Trees in various sciences

IISER, Pune
17 February 2015

Xavier Viennot
CNRS, LaBRI, Bordeaux
France

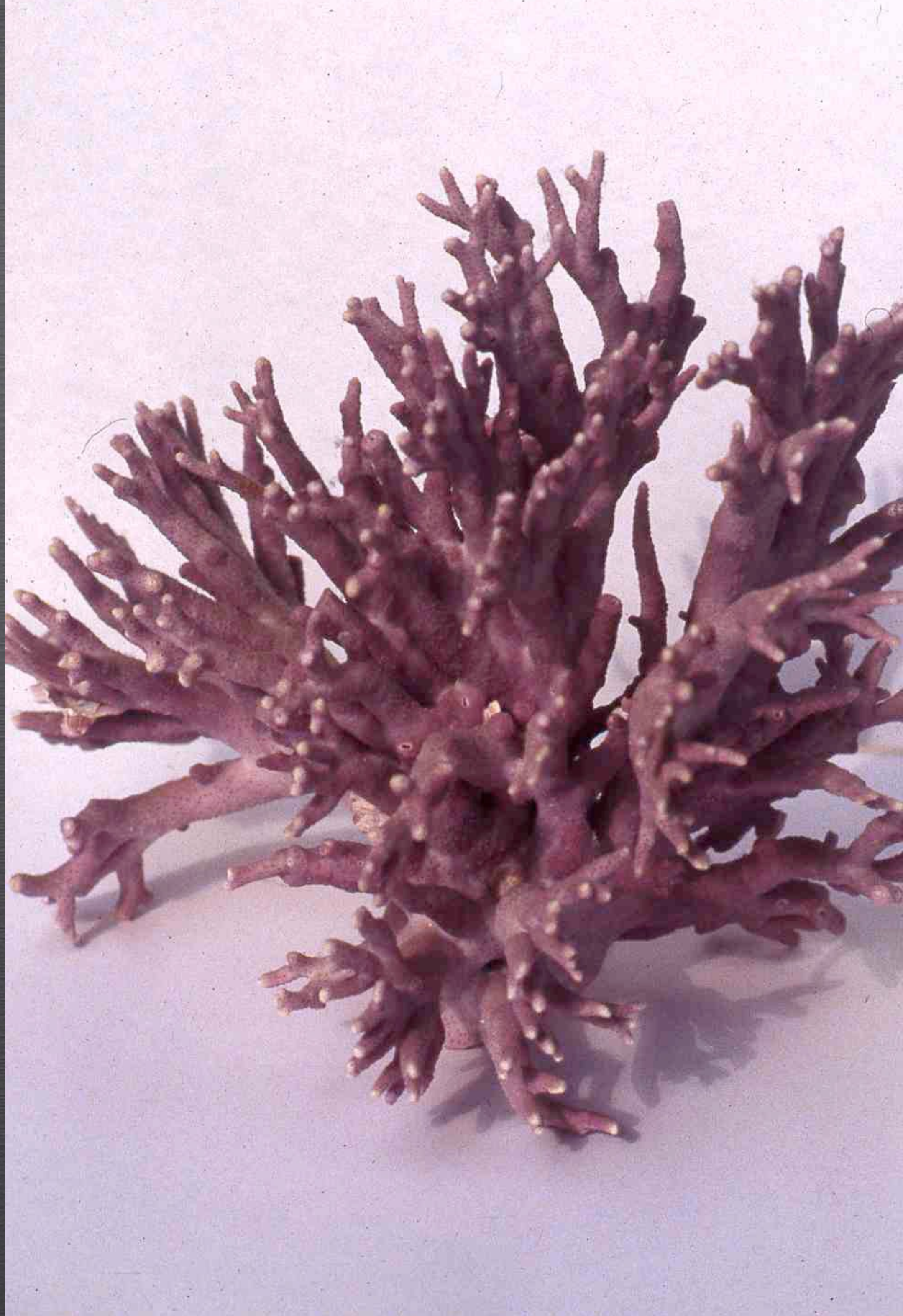
The mathematical beauty of trees

The renaissance of combinatorics
and visual mathematics

From classical physics to quantum gravity

Trees in nature ...

trees everywhere



Coral

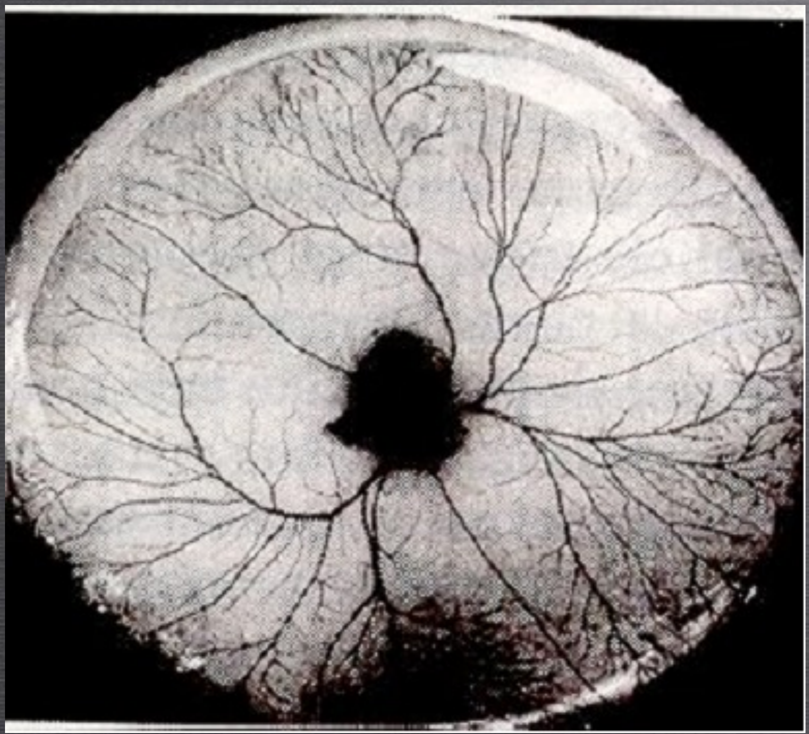

Electrical discharge

electrolysis Deposits

Vincent Fleury


VISCOUS FINGERING
INJECTING OIL BETWEEN TWO PLATES

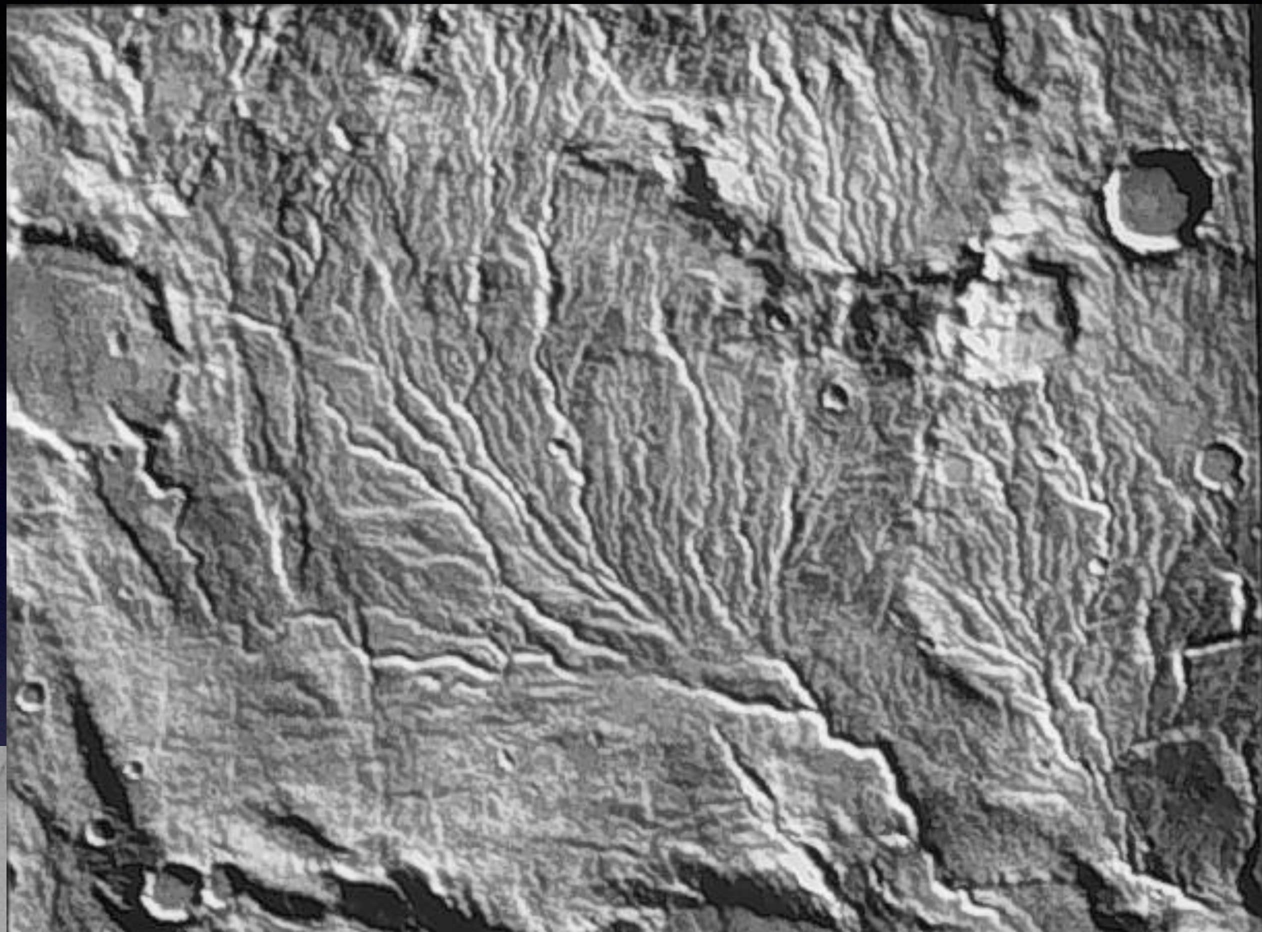
LUNG

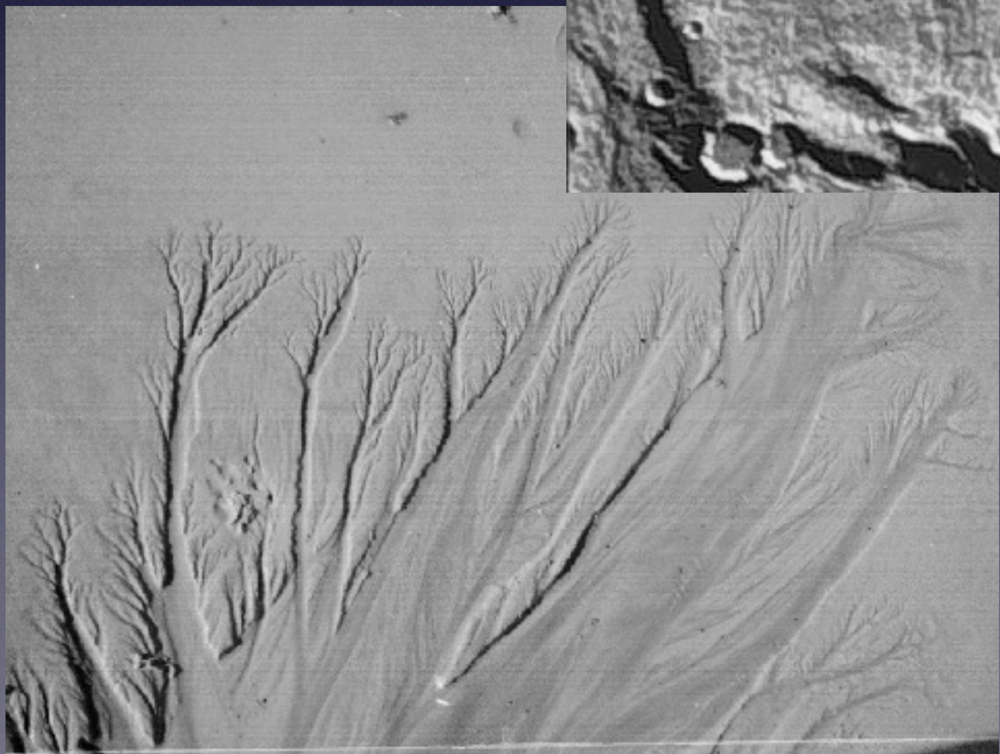


EGG


National Geographic

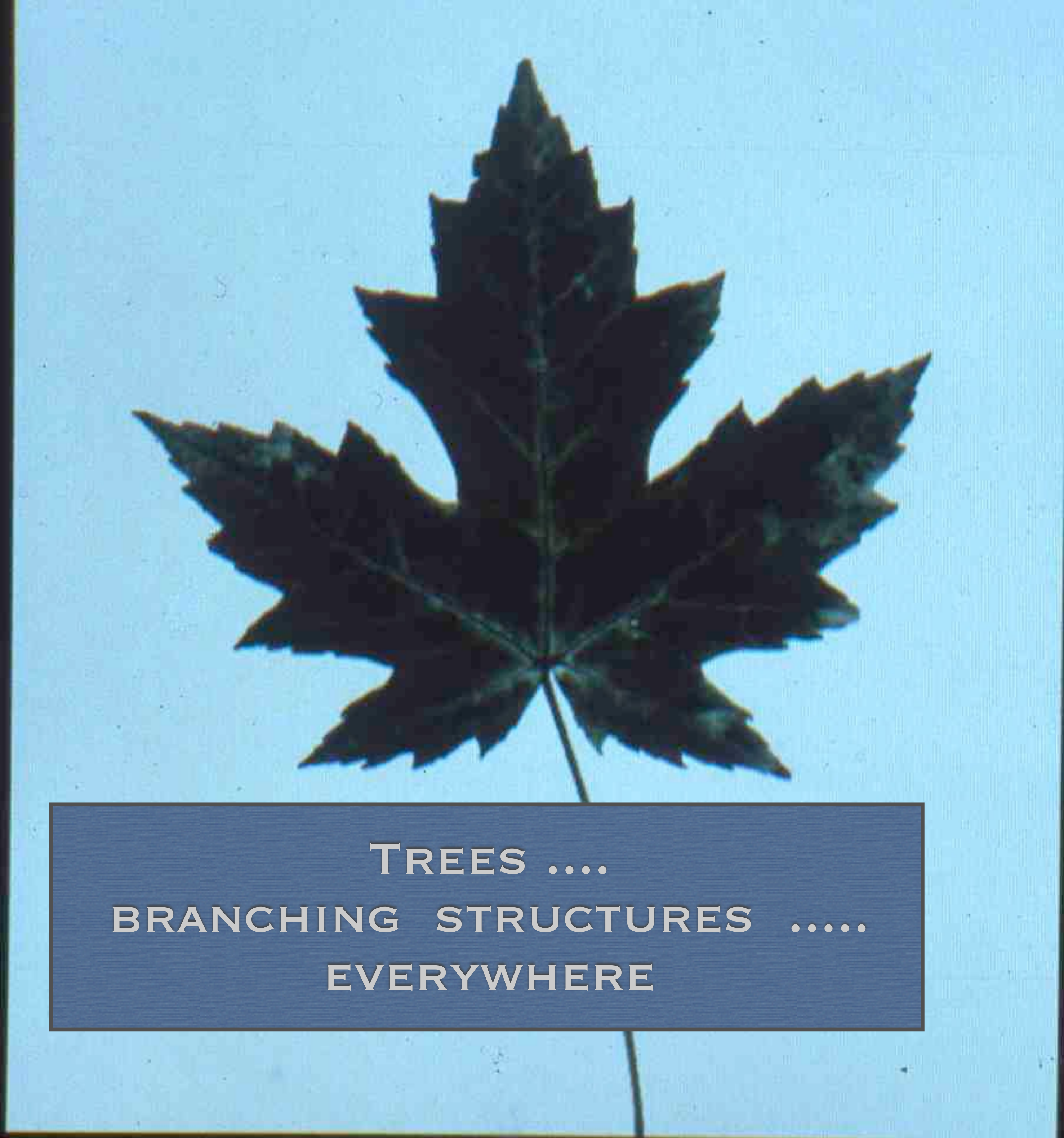
on Mars





on Earth

on a beach


Trees ....
branching structures .....
everywhere

The Tree of knowledge

IIT Bombay, Powai, Mumbai

Trees in the stars,

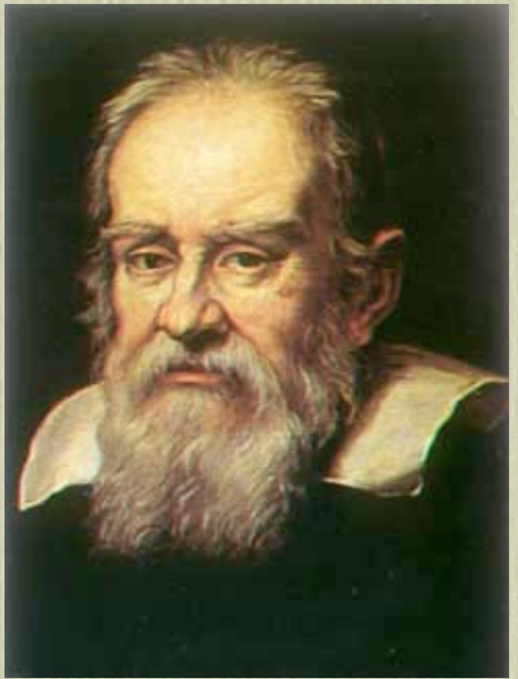
trees in the particles of light ...

Trees in the stars ?

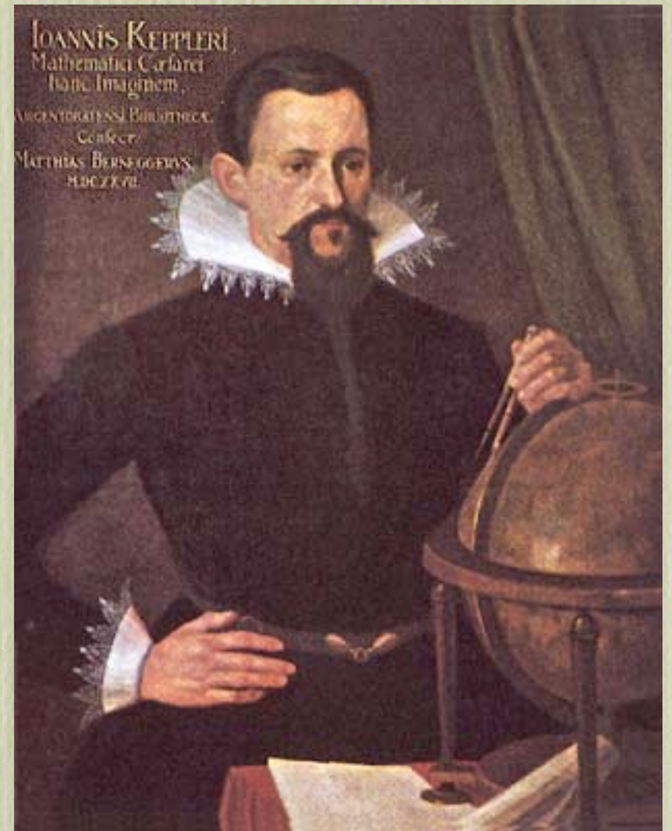
The infinitely large ..

The stars, the planets, the galaxies,
the universe, its birth and history,
space, time, mater, ...

understanding the universe with mathematics



Galileo Galilei
1564-1642



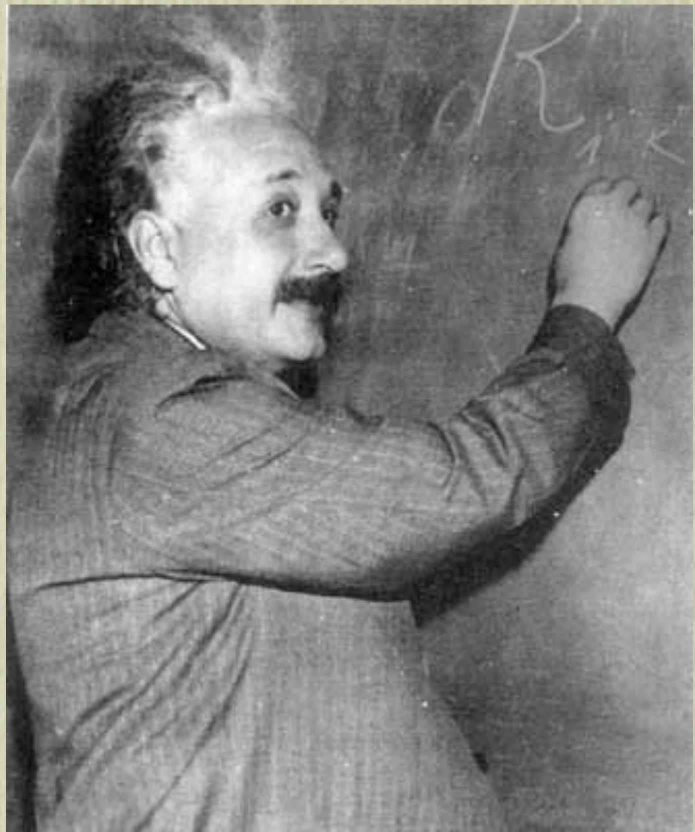
Johannes Kepler
1571 - 1630

Isaac Newton
1643-1727

classical
geometry

classical
mechanics

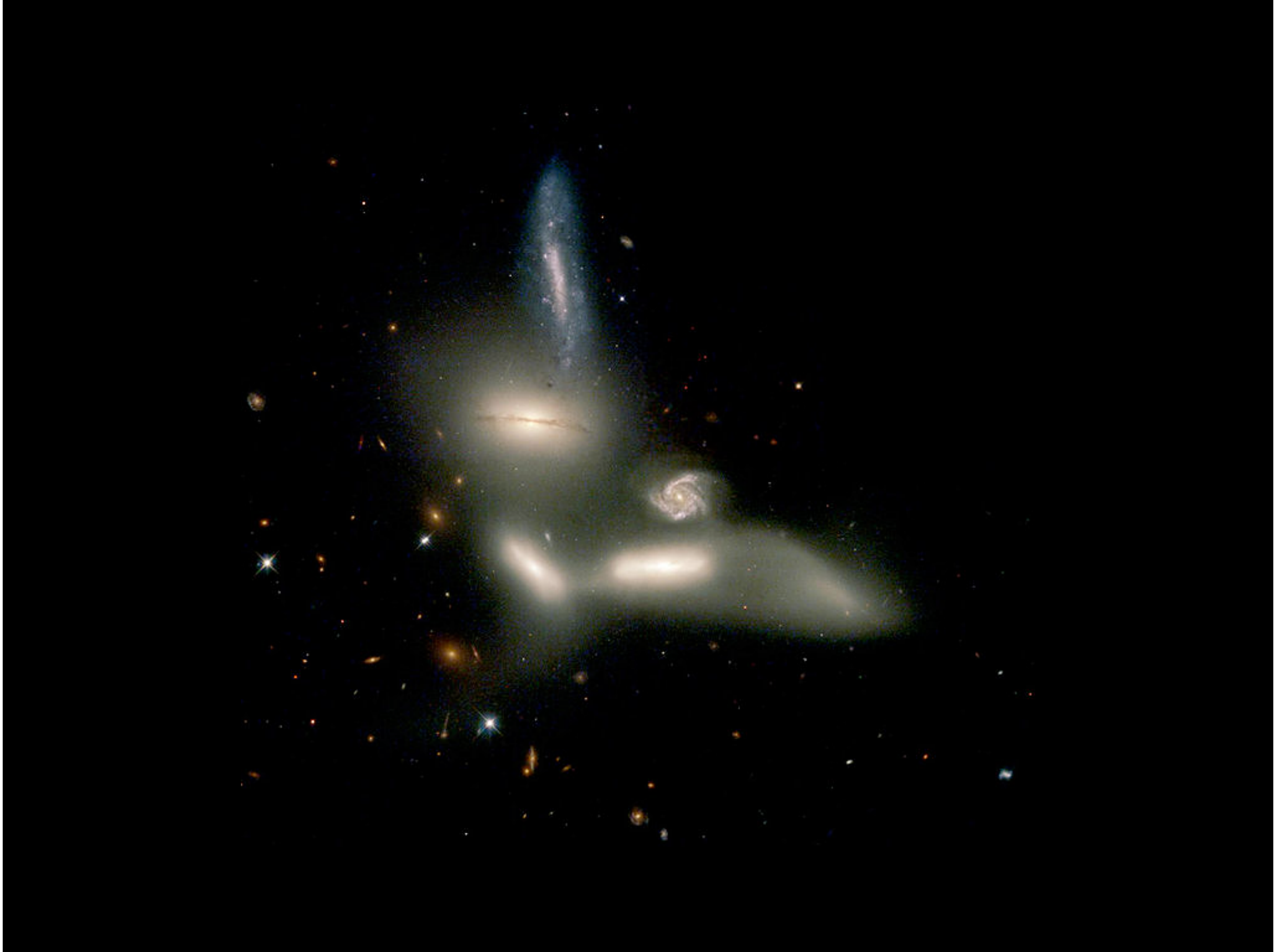
euclidian geometry



Albert Einstein
1879-1955

Relativity theory
restricted
general

gravitation



Trees in the particules of light ?

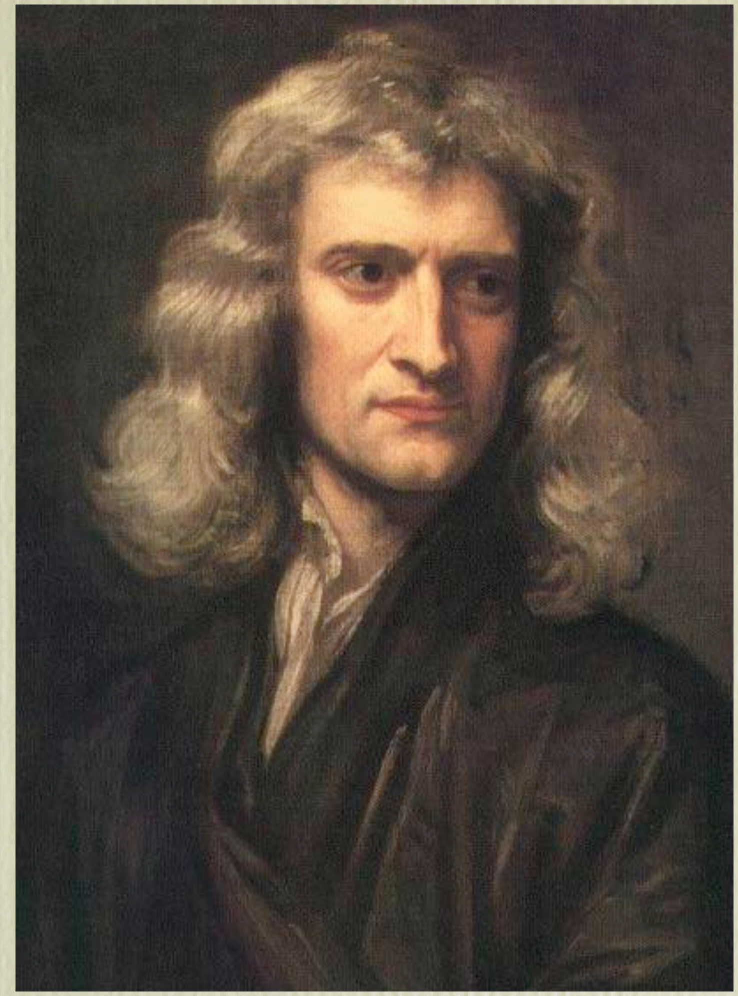
collégiale Notre-Dame Vernon


Daniel B. Holeman

The infinity small ...

the atoms, the electrons
the particles of mater, of light,
the photons, ...

Christian Huygens
1629-1695

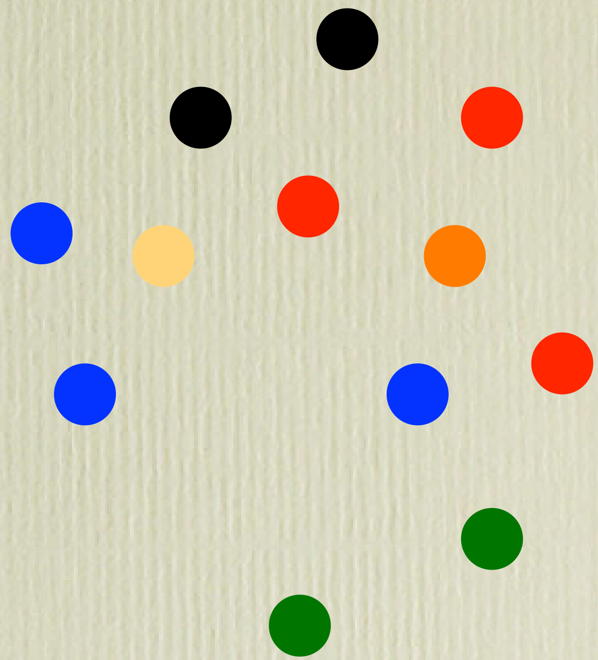


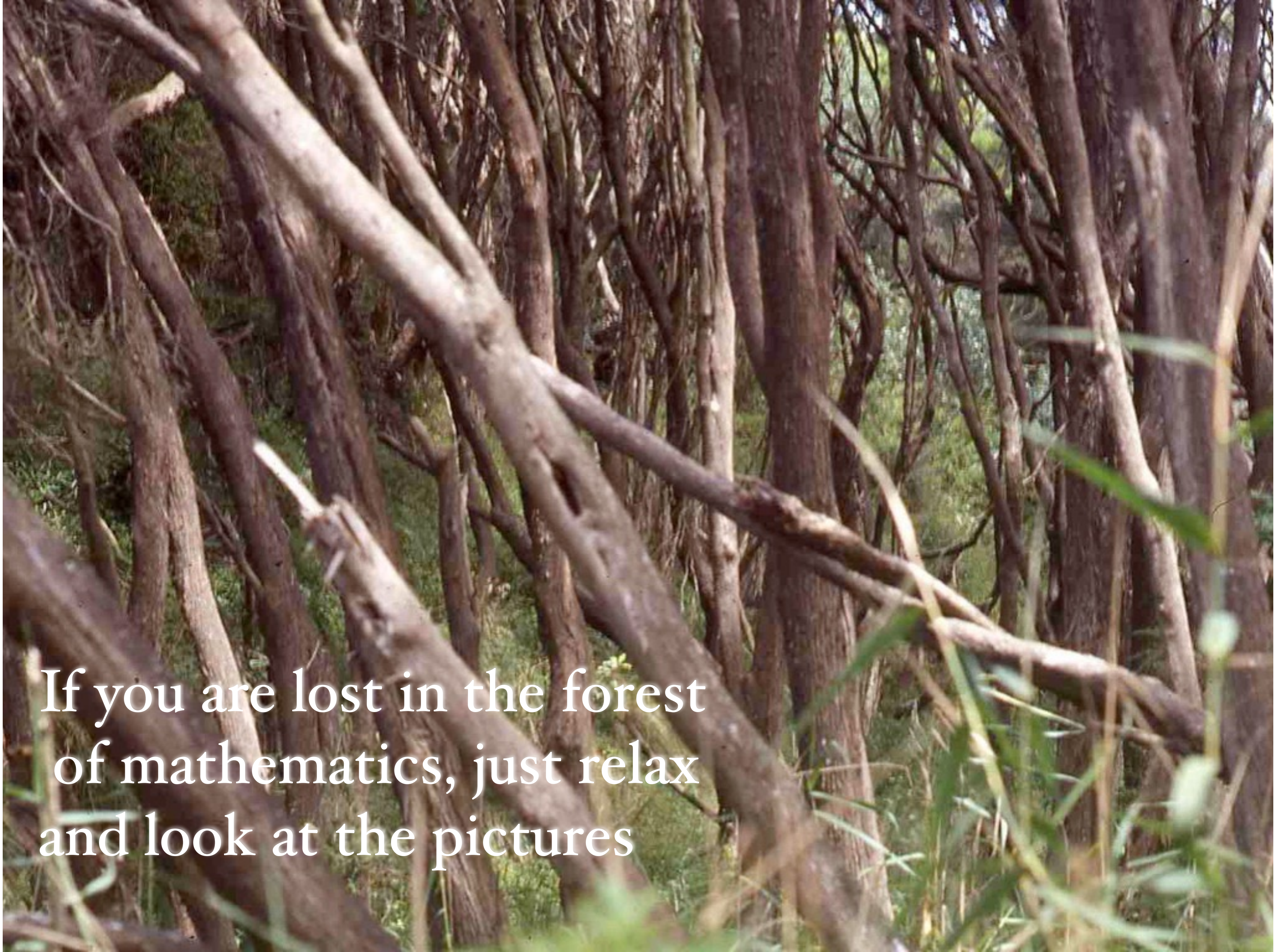
Isaac Newton
1643-1727

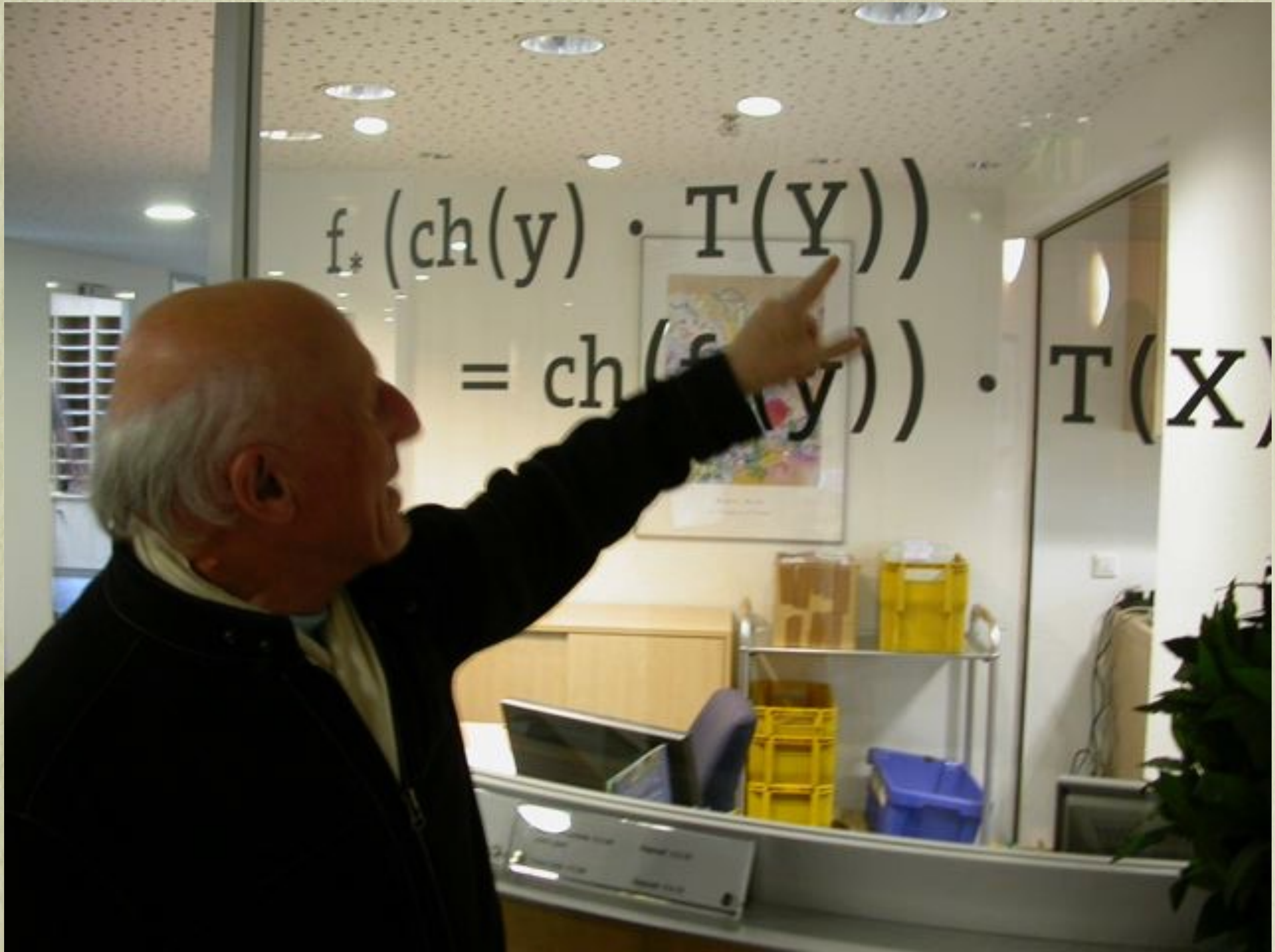
the light:

vibration ?

or particles of mater ?




If you are lost in the forest
of mathematics, just relax
and look at the pictures



look at a mathematical formula
as some abstract art

Rogers - Ramanujan identities

R_I

$$\sum_{n\geq 0} \frac{q^{n^2}}{(1-q)(1-q^2)\cdots(1-q^n)} = \prod_{\substack{i\equiv 1,4 \\ \text{mod } 5}} \frac{1}{(1-q^i)}$$

R_{II}

$$\sum_{n\geq 0} \frac{q^{n^2+n}}{(1-q)(1-q^2)\cdots(1-q^n)} = \prod_{\substack{i\equiv 2,3 \\ \text{mod } 5}} \frac{1}{(1-q^i)}$$

Srinivasan
Ramanujan
(1887-1920)

The langage of mathematics
is like the langage used to write musics.

But mathematics are musics !

Usually, in school you only learn how to write mathematics,
but it is difficult to hear the beauty of mathematics.

Subsublemma 1.1.3:

$$\sum_{\pi\in \mathcal{S}_k} \mathrm{sgn}(\pi)\cdot\pi\left[\frac{x_1x_2^2\dots x_k^k}{(1-x_k)(1-x_kx_{k-1})\cdots(1-x_kx_{k-1}\dots x_1)}\right] = \frac{x_1\cdots x_k\prod_{1\le i<j\le k}(x_j-x_i)}{\prod_{i=1}^k(1-x_i)\prod_{1\le i<j\le k}(1-x_ix_j)} \quad . \qquad (Issai)$$

[Type 'S113(k);' in ROBBINS, for specific k.]

Proof : See [PS], problem VII.47. Alternatively, (Issai) is easily seen to be equivalent to Schur's identity that sums all the Schur functions ([Ma], ex I.5.4, p. 45). This takes care of subsublemma 1.1.3. □

Inserting (*Issai*) into (*Stanley*), expanding $\prod_{1\le i<j\le k}(x_j-x_i)$ by Vandermonde's expansion,

$$\sum_{\pi\in \mathcal{S}_k} \mathrm{sgn}(\pi)\cdot\pi(x_1^0x_2^1\cdots x_k^{k-1}) \quad ,$$

using the antisymmetry of Δ_k once again, and employing crucial fact $\aleph_4$, we get the following string of equalities:

$$b_k(n) = \frac{1}{k!}CT_{x_1,\dots,x_k}\left\{\frac{\Delta_k(x_1,\dots,x_k)}{\prod_{i=1}^k(\bar{x}_i)^{k+n}x_i^{n+k-1}}\left(\frac{x_1\cdots x_k\prod_{1\le i<j\le k}(x_j-x_i)}{\prod_{i=1}^k(1-x_i)\prod_{1\le i<j\le k}(1-x_ix_j)}\right)\right\}$$

$$= \frac{1}{k!}CT_{x_1,\dots,x_k}\left\{\frac{\Delta_k(x_1,\dots,x_k)}{\prod_{i=1}^k(\bar{x}_i)^{k+n+1}x_i^{n+k-2}\prod_{1\le i<j\le k}(1-x_ix_j)}\left(\sum_{\pi\in \mathcal{S}_k}\mathrm{sgn}(\pi)\cdot\pi(x_1^0x_2^1\dots x_k^{k-1})\right)\right\}$$

$$= \frac{1}{k!}\sum_{\pi\in \mathcal{S}_k} CT_{x_1,\dots,x_k}\left\{\pi\left[\frac{\Delta_k(x_1,\dots,x_k)}{\prod_{i=1}^k(\bar{x}_i)^{k+n+1}x_i^{n+k-2}\prod_{1\le i<j\le k}(1-x_ix_j)}\left(\prod_{i=1}^k x_i^{i-1}\right)\right]\right\}$$

$$= \frac{1}{k!}\sum_{\pi\in \mathcal{S}_k} CT_{x_1,\dots,x_k}\left\{\pi\left[\frac{\Delta_k(x_1,\dots,x_k)}{\prod_{i=1}^k(\bar{x}_i)^{k+n+1}x_i^{n+k-i-1}\prod_{1\le i<j\le k}(1-x_ix_j)}\right]\right\}$$

$$= \frac{1}{k!}\sum_{\pi\in \mathcal{S}_k} CT_{x_1,\dots,x_k}\left\{\frac{\Delta_k(x_1,\dots,x_k)}{\prod_{i=1}^k(\bar{x}_i)^{k+n+1}x_i^{n+k-i-1}\prod_{1\le i<j\le k}(1-x_ix_j)}\right\}$$

$$= \frac{1}{k!}\left(\sum_{\pi\in \mathcal{S}_k}1\right) CT_{x_1,\dots,x_k}\left\{\frac{\Delta_k(x_1,\dots,x_k)}{\prod_{i=1}^k(\bar{x}_i)^{k+n+1}x_i^{n+k-i-1}\prod_{1\le i<j\le k}(1-x_ix_j)}\right\}$$

$$= CT_{x_1,\dots,x_k}\left\{\frac{\Delta_k(x_1,\dots,x_k)}{\prod_{i=1}^k(\bar{x}_i)^{k+n+1}x_i^{n+k-i-1}\prod_{1\le i<j\le k}(1-x_ix_j)}\right\}, \qquad (George''')$$

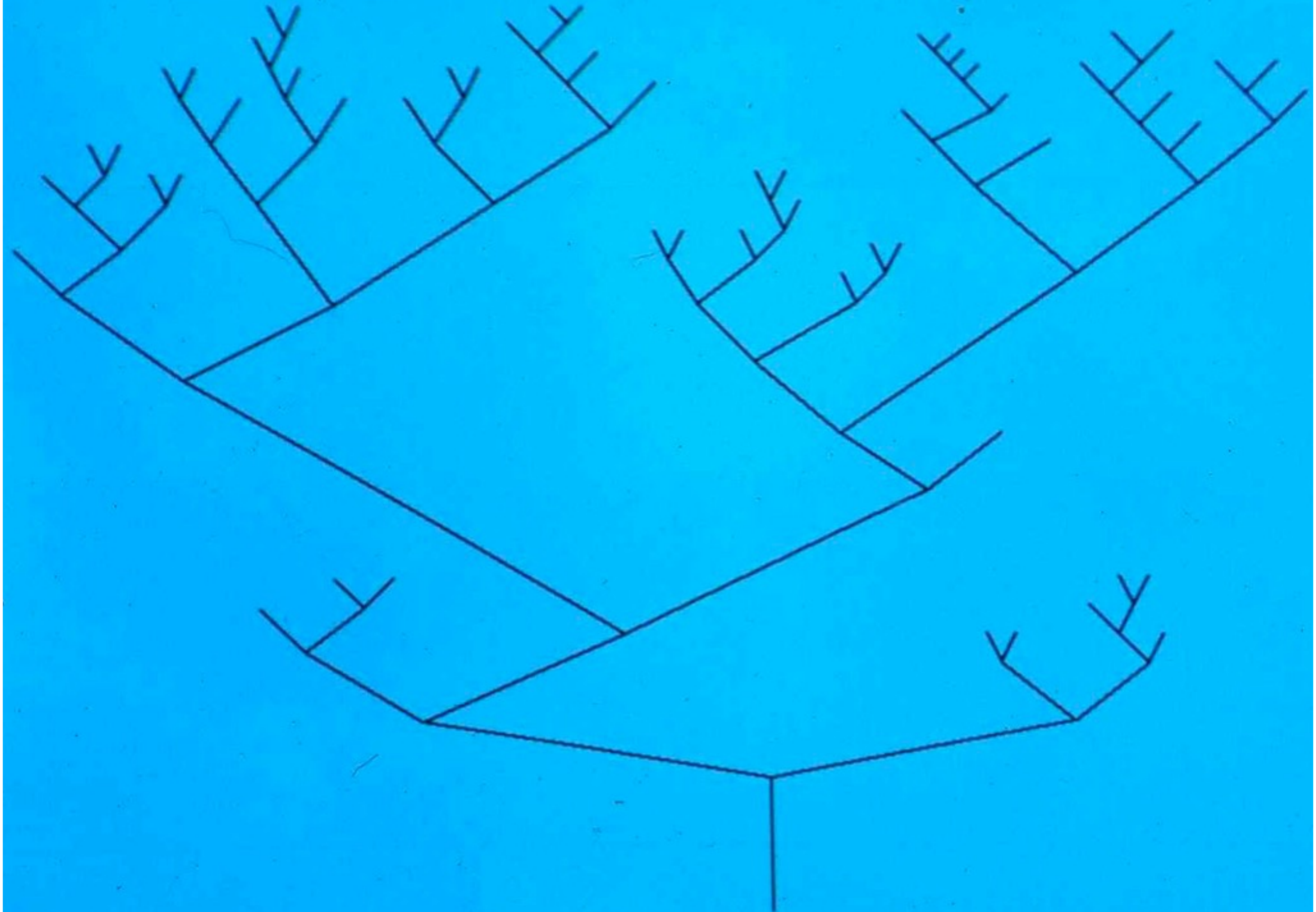
where in the last equality we have used Levi Ben Gerson's celebrated result that the number of elements in $\mathcal{S}_k$ (the symmetric group on k elements,) equals $k!$. The extreme right of ($George'''$) is exactly the right side of ($MagogTotal$). This completes the proof of sublemma 1.1. □

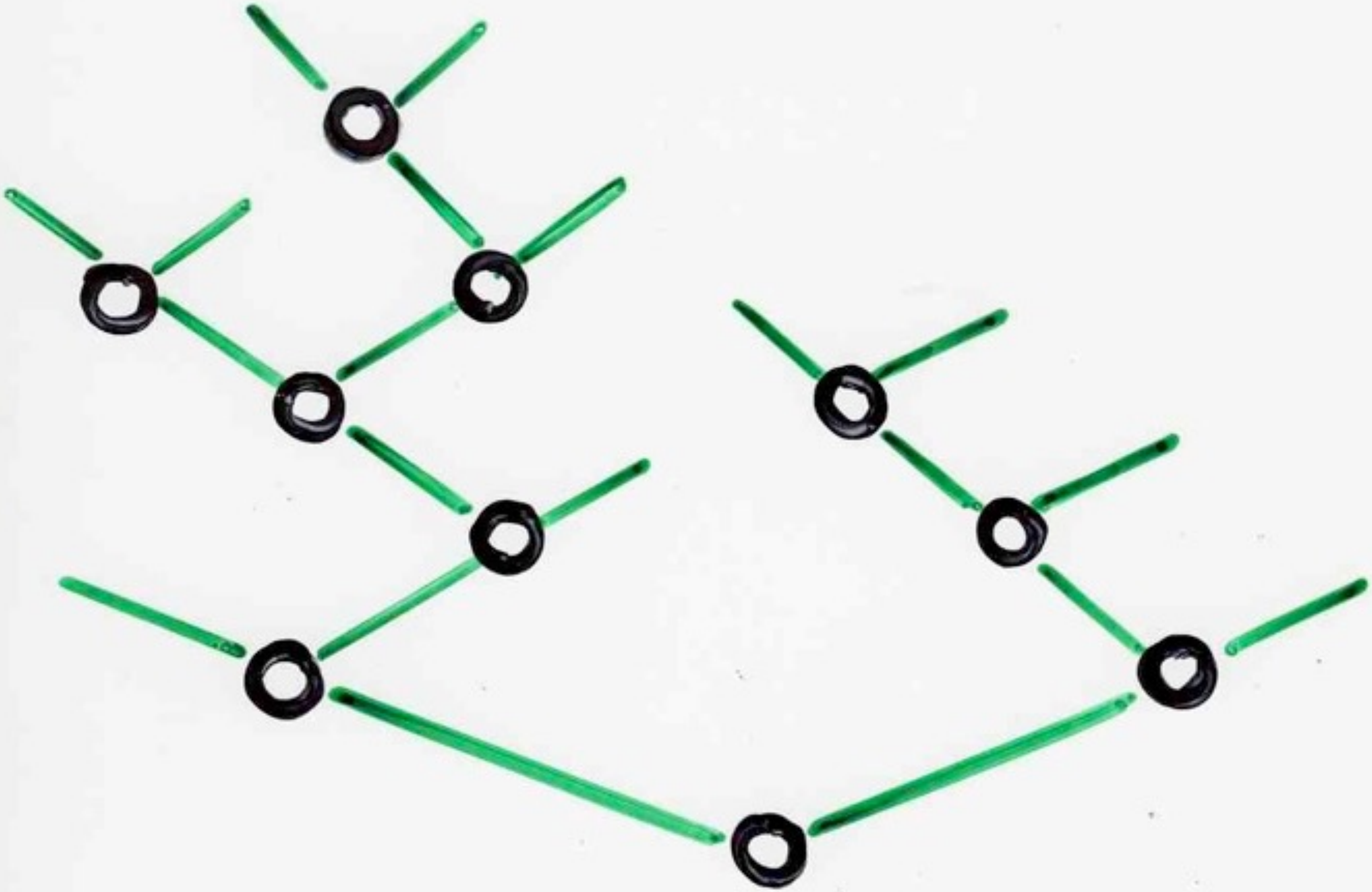

Paris

An example of mathematical object:
binary trees or mathematical trees

giving an abstraction of the trees
in the world around us

From trees in nature...
to mathematical trees

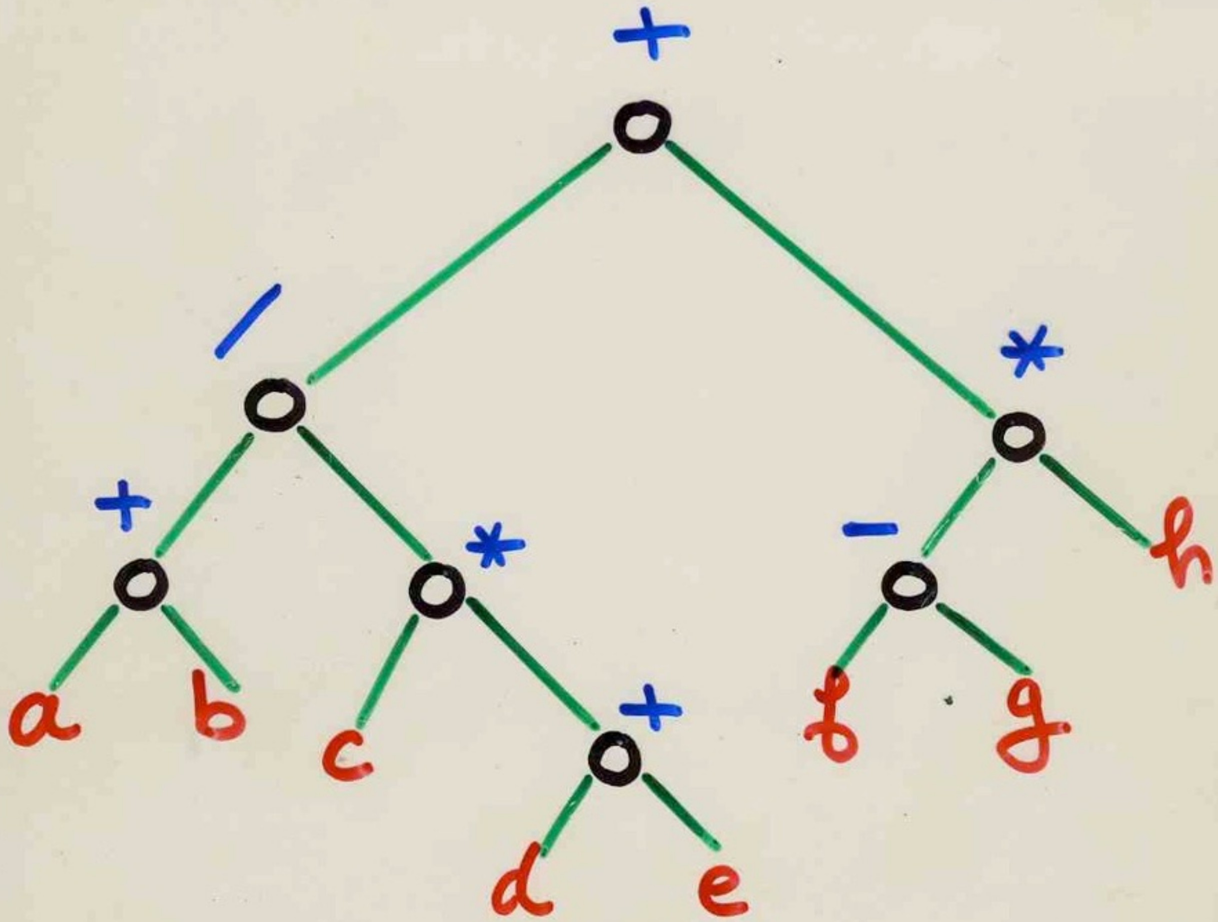




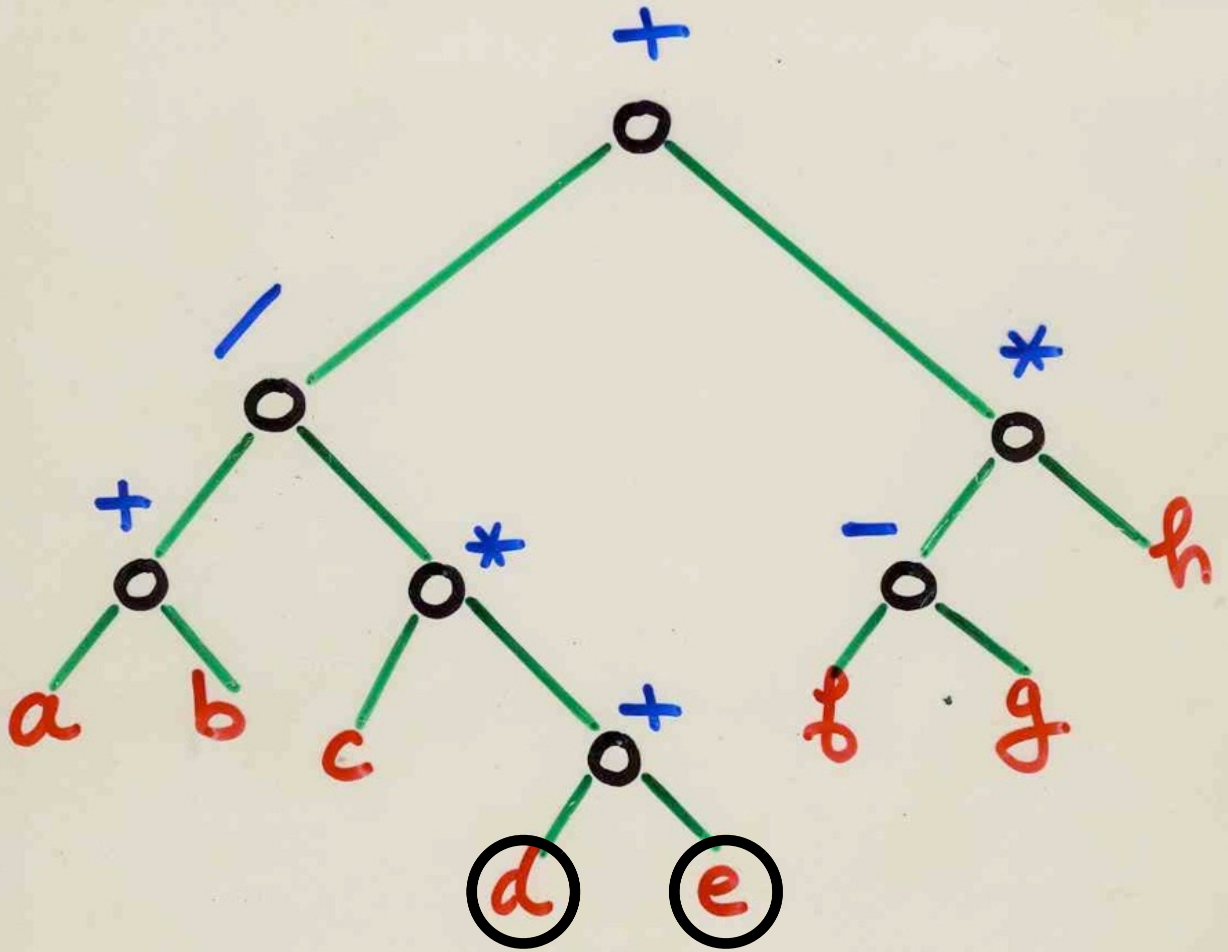
Trees in computers ...

computing an arithmetical expression

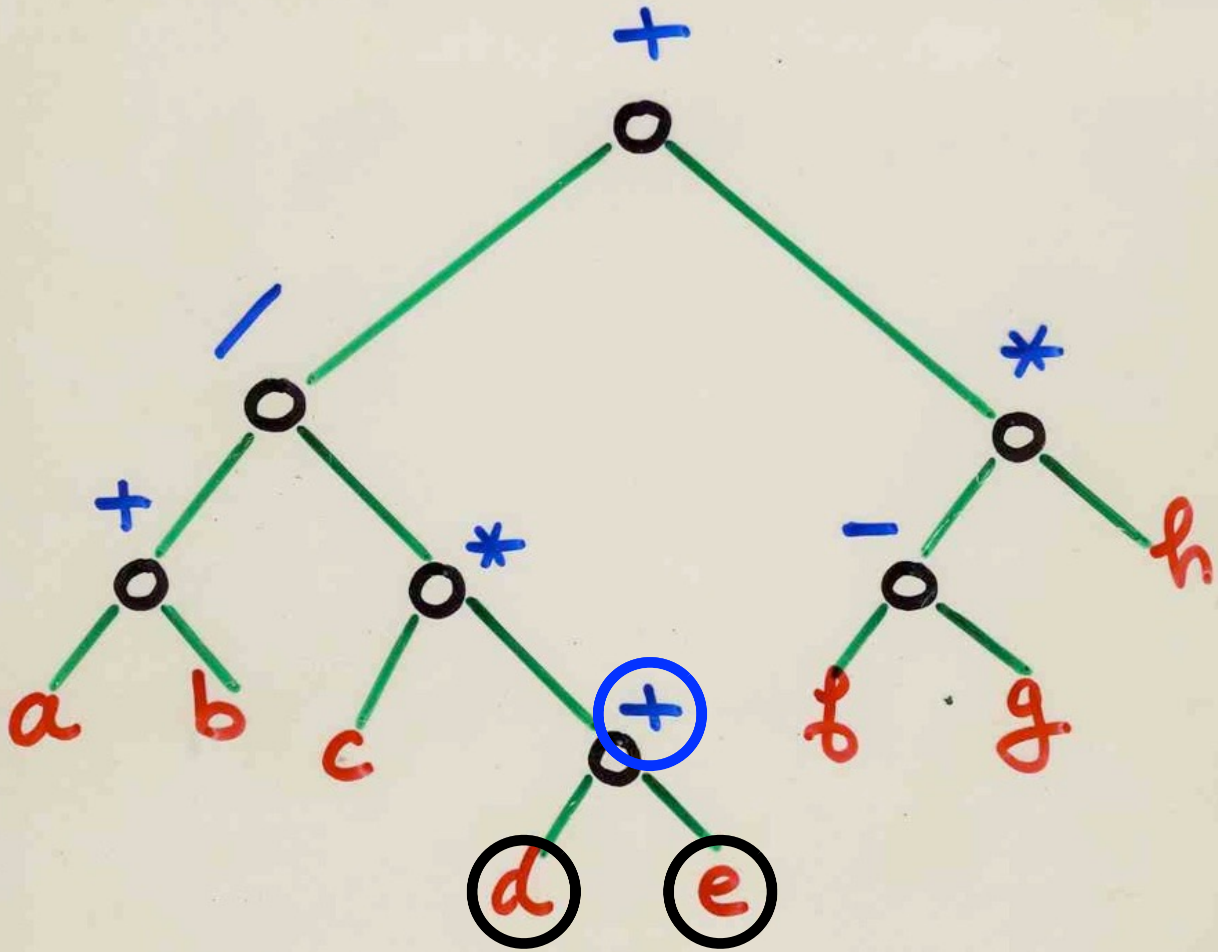
$$\frac{(a+b)}{c\,(d+e)} + (f-g)h$$



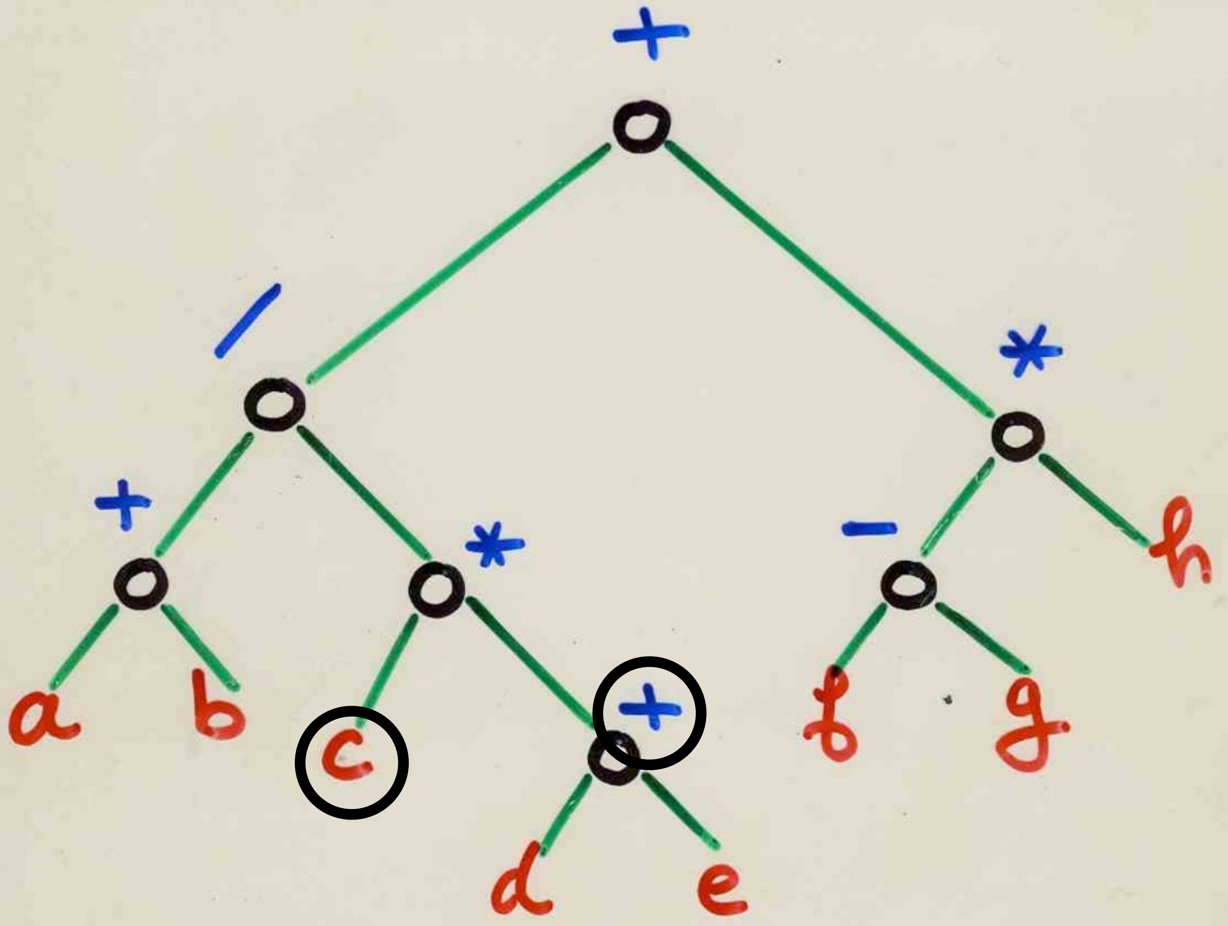
$$\frac{(a+b)}{c\,(d+e)} + (f-g)h$$



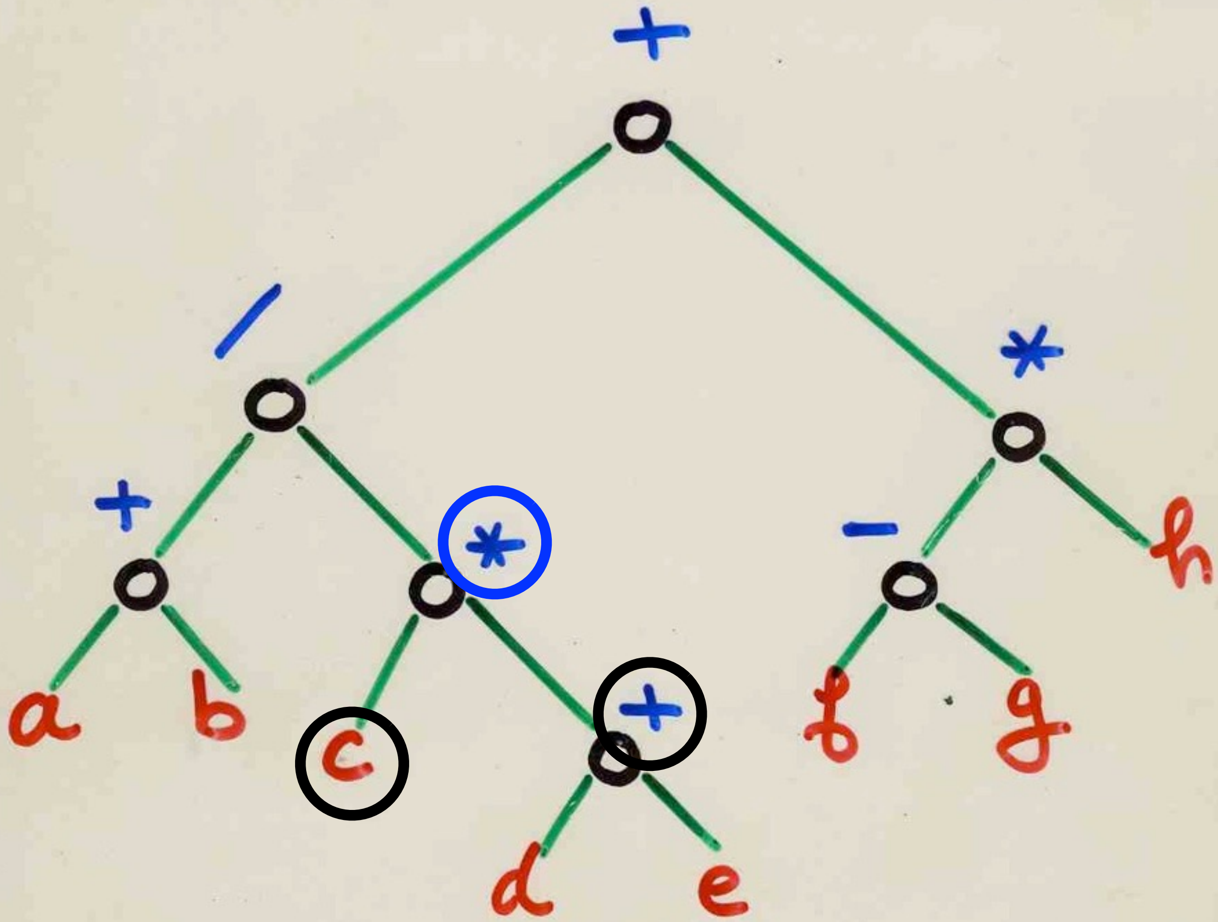
$$\frac{(a+b)}{c\,(d+e)} + (f-g)\,h$$



$$\frac{(a+b)}{c\,(d+e)} + (f-g)h$$



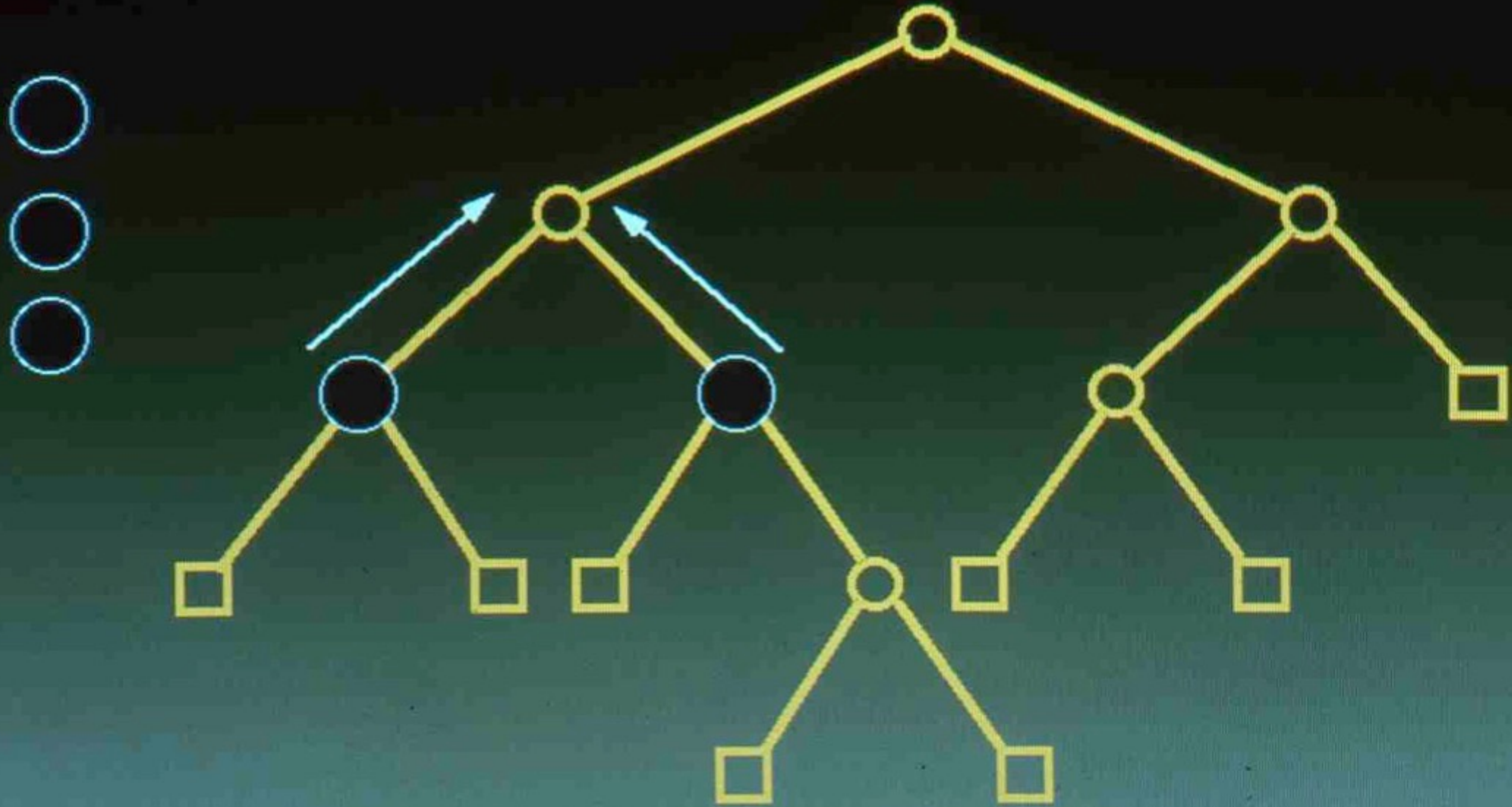
$$\frac{(a+b)}{c\,(d+e)} + (f-g)\,h$$

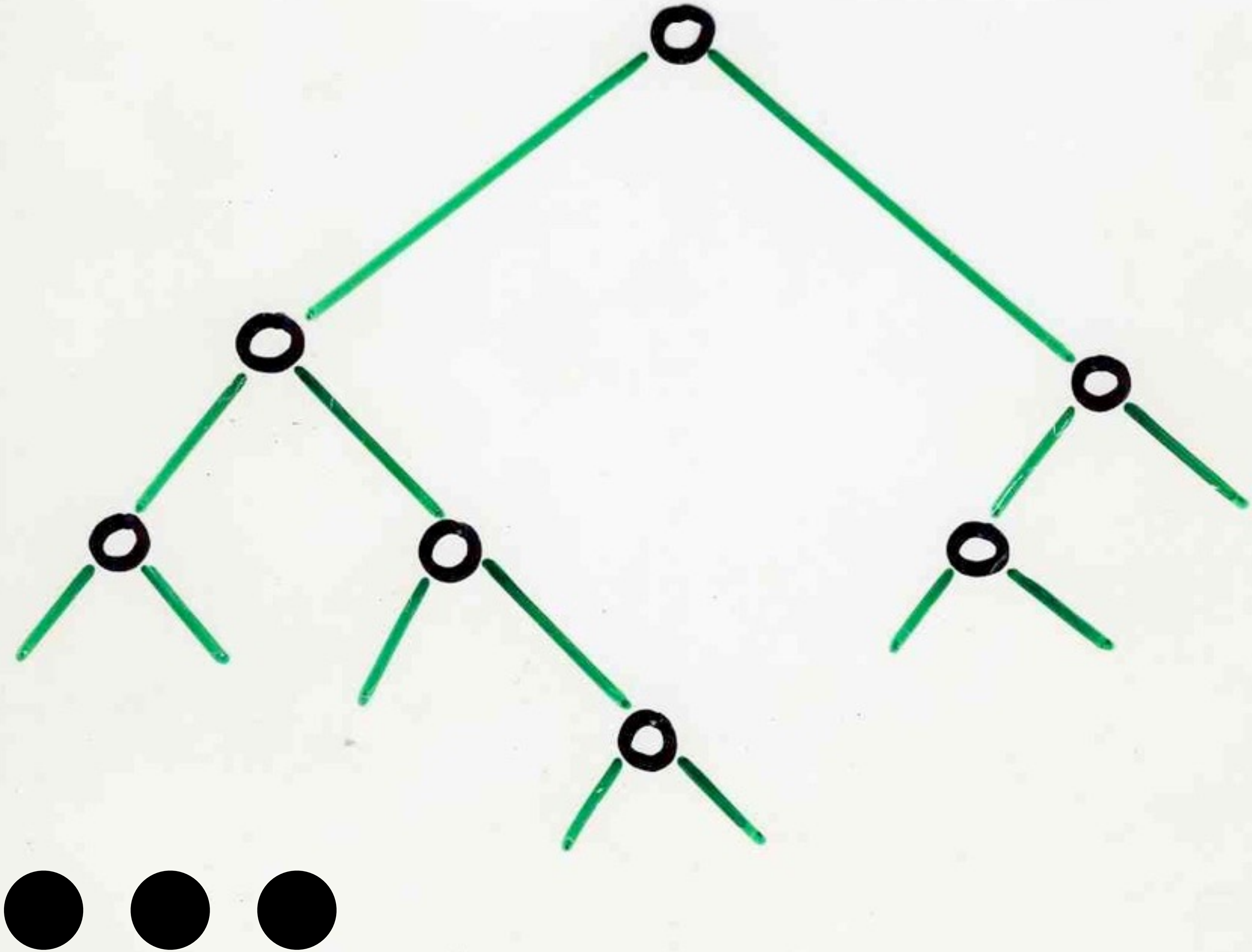


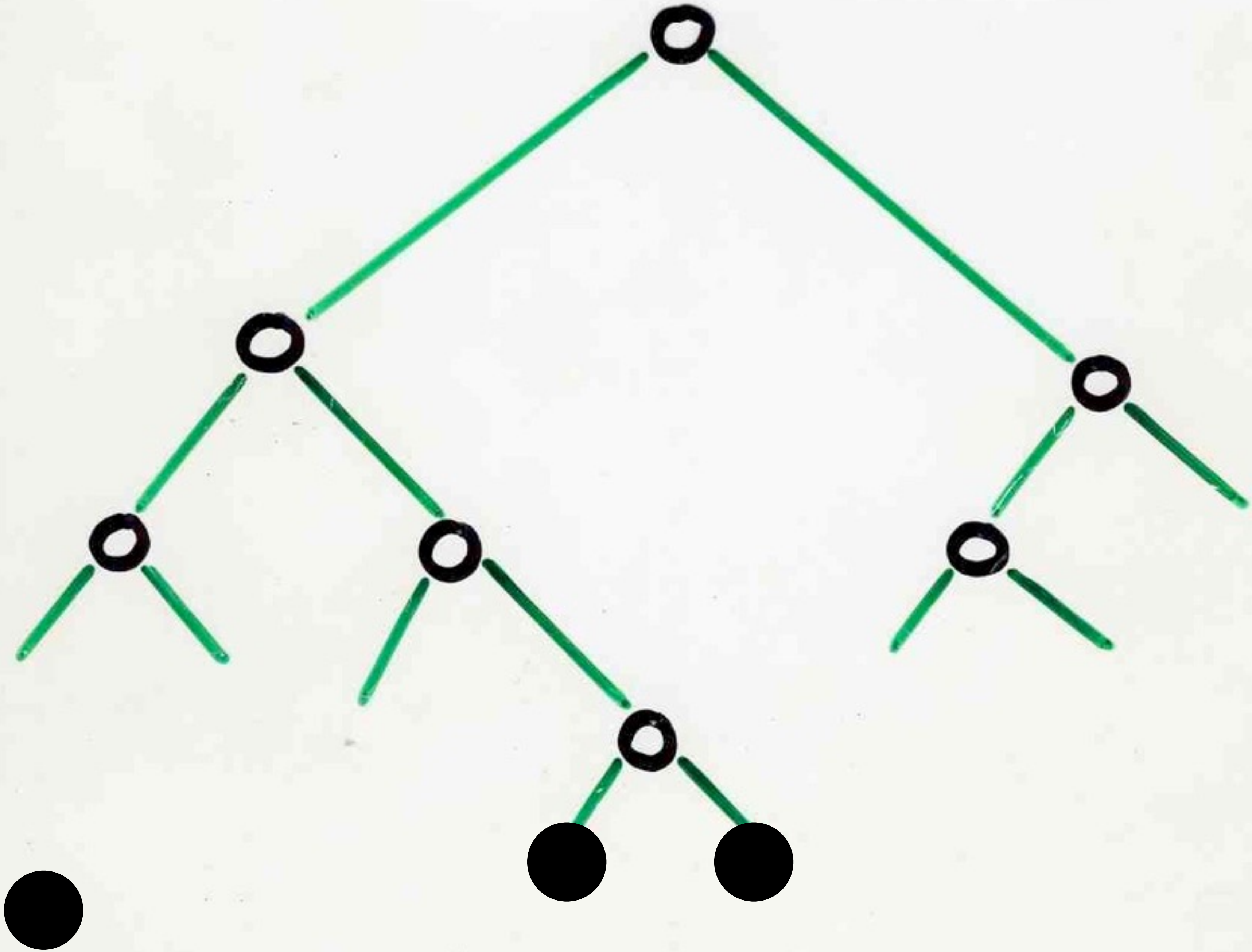
$$\frac{(a+b)}{c\,(d+e)} + (f-g)\,h$$

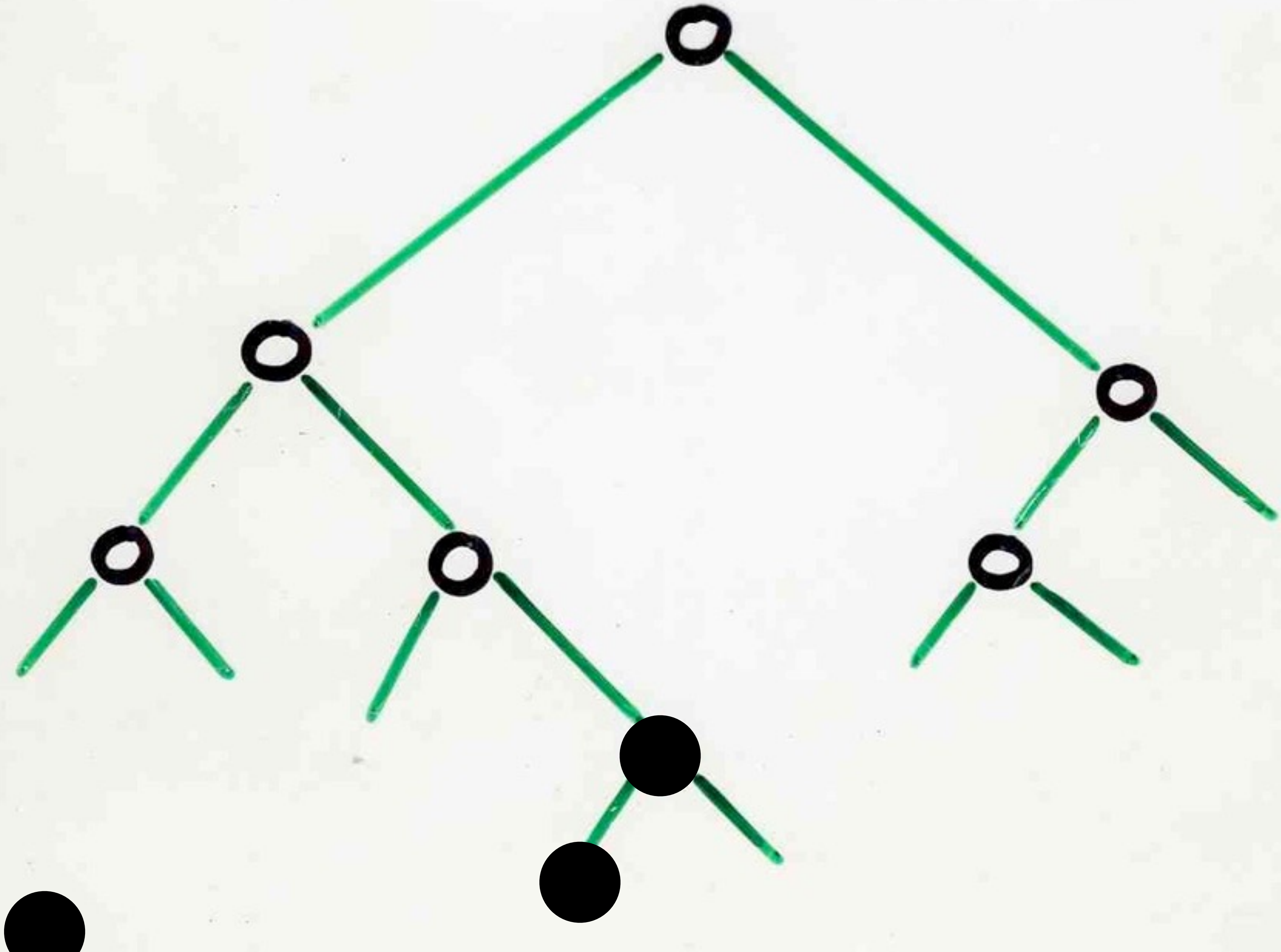
minimum number of registers needed to compute an arithmetical expression

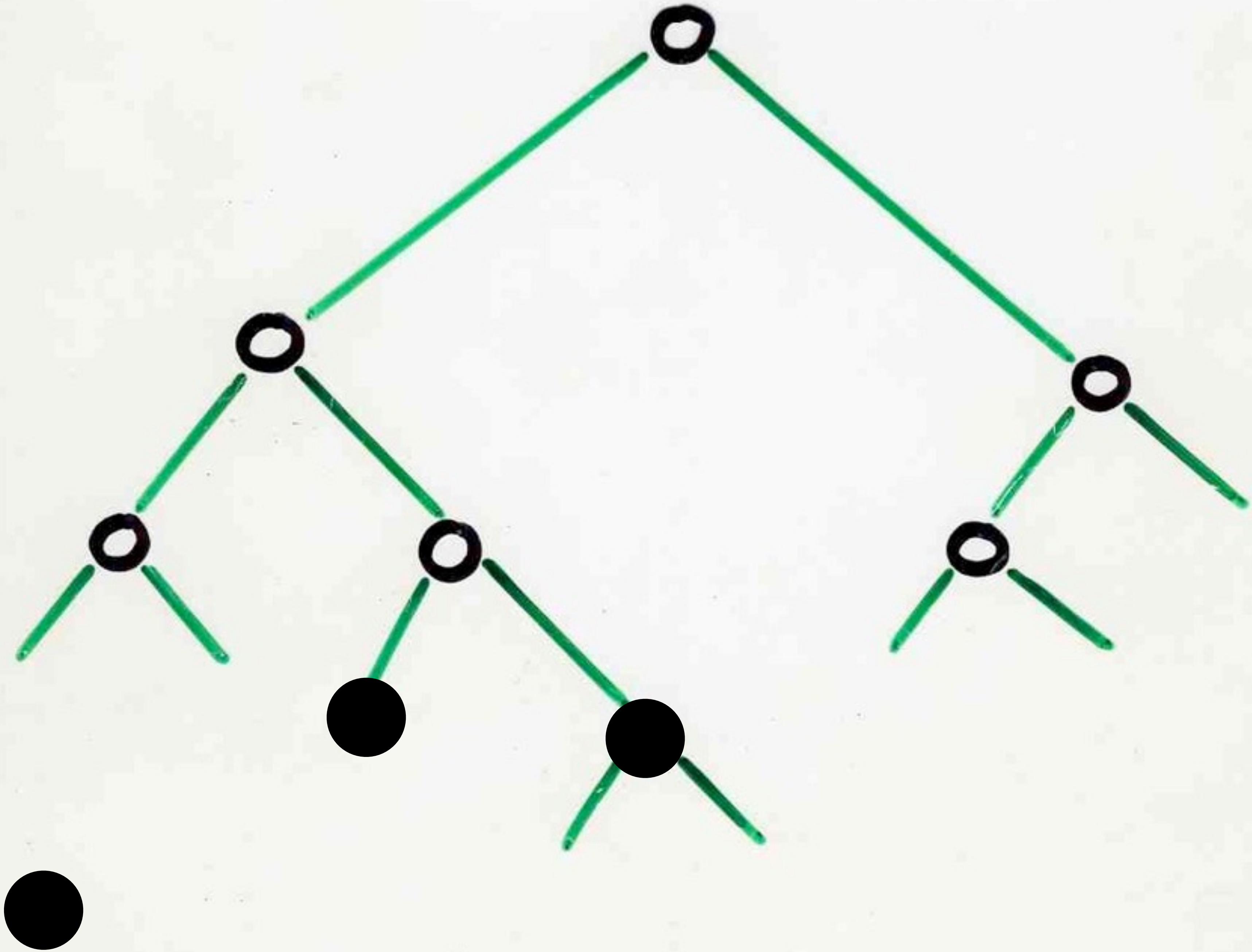
Pebbles problem

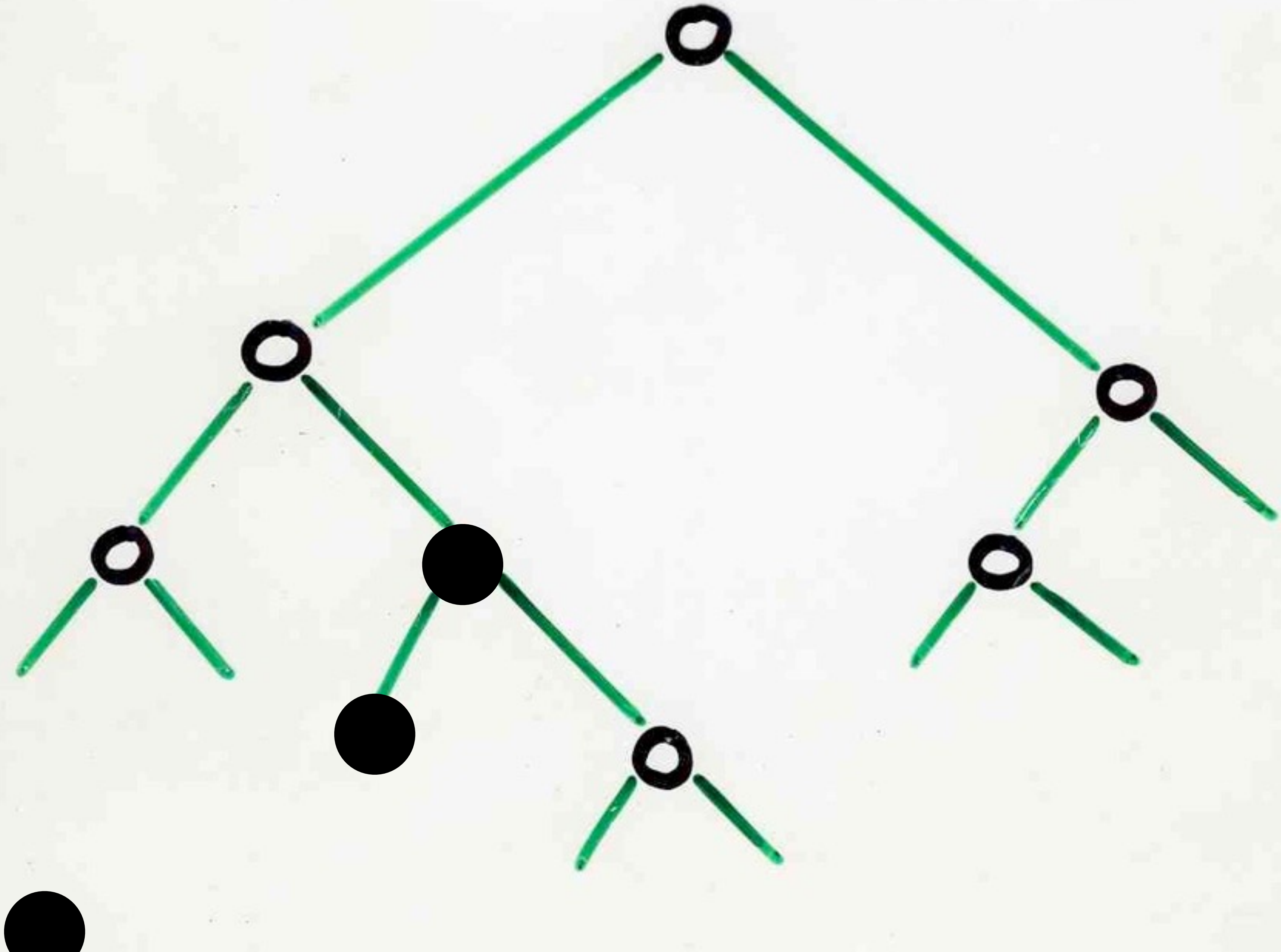


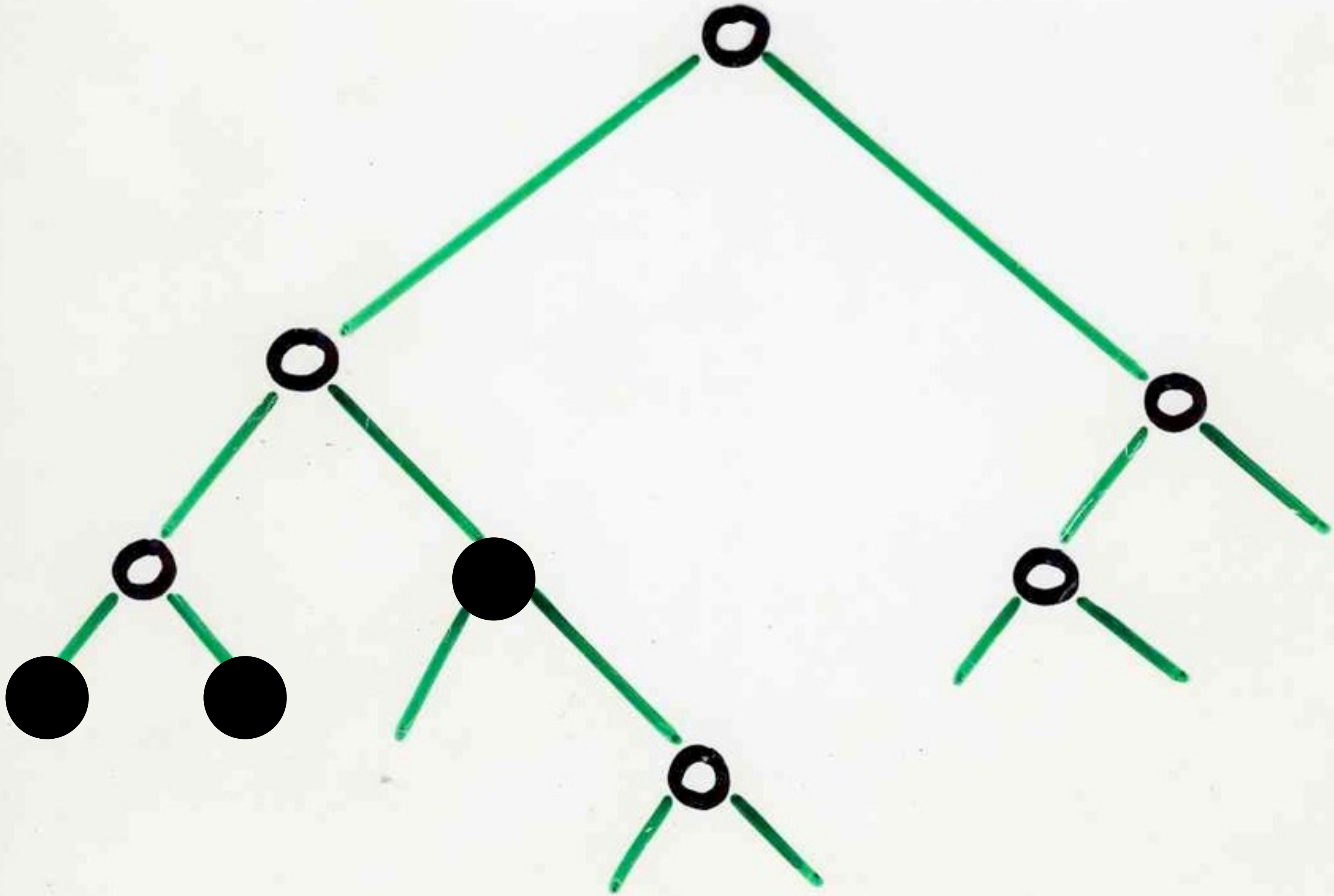


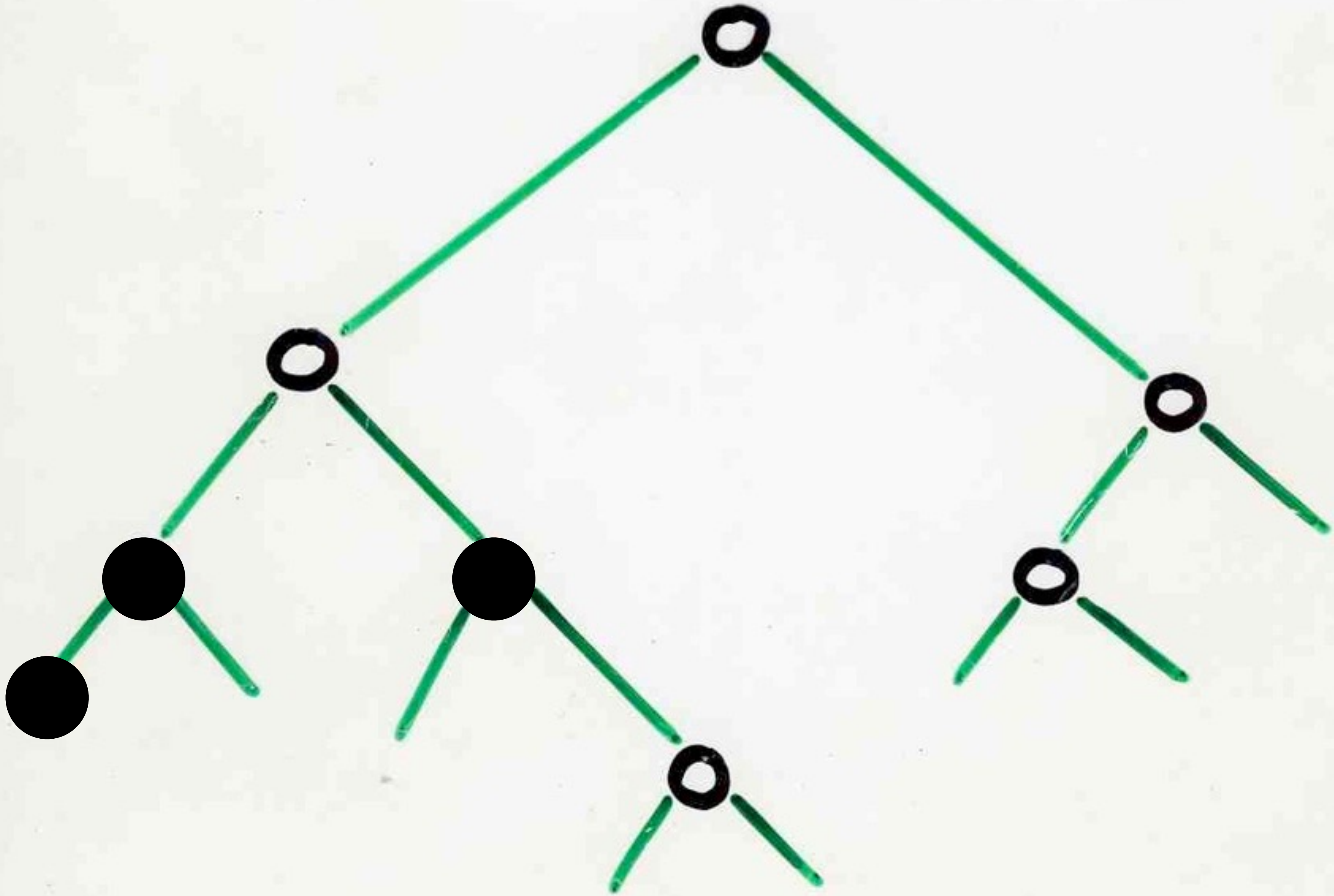


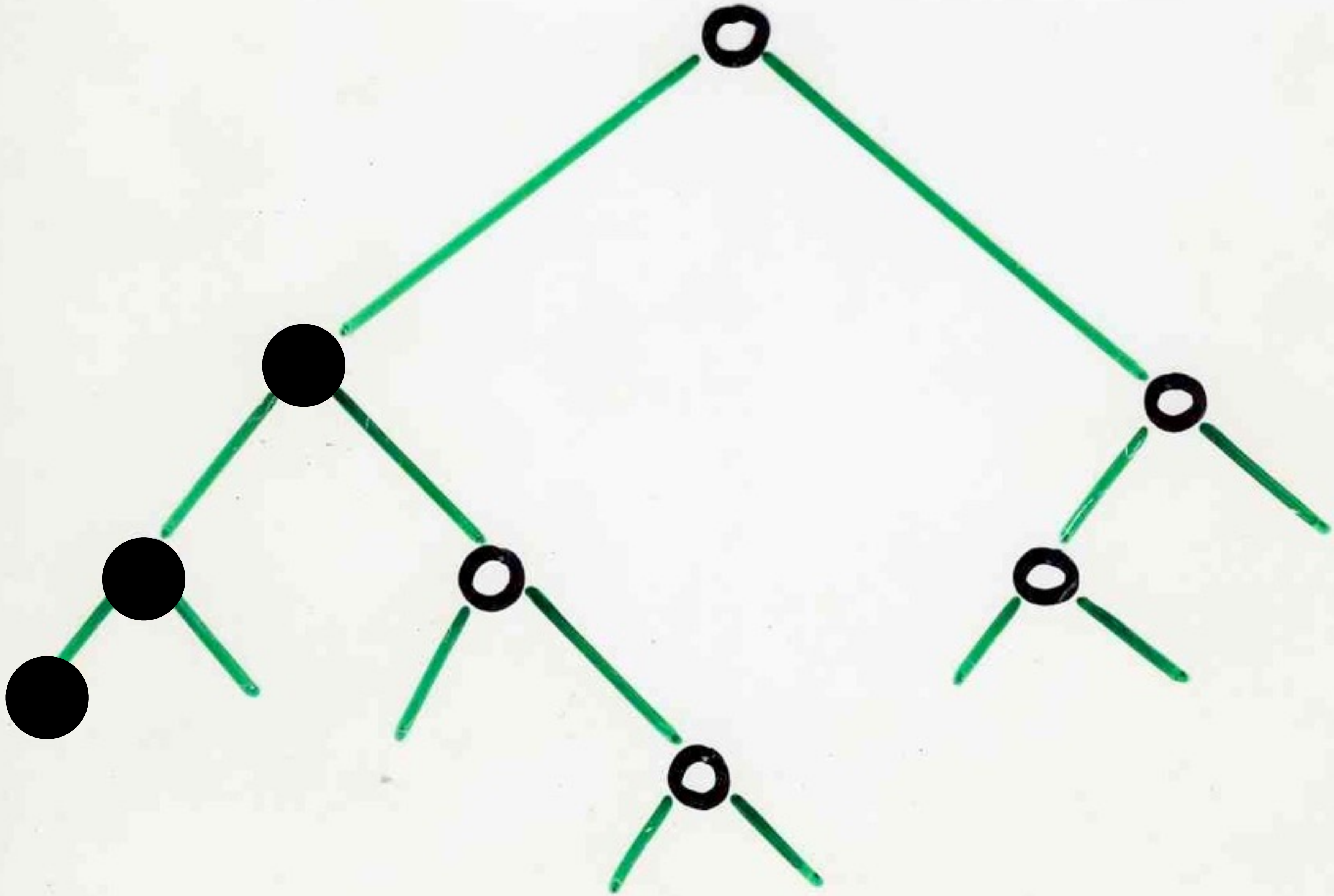


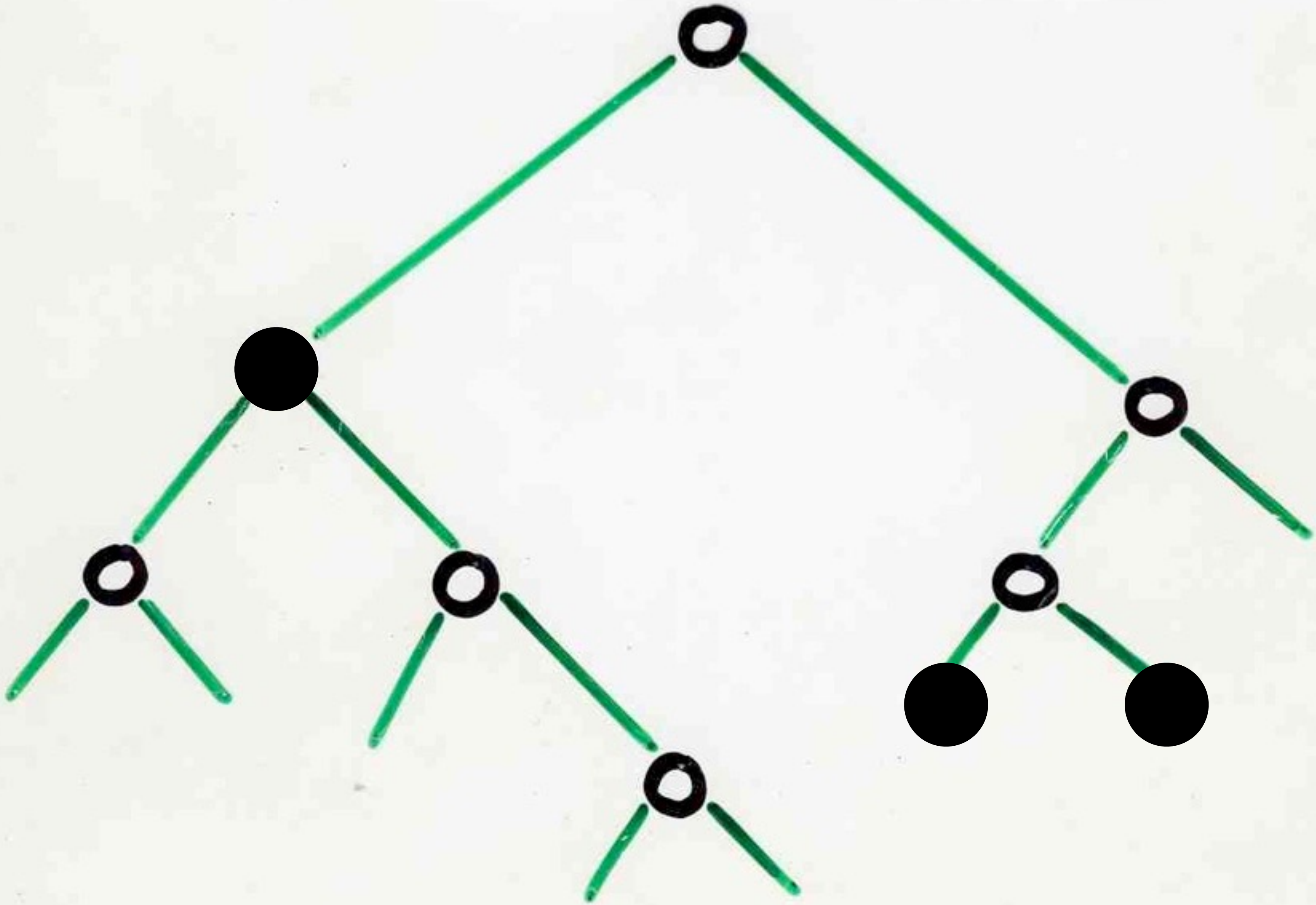


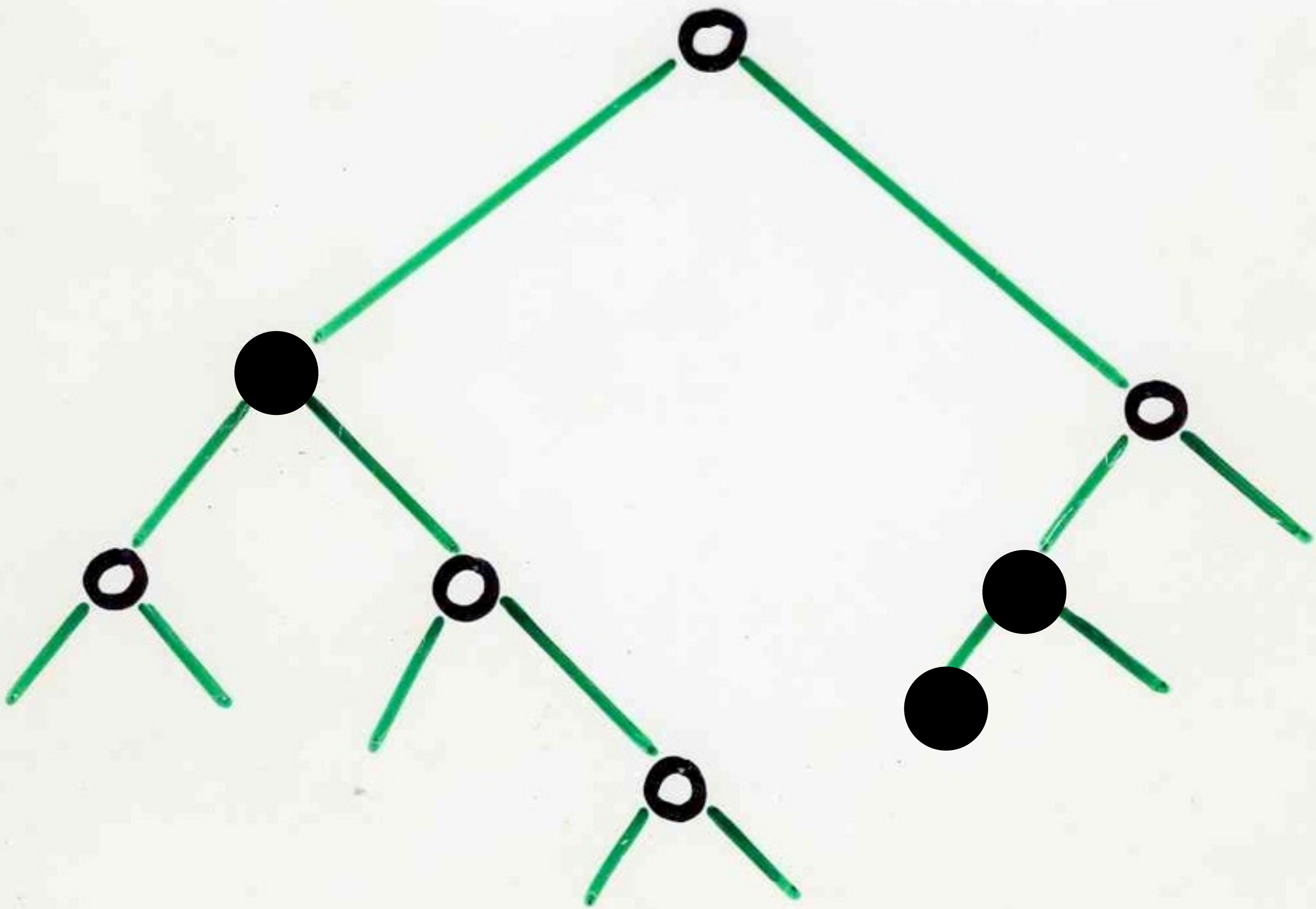


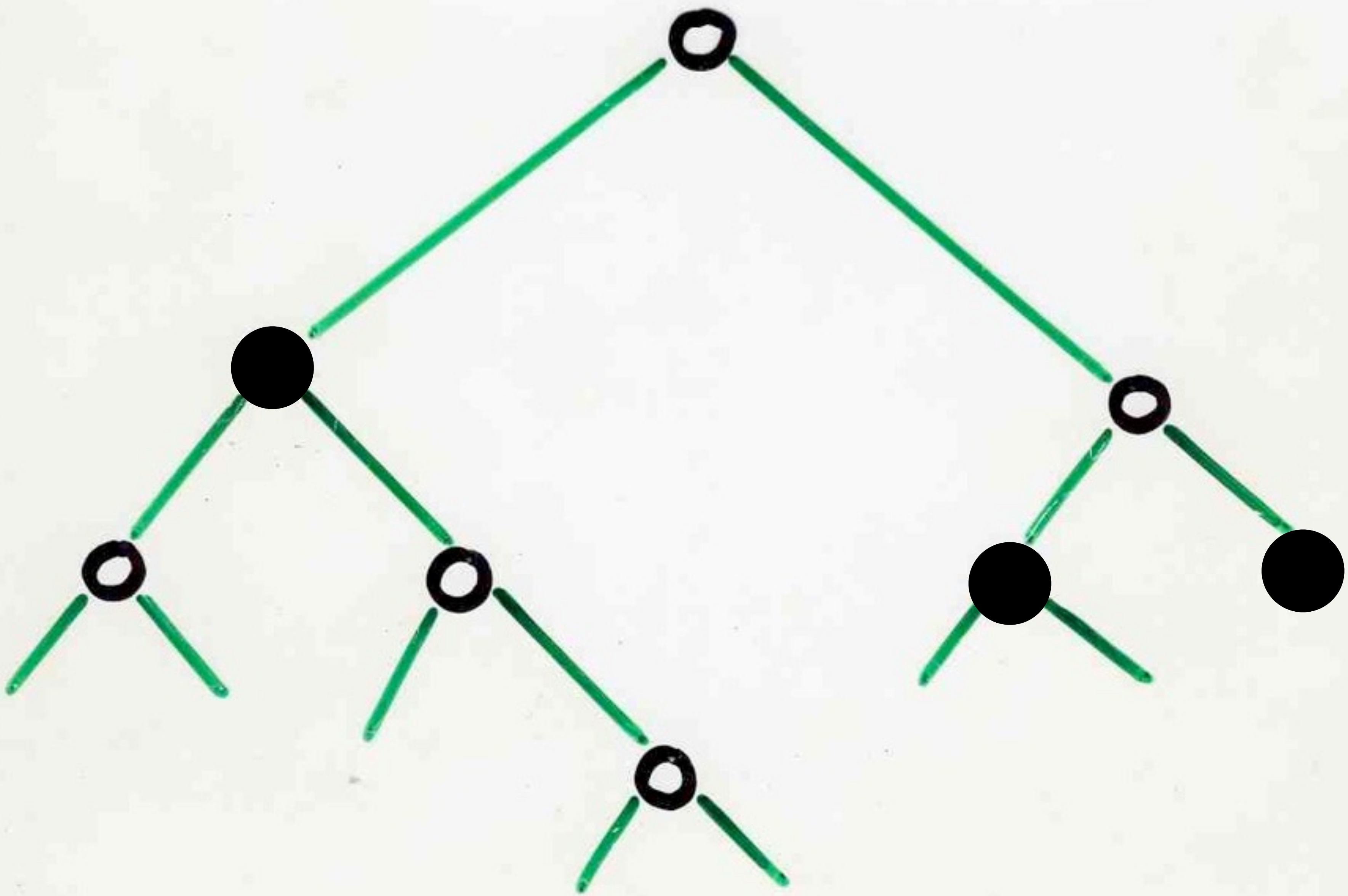


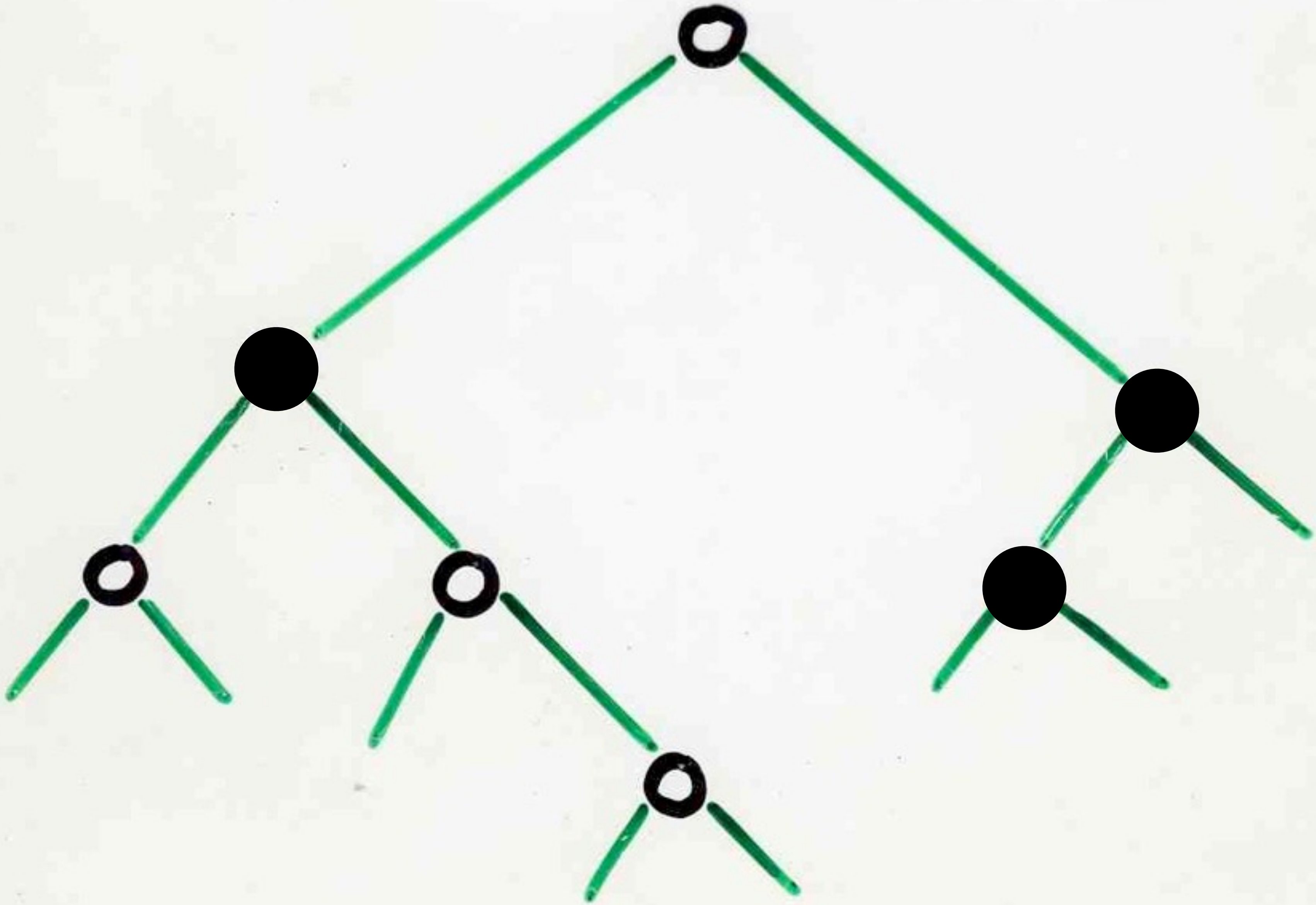


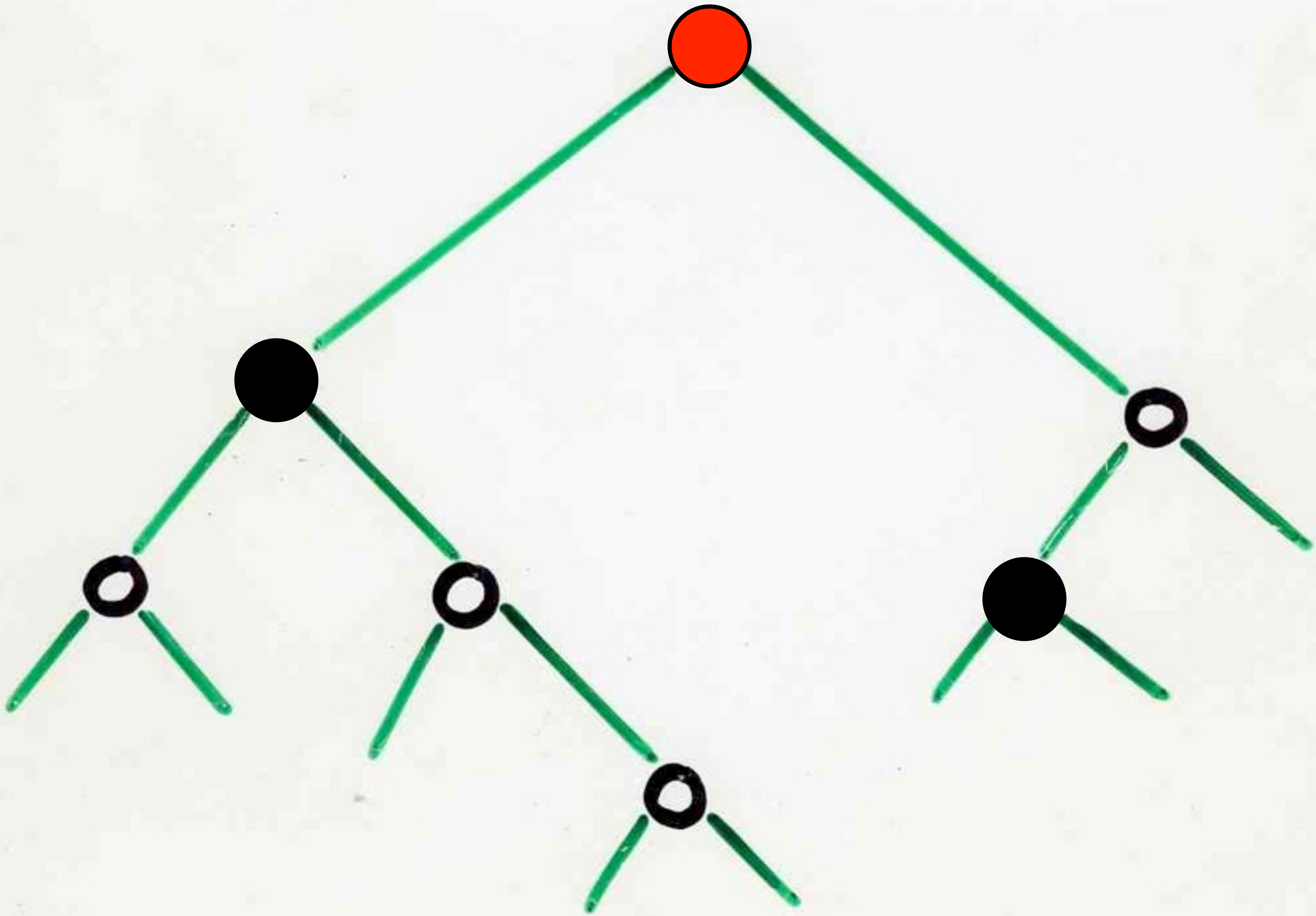












From trees to rivers ....

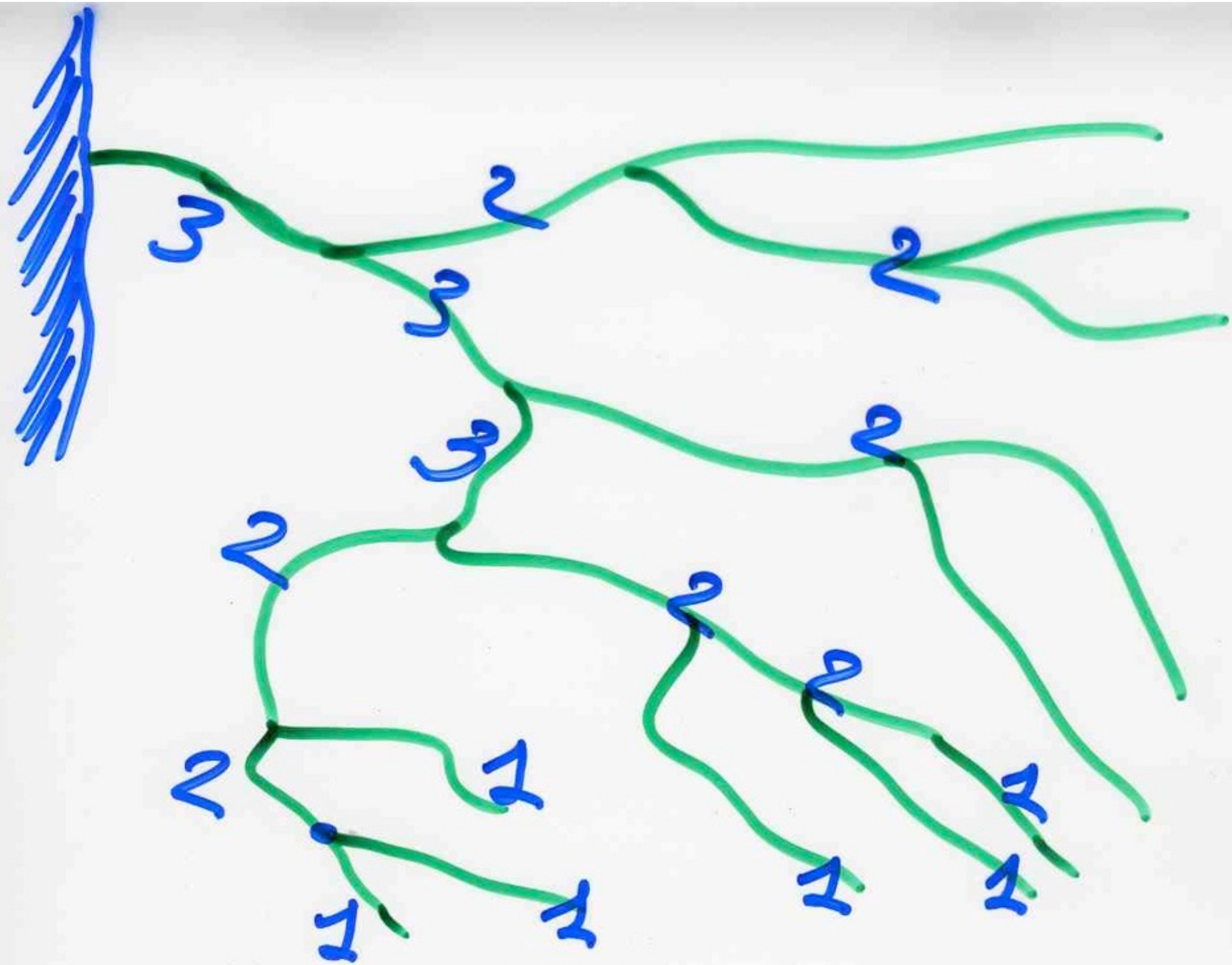
Horton (1945)

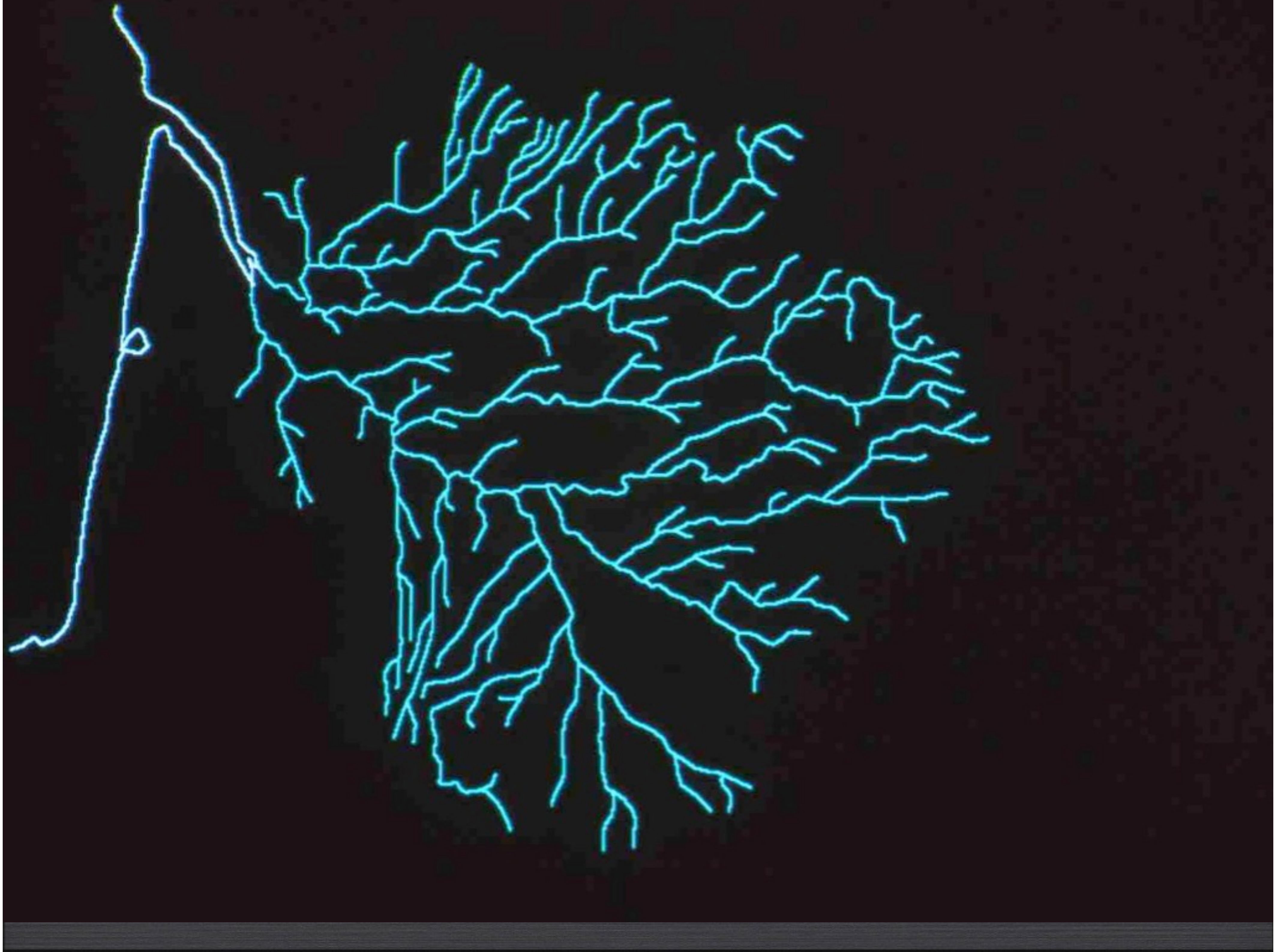
Strahler (1952)

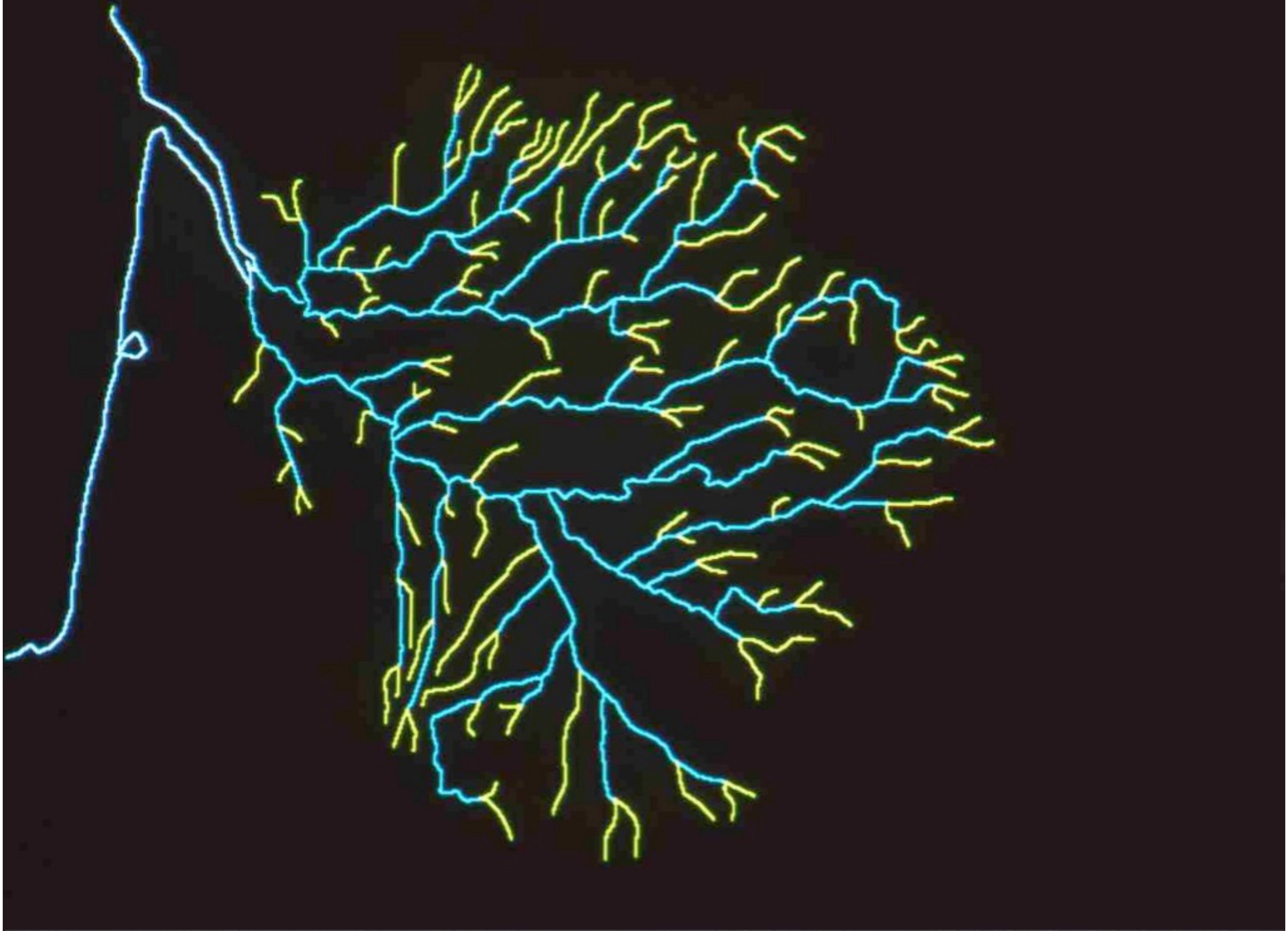
Hydrogeology

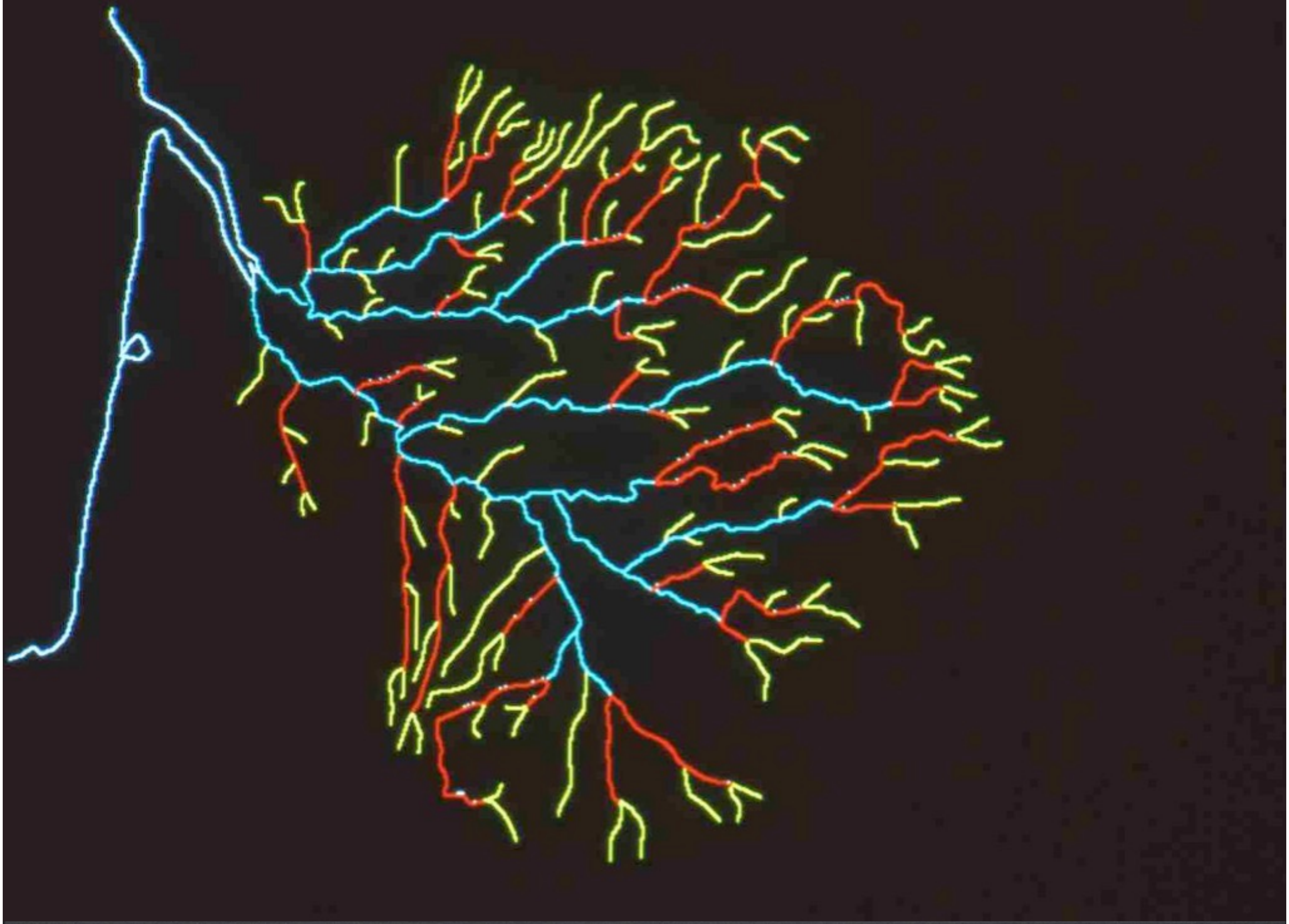
morphology of rivers network

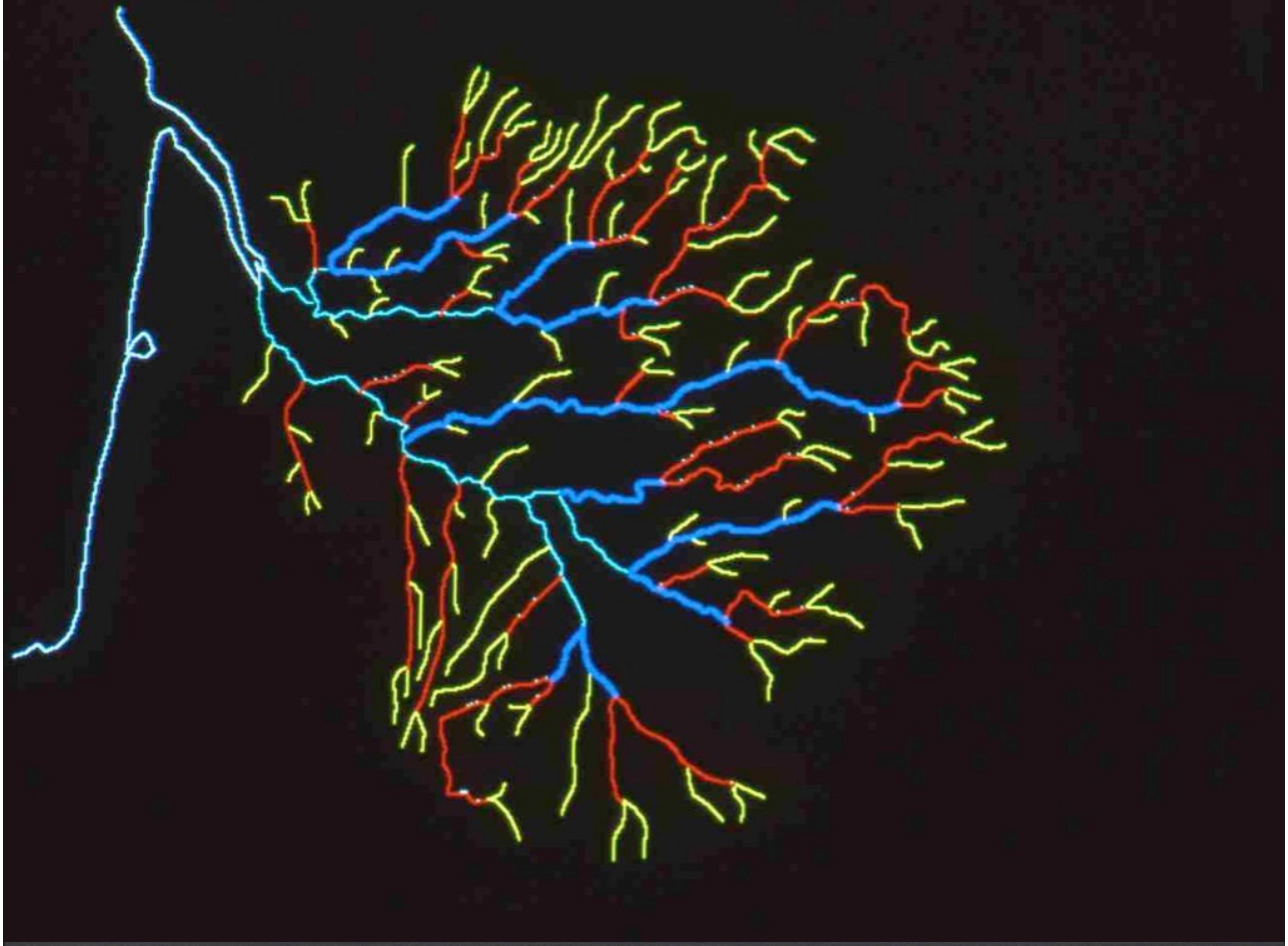
Order of a river

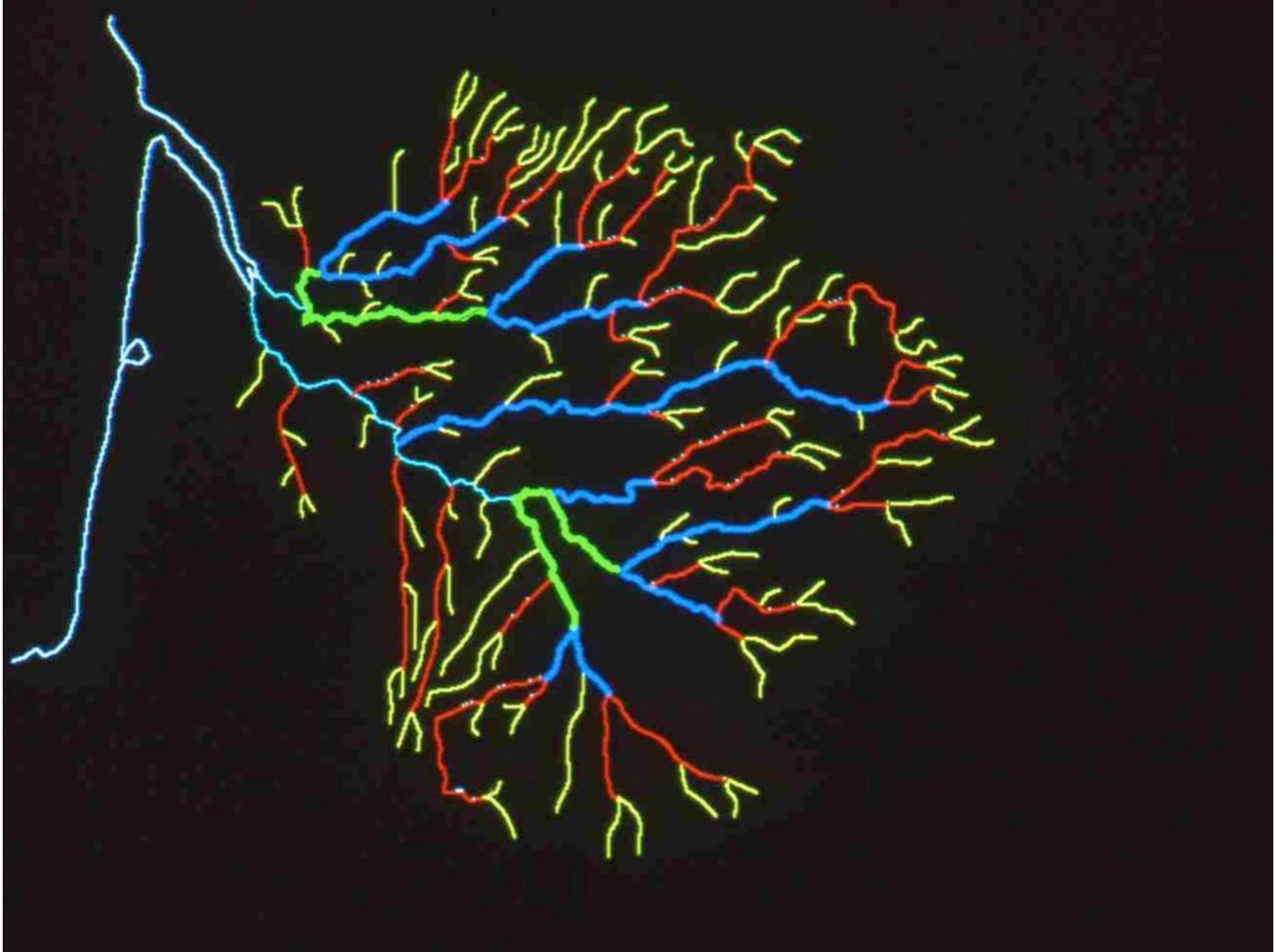

3
2
2
3
3
2
2
2
2
2
1
1
1
1
1
1

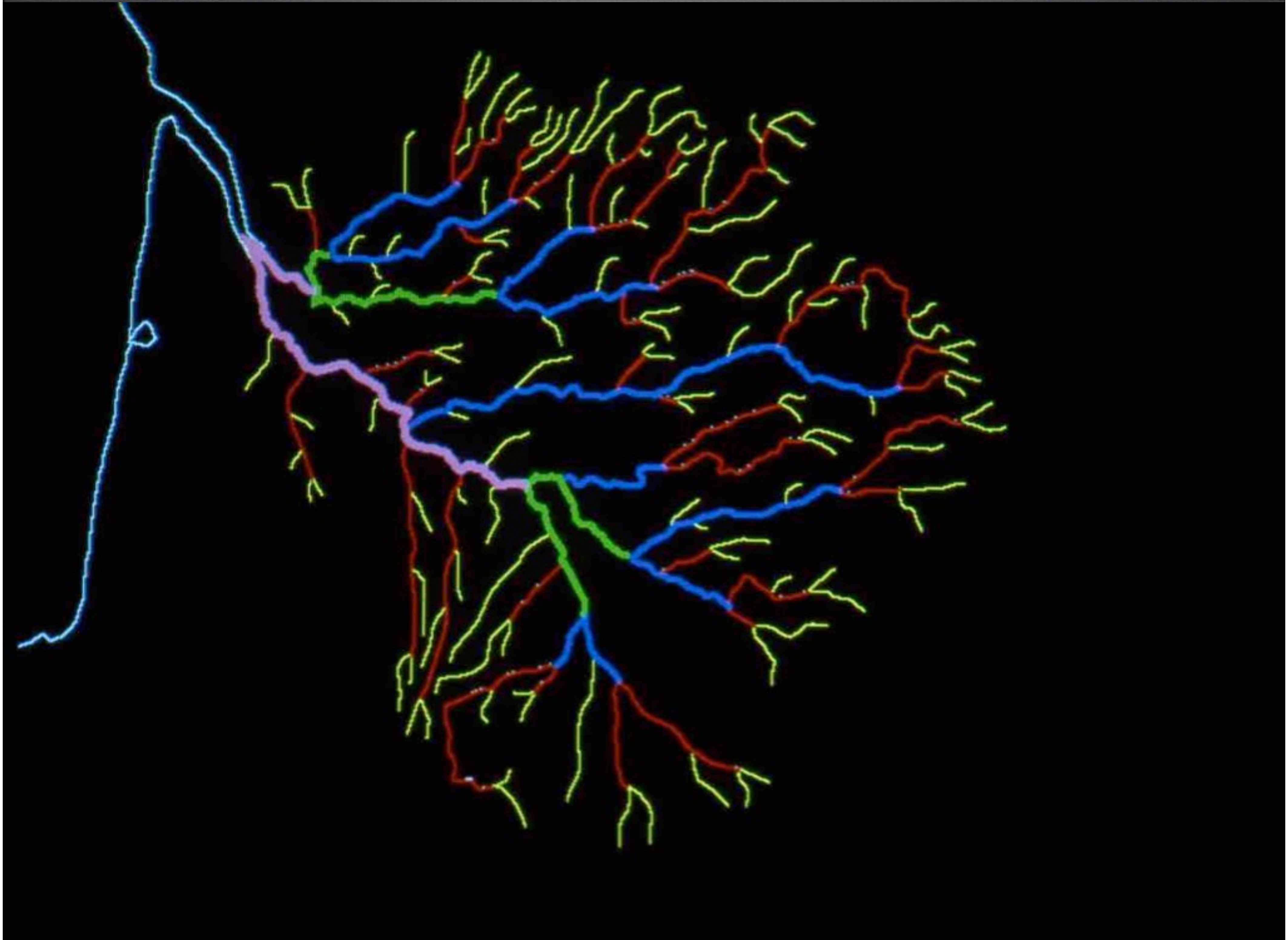


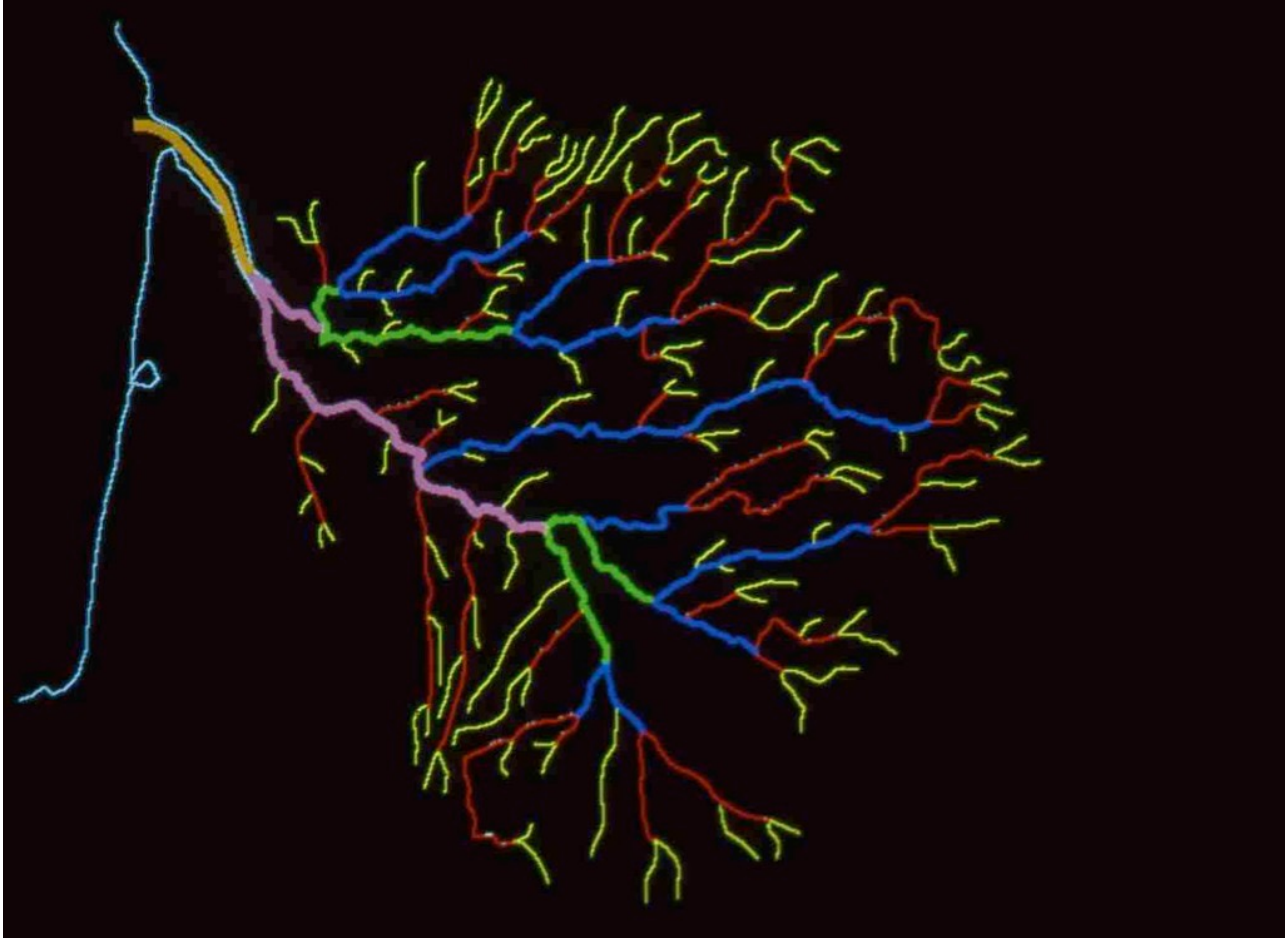


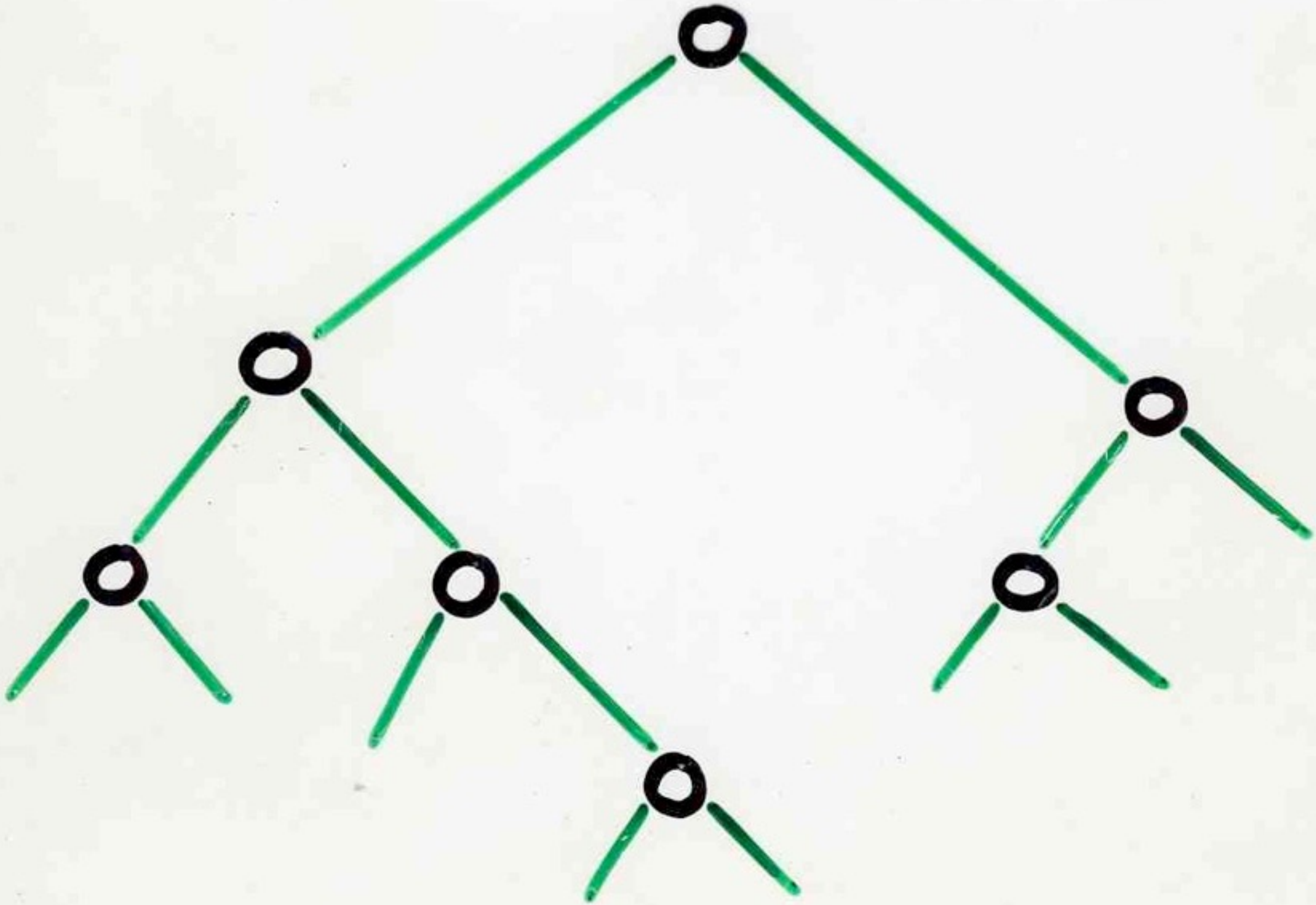


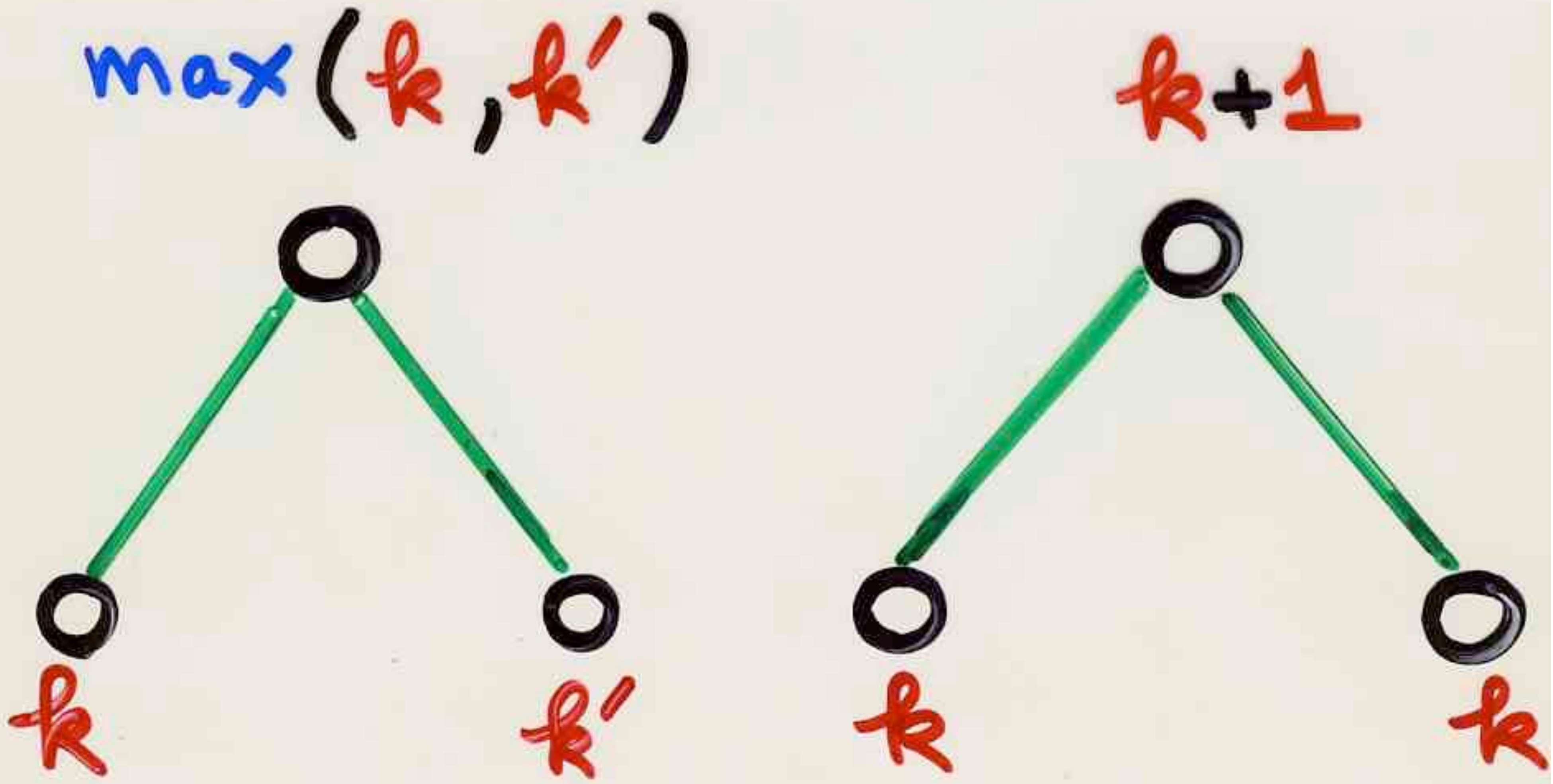






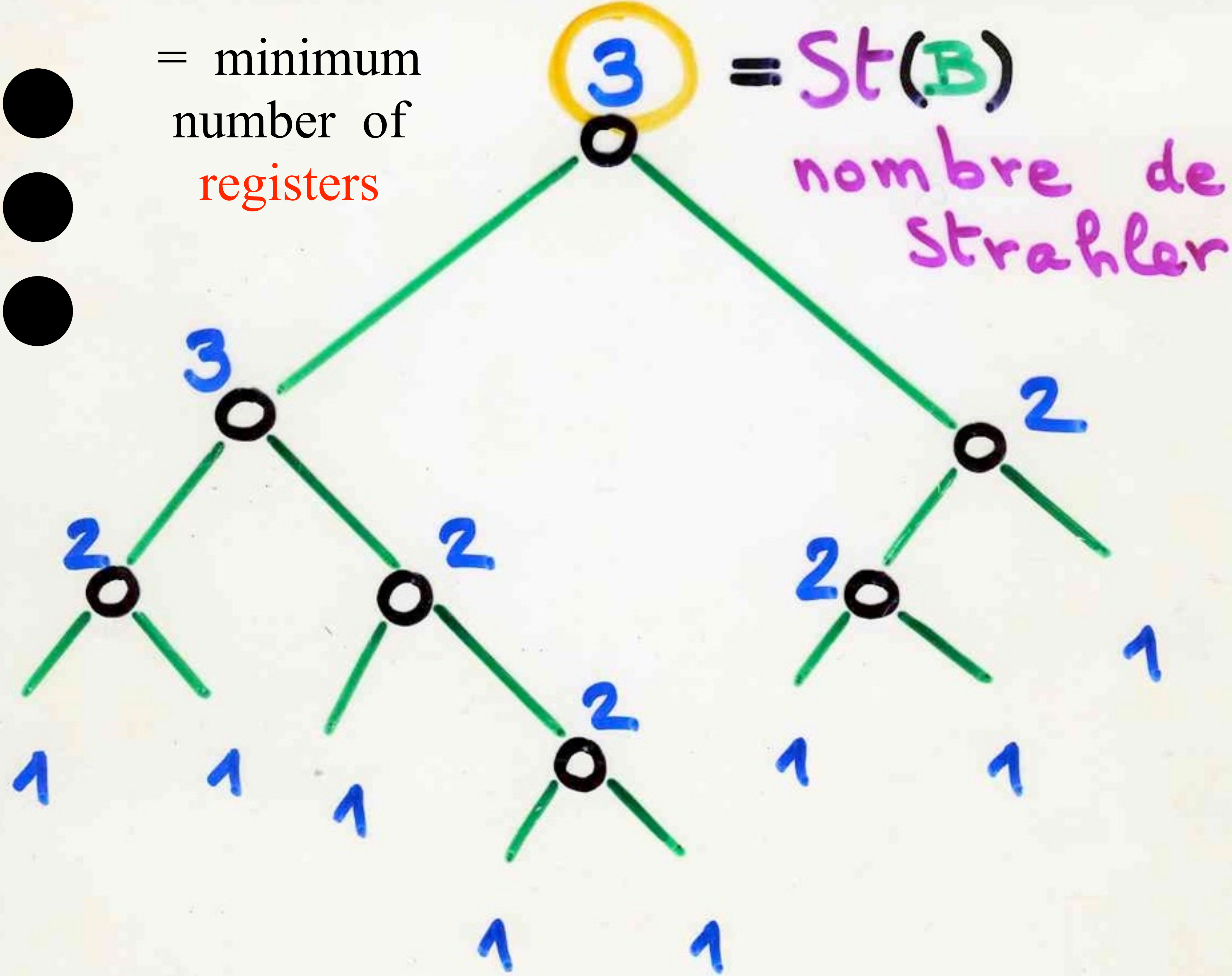




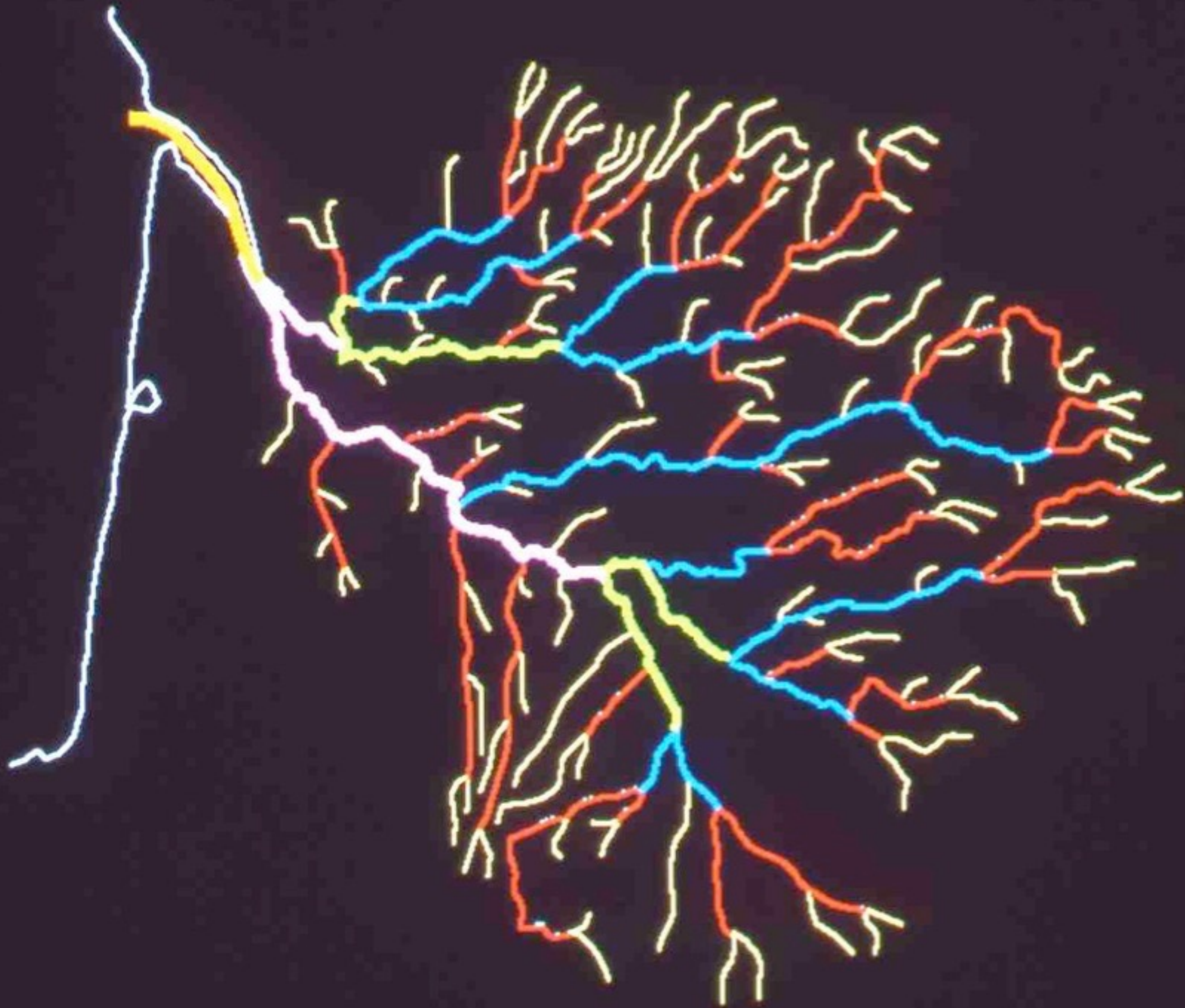


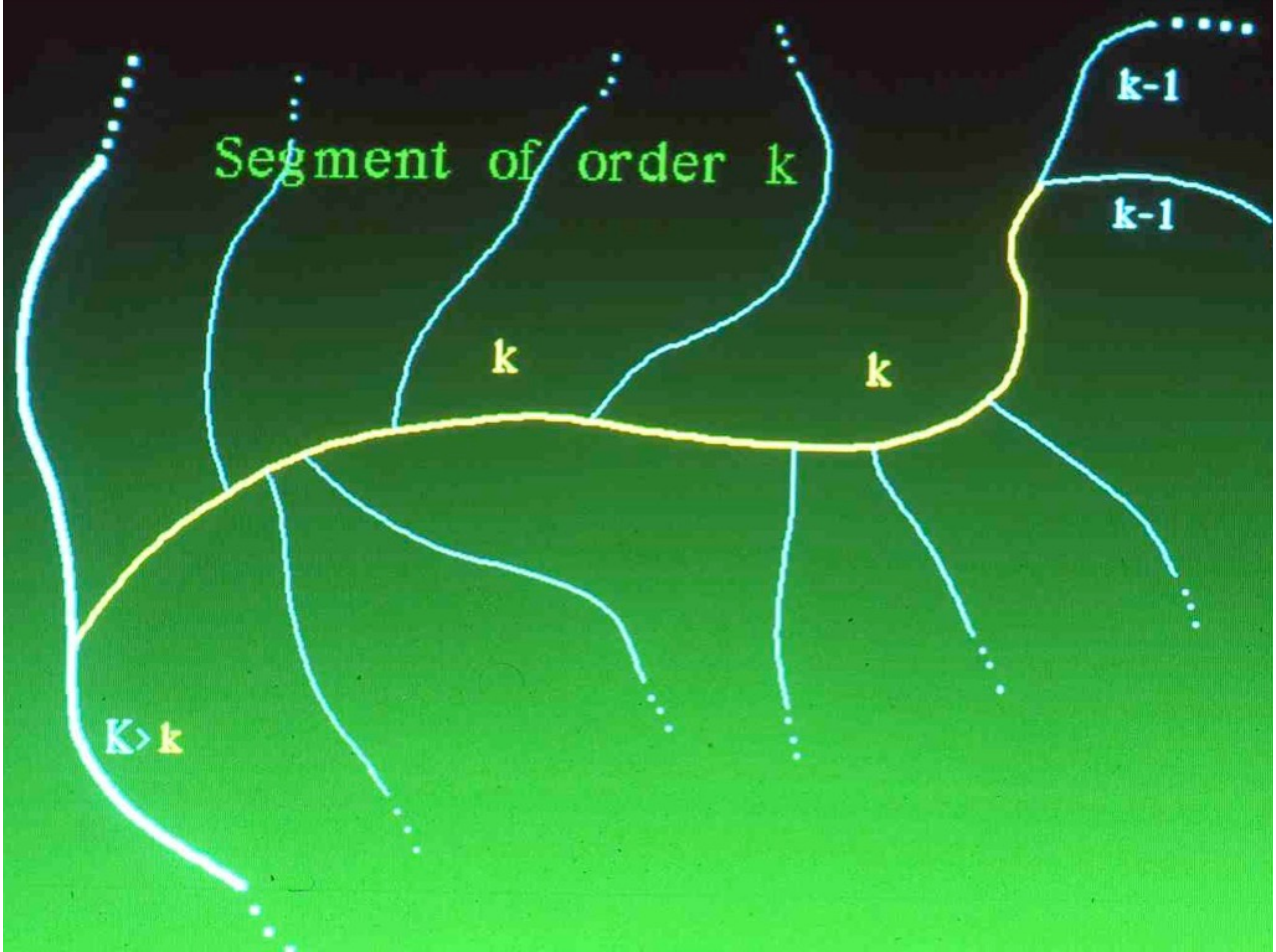
max(k, k')
k+1
k
k'
k
k

- = minimum number of registers
-
-



river or segment or order k





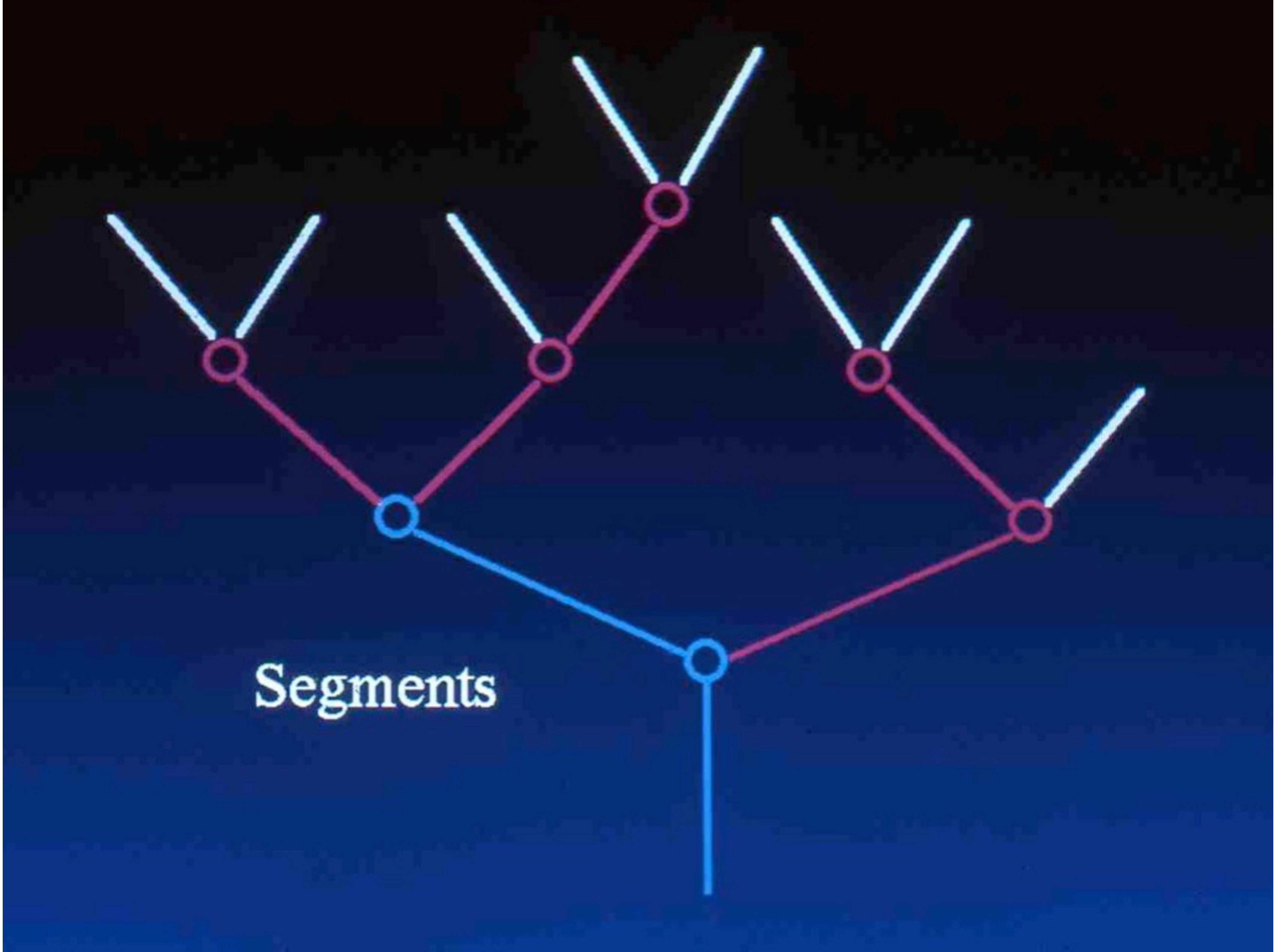
Segment of order k
k-1
k-1
k
k
K>k

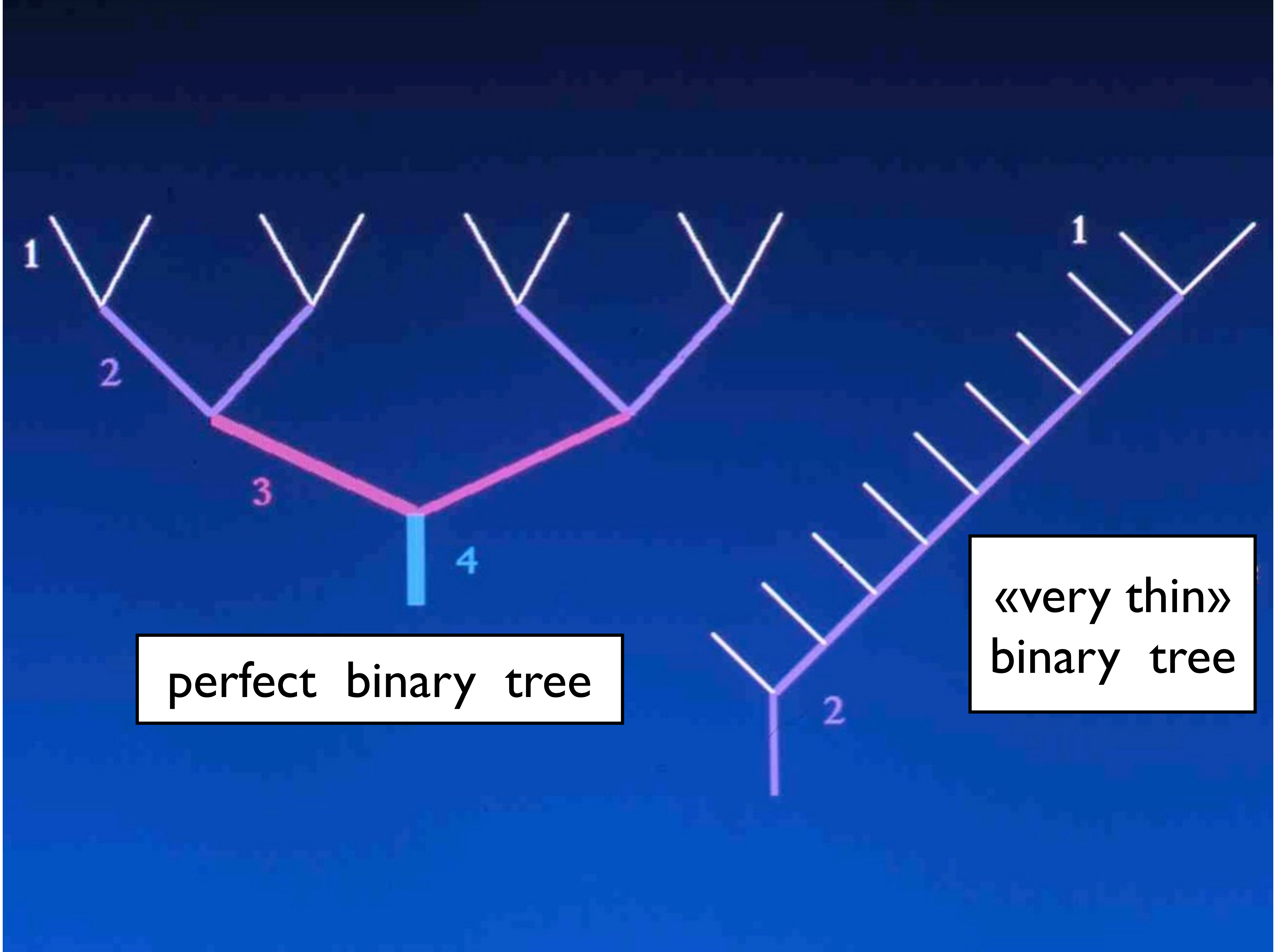
bifurcation ratio

$$3 < \beta_k = \beta < 5$$

$$\beta_k = \frac{b_k}{b_{k+1}}$$

b_k = number of segments of order k


Segments


1
2
3
4
perfect binary tree
1
2
«very thin» binary tree

correlation between the «shape» of the river network
and
the structure of the deep underground

Prud'homme, Nadeau, Vigneaux, 1970, 1980

computer graphics

ramification matrix of a binary tree

Arquès, Eyrolles, Janey, X.V.

SIGGRAPH'89, IMAGINA' 90

Synthetic images of trees, leaves, landscapes ...

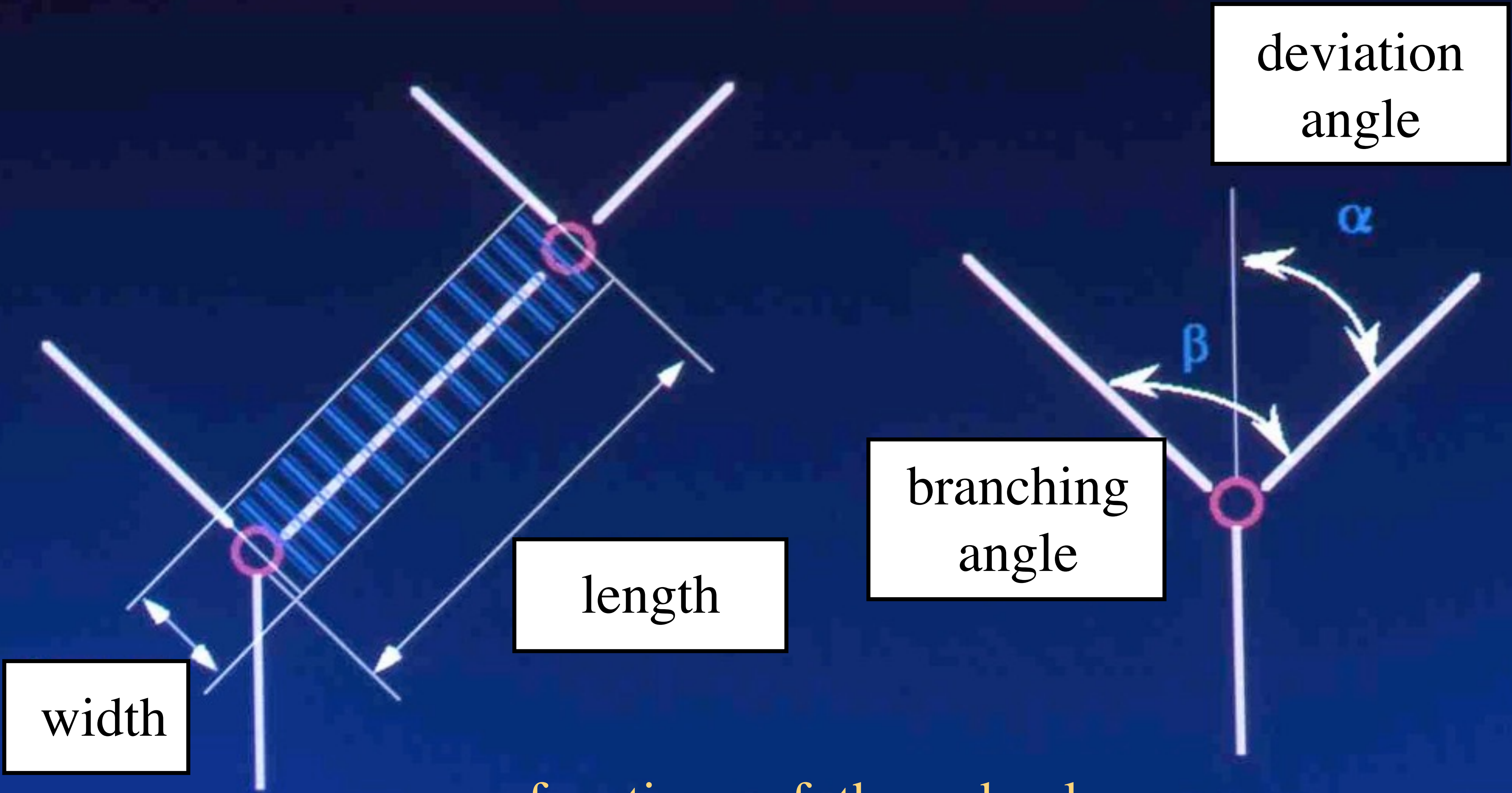
Arquès, Eyrolles, Janey, X.V.

Ramification matrix

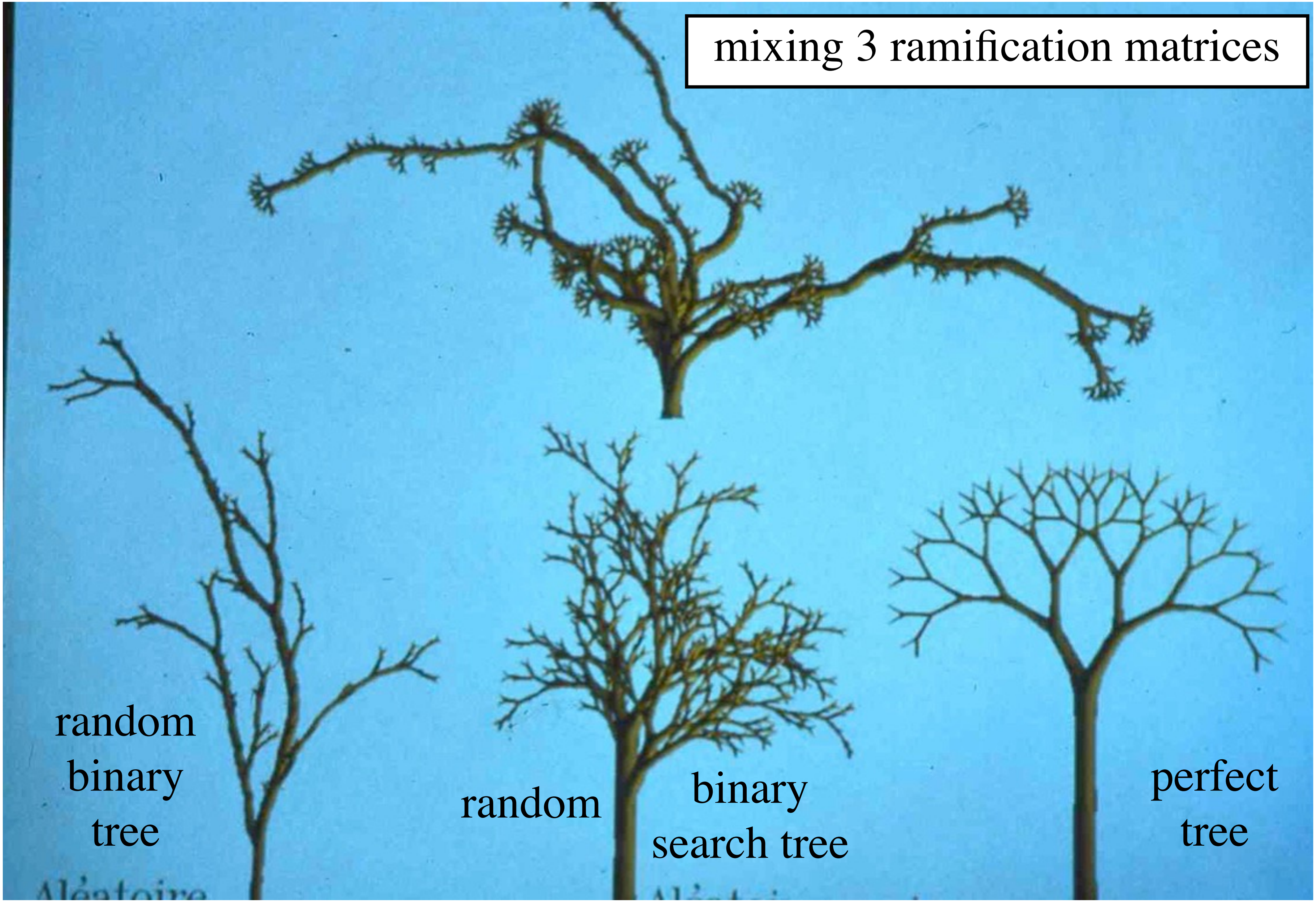
matrix of probabilities

$$P_{k,i} = \frac{b_{k,i}}{a_k}$$

biorder (k,i)



functions of the order k
and of the biorder (k,i)


mixing 3 ramification matrices
random
binary
tree
random
binary
search tree
perfect
tree

Synthetic images of trees, leaves, landscapes ...

Arquès, Eyrolles, Janey, X.V.

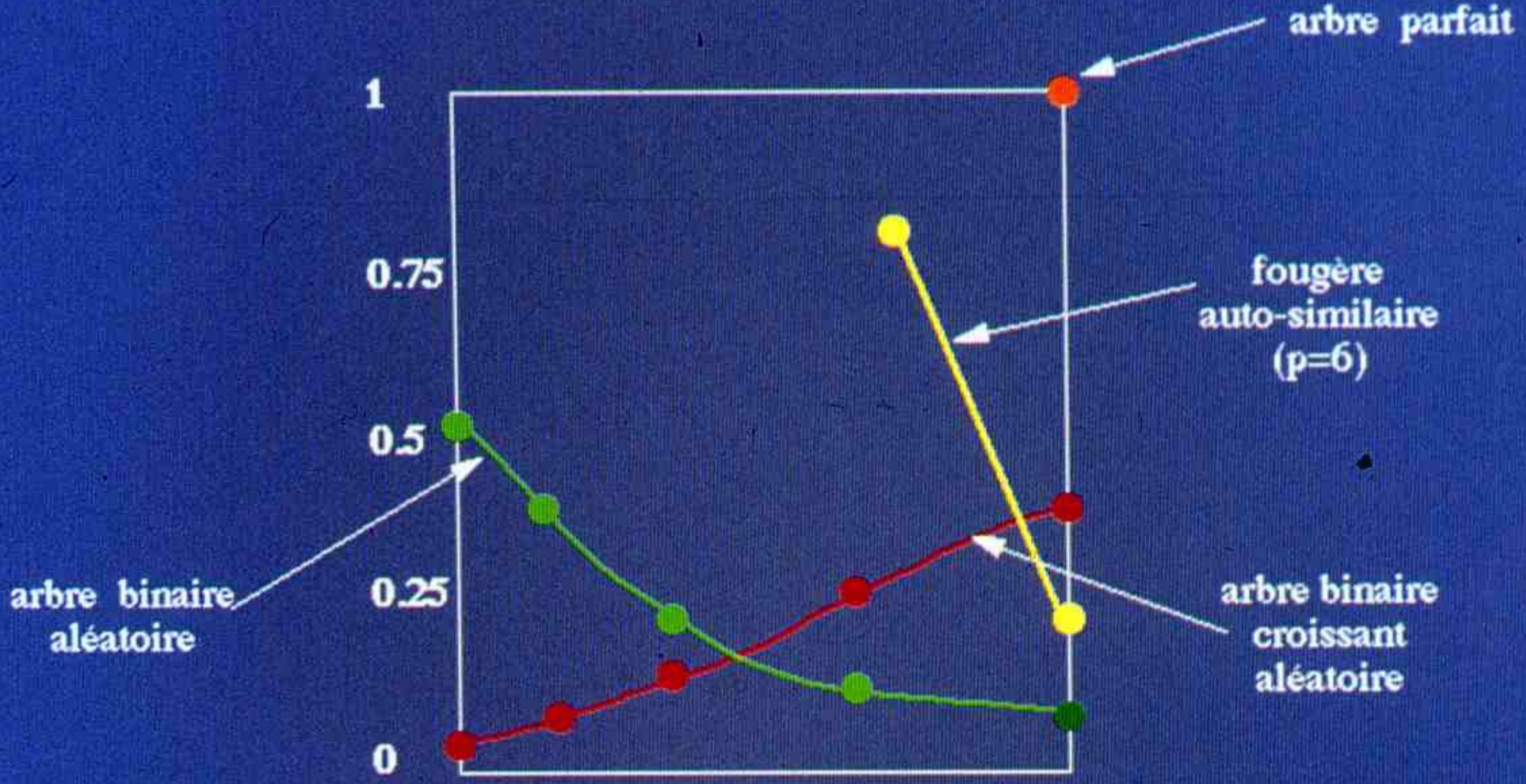
A$\mathfrak{S}$A

Ramification matrix

matrix of probabilities

$$P_{k,i} = \frac{b_{k,i}}{a_k}$$

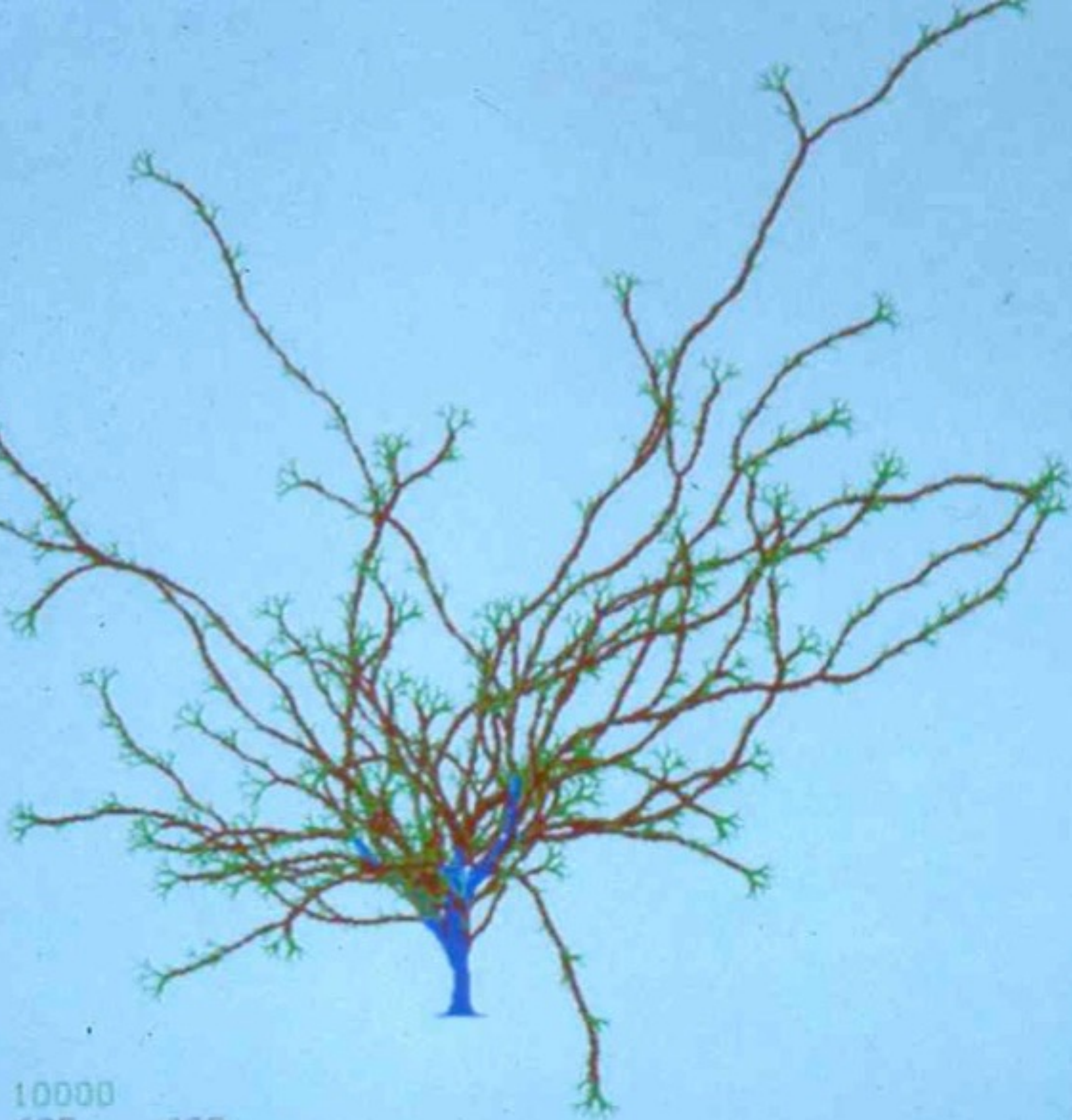
biorder (k,i)



matrices de ramification auto-similaires

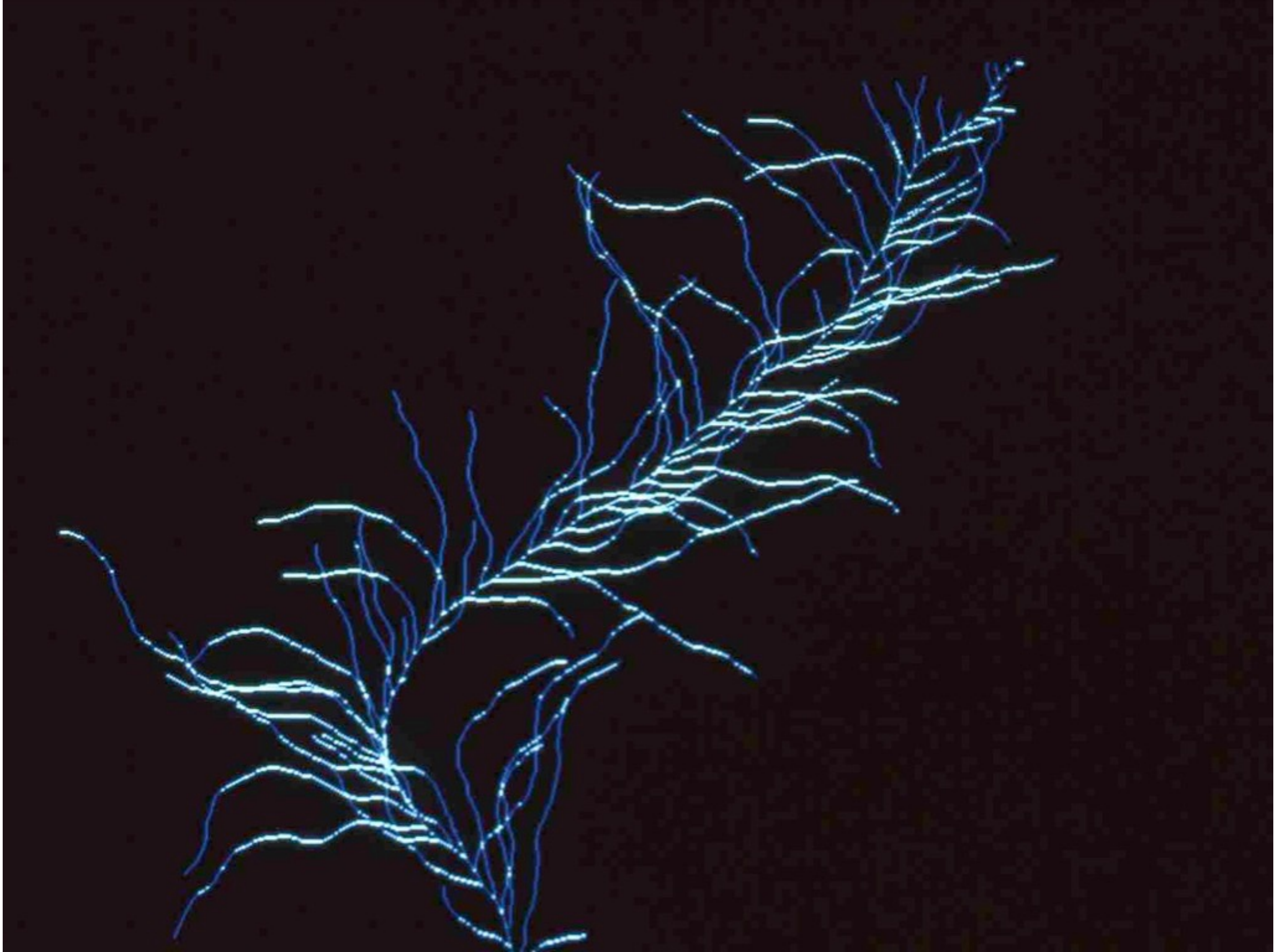
mixing
3 ramification
matrices

3 «shapes»



2 :	0	10000								
3 :	0	0	10000							
4 :	0	0	0	10000						
5 :	5000	2500	1250	625	625					
6 :	5000	2500	1250	625	313	312				
7 :	125	250	500	1000	2000	3000	3125			
8 :	63	125	250	500	1000	2000	3000	3062		
9 :	31	63	125	250	500	1000	2000	3000	3031	
10:	15	31	63	125	250	500	1000	2000	3000	3016
11 :	7	15	31	63	250	125	500	1000	2000	3000	3009


ASA



If there exist some beauty in these synthetic images of trees,
it is only the pale reflection of the extraordinary beauty of the mathematics hidden behind the algorithms generating these images

average Strahler number over binary trees n vertices

$$St_n = \log_4 n + f(\log_4 n) + O(1)$$

Flajolet, Raoult, Vuillemin (1979)
Kemp

f periodic

Numbers theory

$T(n)$ = number of 1's in the binary expansion of 1, 2, .., (n-1)

generating function

$S_{n,k}$ = nb of (complete) binary trees B (with) n (internal) vertices (and) Strahler nb $St(B) = k$

$$S_k(t) = \sum_{k \geq 0} S_{n,k}\, t^n$$

formal power series

$$S_1 = 1$$

$$S_2 = \frac{t}{1-2t}$$

$$S_3 = \frac{t^3}{1-6t+10t^2-4t^3}$$

$$S_4 = \frac{t^7}{1-14t+78t^2-220t^3+330t^4-252t^5+84t^6-8t^7}$$

Pafnuty Chebyshev
(1887-1920)

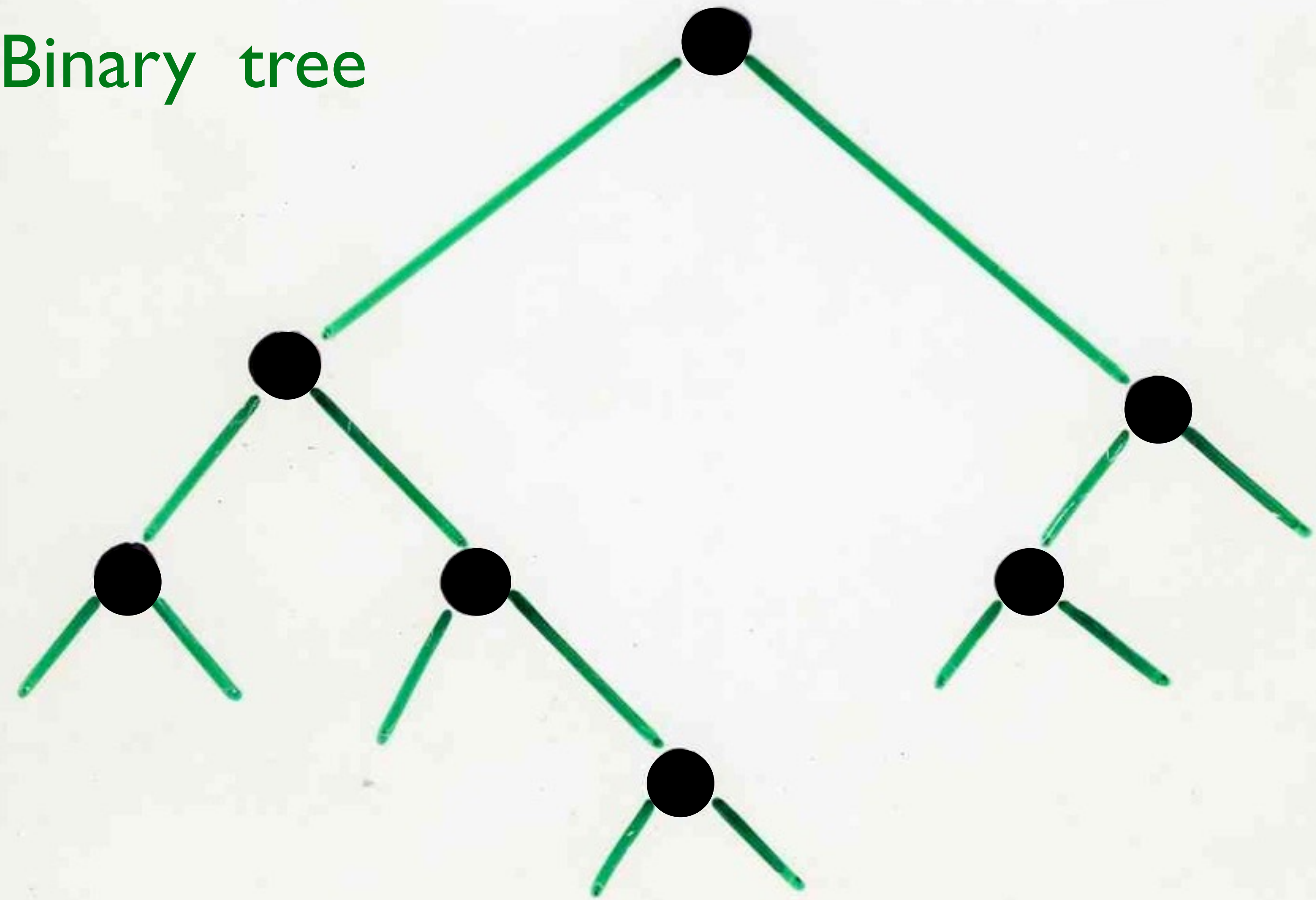
Chebyshev polynomials

trigonometry

$$\sin(n+1)\theta = (\sin\theta)\, U_n(\cos\theta)$$

Counting trees ...

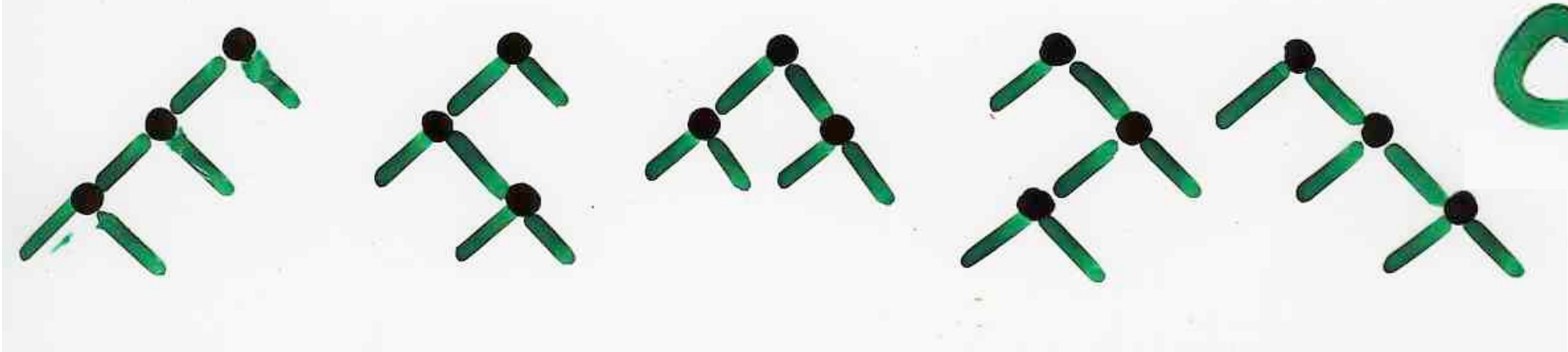
Binary tree



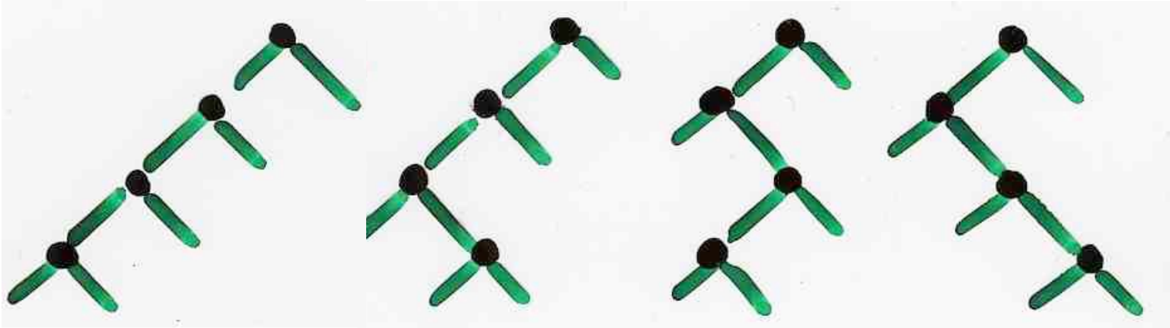
number of binary trees
having n internal vertices ● n=7
(or n+1) leaves (external vertices)

$$C_1 = 1$$

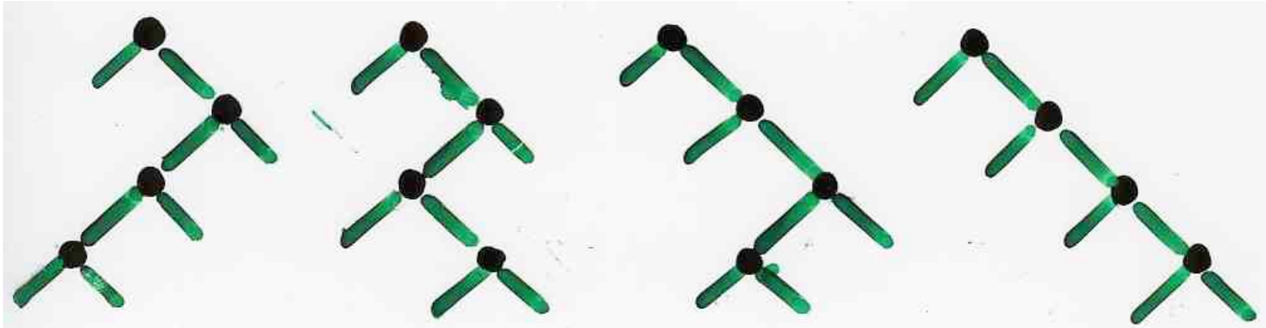
$$C_2 = 2$$

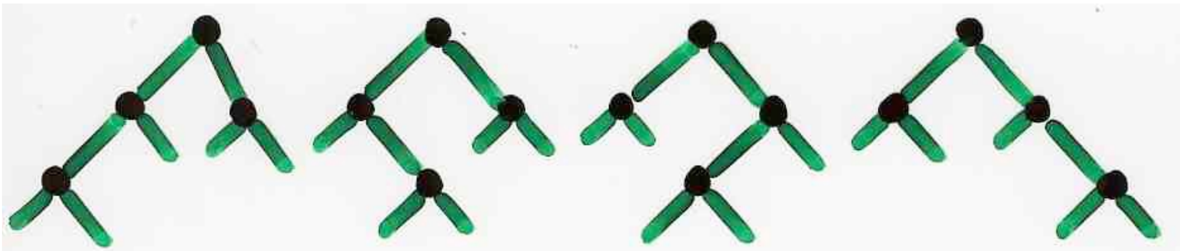


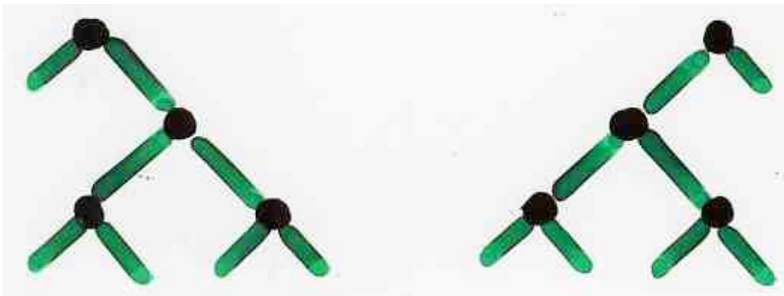
$$C_3 = 5$$



$C_4 = 14$







Catalan number

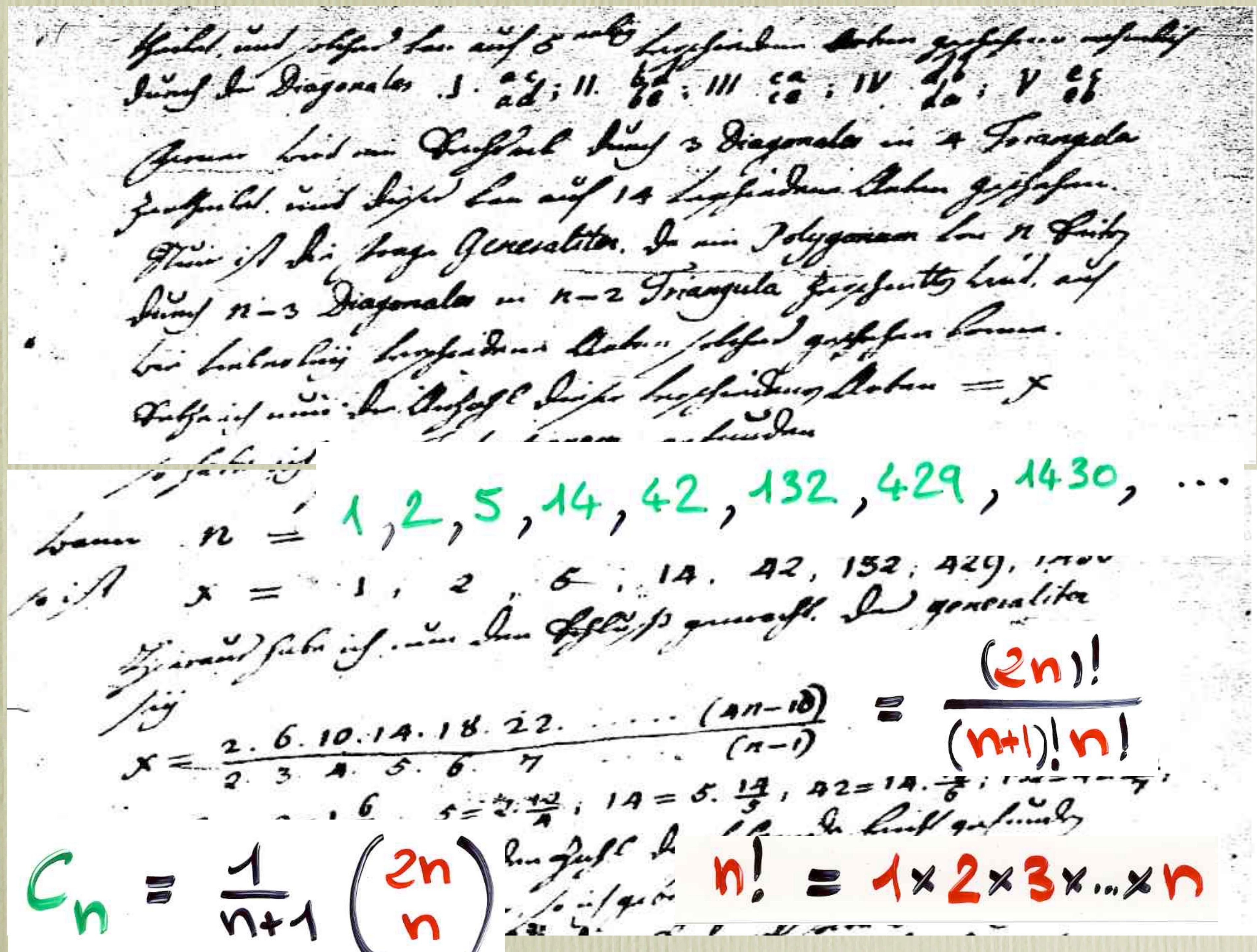
$$C_n = \frac{1}{n+1}\binom{2n}{n}$$

$$= \frac{(2n)!}{(n+1)!\;n!}$$

$$n! = 1 \times 2 \times \dots \times n$$

Catalan numbers

E. Catalan
(1814-1894)

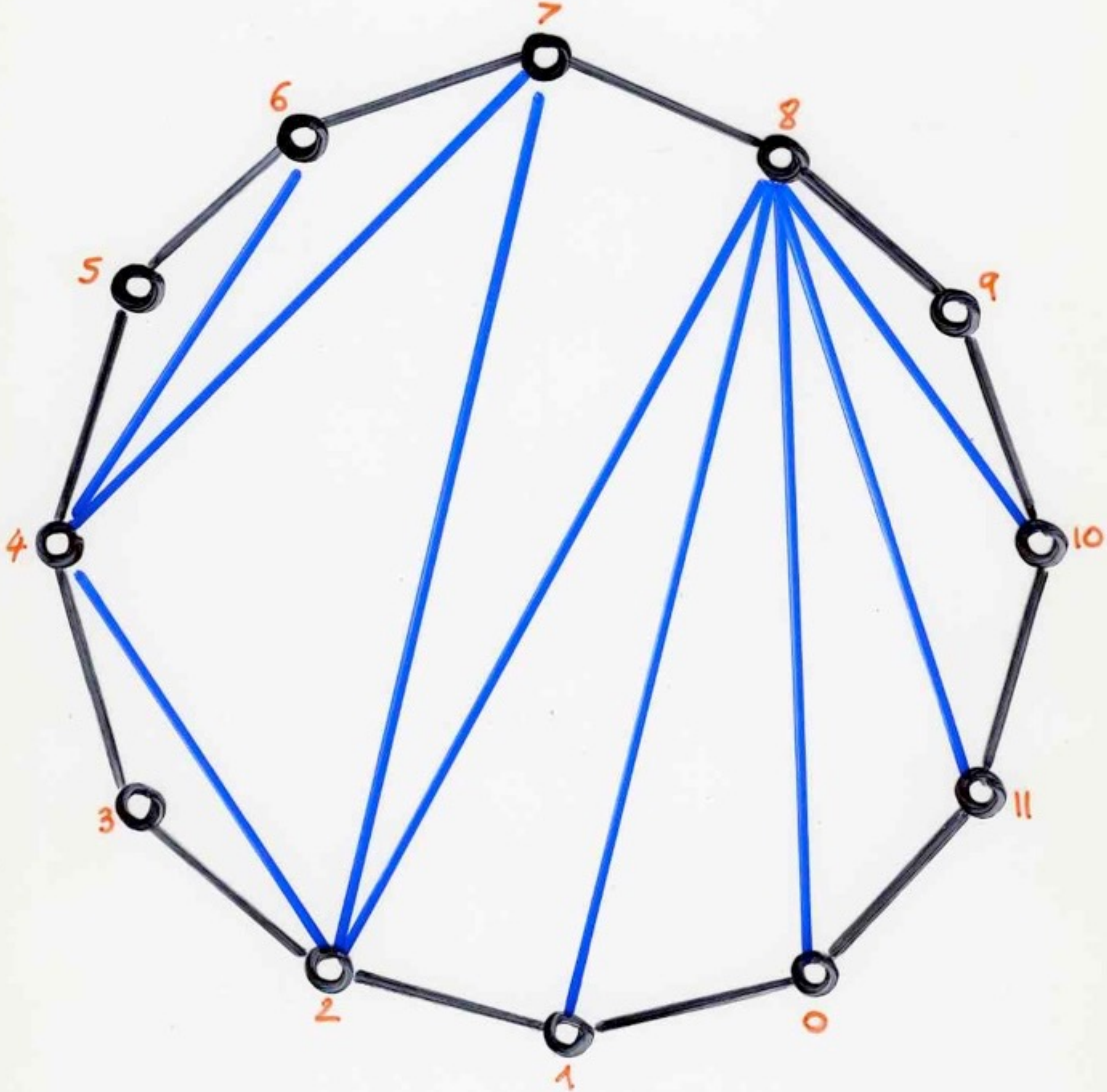


I. ac ad ; II. bd be ; III. ca ce ; IV. da db ; V. ec eb
n − 3
n − 2
= x
n =
1, 2, 5, 14, 42, 132, 429, 1430, ...
x = 1, 2, 5, 14, 42, 132, 429, 1430
x = 2. 6. 10. 14. 18. 22. .... (4n−10) / 2. 3. 4. 5. 6. 7 .... (n−1)
= (2n)! / (n+1)! n!
5 = 2 · 10/4 ; 14 = 5 · 14/5 ; 42 = 14 · 18/6 ;
C_n = 1/(n+1) (2n n)
n! = 1×2×3×...×n

A letter from Leonhard Euler to Christian Goldbach ....

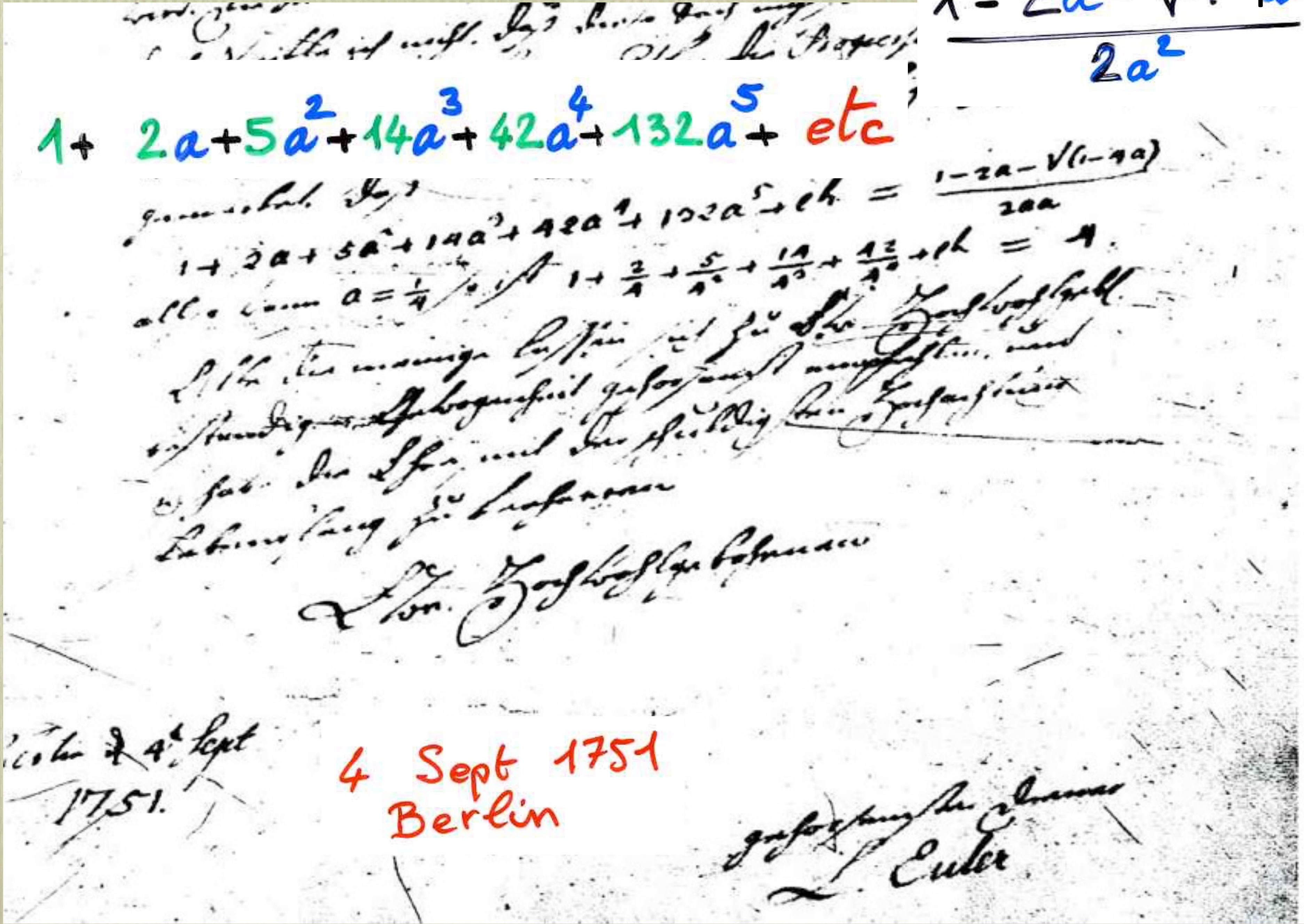
Berlin, 4 September 1751

Leonhard
Euler
1707 - 1783


7
6
8
5
9
4
10
3
11
2
0
1

$$\frac{1-2a-\sqrt{1-4a}}{2a^2}$$

$$1 + 2a + 5a^2 + 14a^3 + 42a^4 + 132a^5 + \text{etc}$$



$$1+2a+5a^2+14a^3+42a^4+132a^5+\text{etc} = \frac{1-2a-\sqrt{(1-4a)}}{2aa}$$

$$1+2a+5a^2+14a^3+42a^4+\text{etc} = 4.$$

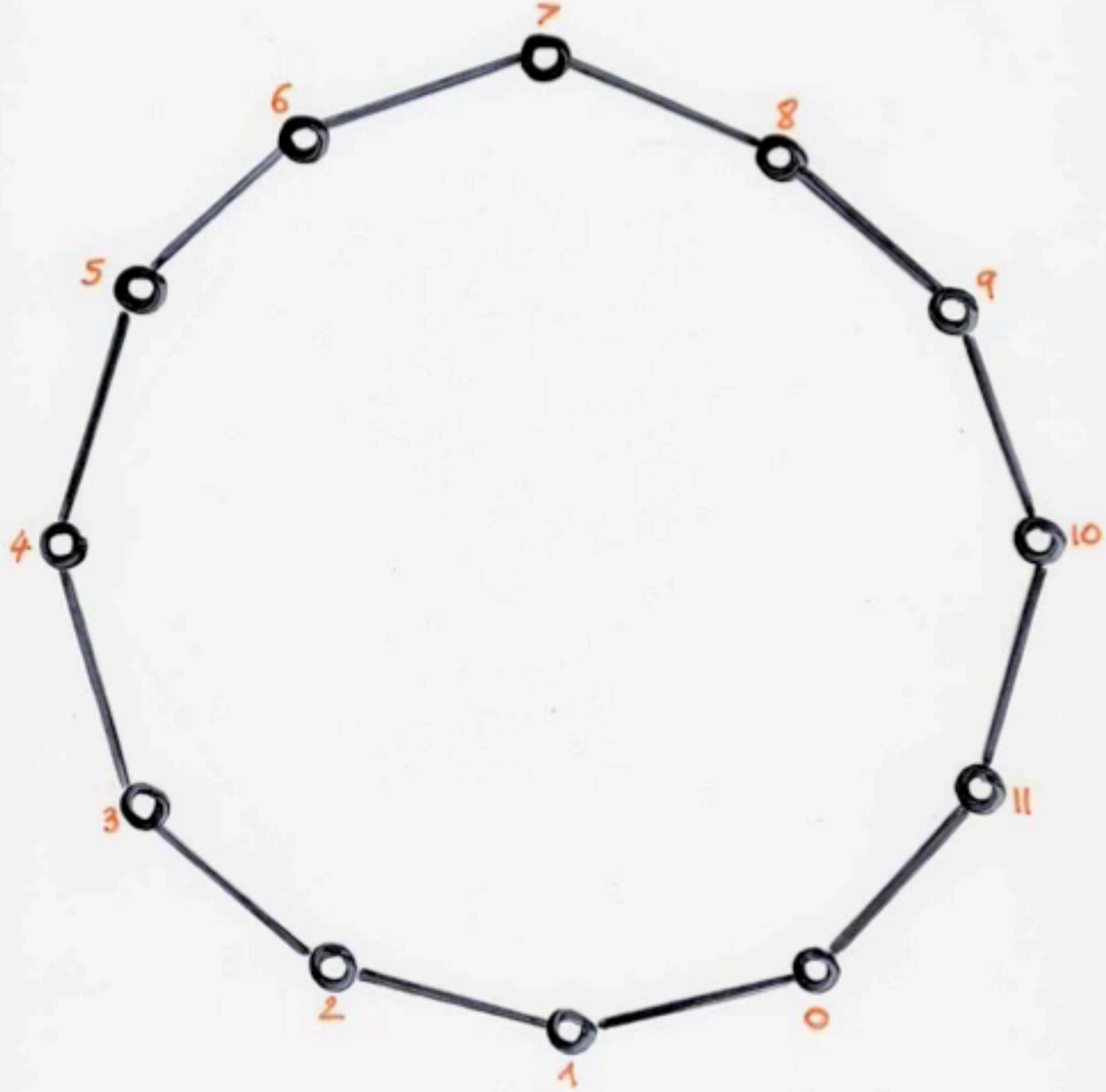
$$a = \tfrac{1}{4} \quad 1+\tfrac{2}{4}+\tfrac{5}{4^2}+\tfrac{14}{4^3}+\tfrac{42}{4^4}+\text{etc}$$

L. Euler

d. 4. Sept 1751.

4 Sept 1751
Berlin

from triangulations
to binary trees

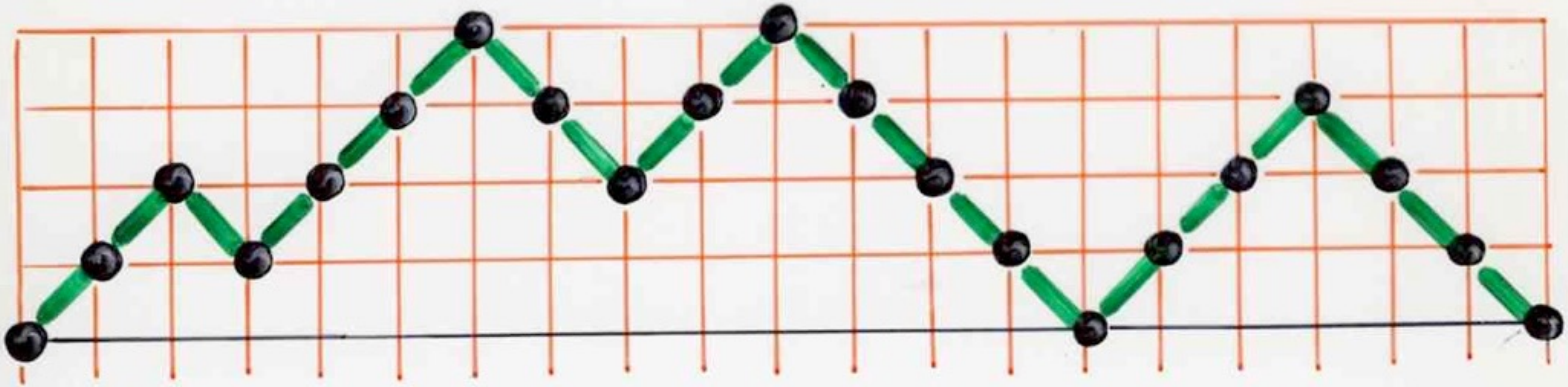

7
6
8
5
9
4
10
3
11
2
0
1

How to prove the relation
between the distribution of Strahler numbers
and Chebyshev polynomials?

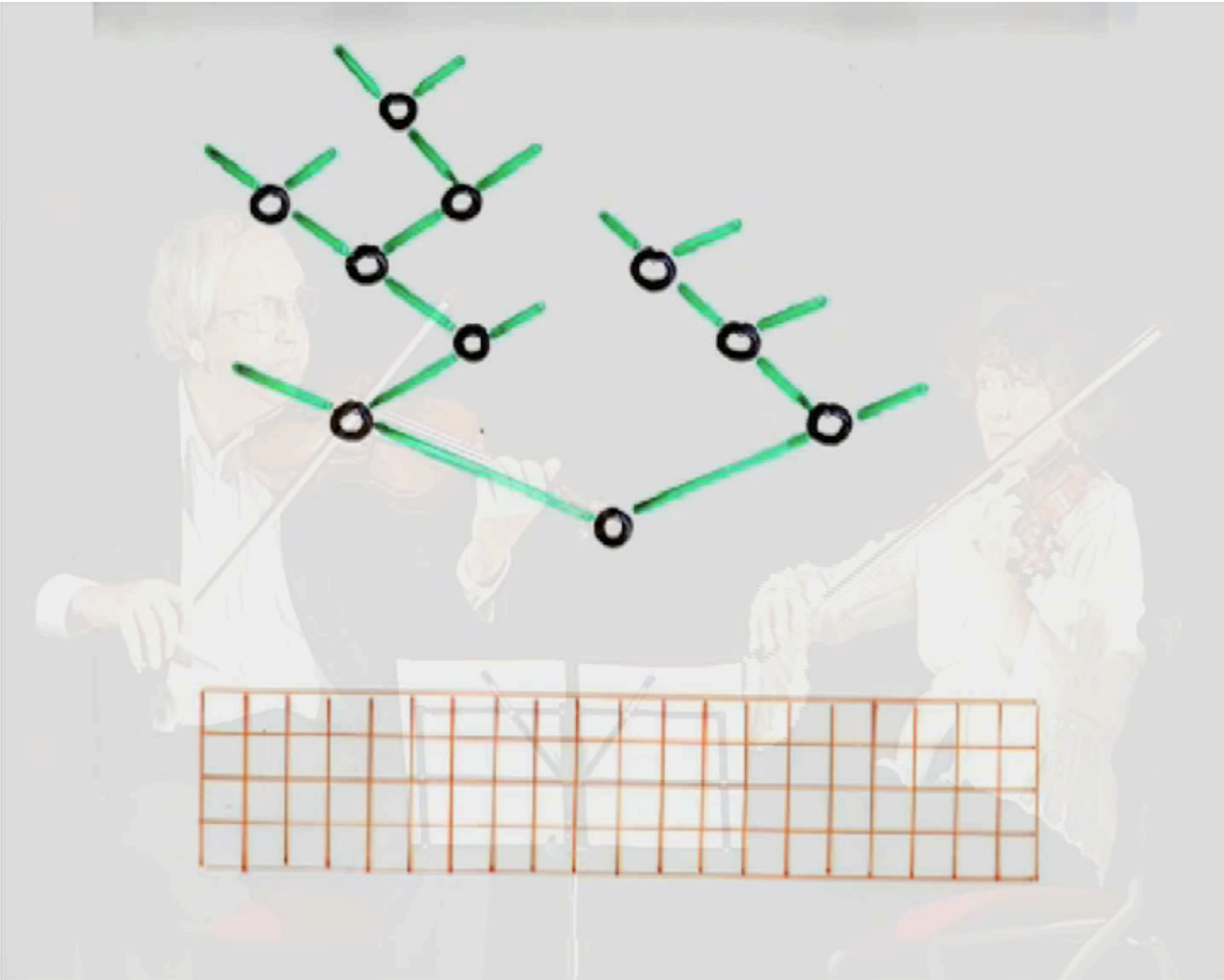
$$S_k(t) = \sum_{k \geq 0} S_{n,k}\, t^n$$

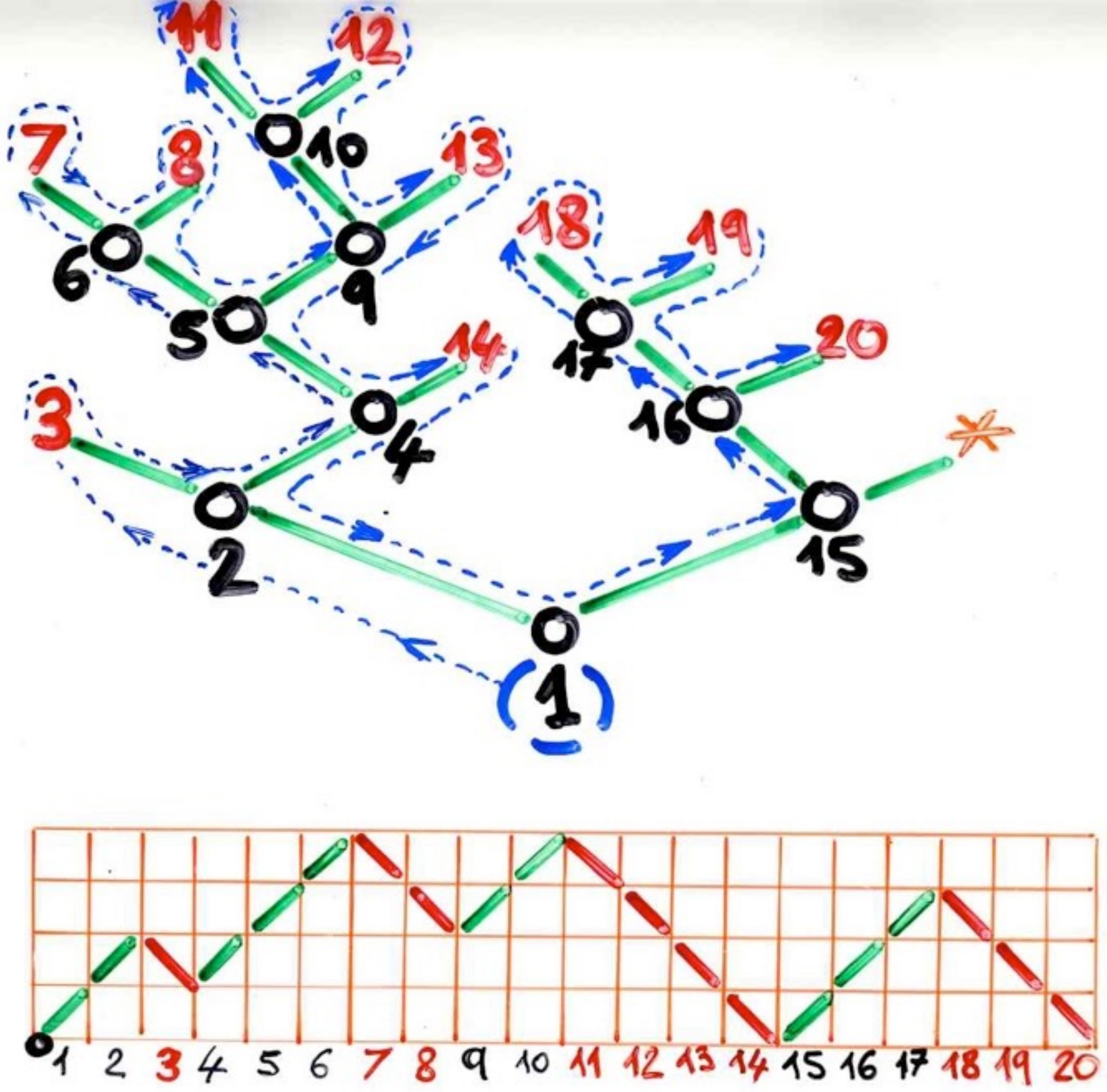
$$\sin(n+1)\theta = (\sin\theta)\, U_n(\cos\theta)$$

Dyck path



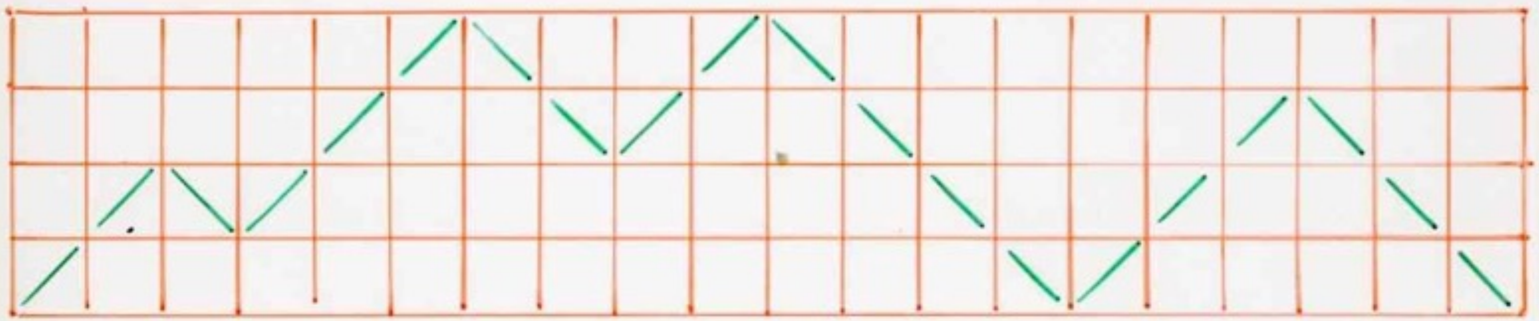
from binary trees
to Dyck paths ....

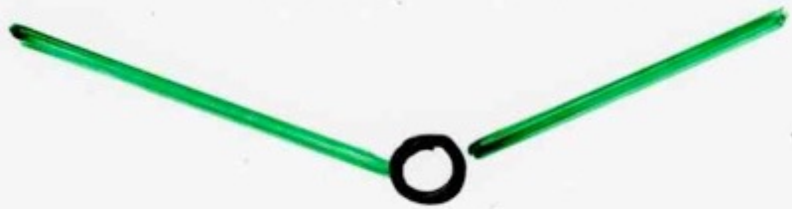


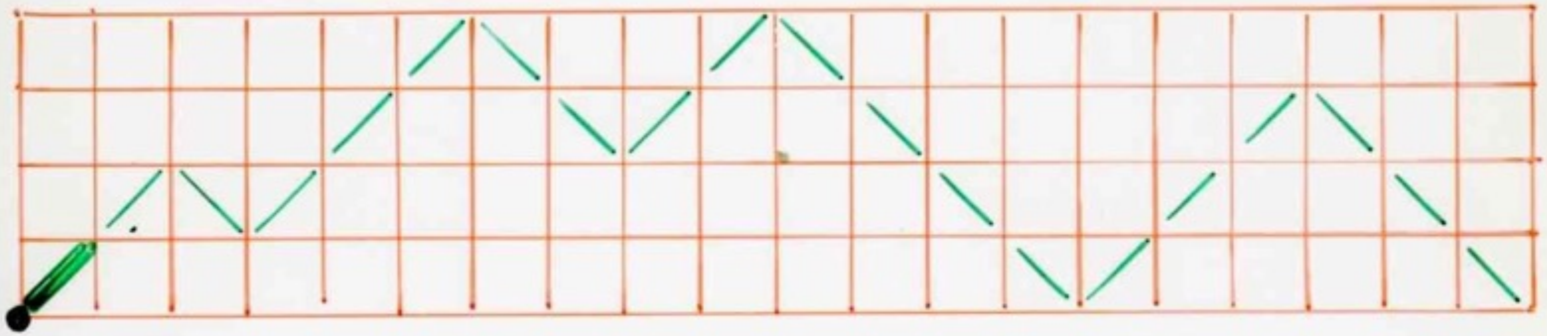


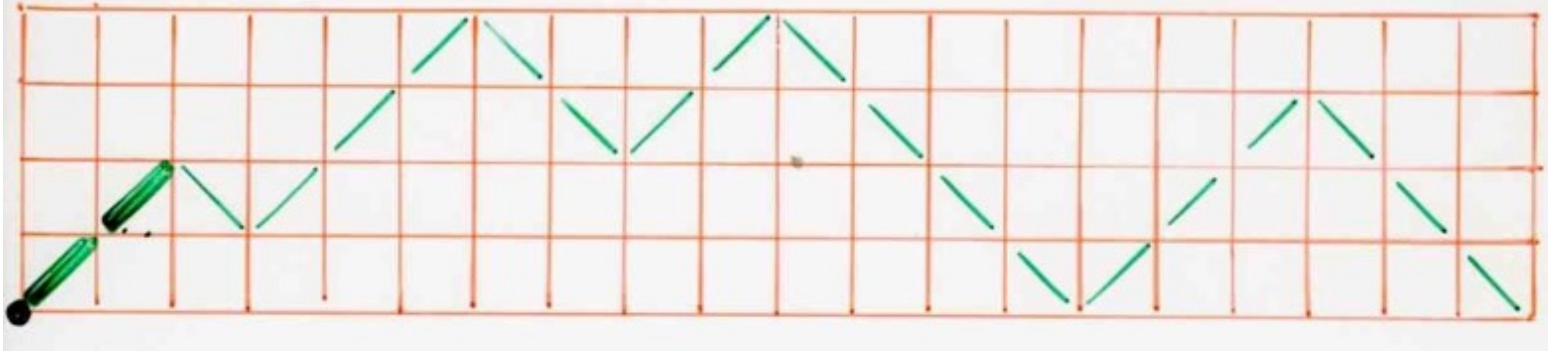
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

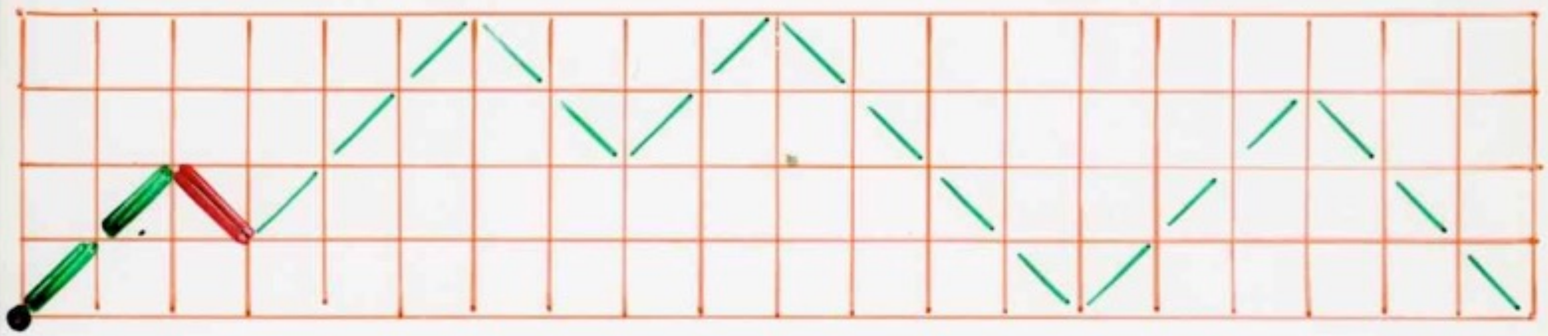
reciprocal bijection

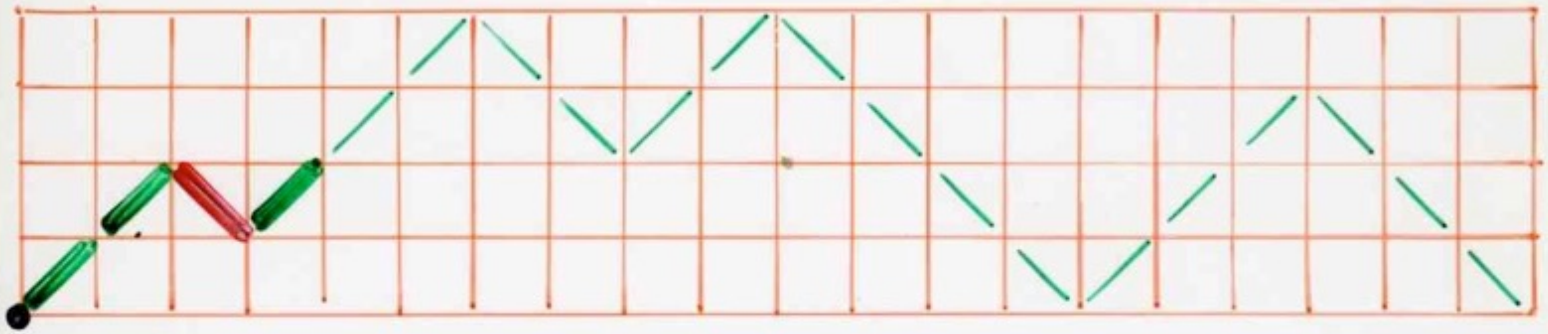


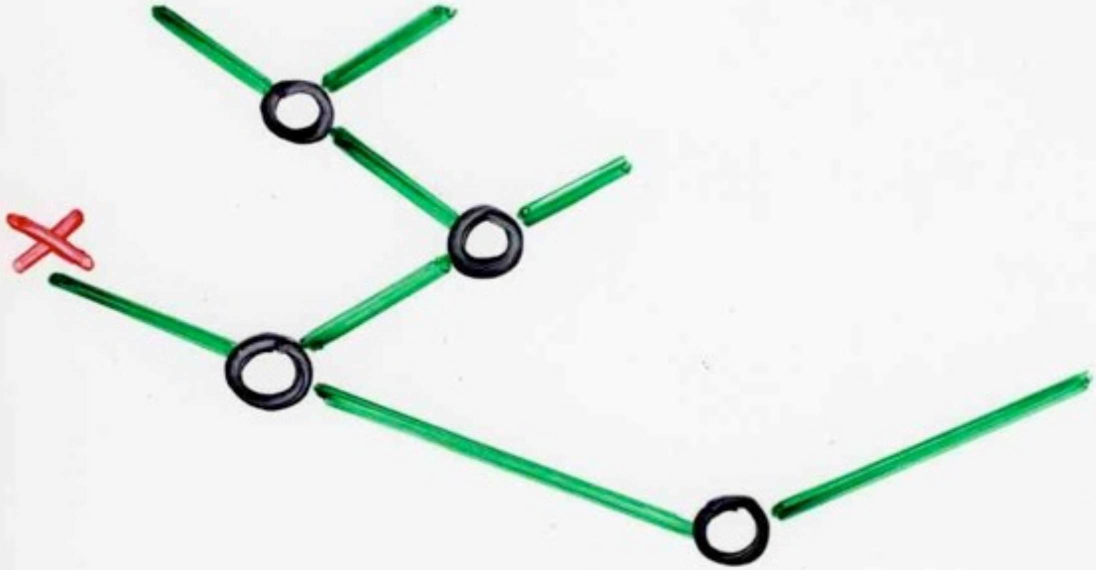


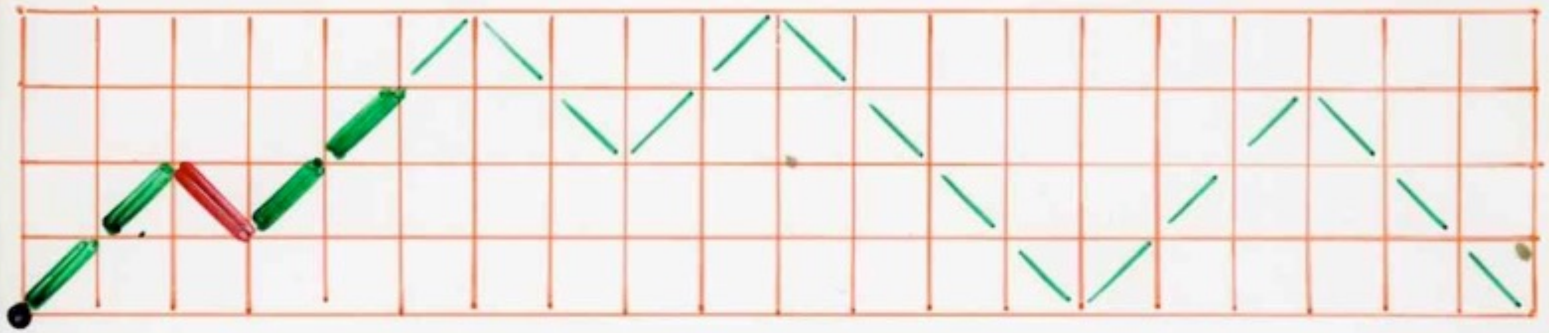


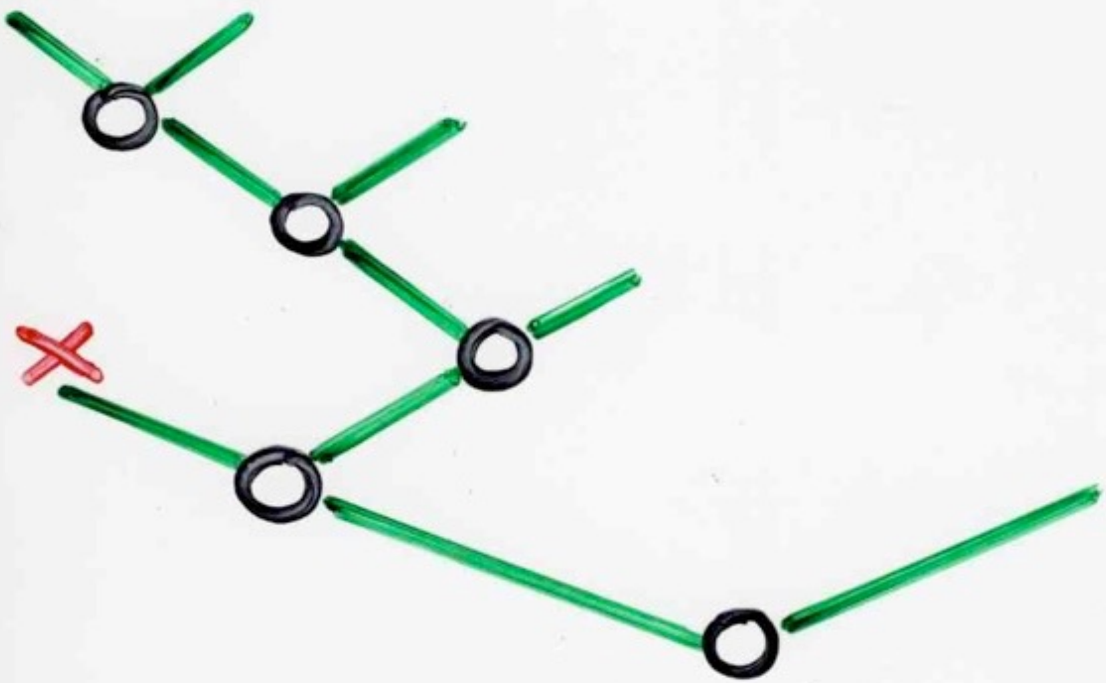


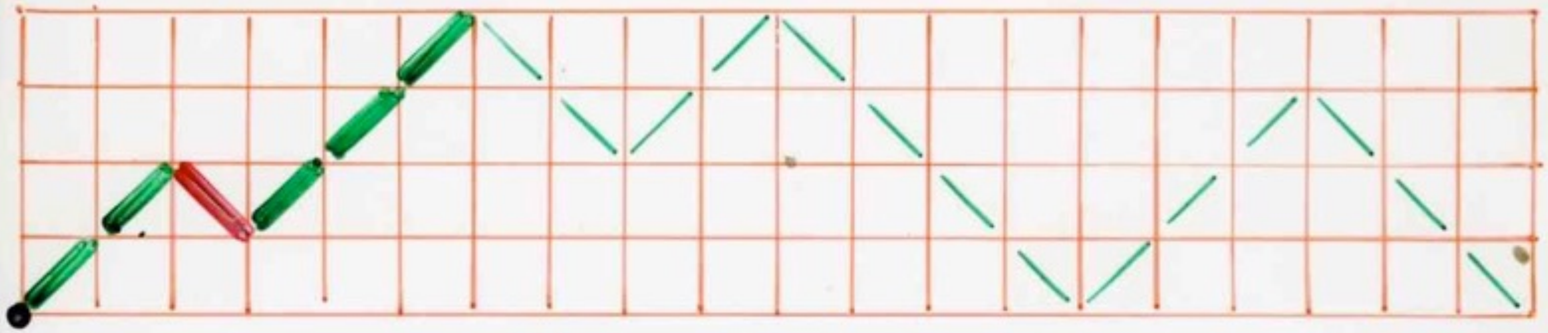


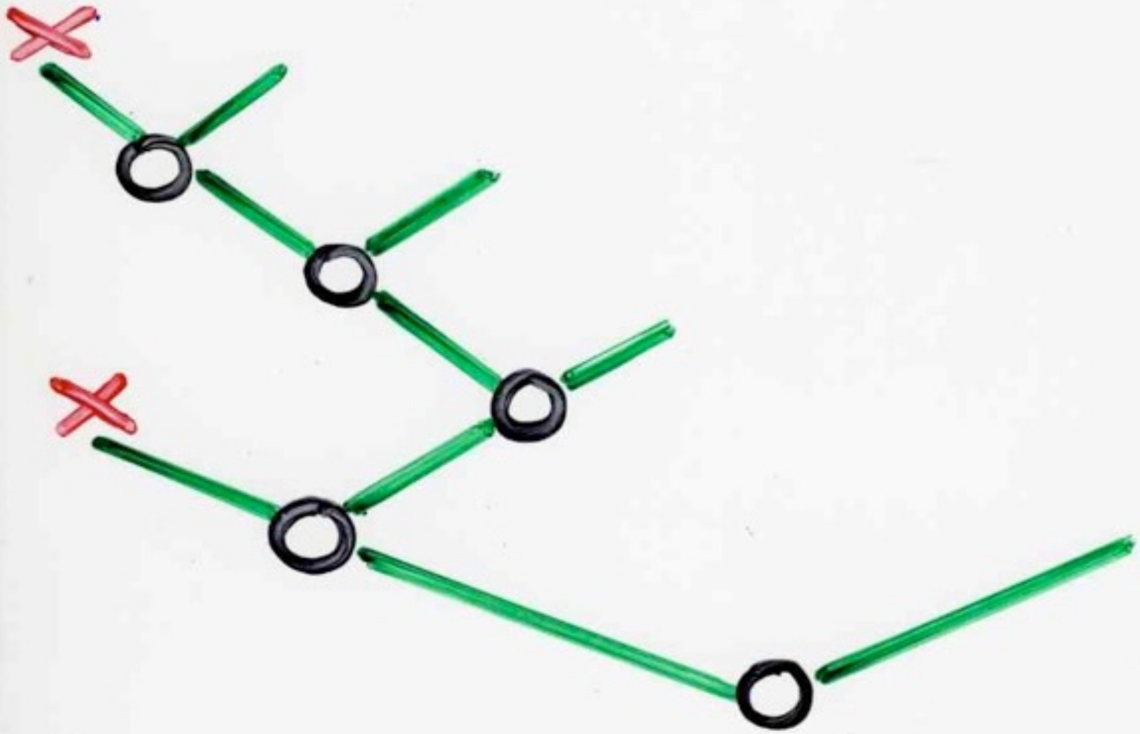


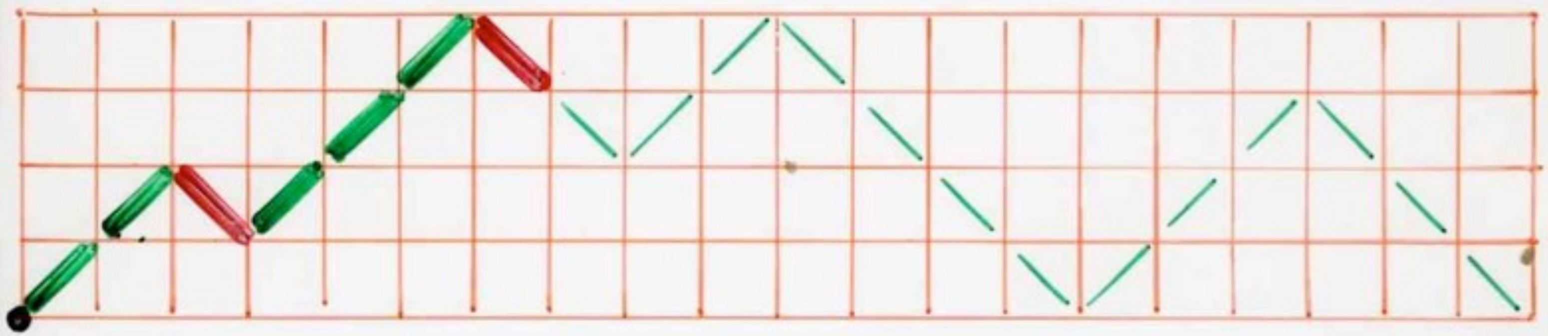


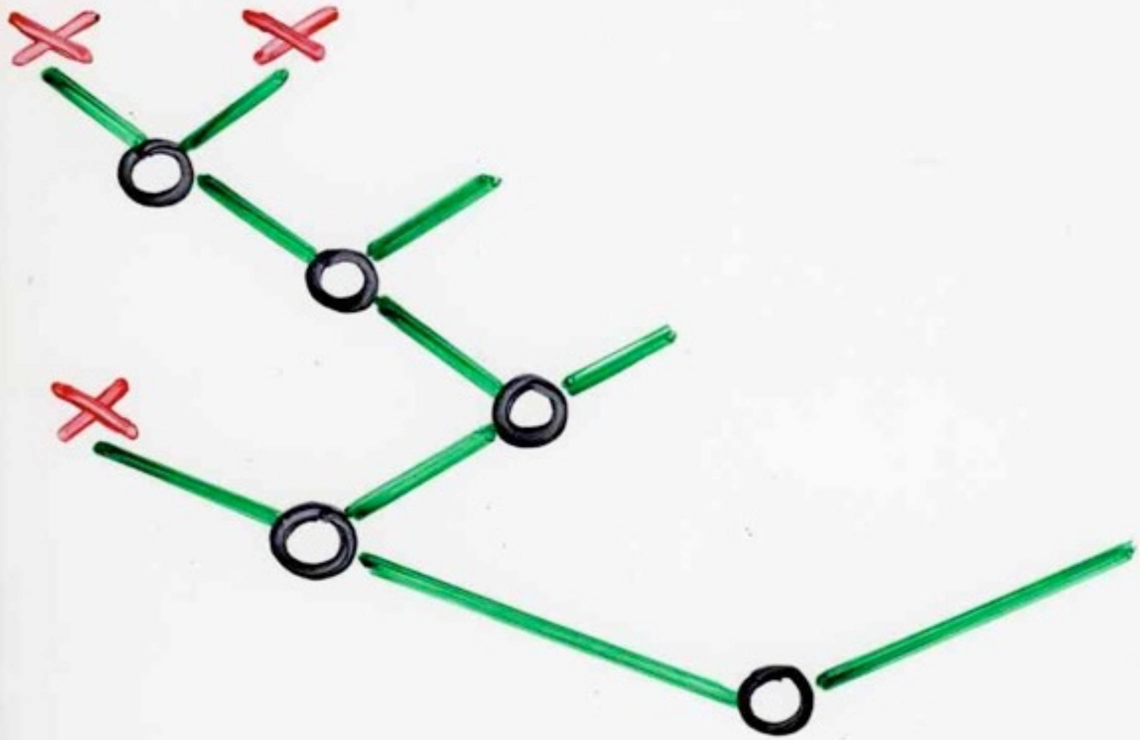


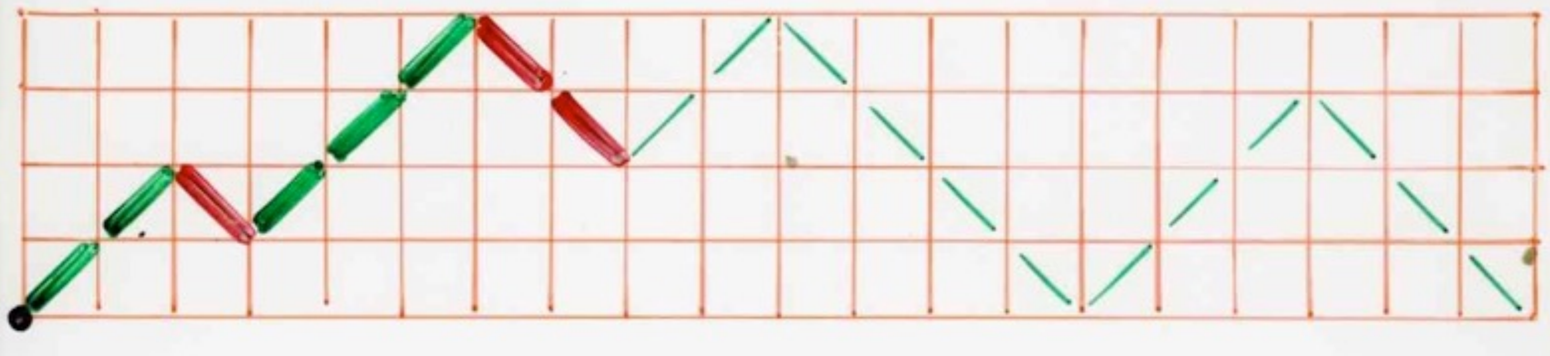


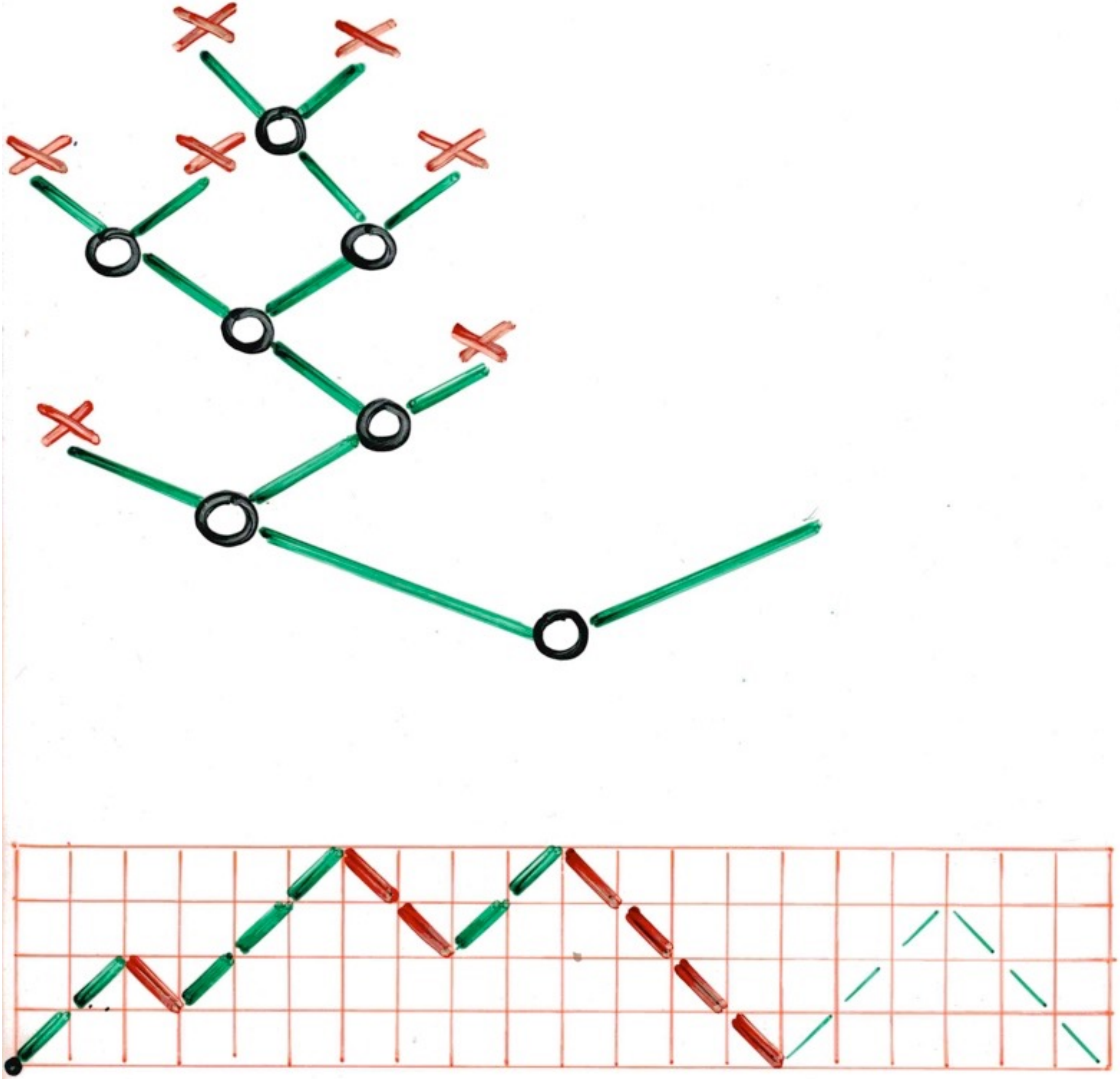


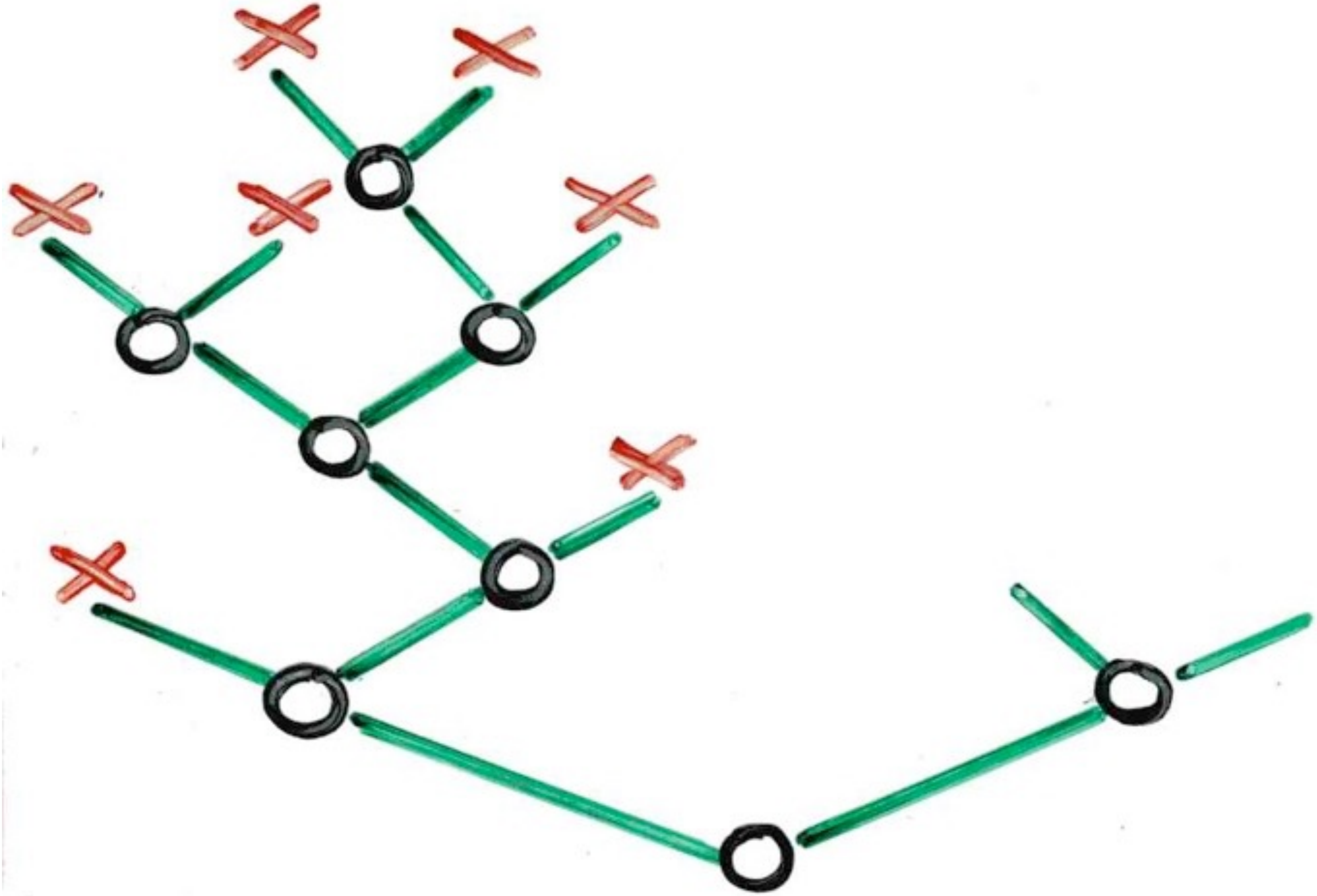


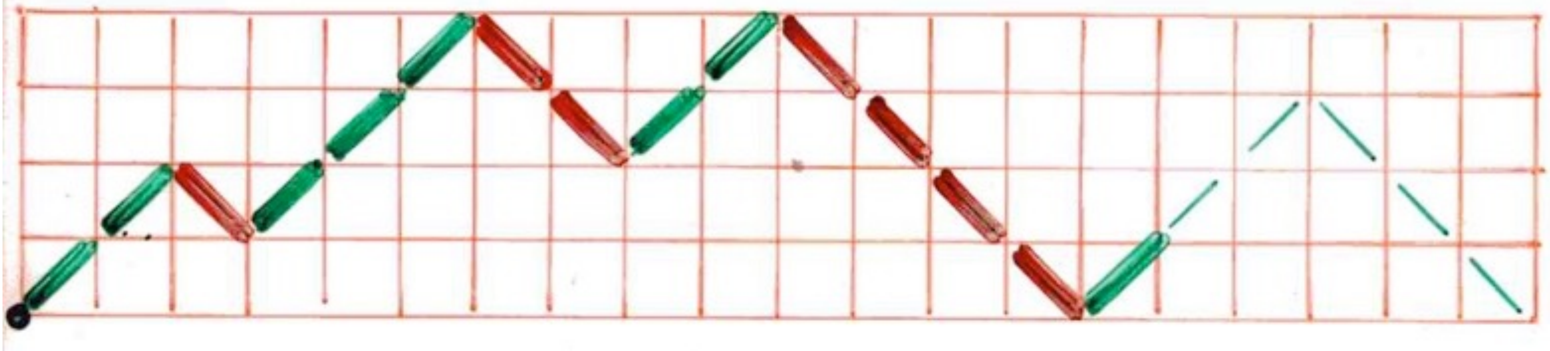


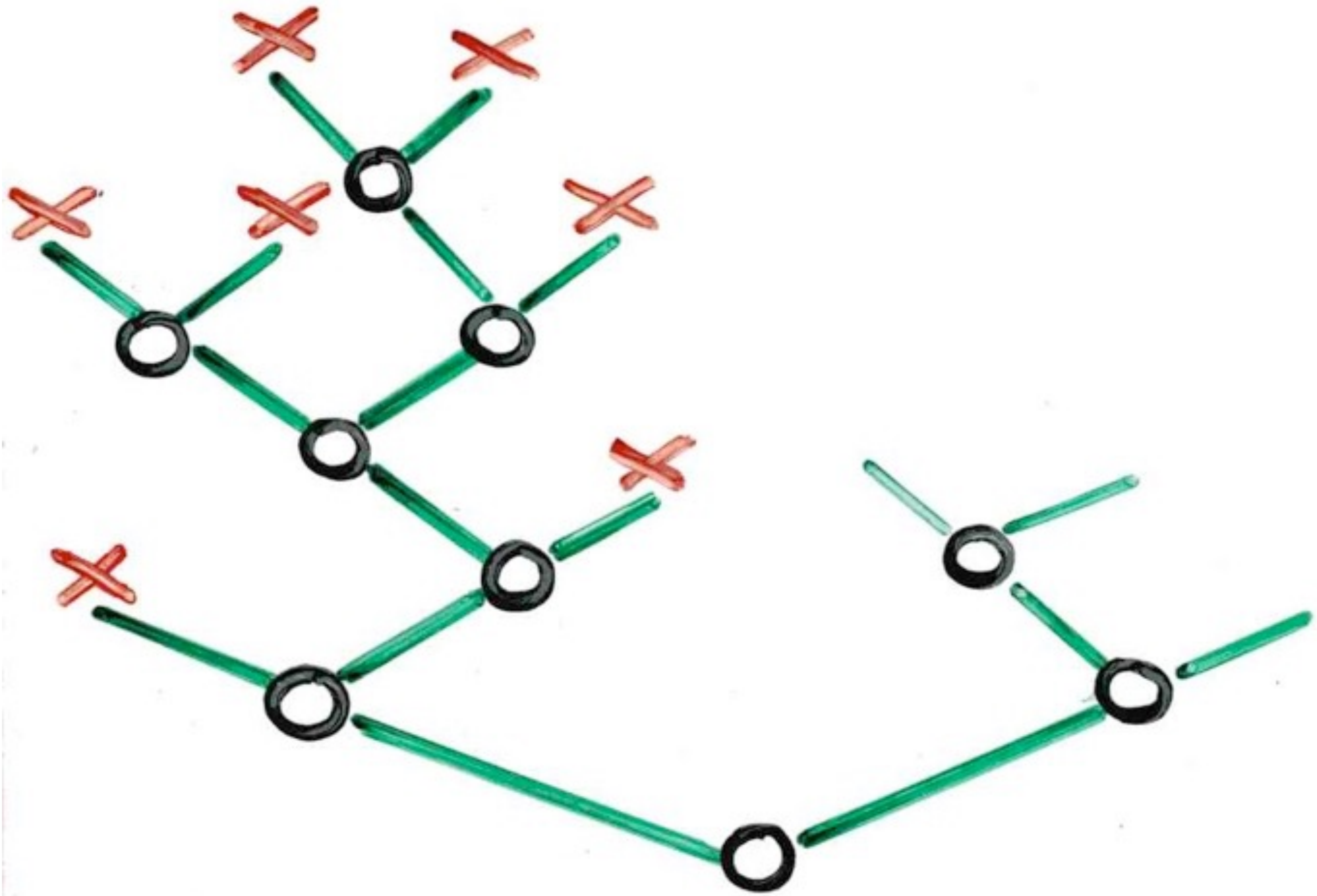


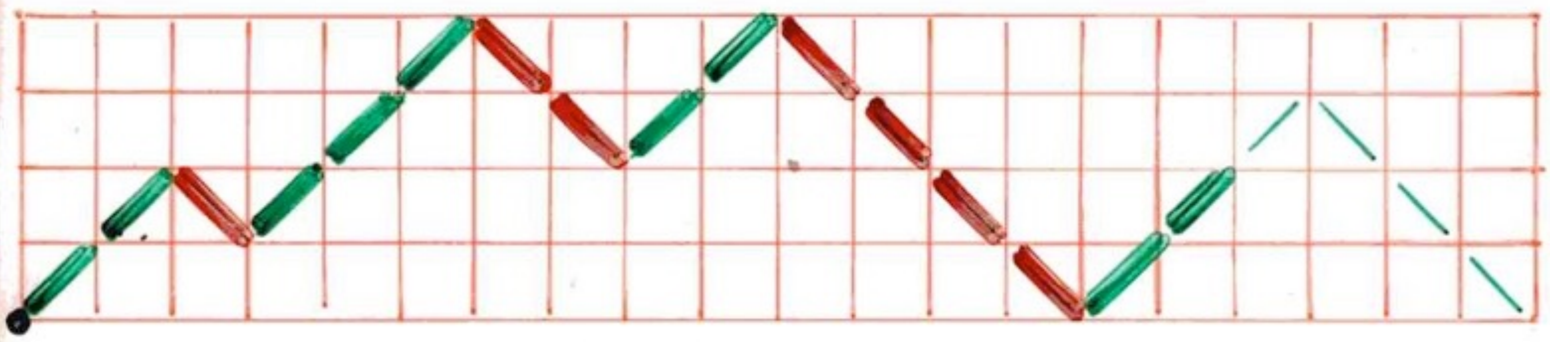


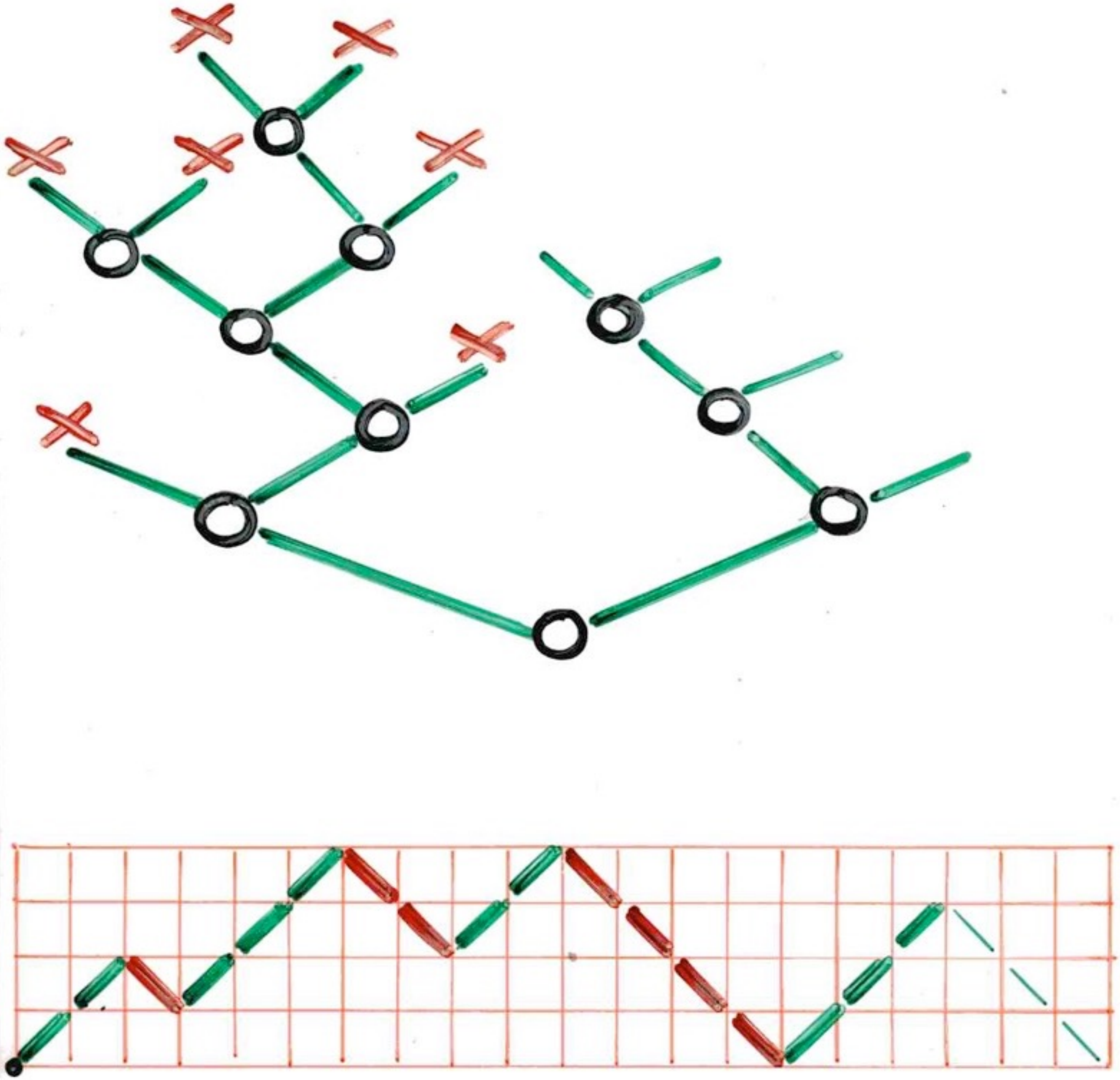


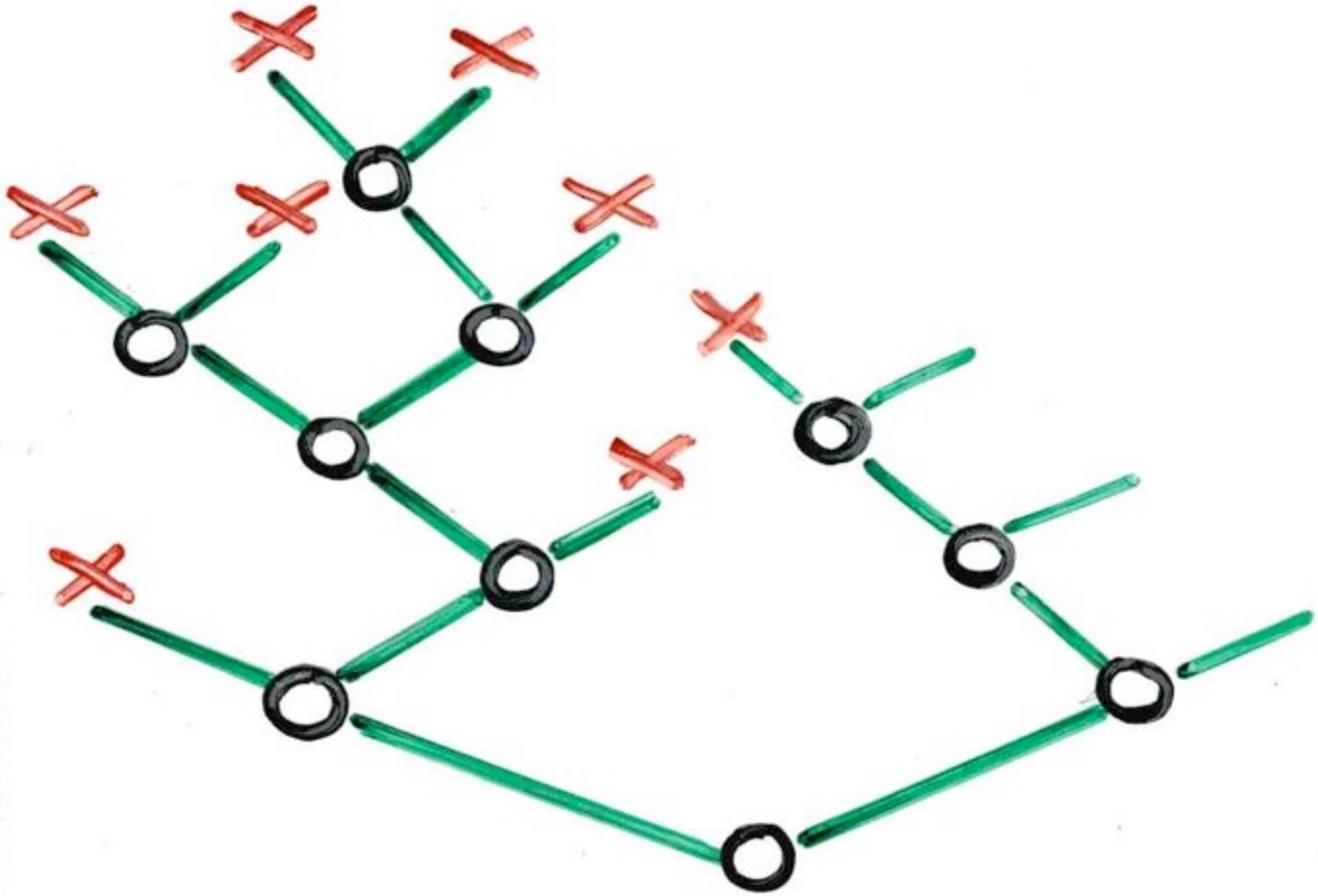


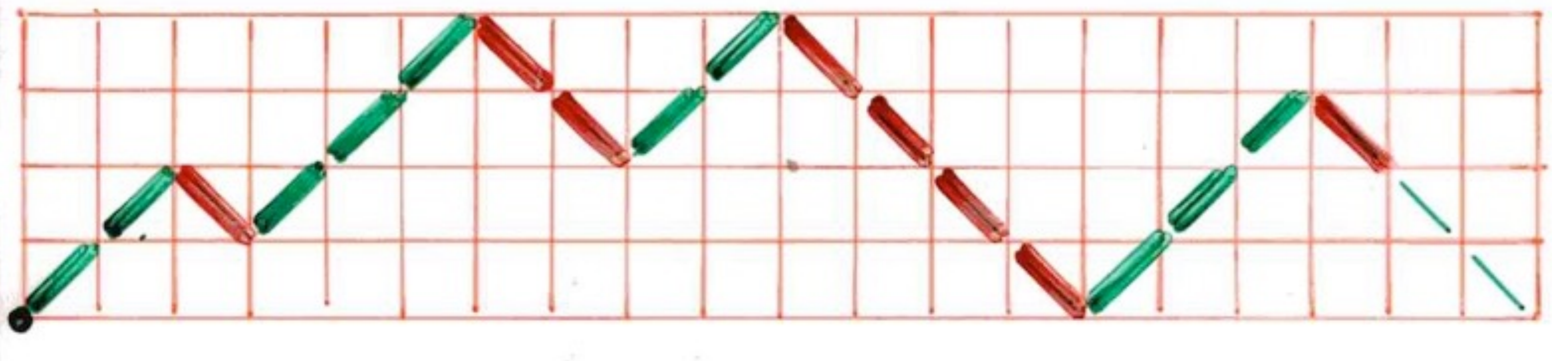


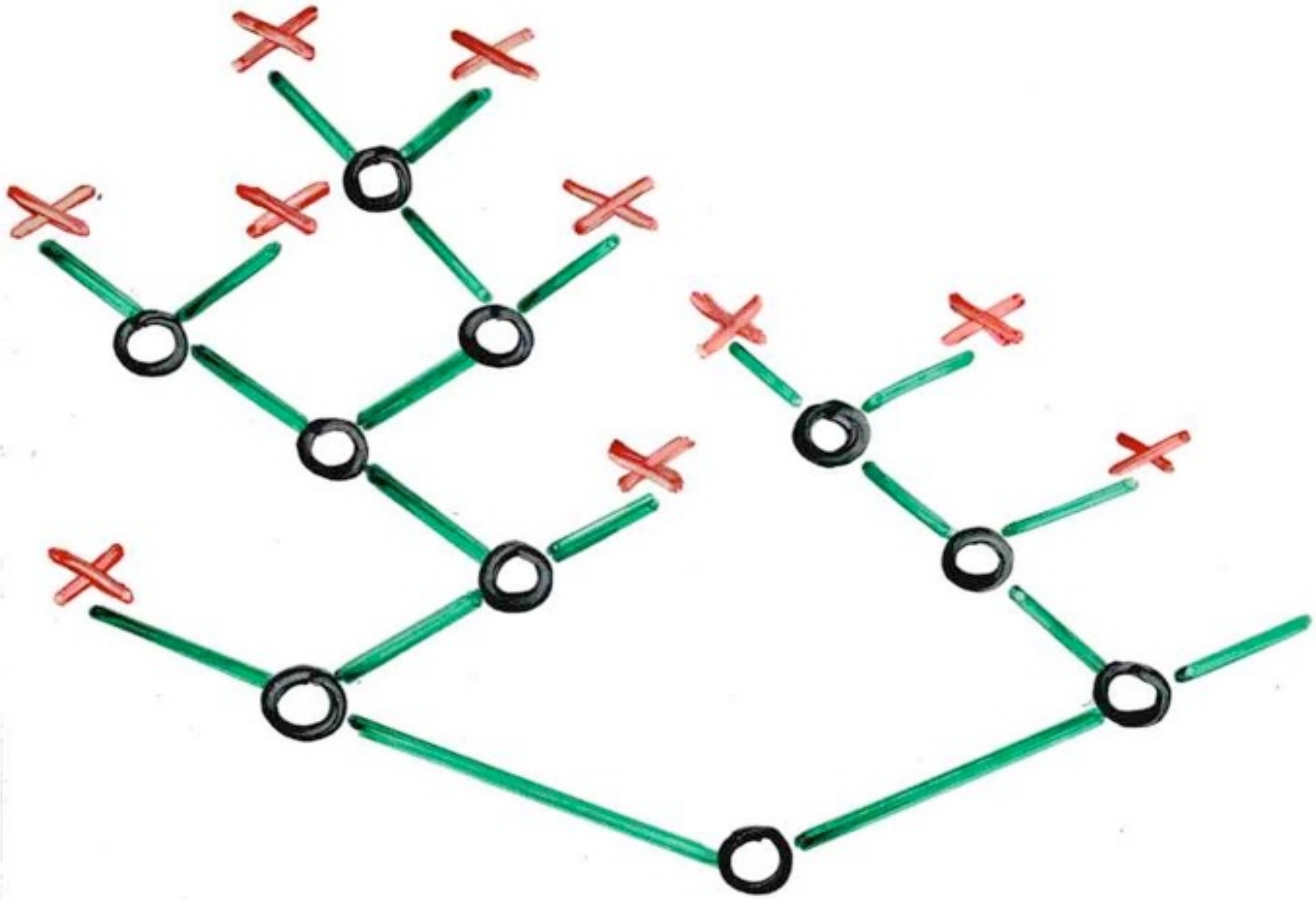


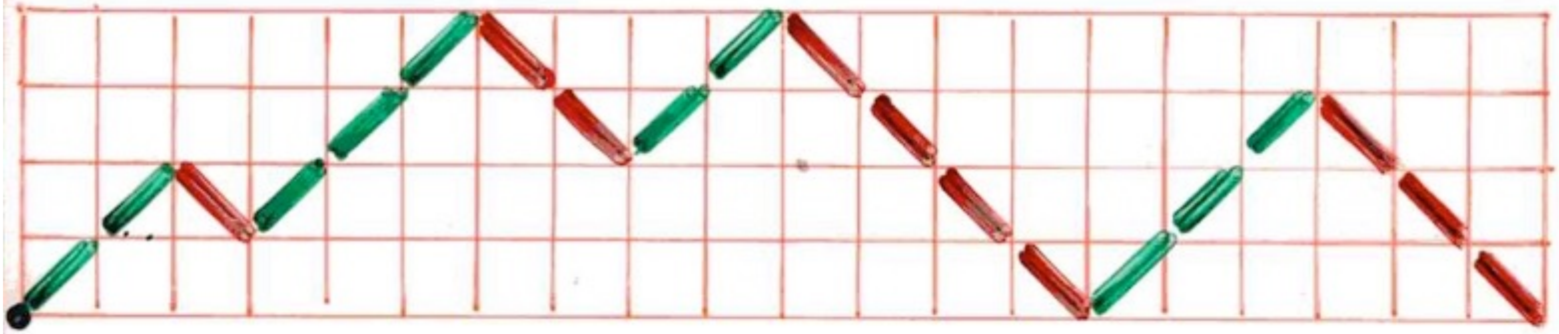


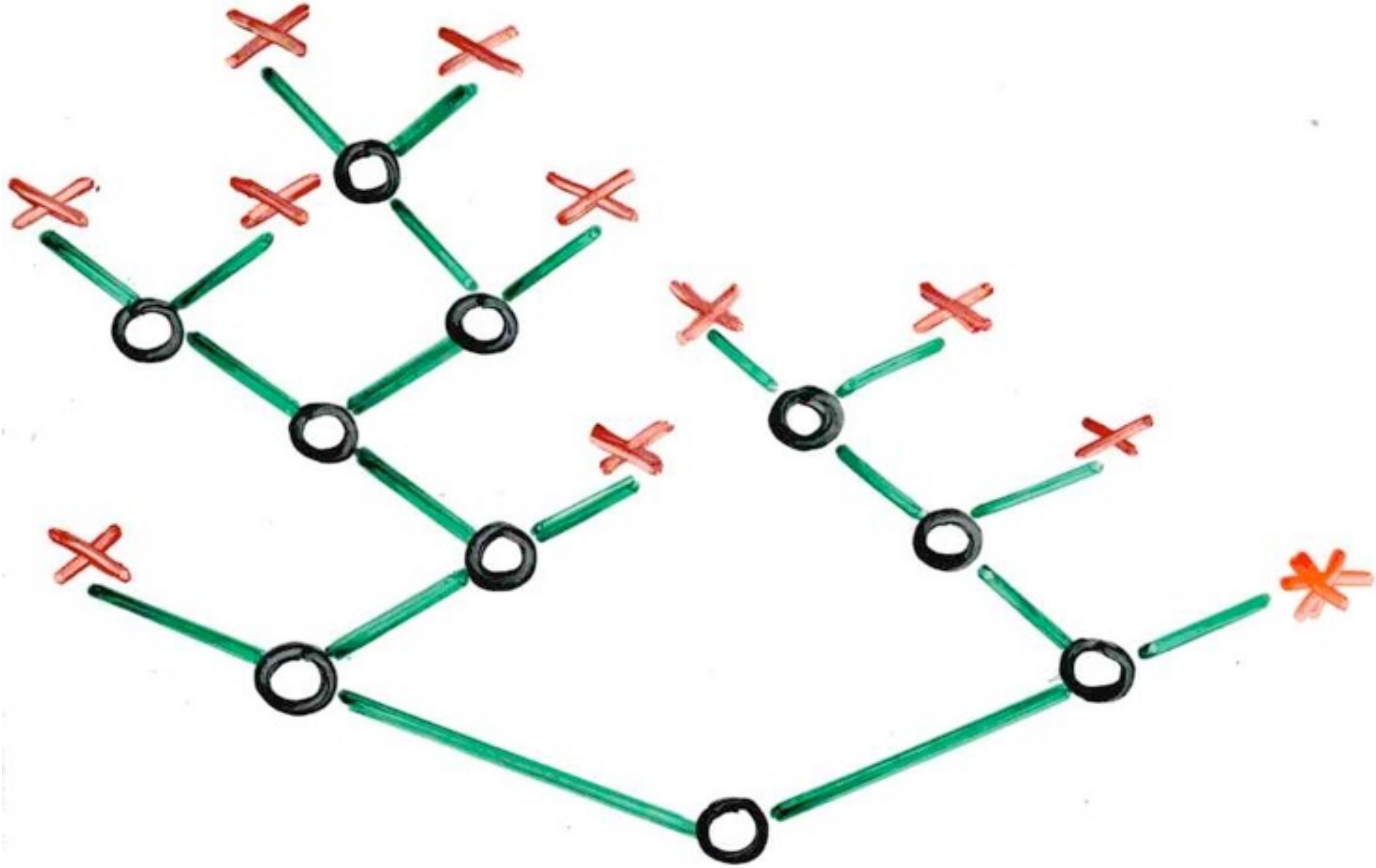


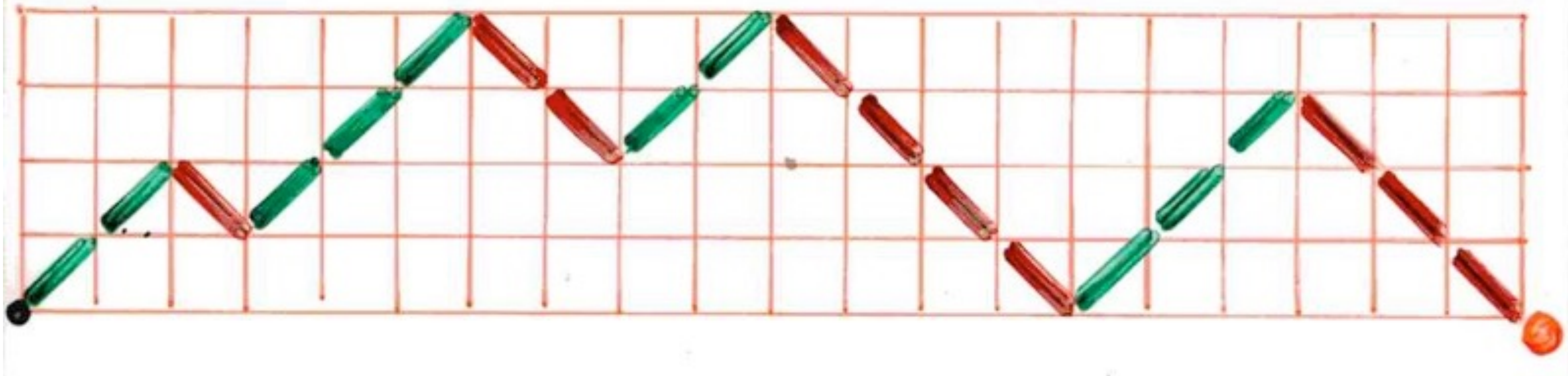








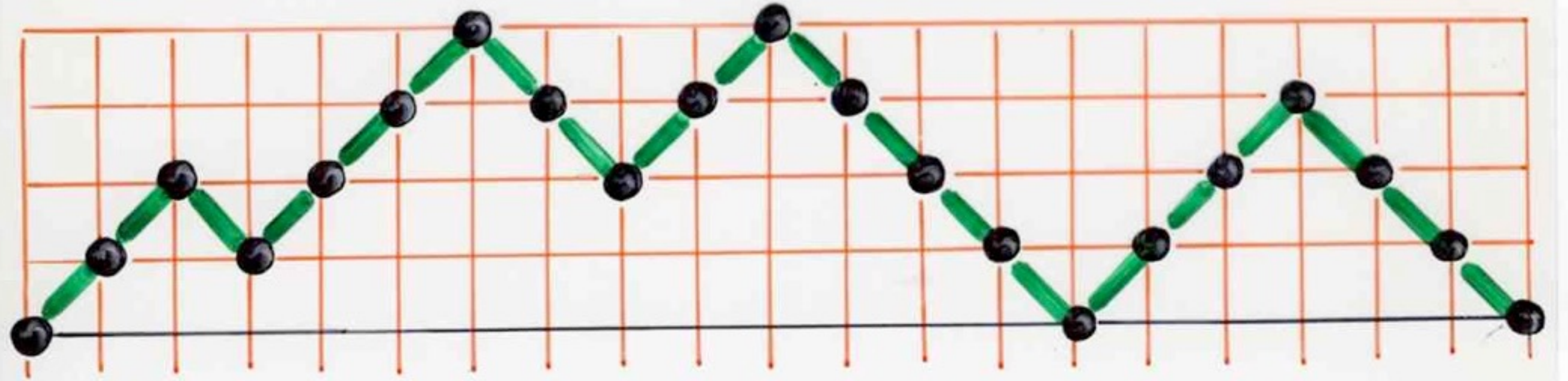




logarithmic height

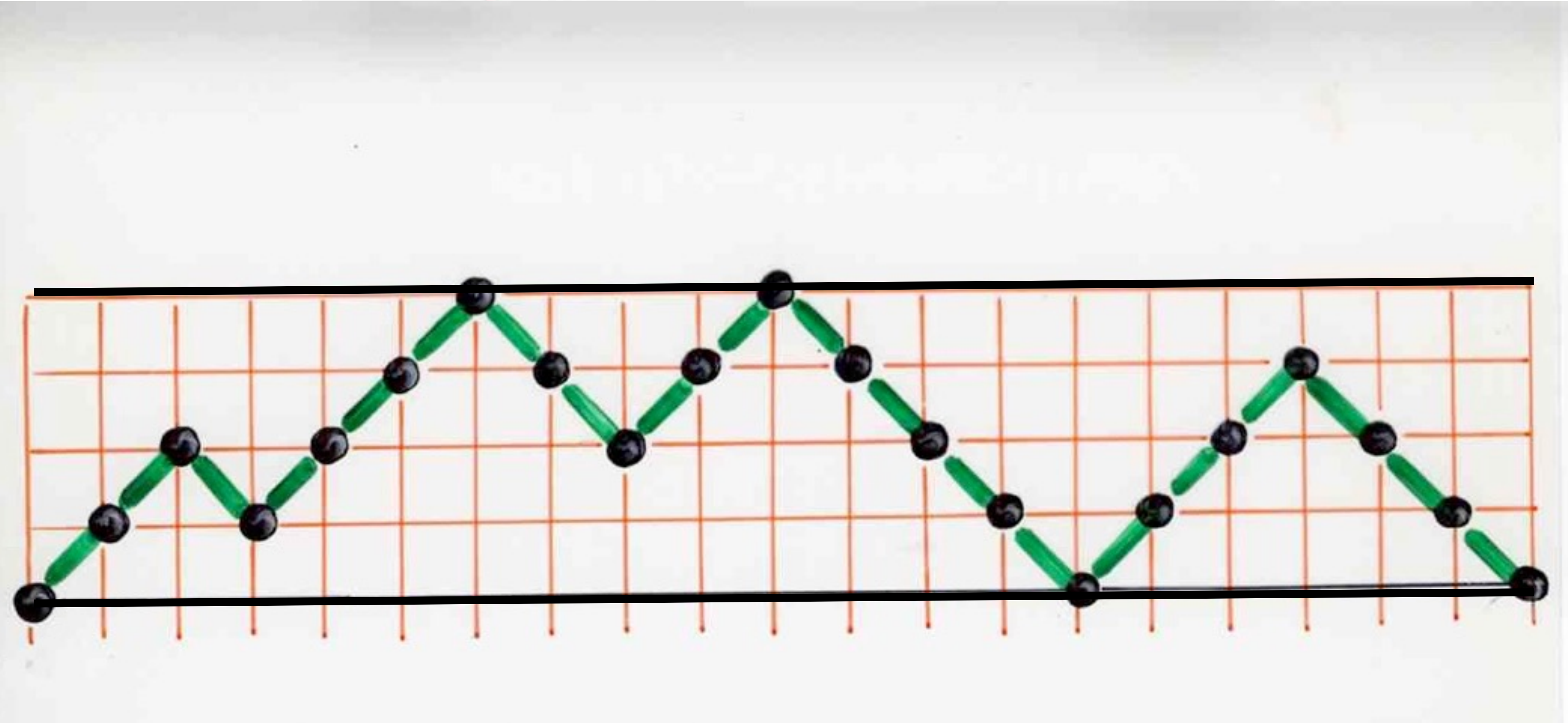
Dyck path w

Height $h(w)$



Dyck path w

Height $h(w) = 4$

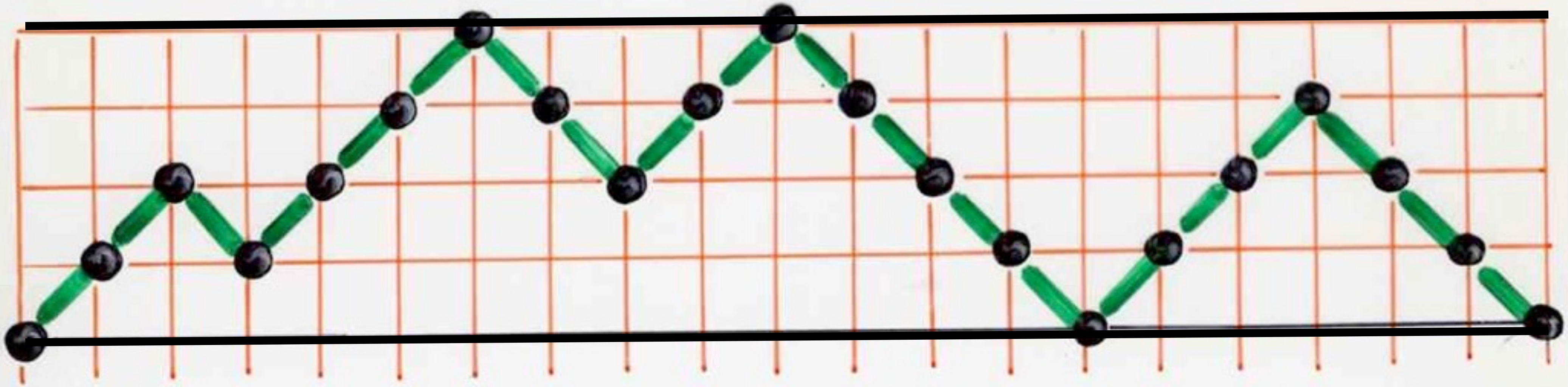


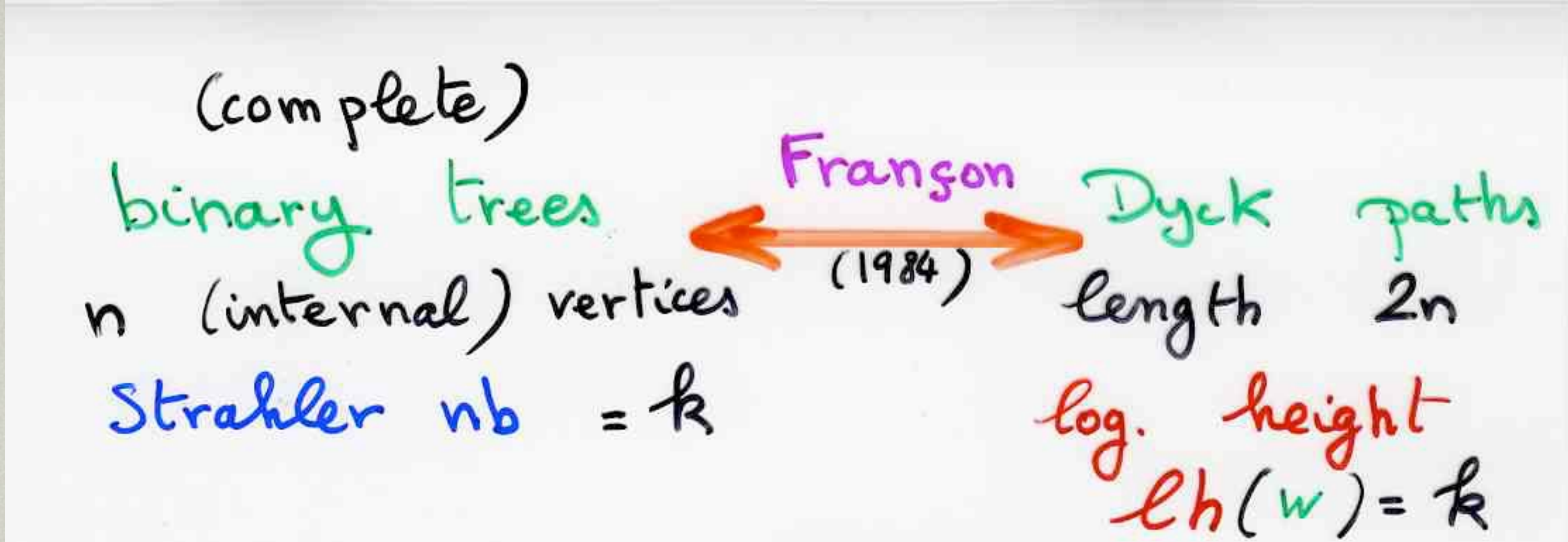
Dyck path w

Height $h(w)$

logarithmic height $lh(w)$

$= \lfloor \log_2(1+h(w)) \rfloor$





same distribution !

average Strahler number

over binary trees n vertices

$$St_n = \log_4 n + f(\log_4 n) + o(1)$$

Flajolet, Raoult, Vuillemin
Kemp (1979)

periodic

ramification matrices

or

mathematical analysis for the shape of a branching structures

How to «measure» the shape of a tree ?

Bernard Gantner

ARBRES aux CORBEAUX Louvre museum

ramification

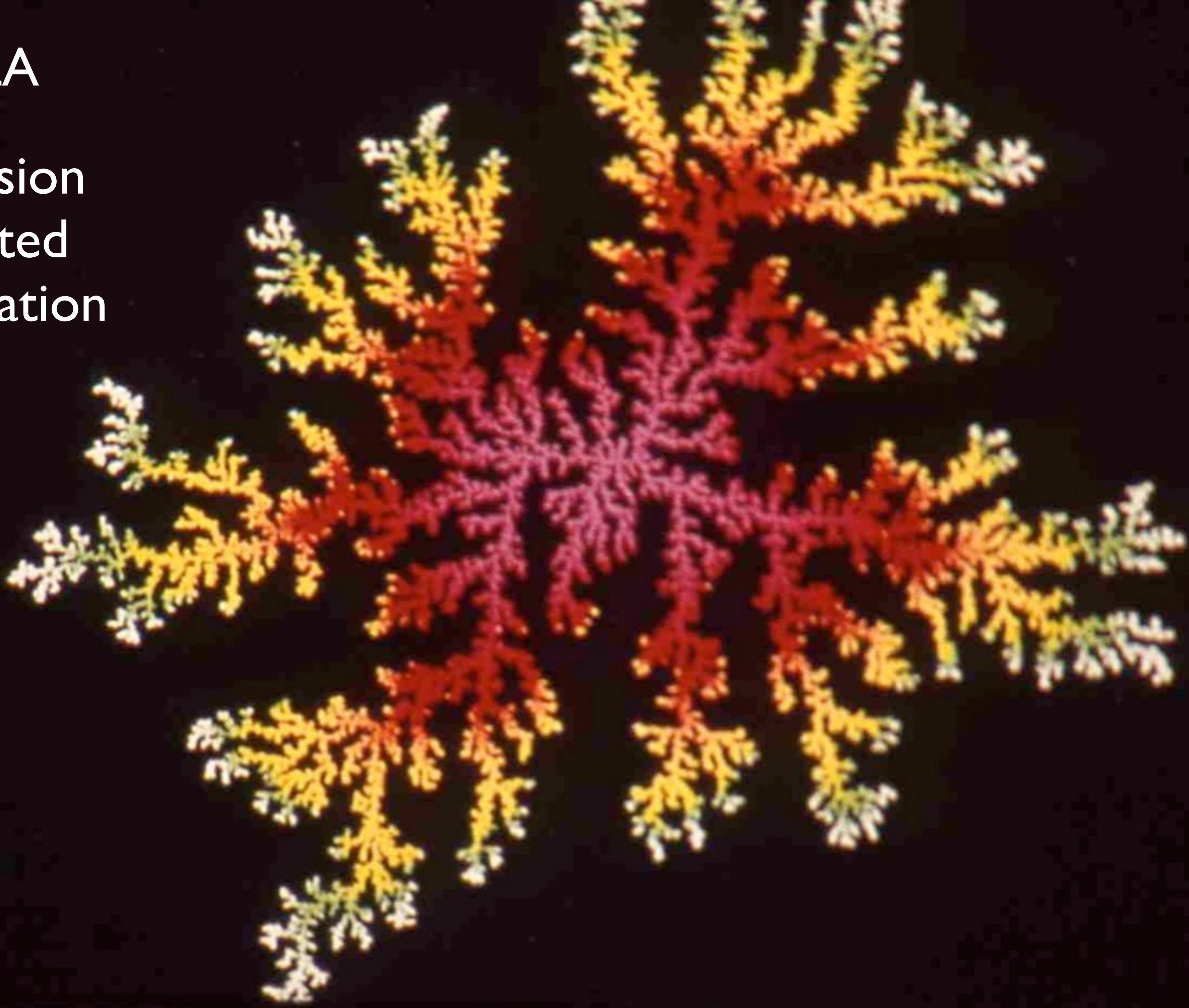
matrices

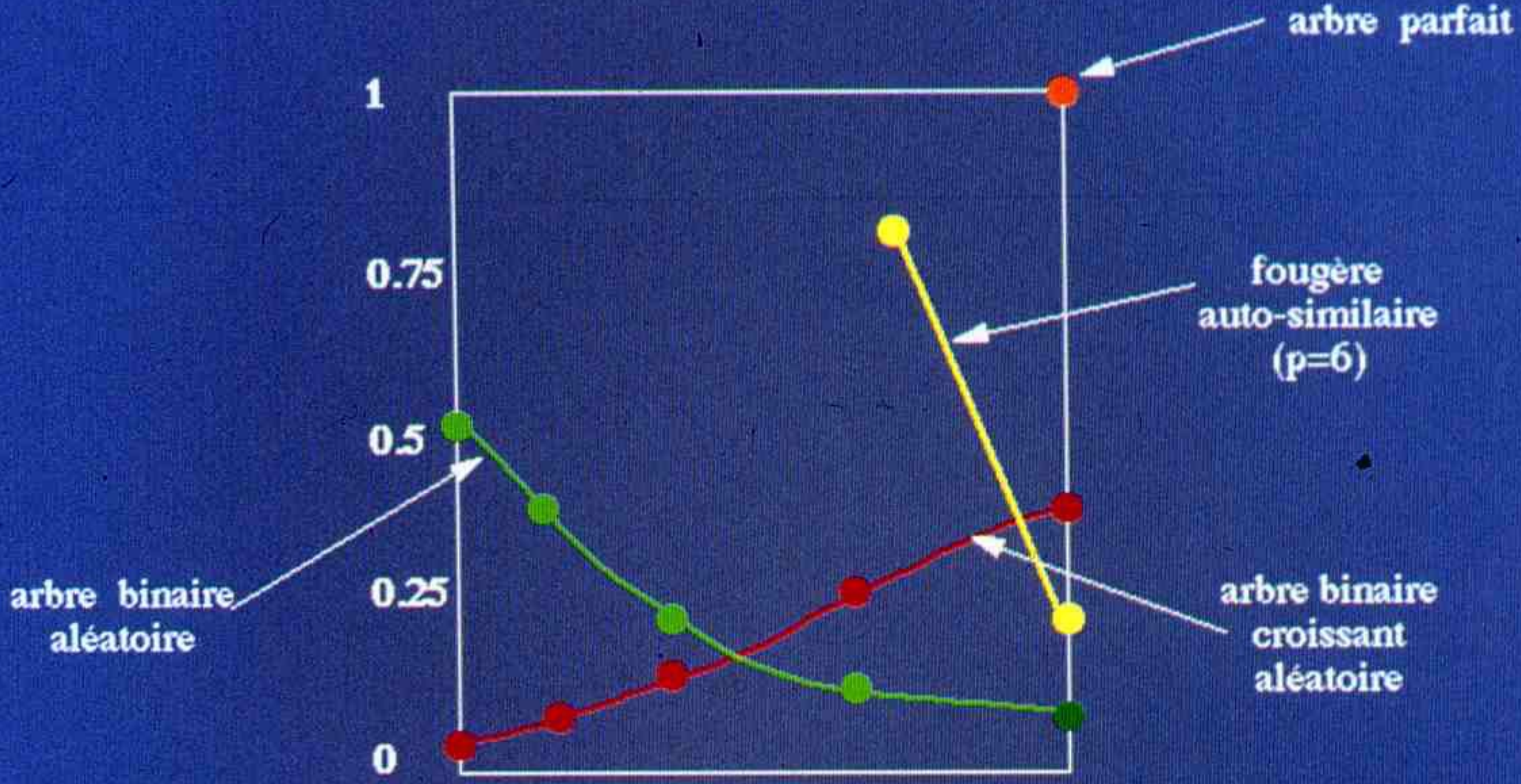
in physics


digitous
fingering

DLA

Diffusion
Limited
Agregation





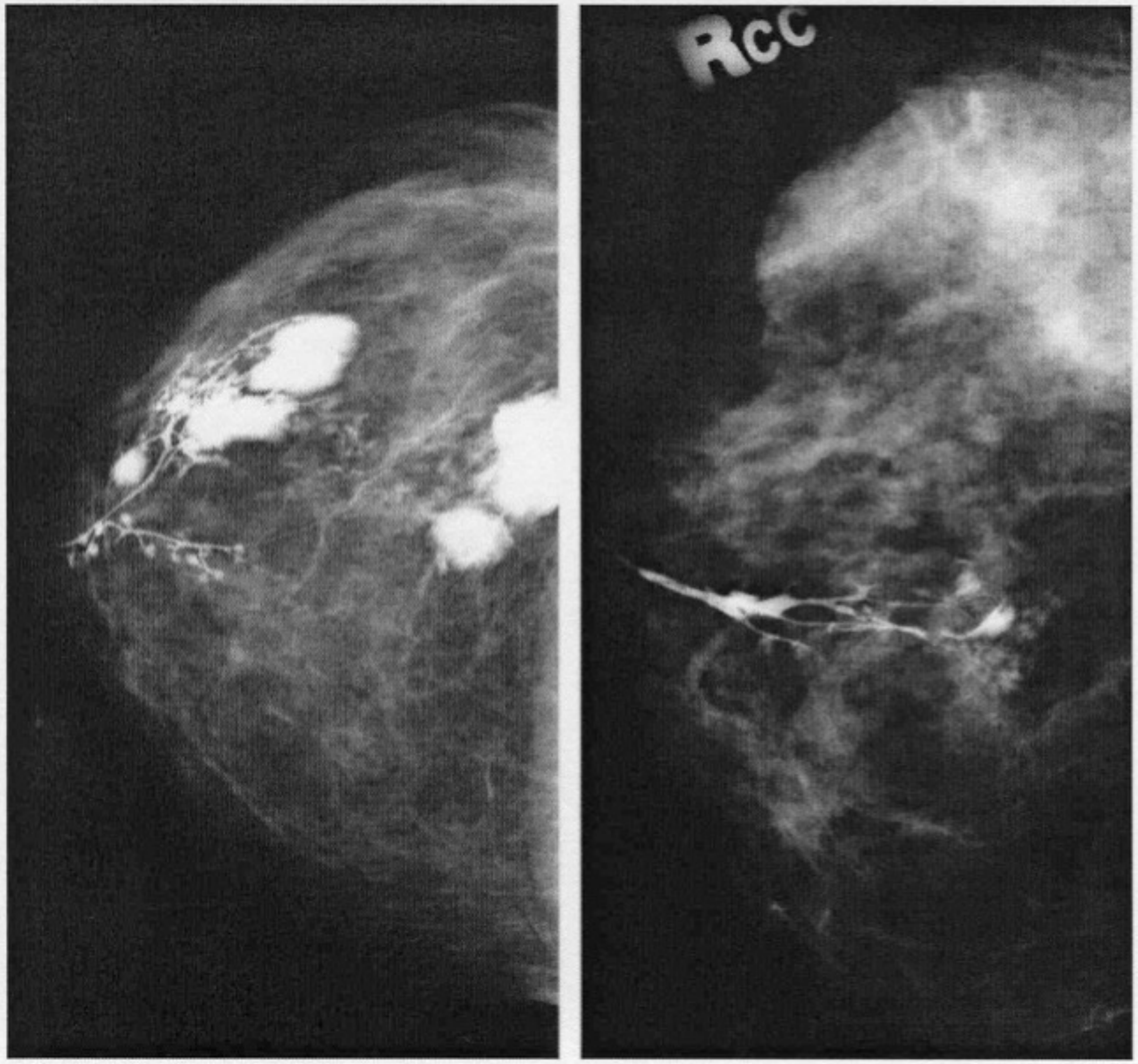
matrices de ramification auto-similaires

- Classification of Galactograms with ramification matrices

P. Bakic, M. Albert, A. Maidment

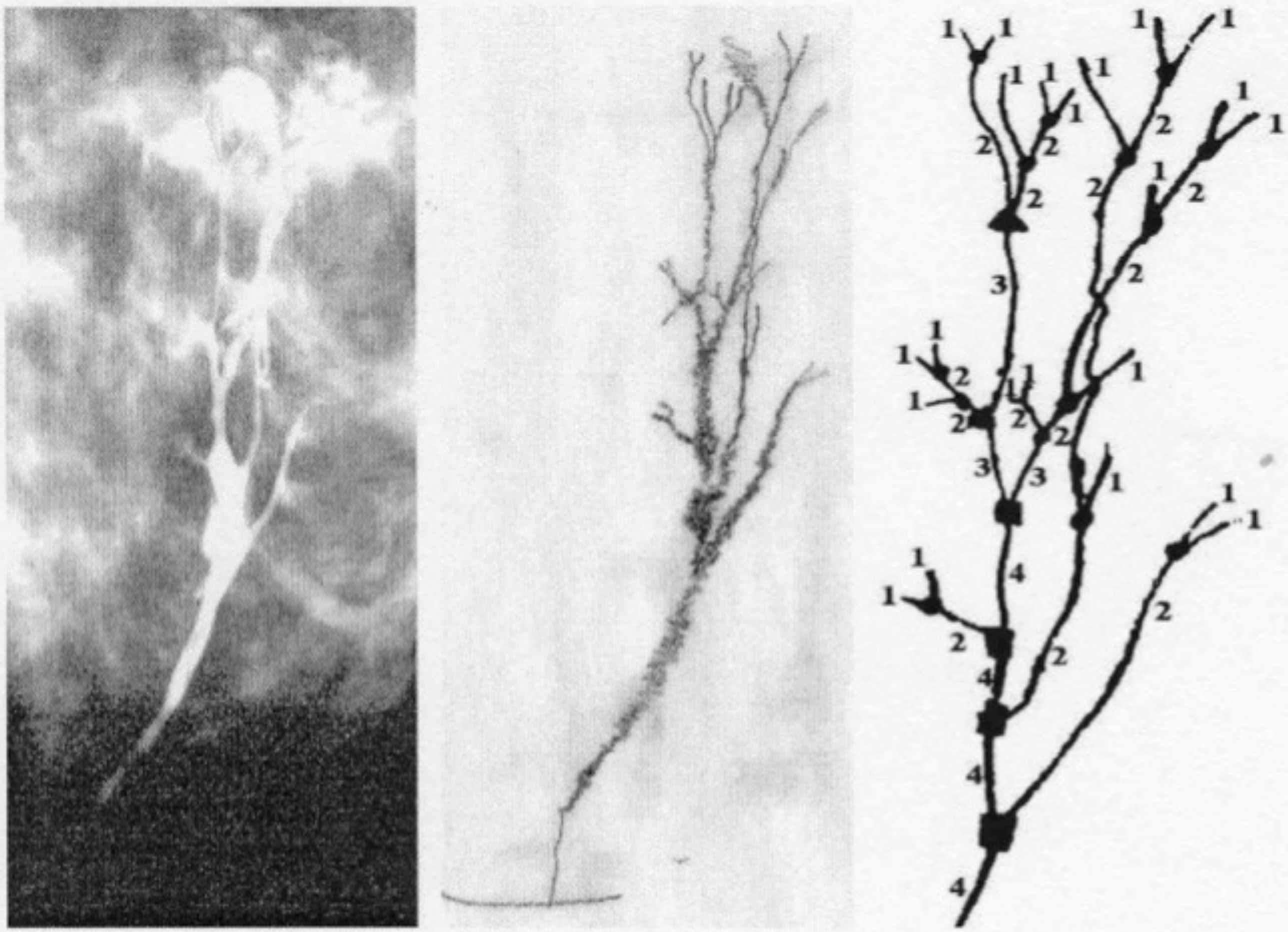
(2003)

Digital mammography



a. b.

Figure 4. Two examples of galactograms that have been correctly classified by means of R matrices. **(a)** Galactogram with no reported findings (patient age, 45 years; right CC view; $r_{3,2} = 0.5$ and $r_{3,3} = 0.19$). (Large bright regions seen in this galactogram are due to extravasation, which did not affect the segmentation of the ductal tree.) **(b)** Galactogram with a reported finding of cysts (patient age, 55 years; right CC view; $r_{3,2} = 0.33$ and $r_{3,3} = 0.67$).



a. b. c.

$$R = \begin{bmatrix} r_{2,1} & r_{2,2} & . & . \\ r_{3,1} & r_{3,2} & r_{3,3} & . \\ r_{4,1} & r_{4,2} & r_{4,3} & r_{4,4} \end{bmatrix} = \begin{bmatrix} 0.43 & 0.57 & . & . \\ 0 & 0.33 & 0.67 & . \\ 0 & 0.75 & 0 & 0.25 \end{bmatrix}$$

d.

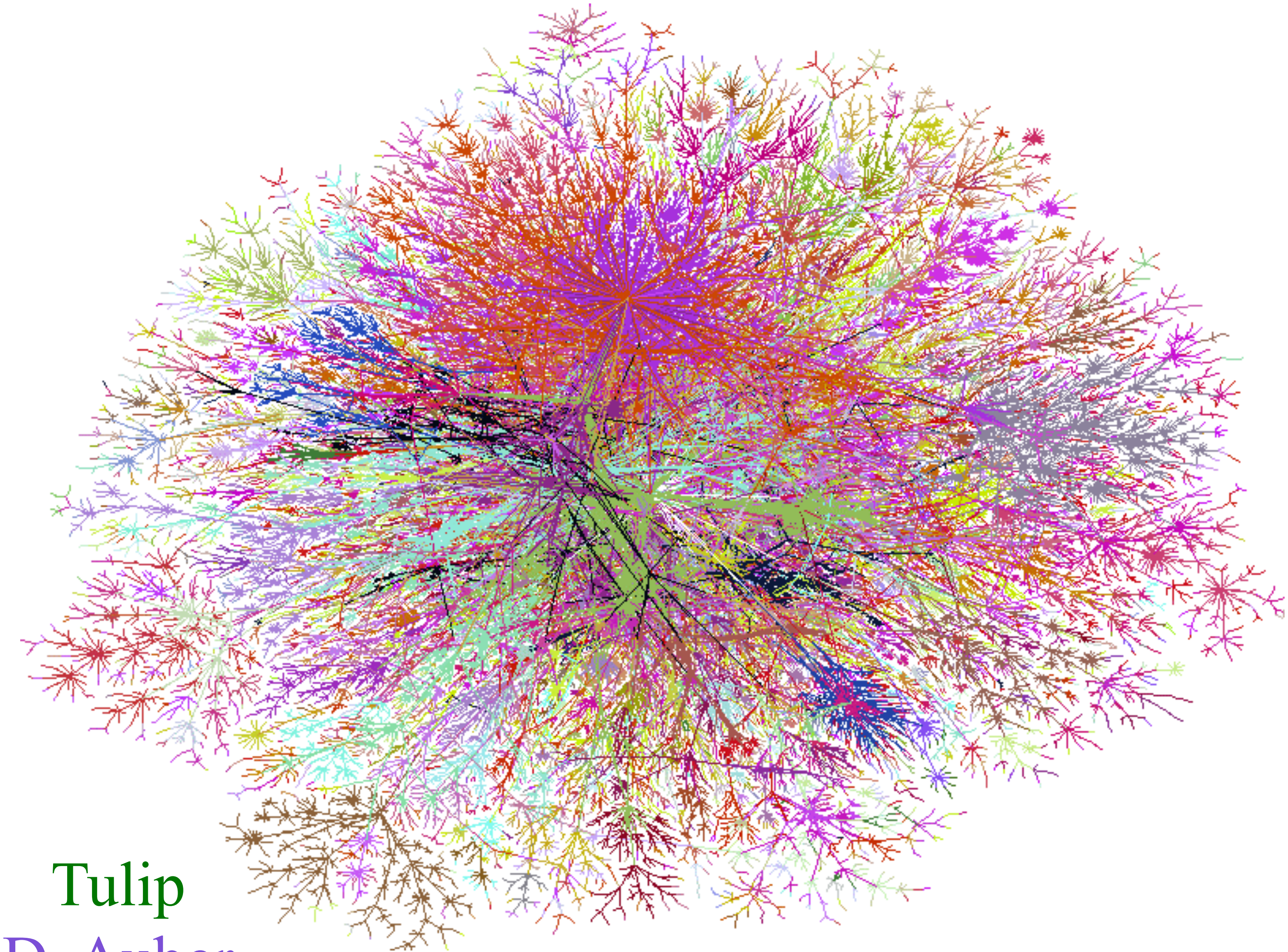
Figure 1. Segmentation of a ductal tree, showing **(a)** part of a galactogram with a contrast–enhanced ductal network, **(b)** the manually traced network of larger ducts from the contrast–enhanced portion of the galactogram, **(c)** numeric labeling of branches in the ductal network, and **(d)** the R matrix computed from the branching pattern. The dots, triangles, and squares denote branching points of different levels of the tree.

visualization of information

Visualization of information for very large graphs

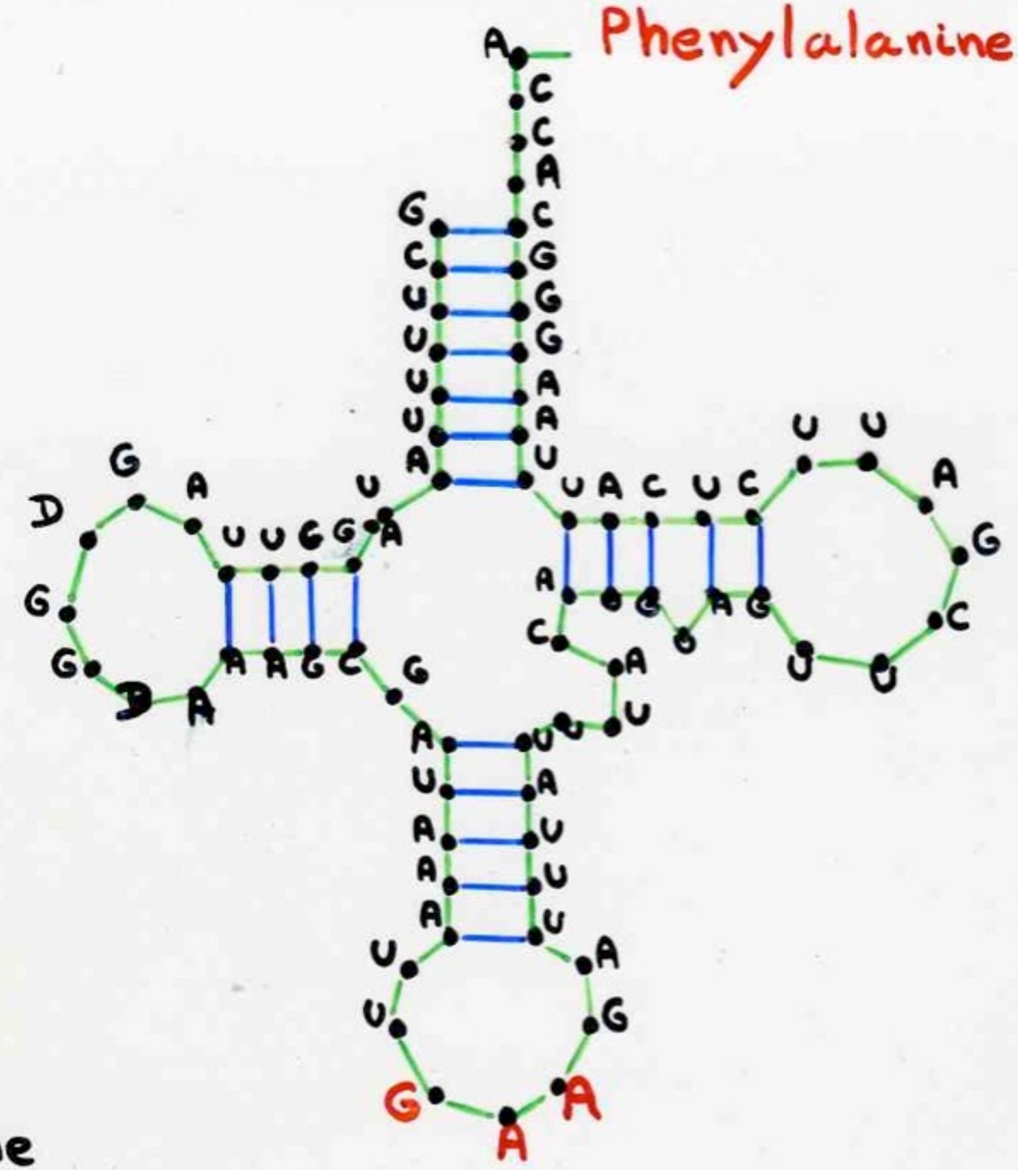
D. Auber, M. Delest
Y. Chiricota, G. Melançon, J.M. Fedou

extension of Horton-Strahler analysis for graphs



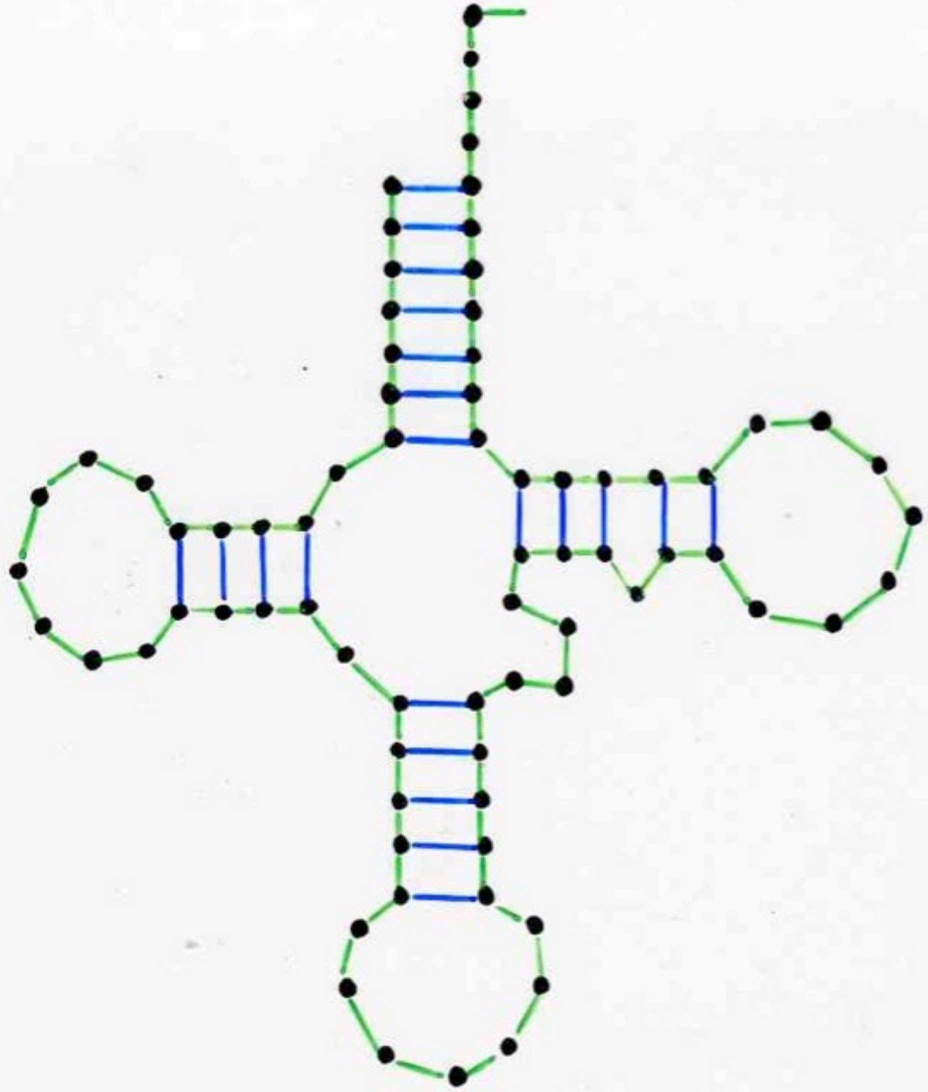
Tulip
D. Auber

trees in molecules ....

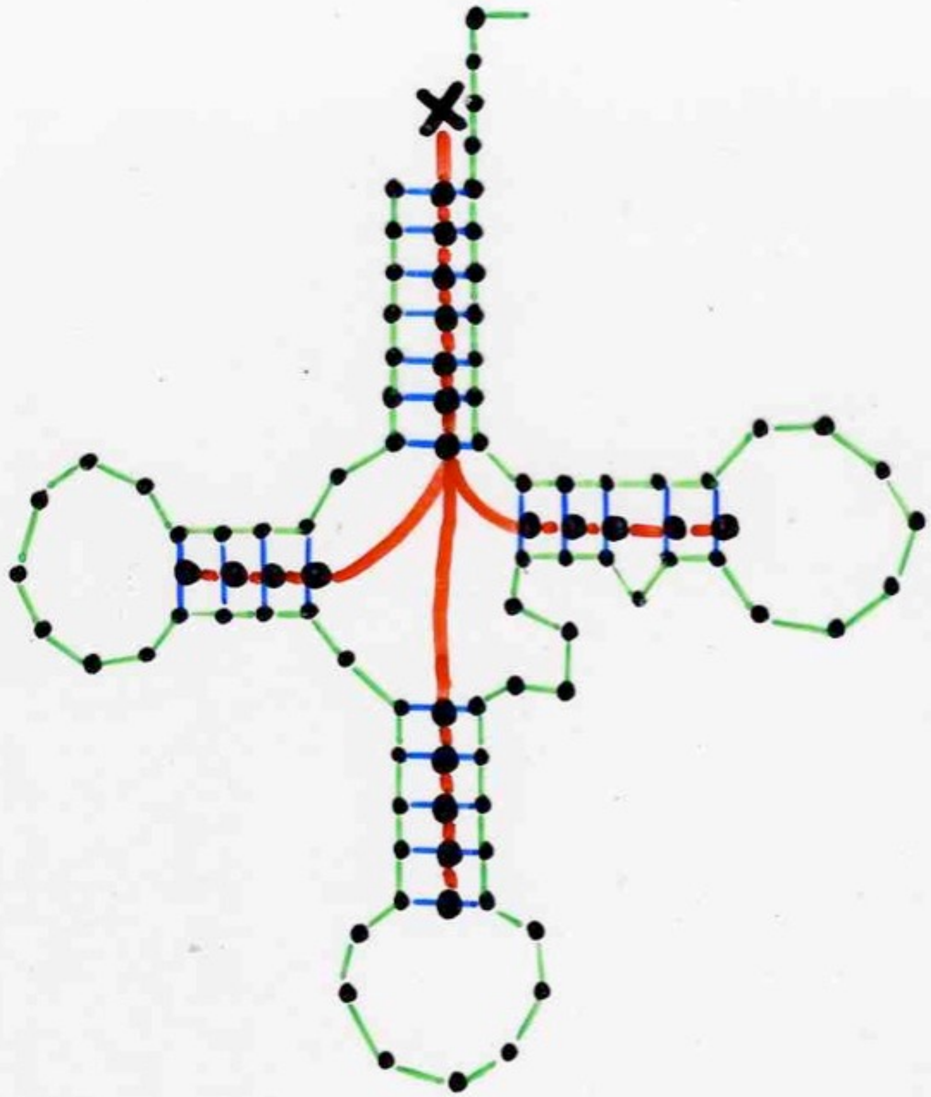


Adenine
Uracyle
Guanine
Cytosine

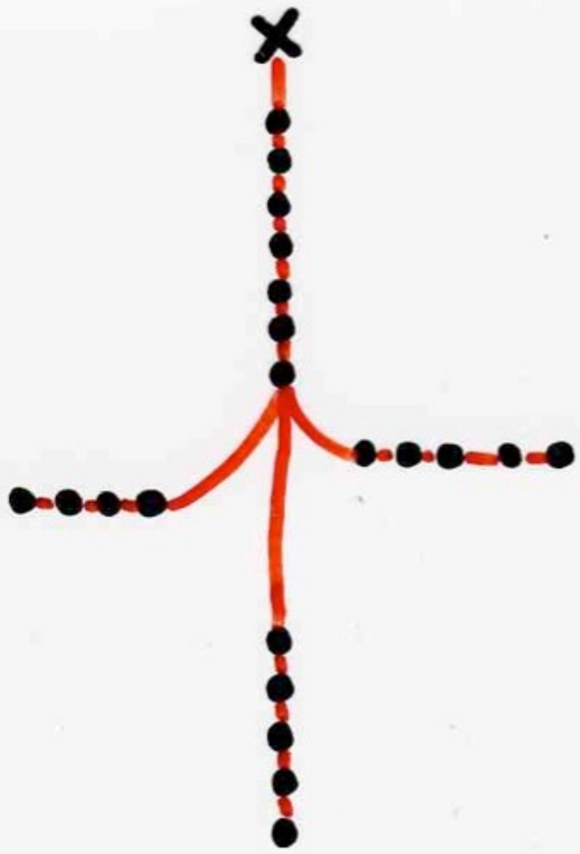
tARN ^{Phe}



tARN ^{Phe}



tARN^{Phe}



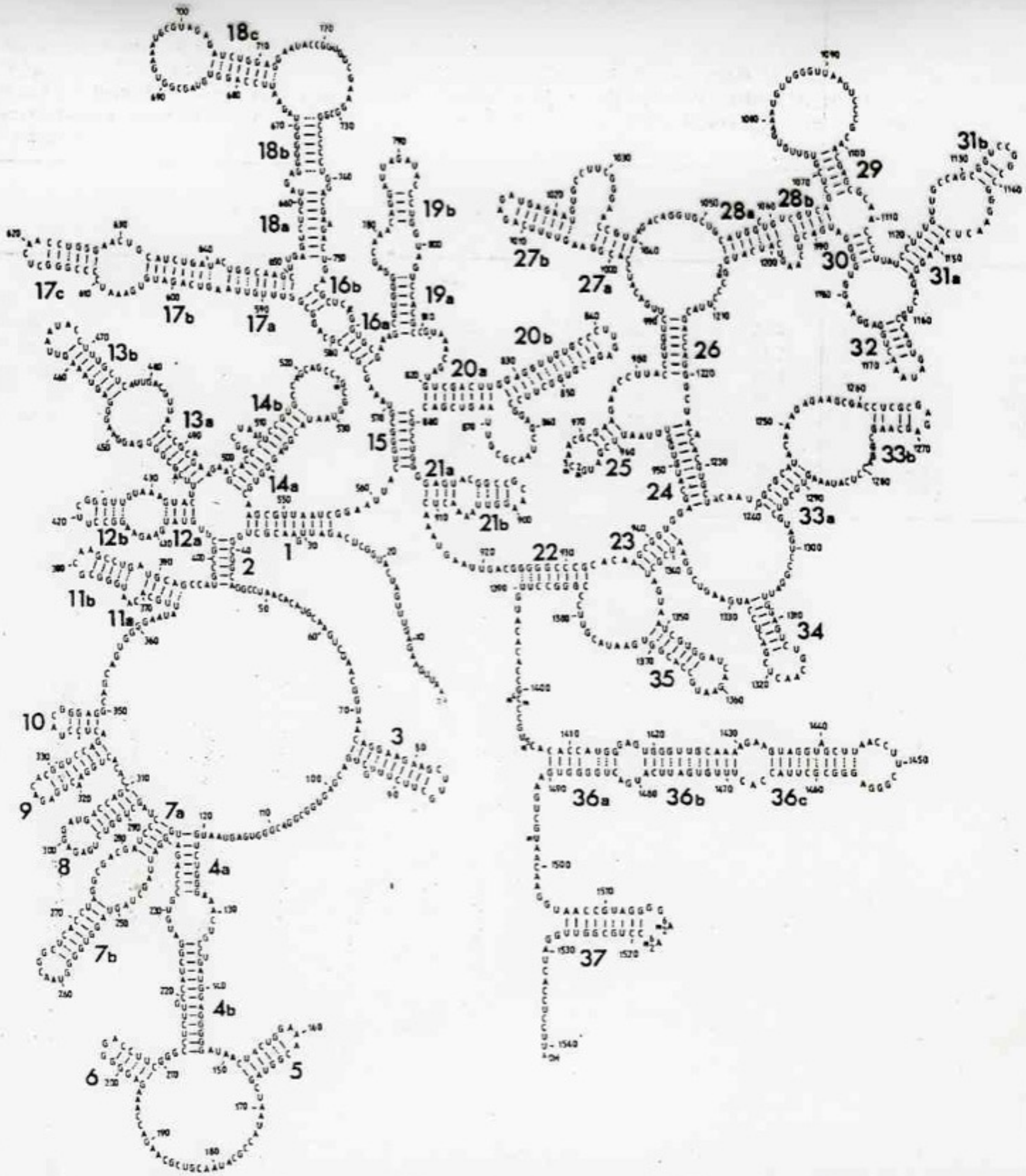


Fig. 1. *Secondary structure model of the 16-S RNA from* E. coli. This model has been fully described elsewhere [18]. The various secondary structure motifs are numbered for reference. Base-pairings 2 and 23 are included in this up-dated scheme and slight modifications have been introduced into helices 18b and 33b

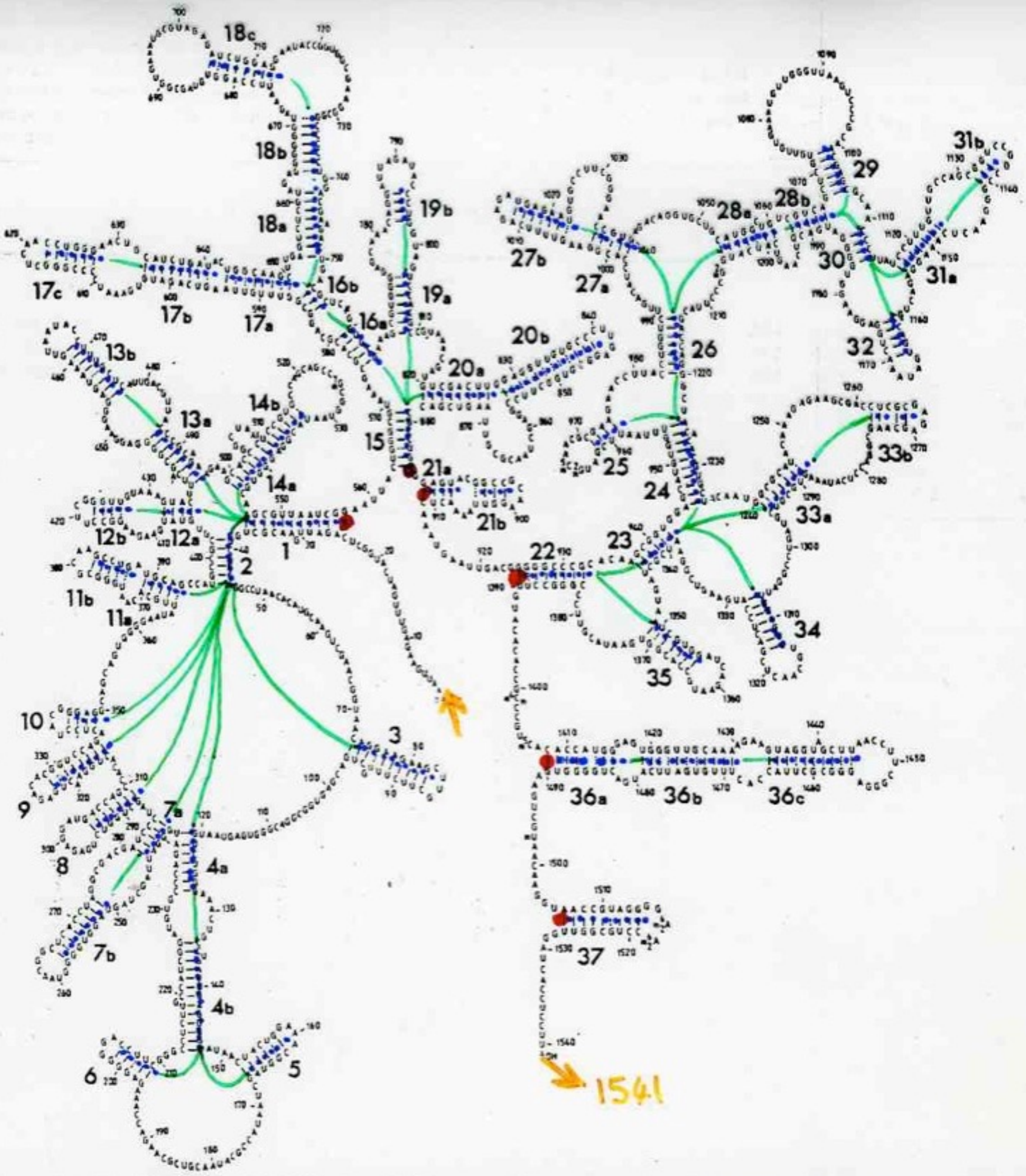
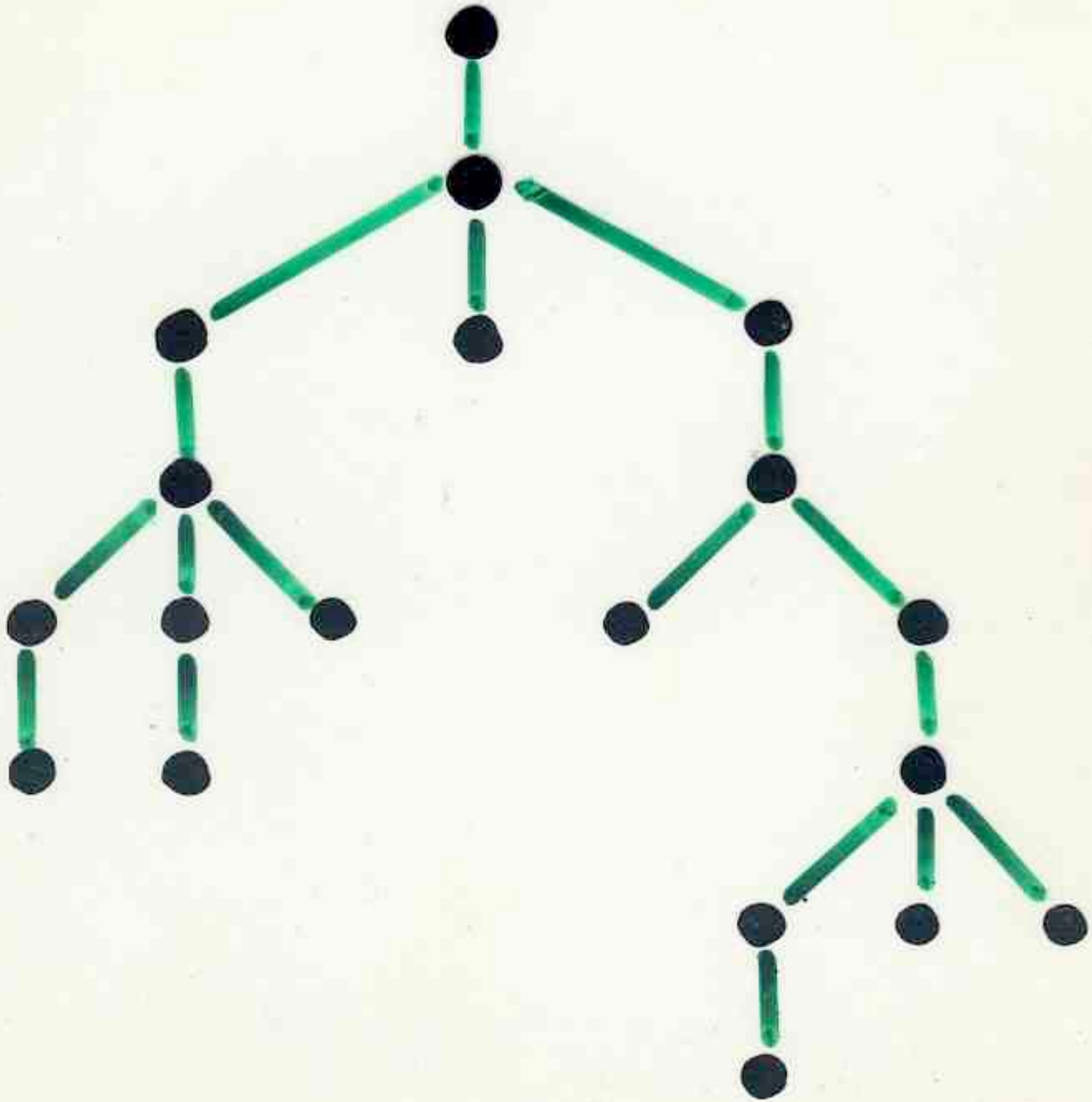
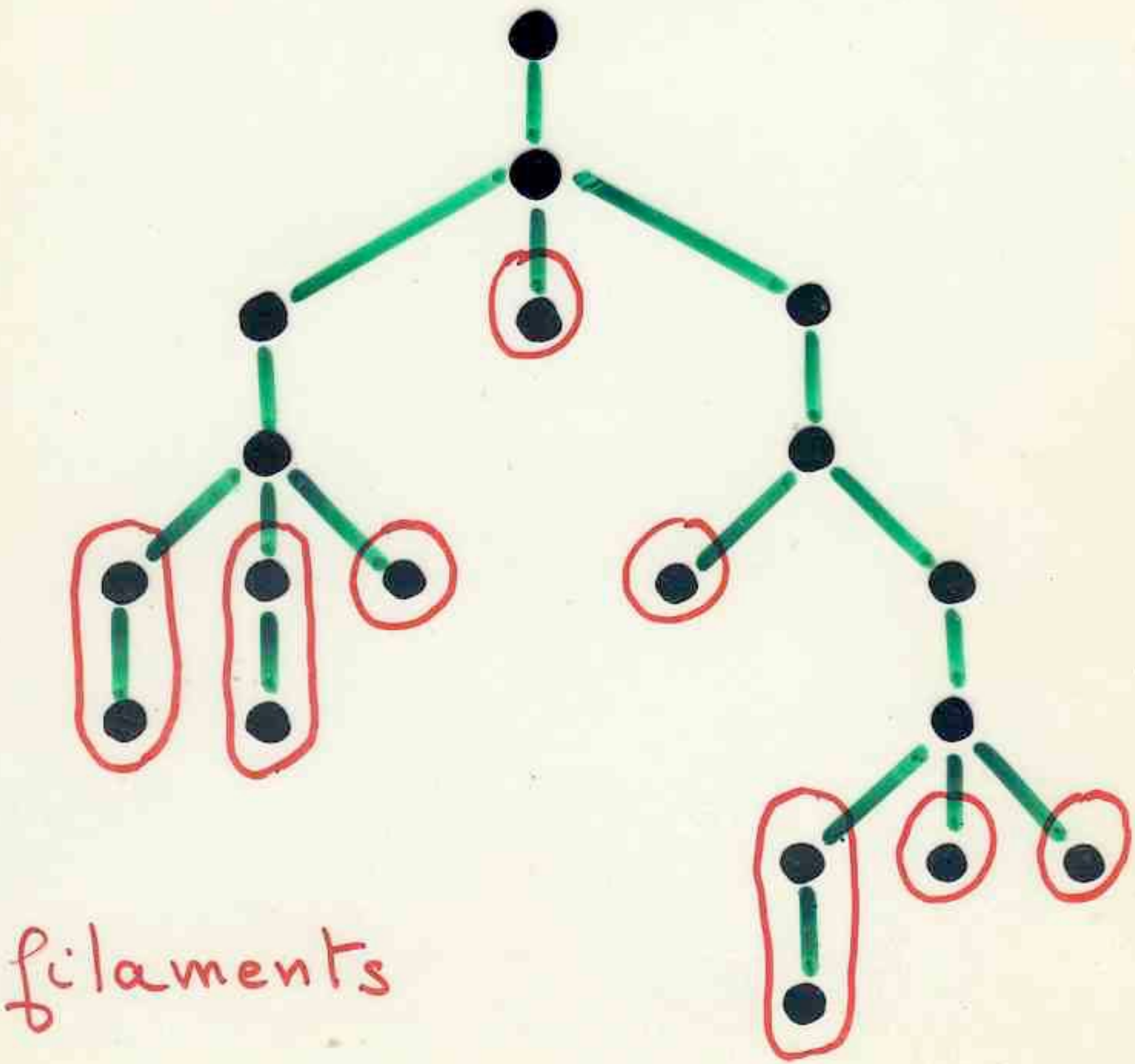


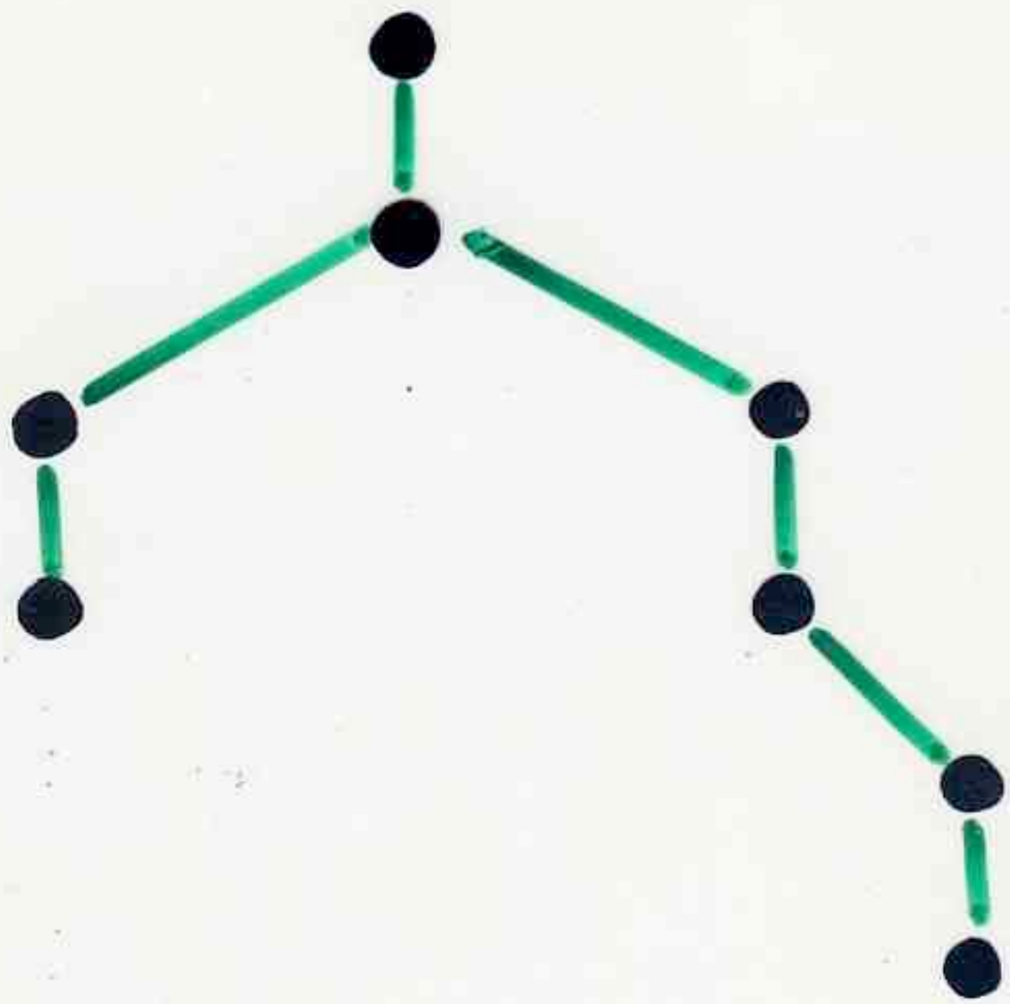
Fig. 1. *Secondary structure model of the 16-S RNA from* E. coli. This model has been fully described elsewhere [18]. The various secondary structure motifs are numbered for reference. Base-pairings 2 and 23 are included in this up-dated scheme and slight modifications have been introduced into helices 18b and 33b

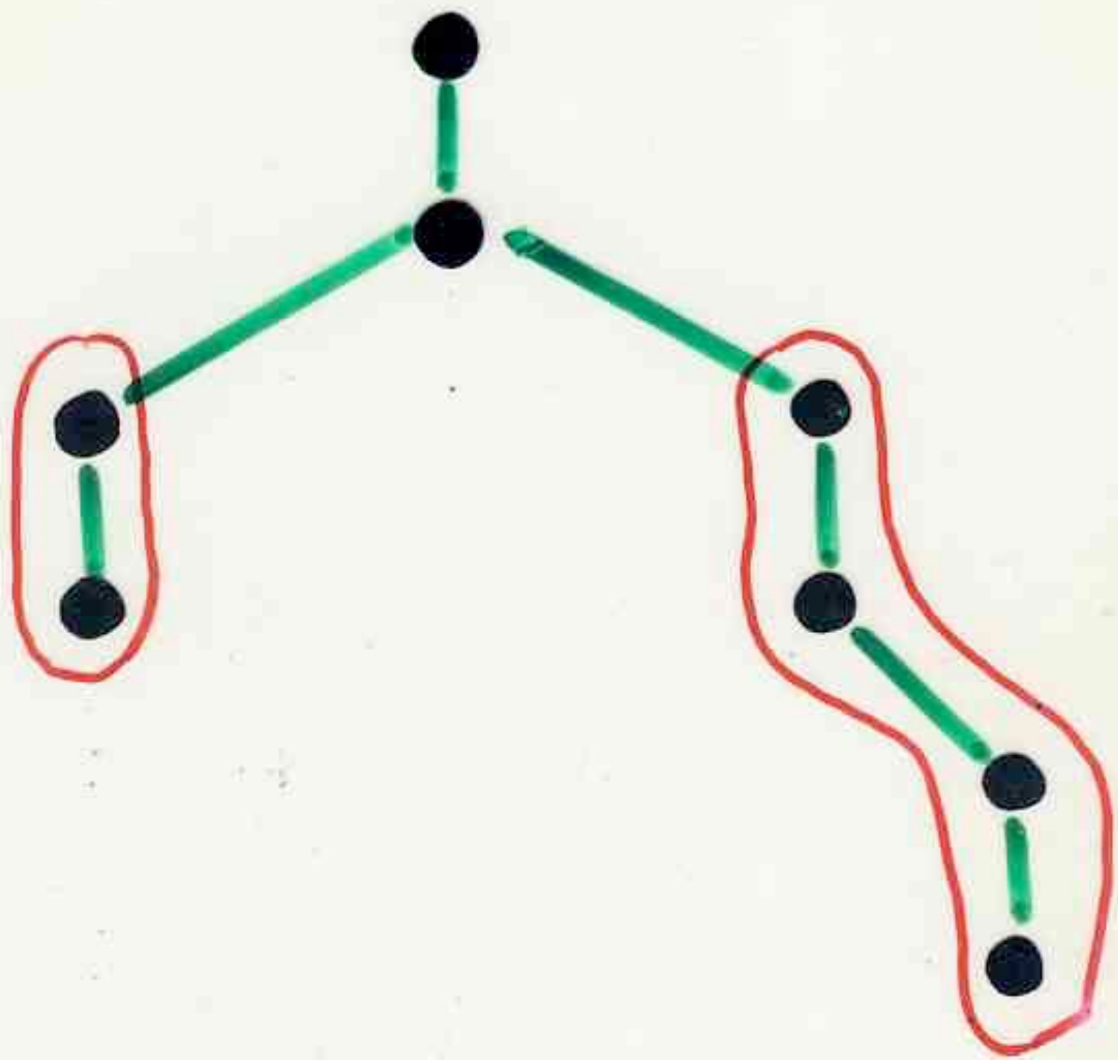
«complexity» or «order» of a molecule

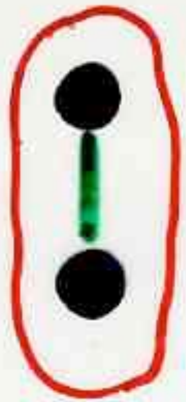
M. Waterman




filaments







$F_{n,k}$ = number of forest of trees with n vertices and order k

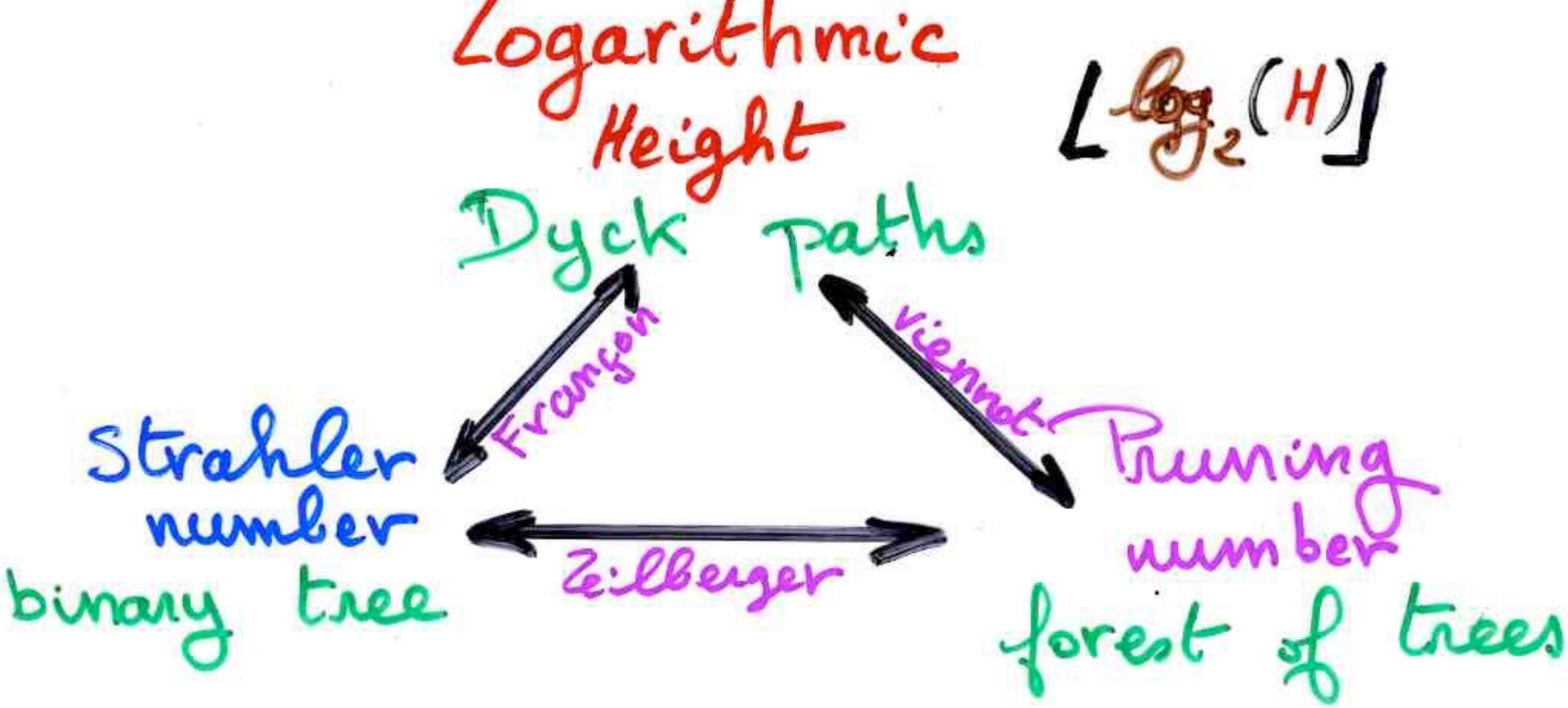
$F_{n,k}$ = number of forest of trees with n vertices and order k

$= S_{n,k}$

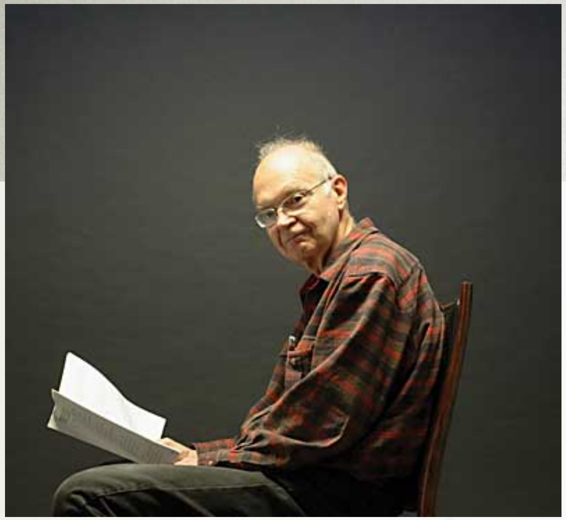
again same distribution !

Vauchaussade de Chaumont X. V. (1985) (2001)

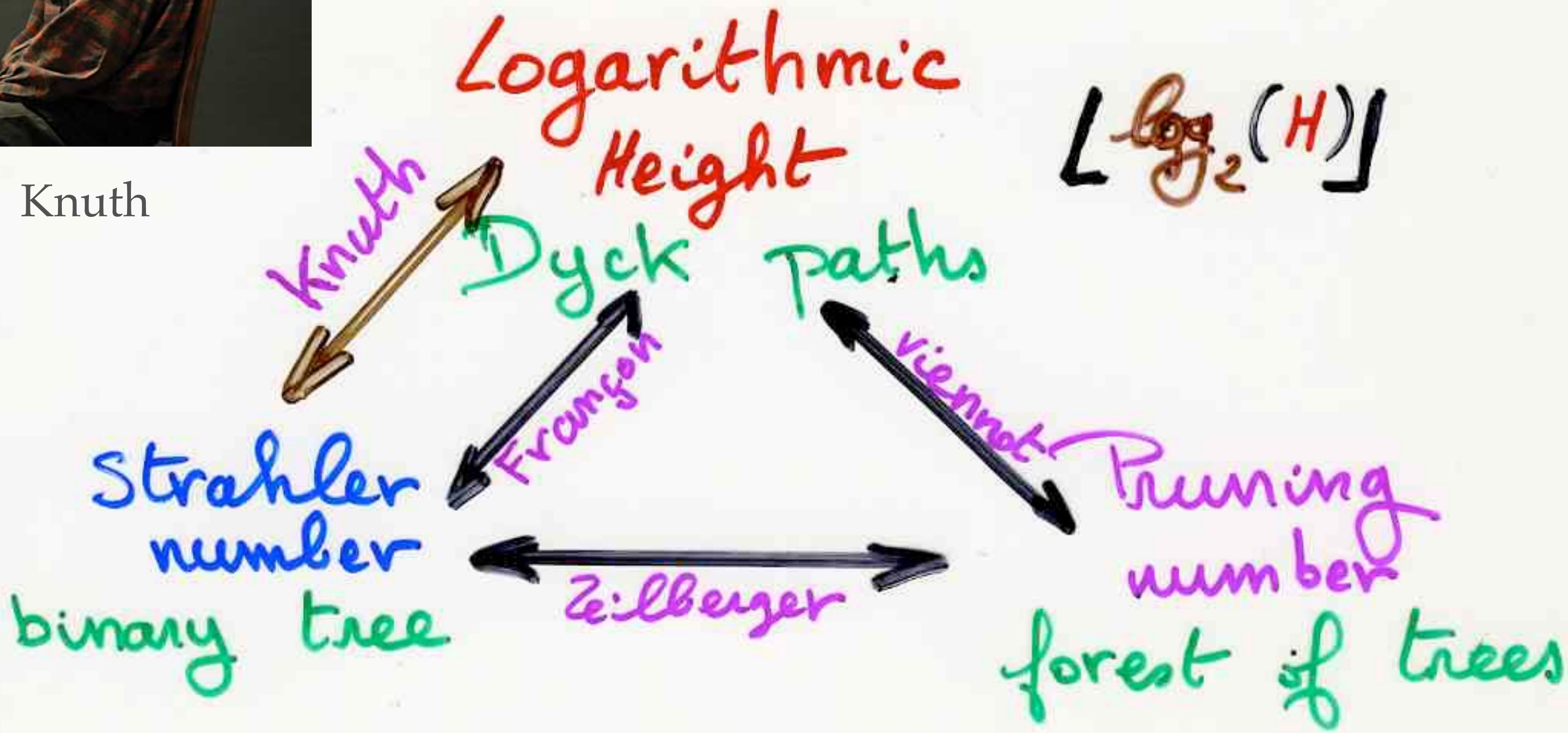
D. Zeilberger (1985)



Logarithmic
Height
$\lfloor \log_2(H) \rfloor$
Dyck paths
Françon
Viennot
Strahler
number
binary tree
Zeilberger
Pruning
number
forest of trees



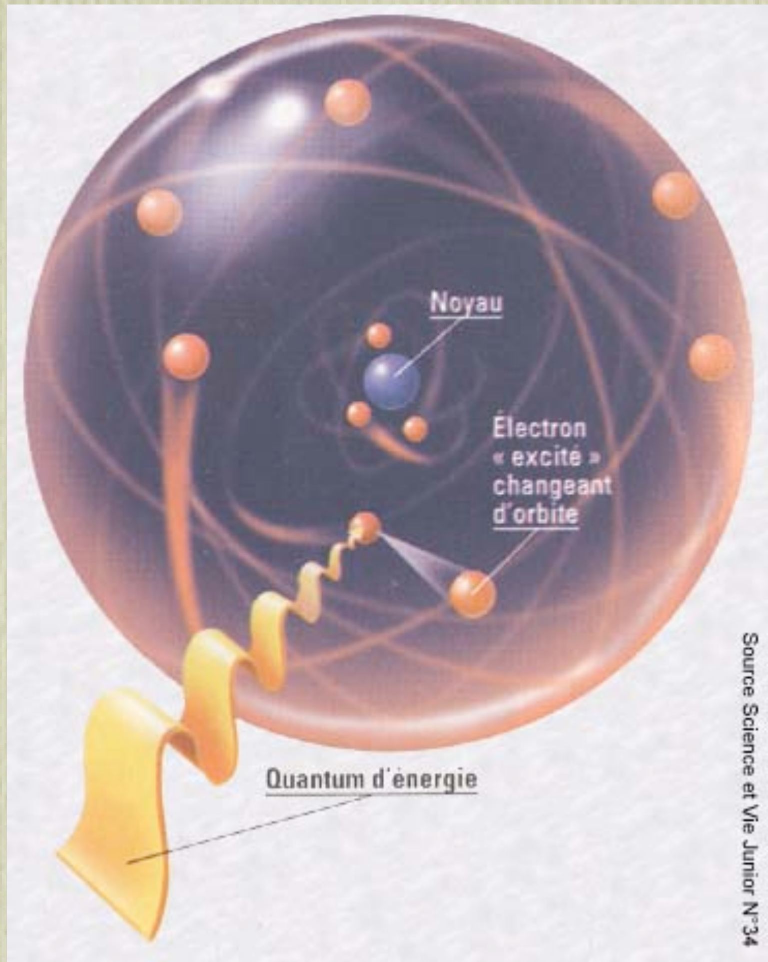
Donald Knuth



The infinitely small

trees in the particles of light ?

the quantum world

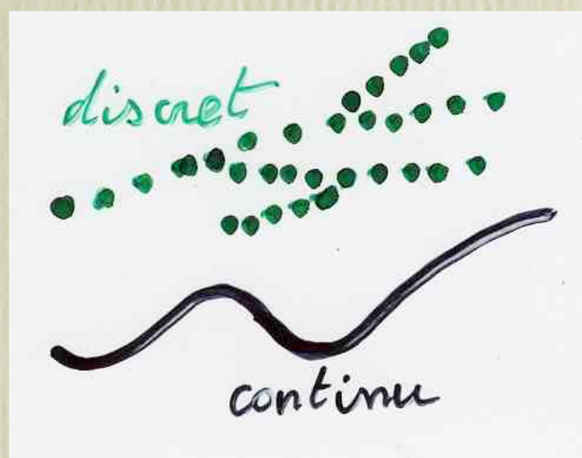


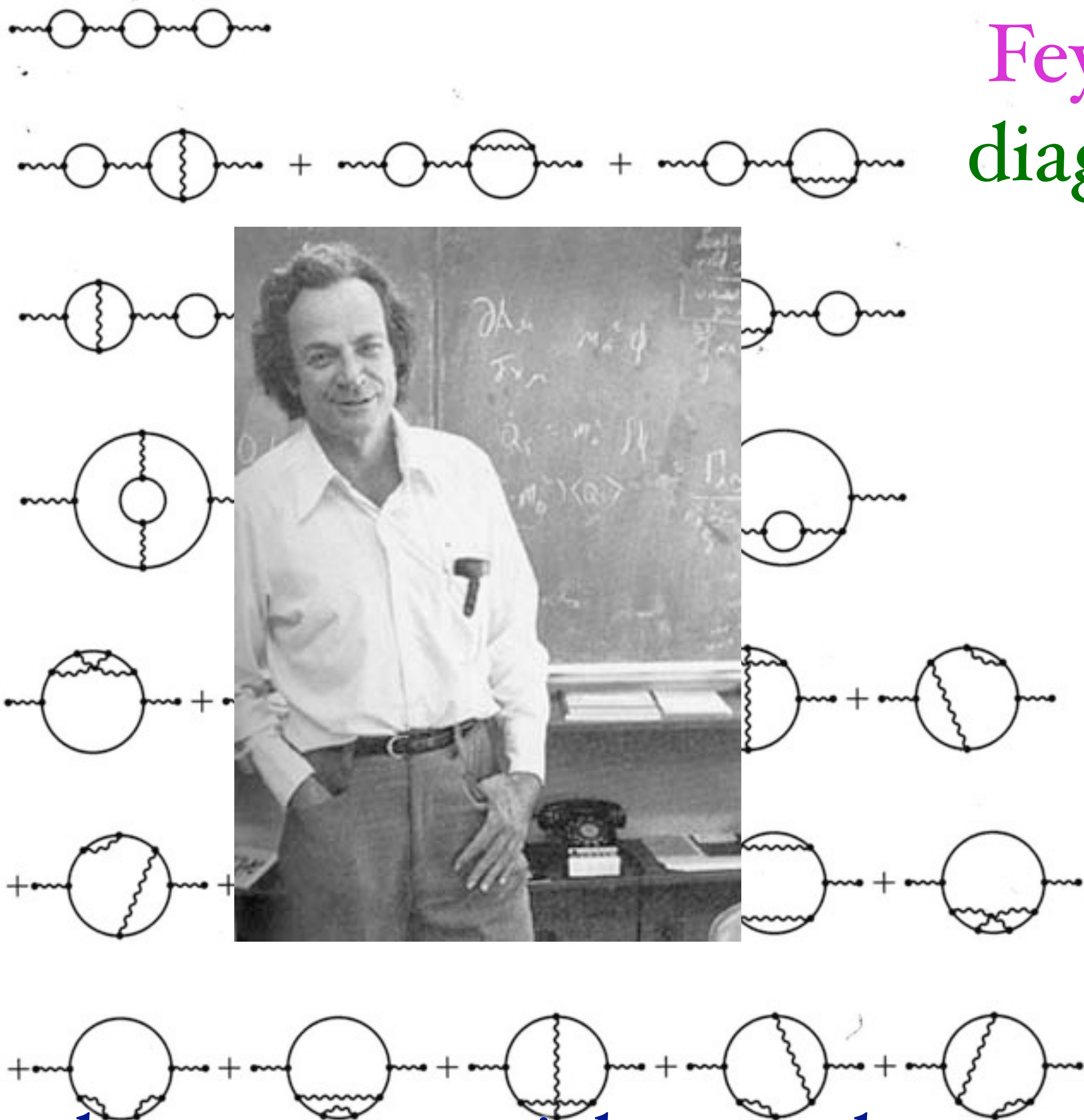
quantum mechanics
very far from common intuition

particle: tendancy to exist ...

the famous Schrödinger cat, dead and alive at the same time

space, time, mater, energy: continuous or discrete ?





Feyman diagrams

interactions between particles, photons

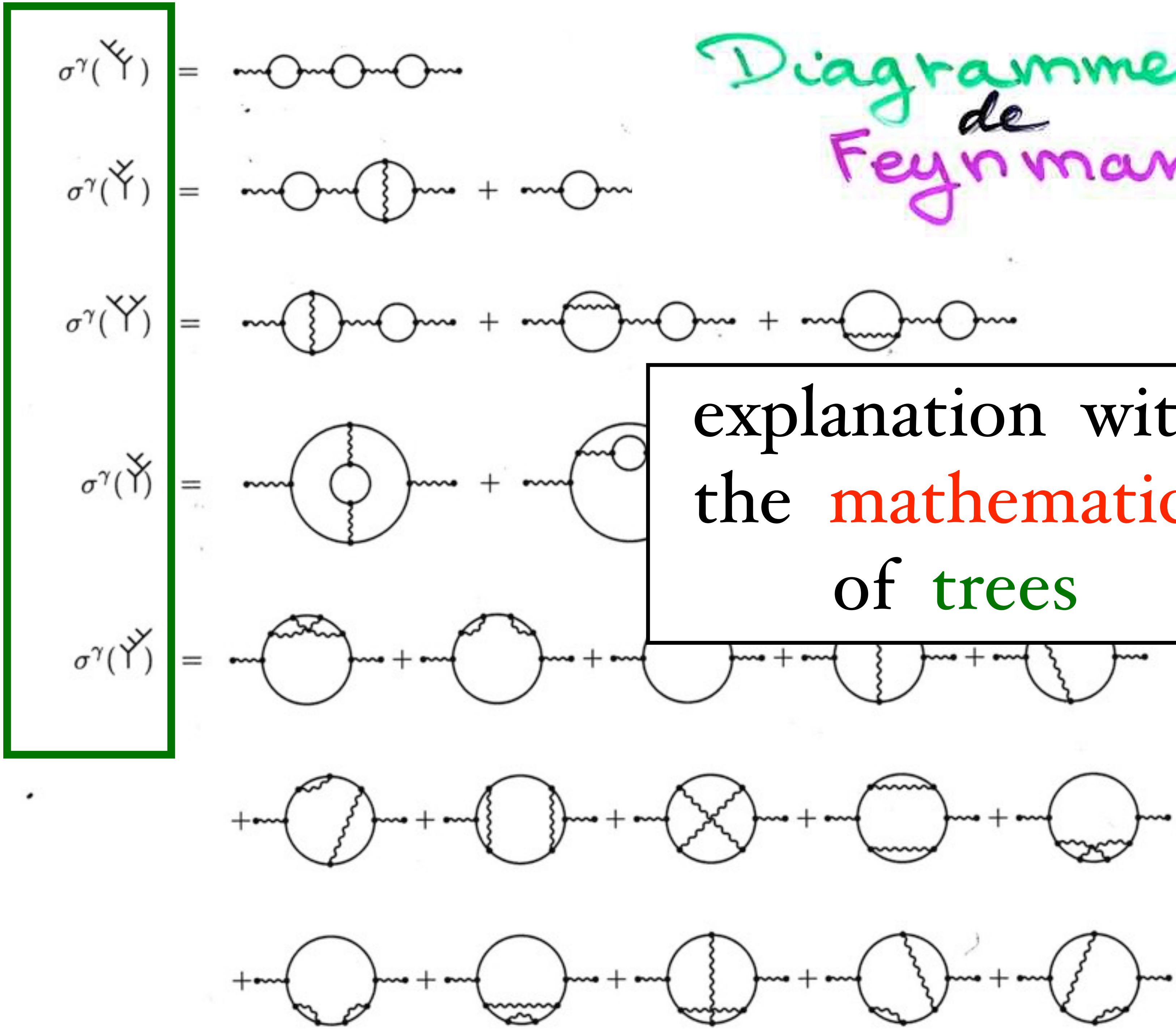
infinite sums of infinite quantities ?!?

deleting the double infinite ...

quantum renormalization

recipe for cooking

Diagrammes de Feynman

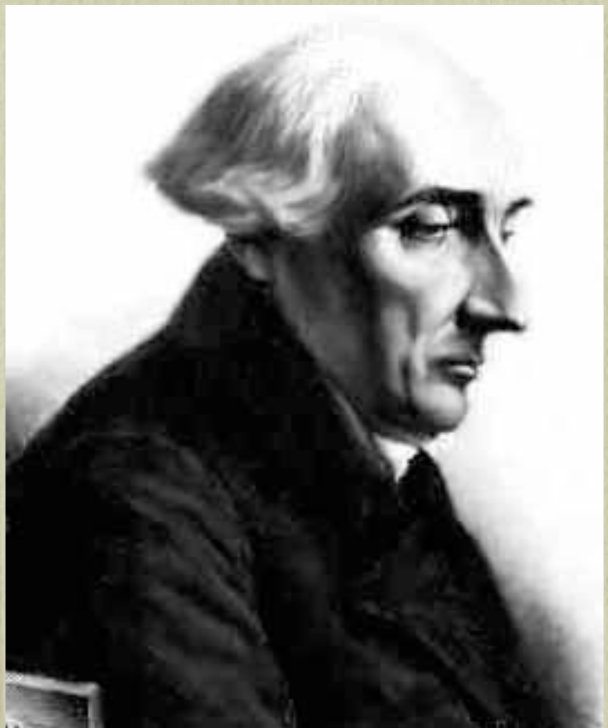


explanation with the mathematics of trees

Alain Connes

Euclide mathematics, many figures until Newton
after, elimination of figures

Lagrange, treatise on mechanics: not a single figure
equations, identities, pure abstraction



Joseph-Louis Lagrange
1736 - 1813

AVERTISSEMENT

DE LA DEUXIÈME ÉDITION.

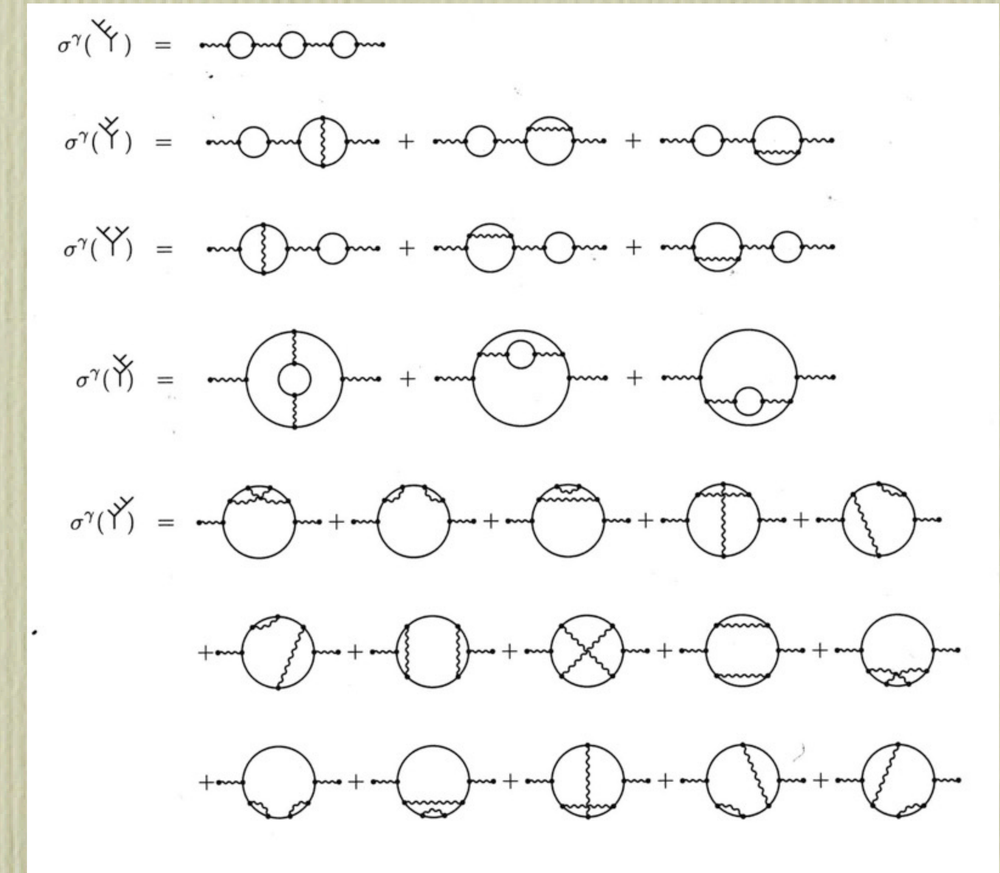
On a déjà plusieurs Traités de Mécanique, mais le plan de celui-ci est entièrement neuf. Je me suis proposé de réduire la théorie de cette Science, et l'art de résoudre les problèmes qui s'y rapportent, à des formules générales, dont le simple développement donne toutes les équations nécessaires pour la solution de chaque problème.

Cet Ouvrage aura d'ailleurs une autre utilité : il réunira et présentera sous un même point de vue les différents principes trouvés jusqu'ici pour faciliter la solution des questions de Mécanique, en montrera la liaison et la dépendance mutuelle, et mettra à portée de juger de leur justesse et de leur étendue.

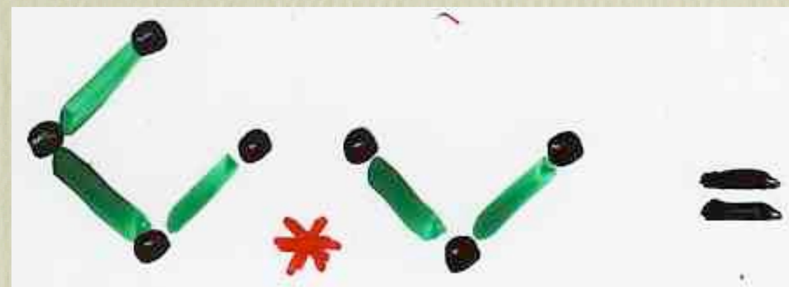
Je le divise en deux Parties : la Statique ou la Théorie de l'Équilibre, et la Dynamique ou la Théorie du Mouvement; et, dans chacune de ces Parties, je traite séparément des corps solides et des fluides.

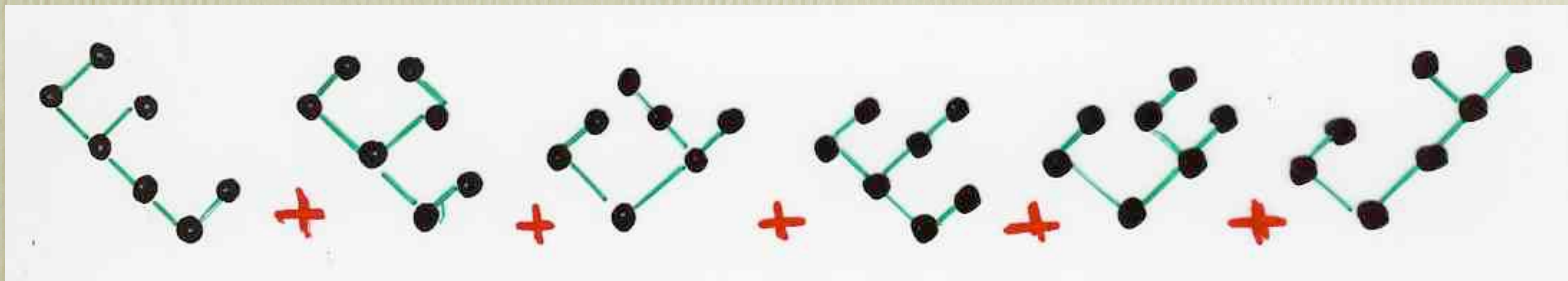
On ne trouvera point de Figures dans cet Ouvrage. Les méthodes que j'y expose ne demandent ni constructions, ni raisonnements géométriques ou mécaniques, mais seulement des opérations algébriques, assujetties à une marche régulière et uniforme. Ceux qui aiment l'Analyse verront avec plaisir la Mécanique en devenir une nouvelle branche, et me sauront gré d'en avoir étendu ainsi le domaine.

today, apparition of «figures», but on another level



product of two binary trees





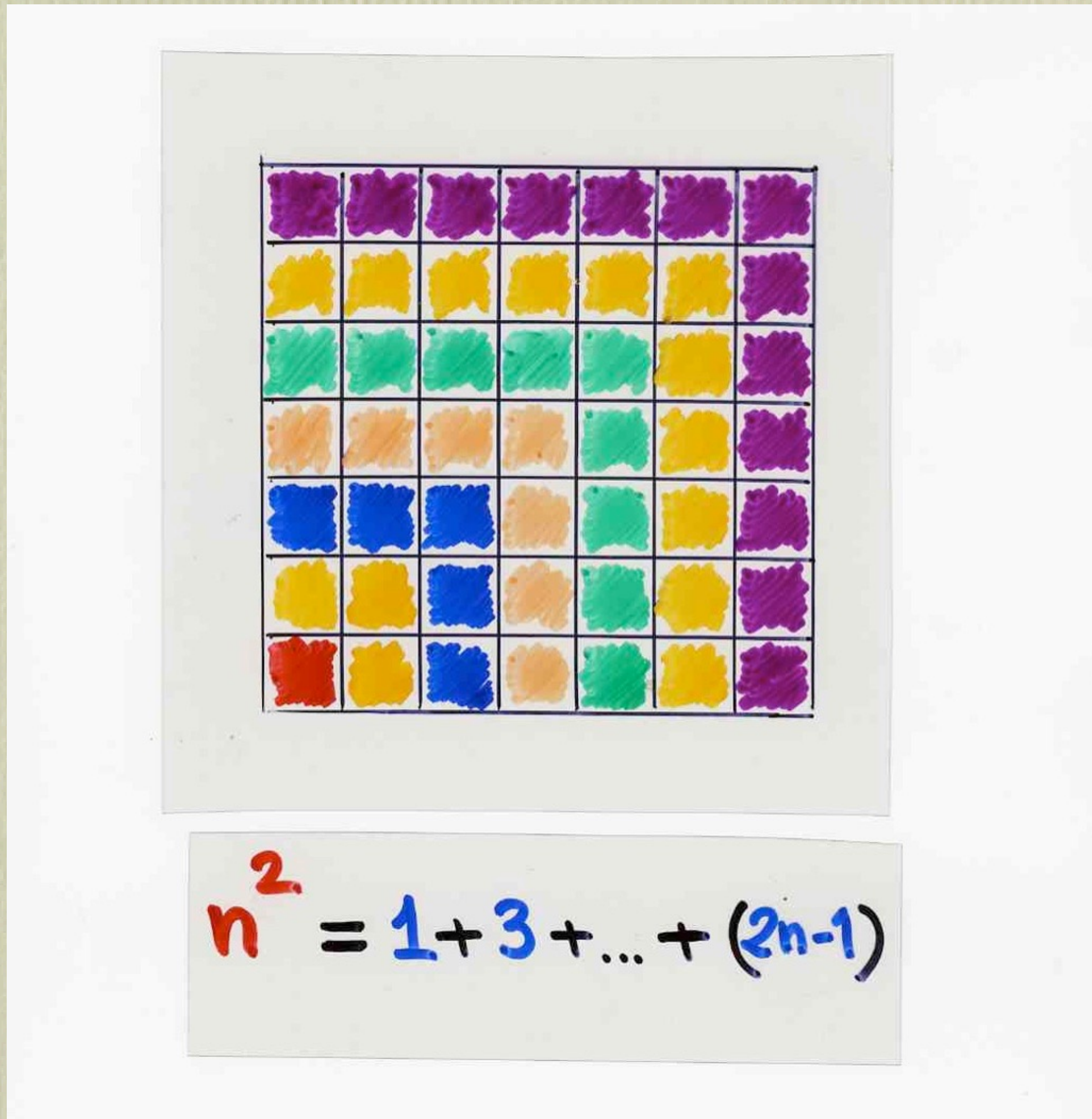
proofs with «figures»

Combinatorial
proofs

«combinatorial proof» of some identities

with bijections, correspondences

combinatorial interpretations



$$\sum_{m\geq 1} \frac{q^{m^2}}{\left[(1-q)(1-q^2)\cdots(1-q^m)\right]^2} = \prod_{i\geq 1} \frac{1}{(1-q^i)}$$

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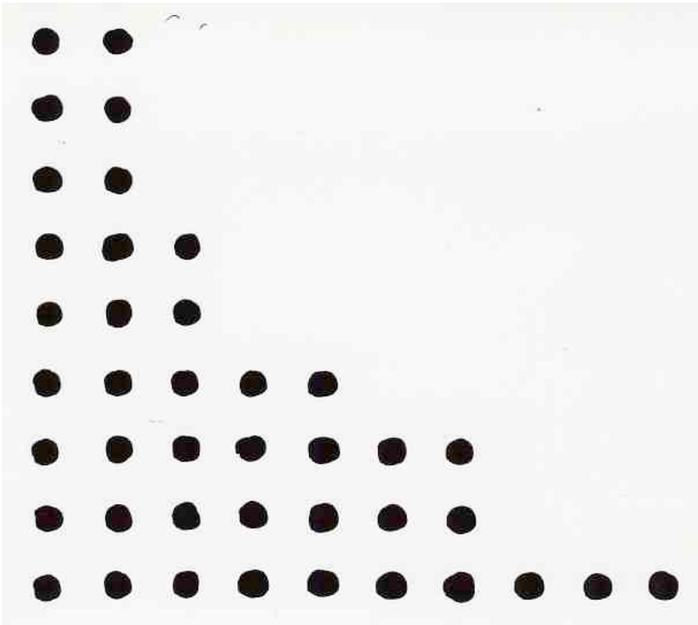
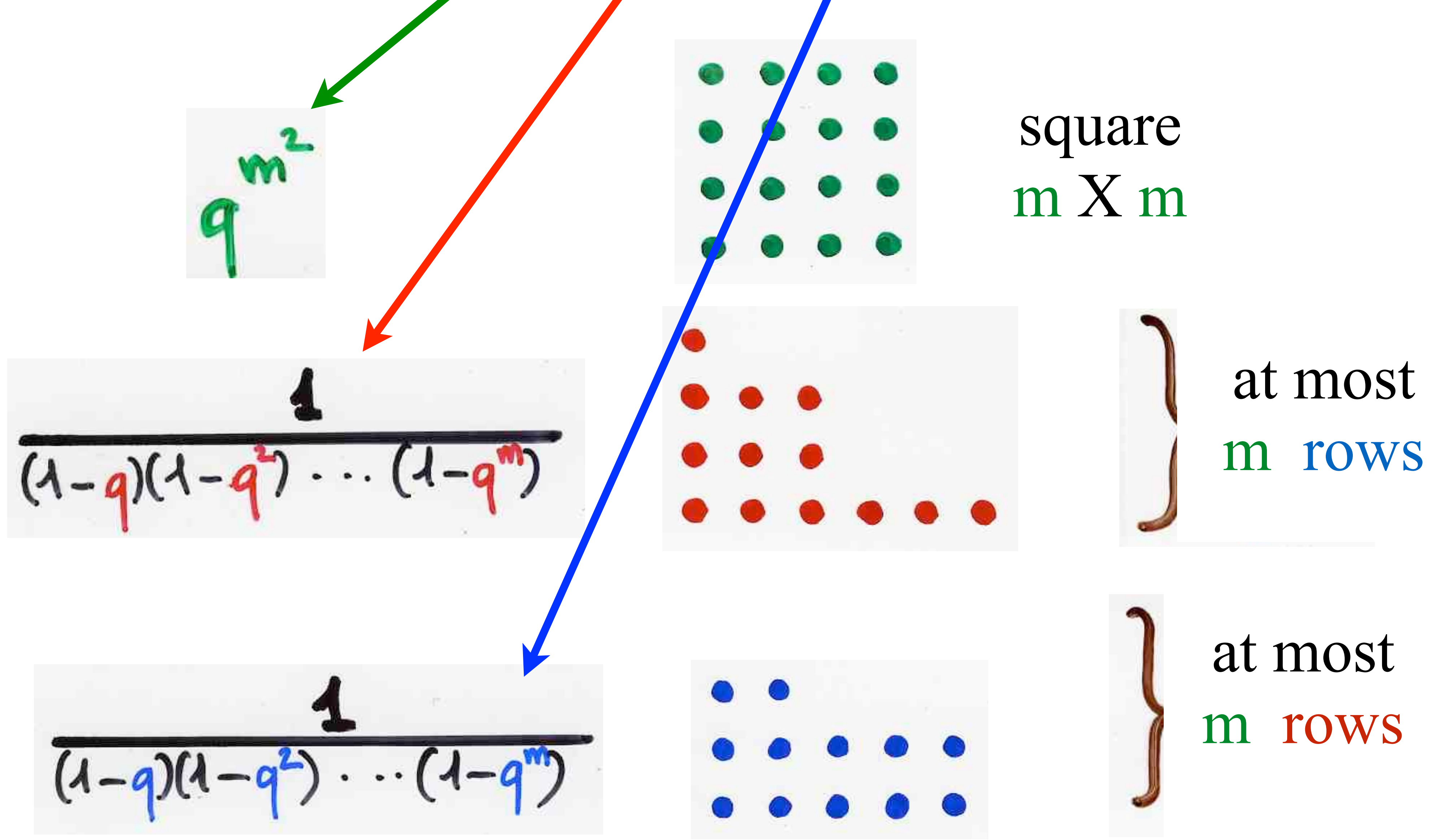
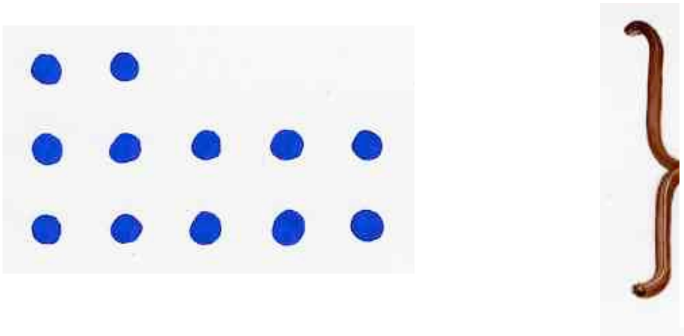


$$= \prod_{i\geq 1} \frac{1}{(1-q^i)}$$

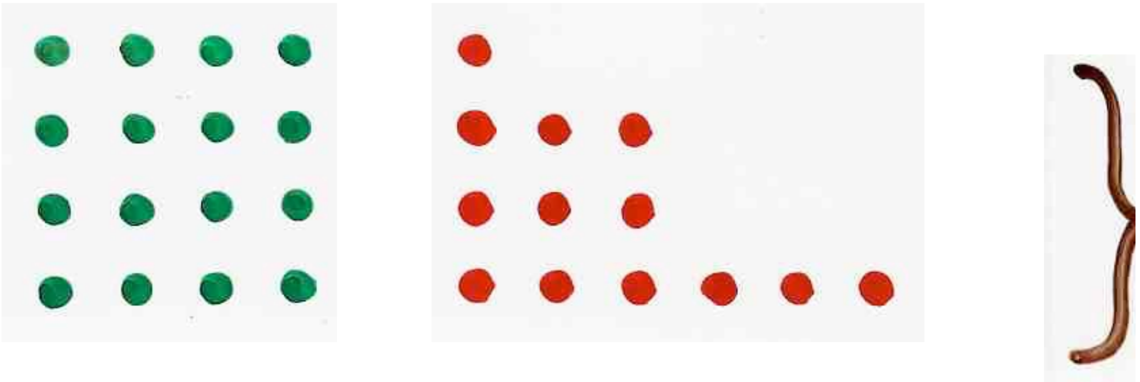
$$\sum_{m\geq 1} \frac{q^{m^2}}{[(1-q)(1-q^2)\cdots(1-q^m)]^2} = \prod_{i\geq 1} \frac{1}{(1-q^i)}$$



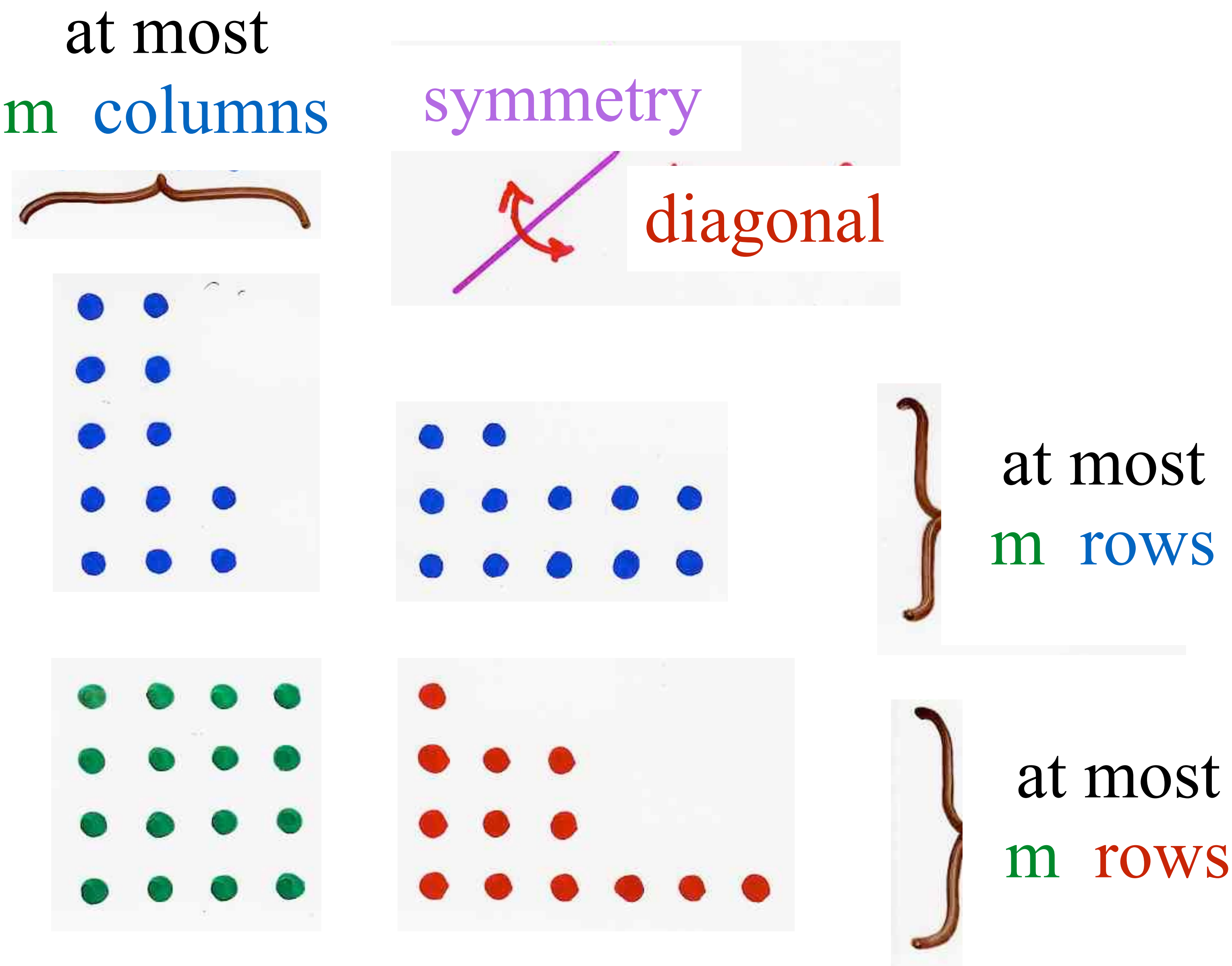
square
m X m

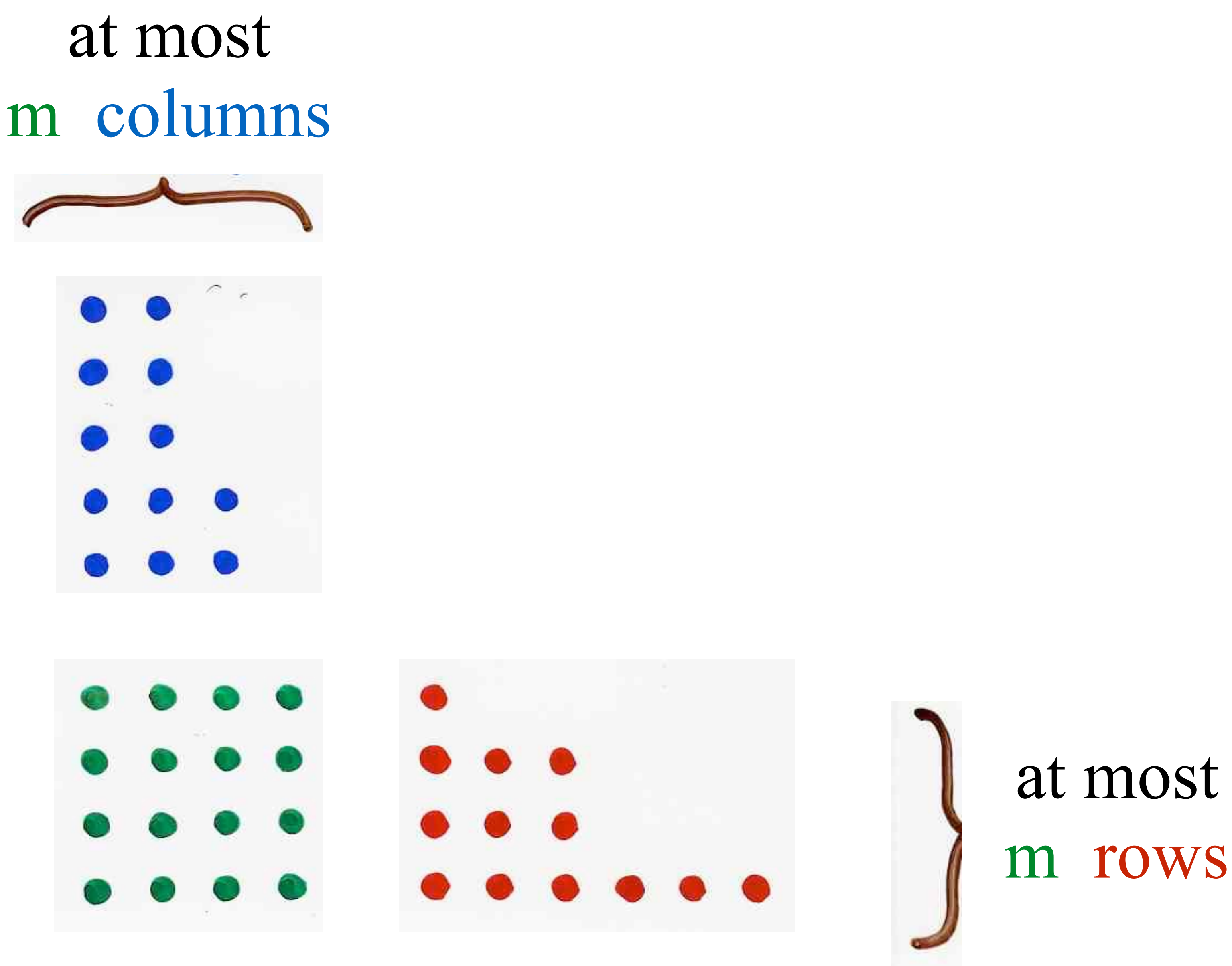


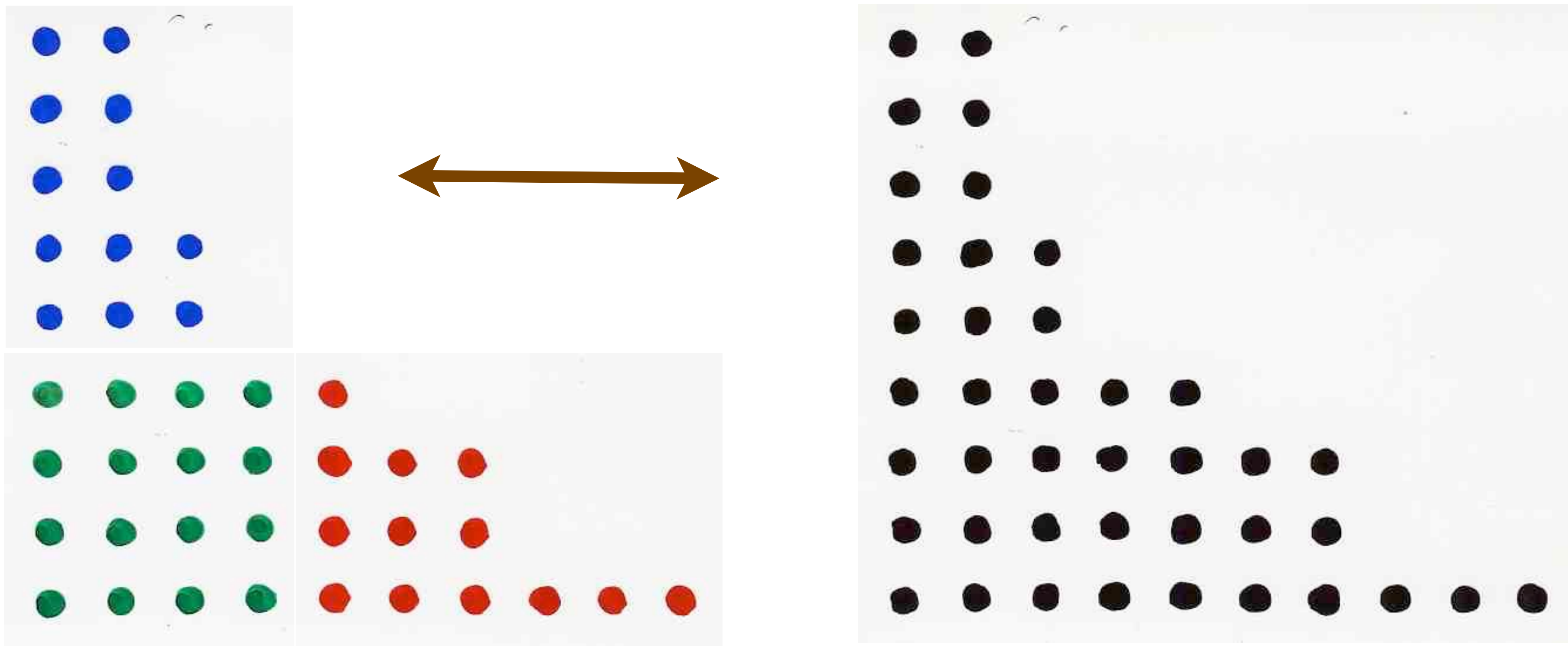
at most
m rows



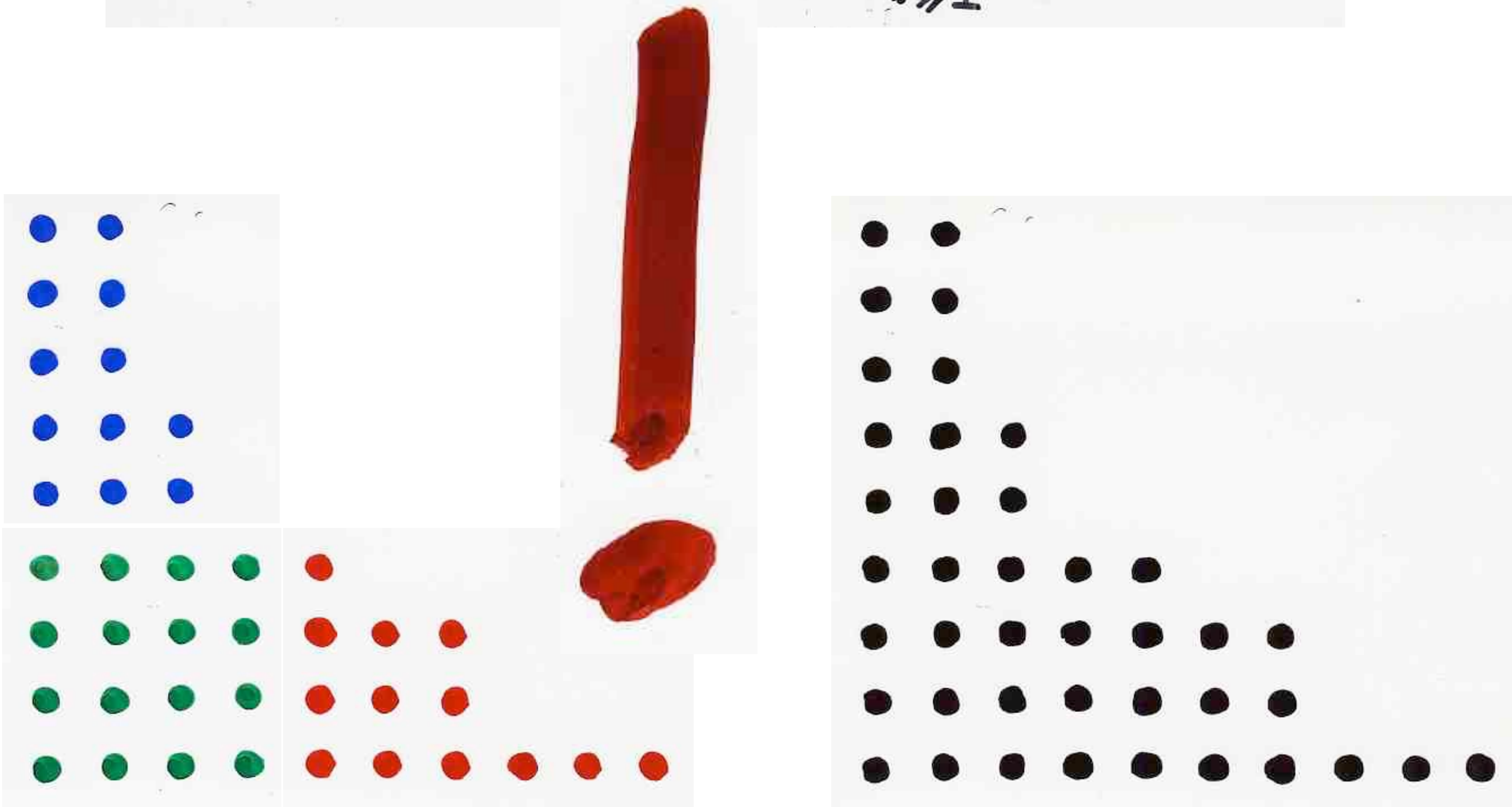
at most
m rows


at most
m columns
symmetry
diagonal
at most
m rows
at most
m rows


at most
m columns
at most
m rows



$$\sum_{m\geq 1} \frac{q^{m^2}}{\left[(1-q)(1-q^2)\cdots(1-q^m)\right]^2} = \prod_{i\geq 1} \frac{1}{(1-q^i)}$$



drawing calculus

...

computing drawings

better understanding

Rogers - Ramanujan identities

$$R_I \quad \sum_{n \geq 0} \frac{q^{n^2}}{(1-q)(1-q^2)\cdots(1-q^n)} = \prod_{\substack{i \equiv 1,4 \\ \text{mod } 5}} \frac{1}{(1-q^i)}$$

$$R_{II} \quad \sum_{n \geq 0} \frac{q^{n^2+n}}{(1-q)(1-q^2)\cdots(1-q^n)} = \prod_{\substack{i \equiv 2,3 \\ \text{mod } 5}} \frac{1}{(1-q^i)}$$

Srinivasan
Ramanujan
(1887-1920)

"La fraction continue" de Ramanujan

$$\cfrac{1}{1+\cfrac{q}{1+\cfrac{q^2}{1+\cfrac{q^3}{\cfrac{\cdots\cdots}{1+\cfrac{q^k}{\cdots\cdots}}}}}} = \frac{\displaystyle\sum_{n\geqslant 0} \frac{q^{n^2+n}}{(1-q)(1-q^2)\cdots(1-q^n)}}{\displaystyle\sum_{n\geqslant 0} \frac{q^{n^2}}{(1-q)(1-q^2)\cdots(1-q^n)}}$$

$$R(q) = \prod_{n \geq 0} \frac{(1-q^{5n+1})(1-q^{5n+4})}{(1-q^{5n+3})(1-q^{5n+2})} = \frac{R_{II}}{R_{I}}$$

$$R(q) = \prod_{n \geqslant 0} \frac{(1-q^{5n+1})(1-q^{5n+4})}{(1-q^{5n+3})(1-q^{5n+2})} = \frac{R_{II}}{R_{I}}$$

$$t = -q\left[R(q)\right]^{5}$$

$$R(q) = \prod_{n\geq 0} \frac{(1-q^{5n+1})(1-q^{5n+4})}{(1-q^{5n+3})(1-q^{5n+2})} = \frac{R_{II}}{R_{I}}$$

$$t = -q\left[R(q)\right]^5$$

$$Y(q) = \prod_{n\geq 0} \frac{(1-q^{6n+2})(1-q^{6n+3})^2(1-q^{6n+4})(1-q^{5n+1})^2(1-q^{5n+4})^2(1-q^{5n})^2}{(1-q^{6n+1})(1-q^{6n+5})\,(1-q^{6n})^2\,(1-q^{5n+2})^3(1-q^{5n+3})^3}$$

$$R(q) = \prod_{n \geq 0} \frac{(1-q^{5n+1})(1-q^{5n+4})}{(1-q^{5n+3})(1-q^{5n+2})} = \frac{R_{II}}{R_{I}}$$

$$t = -q\,[R(q)]^{5}$$

$$Y(q) = \prod_{n \geq 0} \frac{(1-q^{6n+2})(1-q^{6n+3})^{2}(1-q^{6n+4})(1-q^{5n+1})^{2}(1-q^{5n+4})^{2}(1-q^{5n})^{2}}{(1-q^{6n+1})(1-q^{6n+5})(1-q^{6n})^{2}(1-q^{5n+2})^{3}(1-q^{5n+3})^{3}}$$

$$Z(t) = Y(q(t))$$

$$y\left(1+14t+97t^{2}+415t^{3}+1180t^{4}+2321t^{5}+3247t^{6}+3300t^{7}+2475t^{8}+1375t^{9}+550t^{10}+143t^{11}+18t^{12}\right)+$$

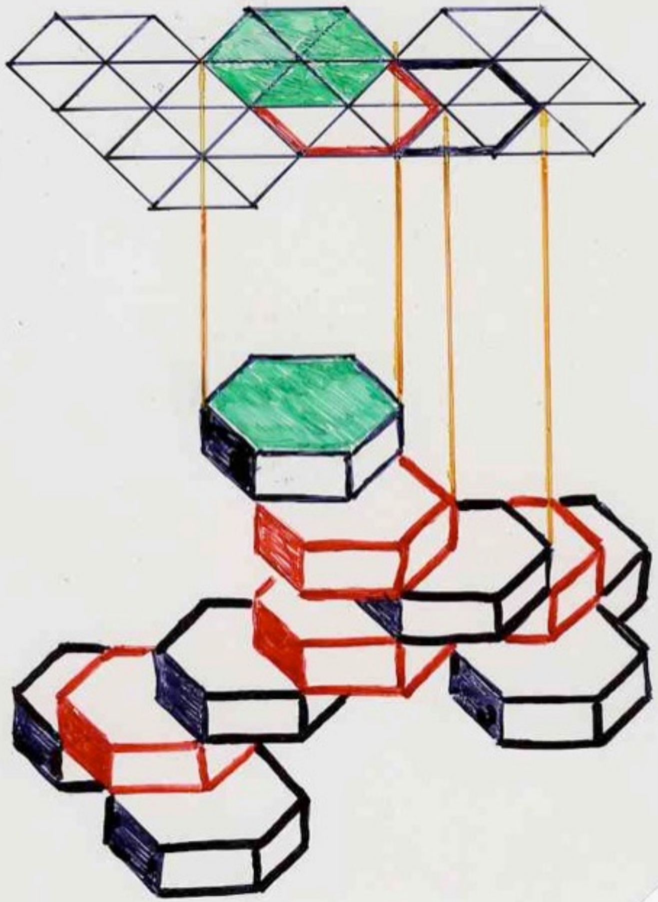
$$y^{2}\left(1+17t+83t^{2}+601t^{3}+1647t^{4}+4606t^{5}+7809t^{6}+710t^{7}+124t^{8}-608t^{9}-440t^{10}-92t^{11}-36t^{12}\right)+$$

$$y^{3}\left(3+50t+381t^{2}+1715t^{3}+5040t^{4}+10130t^{5}+14062t^{6}+13002t^{7}+6930t^{8}+715t^{9}-1595t^{10}-488t^{11}-198t^{12}\right)+$$

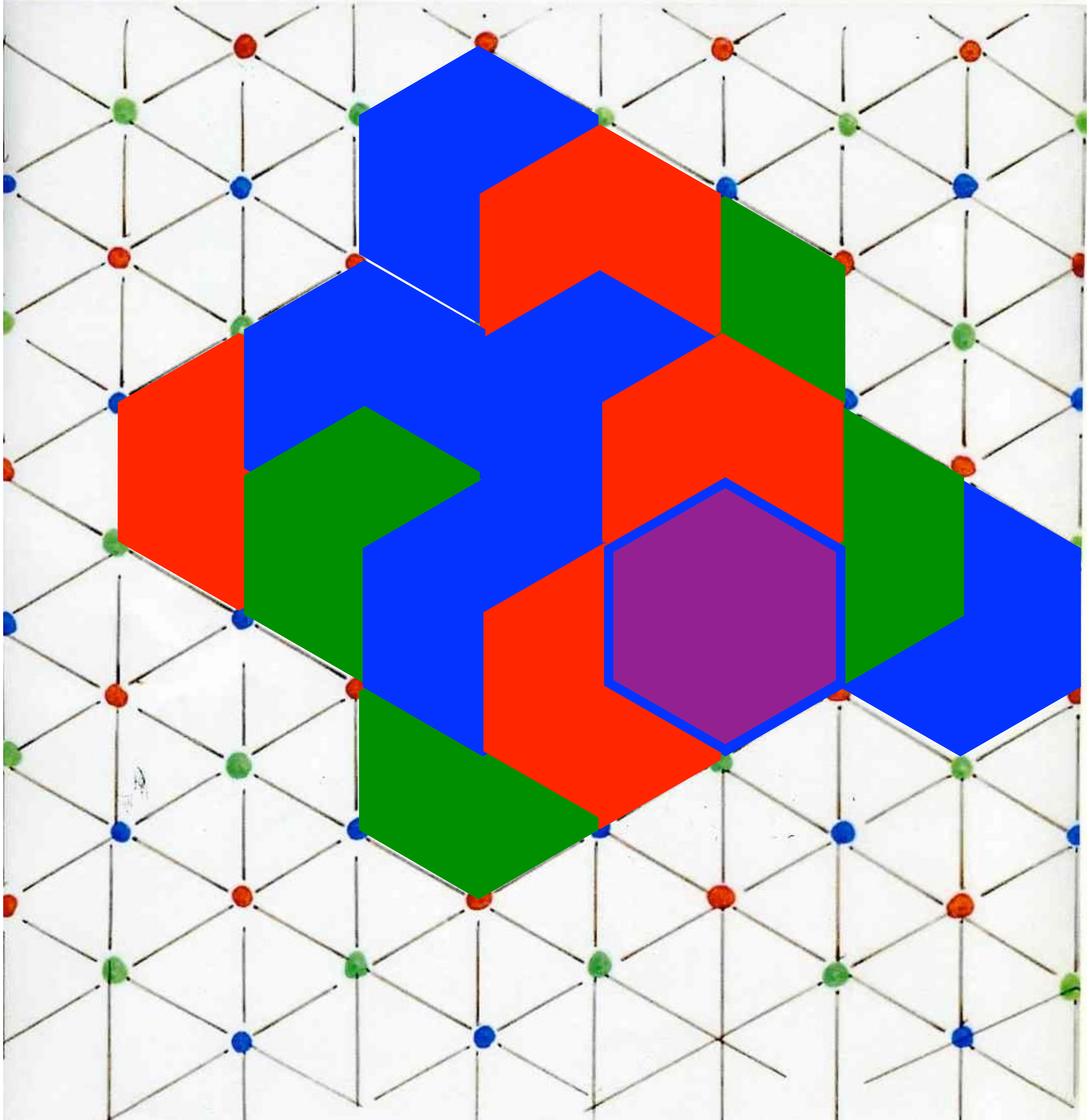
$$y^{4}\left(1+17t+131t^{2}+595t^{3}+1765t^{4}+3574t^{5}+4939t^{6}+4356t^{7}+1815t^{8}-605t^{9}-1210t^{10}-616t^{11}-126t^{12}\right)$$

$$=\left(t+11t^{2}+55t^{33}+165t^{4}+330t^{5}+462t^{6}+462t^{7}+330t^{8}+165t^{9}+55t^{10}+11t^{11}+t^{12}\right)$$

$$-p(-t) = y$$



The idea of heaps of pieces



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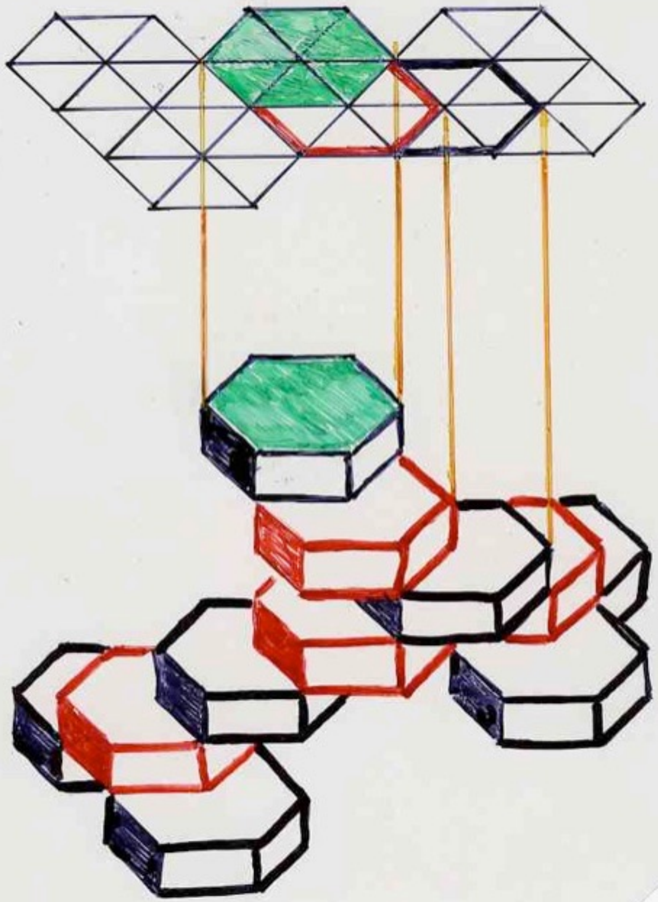
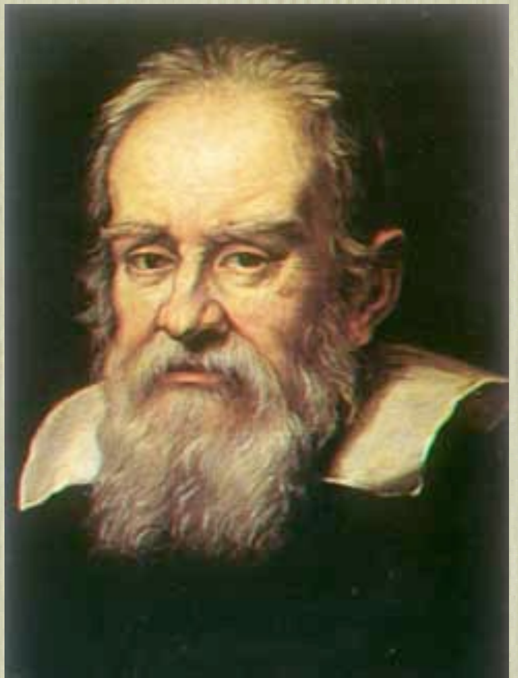
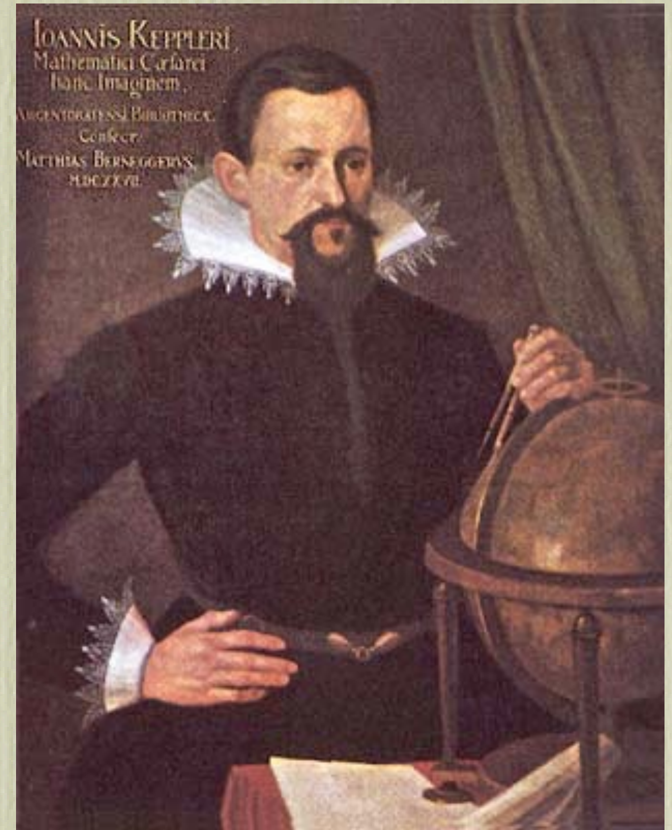


Combinatorial Physics

The infinitely large

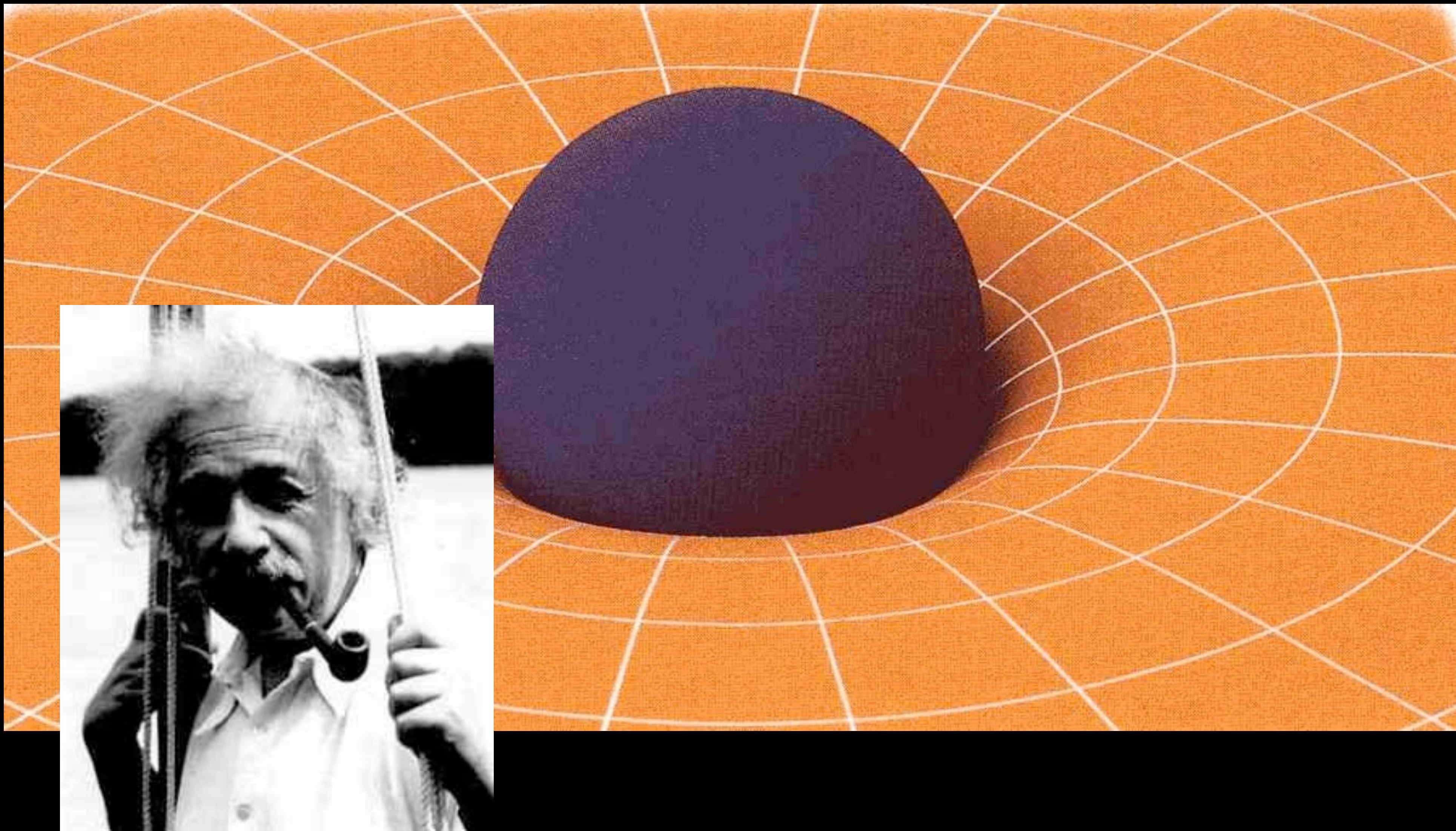
Trees in the stars?

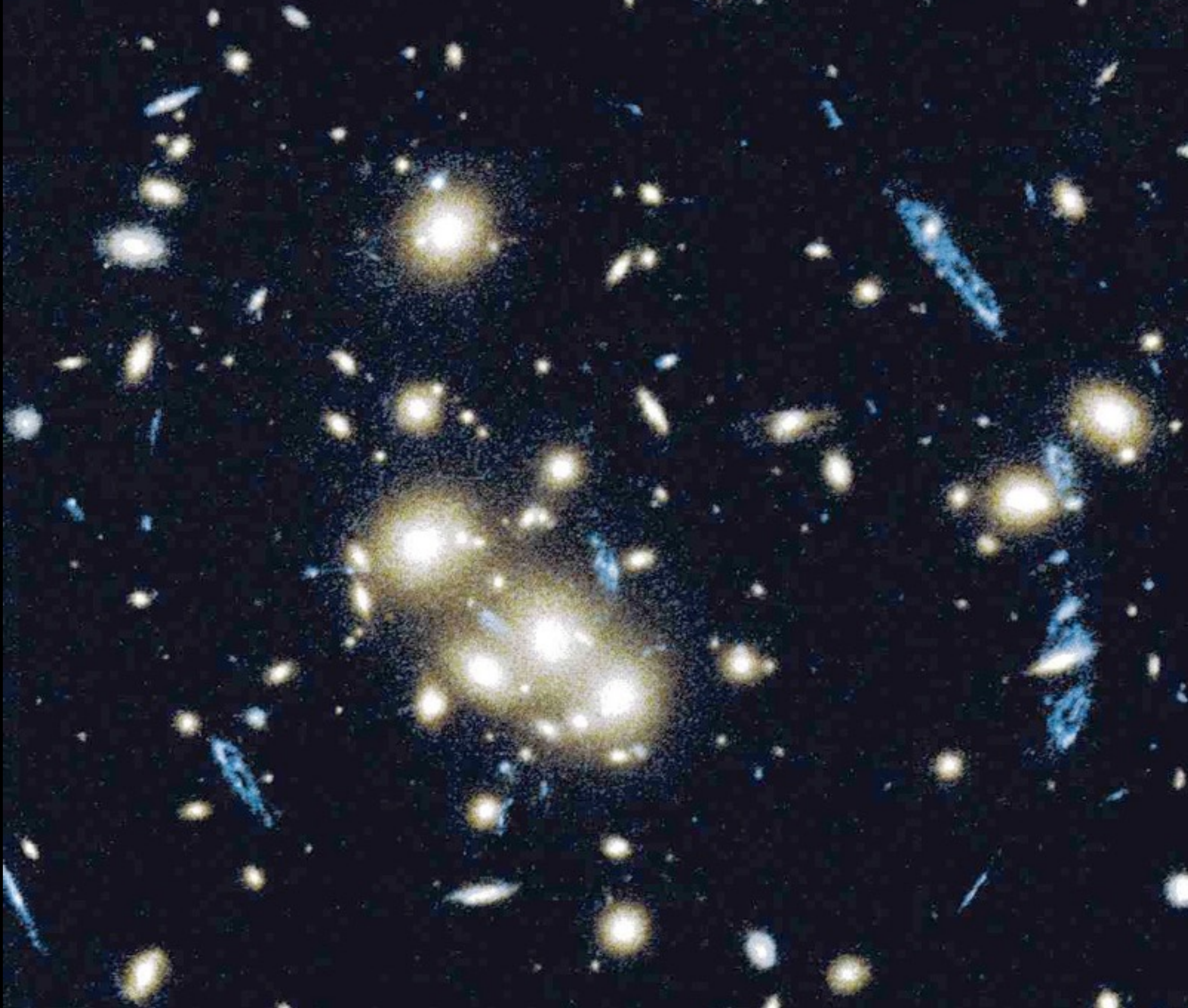




classical geometry and mechanics
Galileo, Kepler, Newton,...

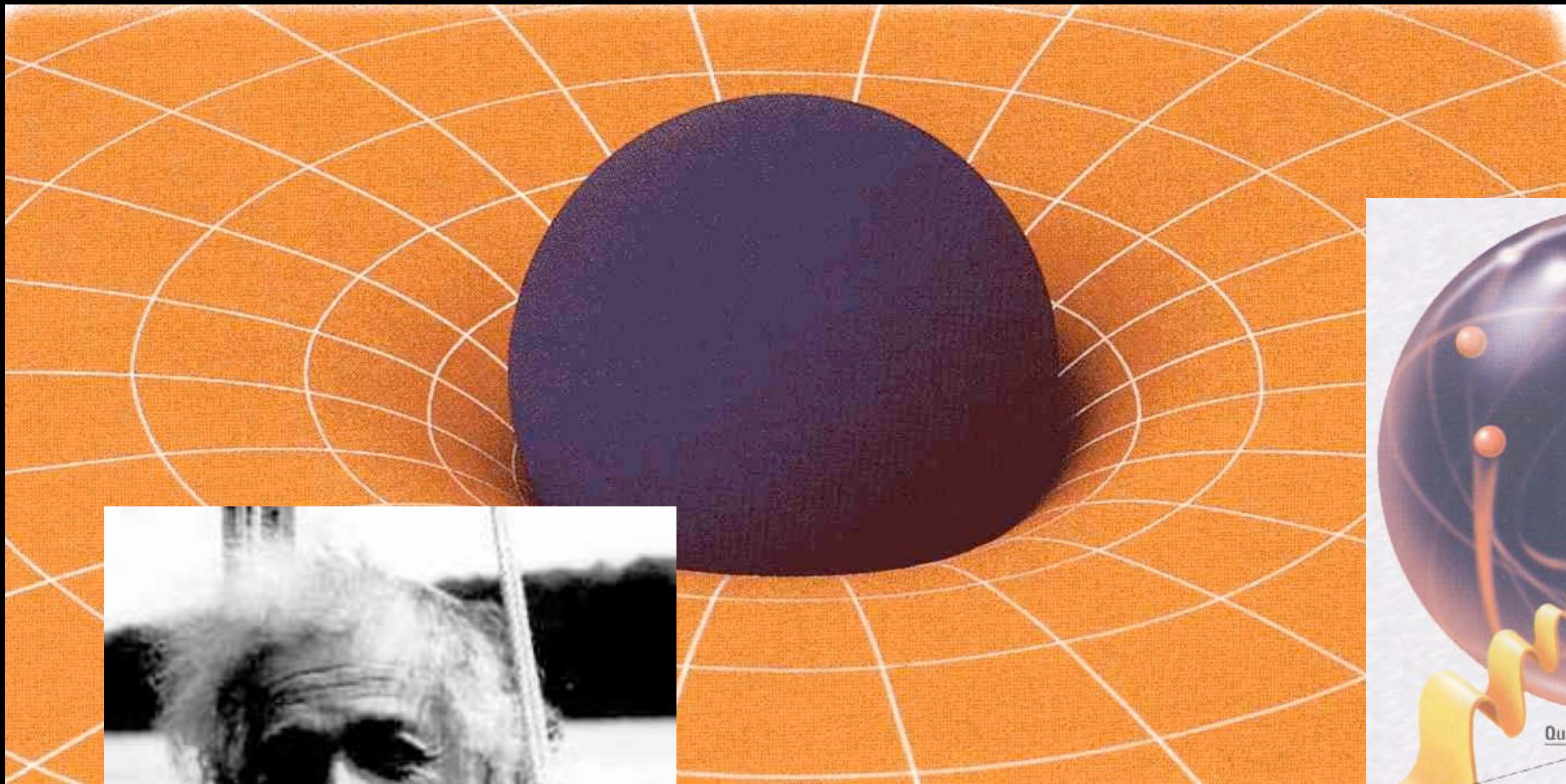
general relativity

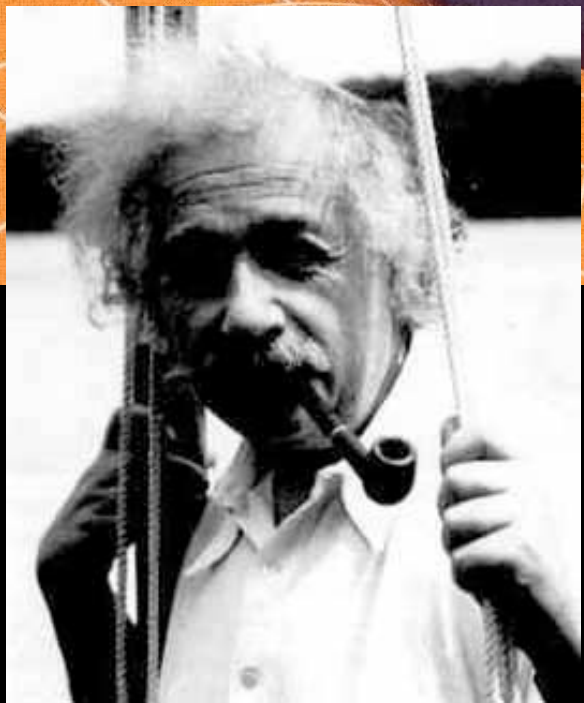


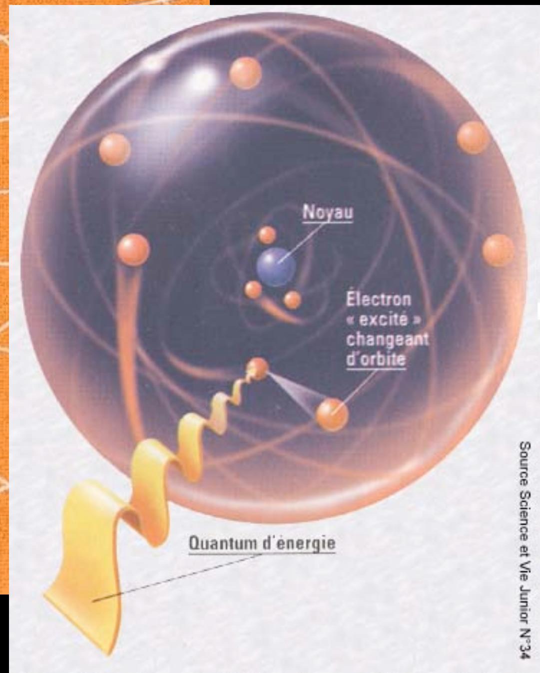


general relativity

quantum mechanics







quantum gravity ?

strings theory

particle as a violin chord ... ?
each frequency corresponds to a particle.... ?

Catalan numbers ?


non-commutative geometry
Alain Connes

loop quantum gravity

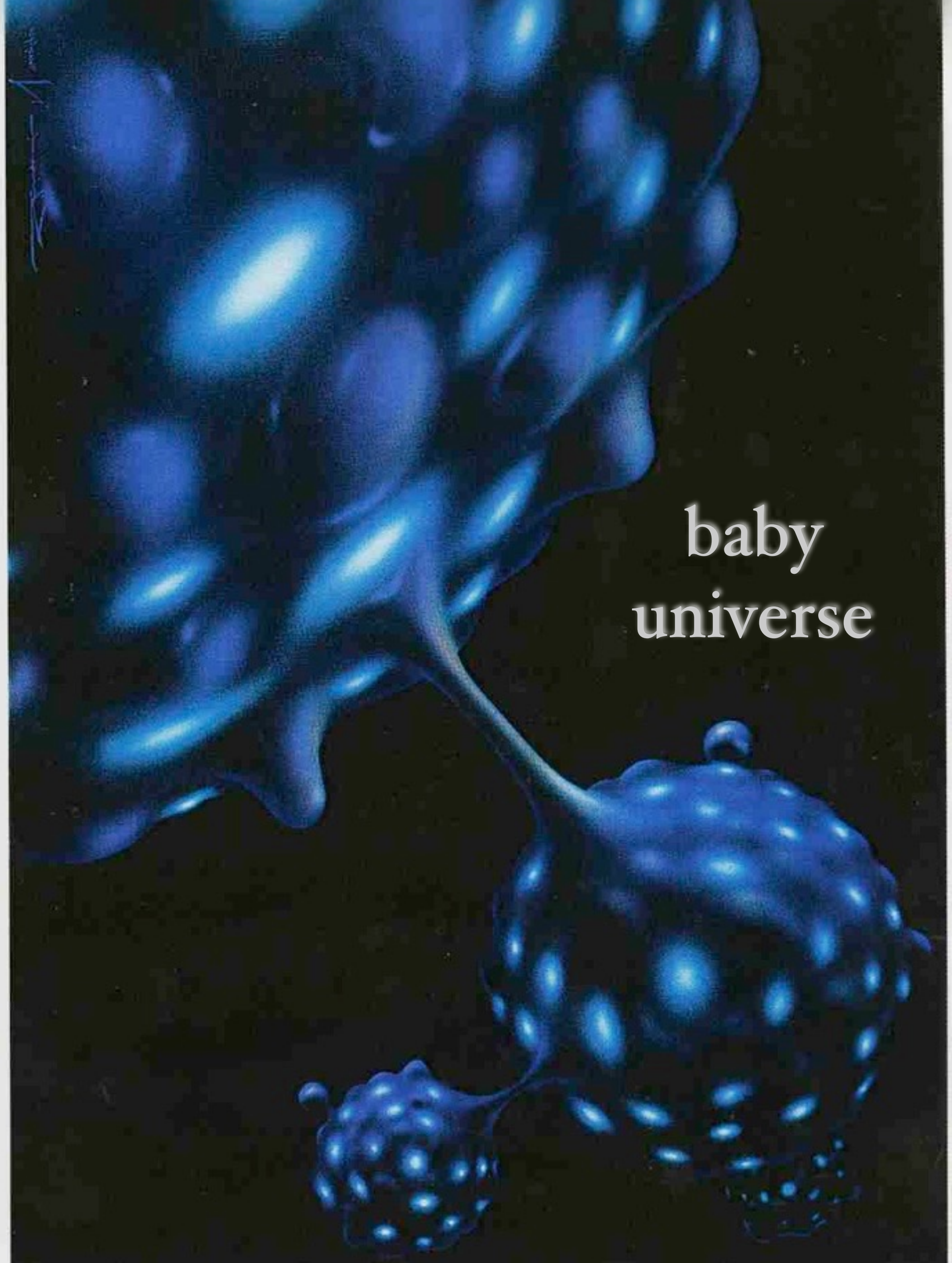
Carlo Rovelli

May be time does not exists ?


foam of the
space-time

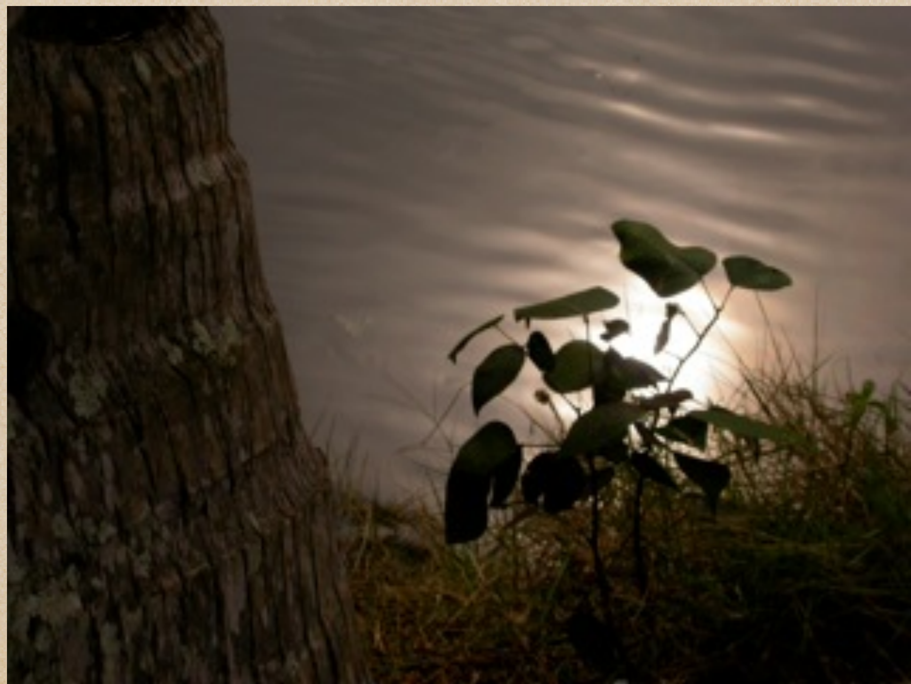
Drawing
S. Numazawa

Ciel & Espace



quantum gravity

causal dynamical triangulations



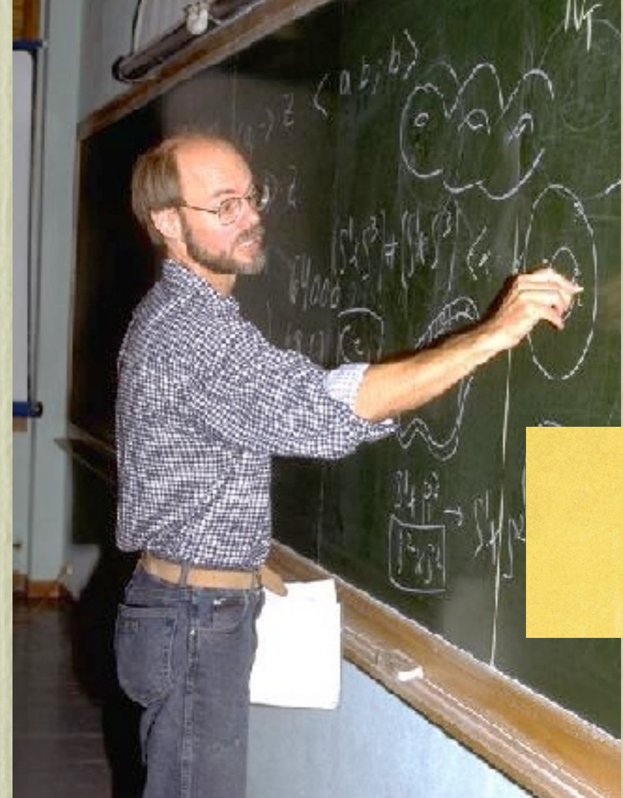
Deepak Dhar
TIFR Bombay

Xavier, you should have a look at these papers:

- J. Ambjørn, R. Loll, "Non-perturbative Lorentzian quantum gravity and topology change", Nucl. Phys. B536 (1998) 407-434
  arXiv: hep-th/9805108

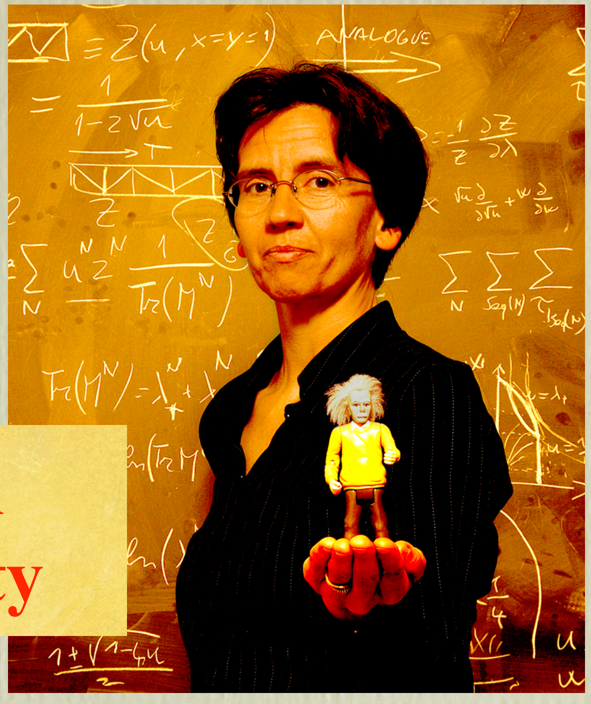
- P. Di Francesco, E. Guitter, C. Kristjansen, Integrable 2D Lorentzian gravity and random walks", Nucl. Phys. B567 (2000) 515-553
  arXiv: hep-th/9907084

gravitation quantique



J.Ambjørn

2D Lorentzian quantum gravity

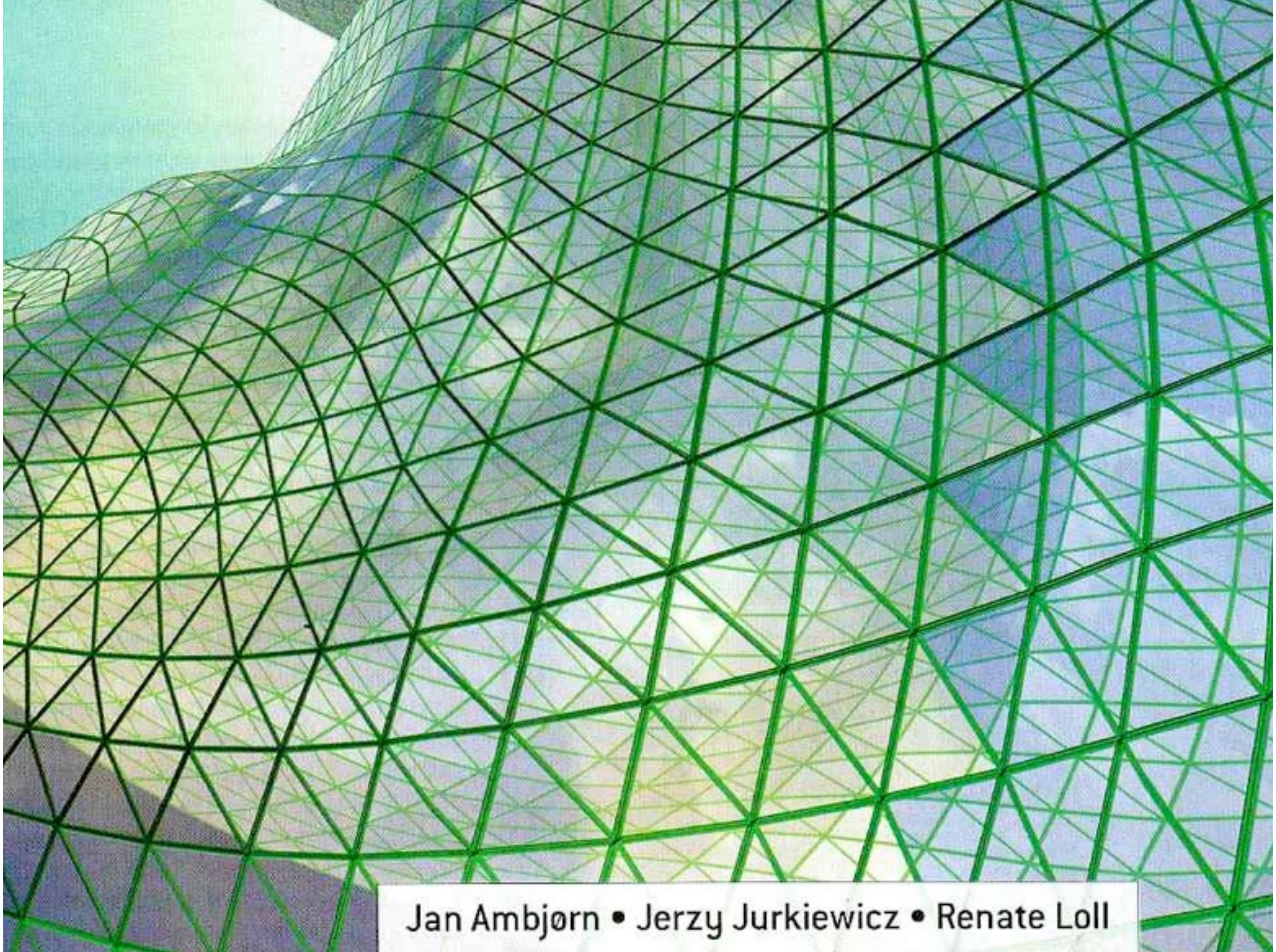


R. Loll

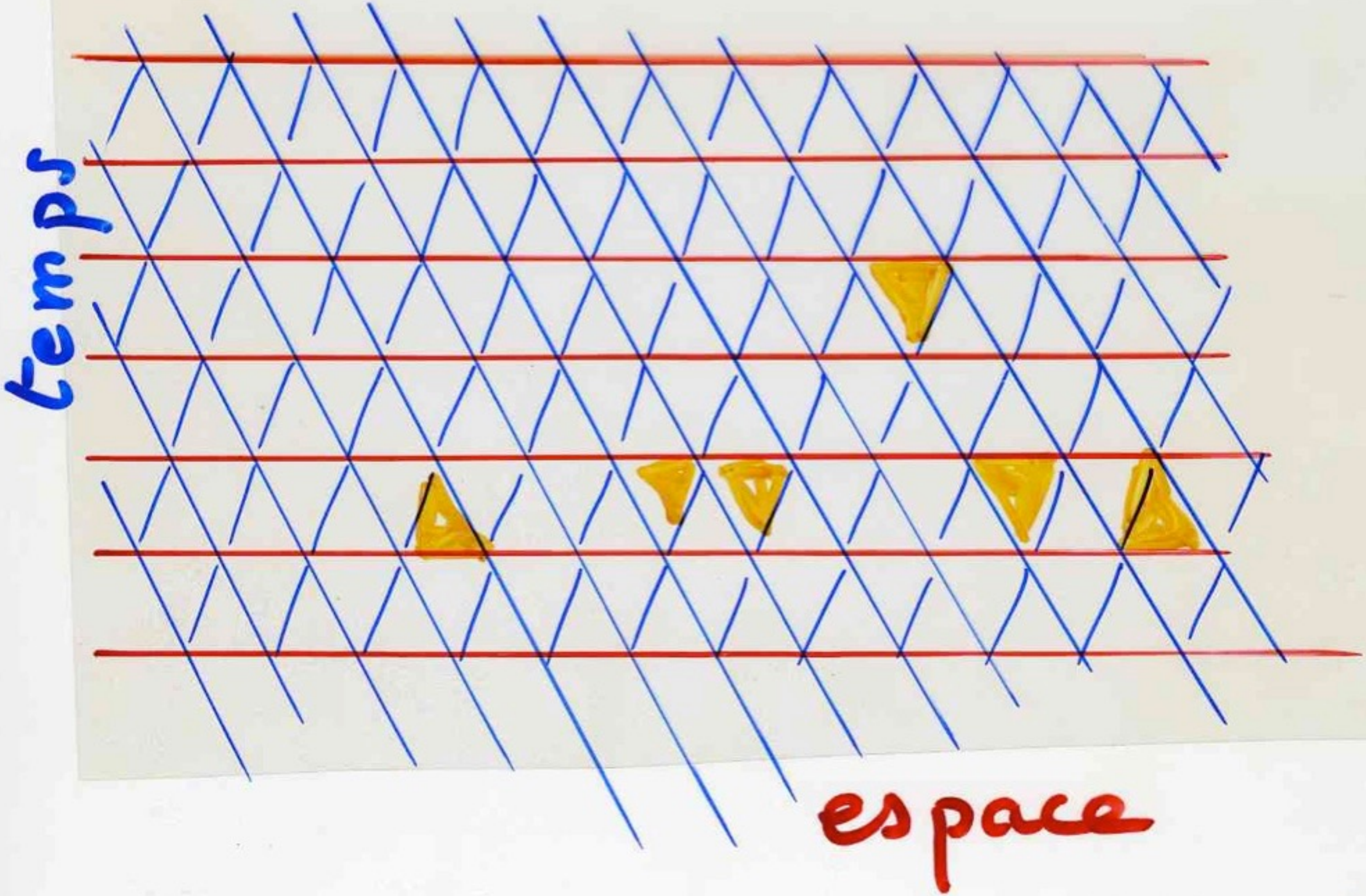
P. Di Francesco

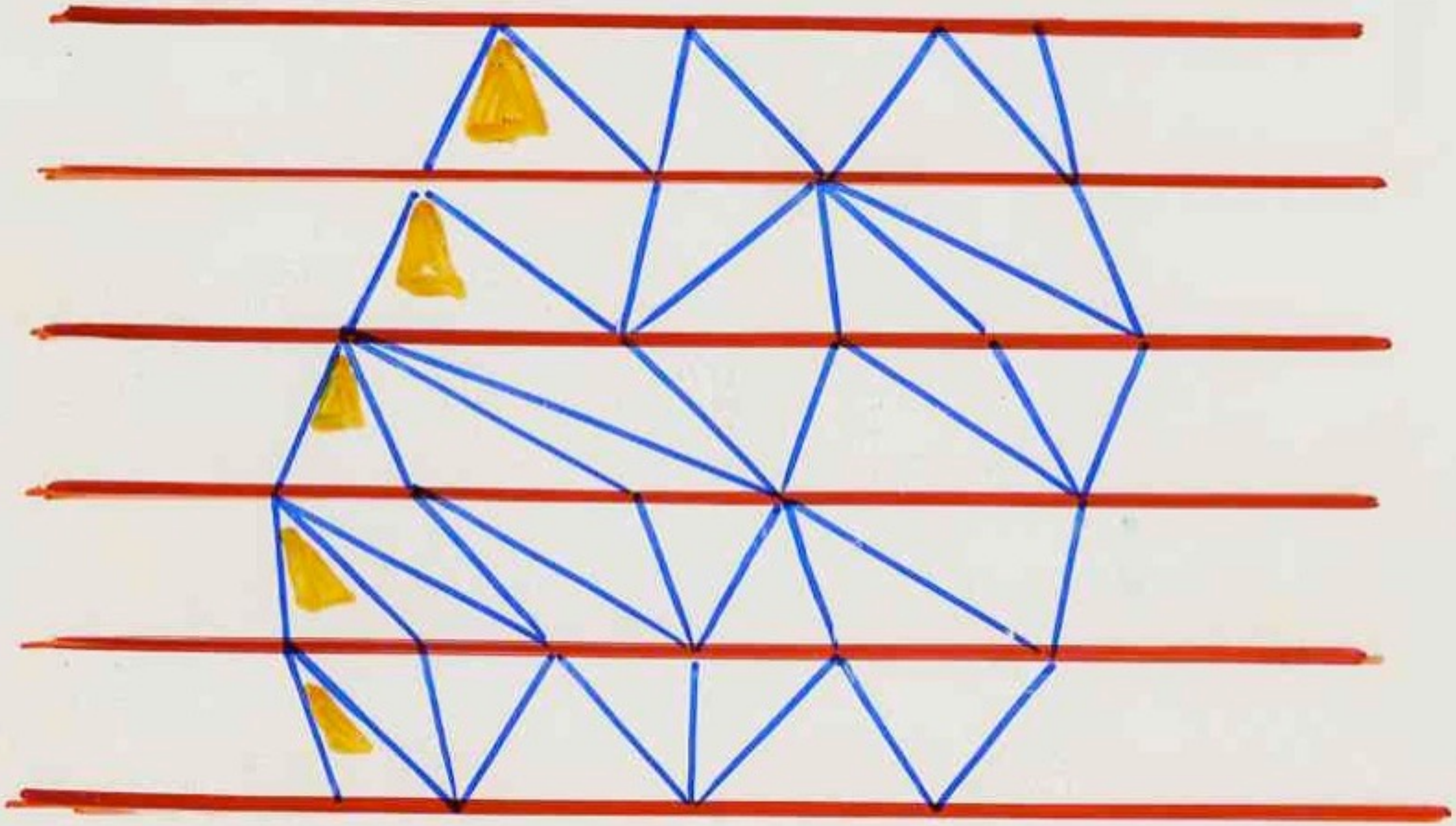
E.Guitter

C. Kristjansen



Jan Ambjørn • Jerzy Jurkiewicz • Renate Loll

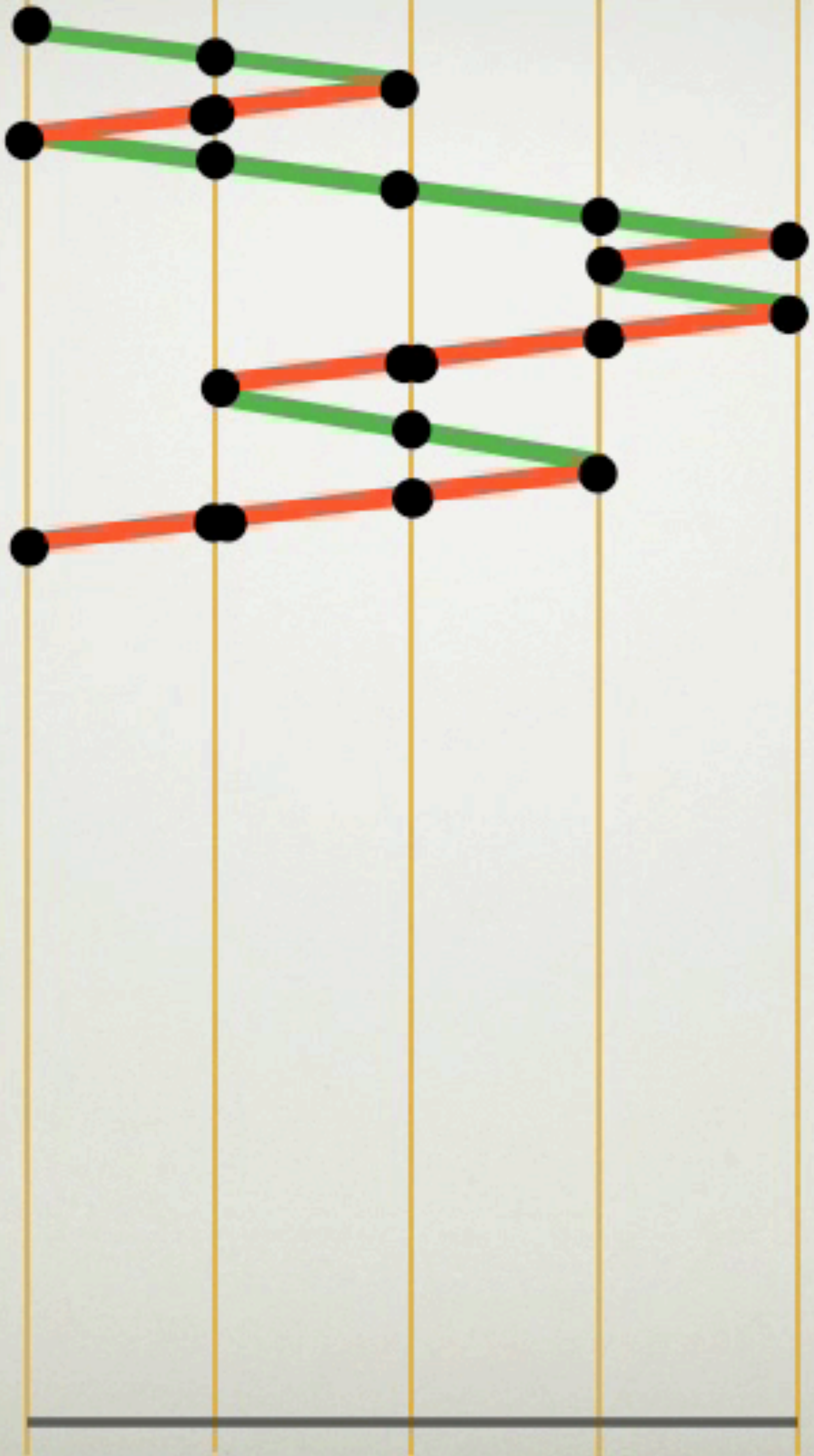

temps
espace



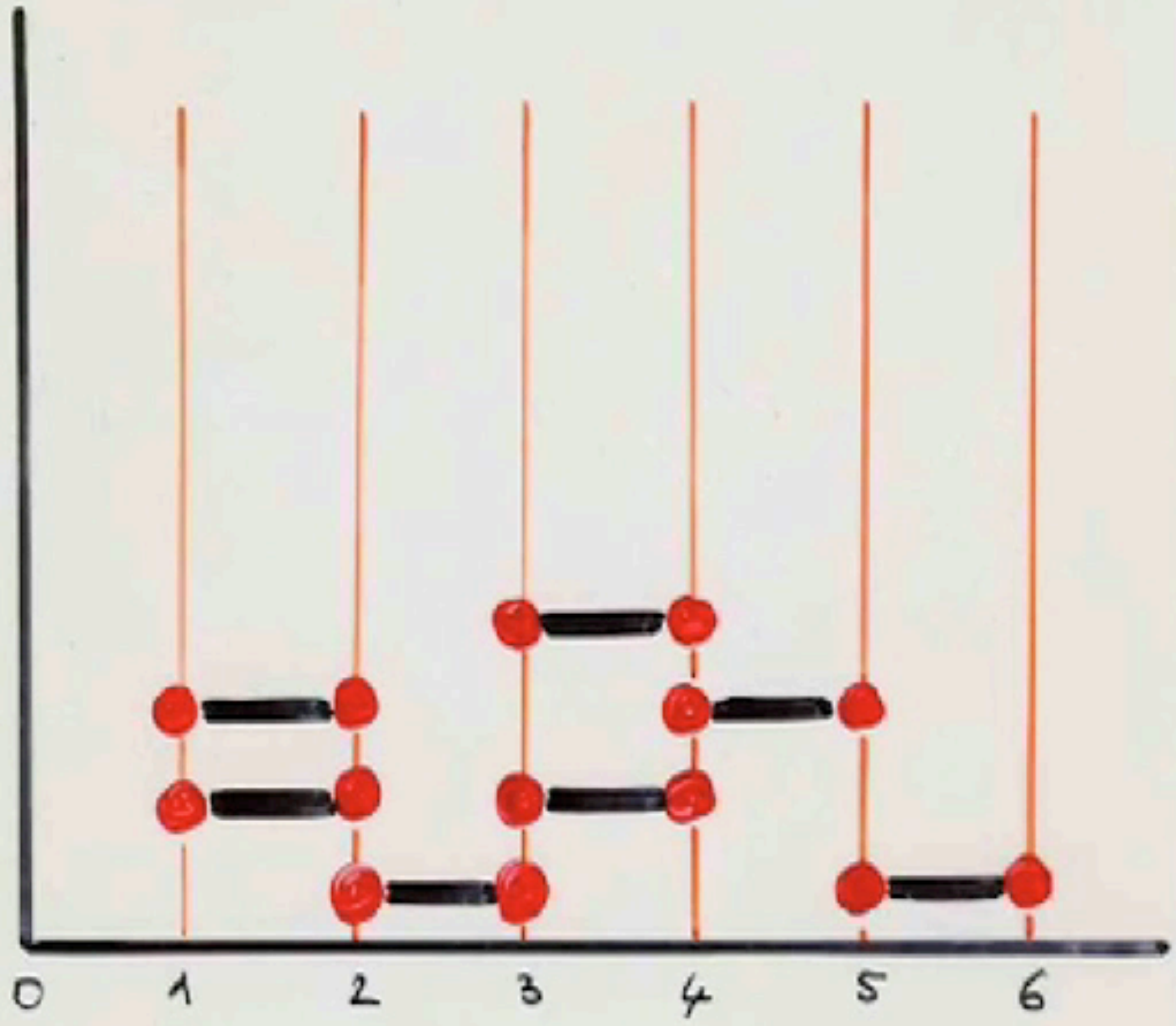
Catalan number !

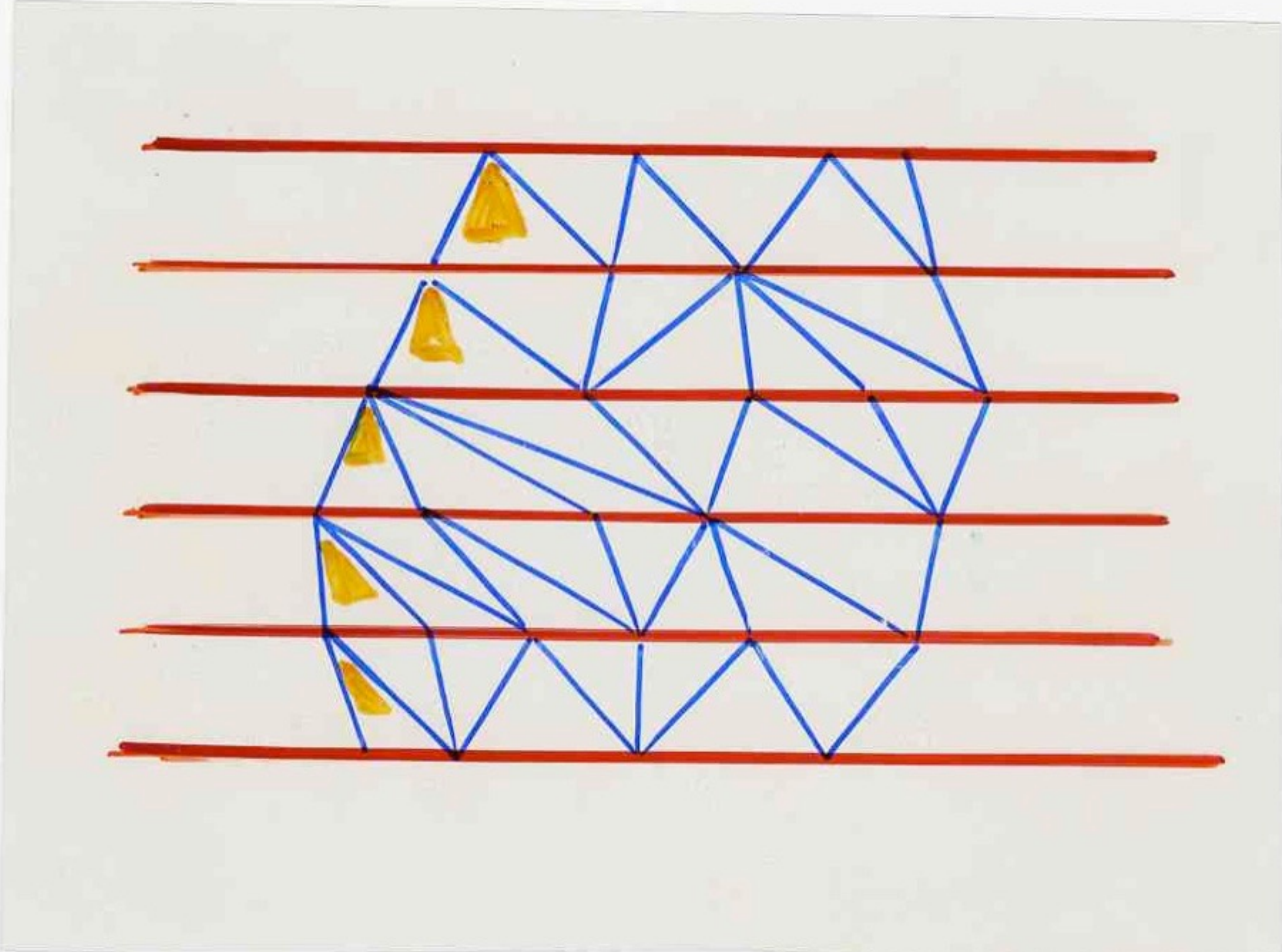
$$C_n = \frac{1}{(n+1)} \binom{2n}{n}$$

from Dyck paths
to heaps of dimers



From heaps of dimers to Lorentzian triangulations


0
1
2
3
4
5
6



metamorphosis:

Euler triangulations

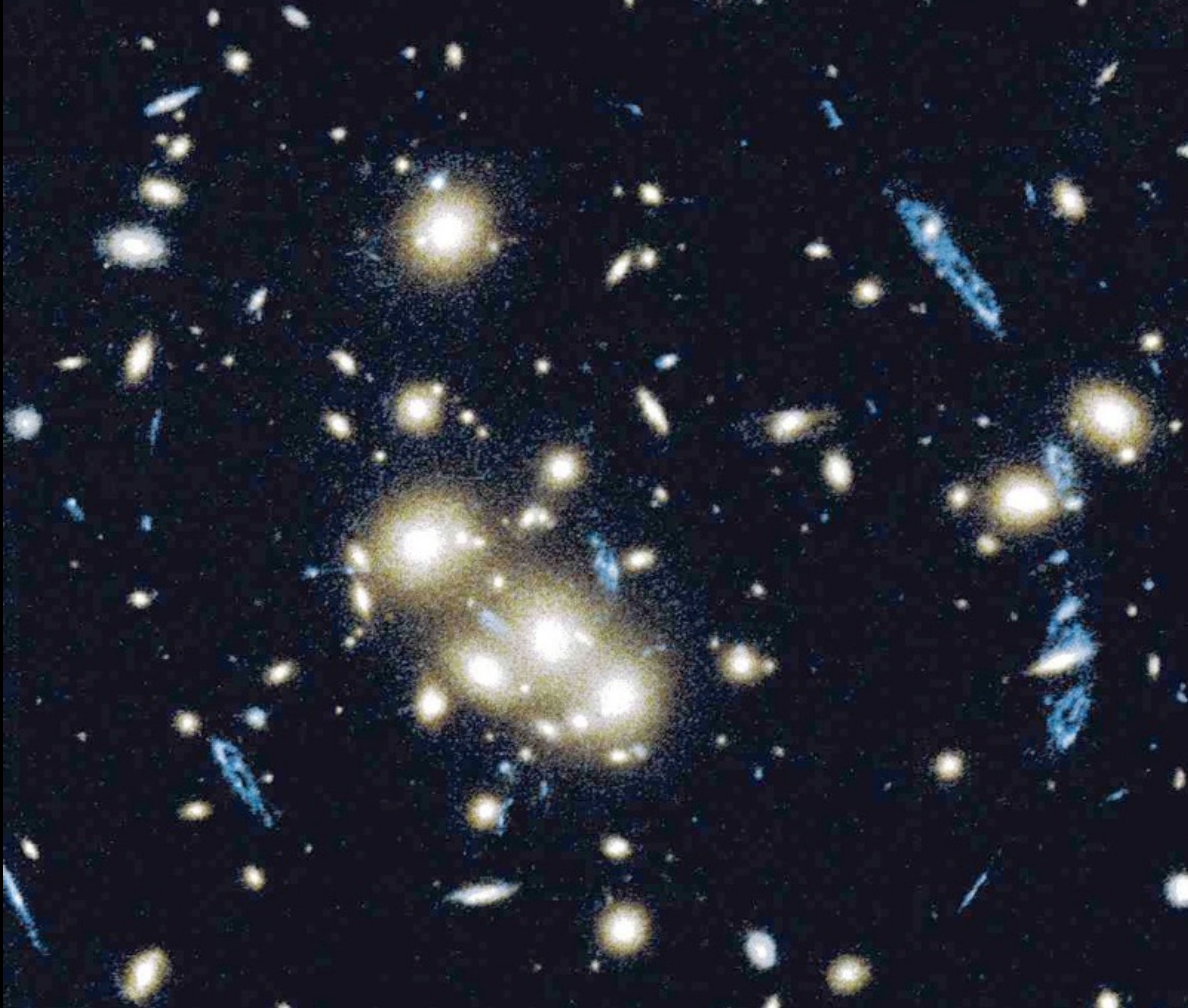
binary trees

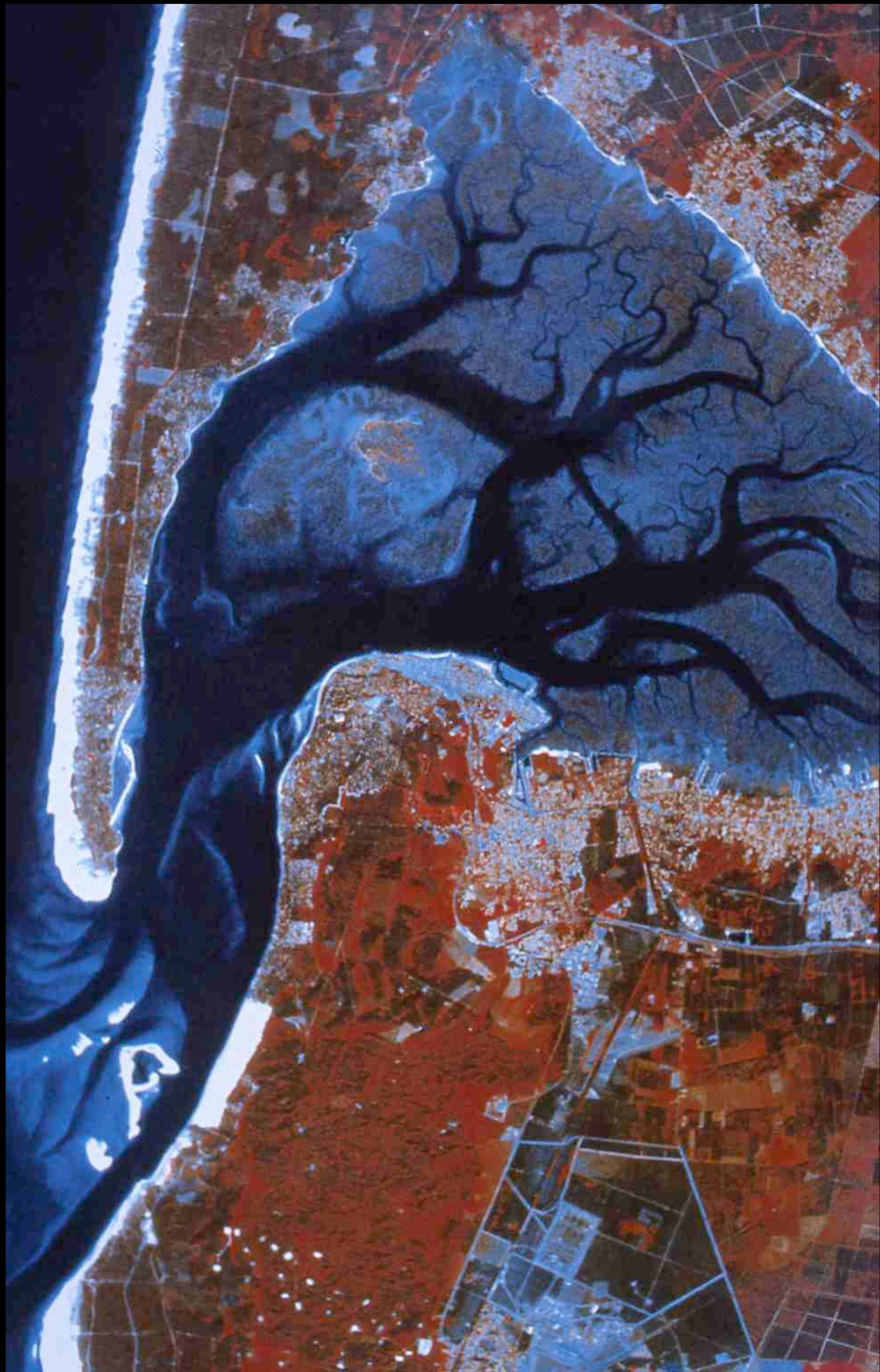
Dyck paths

heaps of dimers

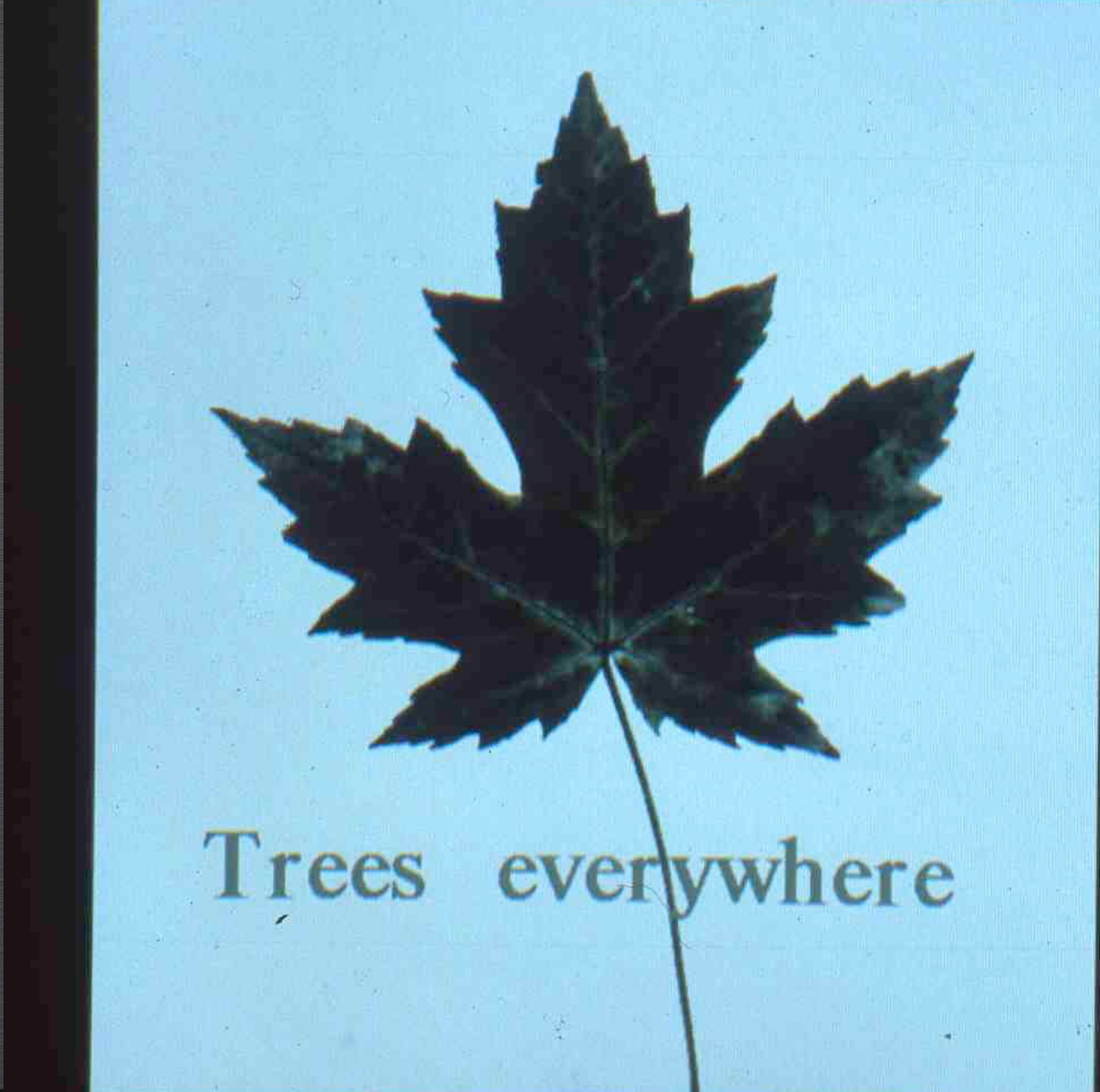
Lorentzian triangulations

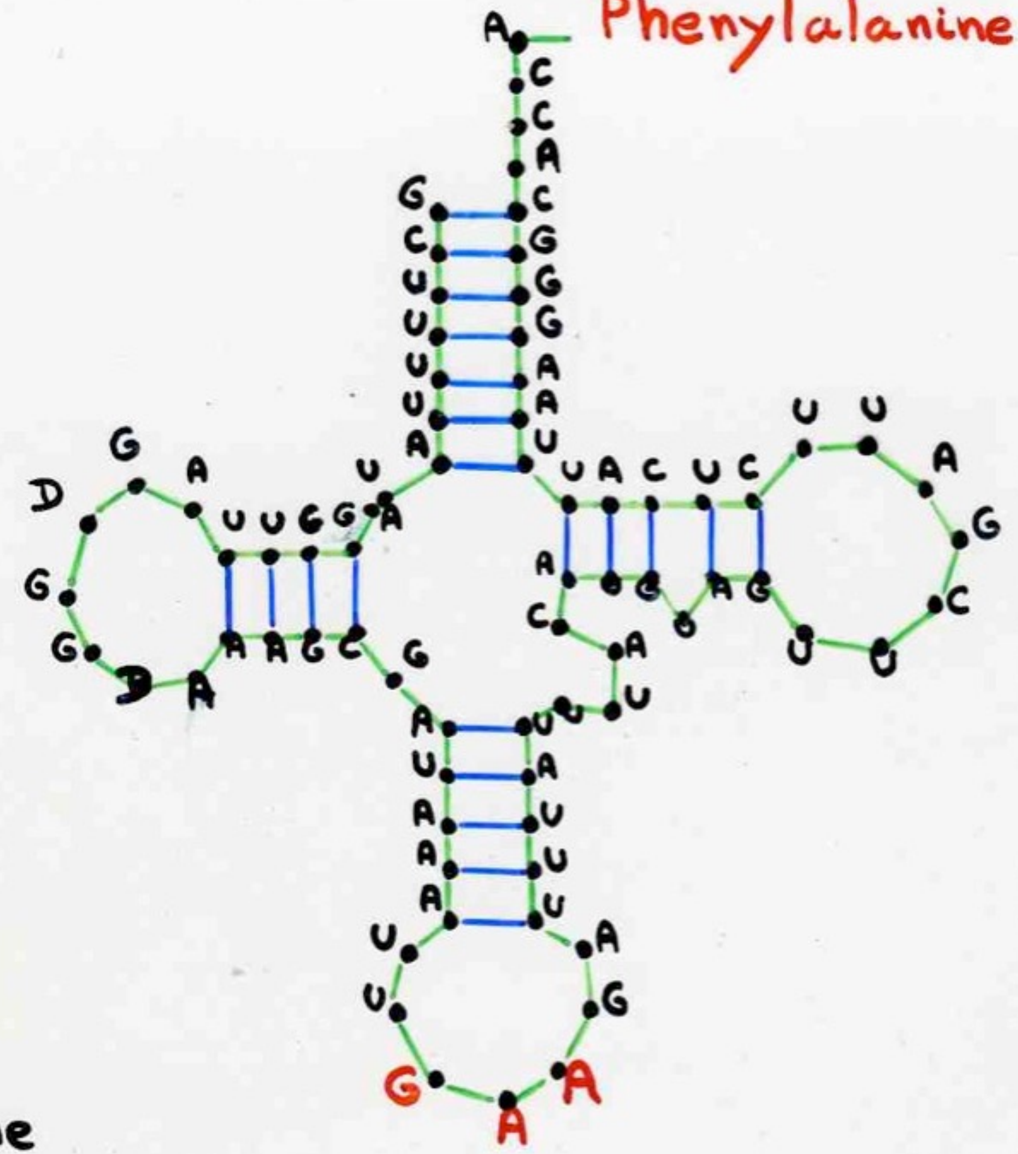
Epilogue





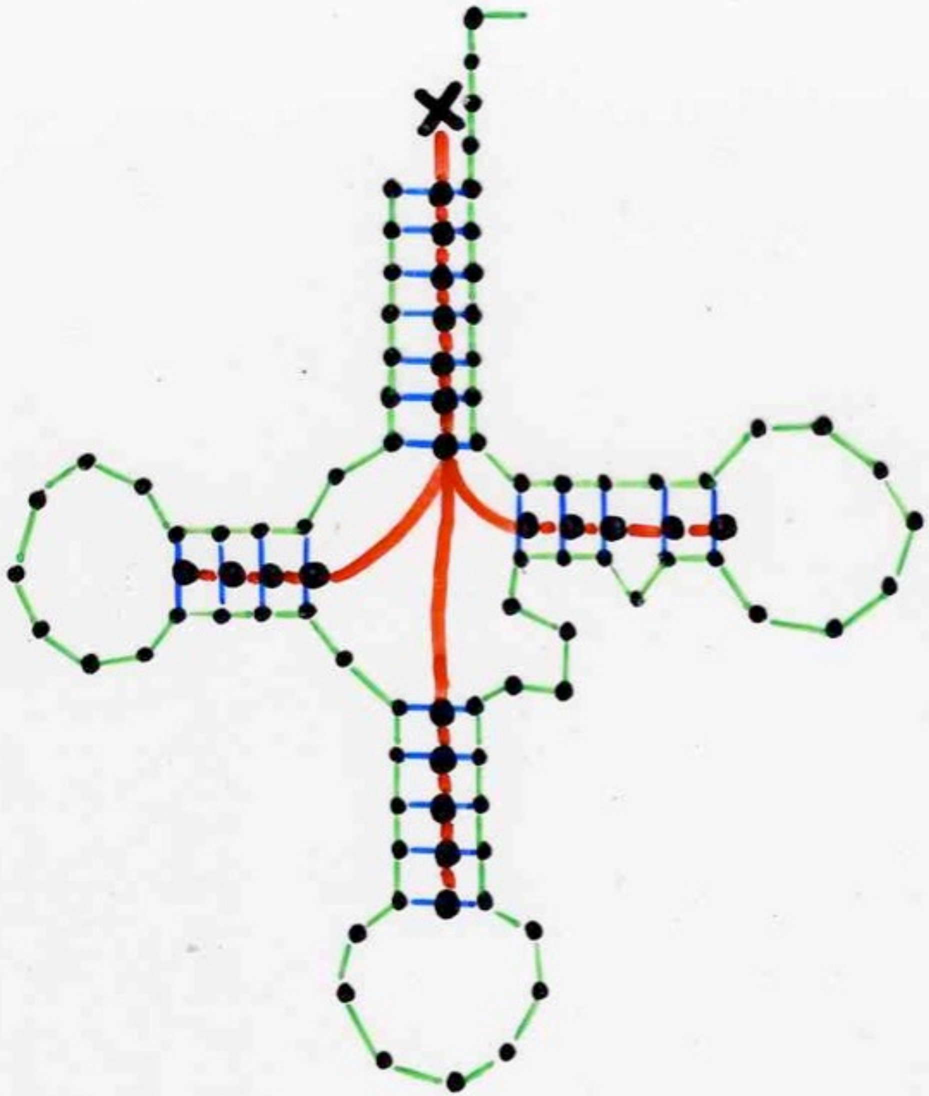

National Geographic


Trees everywhere

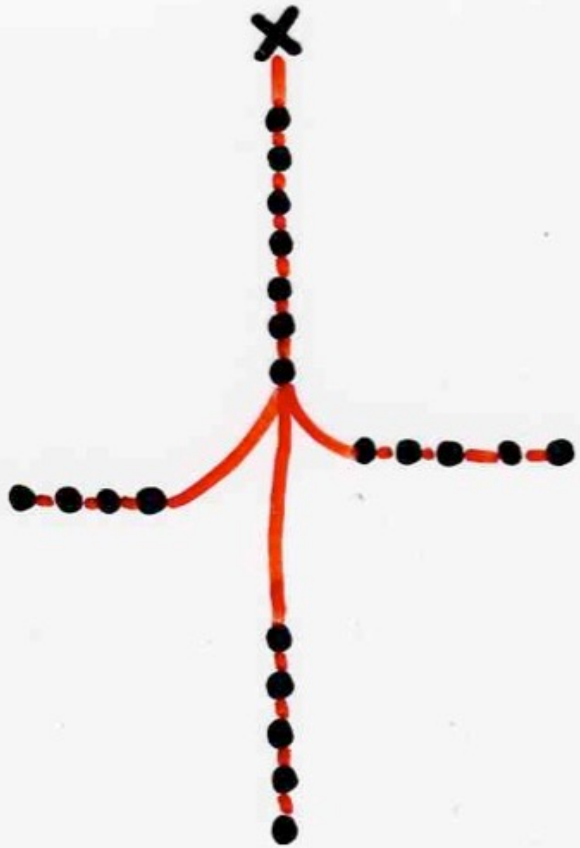


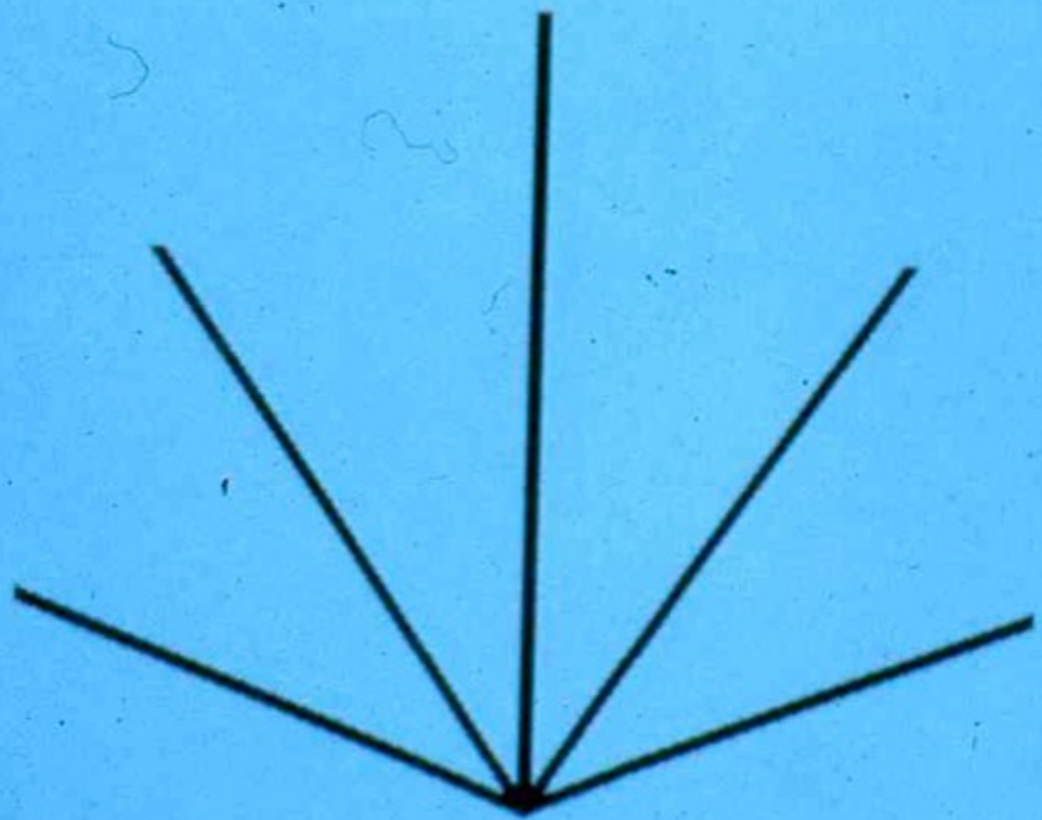
Adenine
Uracyle
Guanine
Cytosine

tARN ^{Phe}



$tARN^{Phe}$





Il y a des arbres dans les étoiles,
des arbres dans les grains de lumière.

There are trees in the stars
trees in the particles of lights.

Les théories mathématiques s’interpellent,
s’entrecroisent, renaissent, se fondent entre elles.

Mathematical theories call each other,
intercross, are born again, merge in themselves.

Les grands Maîtres se parlent à travers les siècles
dans le jardin merveilleux des Mathématiques.

The great Masters talk each other through
centuries in the wonderful garden of mathematics.


The end
thank you everyone !

space-time text:
Marcia Pig Lagos

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Gérard H.E. Duchamp
Mariette Freudentheil

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Association
Cont'Science

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ॐ सरस्वत्यै नमः।