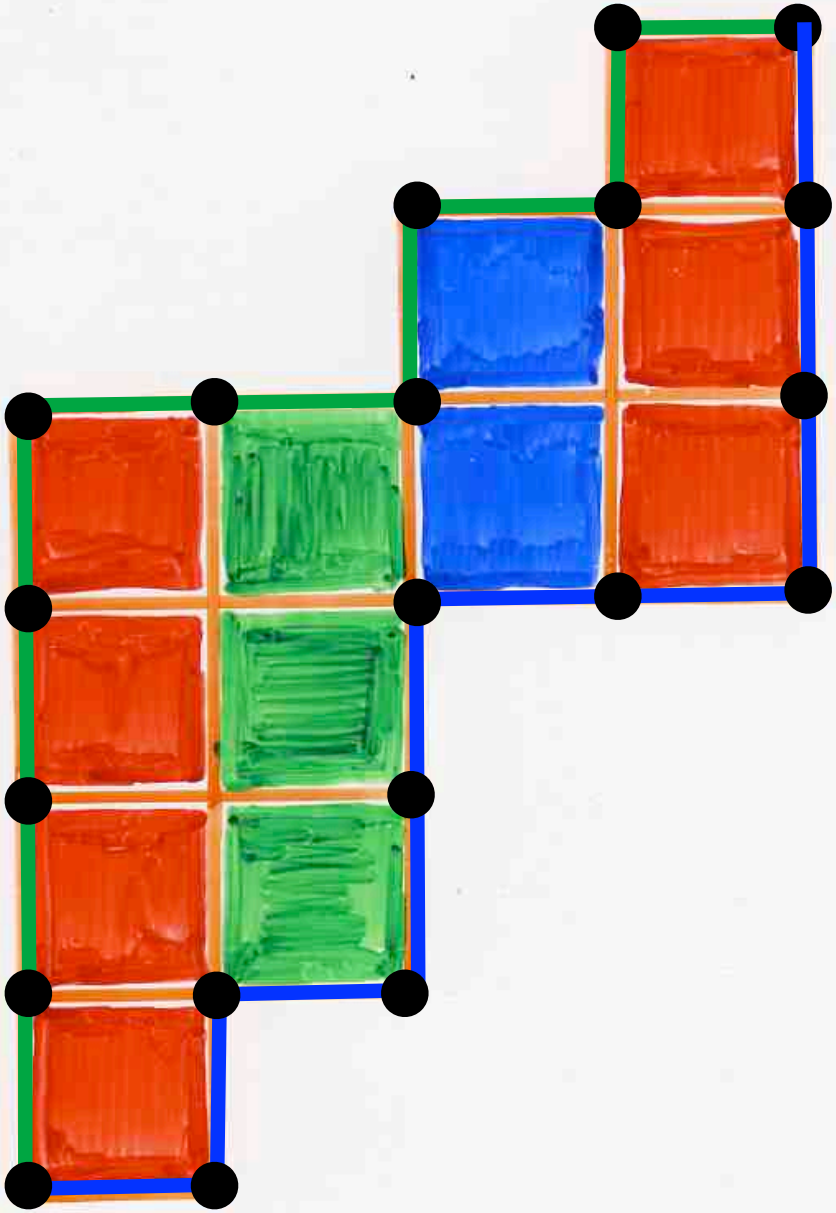


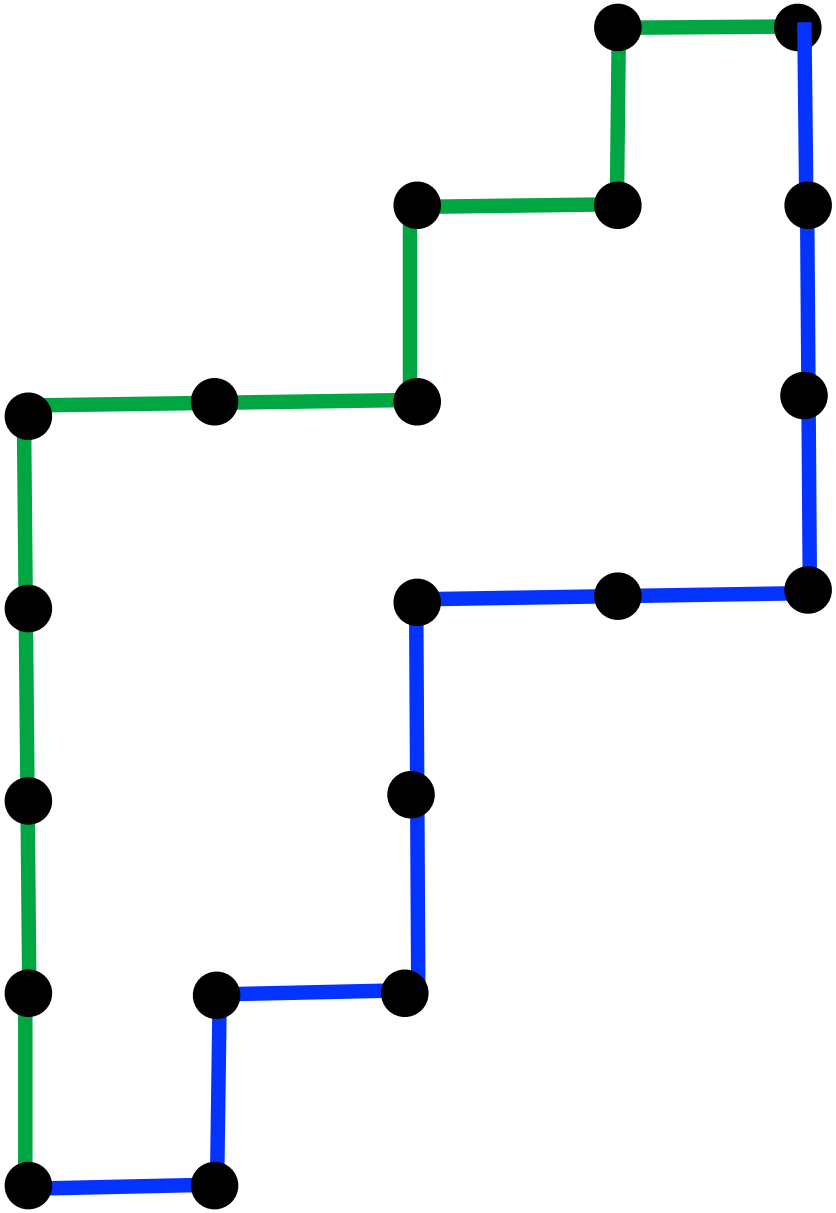

4
3
2
3

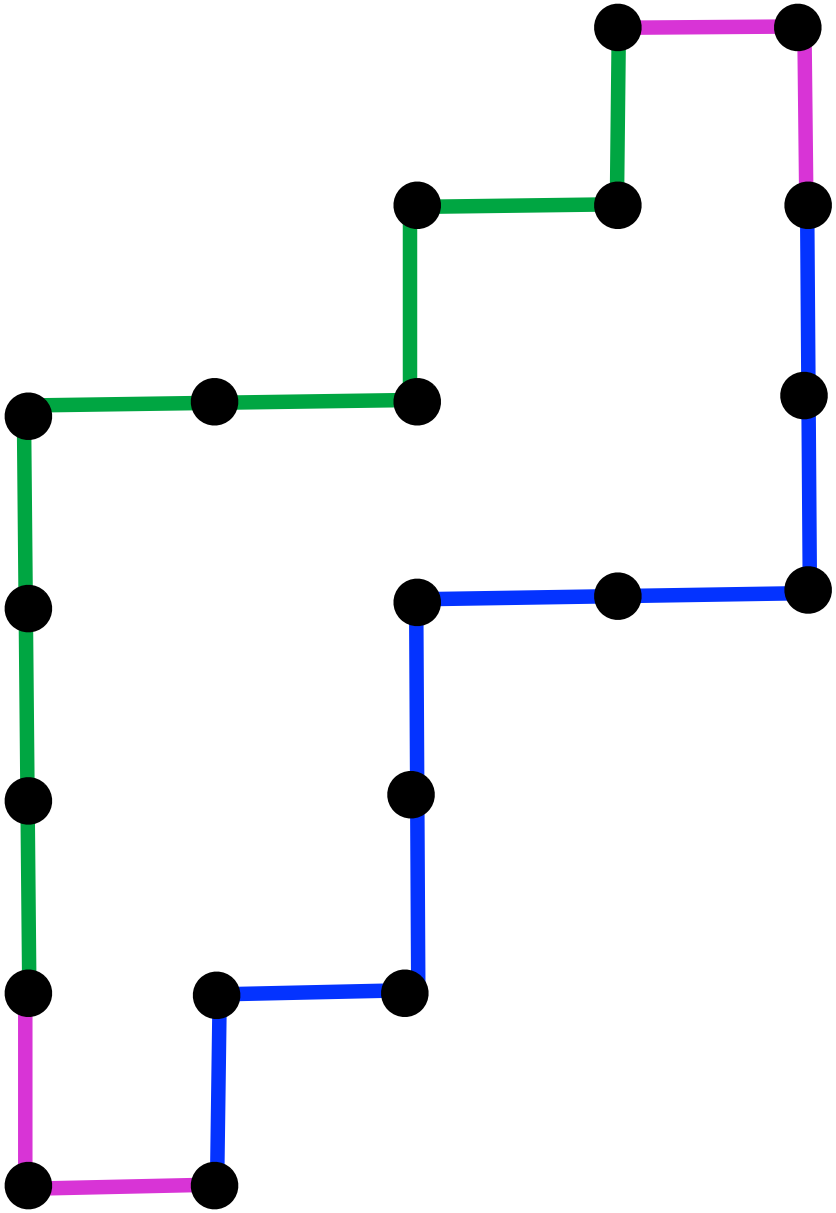

4
3
3
1
2
2
3

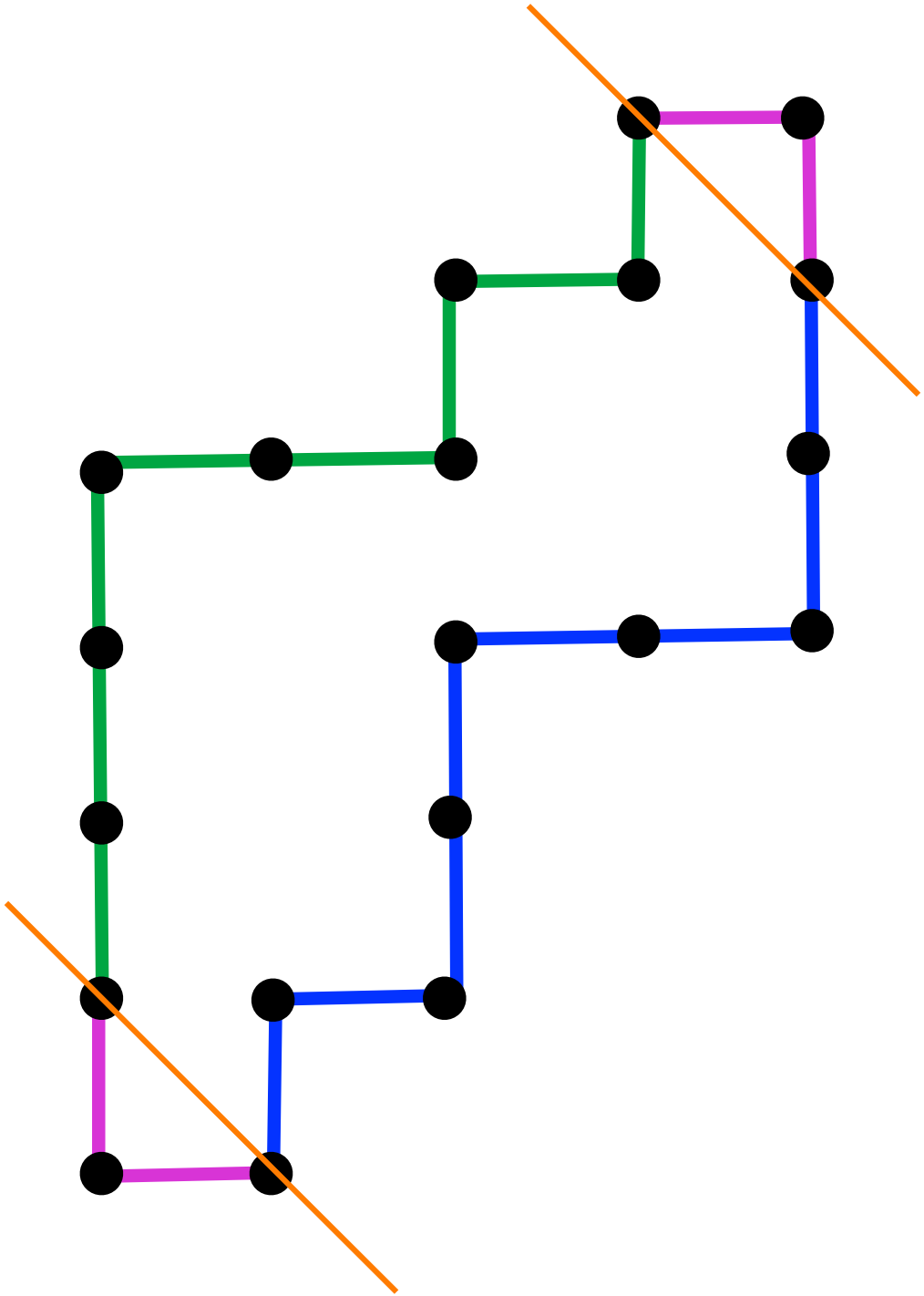


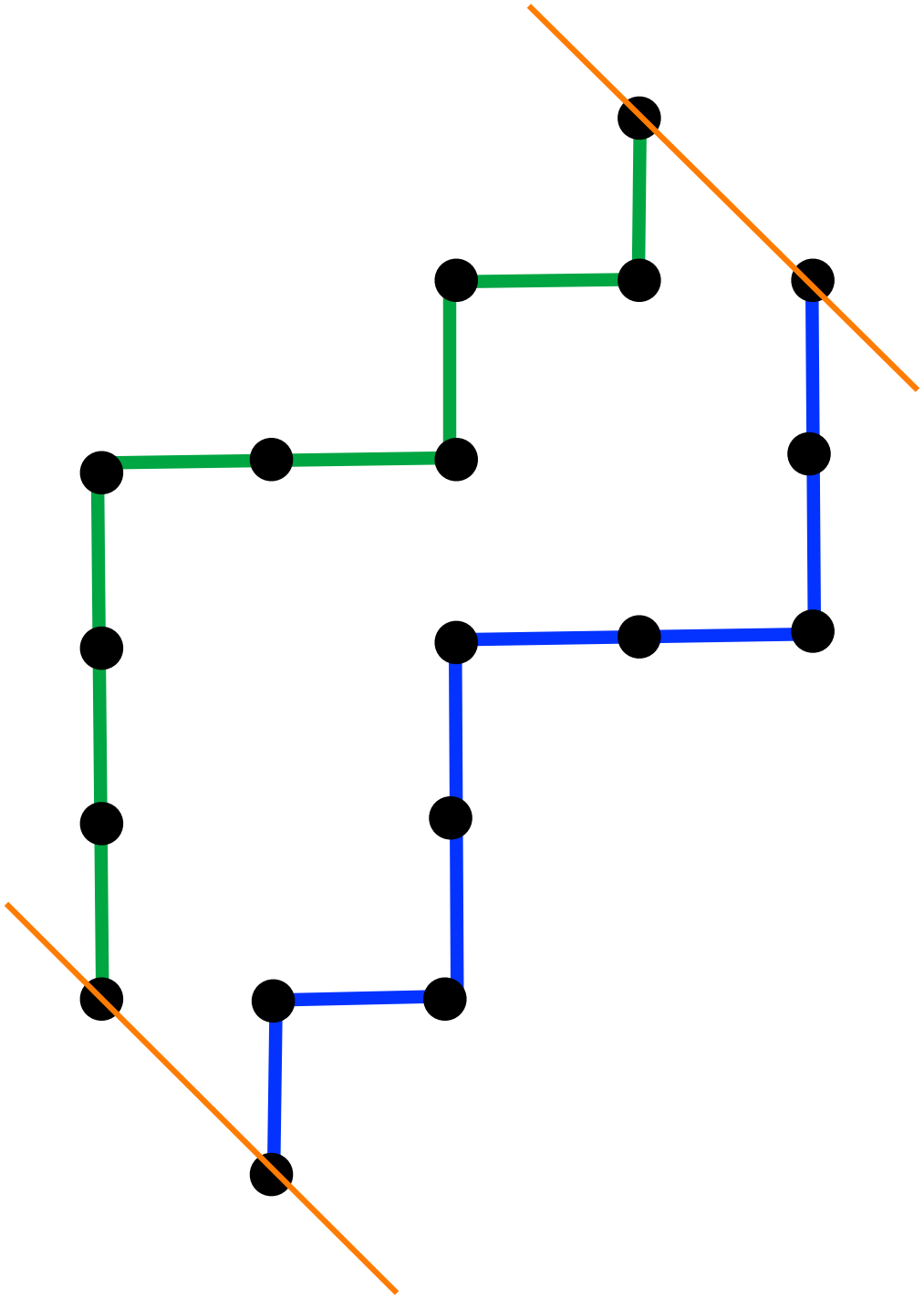


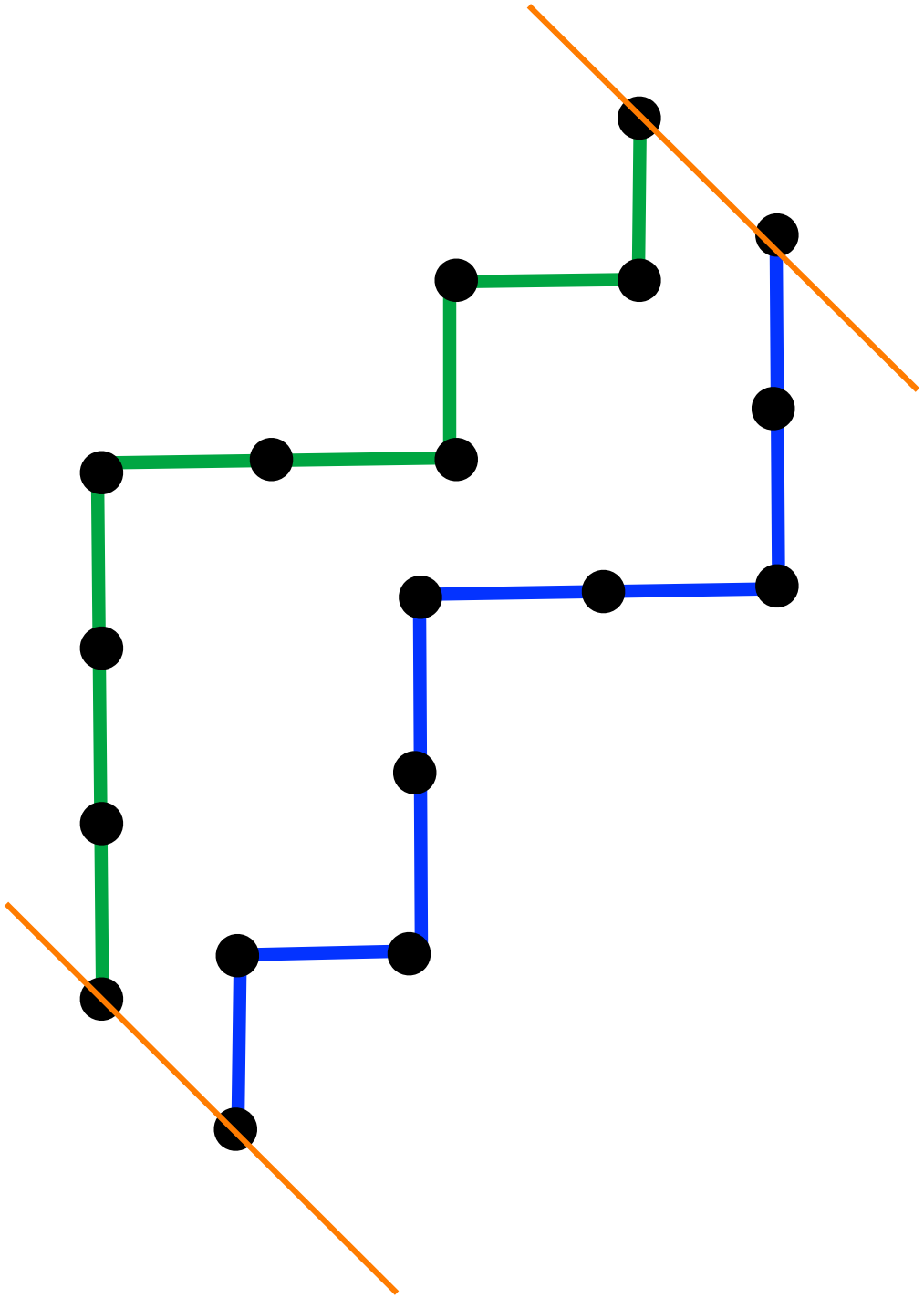


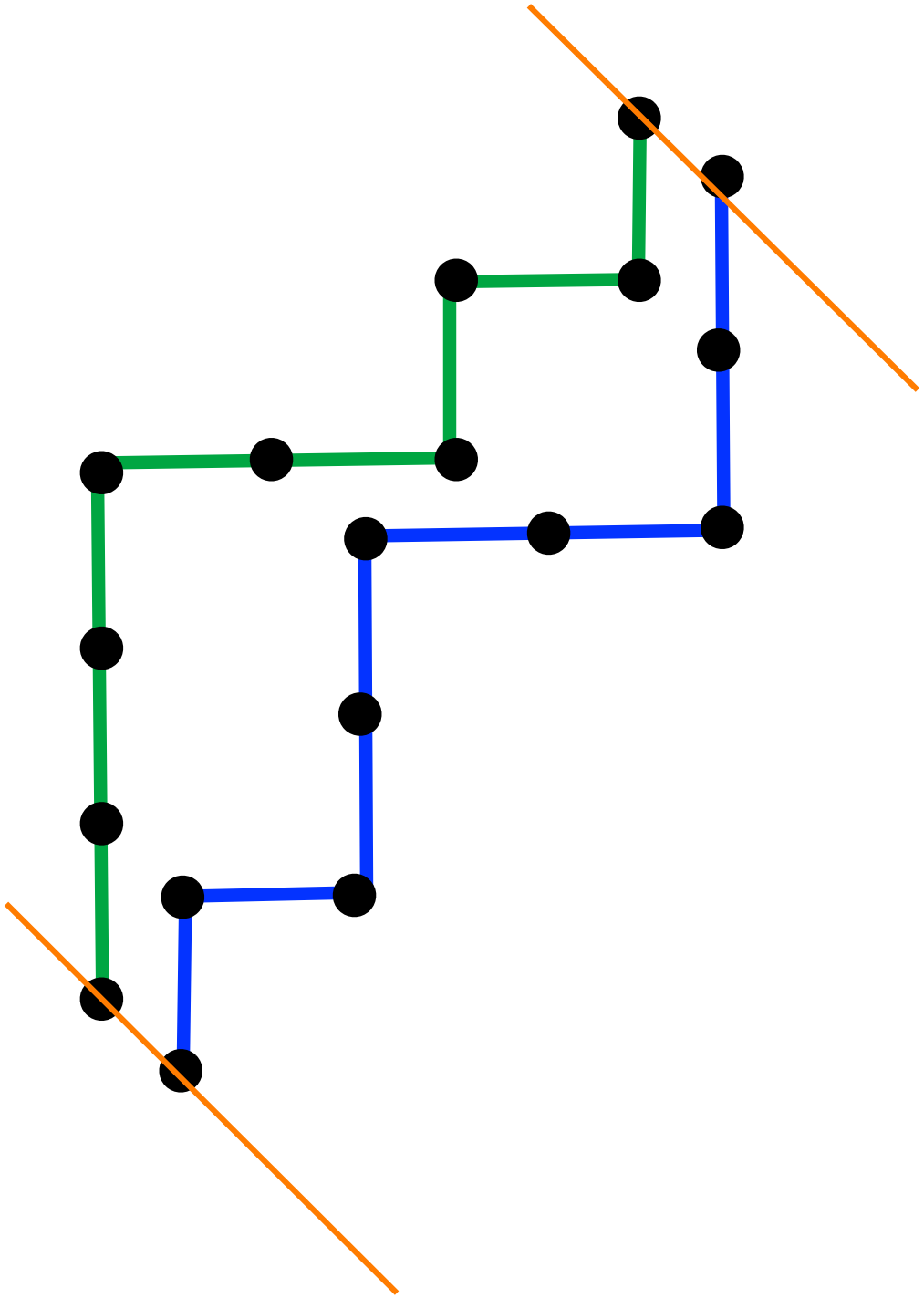


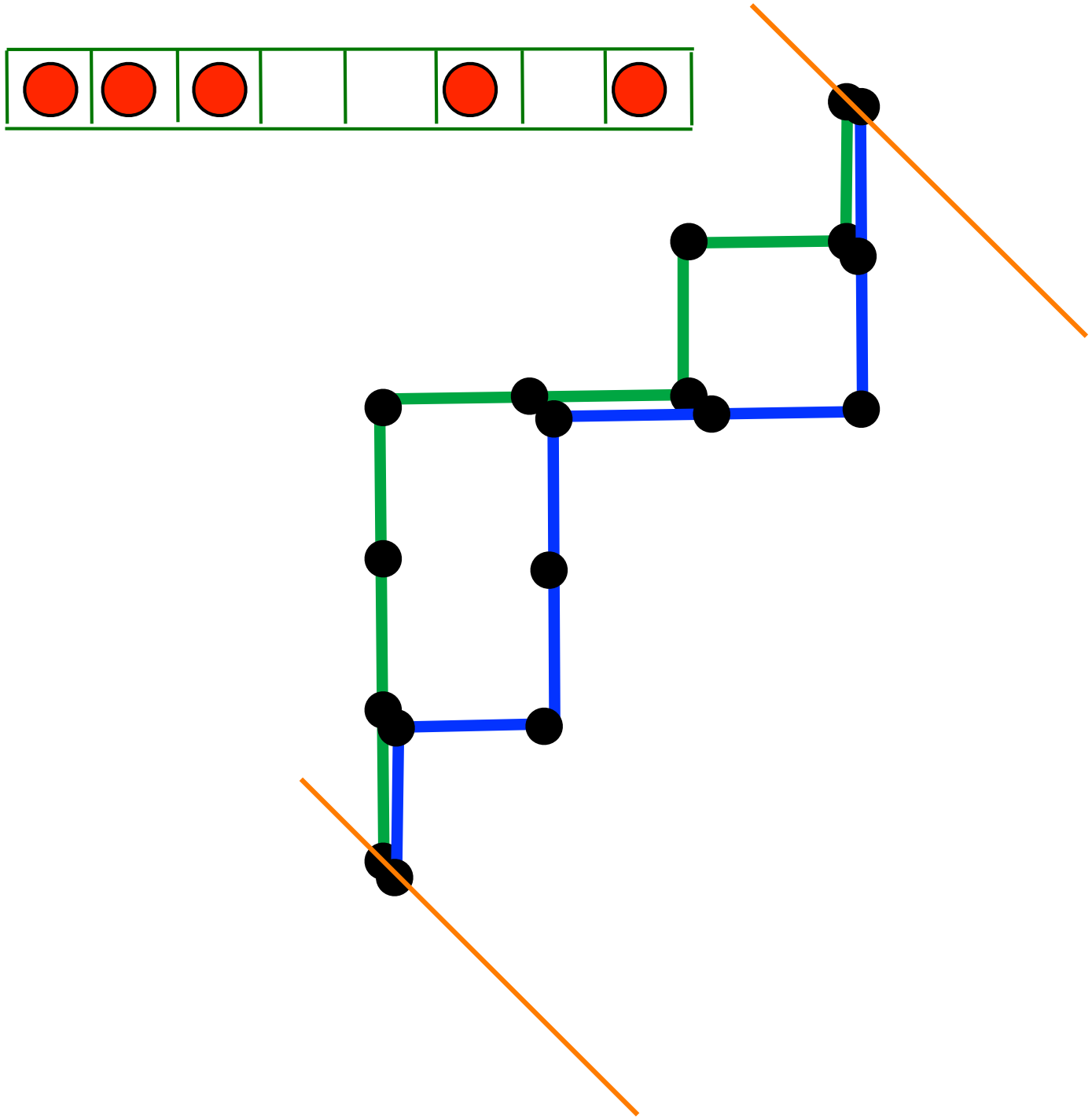


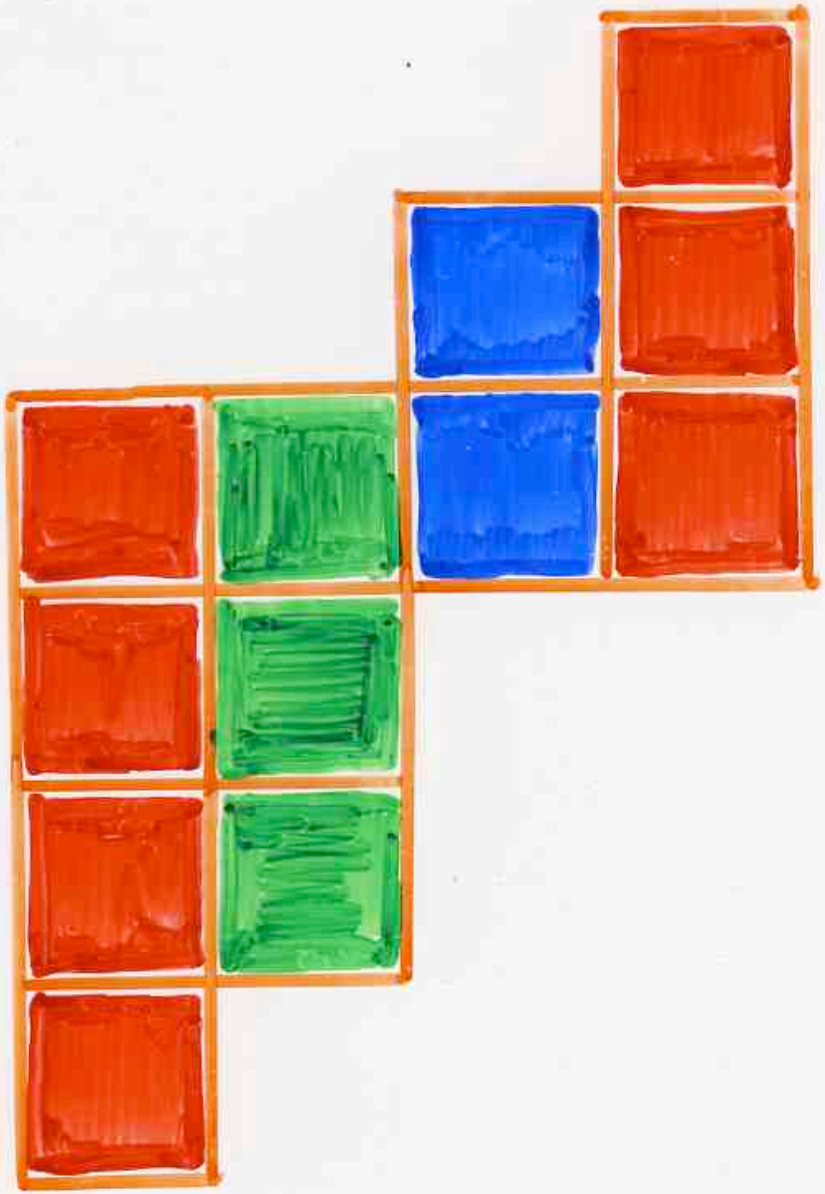


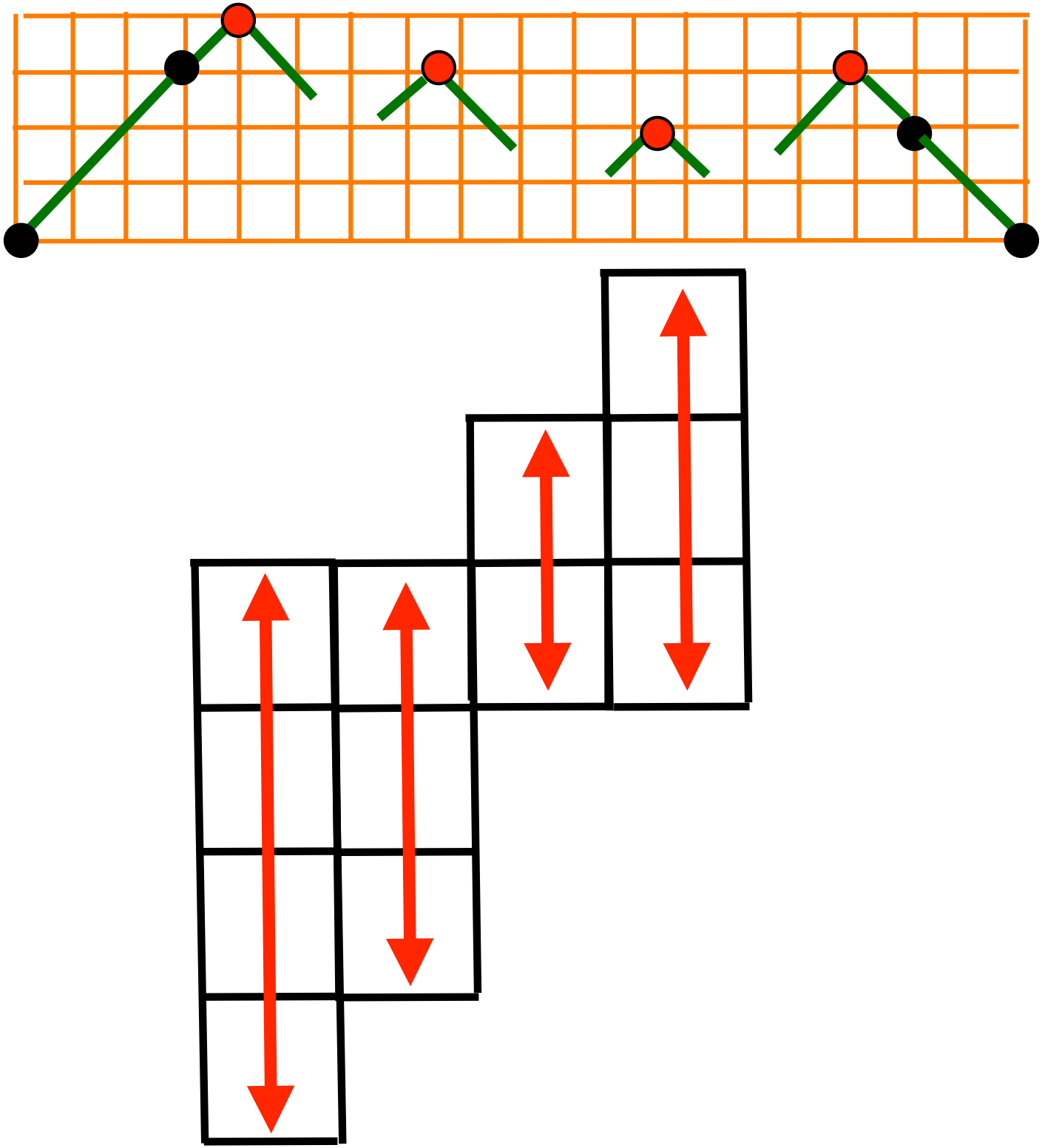


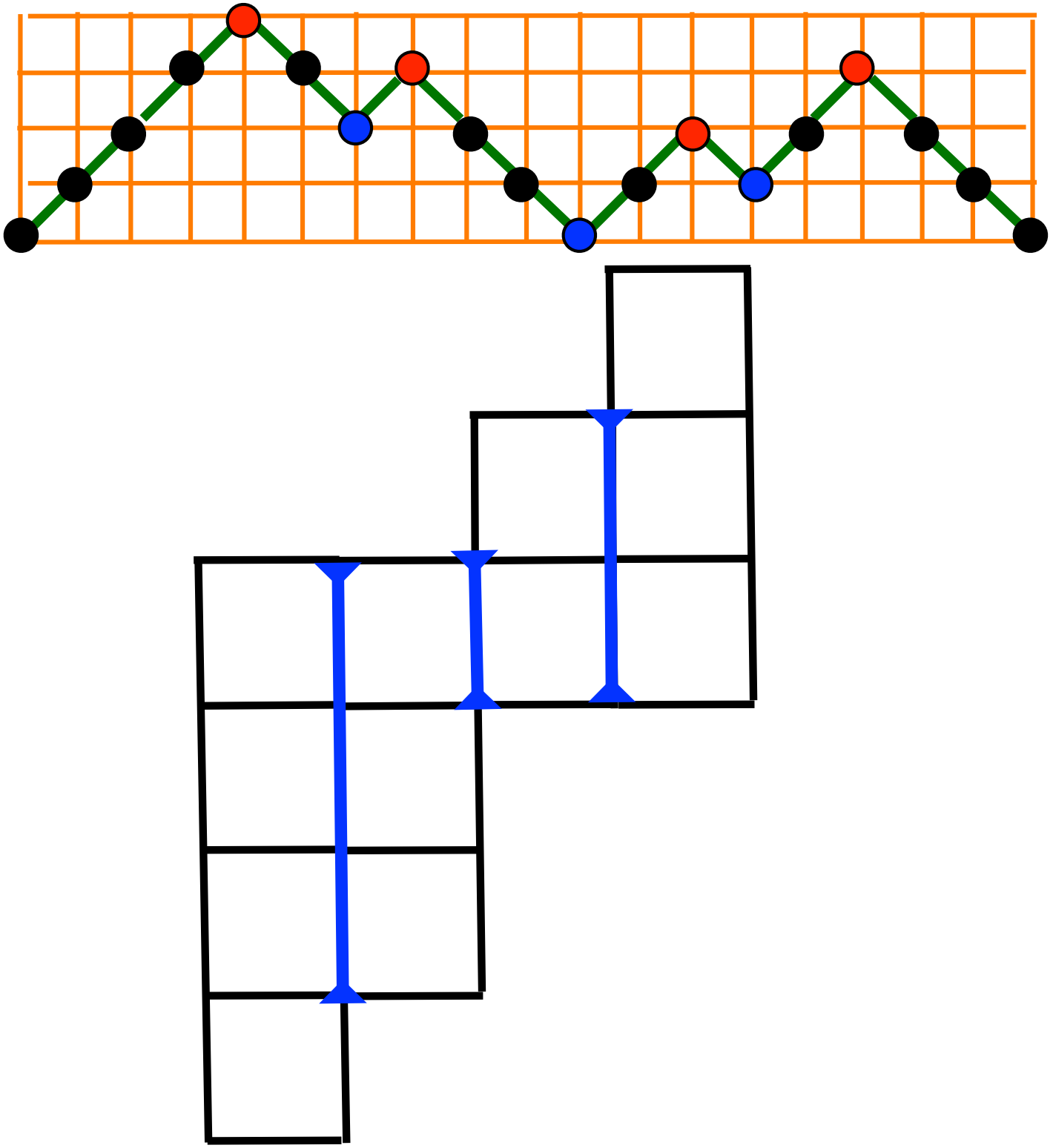






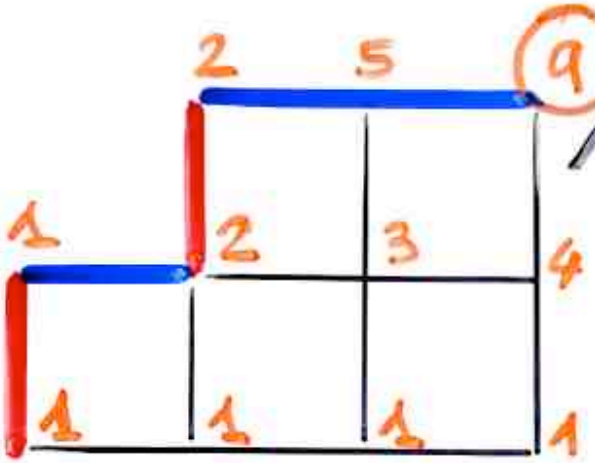




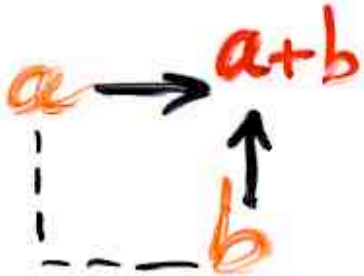


computing the stationnary probabilities

$\Delta = (1,0,1,0,0)$ $\lambda = (1,2,2)$



$/C_6 = P(1,0,1,0,0)$



probabilité

k particules 

$$= \frac{1}{C_{n+1}} \underbrace{\frac{1}{n+1}\binom{n+1}{k}\binom{n+1}{k+1}}_{\text{nb Narayana}}$$