Some complements

This set of slides is in complement of the talk

"Alternative tableaux, permutations and partially asymmetric exclusion process"

given at the Isaac Newton Institute, Cambridge, 23 April 2008.

v1: 26 May 2008

v2: 4 June 2008

xavier viennot

LaBRI, CNRS

Université Bordeaux 1

Some of these complementary slides comes from the preliminary slides of a series of 3 talks introducing alternative tableaux given in Bordeaux at the combinatorial "groupe de travail" , 1, 8, 15 February 2008. Other slides has been added later, following some electronic communications with L. Williams and oral discussions with P. Nadeau and Olivier Bernardi at ESI in Vienna. Many thanks to Olivier and Lauren and particularly to Philippe.

In particular we give the main ideas which are behind the proof of the fact that the two bijections between alternative tableaux and permutations given by the "exchange-delete" algorithm and by the "local rules" (derived from a representation of the operators D, E with commutation relation DE = ED + E + D), are the same, up to a change of the permutation into its inverse.

P. Nadeau notice that , the (first) bijection described by him and S.Corteel (to be published in Electronic J. of Combinatorics) between permutation tableaux and permutations, is equivalent to a "column insertion" in the algorithm presented here with "local rules", up to transforming permutation tableaux into alternating tableaux and taking complements mirror image of the permutation constructed by "local rules" (which is the inverse of the permutation used in the "exchange-delete" algorithm).

The bijection presented at Tienjin FPSAC'07 between binary trees and "Catalan permutation tableaux", once rewritten in term in terms of "Catalan alternating tableaux" (which is immediate to do), can be viewed as a particular case of the inverse of the "exchange-fusion" algorithm.

§1 The "exchange-fusion" algorithm

§2 Some Parameters

Peaks, Valleys, DR, DD and "q-Laguerre" histories

§3 alternative "jeu de taquin"

§4 (idea) of the proof of the main theorem $\sigma = \tau^{-1}$

§5 the "binary trees sliding algorithm" for "Catalan alternative tableaux"

§6 "Data structure histories" and operators D and E

§7 local RSK and geometric RSK

An alternative description of the bijection
alternative tableaux -- permutations

§1 The "exchange-fusion" algorithm

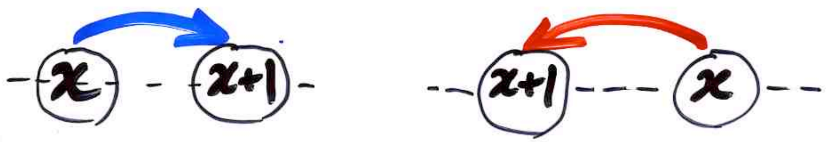
with the same example as for the "exchange-delete" algorithm:
$s = 7\,4\,3\,8\,2\,9\,5\,1\,6$, explanations at the end of the animation.

Def- Permutation $\sigma = \sigma(1) \cdots \sigma(n)$

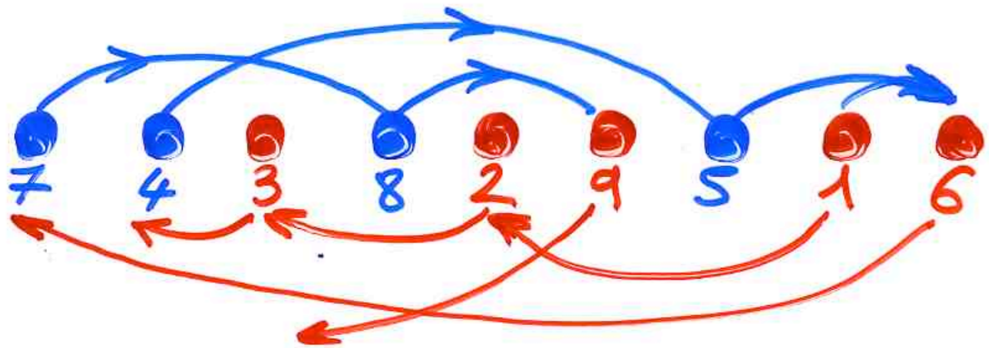
$x = \sigma(i)$, $1 \leq x < n$

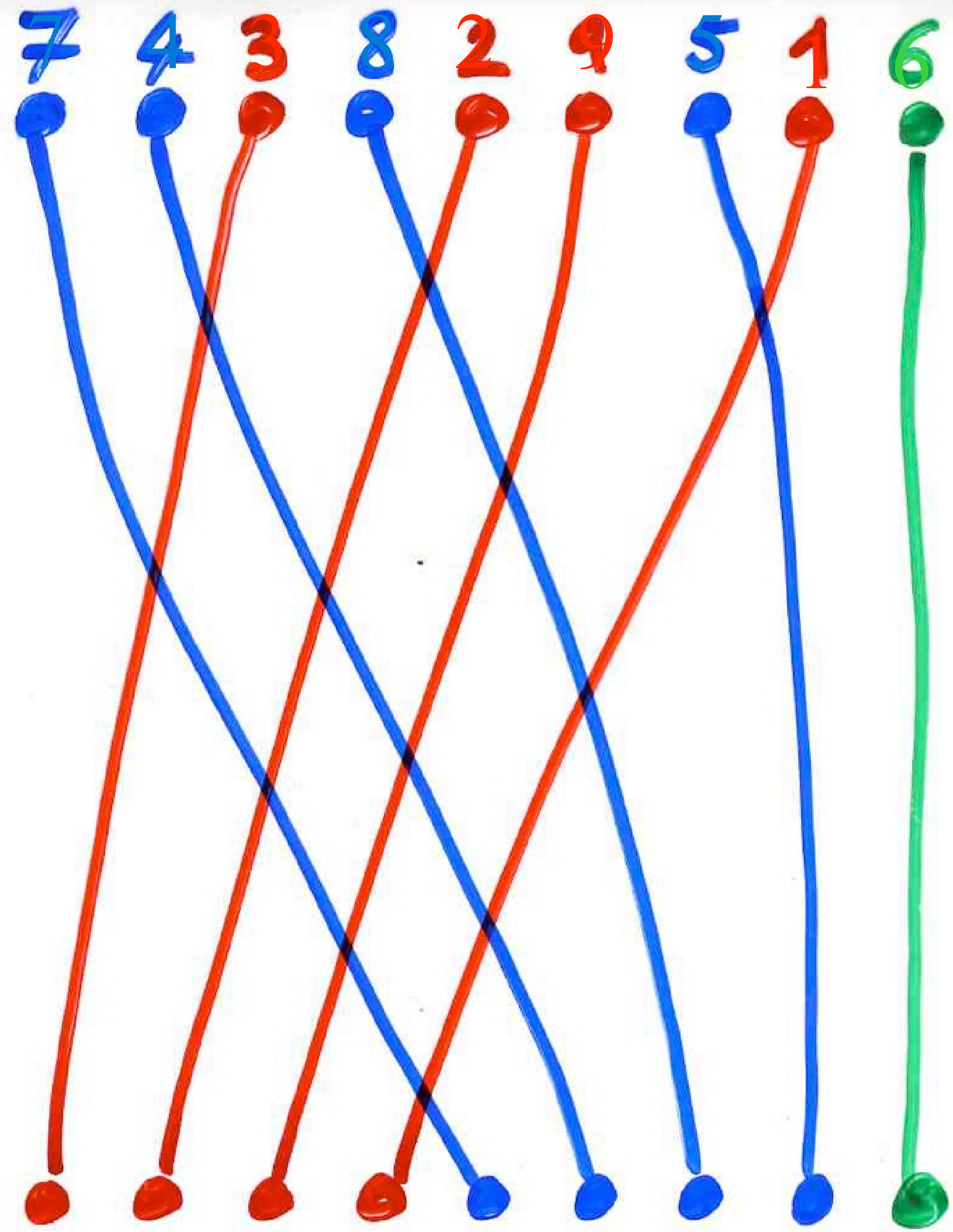
(valeur) x $\begin{cases} \text{avance} \\ \text{recul} \end{cases}$ $x+1 = \sigma(j)$, $\begin{cases} i<j \\ j<i \end{cases}$

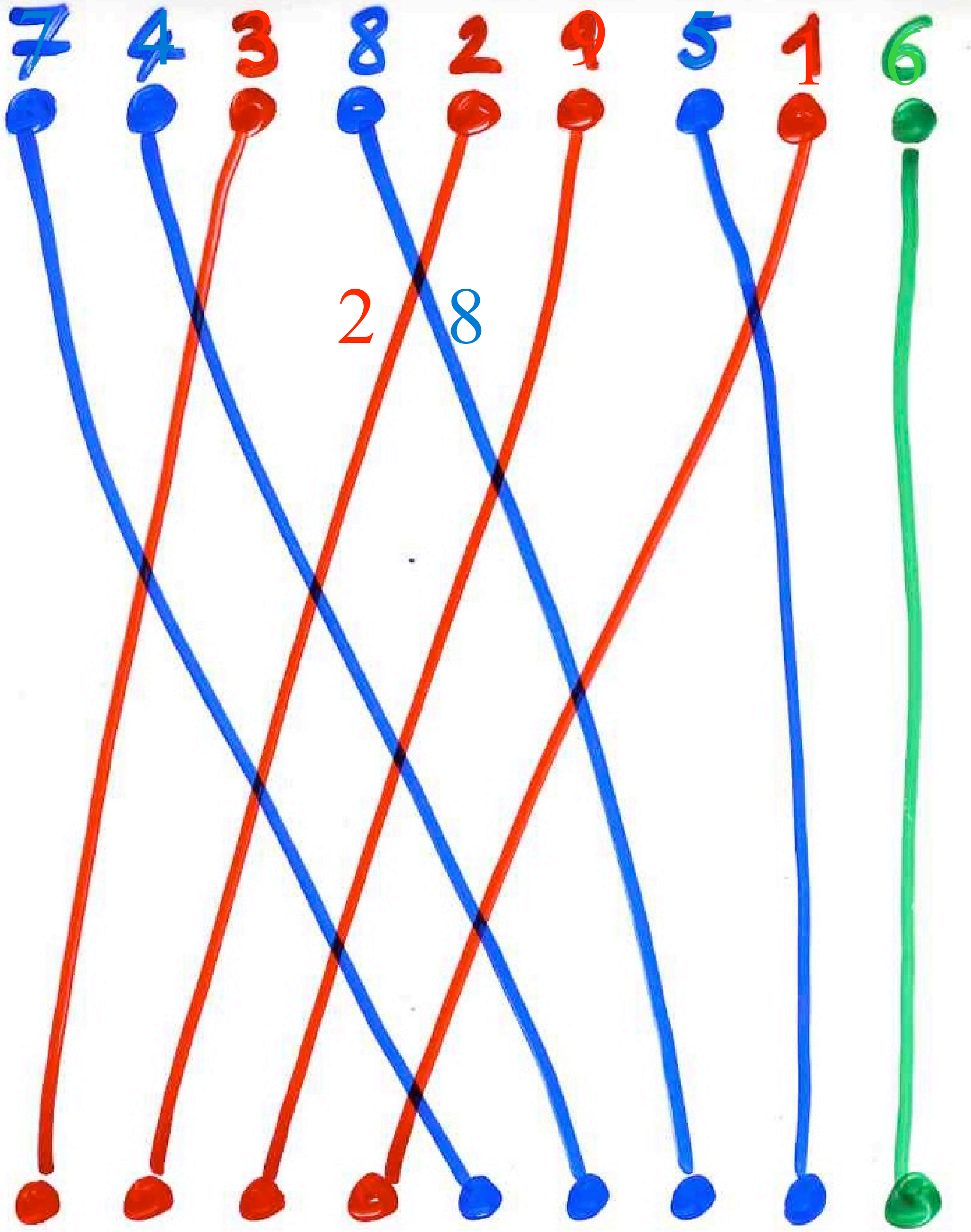
- convention $x = n$ est un recul



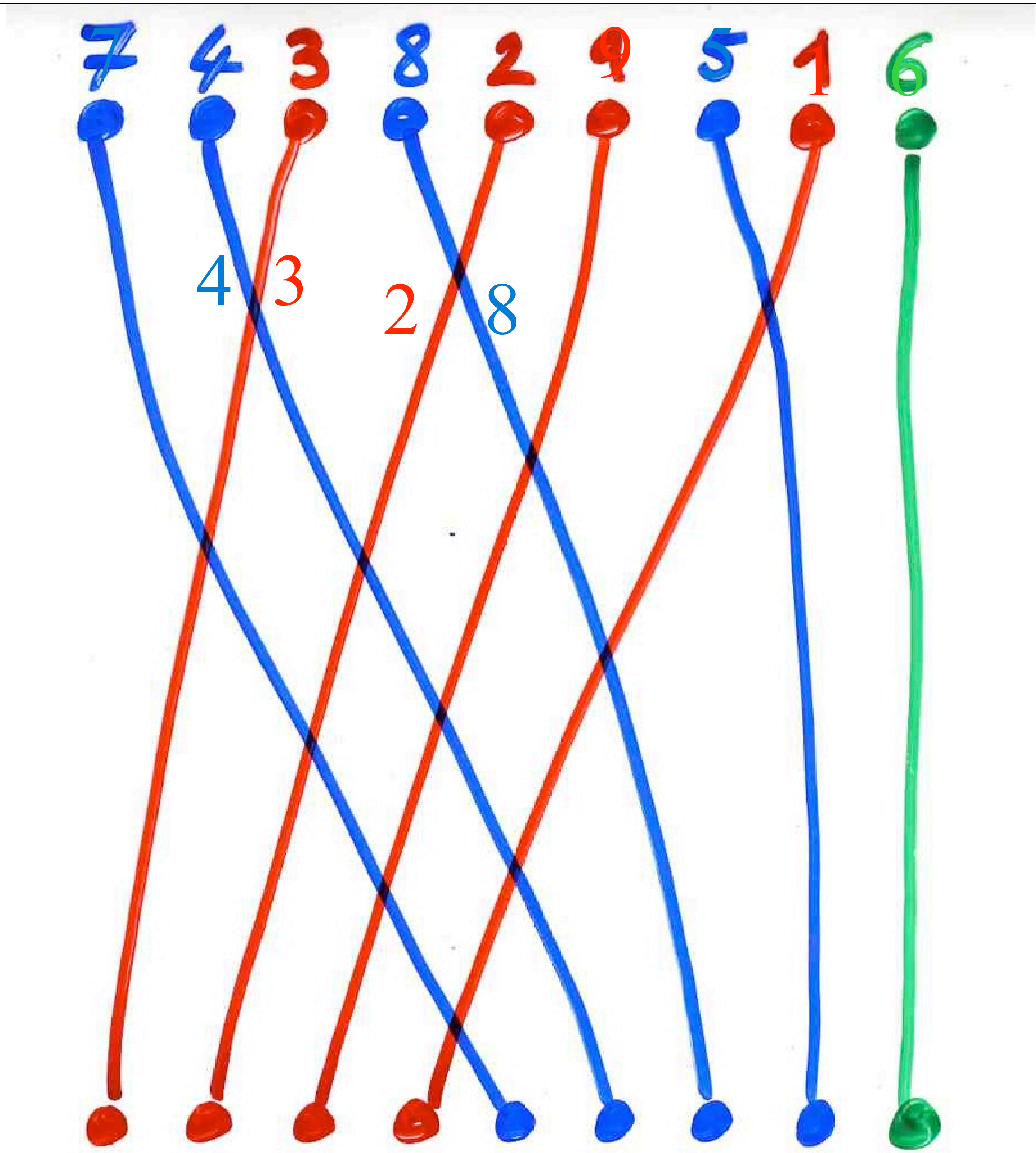
$\sigma = 7\ 4\ 3\ 8\ 2\ 9\ 5\ 1\ 6$

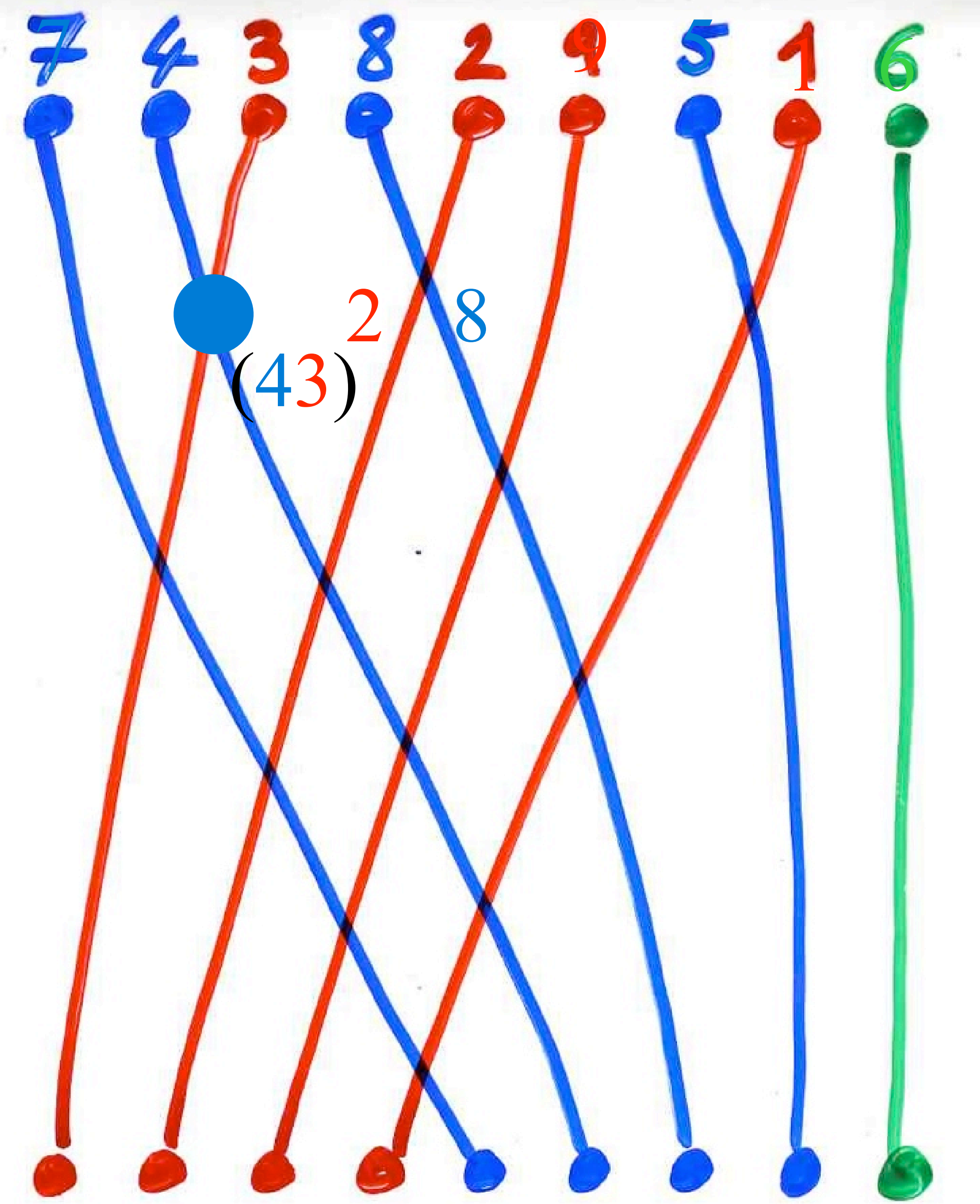


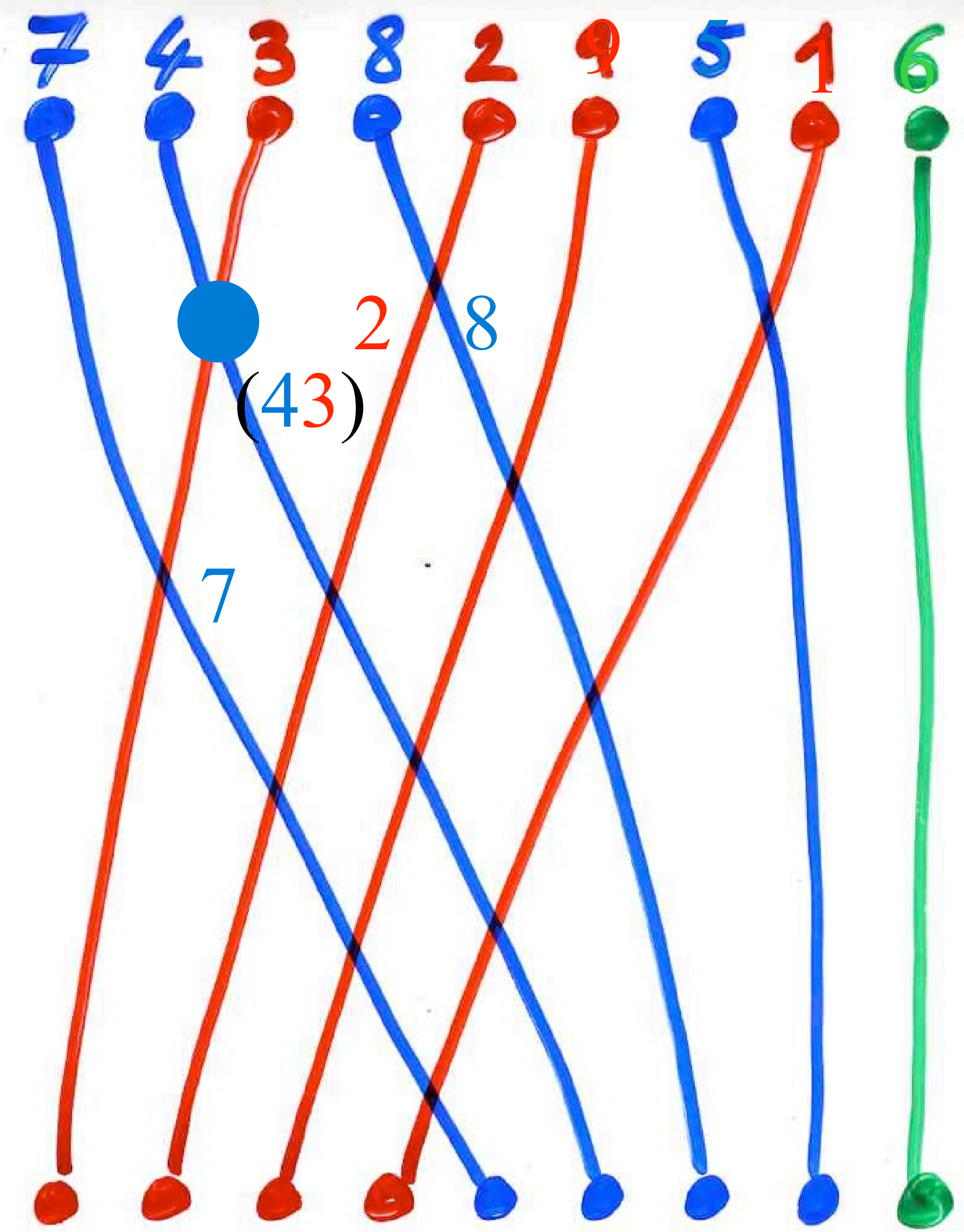

7 4 3 8 2 9 5 1 6



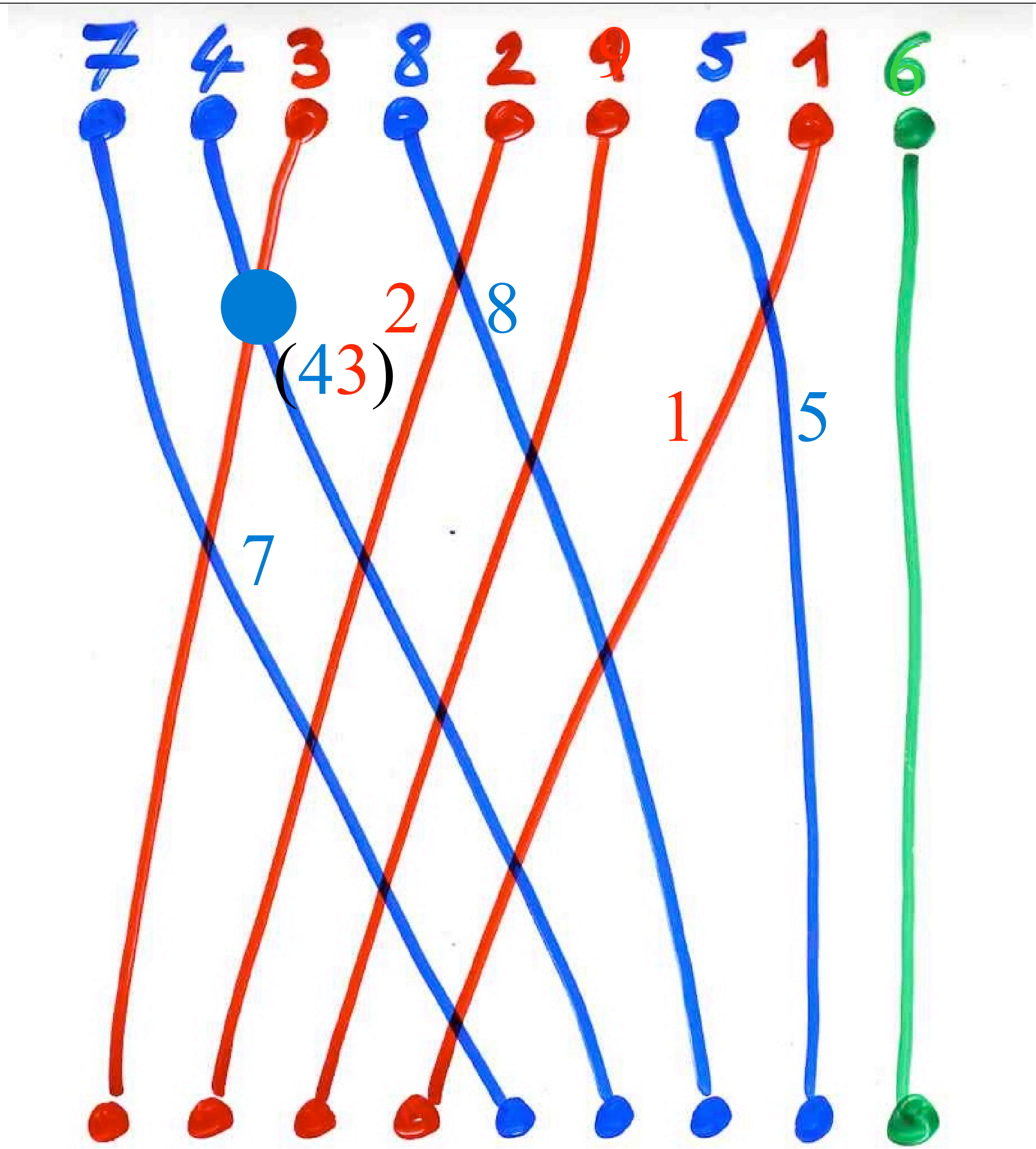
7 4 3 8 2 9 5 1 6
2 8


7 4 3 8 2 9 5 1 6
4 3
2 8

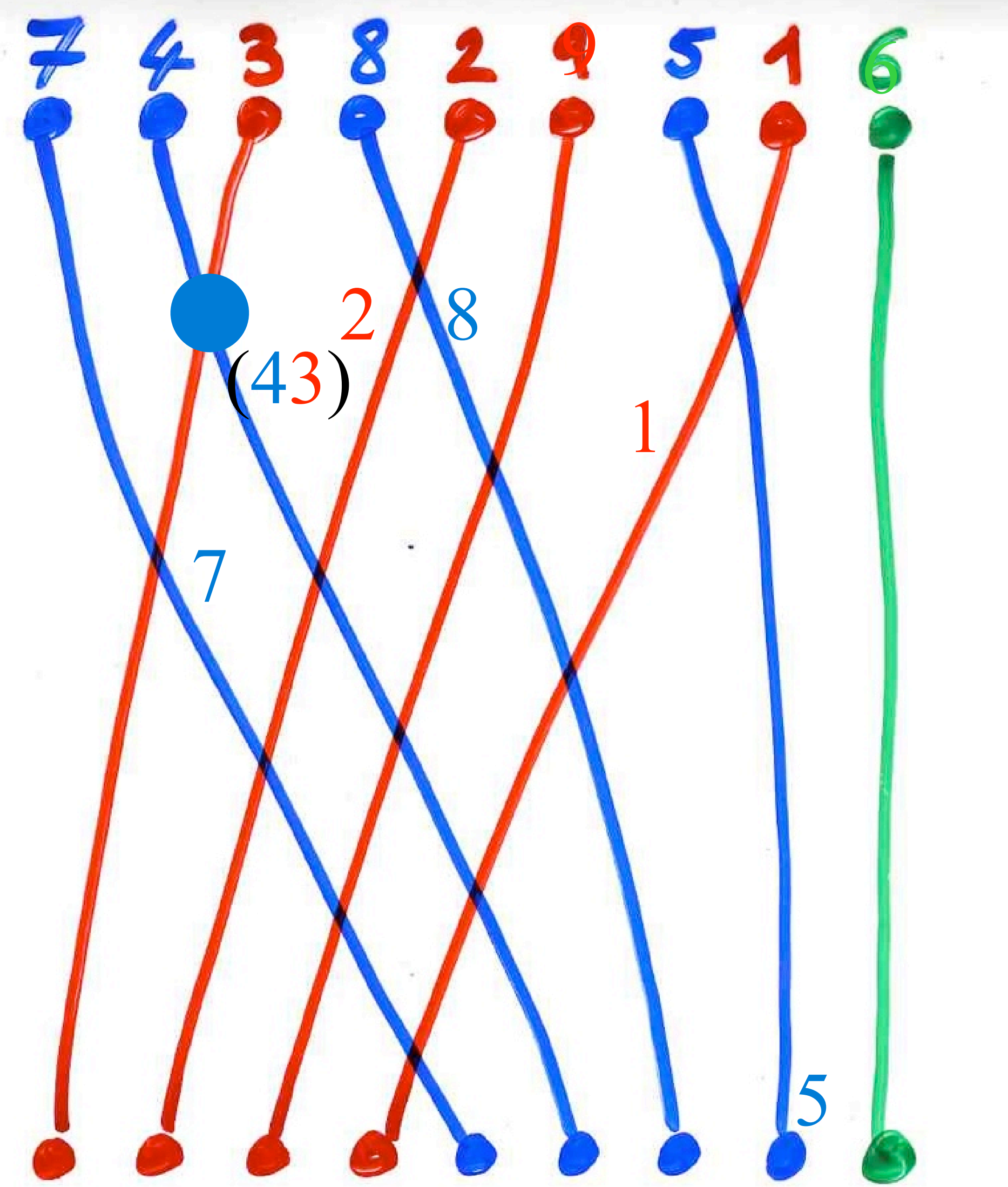

7 4 3 8 2 9 5 1 6
(43)
2
8



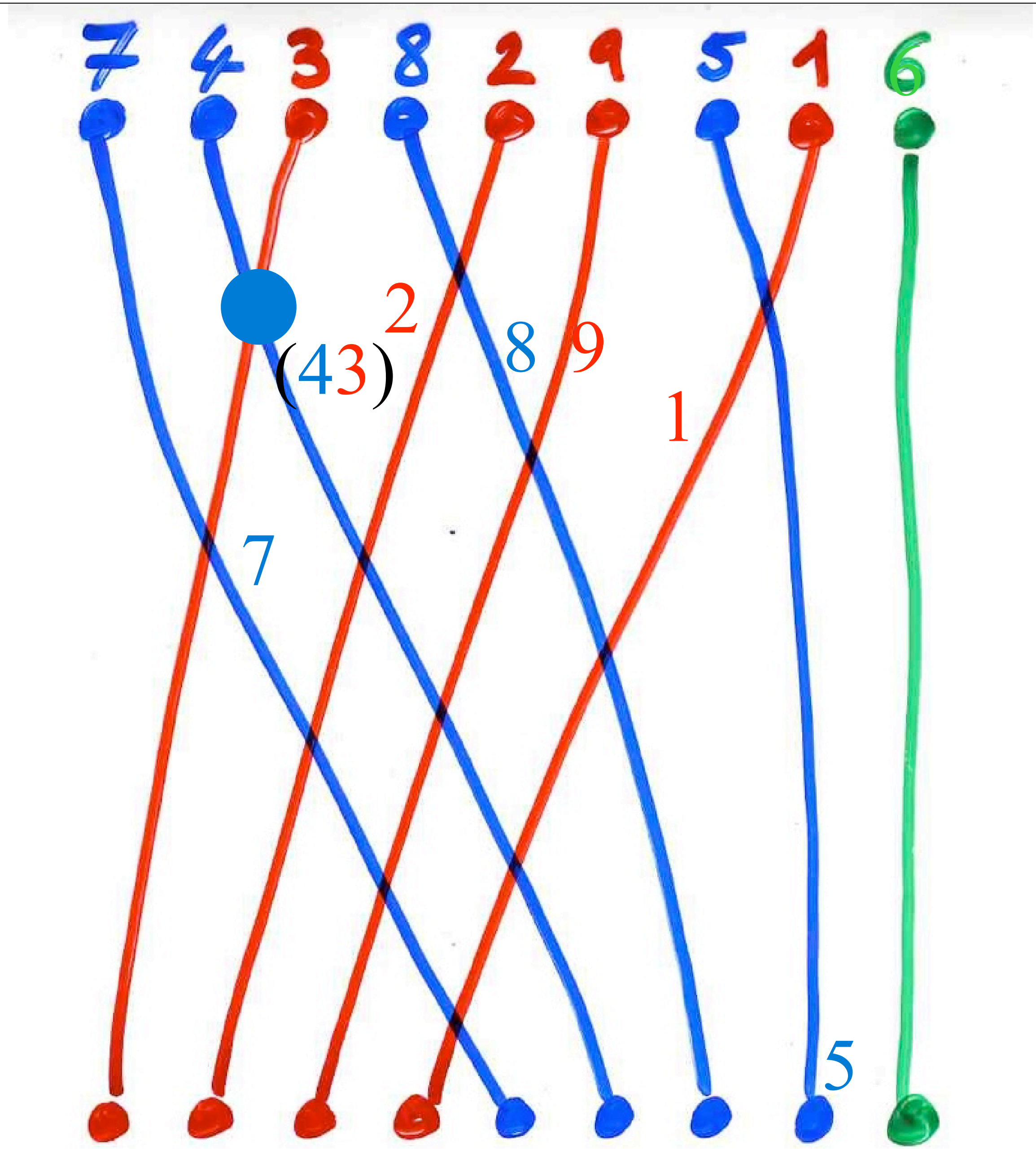
7 4 3 8 2 9 5 1 6
2
8
(43)
7

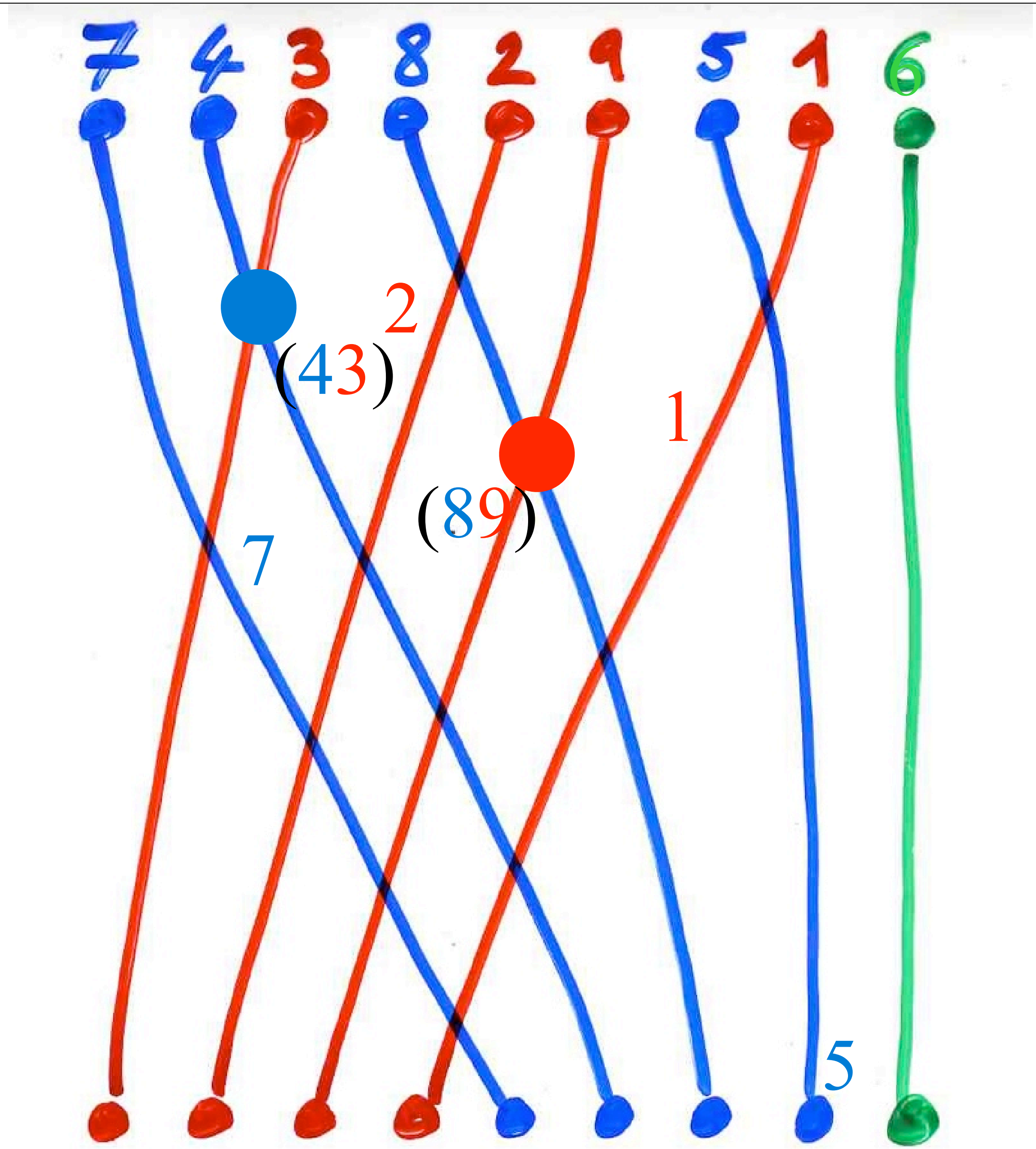


7 4 3 8 2 9 5 1 6
2
8
(43)
1
5
7

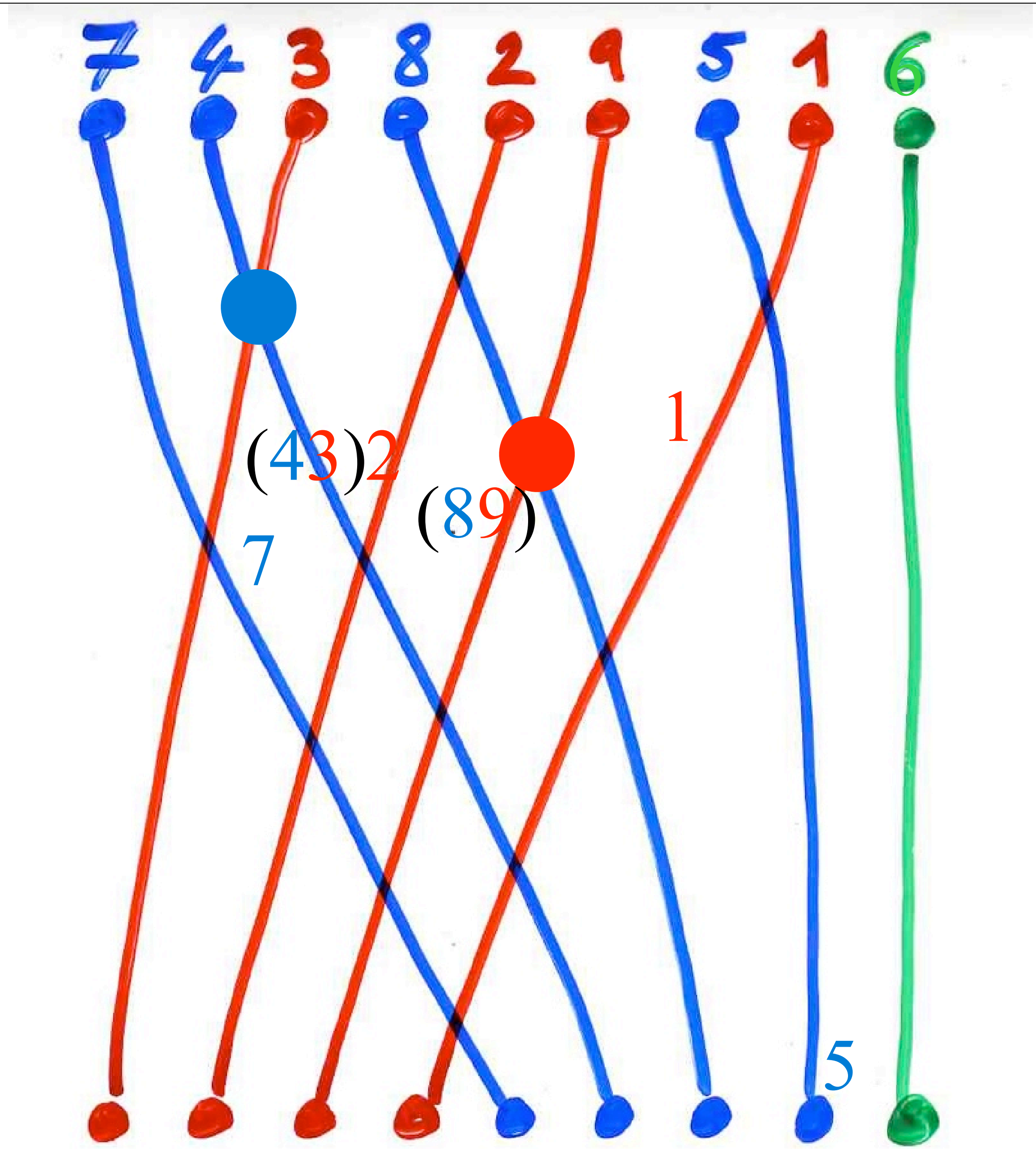


7 4 3 8 2 9 5 1 6
2
8
(43)
1
7
5

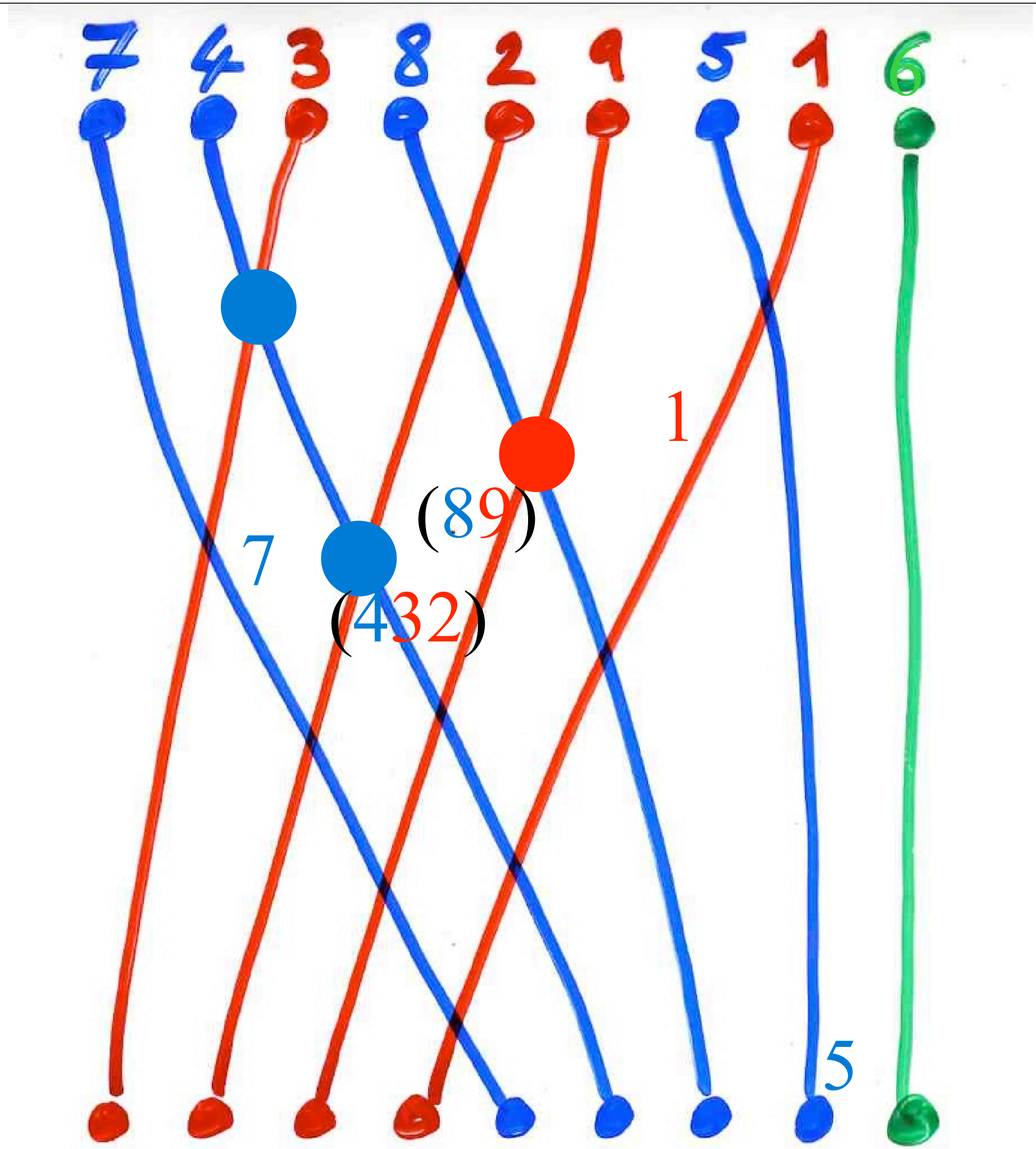

7 4 3 8 2 9 5 1 6
(43)
2
8
9
1
7
5



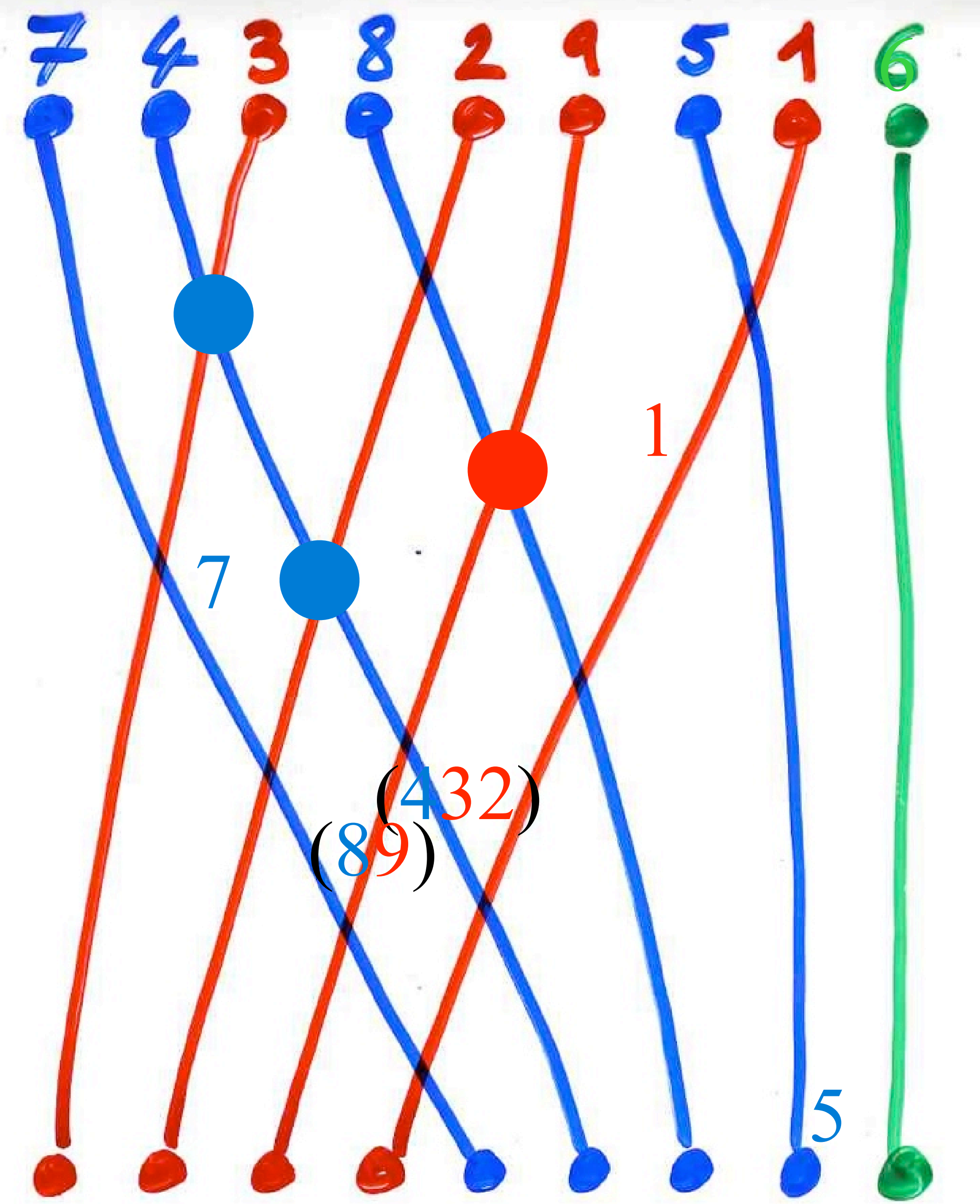
7 4 3 8 2 9 5 1 6
2
(43)
1
(89)
7
5

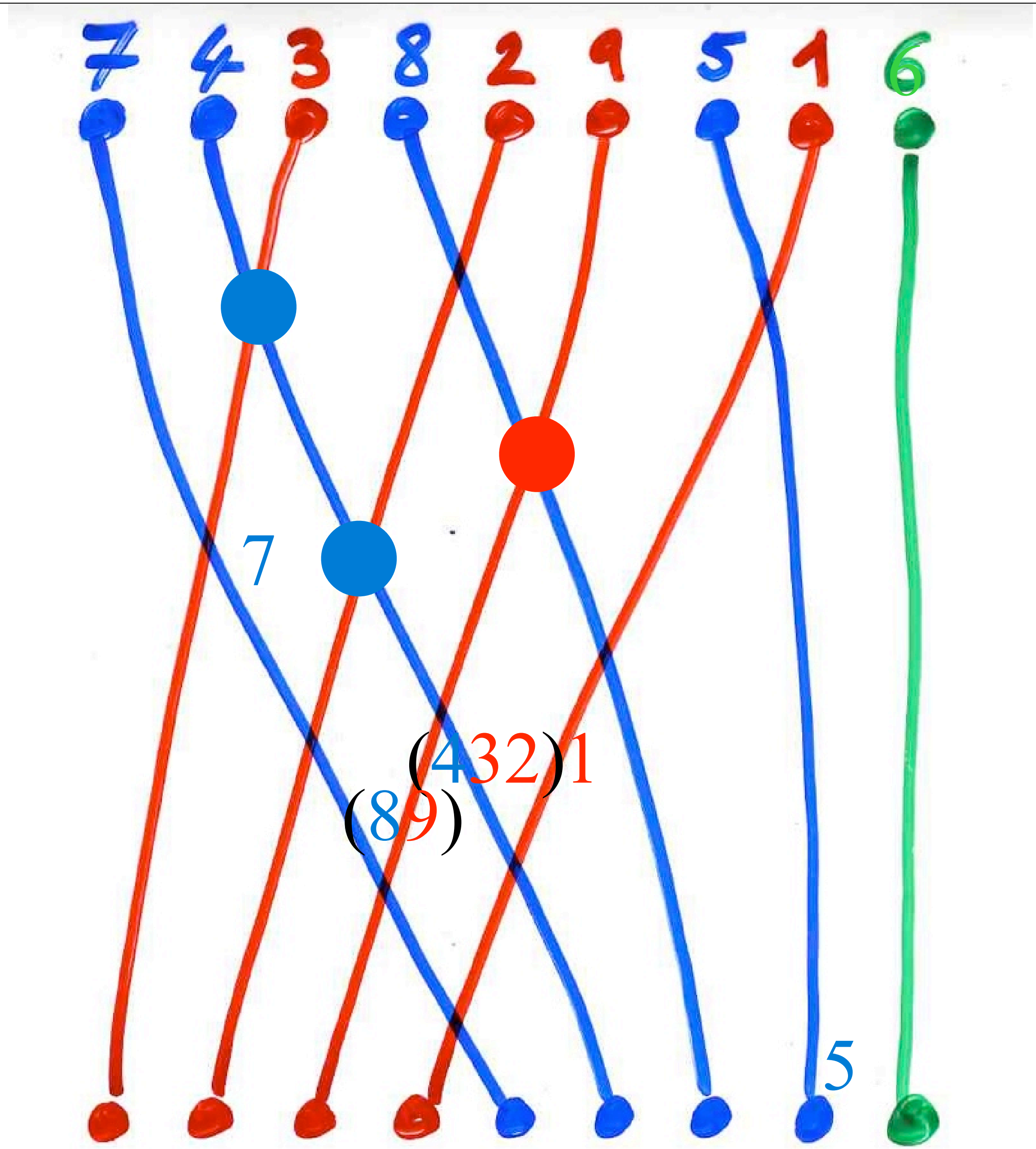


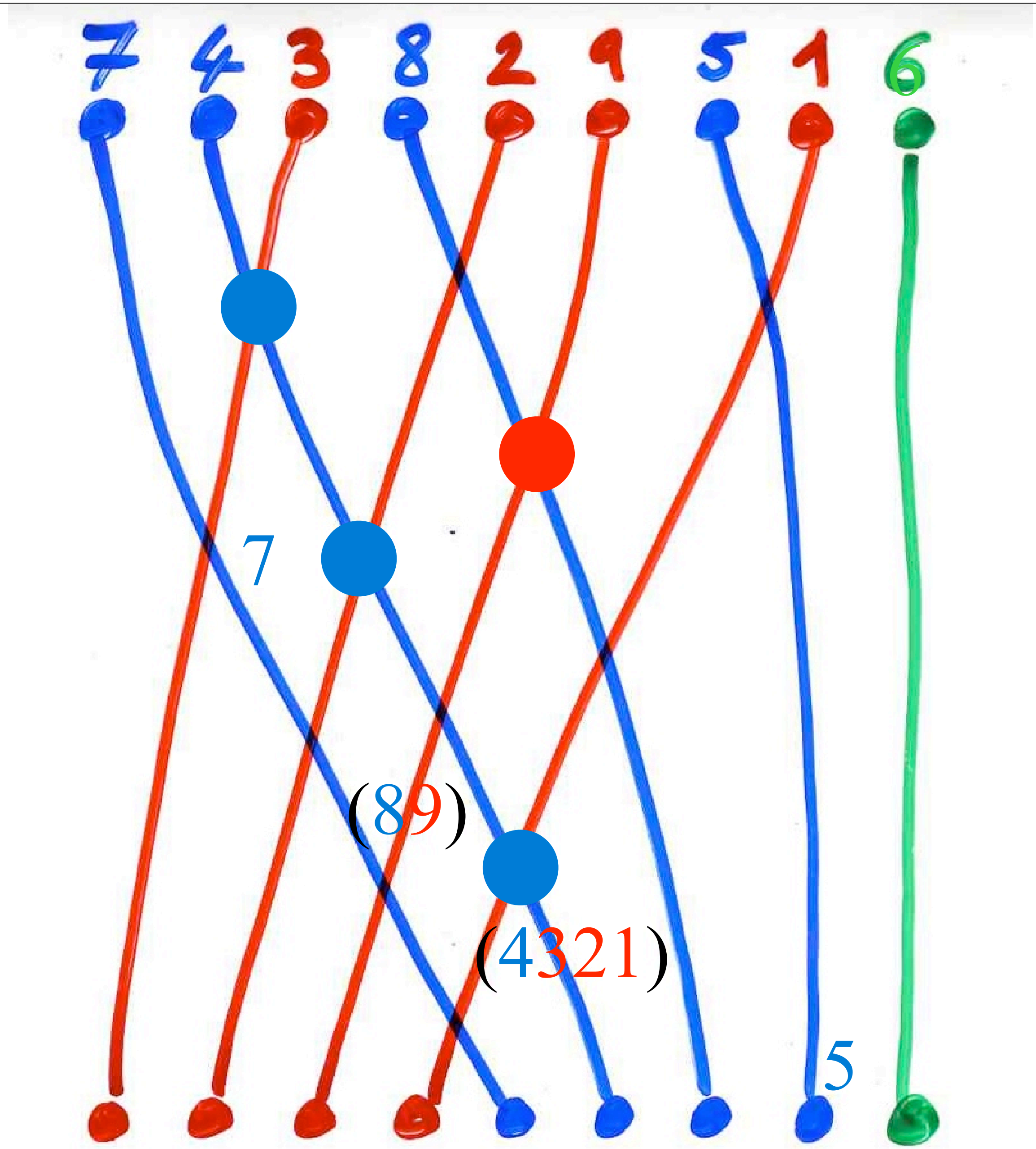
7 4 3 8 2 9 5 1 6
(43)2
(89)
1
7
5



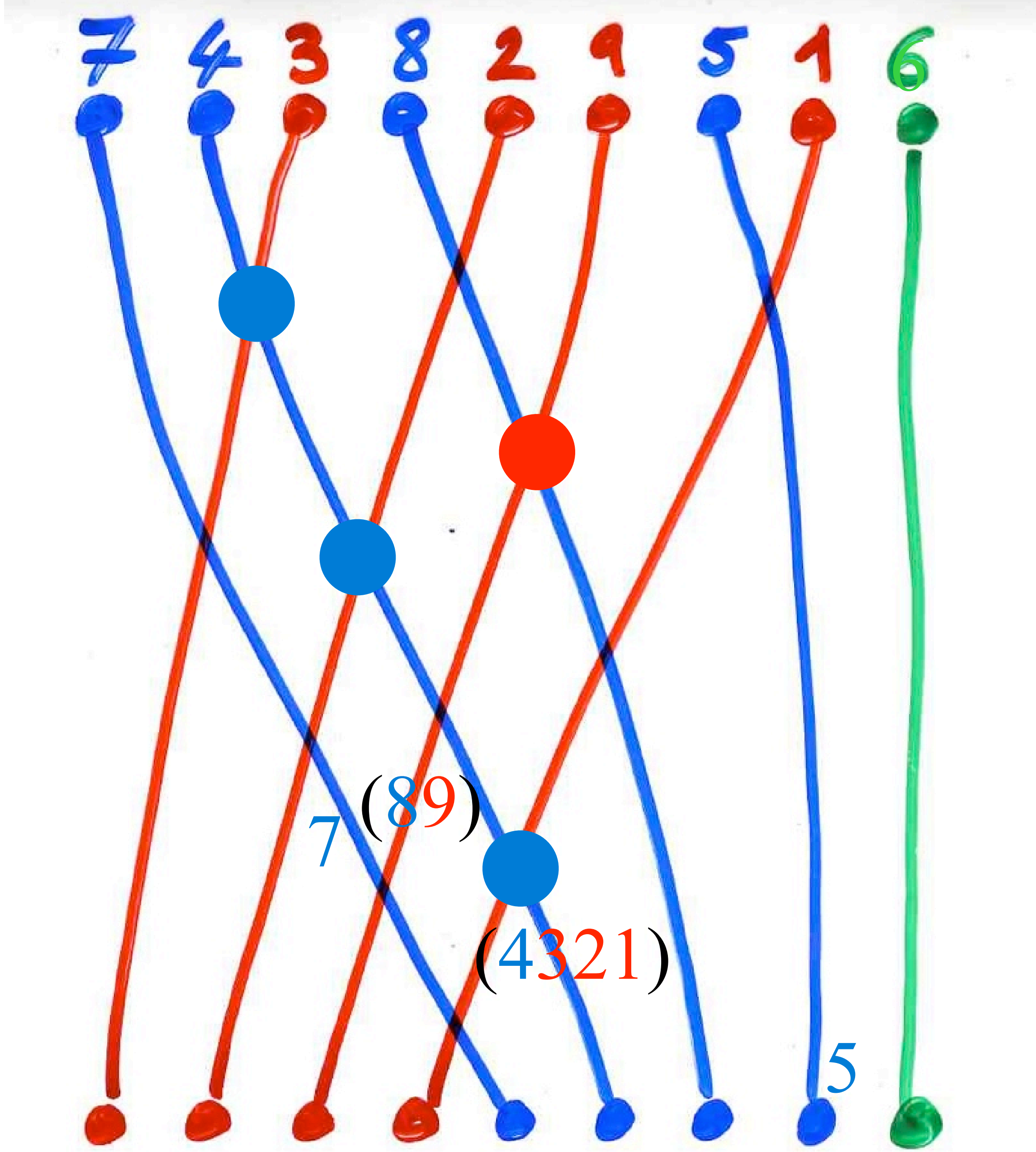
7 4 3 8 2 9 5 1 6
1
(89)
7
(432)
5


7 4 3 8 2 9 5 1 6
1
7
(432)
(89)
5

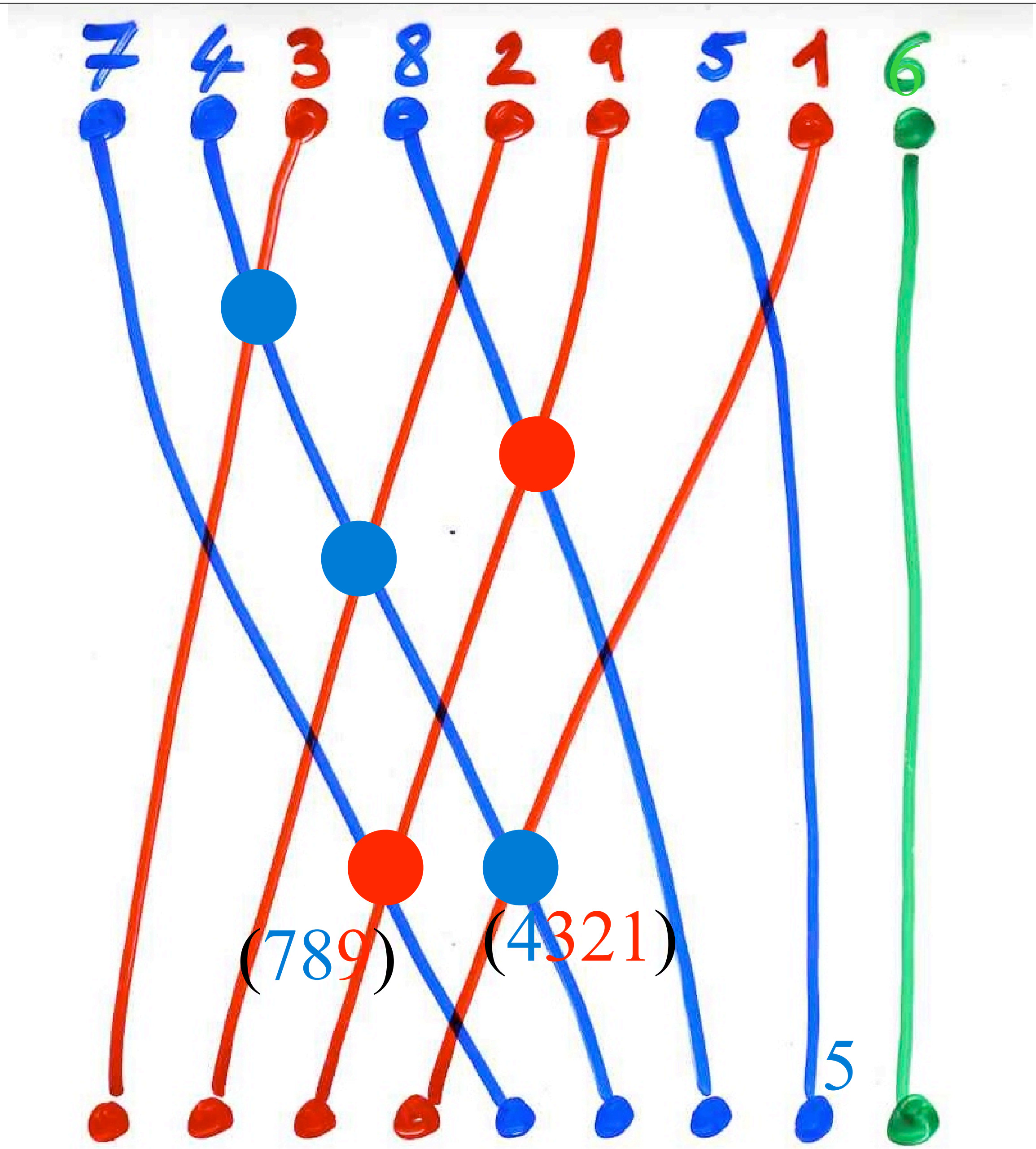

7 4 3 8 2 9 5 1 6
7
(432)1
(89)
5



7 4 3 8 2 9 5 1 6
7
(89)
(4321)
5

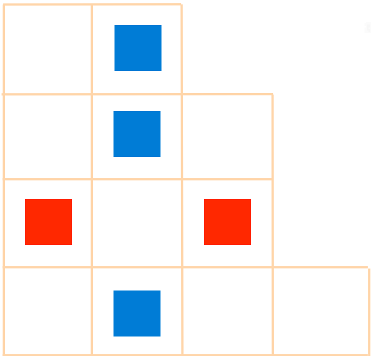


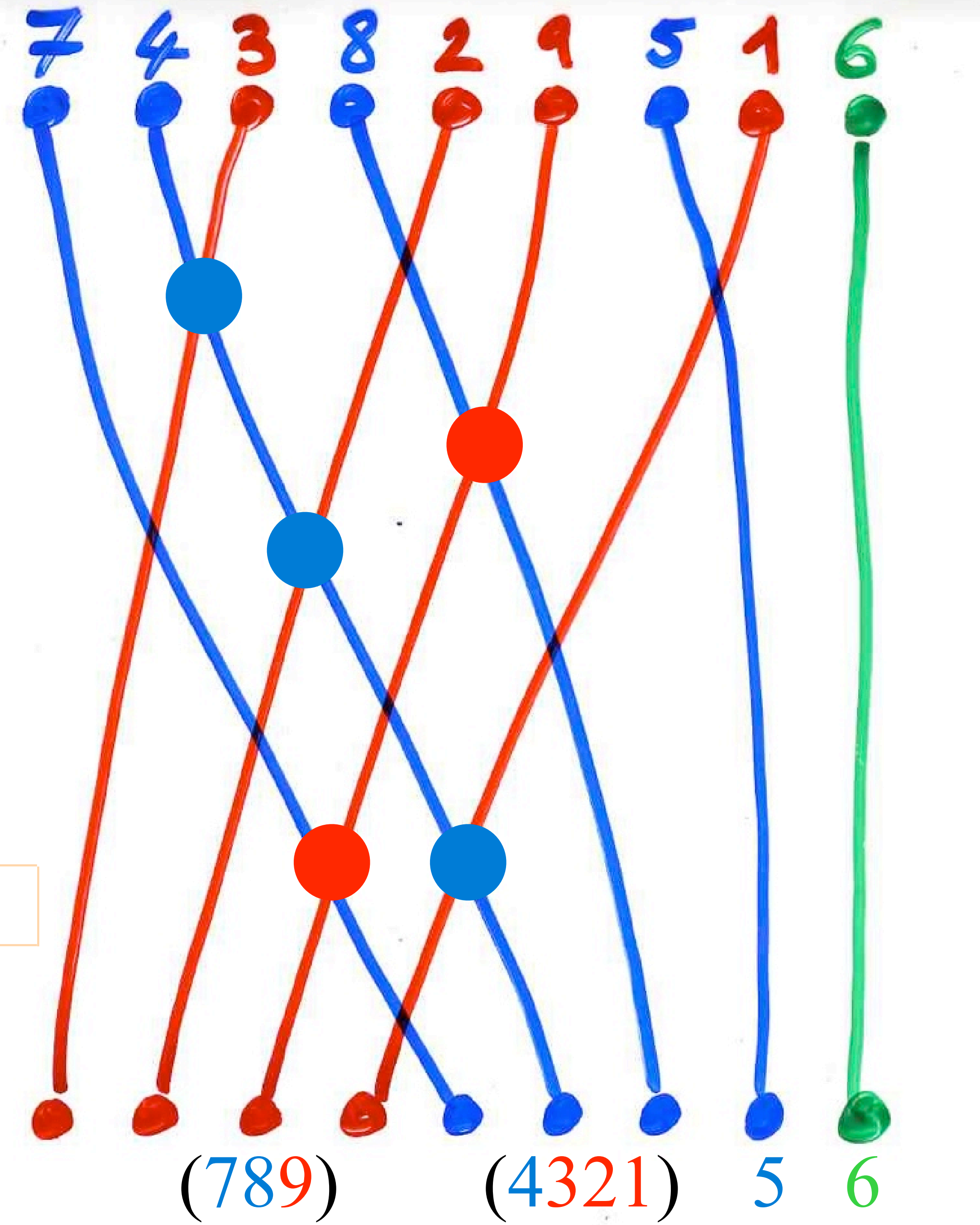
7 4 3 8 2 9 5 1 6
7
(89)
(4321)
5



7 4 3 8 2 9 5 1 6
(789)
(4321)
5

"exchange-fusion" algorithm





Description of the “exchange-fusion” algorithm

In the “exchange-fusion” algorithm, the red and blue blocks are falling down, starting at the beginning where all the blocks have only one letter. Each blocks is formed of consecutive letters.

- When two blocks meet at the crossing of a blue and red thread, if the union of the two blocks is formed with consecutive letters, then the two blocks form a single block by concatenation, and the new block follows the thread of the block having the biggest letters.
- If not, then the two blocks cross and follow their own colored thread.

The proof of the fact that the two algorithms “exchange-delete” and “exchange-fusion” produce exactly the same alternating tableau is based on the following observation:

(key) **observation**
In the “exchange-delete” algorithm, when a blue or a red dot is put on a crossing, that is when the two values x and y which are going to cross are “consecutive”, then all the intermediate values between x and y (which have disappeared) belong to one of the corresponding blocks in the analog crossing which will appears in the “exchange-fusion” algorithm.

A consequence of that is to give an interpretation of the number of red or blue blocks falling on the ground level, that is the number of columns having no red cells and numbers of rows having no blue cells. We call such row or column **“open”.**

§ 2 Some Parameters

The maximum letter of the blocks of letters reaching the ground level are:

- for the columns of T (red threads), the left-to-right maximum elements of the values of the permutation s less than the last letter s(n+1),
- for the rows of T (blue threads), the right-to-left maximum elements of the values of the permutation s bigger than the last letter

(3 proofs comming 3 different methodologies: by P. Nadeau , O.Bernardi and xgv)

This gives an interpretation of the two parameters on alternative tableaux:

- number of “open” columns (i.e. columns without a red cell)
- number of “open” rows (i.e. rows without a blue cell)

and recover previous results of S.Corteel, P. Nadeau and L. Williams about the double distribution of permutation tableaux according to the “number of unrestricted rows” and “number of 1’s in the first row”.

In fact, each block falling on the ground level in the “exchange-fusion” algorithm (corresponding to an open column or row), has an underlying binary tree structure coming from the different fusions (or equivalently the deletions of the “exchange-delete” algorithm) (see a forthcoming paper of P. Nadeau on “alternative trees” and alternative tableaux). In the case of “Catalan alternative tableaux”, these trees are simply related to the binary trees obtained by the bijection I presented in the paper for the Tienjin FPSAC’07.

permutation tableaux

alternative tableaux

$$\left\{\begin{array}{l}\bullet \text{ nb of unrestricted rows} \\ \bullet \text{ nb of 1's in the first row}\end{array}\right\} \longleftrightarrow \left\{\begin{array}{l}\bullet \text{ nb of rows without blue} \\ \bullet \text{ nb of columns without red}\end{array}\right.$$

Corteel (2006)

$$T_n(x,y) = \prod_{i=0}^{n-1} (x+y+i)$$

{ RL- minima
{ LR- minima

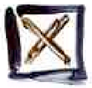
bijection Corteel-Nadeau (2007)

permutation tableaux ⟷ permutation

- profile ⟷ • (rises, descents)
- ~~nb of~~ unrestricted rows ⟷ RL- minimum
- ne of "superflus" 1 ⟷ ne of occurences of (31-2)

↕

alternative tableaux

- profile
- ne of rows without 
- ne of cells ~~without~~ 

Number of "crossings" in the alternative tableaux

This parameter is the number of crossing occurring in the "exchange-delete", or equivalently of the "exchange-fusion" algorithm.Each crossing corresponds to a cell in the alternative tableau (colored) which is above a red cell and at the right of a blue cell. It has the same distribution as the parameter "number of occurrences of the pattern (31-2)" in permutations. (from the bijection of S. Corteel and P. Nadeau or from Steingrimsson and Williams)

This parameter is the natural q-analogue of Laguerre histories, that is the parameter obtained by taking the sum of all the "possibilities choices decreased by one". In other words, if at each step 1, 2, ..., x, ... , n+1, of the construction of the permutation, the (k +1) free positions available to insert the value x are labeled (in a certain way) 0, 1, ..., k, then we put the weight q^i when value x is inserted at position i , and the weight of the Laguerre history is the product of the weight of each individual step. If the labeling is always from left to right, then the q-analogue becomes the number of occurrence of (31-2). (see the next section).

The number of crossings of the alternative tableau has been be characterized by O.Bernardi on the corresponding permutation s.

It is the number of pairs (x,y), x=s(i), y=s(j),1≤ i<j≤n+1,
such that there exist two integers k, l≥ 0 such that:
the set of the values x+1, x+2, ..., x+k, y+1, .., y+l are located between x and y (in the word s), and x+k+1 is located (in s) at the right of y and y+l+1 is located (in s) at the left of x (with the convention of n+2 at the left of all the values).

O.B. deduce the nice corollary:
The permutations s coming from tableaux with no crossing (counted by Catalan numbers) are characterised by the following condition

there is no pair of values (x, y) such that the four values (x,x+1,y,y+1) appear in the following order in the permutation:

s = ... y+1 ..... x ..... y .... x+1 ......

Laguerre histories
Peaks, Valleys,
Double Rises, Double Descents
and   parameter "q-Laguerre"

- convention

$\sigma \in \mathfrak{S}_n$, $\quad \sigma(0) = \sigma(n+1) = 0$

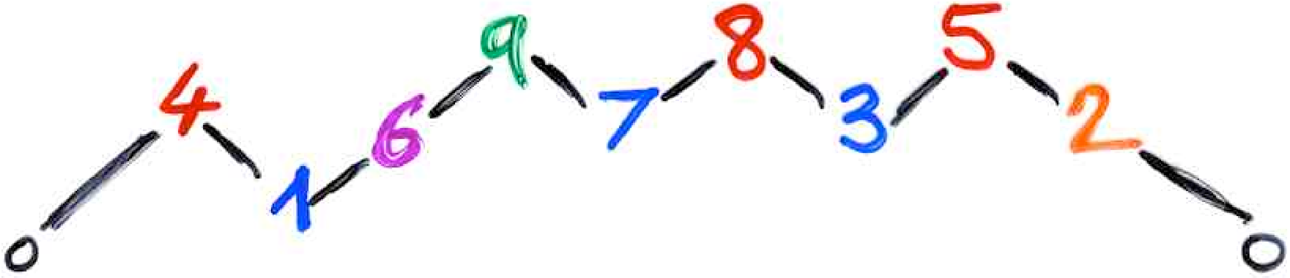
Def-

$x \in [1, n]$

(x valeur
  i indice)

pic	$\sigma(i-1) < x = \sigma(i) > \sigma(i+1)$
creux	$\sigma(i-1) > x = \sigma(i) < \sigma(i+1)$
double montée	$\sigma(i-1) < x = \sigma(i) < \sigma(i+1)$
double descente	$\sigma(i-1) > x = \sigma(i) > \sigma(i+1)$

$\sigma = 4\ 1\ 6\ 9\ 7\ 8\ 3\ 5\ 2$



A through (valley)

S peak

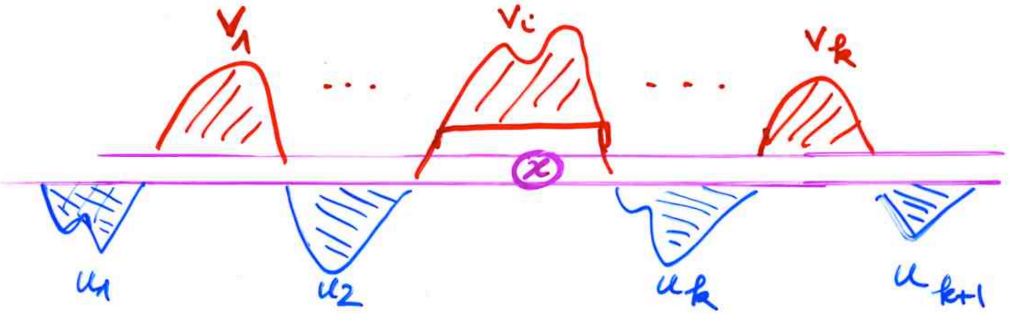
J double rise

K double descent

Def – $\sigma \in \mathfrak{S}_n$, $x \in [1, n]$

x– decomposition

- $\sigma = u_1 v_1 \dots u_k v_k u_{k+1}$ — permutation
- lettres $(u_i) < x$
- lettres $(v_j) \geq x$
- mots $v_1, u_2, \dots, u_k, v_k$ non vides



ex. $\sigma = 4\,1\,6\,9\,7\,8\,3\,5\,2$, $x = 3$

$\underbrace{\ }_{u_1}\ \overbrace{4}^{v_1}\ \underbrace{1}_{u_2}\ \overbrace{6\,9\,7\,8\,\textcircled{3}\,5}^{v_2}\ \underbrace{2}_{u_3}$

Lemme — $h = (\omega_c \,;\, (p_1, \dots, p_n)) \in \mathcal{L}_n$ Laguerre history

$\Pi \circ \Theta^{-1} \downarrow$

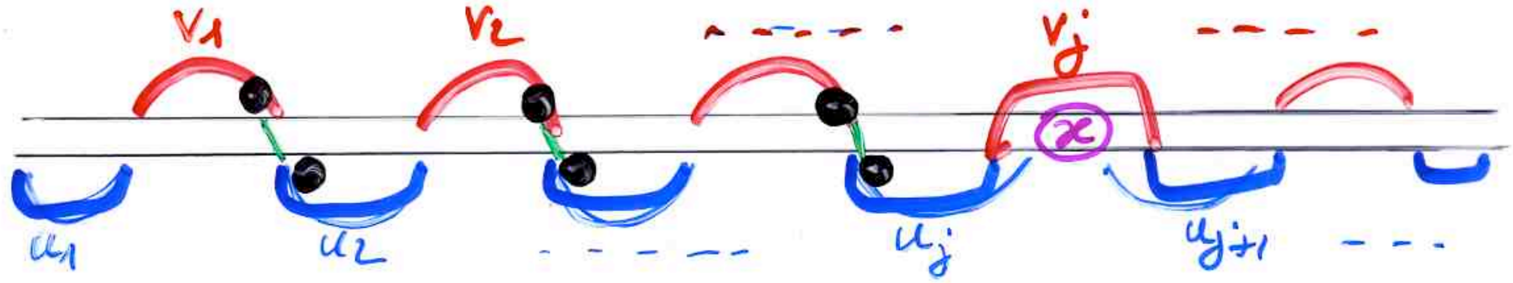
permutation $\sigma \in \mathfrak{S}_{n+1}$

$p_x = j$ est aussi :

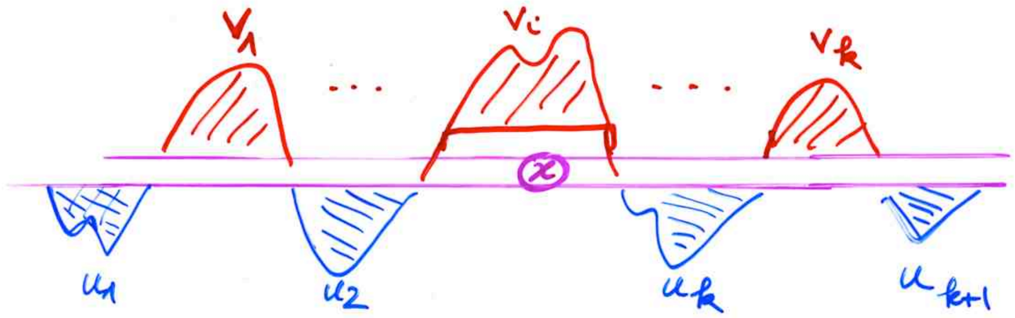
$j = 1 +$ nb de triplets (a, b, x) ayant le "motif" (31-2) c.à.d :

$a = \sigma(i)$, $b = \sigma(i+1)$, $x = \sigma(\ell)$

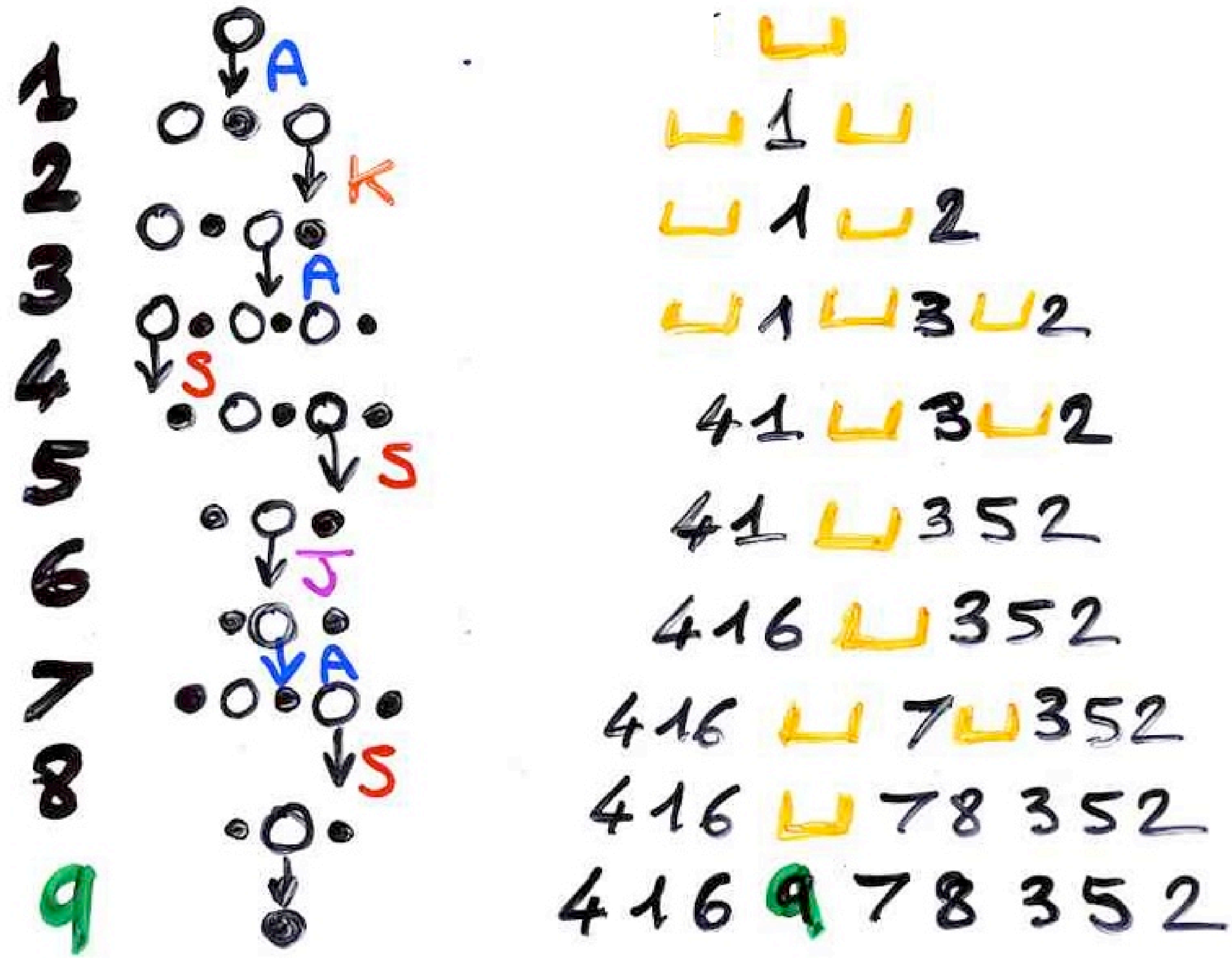
$i < i+1 < \ell$ $\qquad$ $b < x < a$



"q-analogue" of Laguerre histories



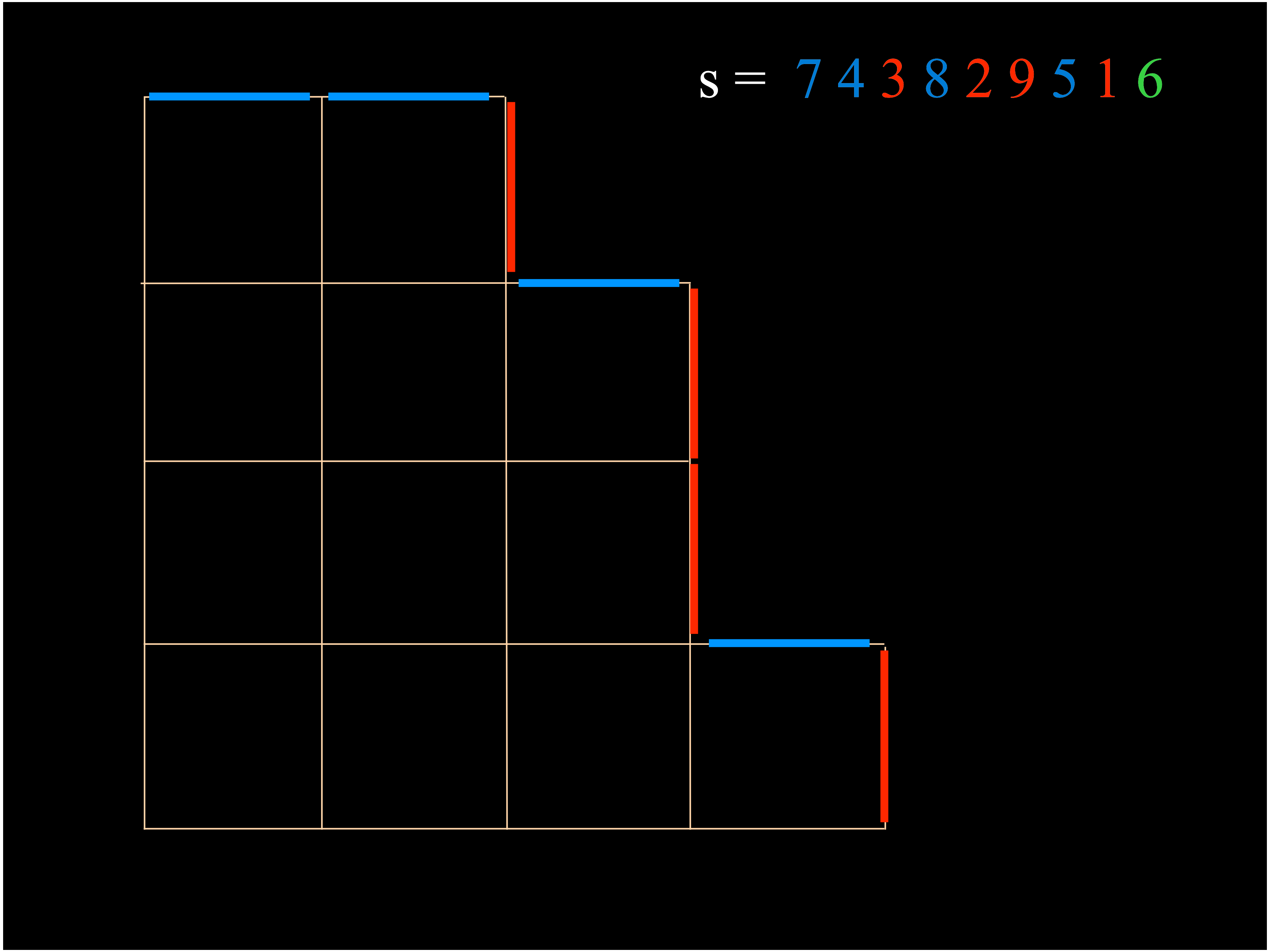
v1
vi
vk
x
u1
u2
uk
uk+1



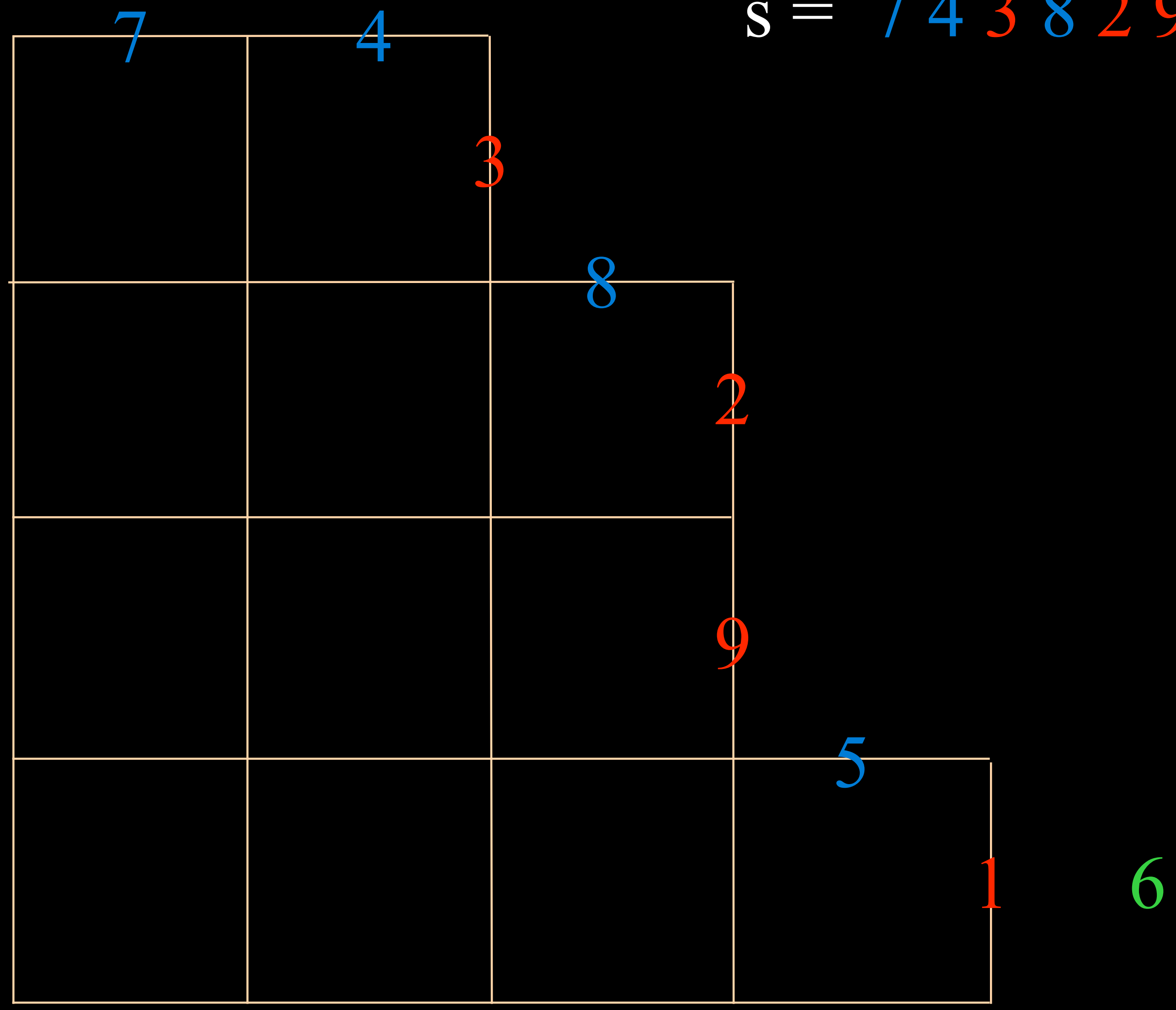
1
2
3
4
5
6
7
8
9
A
K
A
S
S
J
A
S
1
1 2
1 3 2
4 1 3 2
4 1 3 5 2
4 1 6 3 5 2
4 1 6 7 3 5 2
4 1 6 7 8 3 5 2
4 1 6 9 7 8 3 5 2

§3 alternative jeu de taquin

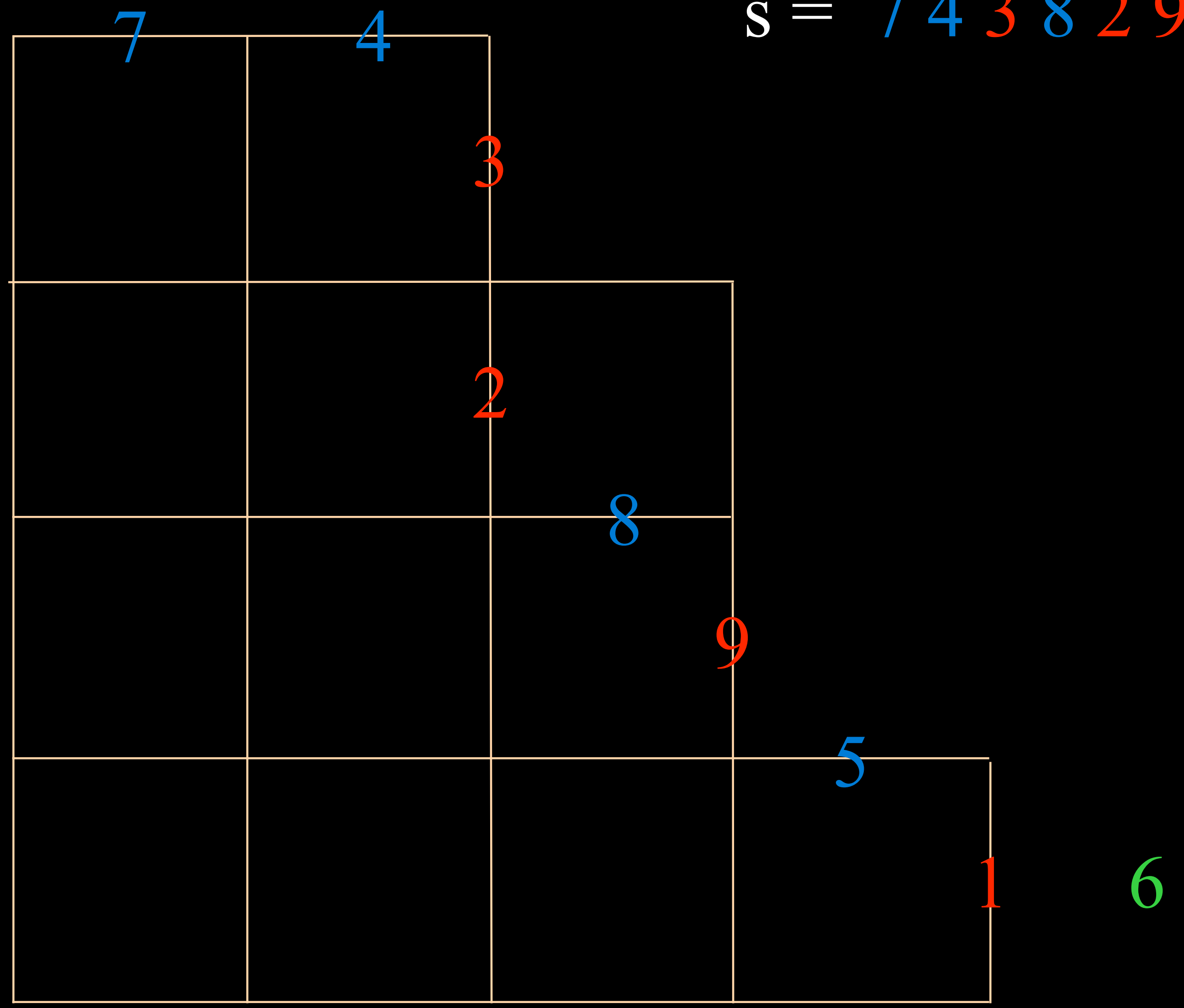
The "exchange-delete" algorithm
can also be rewritten on a grid
and gives an analogue of
Schützenberger 's "jeu de taquin"
for alternative tableau

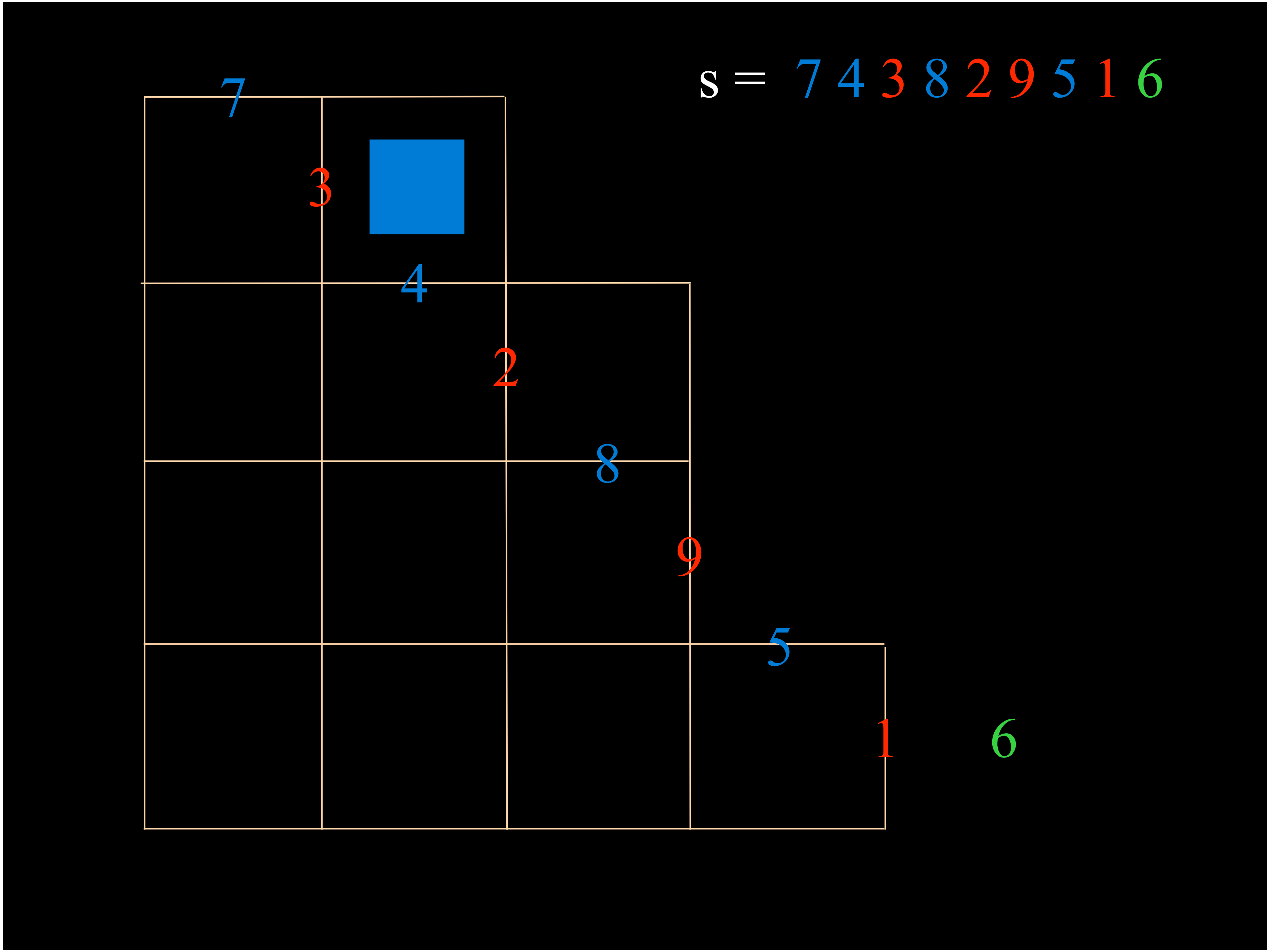

s = 7 4 3 8 2 9 5 1 6

$s = 7\ 4\ 3\ 8\ 2\ 9\ 5\ 1\ 6$



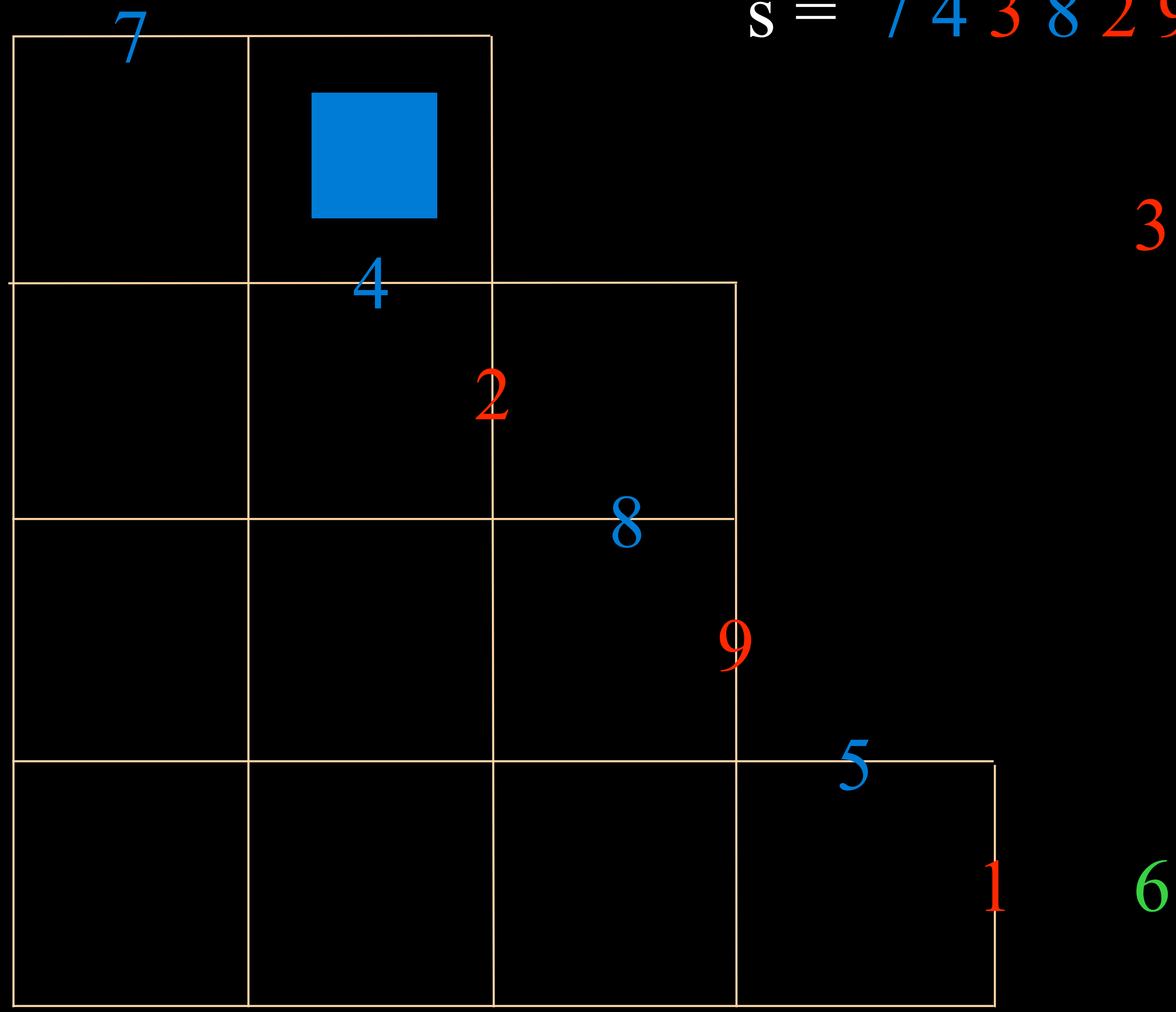
$s = 7\ 4\ 3\ 8\ 2\ 9\ 5\ 1\ 6$



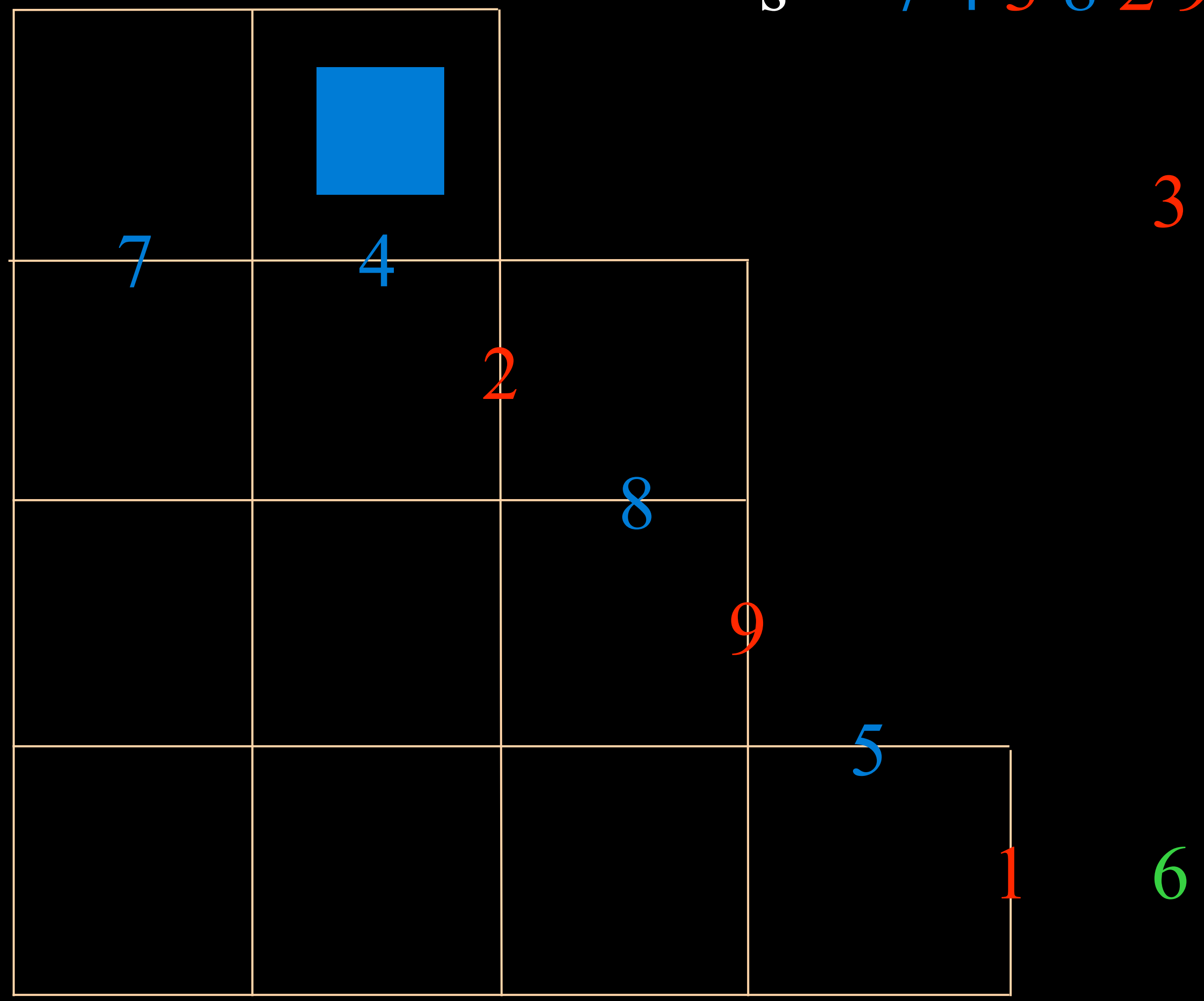


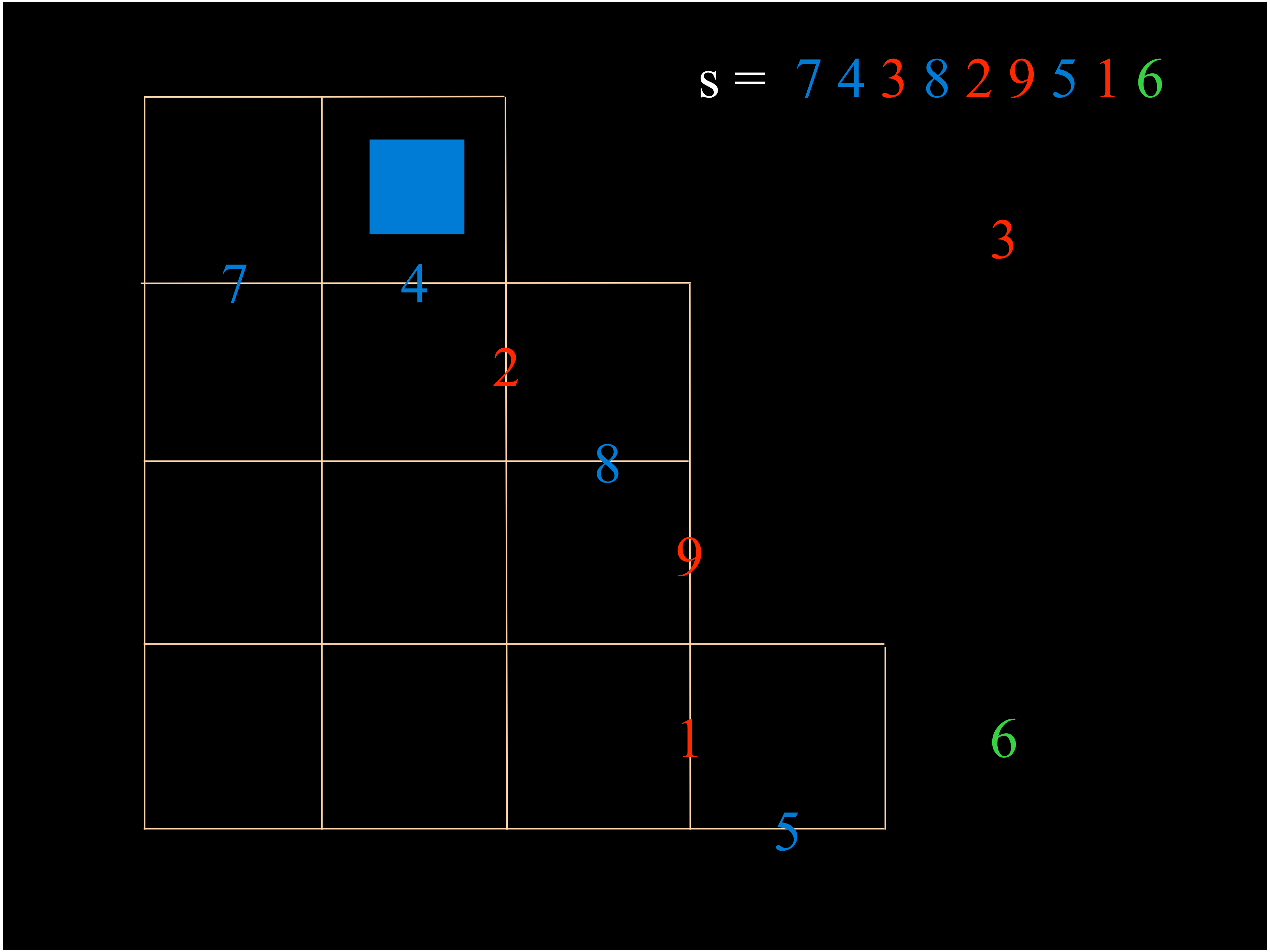
s = 7 4 3 8 2 9 5 1 6
7
3
4
2
8
9
5
1
6

$s = 7\,4\,3\,8\,2\,9\,5\,1\,6$

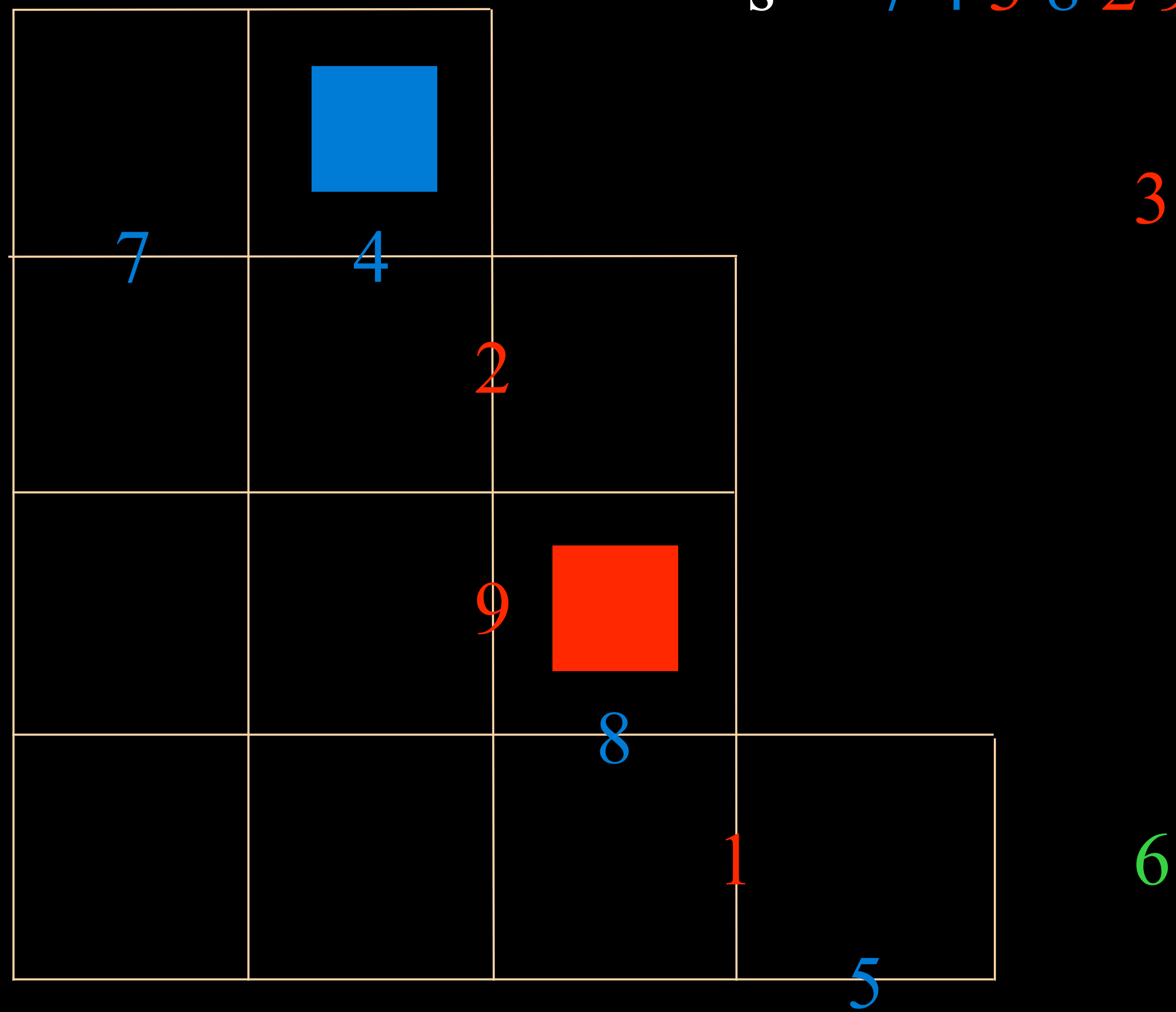


s = 7 4 3 8 2 9 5 1 6

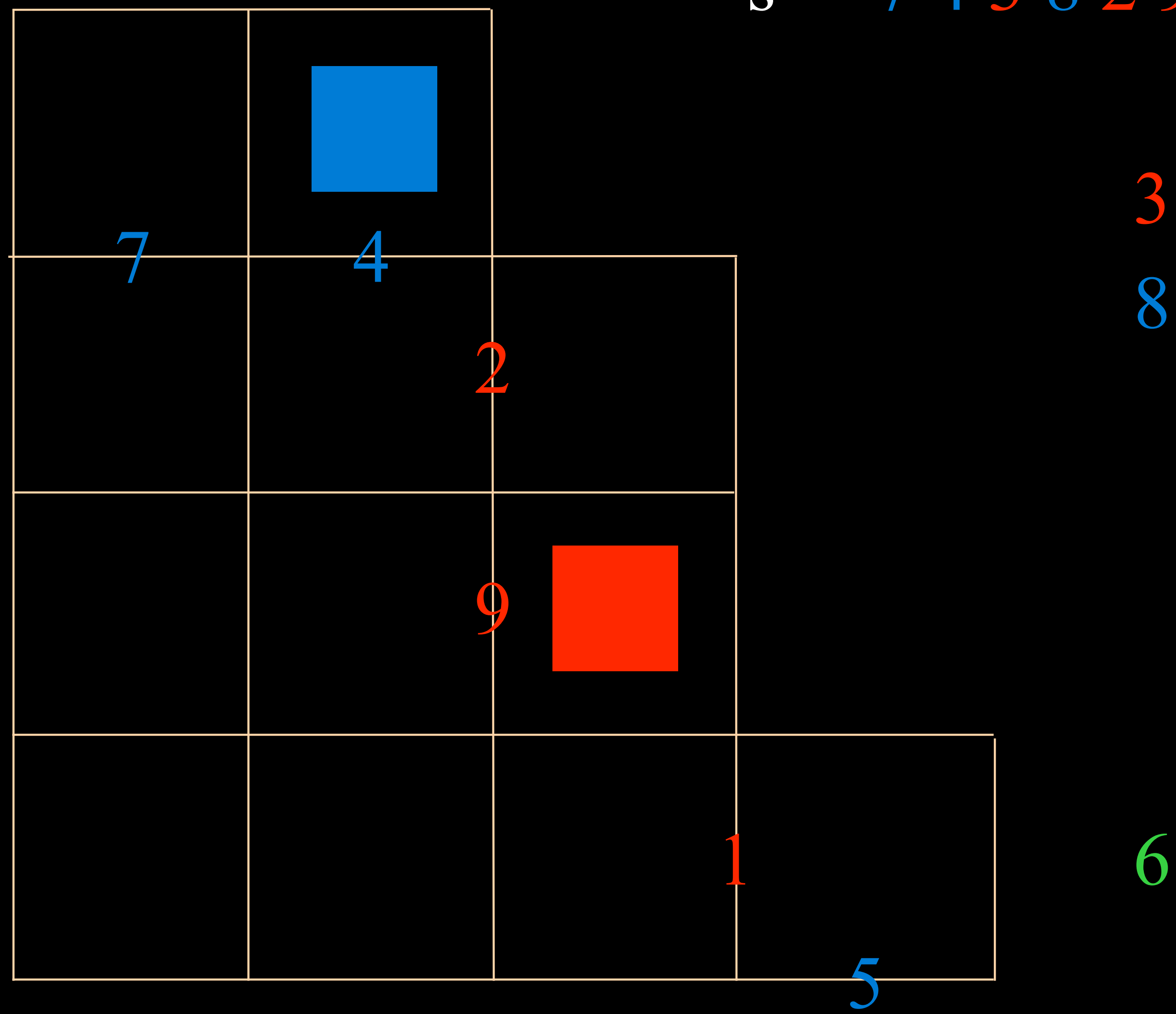



s = 7 4 3 8 2 9 5 1 6
3
7
4
2
8
9
6
1
5

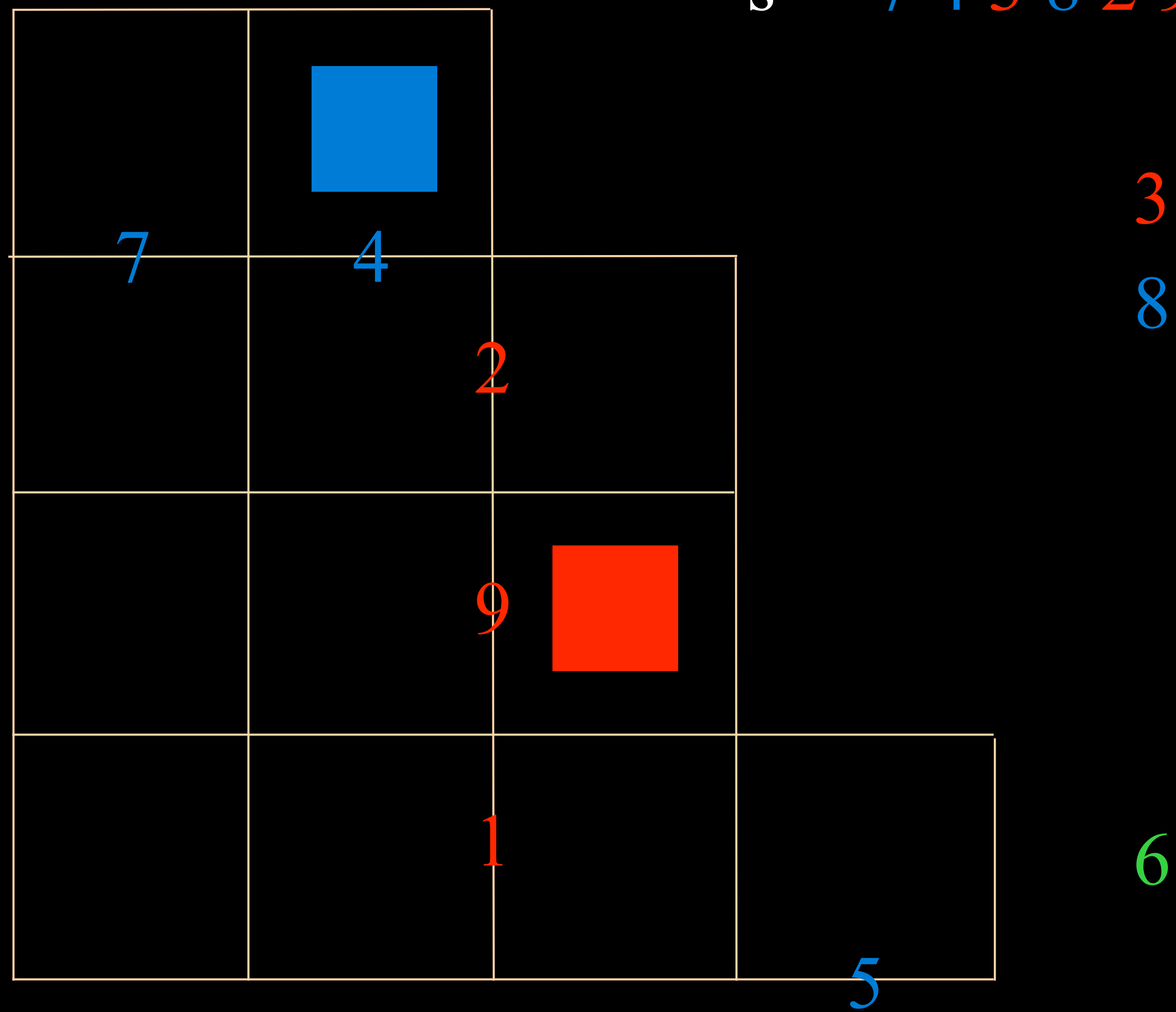
s = 7 4 3 8 2 9 5 1 6



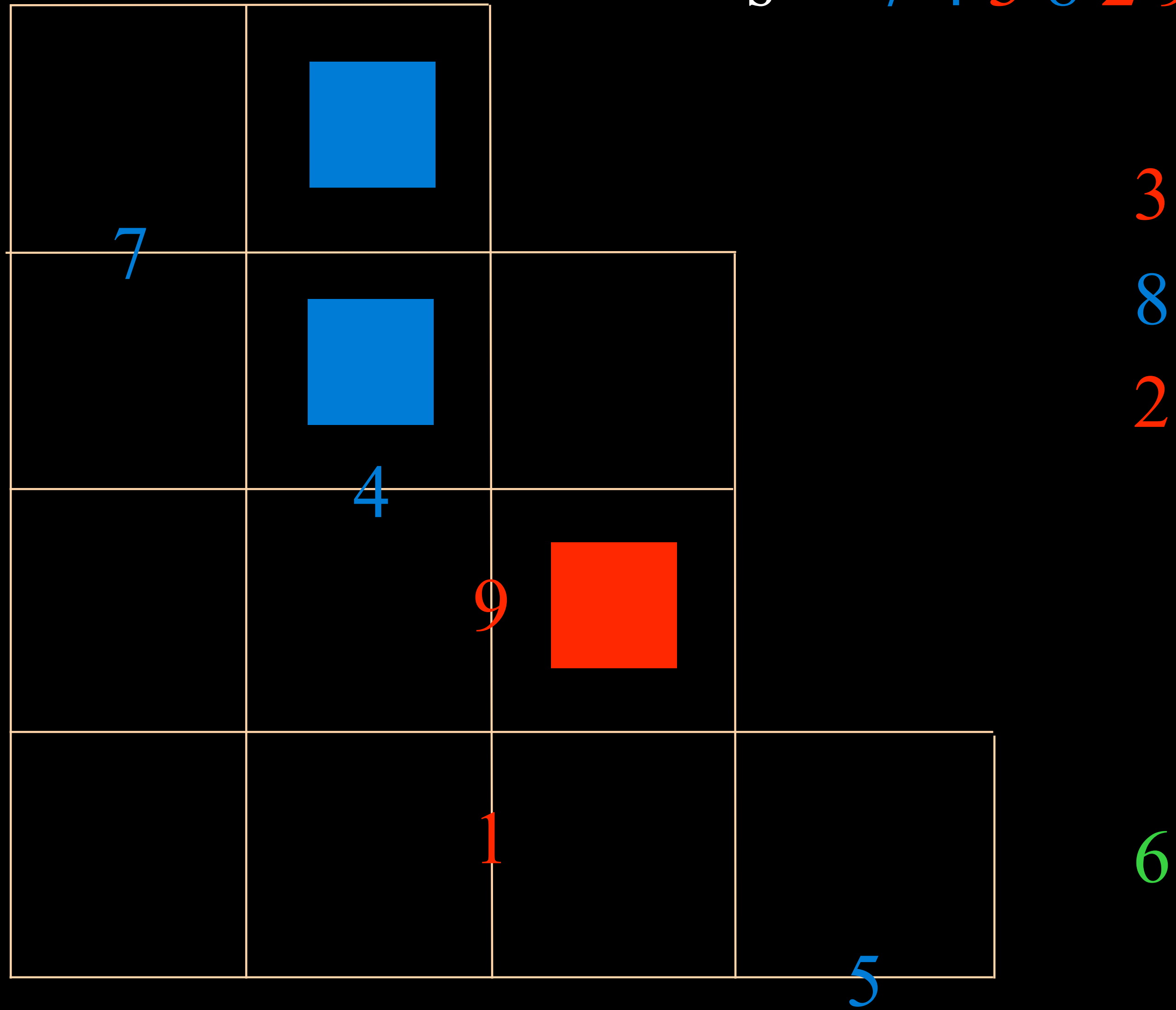
s = 7 4 3 8 2 9 5 1 6



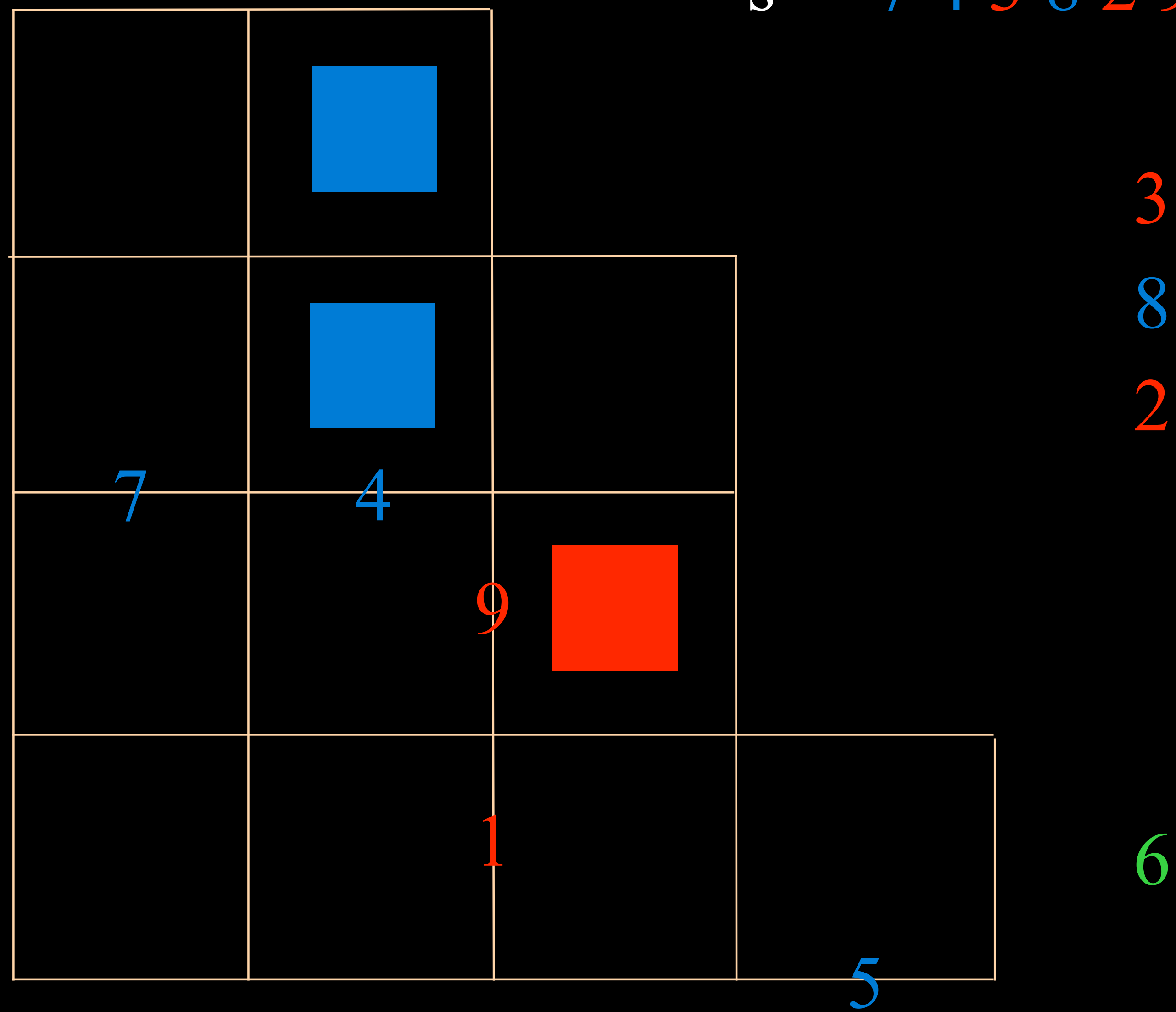
s = 7 4 3 8 2 9 5 1 6

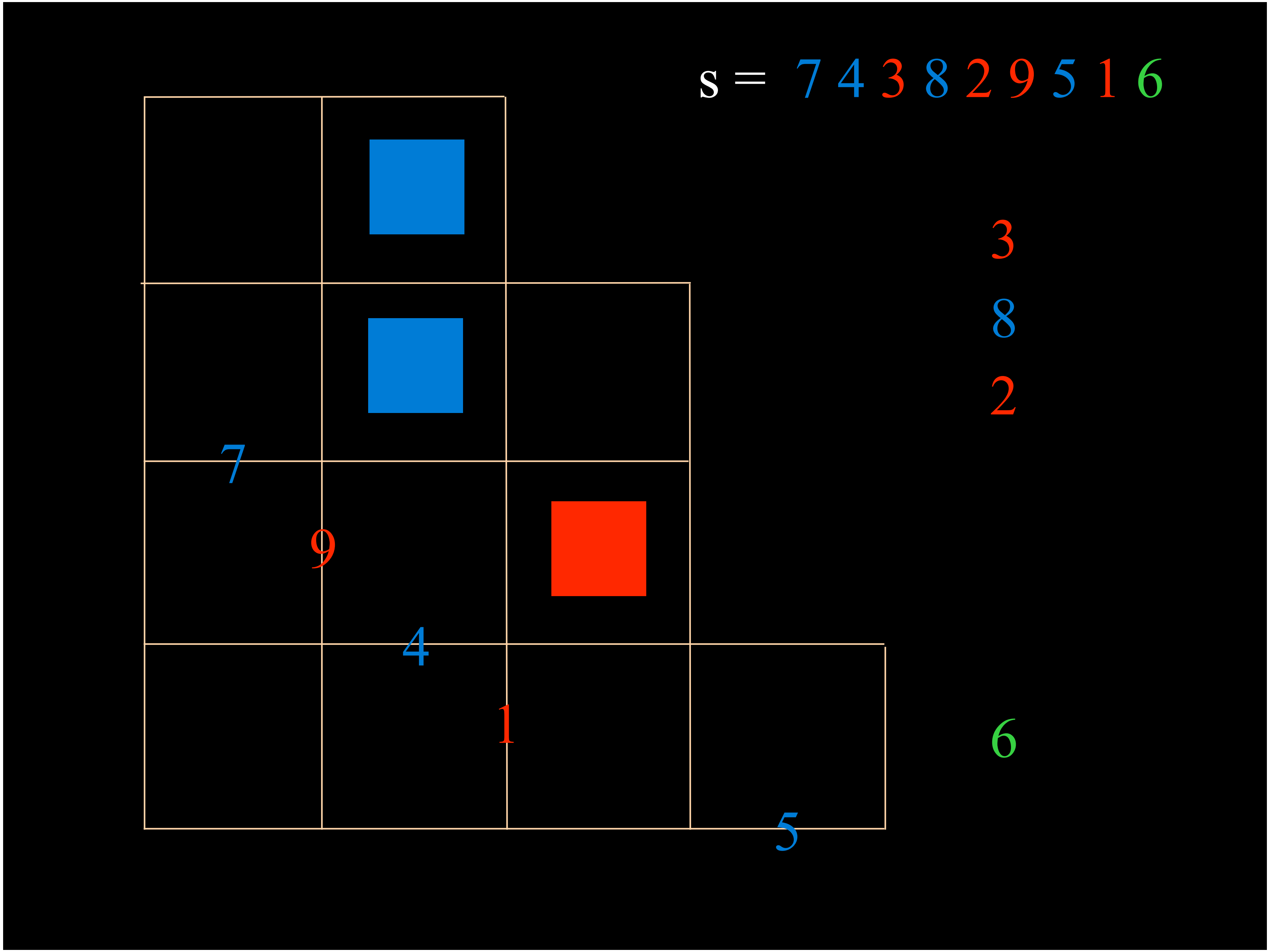


$s = 7\ 4\ 3\ 8\ 2\ 9\ 5\ 1\ 6$

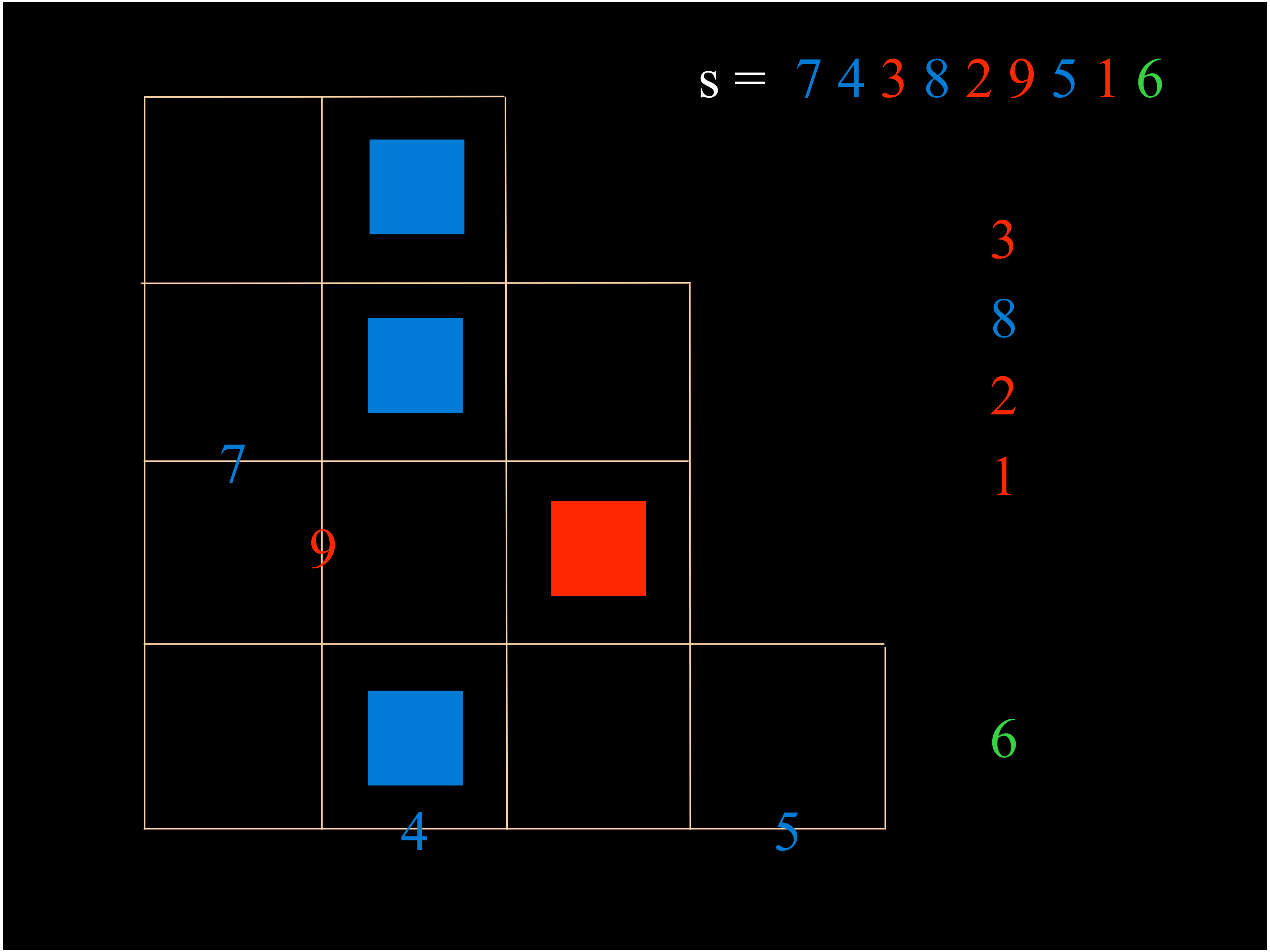


$s = 7\ 4\ 3\ 8\ 2\ 9\ 5\ 1\ 6$



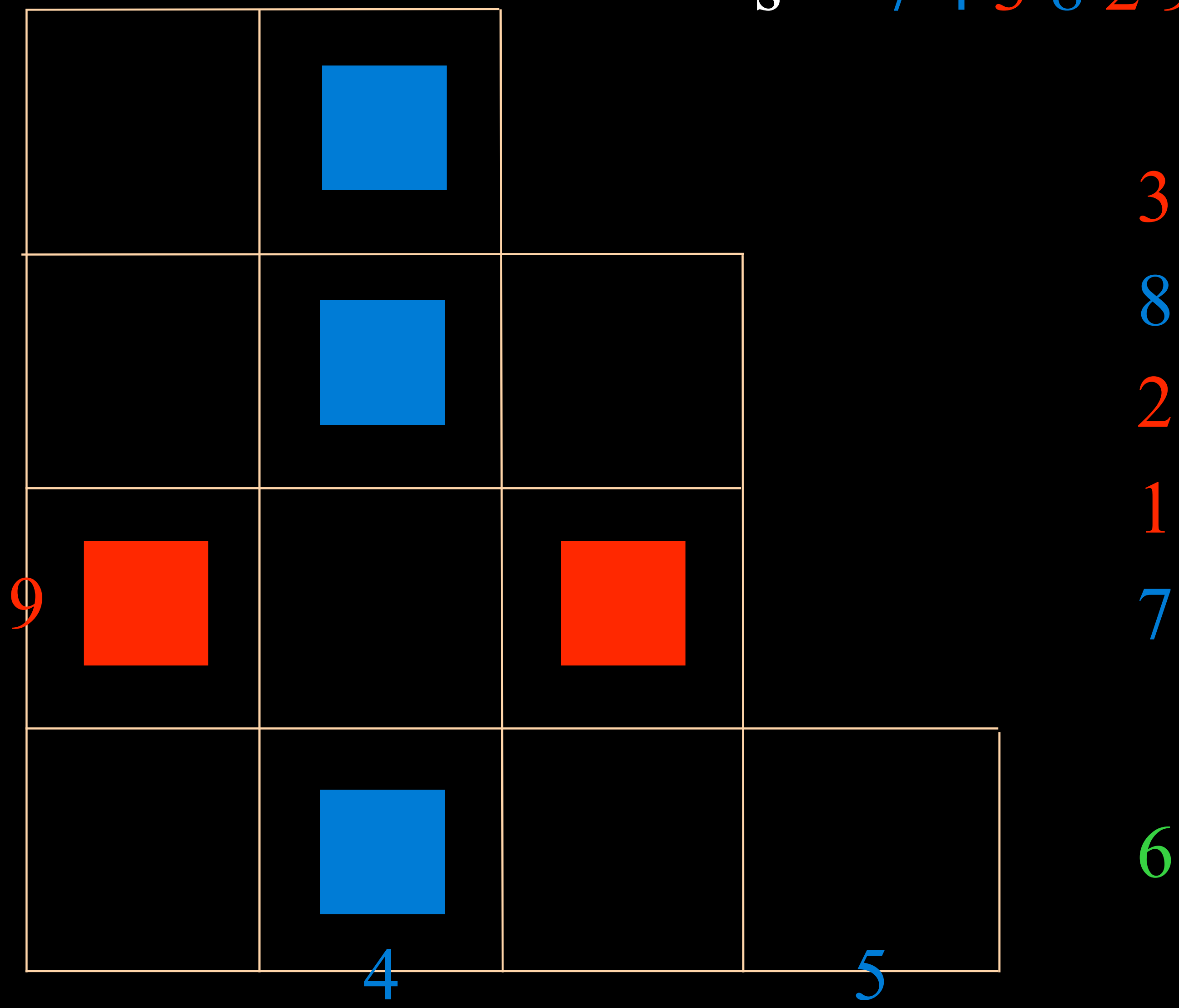


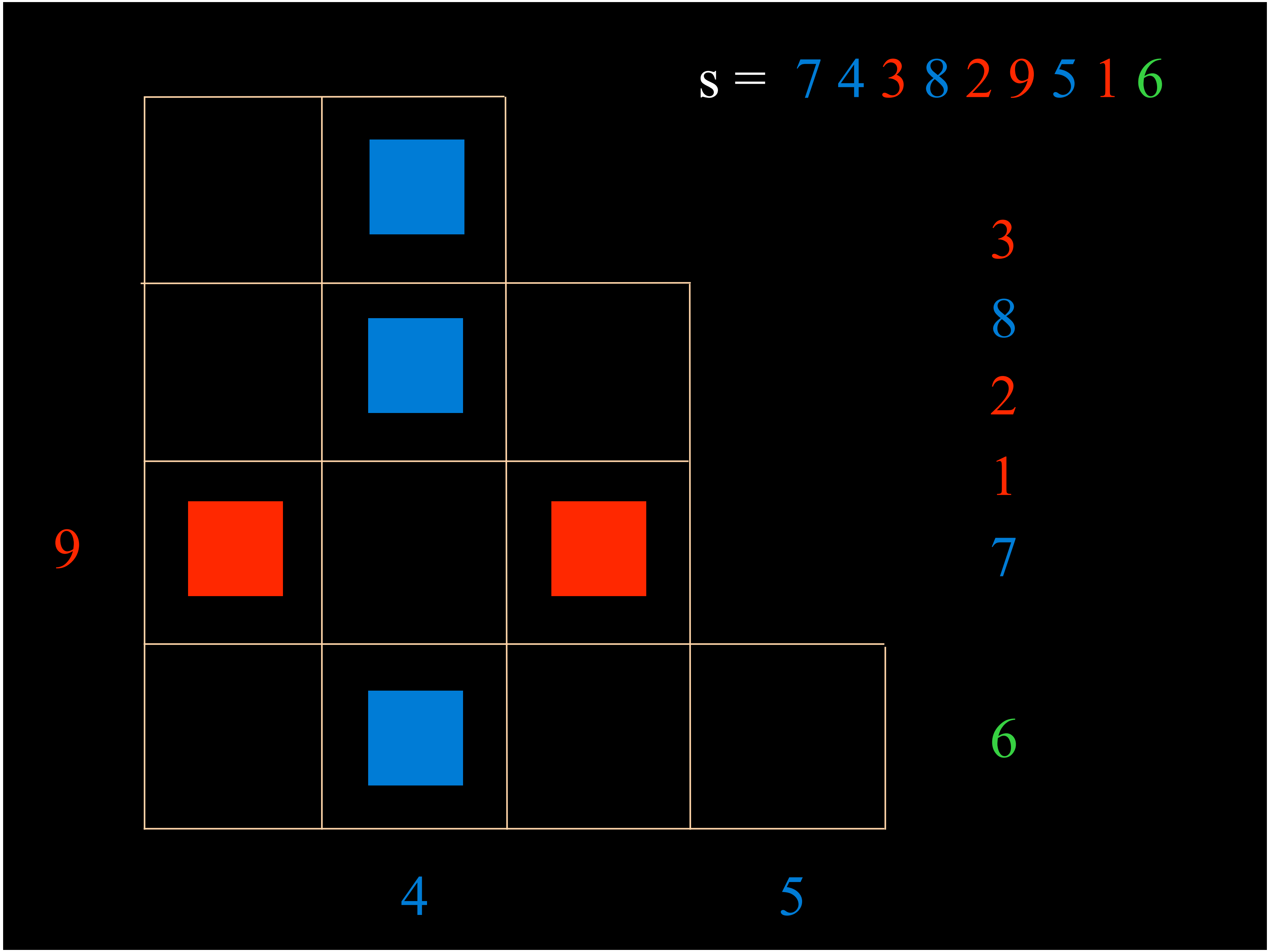
s = 7 4 3 8 2 9 5 1 6
3
8
2
7
9
4
1
6
5



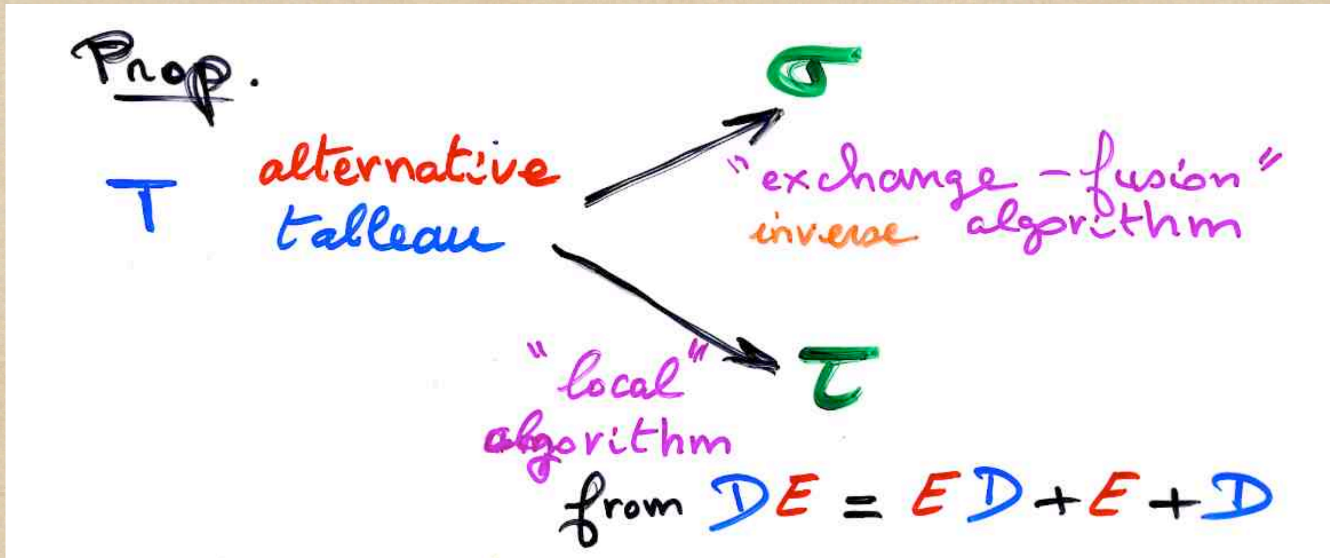
s = 7 4 3 8 2 9 5 1 6
3
8
2
1
7
9
6
4
5

$s = 7\,4\,3\,8\,2\,9\,5\,1\,6$

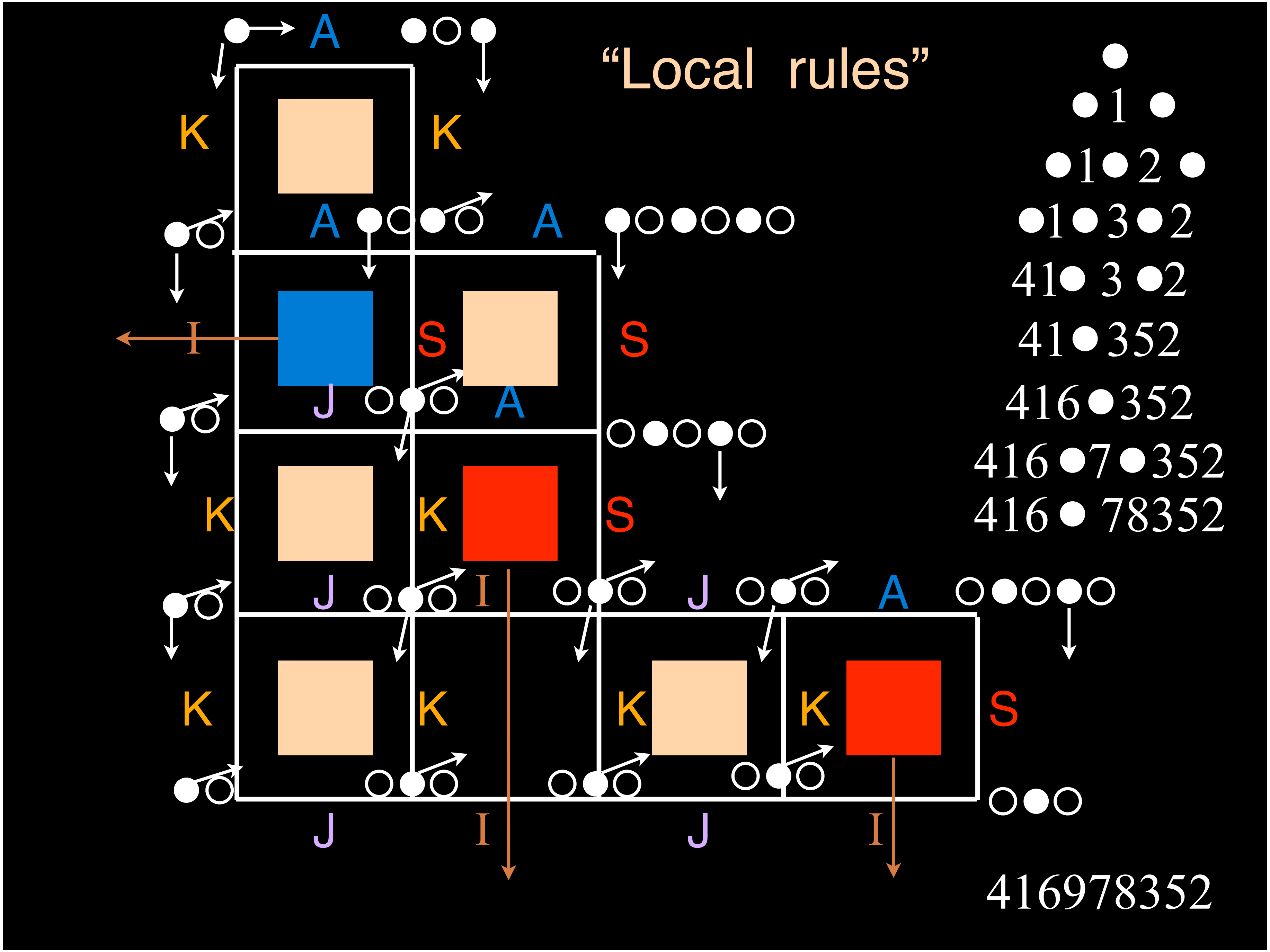



s = 7 4 3 8 2 9 5 1 6
3
8
2
1
7
9
6
4
5

§4 (idea) of the proof of the theorem:



$$\sigma = \tau^{-1}$$


"Local rules"
●
●1 ●
●1●2 ●
●1●3●2
41● 3 ●2
41●352
416●352
416 ●7●352
416 ● 78352
416978352
A
K
K
A
A
S
S
J
A
K
K
S
J
I
J
A
K
K
K
K
S
J
I
J
I

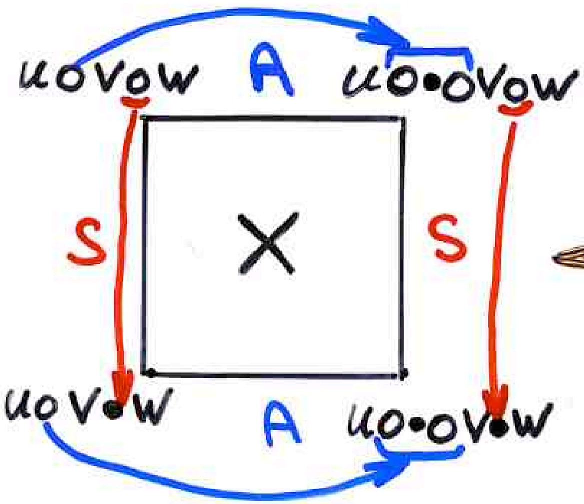
Recall the 8 "commutations diagram"
corresponding to the 8 possible rewritings rules

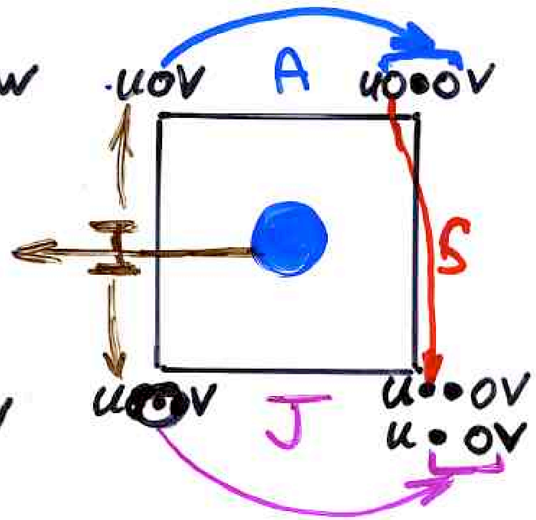
$$AS = SA + J + K$$

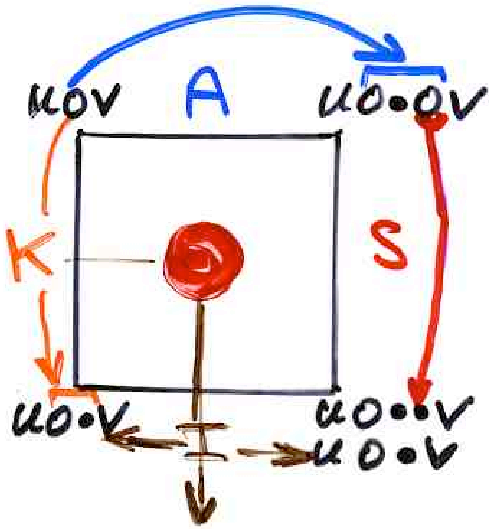
$$AK = KA + A$$

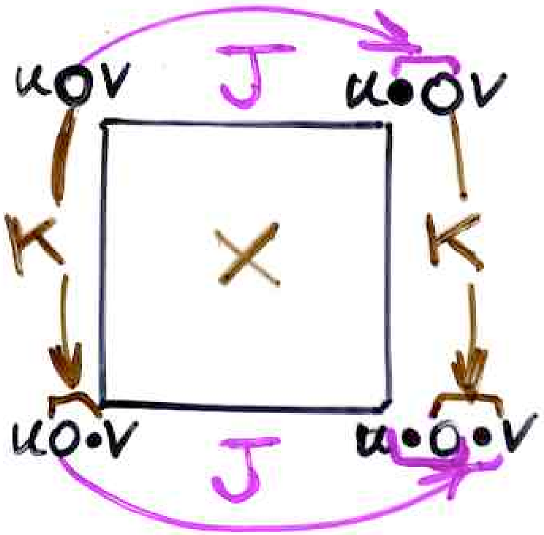
$$JS = SJ + S$$

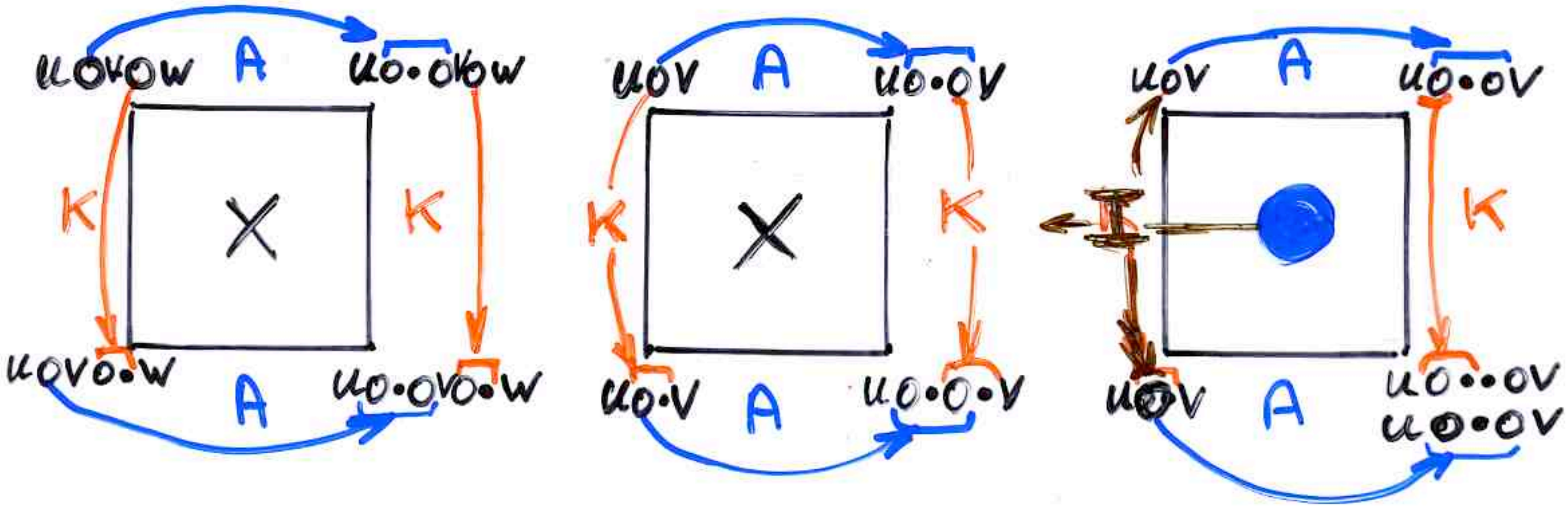
$$JK = KJ$$

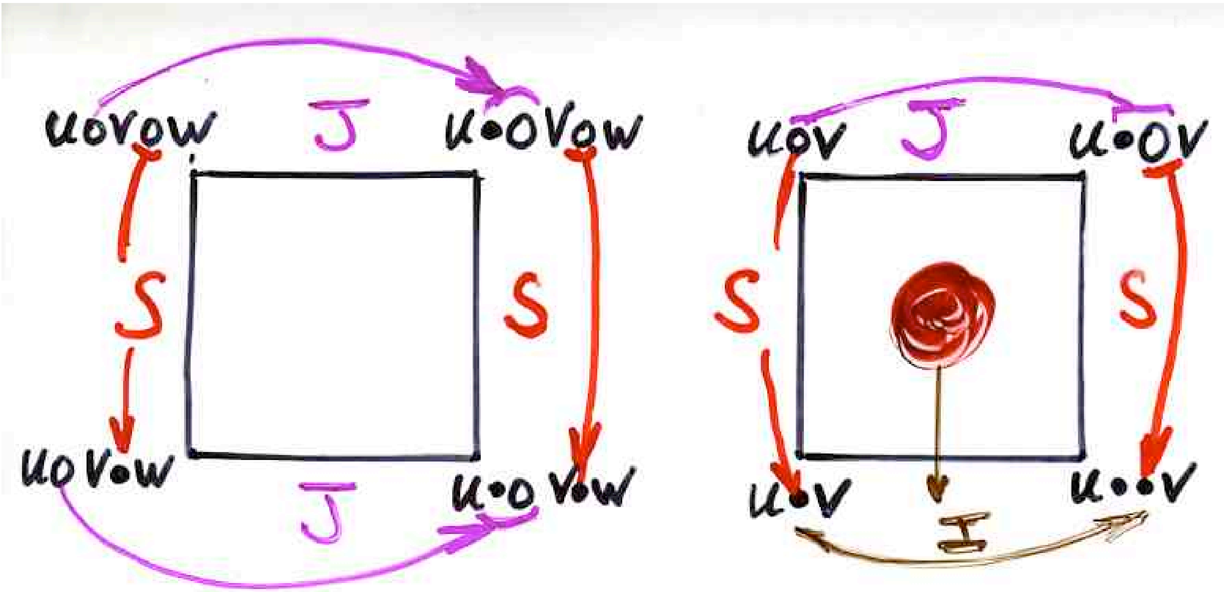

uovow
A
uo•ovow
S
S
uov•w
A
uo•ov•w

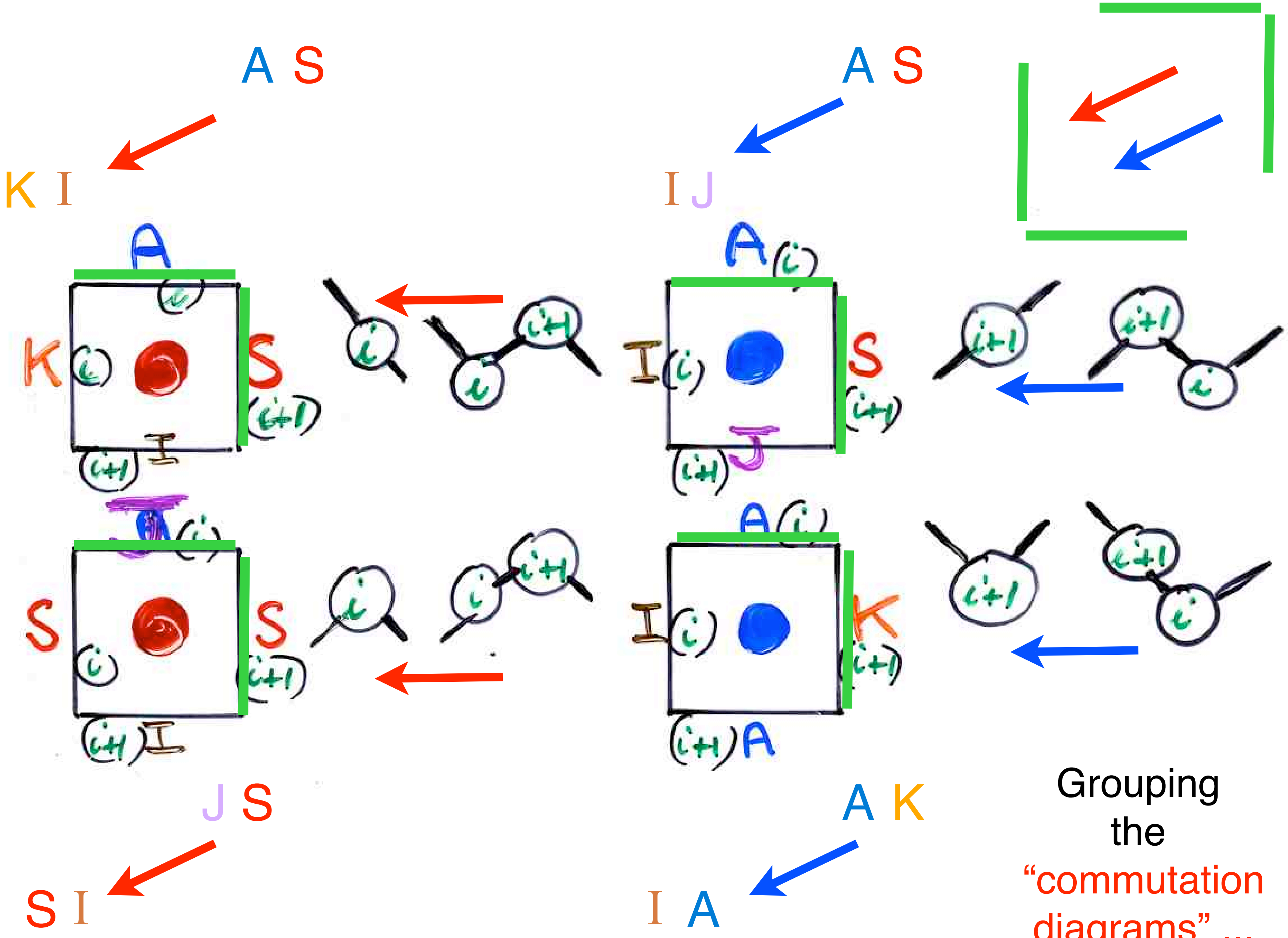

uov
A
uo•ov
S
uo•v
J
u••ov
u•ov


uov
A
uo•ov
K
S
uo•v
uo••v
uo•v

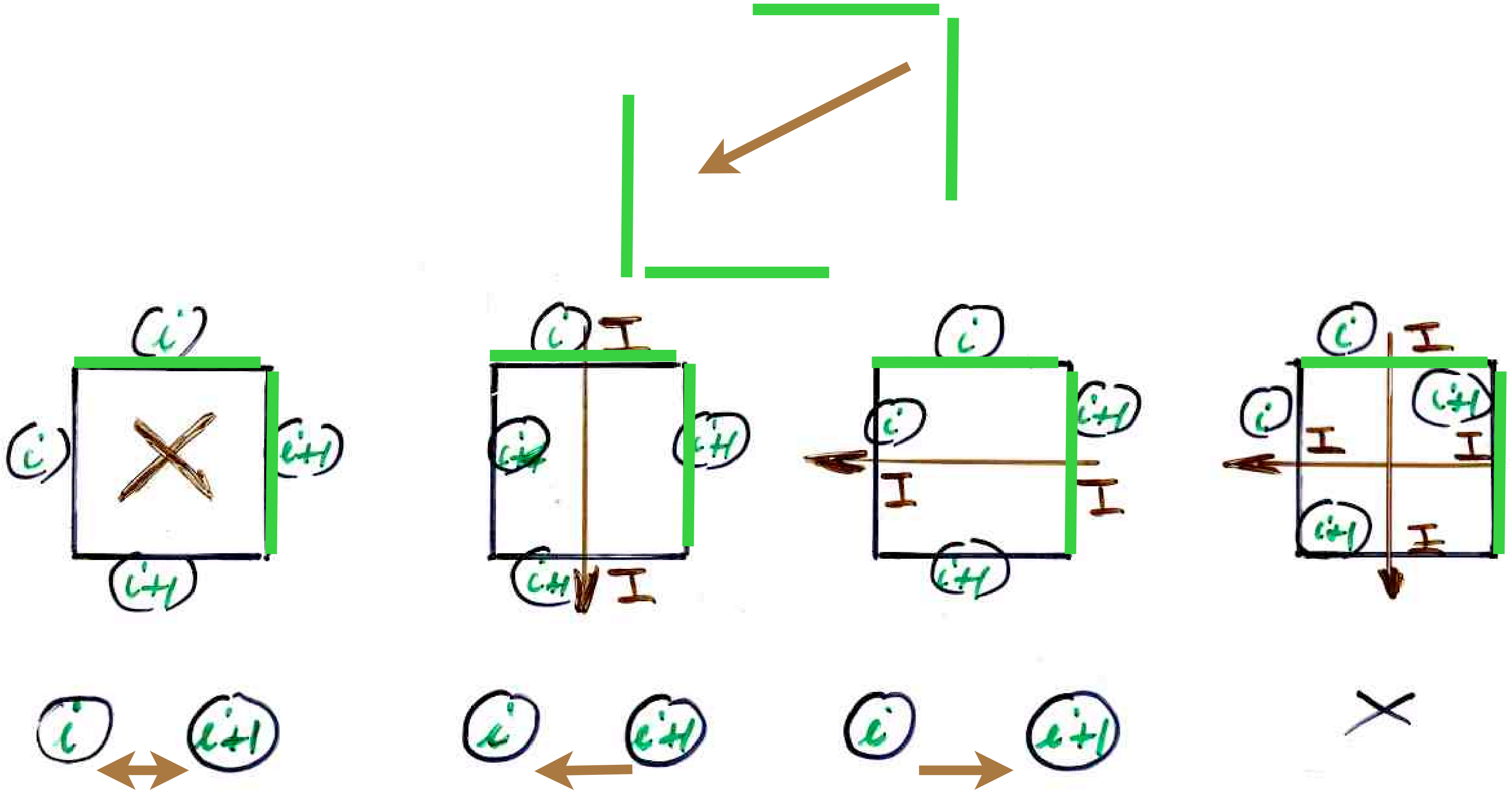

uov
J
u•ov
K
K
uo•v
J
u•o•v


A
K
A
K
A
K
A
K
A
K
A
K

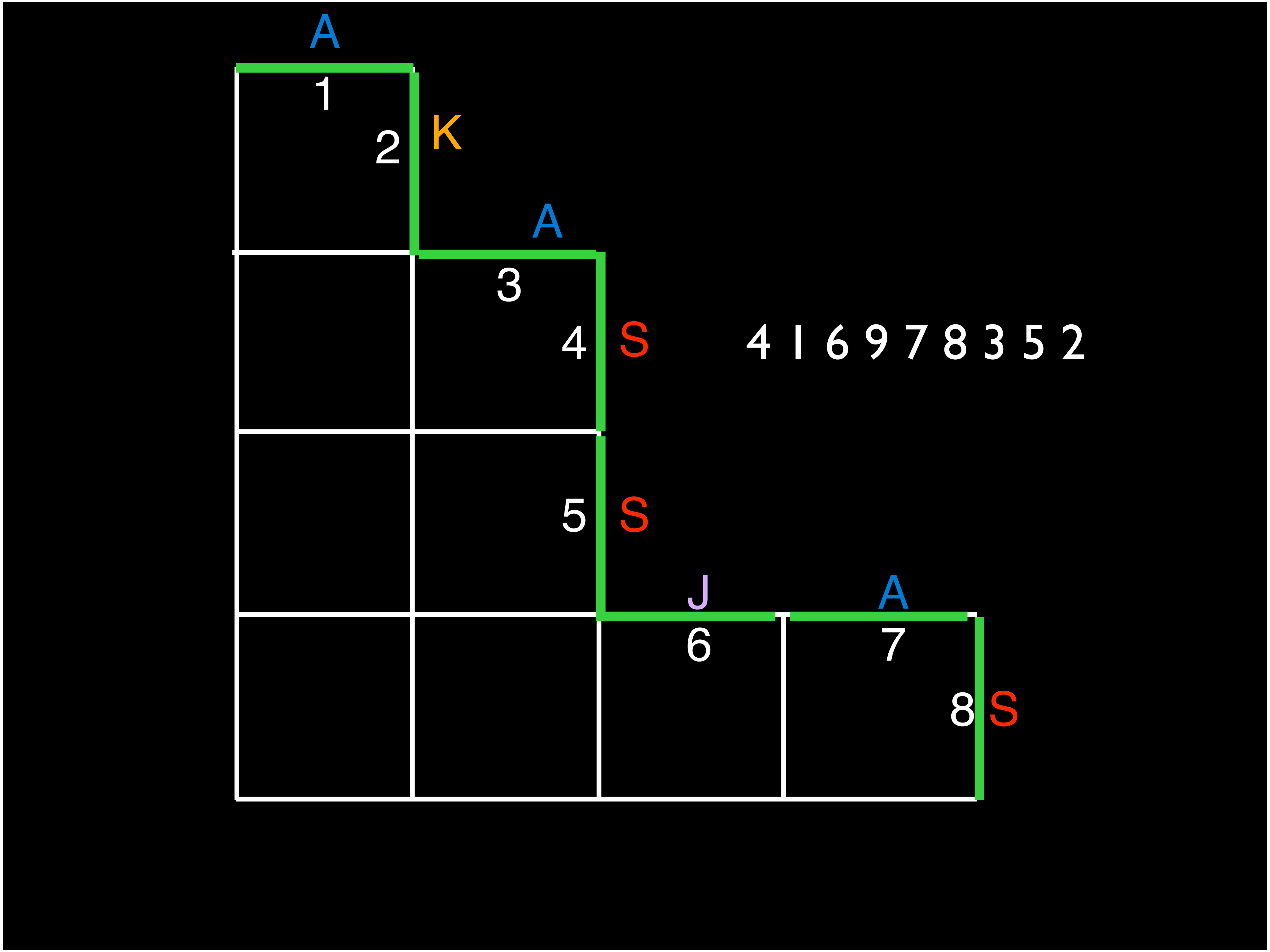

J
S
S
J
J
S
S

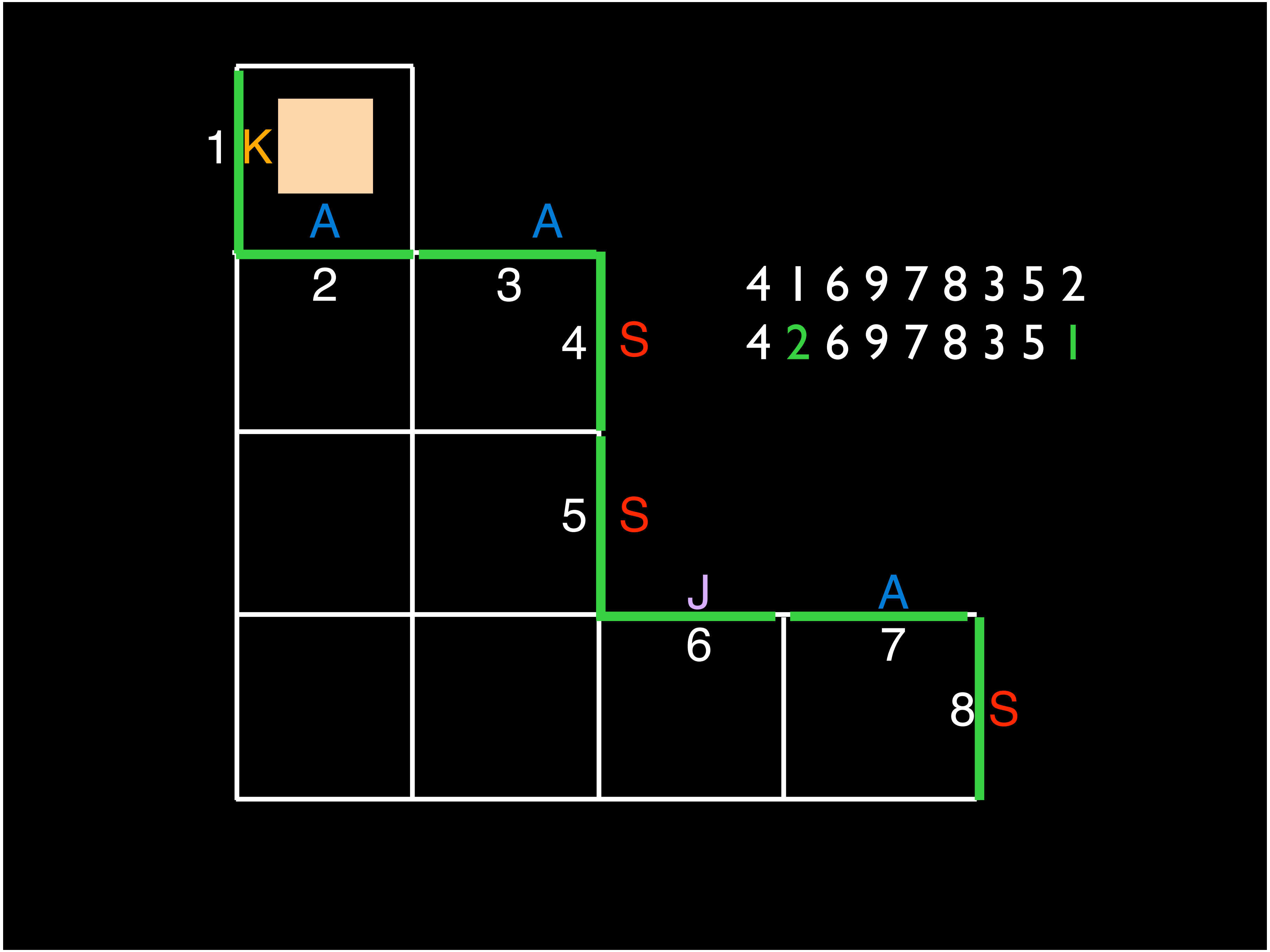


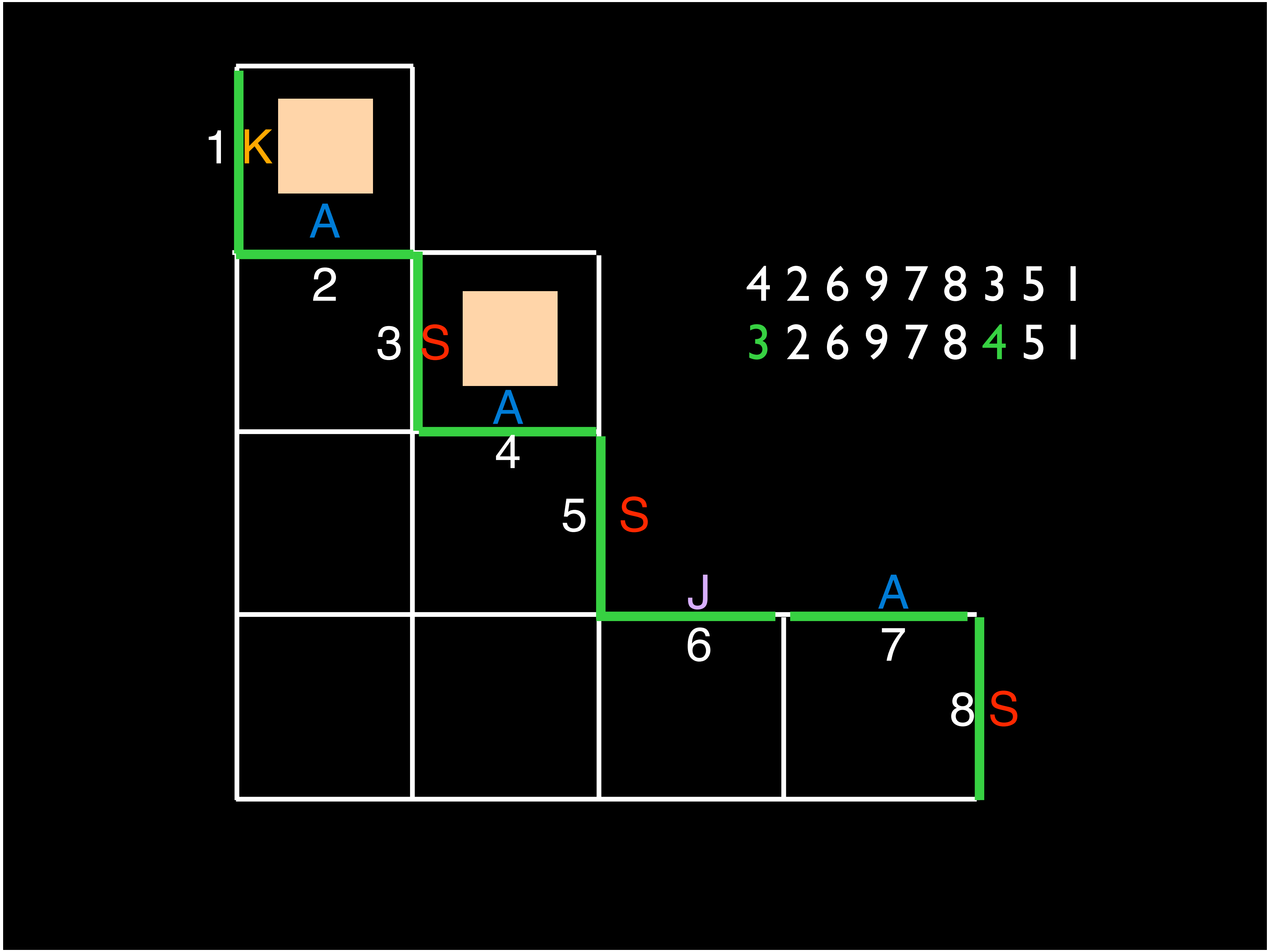
Grouping
the
“commutation
diagrams” ...



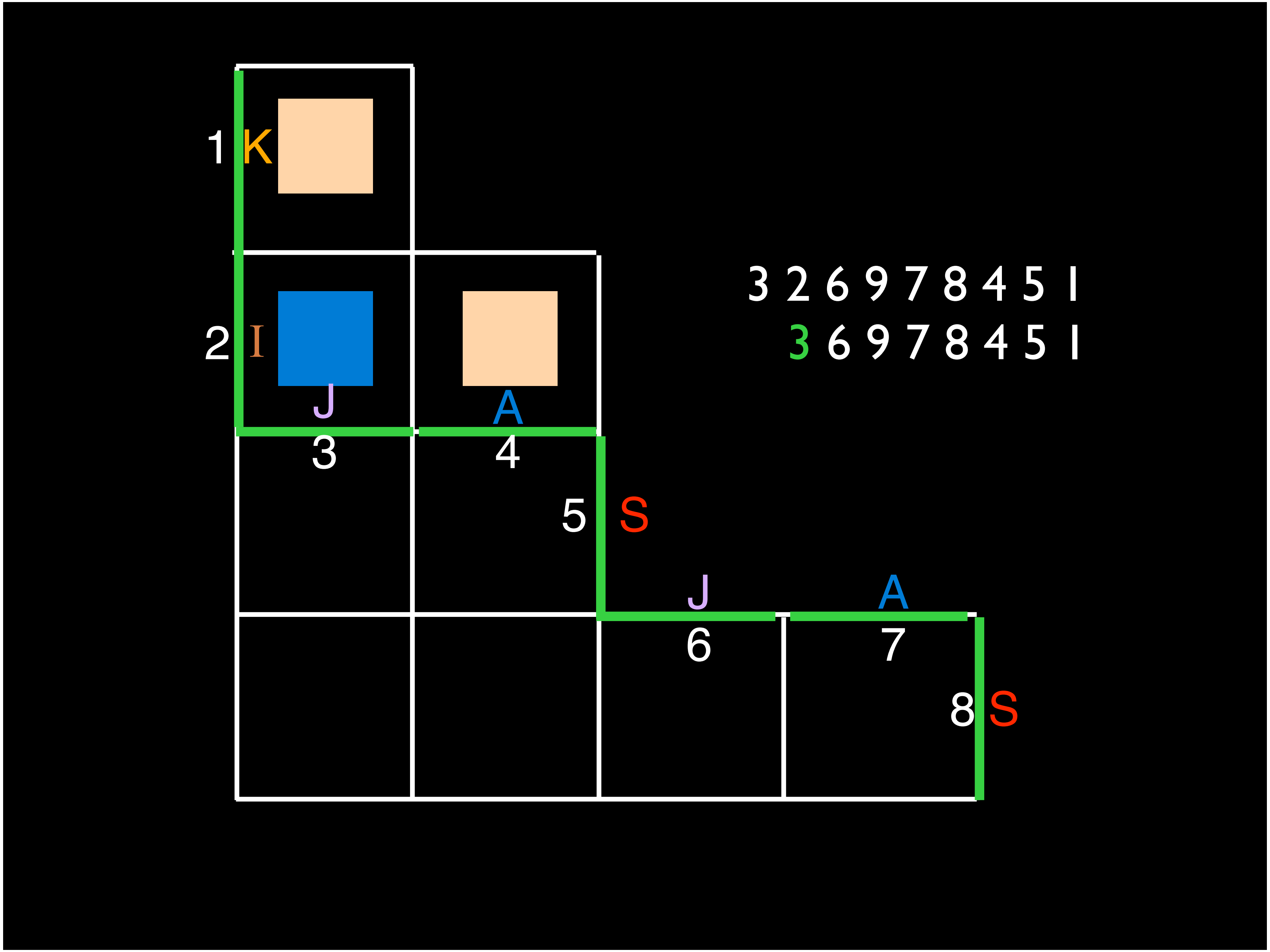
.... in order to give simple description of the "propagation" (the green line) of "local rules".

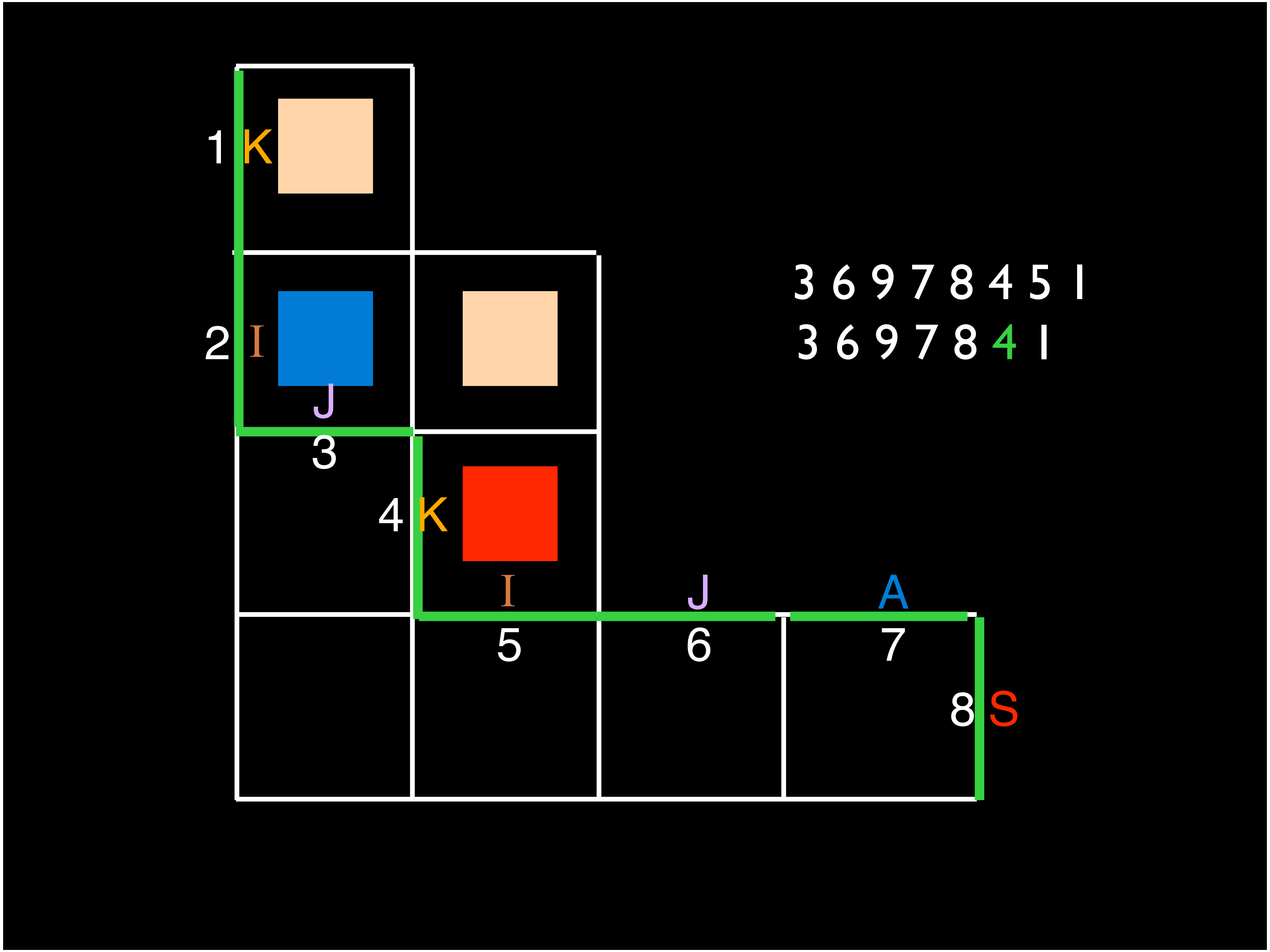

A
1
2
K
A
3
4
S
4 1 6 9 7 8 3 5 2
5
S
J
A
6
7
8
S

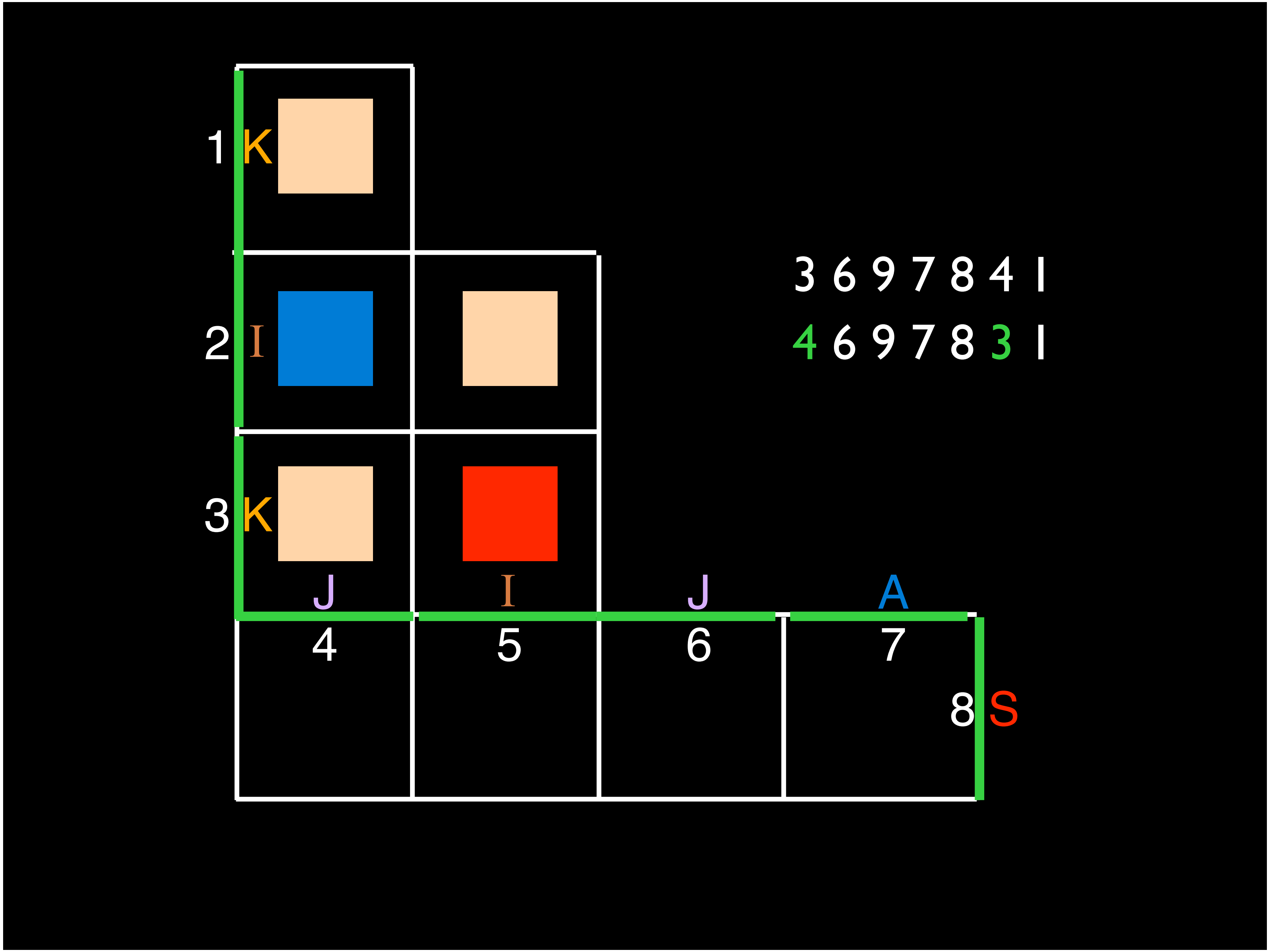

1 K
A
A
2
3
4 S
5 S
J
A
6
7
8 S
4 1 6 9 7 8 3 5 2
4 2 6 9 7 8 3 5 1

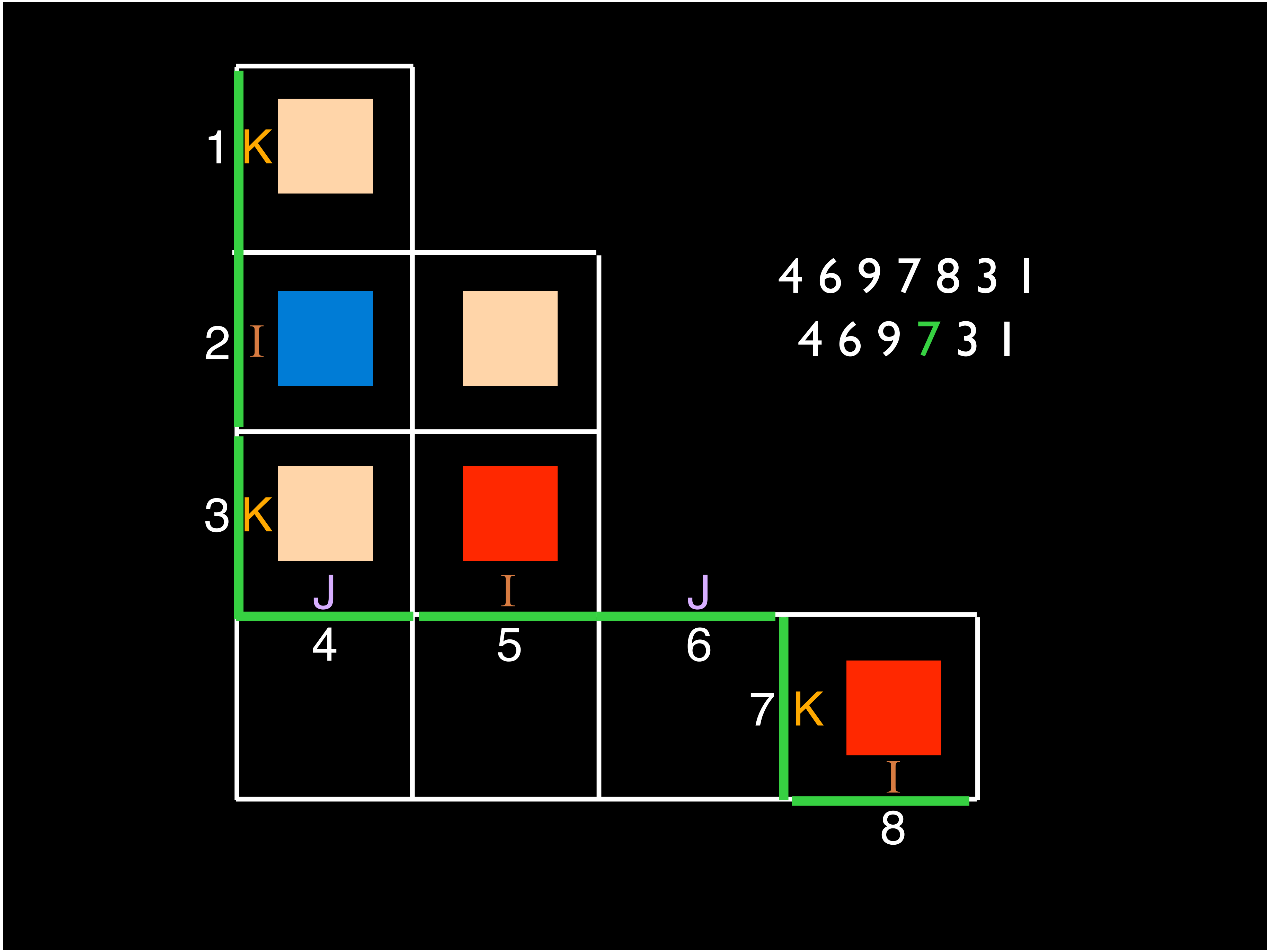


1 K
A
2
3 S
A
4
5 S
J
A
6
7
8 S
4 2 6 9 7 8 3 5 1
3 2 6 9 7 8 4 5 1

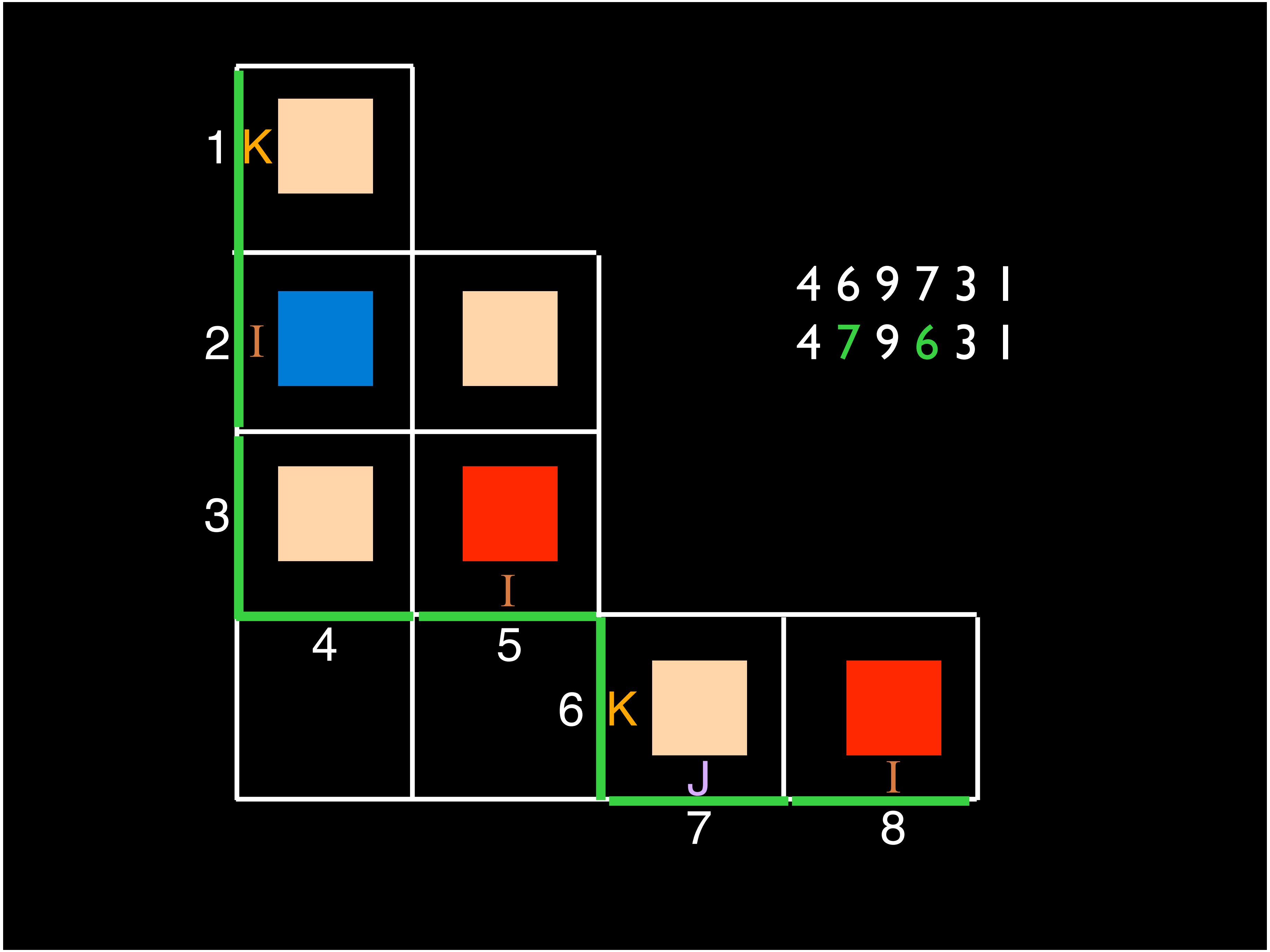

1 K
2 I
J
A
3
4
5 S
J
A
6
7
8 S
3 2 6 9 7 8 4 5 1
3 6 9 7 8 4 5 1

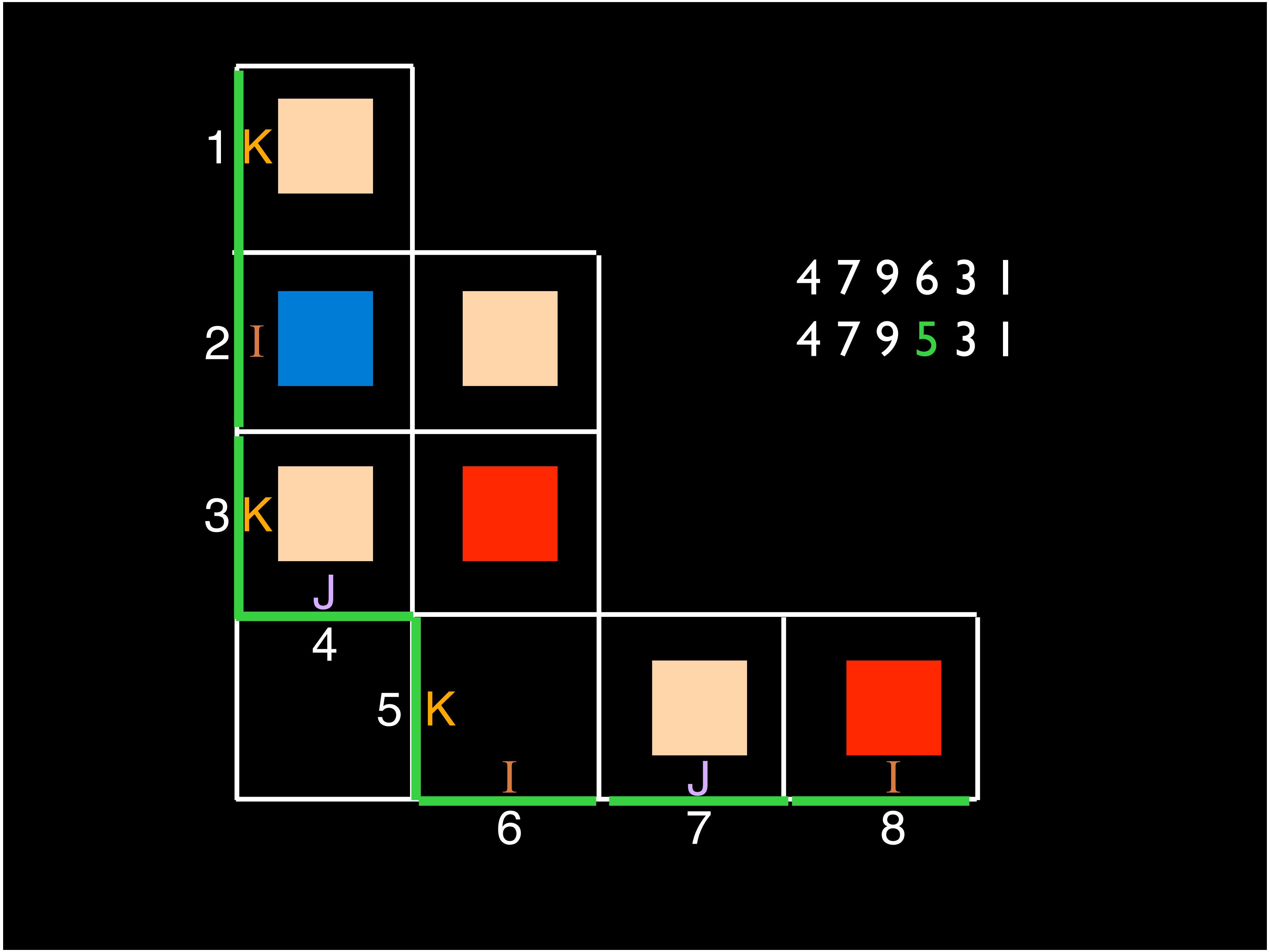

1 K
2 I
J
3
4 K
I
5
J
6
A
7
8 S
3 6 9 7 8 4 5 I
3 6 9 7 8 4 I


1 K
2 I
3 K
J I J A
4 5 6 7
8 S
3 6 9 7 8 4 1
4 6 9 7 8 3 1

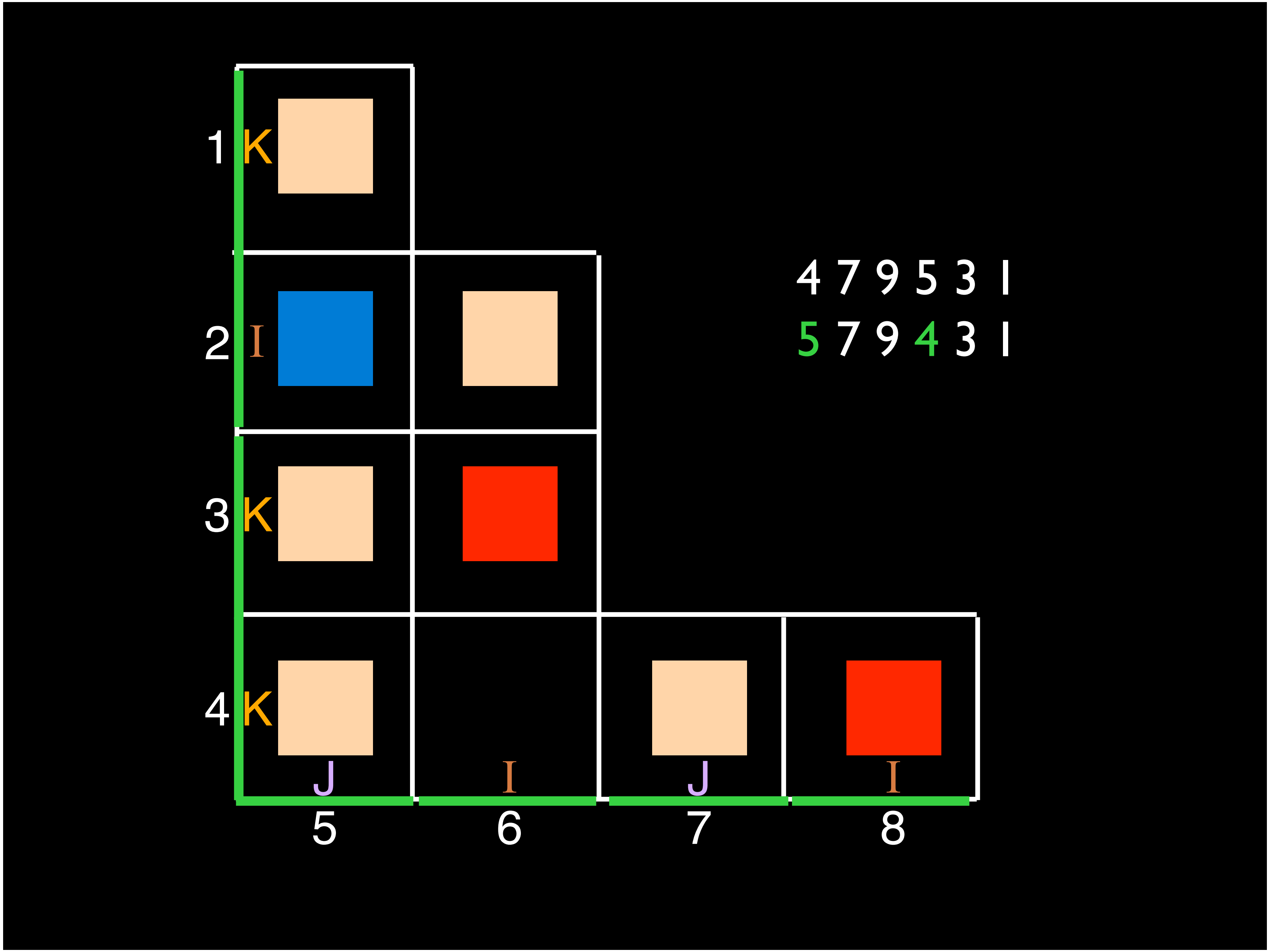


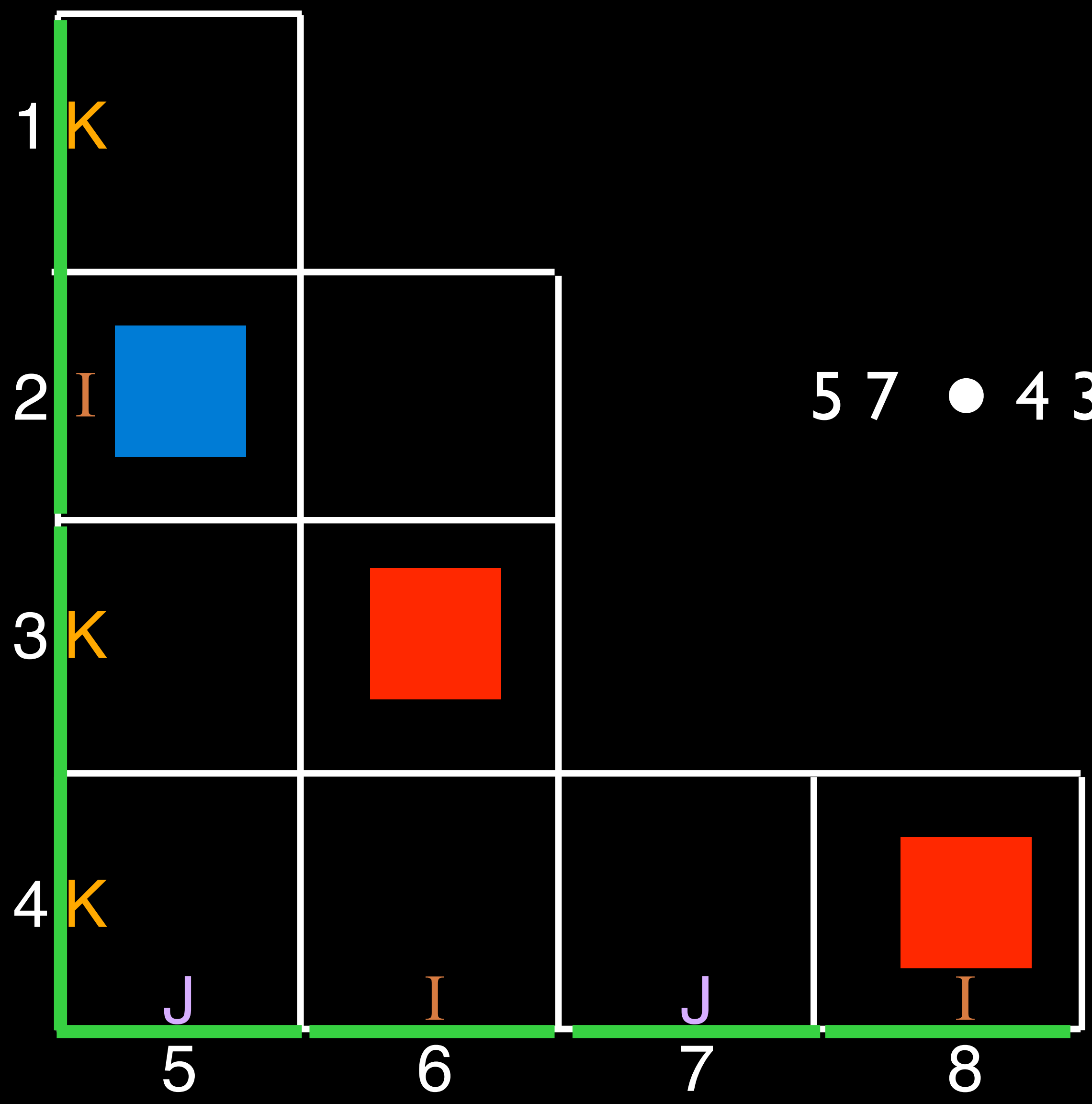
1 K
2 I
3 K
J
I
J
4
5
6
7 K
I
8
4 6 9 7 8 3 1
4 6 9 7 3 1


1 K
2 I
3
4
5
I
6 K
J
7
I
8
4 6 9 7 3 1
4 7 9 6 3 1

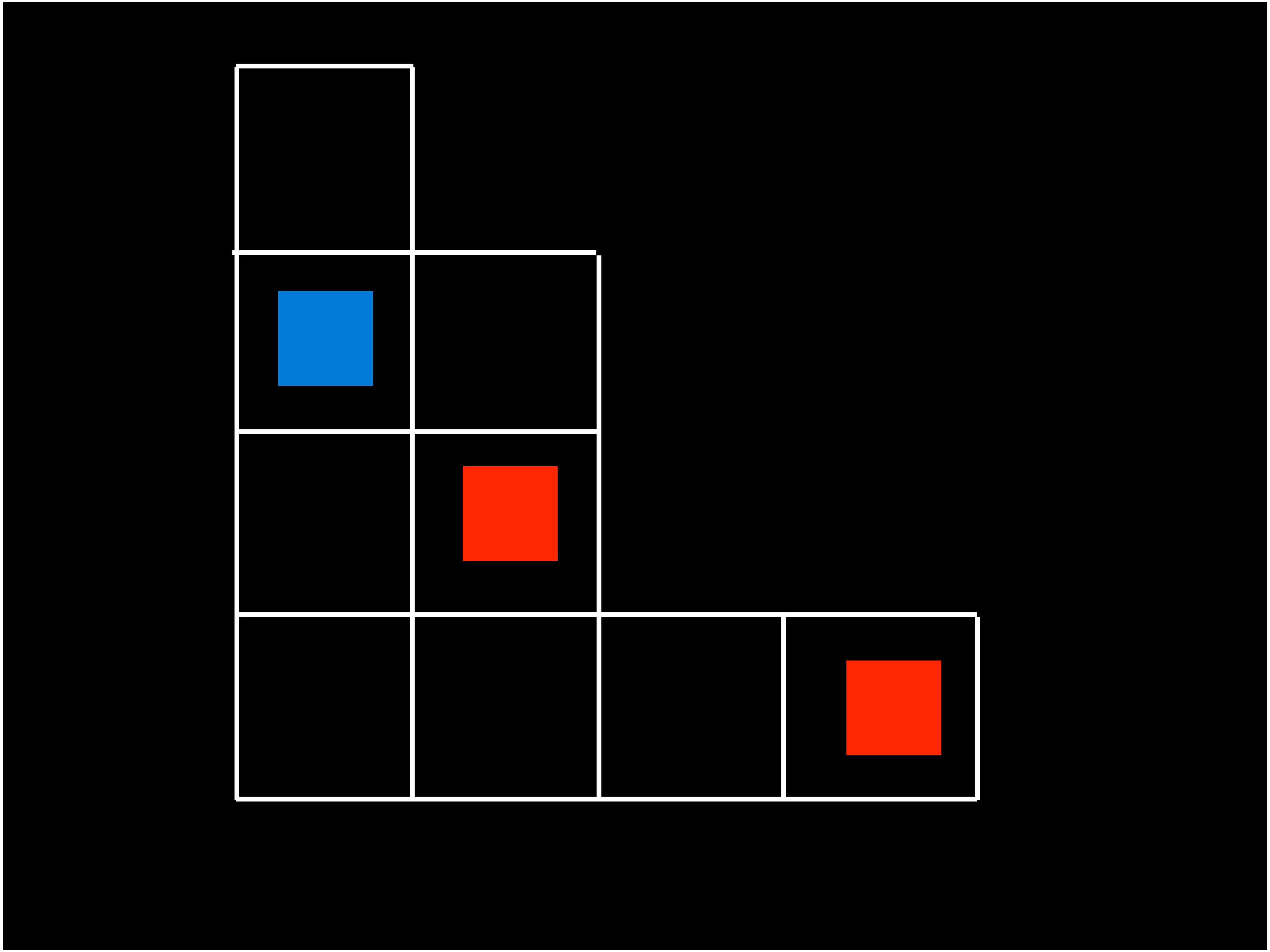


1 K
2 I
3 K
J
4
5 K
I
J
I
6
7
8
4 7 9 6 3 1
4 7 9 5 3 1

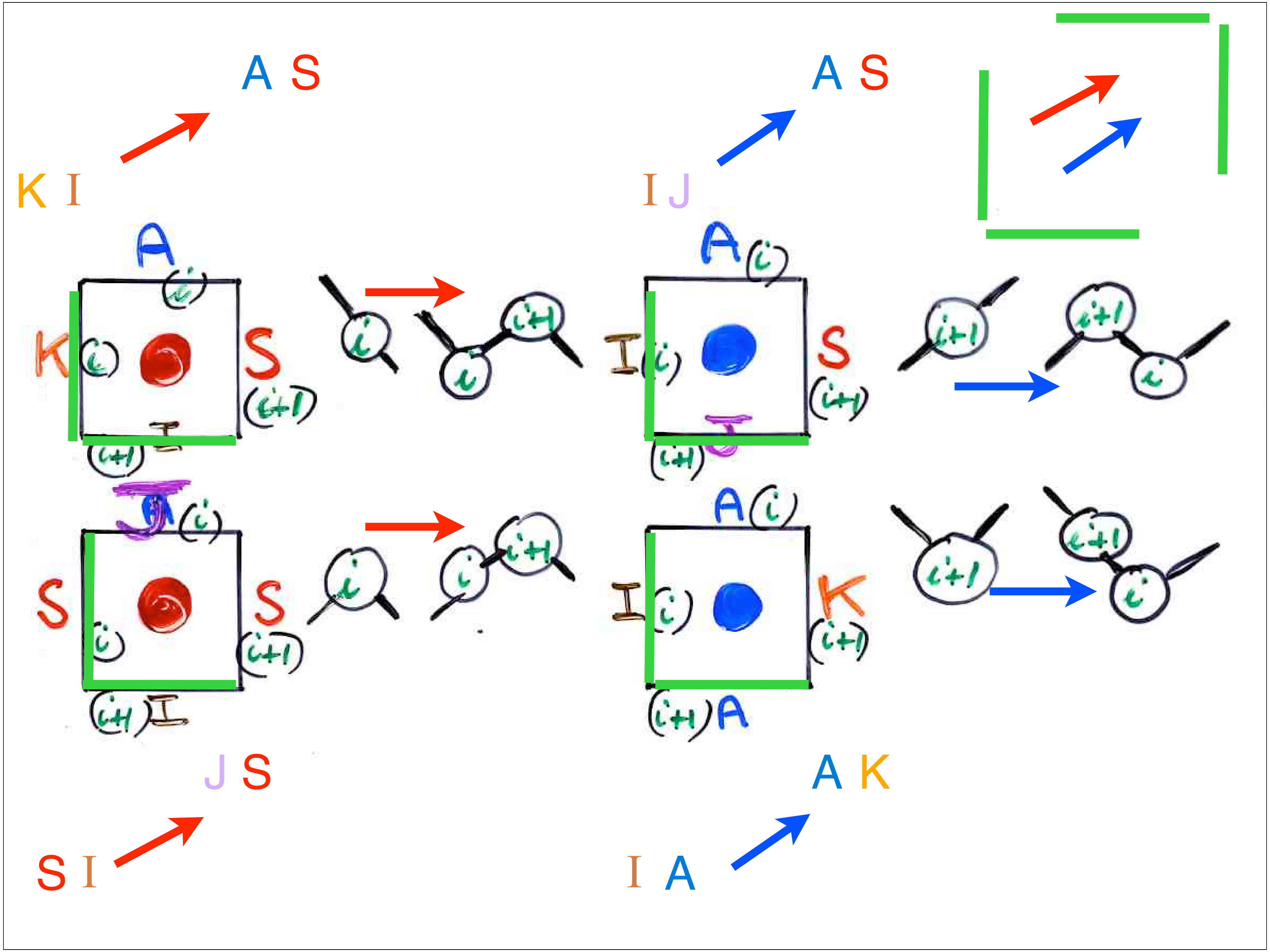

1 K
2 I
3 K
4 K
J I J I
5 6 7 8
4 7 9 5 3 1
5 7 9 4 3 1



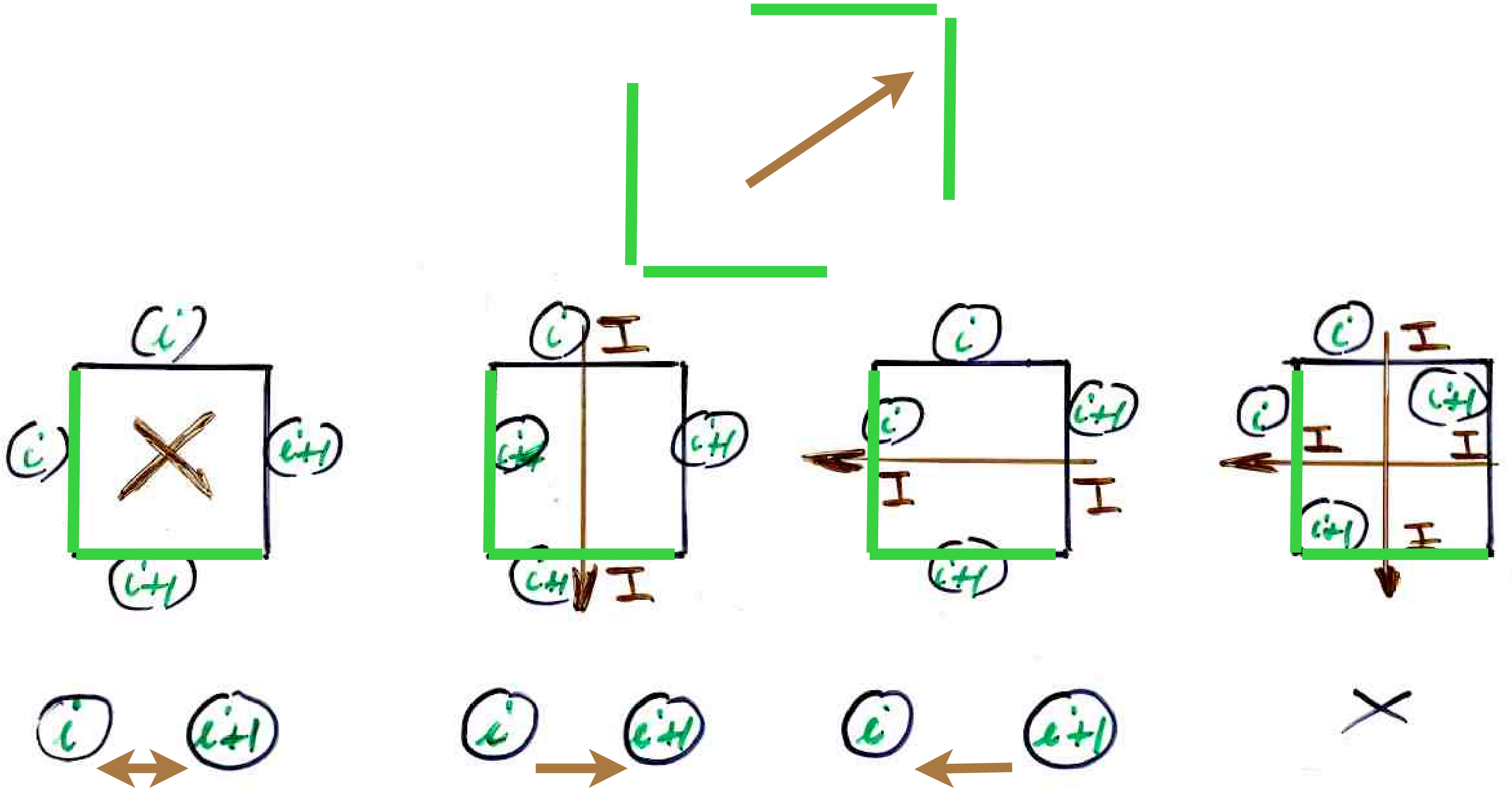
5 7 • 4 3 1

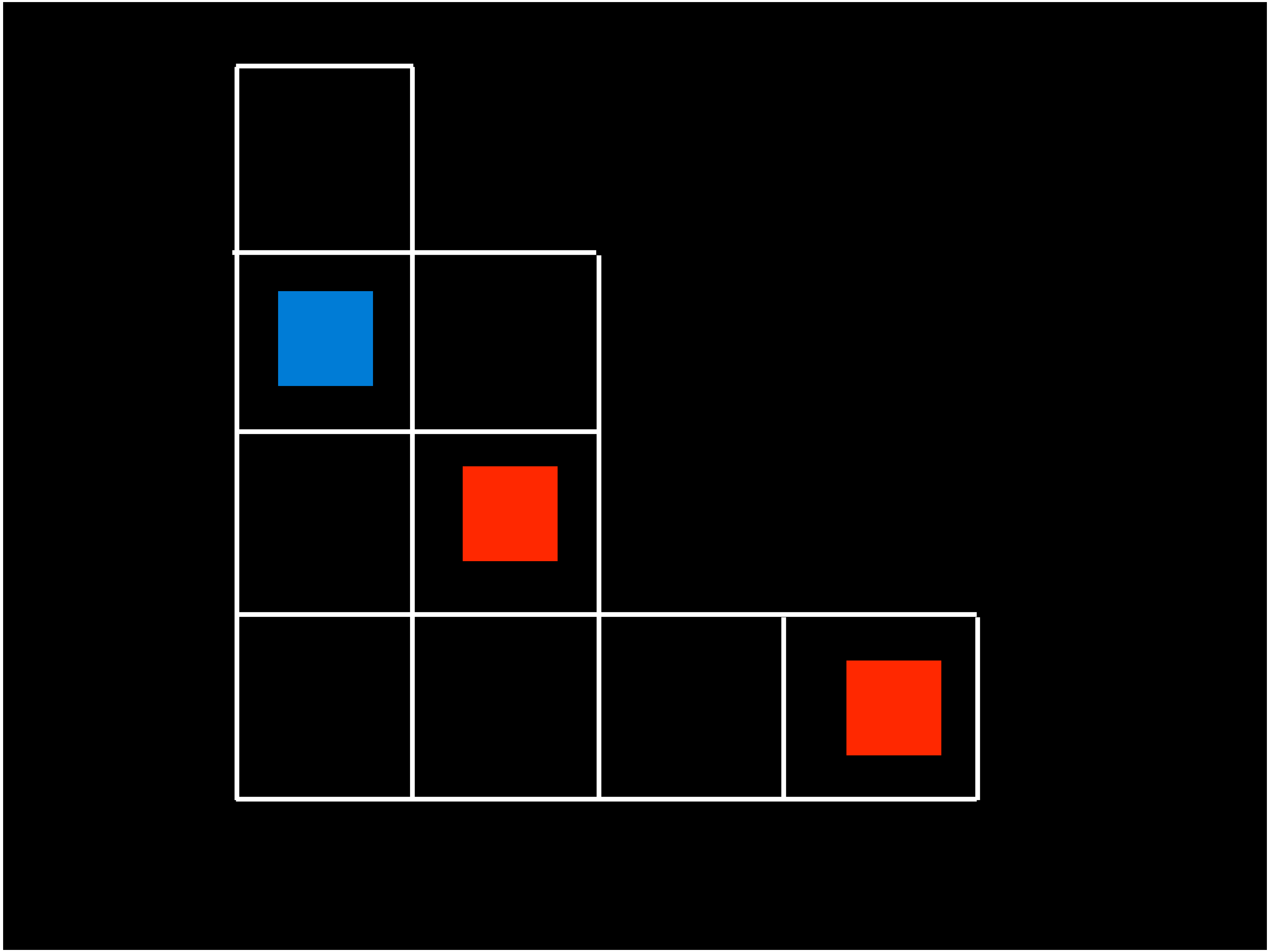


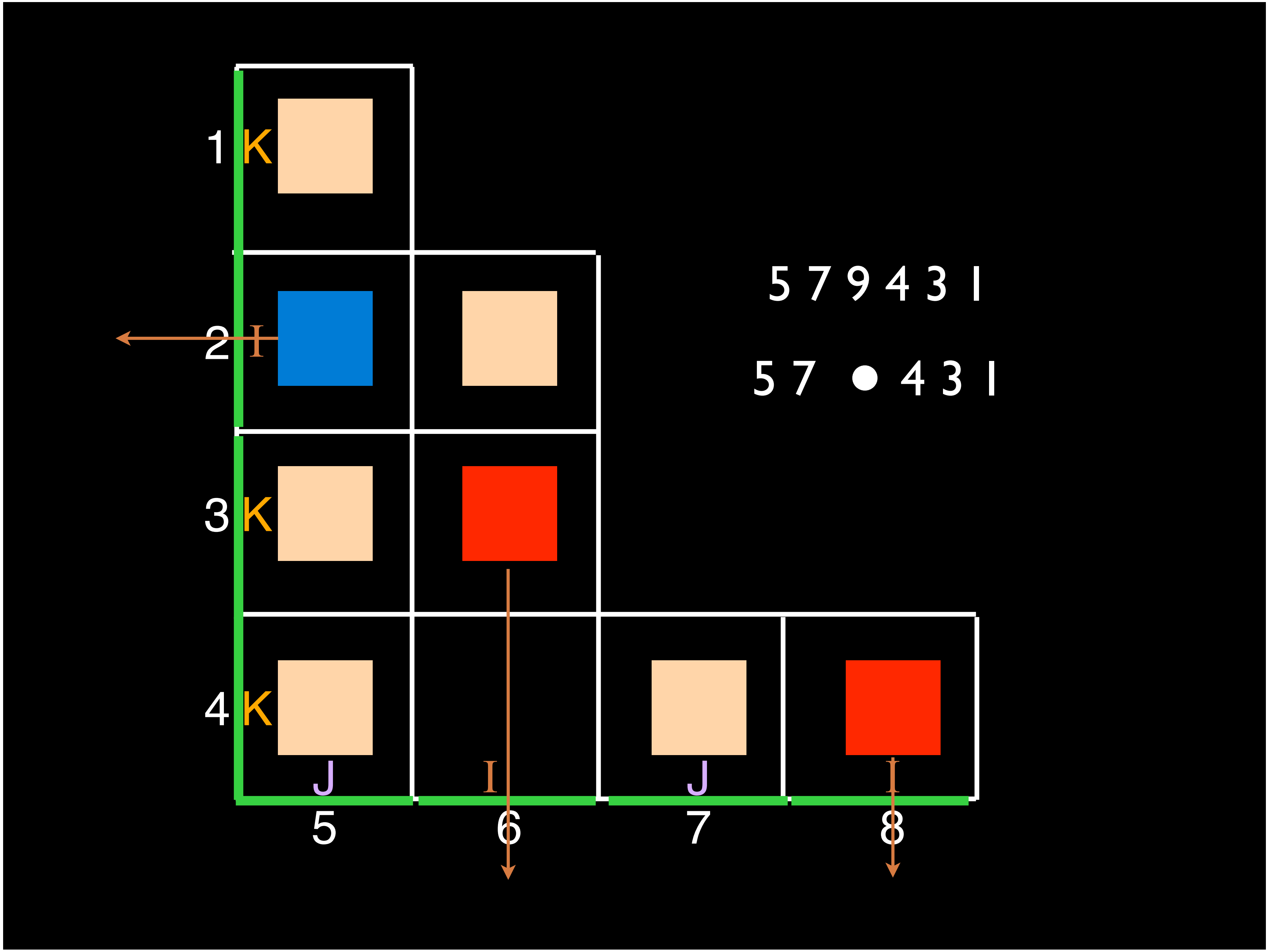
the inverse algorithm



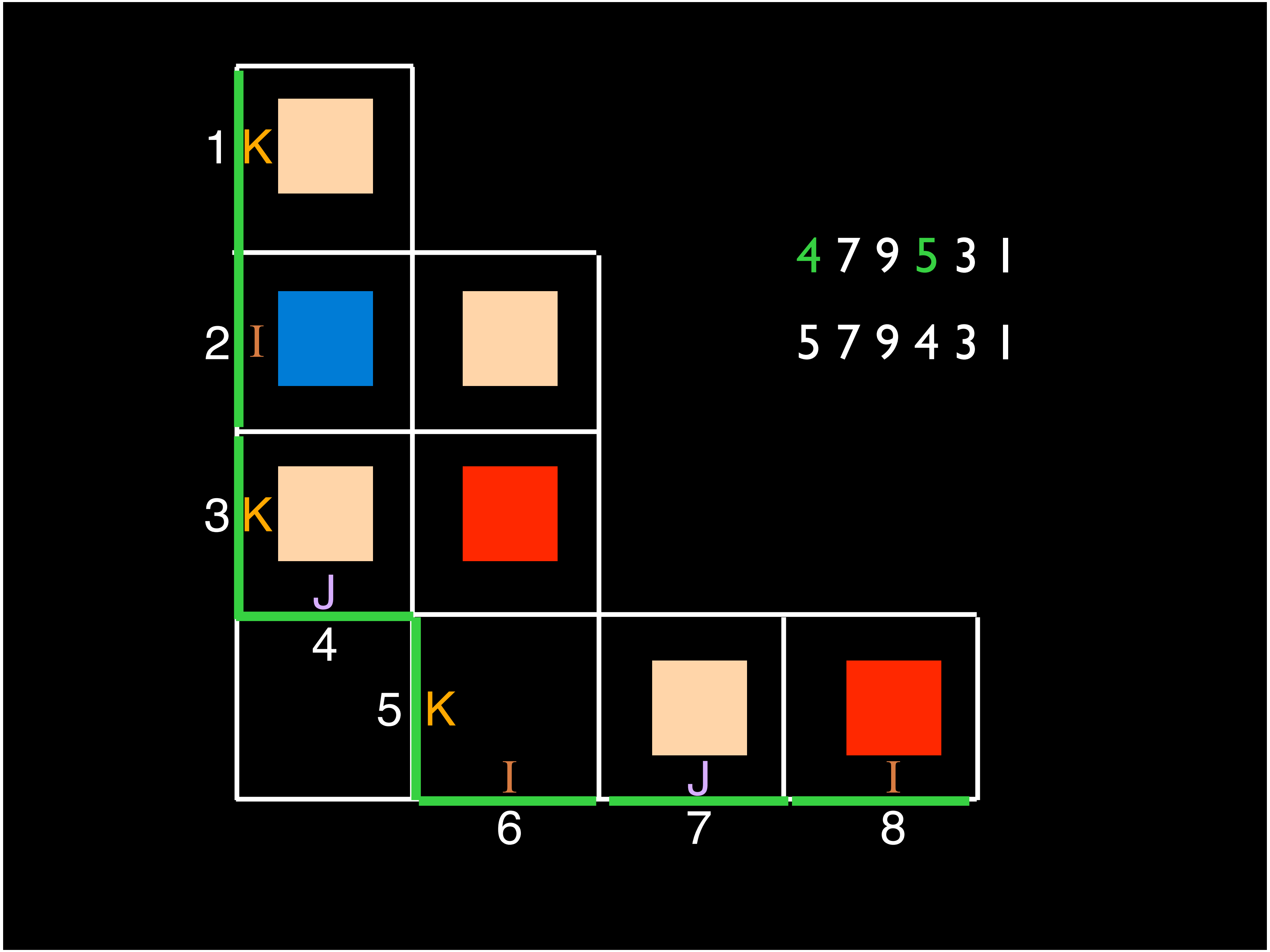
A S
K I
A S
I J
J S
S I
A K
I A


i
i+1
I

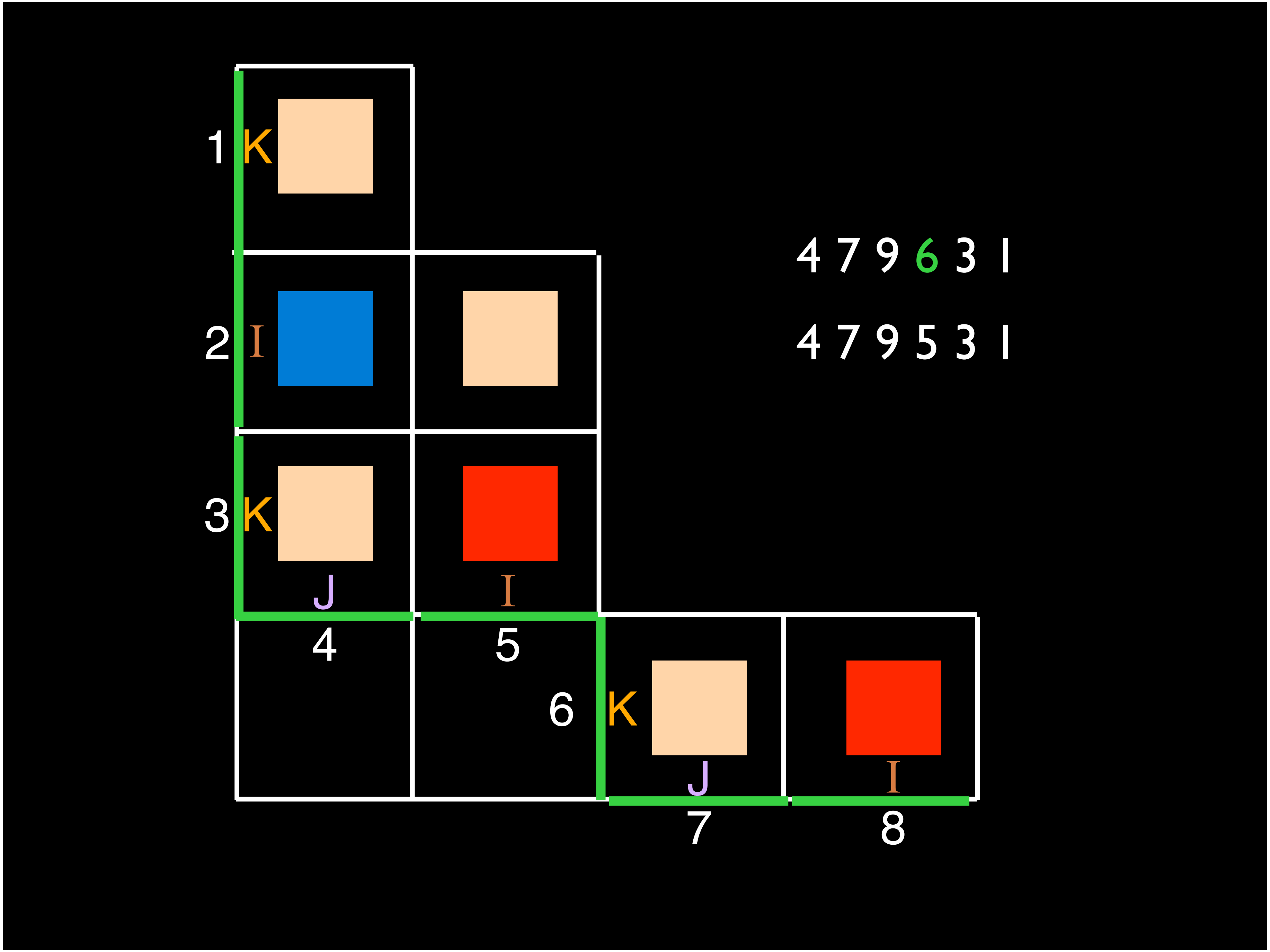




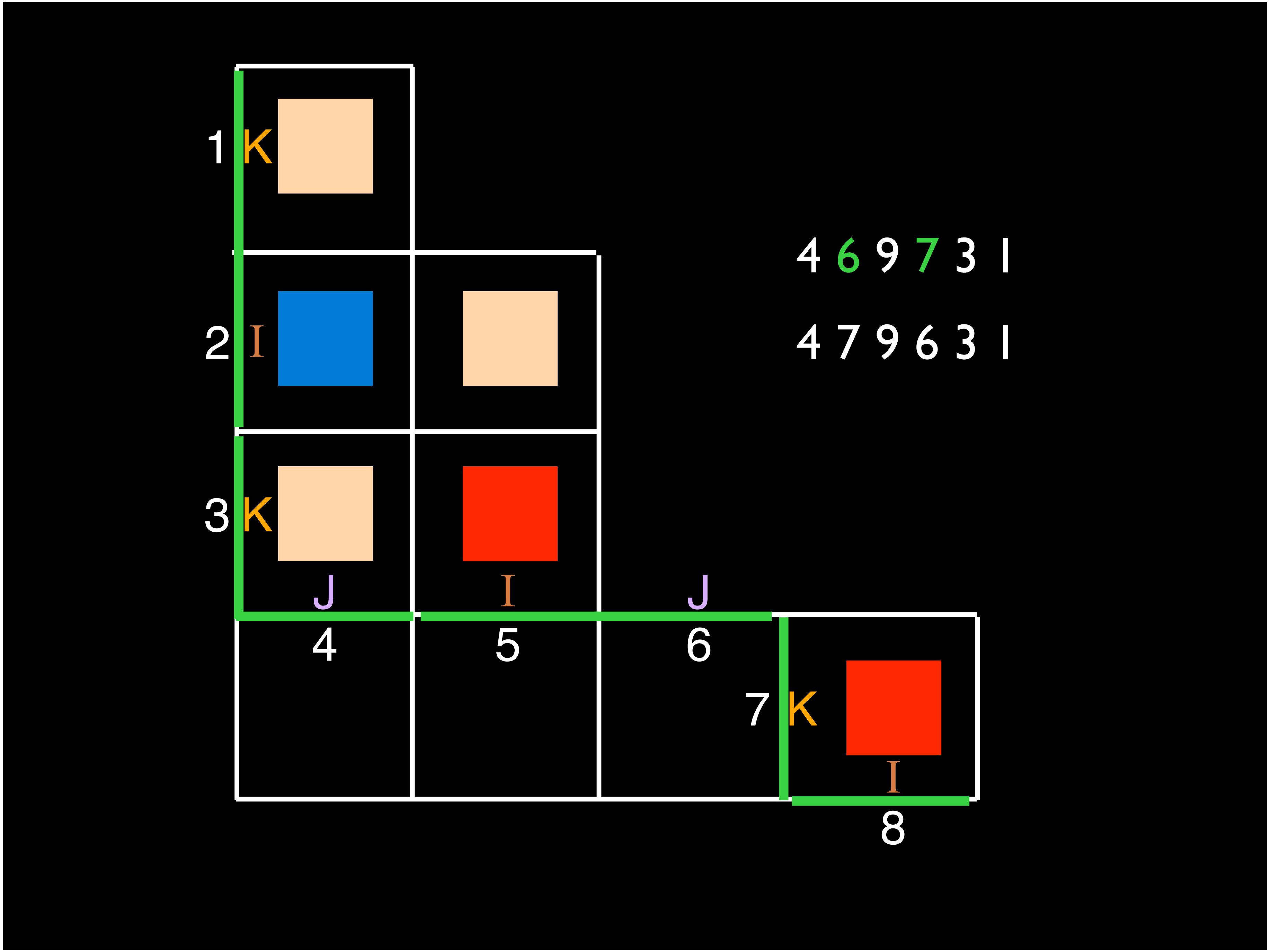
1 K
2 I
3 K
4 K
J I J I
5 6 7 8
5 7 9 4 3 1
5 7 ● 4 3 1



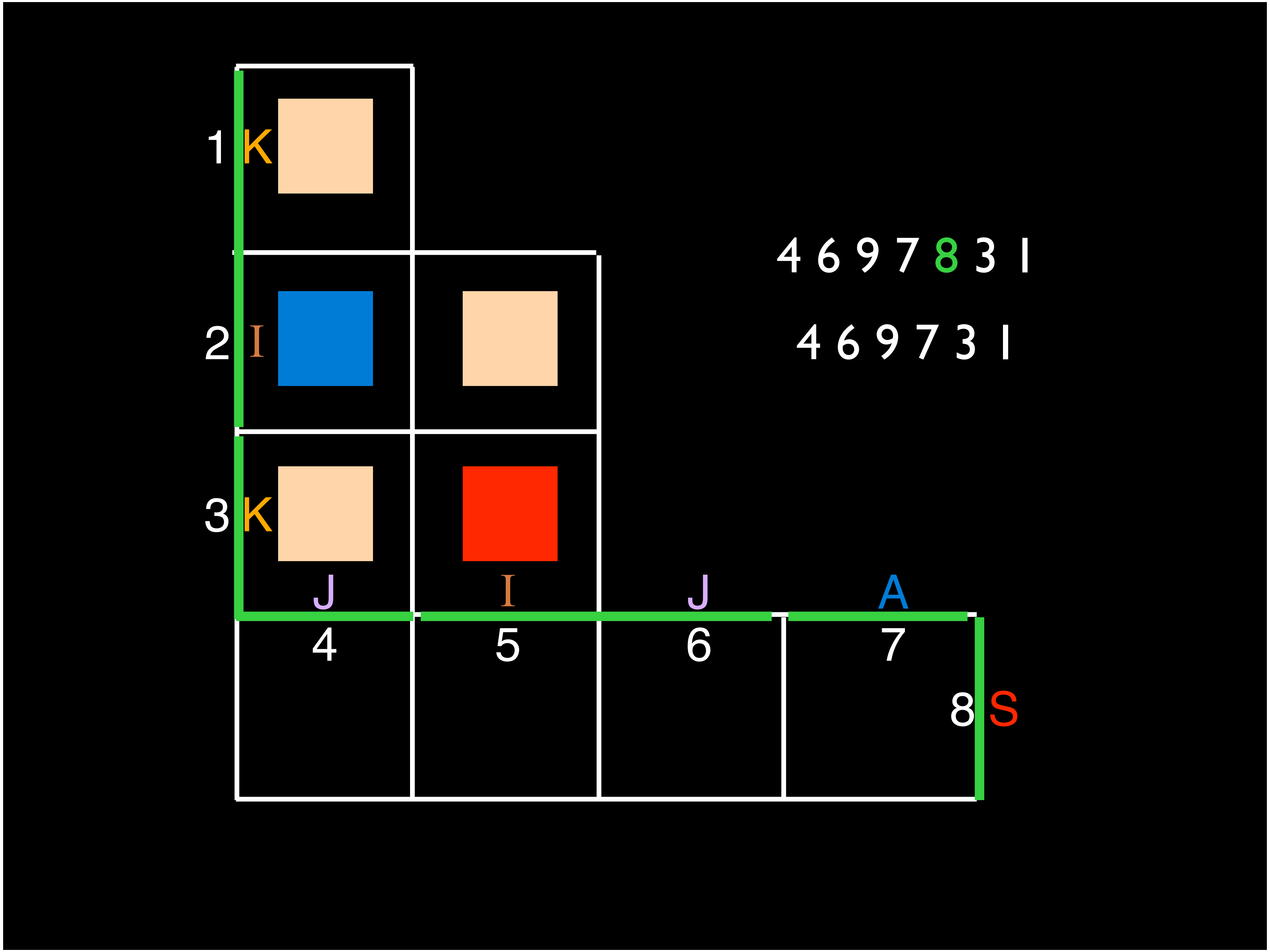
1 K
2 I
3 K
J
4
5 K
I
J
I
6
7
8
4 7 9 5 3 1
5 7 9 4 3 1

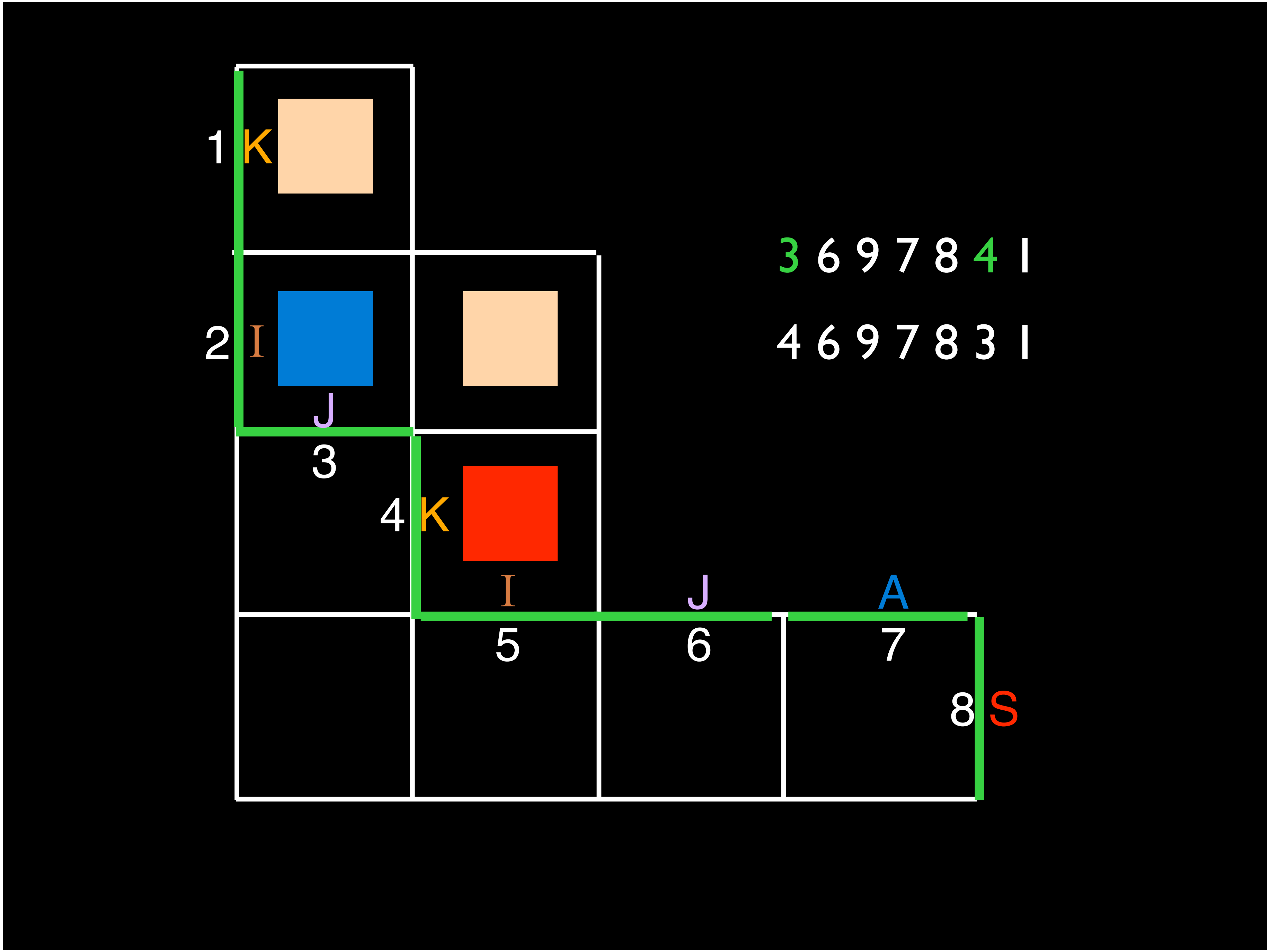


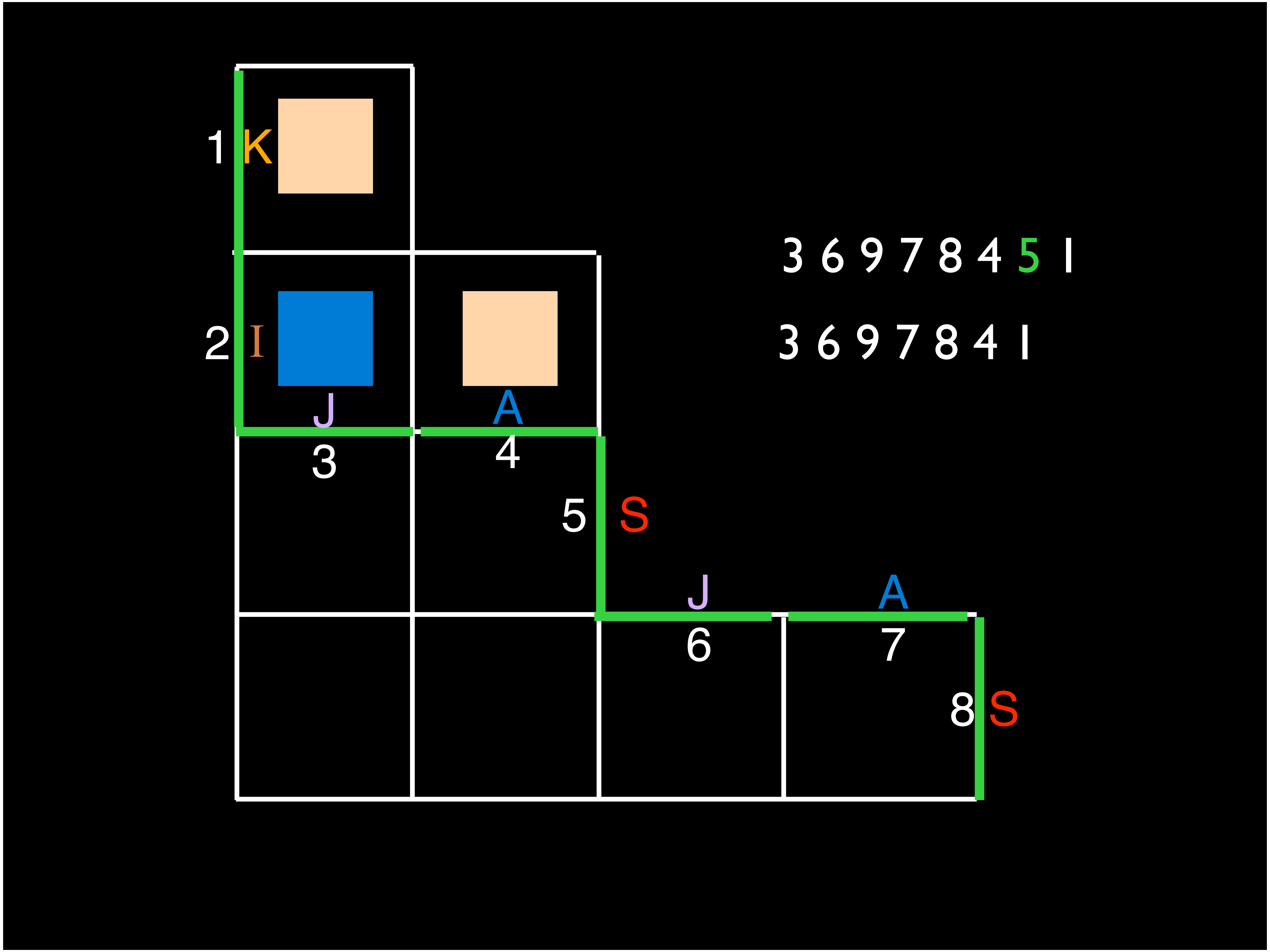
1 K
2 I
3 K
J
I
4
5
6 K
J
I
7
8
4 7 9 6 3 1
4 7 9 5 3 1



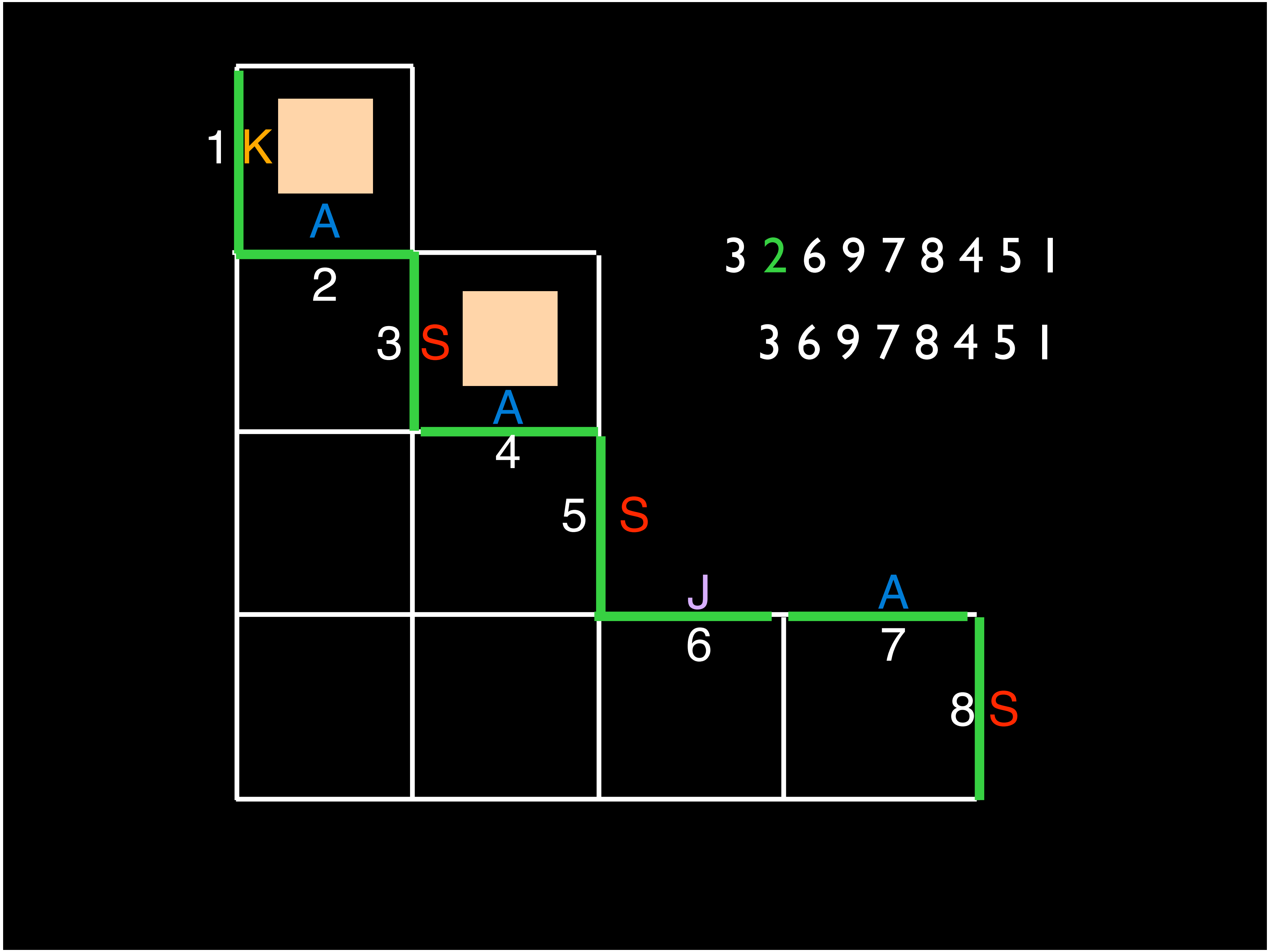
1 K
2 I
3 K
J
I
J
4
5
6
7 K
I
8
4 6 9 7 3 1
4 7 9 6 3 1

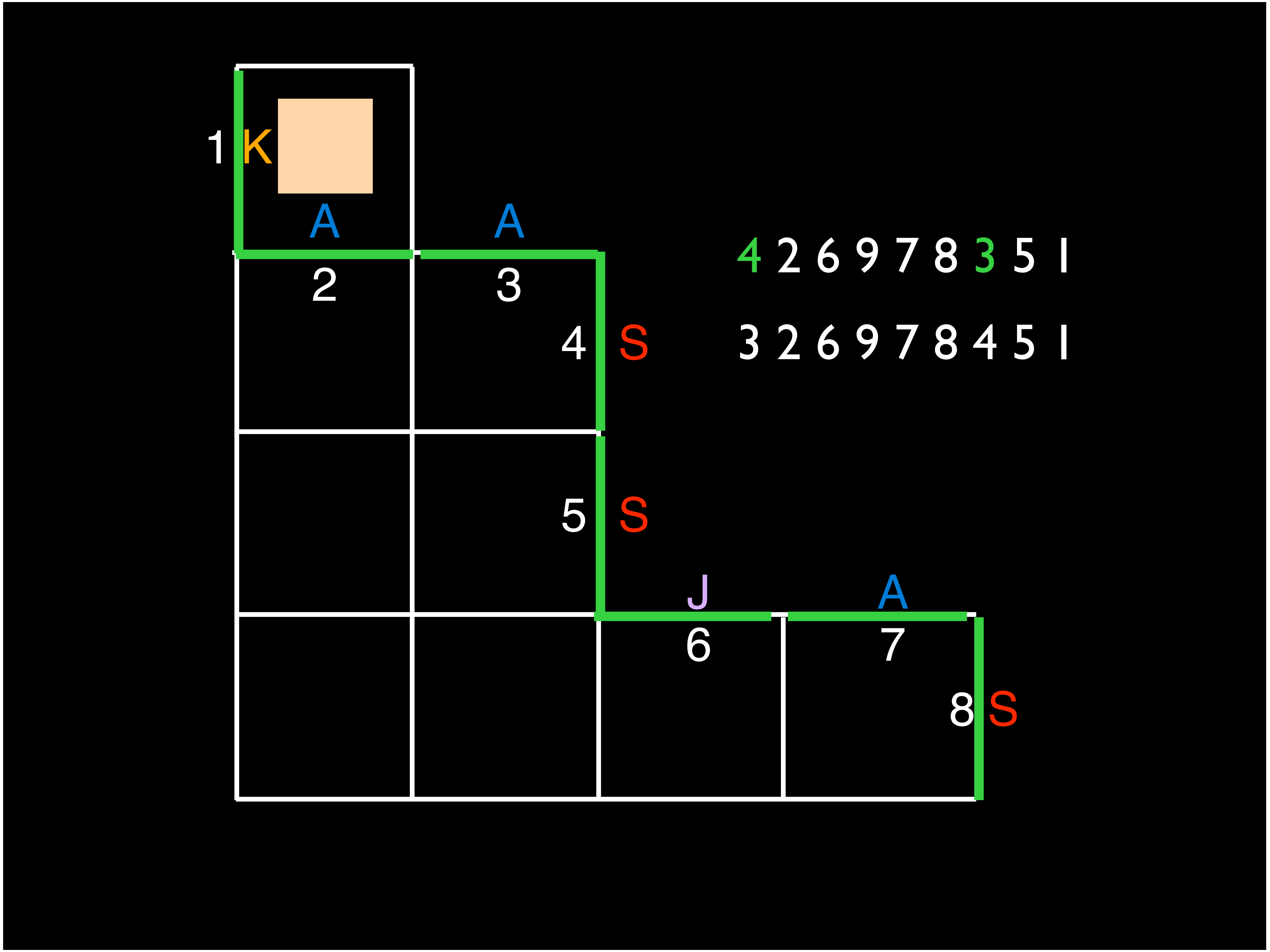

1 K
2 I
3 K
J
I
J
A
4
5
6
7
8 S
4 6 9 7 8 3 1
4 6 9 7 3 1

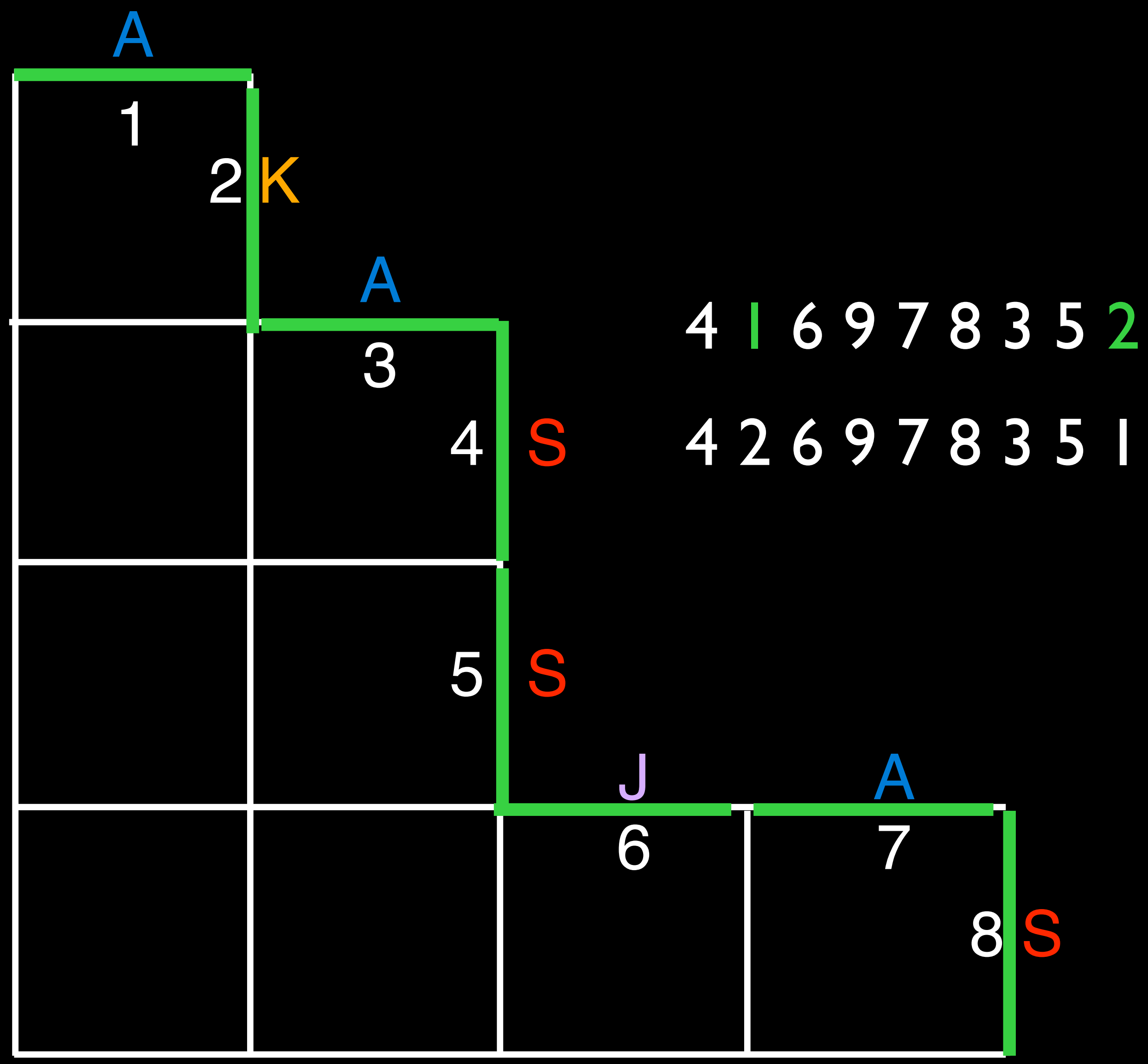

1 K
2 I
J
3
4 K
I
J
A
5
6
7
8 S
3 6 9 7 8 4 1
4 6 9 7 8 3 1



1 K
2 I
J
A
3
4
5 S
J
A
6
7
8 S
3 6 9 7 8 4 5 1
3 6 9 7 8 4 1


1 K
A
2
3 S
A
4
5 S
J
A
6
7
8 S
3 2 6 9 7 8 4 5 1
3 6 9 7 8 4 5 1


1 K
A
A
2
3
4 S
5 S
J
A
6
7
8 S
4 2 6 9 7 8 3 5 1
3 2 6 9 7 8 4 5 1



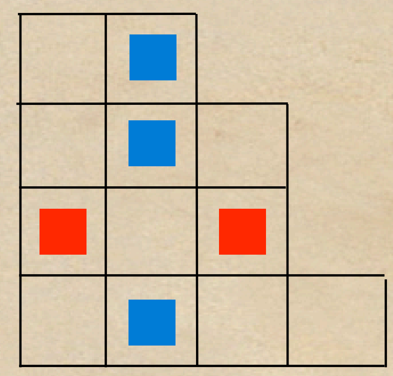
A
1
2 K
A
3
4 S
5 S
J
A
6
7
8 S
4 1 6 9 7 8 3 5 2
4 2 6 9 7 8 3 5 1

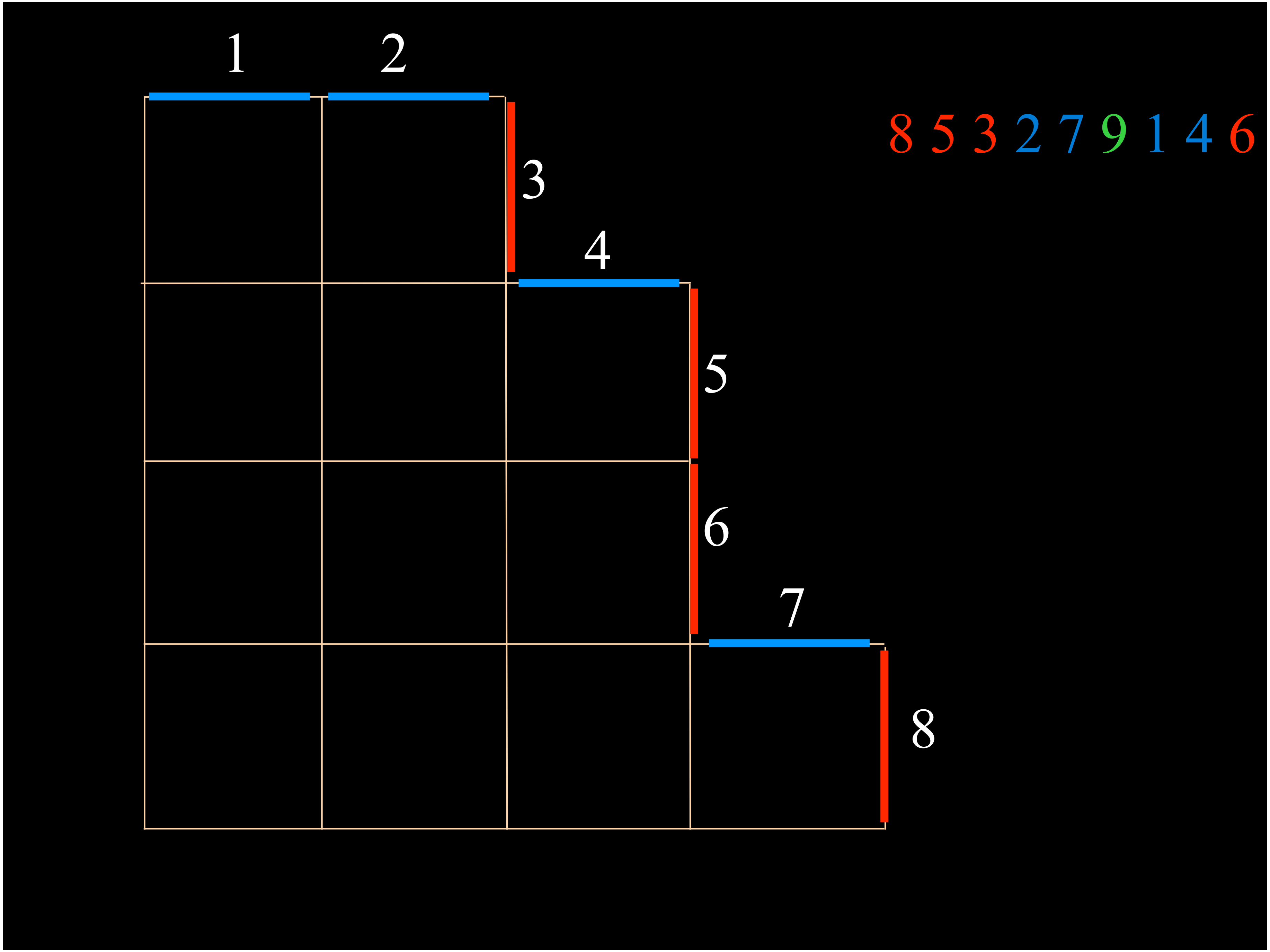
A description of the algorithm
going from a permutation to an alternative tableau
using "local rules" with a variant:
keeping track of the deleted values

with the permutation of the initial example:

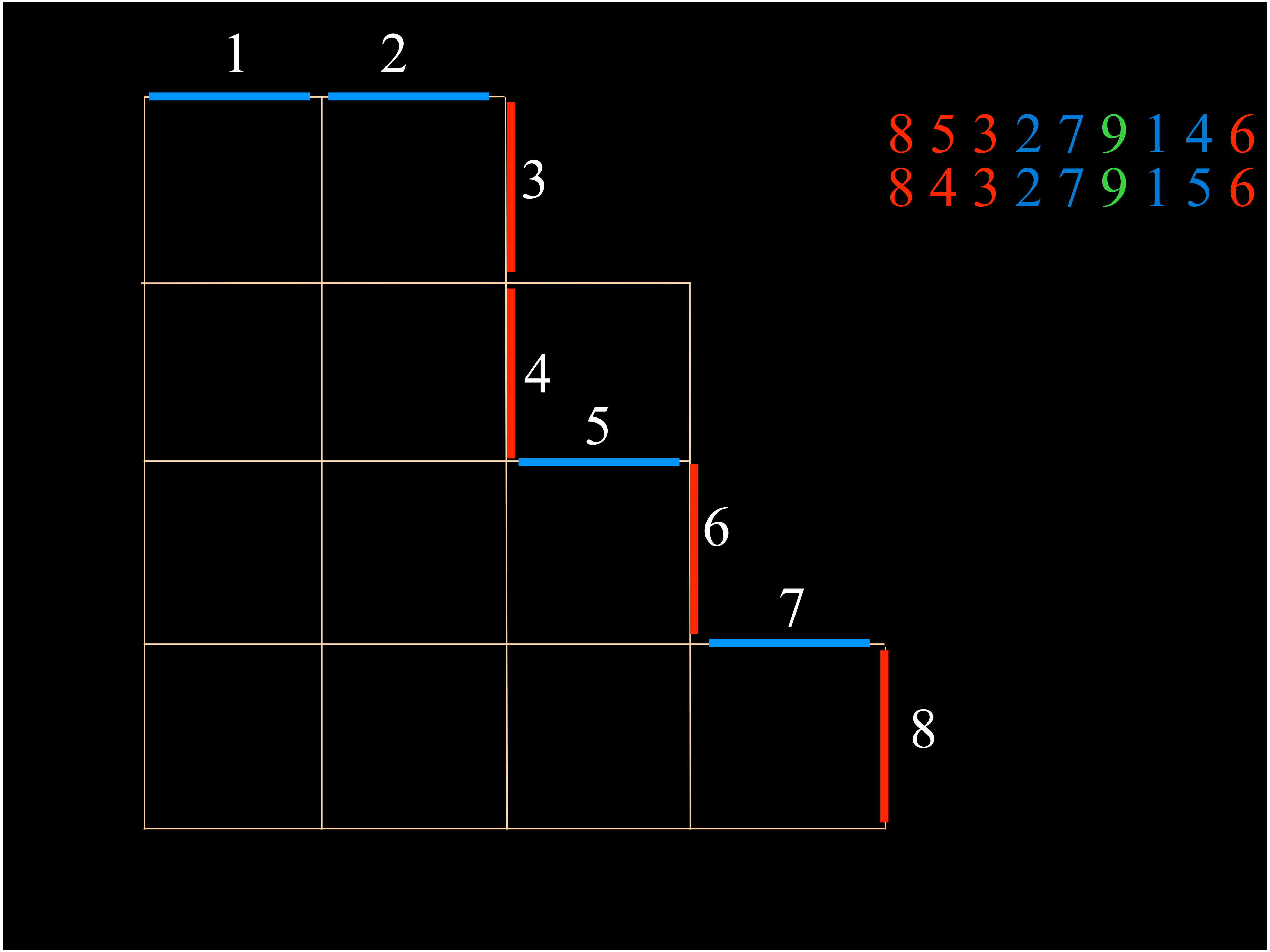
$$\begin{matrix} & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ s = & 7 & 4 & 3 & 8 & 2 & 9 & 5 & 1 & 6 \end{matrix}$$

$$\begin{matrix} & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ s^{-1} = & 8 & 5 & 3 & 2 & 7 & 9 & 1 & 4 & 6 \end{matrix}$$

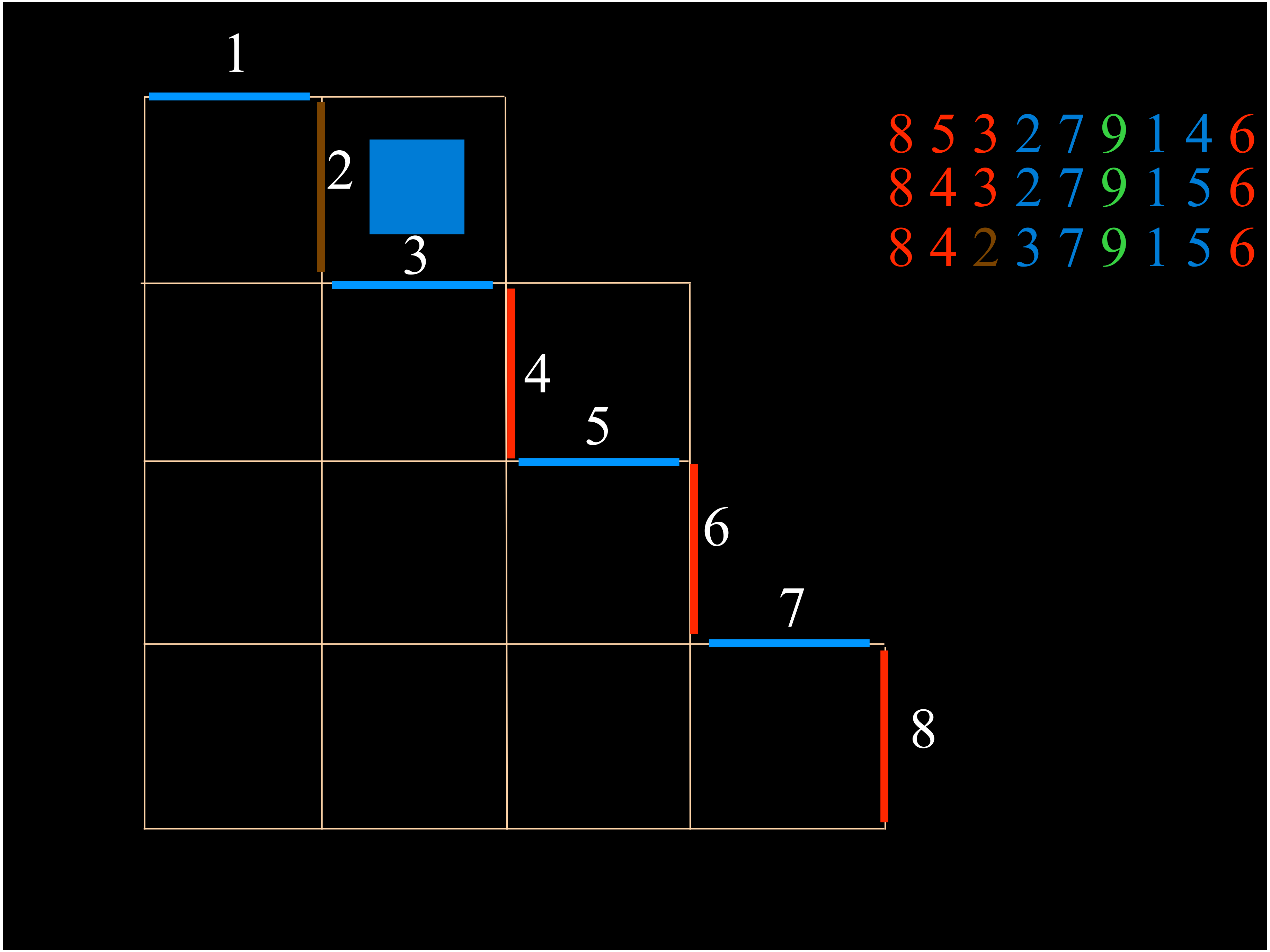


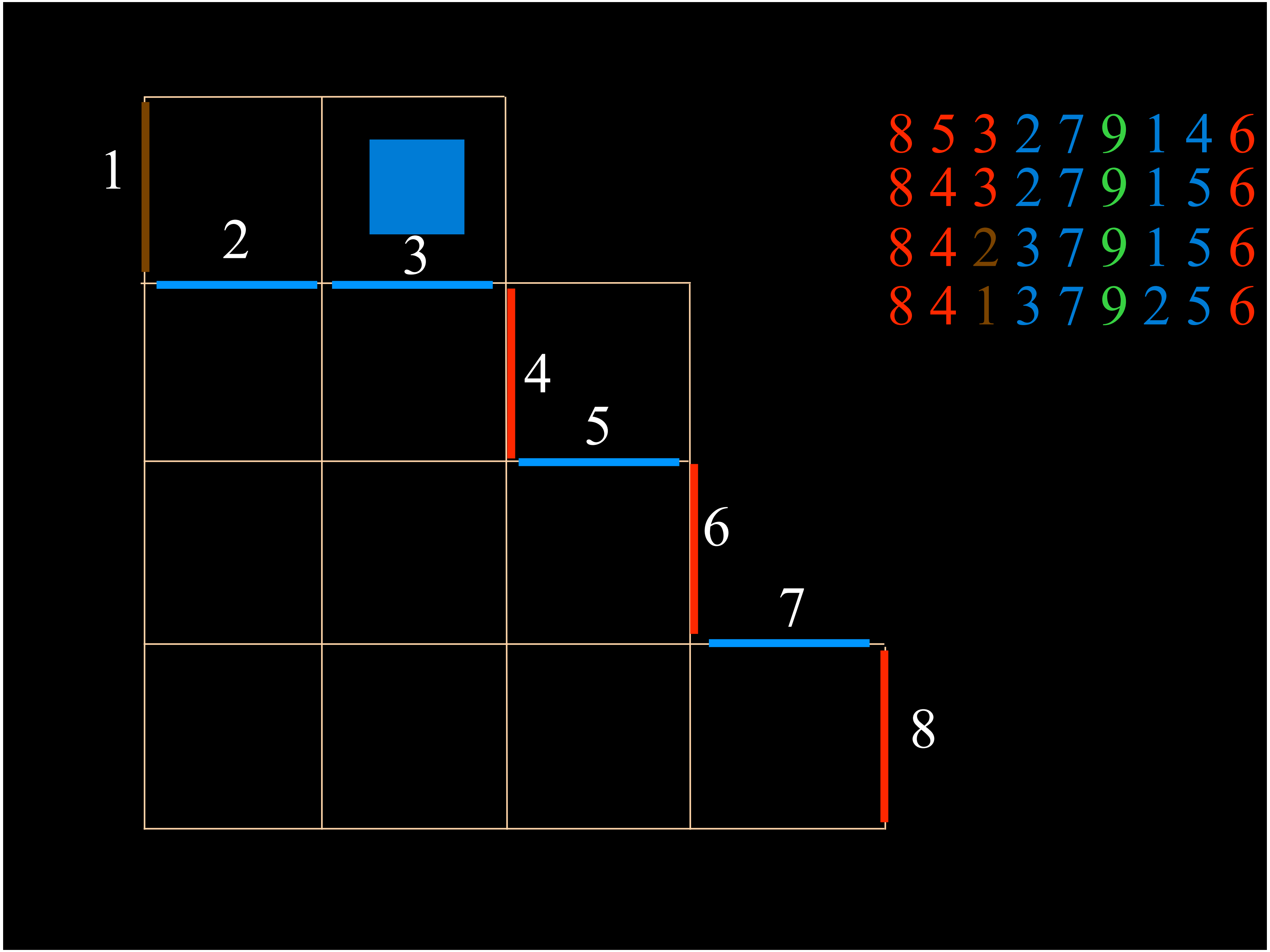


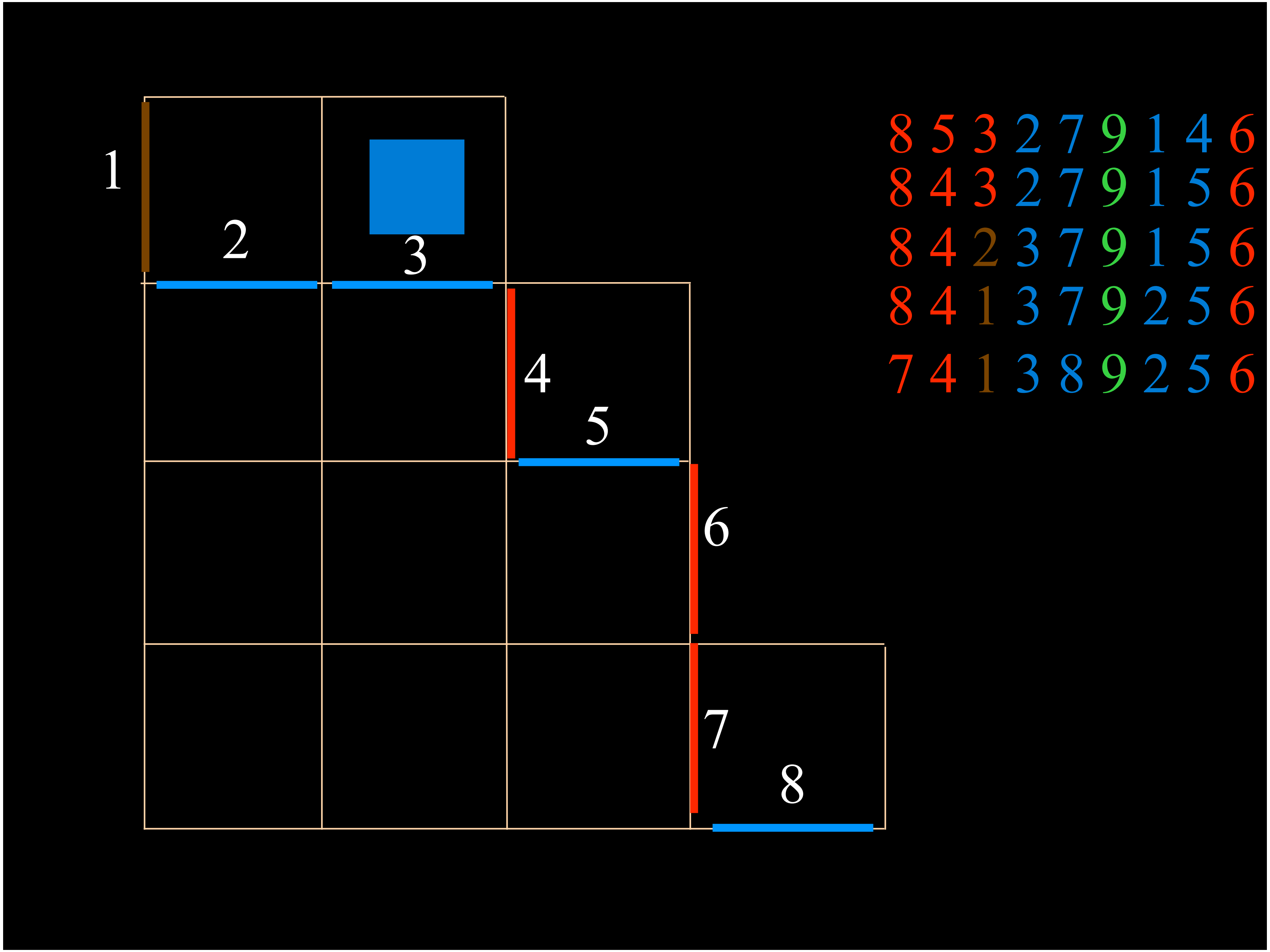
1
2
3
4
5
6
7
8
8 5 3 2 7 9 1 4 6

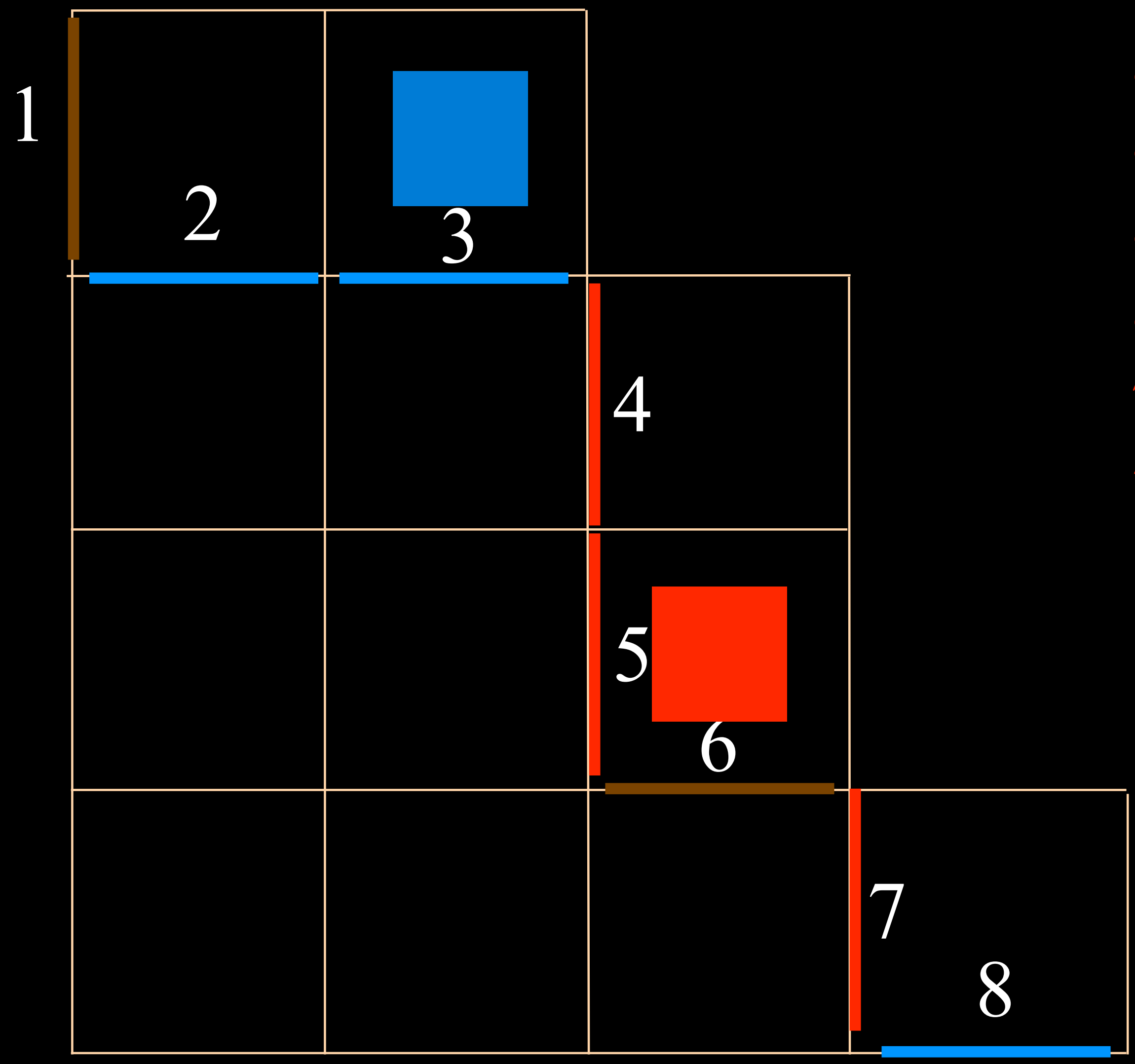


1
2
3
4
5
6
7
8
8 5 3 2 7 9 1 4 6
8 4 3 2 7 9 1 5 6


1
2
3
4
5
6
7
8
8 5 3 2 7 9 1 4 6
8 4 3 2 7 9 1 5 6
8 4 2 3 7 9 1 5 6


1
2
3
4
5
6
7
8
8 5 3 2 7 9 1 4 6
8 4 3 2 7 9 1 5 6
8 4 2 3 7 9 1 5 6
8 4 1 3 7 9 2 5 6


1
2
3
4
5
6
7
8
8 5 3 2 7 9 1 4 6
8 4 3 2 7 9 1 5 6
8 4 2 3 7 9 1 5 6
8 4 1 3 7 9 2 5 6
7 4 1 3 8 9 2 5 6



8 5 3 2 7 9 1 4 6

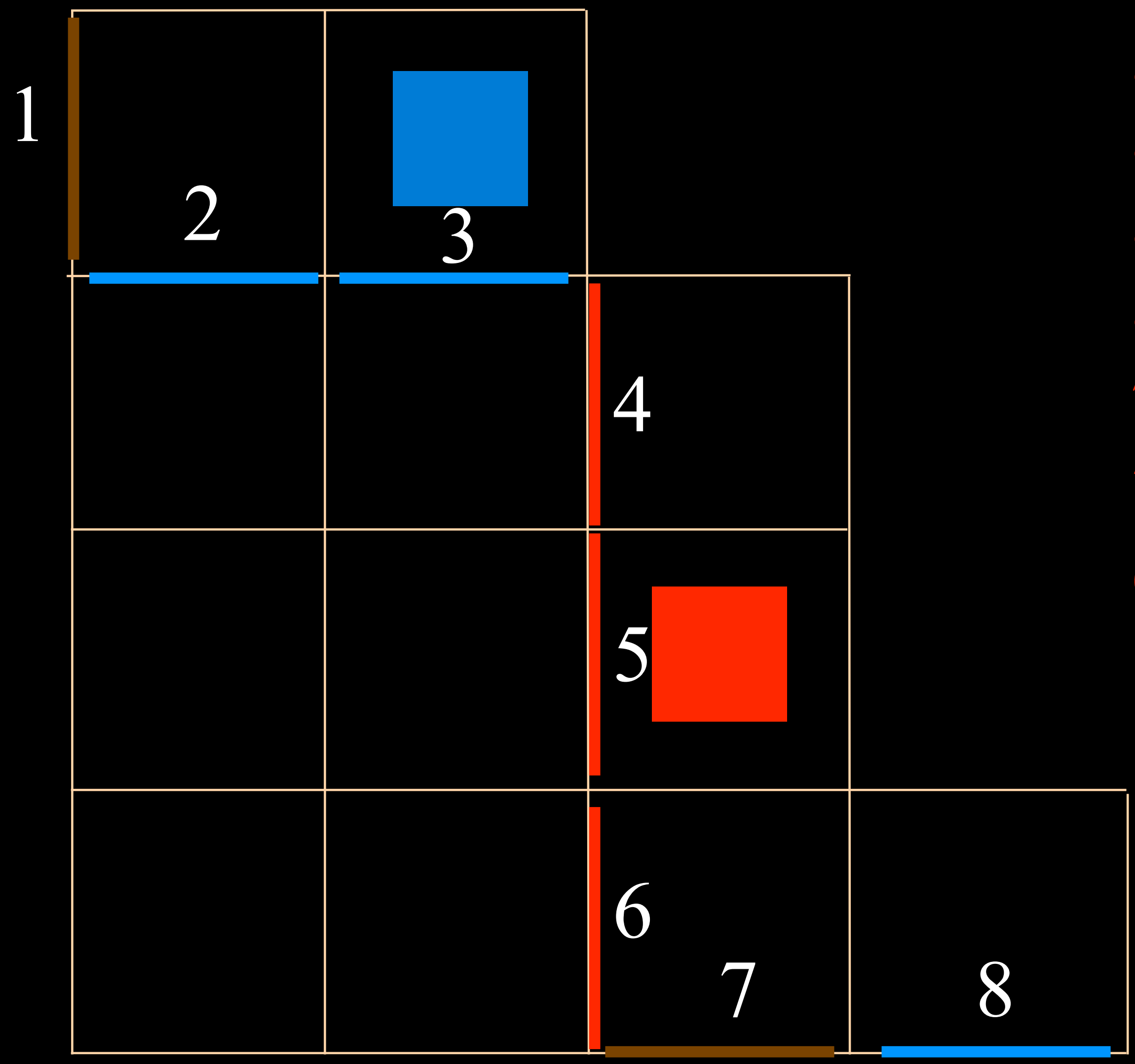
8 4 3 2 7 9 1 5 6

8 4 2 3 7 9 1 5 6

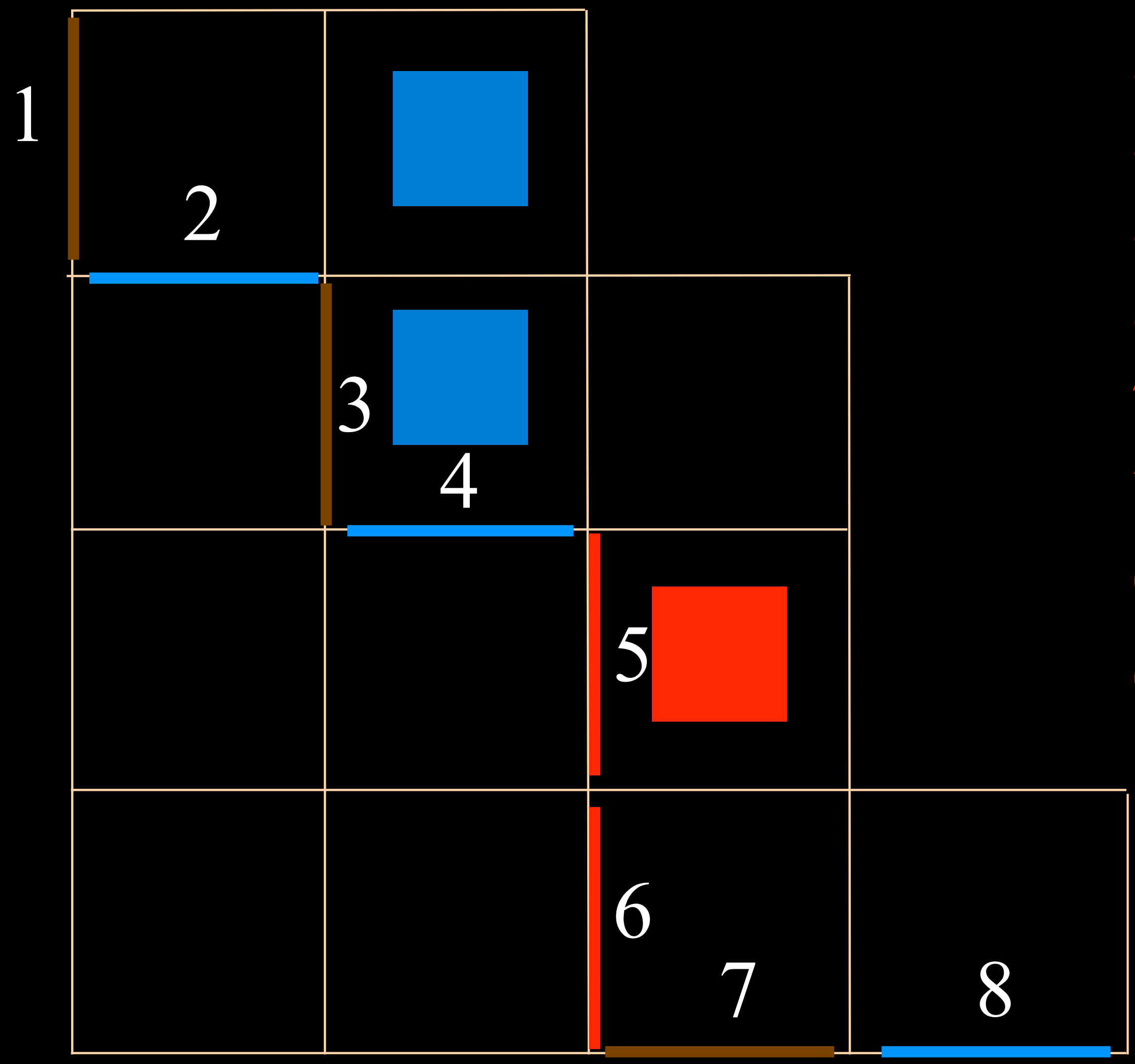
8 4 1 3 7 9 2 5 6

7 4 1 3 8 9 2 5 6

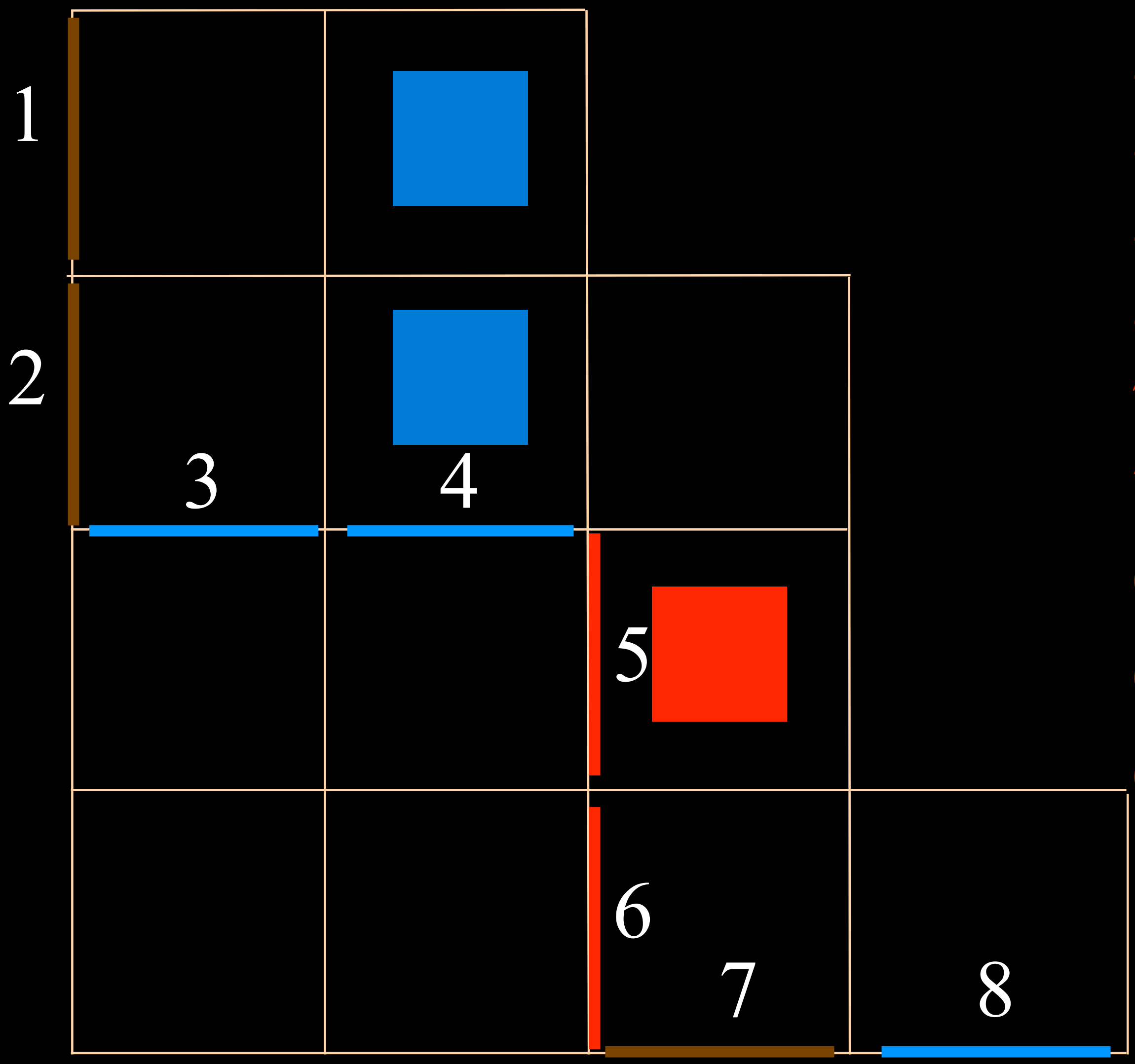
7 4 1 3 8 9 2 6 5



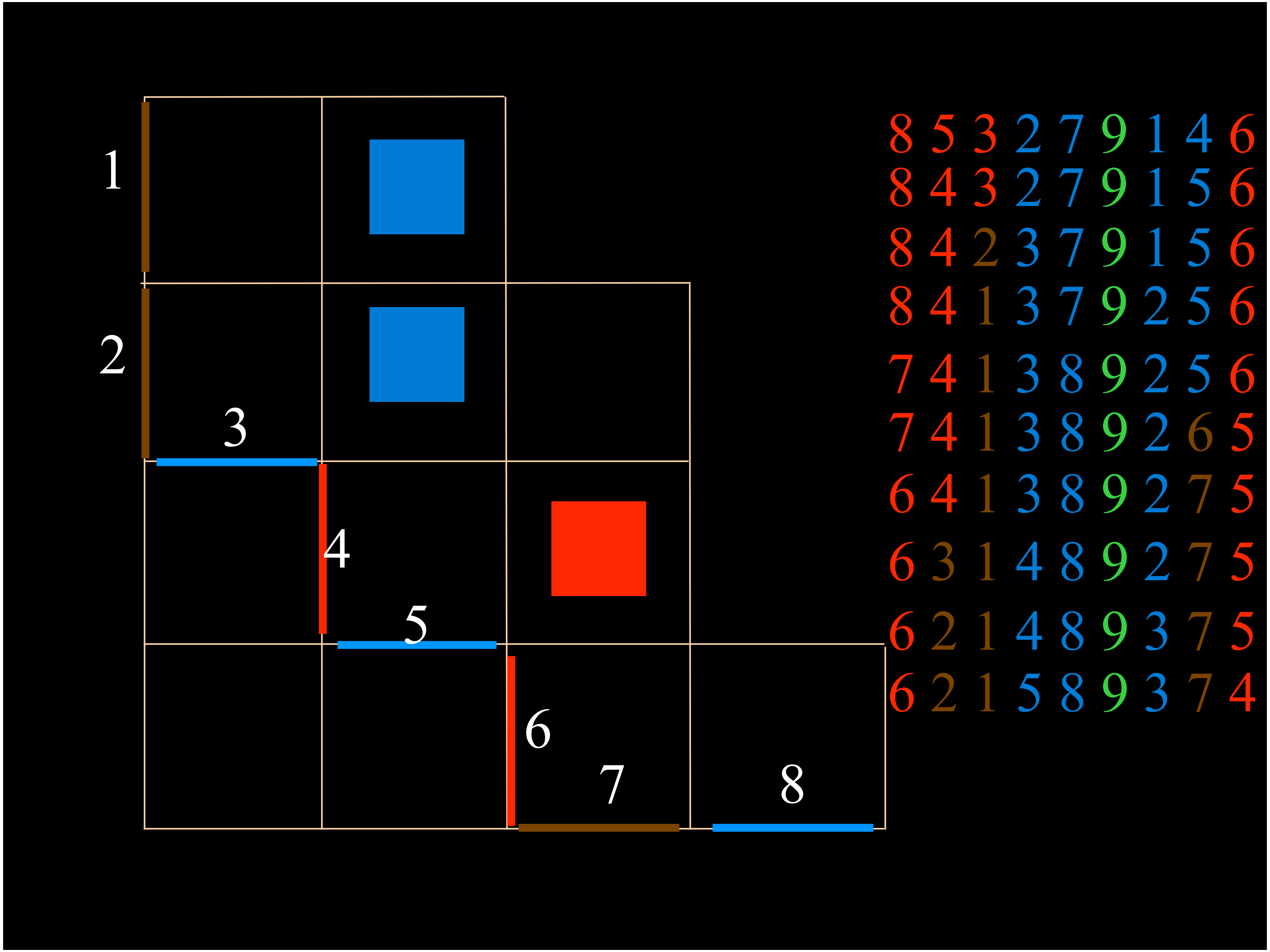
8	5	3	2	7	9	1	4	6
8	4	3	2	7	9	1	5	6
8	4	2	3	7	9	1	5	6
8	4	1	3	7	9	2	5	6
7	4	1	3	8	9	2	5	6
7	4	1	3	8	9	2	6	5
6	4	1	3	8	9	2	7	5

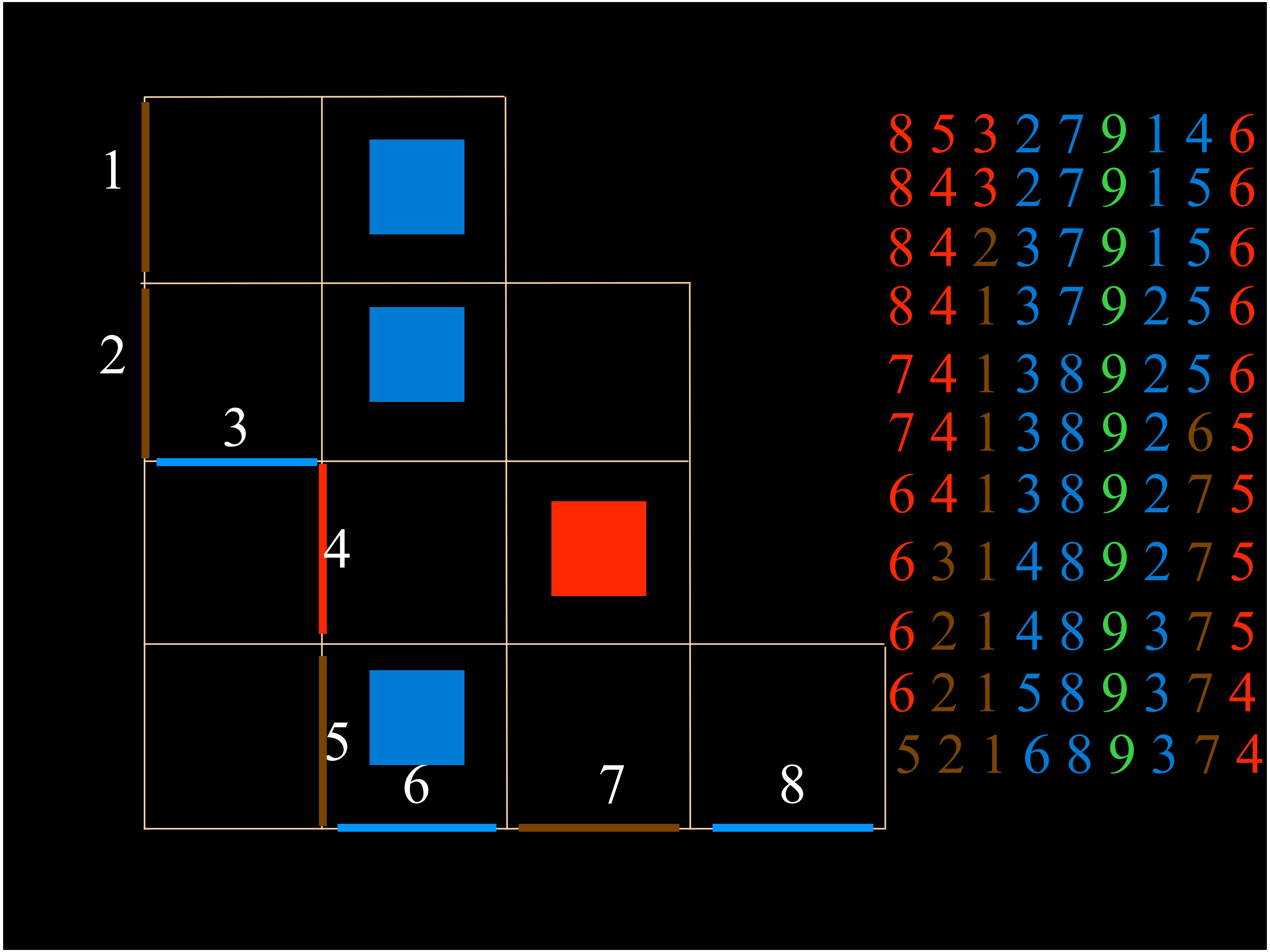


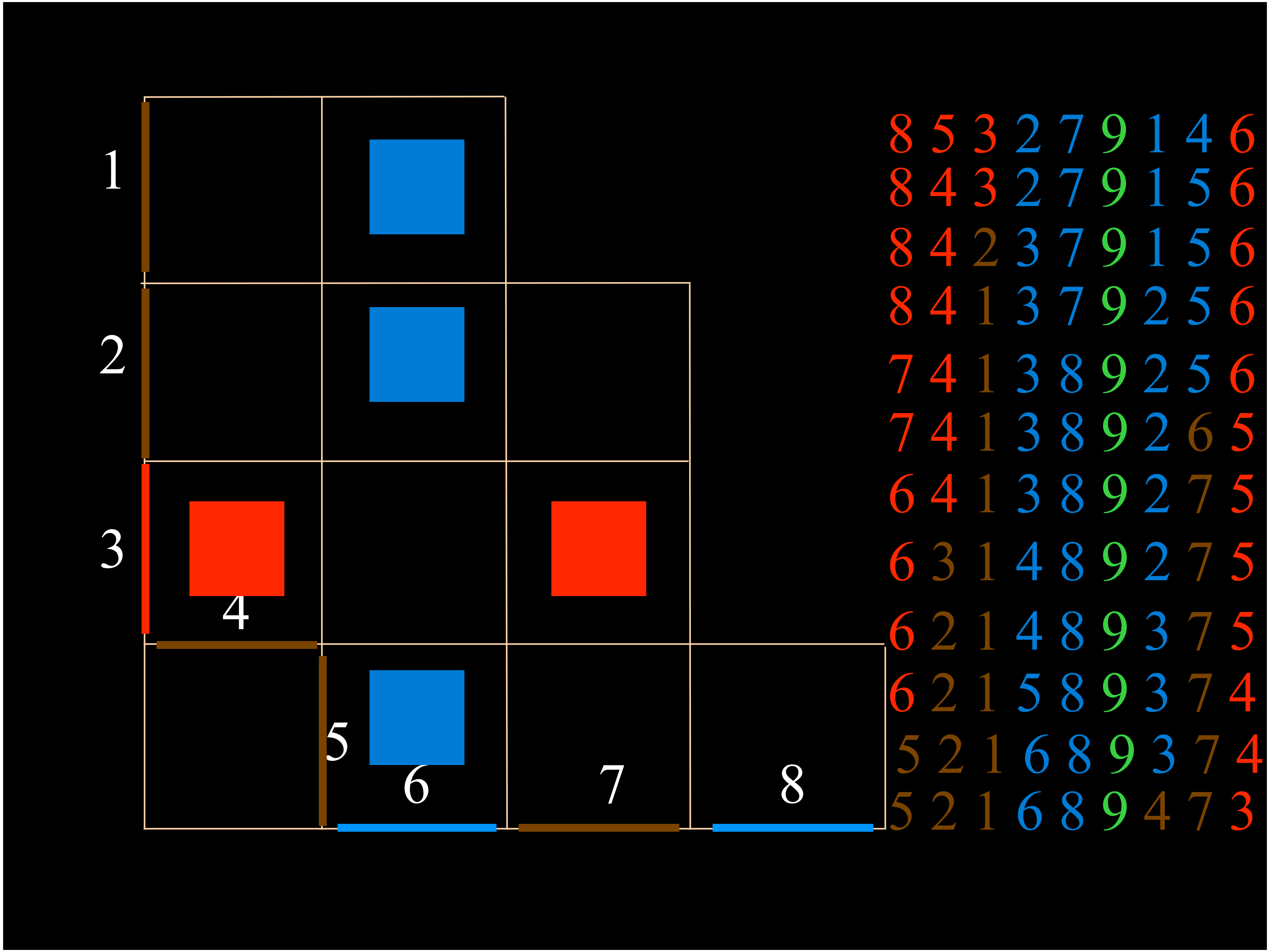
8 5 3 2 7 9 1 4 6
8 4 3 2 7 9 1 5 6
8 4 2 3 7 9 1 5 6
8 4 1 3 7 9 2 5 6
7 4 1 3 8 9 2 5 6
7 4 1 3 8 9 2 6 5
6 4 1 3 8 9 2 7 5
6 3 1 4 8 9 2 7 5

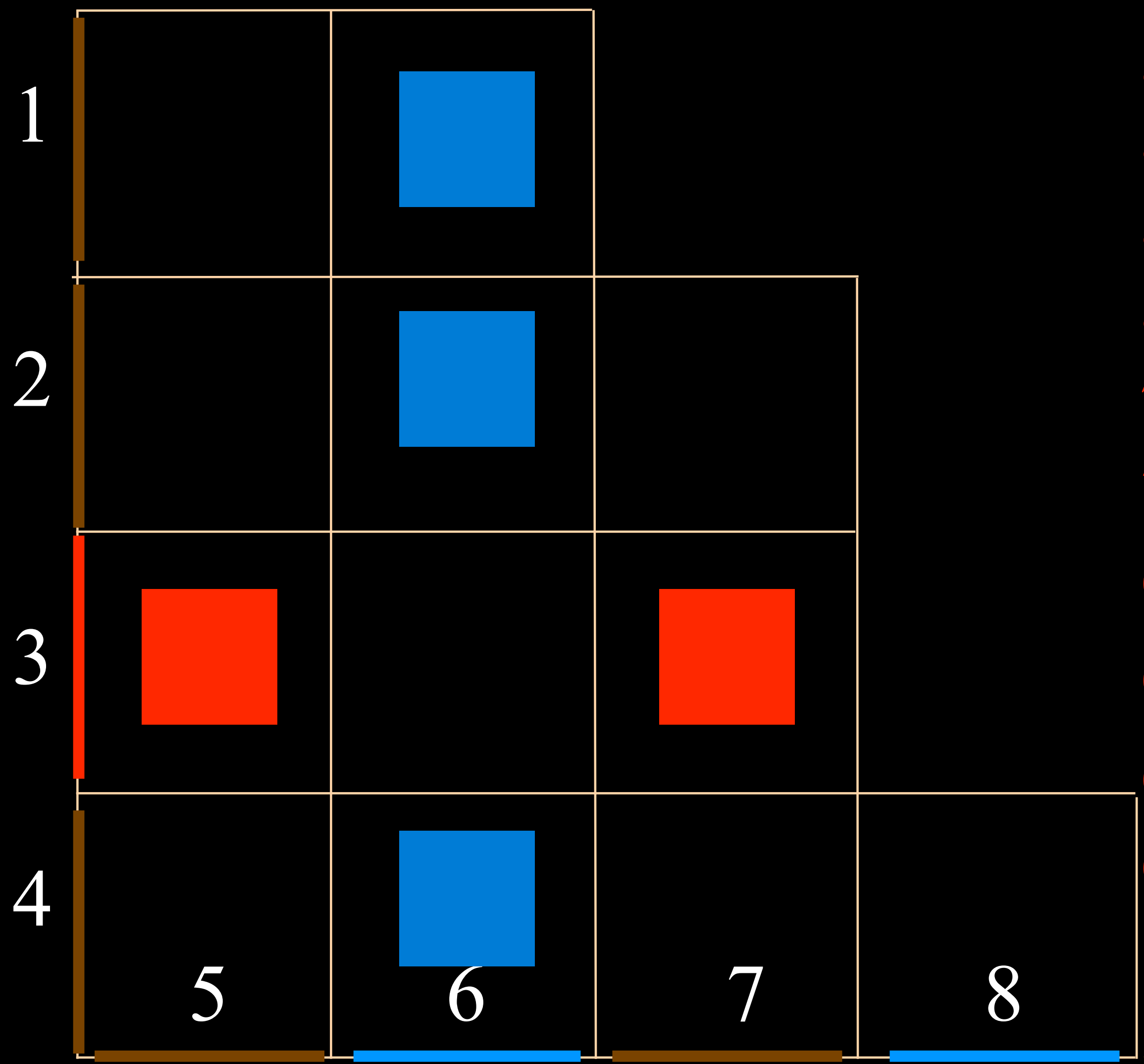


8	5	3	2	7	9	1	4	6
8	4	3	2	7	9	1	5	6
8	4	2	3	7	9	1	5	6
8	4	1	3	7	9	2	5	6
7	4	1	3	8	9	2	5	6
7	4	1	3	8	9	2	6	5
6	4	1	3	8	9	2	7	5
6	3	1	4	8	9	2	7	5
6	2	1	4	8	9	3	7	5

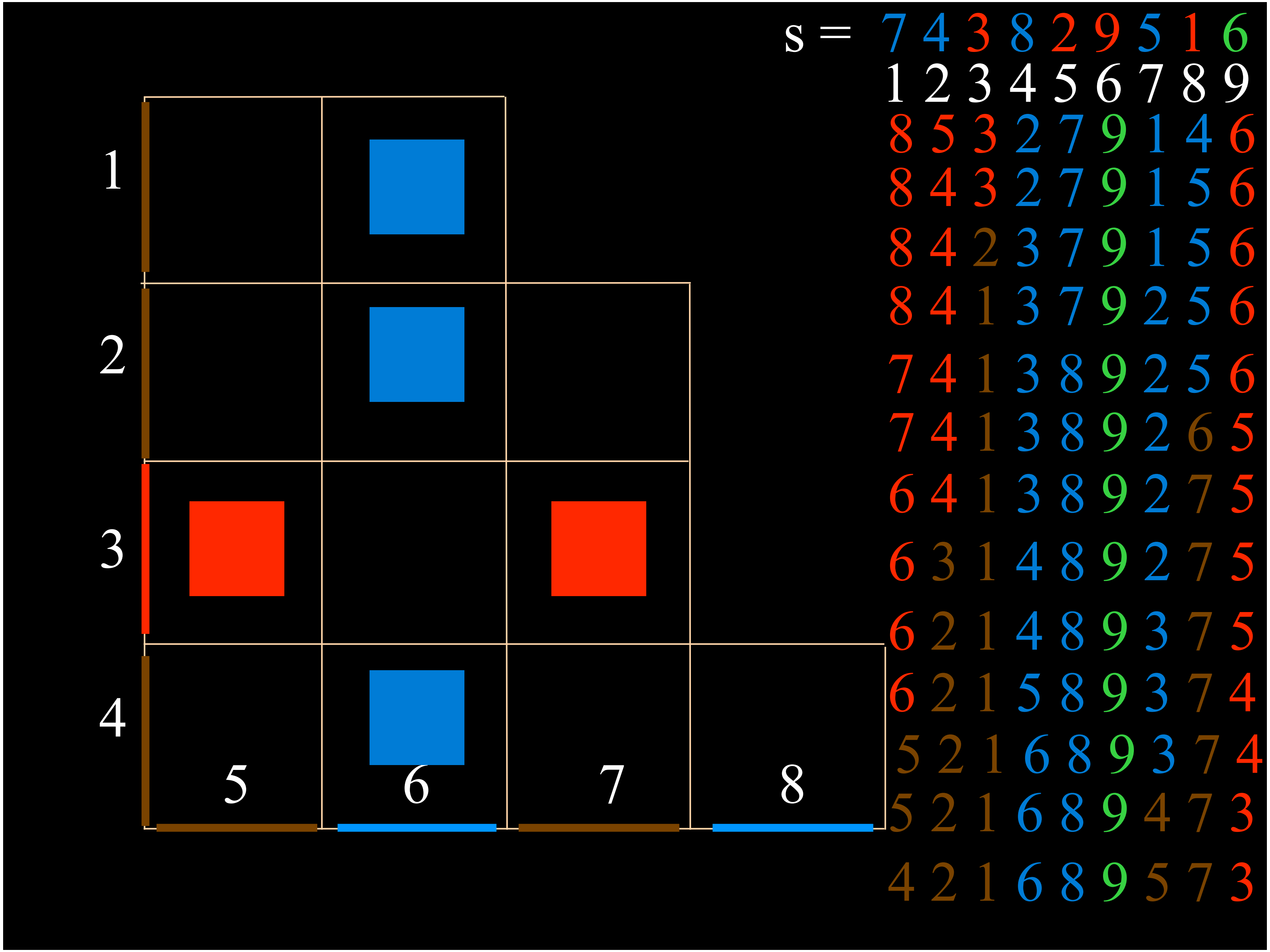

1
2
3
4
5
6
7
8
8 5 3 2 7 9 1 4 6
8 4 3 2 7 9 1 5 6
8 4 2 3 7 9 1 5 6
8 4 1 3 7 9 2 5 6
7 4 1 3 8 9 2 5 6
7 4 1 3 8 9 2 6 5
6 4 1 3 8 9 2 7 5
6 3 1 4 8 9 2 7 5
6 2 1 4 8 9 3 7 5
6 2 1 5 8 9 3 7 4


1
2
3
4
5
6
7
8
8 5 3 2 7 9 1 4 6
8 4 3 2 7 9 1 5 6
8 4 2 3 7 9 1 5 6
8 4 1 3 7 9 2 5 6
7 4 1 3 8 9 2 5 6
7 4 1 3 8 9 2 6 5
6 4 1 3 8 9 2 7 5
6 3 1 4 8 9 2 7 5
6 2 1 4 8 9 3 7 5
6 2 1 5 8 9 3 7 4
5 2 1 6 8 9 3 7 4


1
2
3
4
5
6
7
8
8 5 3 2 7 9 1 4 6
8 4 3 2 7 9 1 5 6
8 4 2 3 7 9 1 5 6
8 4 1 3 7 9 2 5 6
7 4 1 3 8 9 2 5 6
7 4 1 3 8 9 2 6 5
6 4 1 3 8 9 2 7 5
6 3 1 4 8 9 2 7 5
6 2 1 4 8 9 3 7 5
6 2 1 5 8 9 3 7 4
5 2 1 6 8 9 3 7 4
5 2 1 6 8 9 4 7 3



8 5 3 2 7 9 1 4 6
8 4 3 2 7 9 1 5 6
8 4 2 3 7 9 1 5 6
8 4 1 3 7 9 2 5 6
7 4 1 3 8 9 2 5 6
7 4 1 3 8 9 2 6 5
6 4 1 3 8 9 2 7 5
6 3 1 4 8 9 2 7 5
6 2 1 4 8 9 3 7 5
6 2 1 5 8 9 3 7 4
5 2 1 6 8 9 3 7 4
5 2 1 6 8 9 4 7 3
4 2 1 6 8 9 5 7 3


s = 7 4 3 8 2 9 5 1 6
1 2 3 4 5 6 7 8 9
8 5 3 2 7 9 1 4 6
8 4 3 2 7 9 1 5 6
8 4 2 3 7 9 1 5 6
8 4 1 3 7 9 2 5 6
7 4 1 3 8 9 2 5 6
7 4 1 3 8 9 2 6 5
6 4 1 3 8 9 2 7 5
6 3 1 4 8 9 2 7 5
6 2 1 4 8 9 3 7 5
6 2 1 5 8 9 3 7 4
5 2 1 6 8 9 3 7 4
5 2 1 6 8 9 4 7 3
4 2 1 6 8 9 5 7 3
1
2
3
4
5
6
7
8

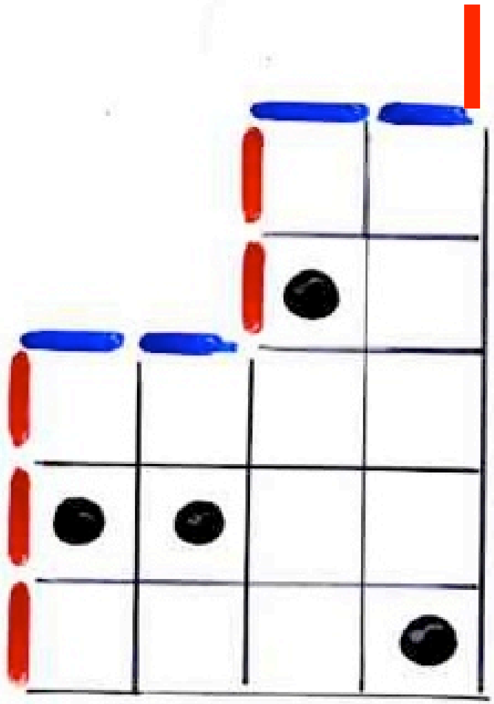
§5 the “binary trees sliding” algorithm

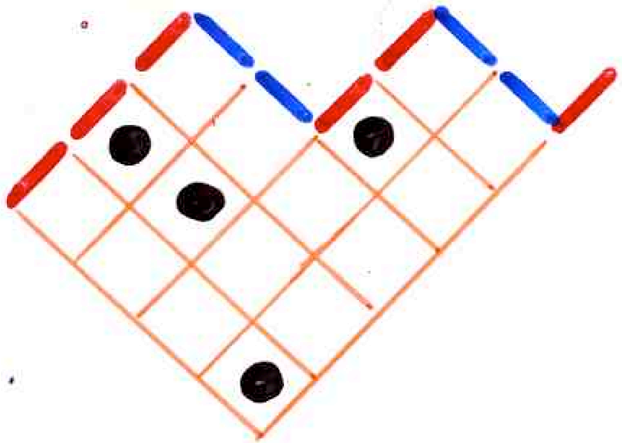
bijection

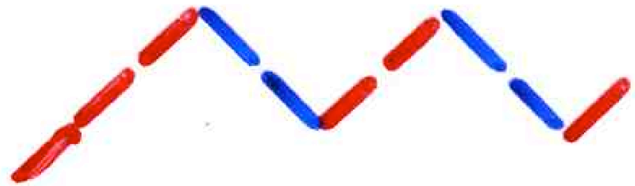
Catalan tableaux $\longleftrightarrow$ binary trees

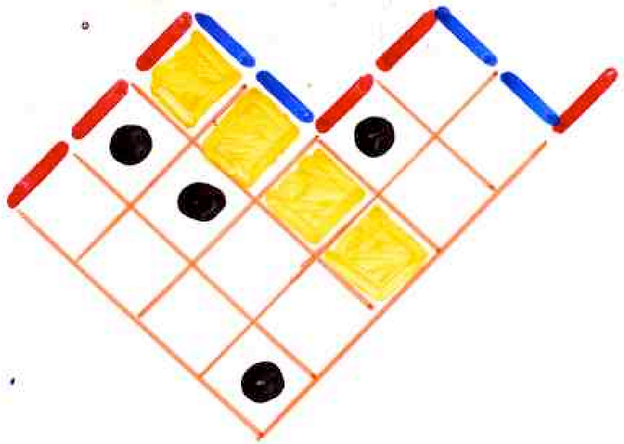
in Proc. FPSAC’07, Tienjin

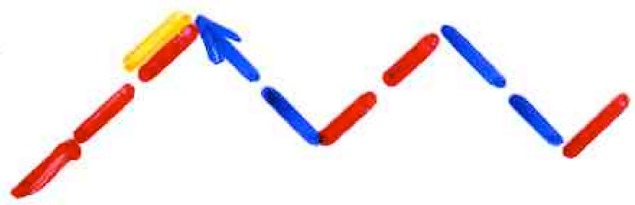
(described in term of permutation tableaux)

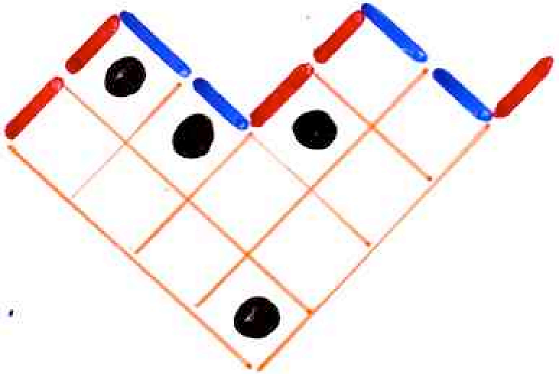


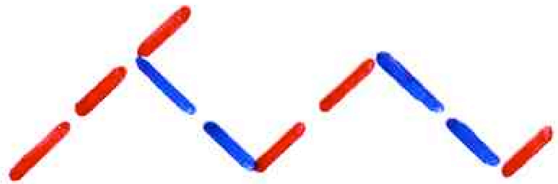


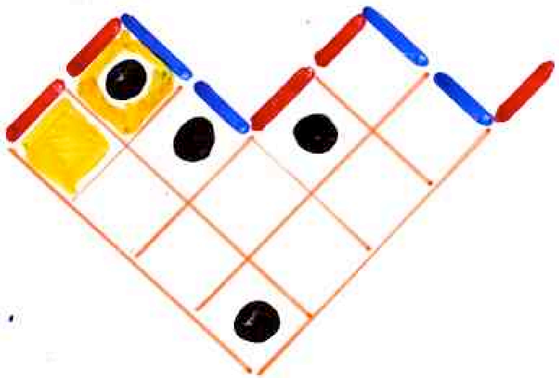


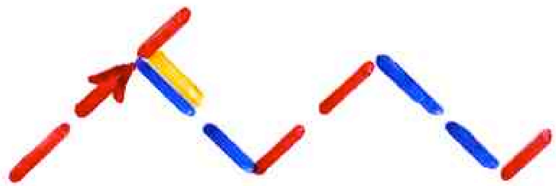


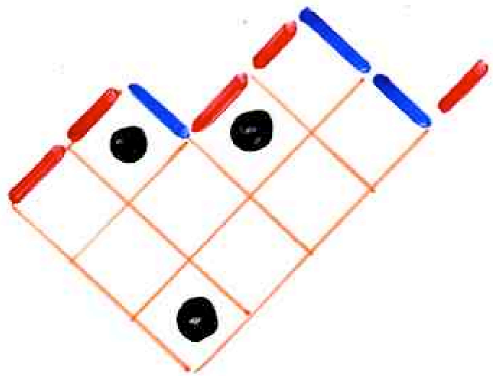


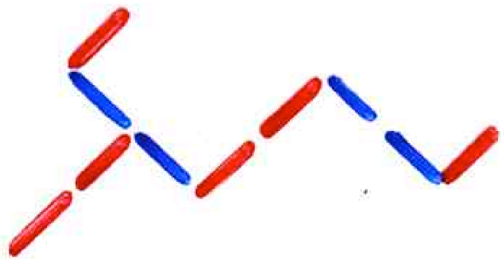


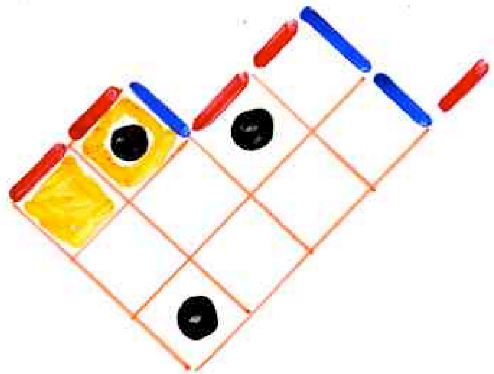


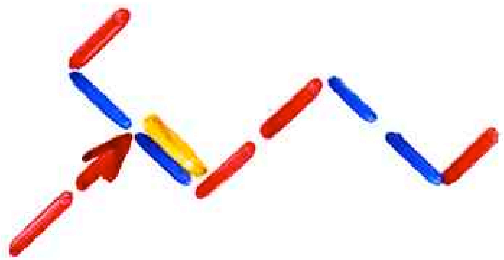


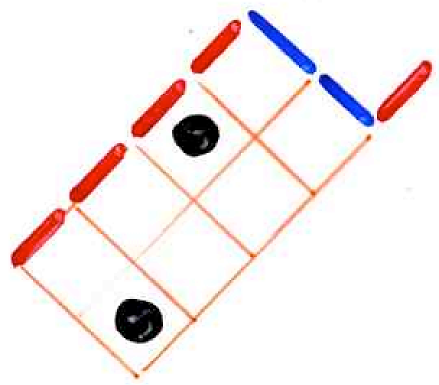


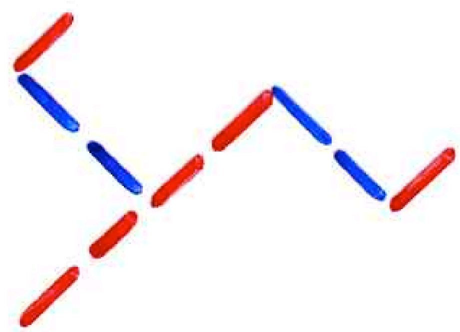


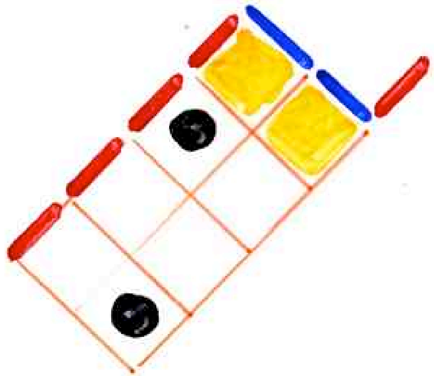


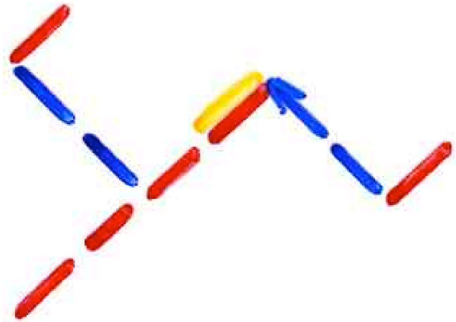


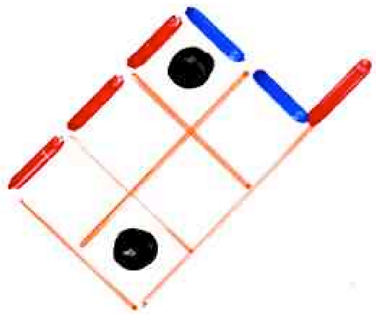


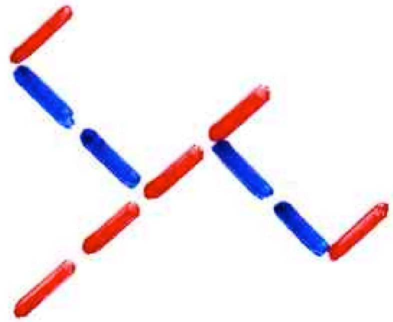


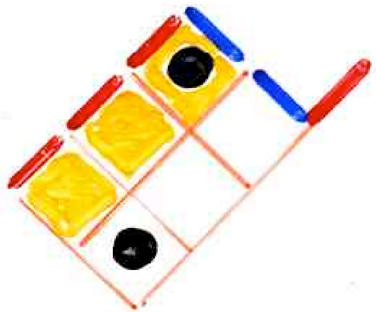


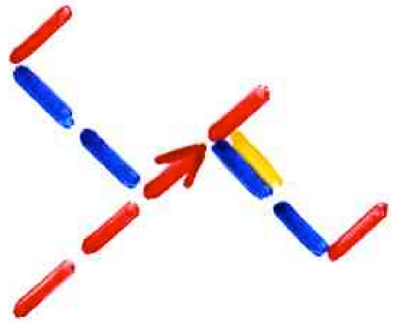


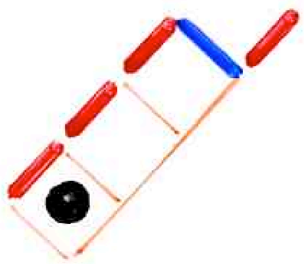


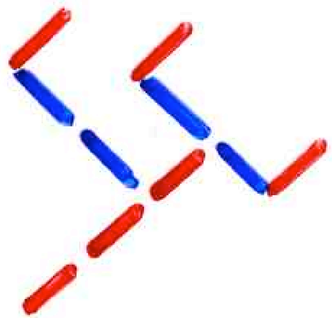


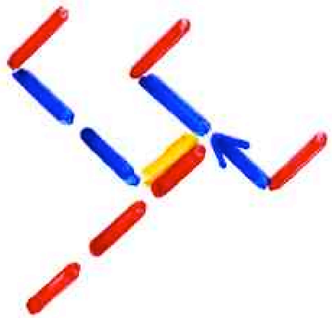


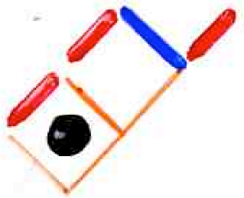


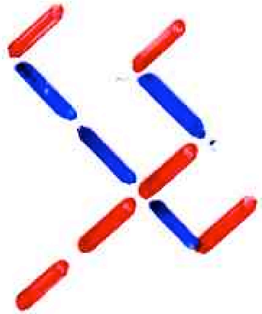


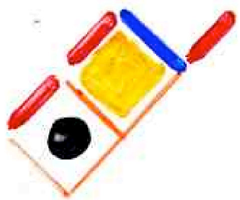


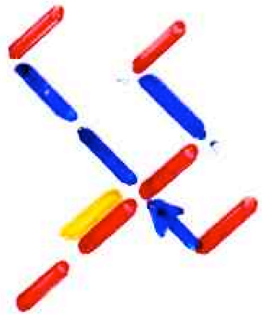


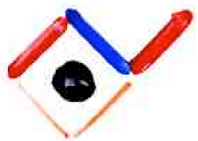


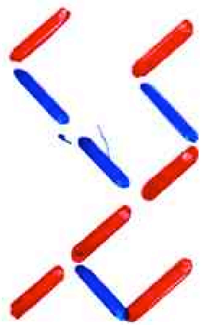


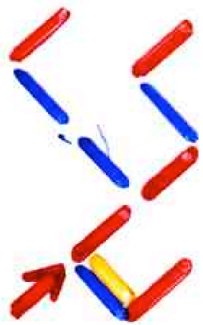


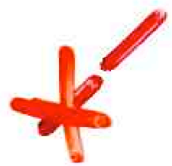


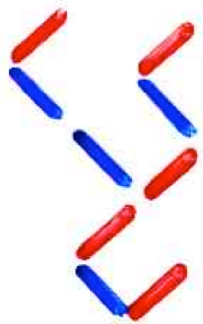












The bijection presented at Tienjin FPSAC'07 between binary trees and "Catalan permutation tableaux", once rewritten in term in terms of "Catalan alternating tableaux" (which is immediate to do), can be viewed as a particular case of the inverse of the "exchange-fusion" algorithm.

This "binary tree sliding algorithm" can be extended to permutations and gives a bijection between alternative tableaux and a new kind of binary trees introduced by P. Nadeau in his forthcoming paper under the name of "alternative binary tree"

§6 Representation of the operators D and E

and

"Data structure histories"

Calcul du coût intégré
d'une structure de donnée
pour une séquence aléatoire
d'opérations primitives

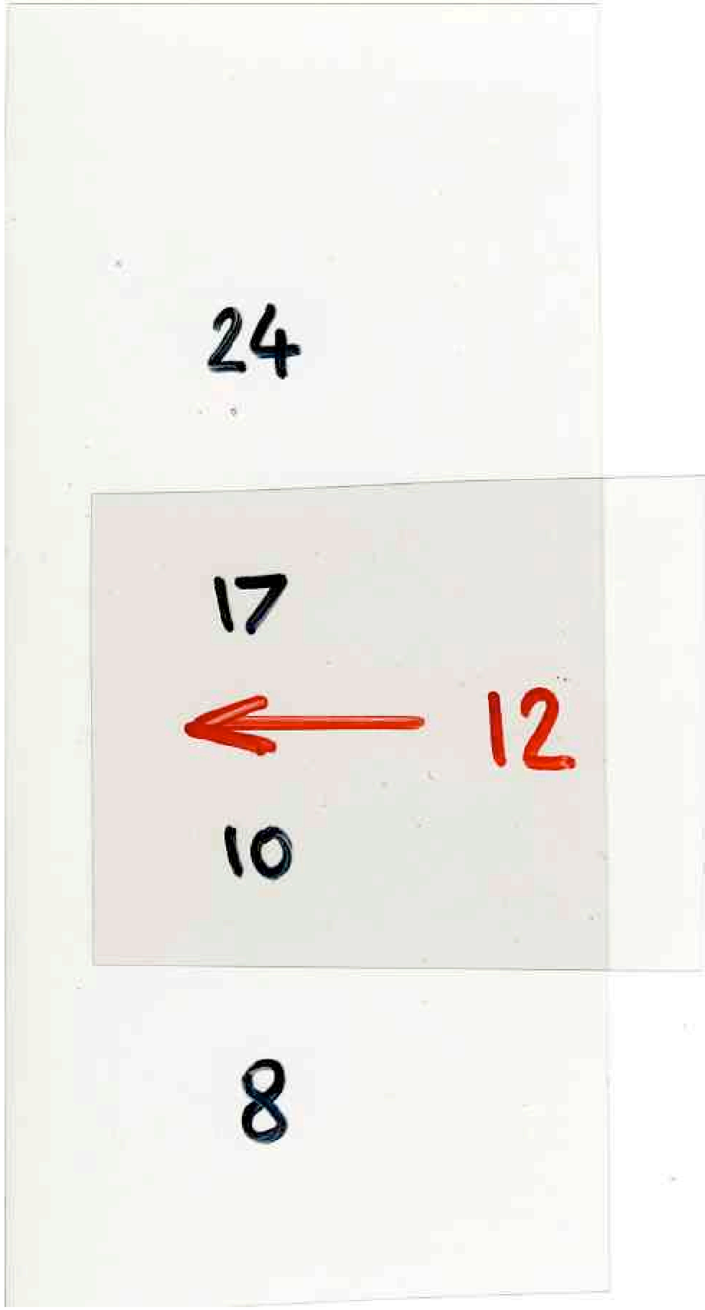
Françon, Flajolet, Vuillemin (1980, ...)
connaissant le coût moyen
d'une opération primitive.

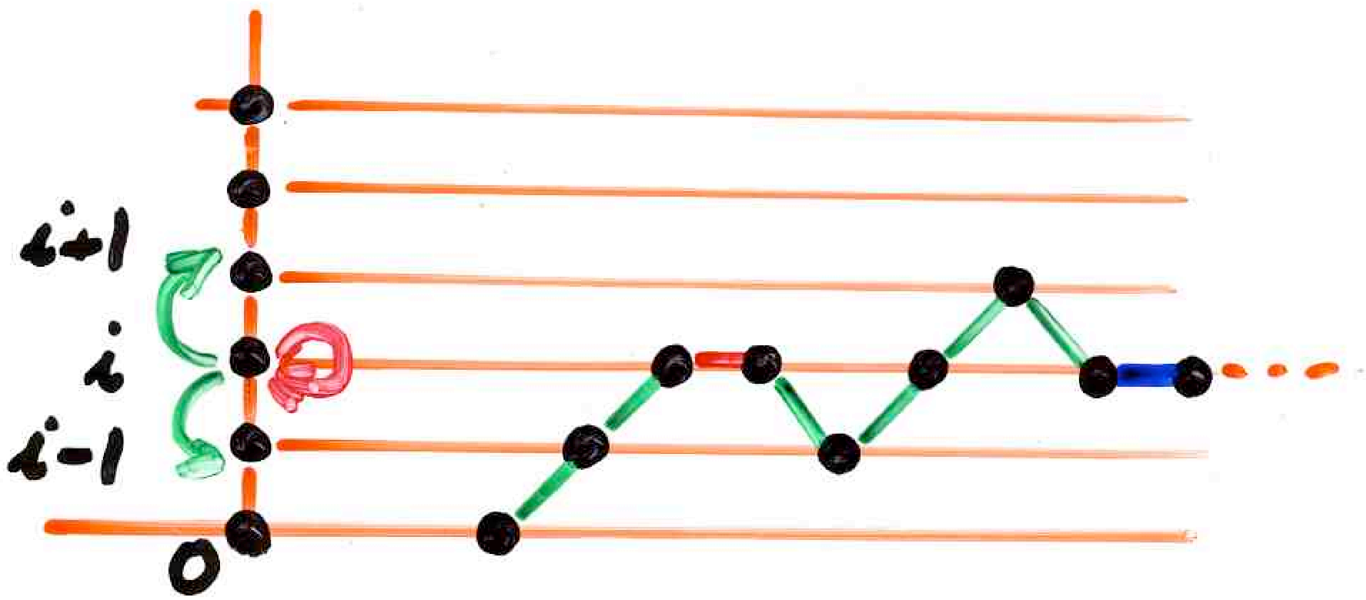
24

17

10

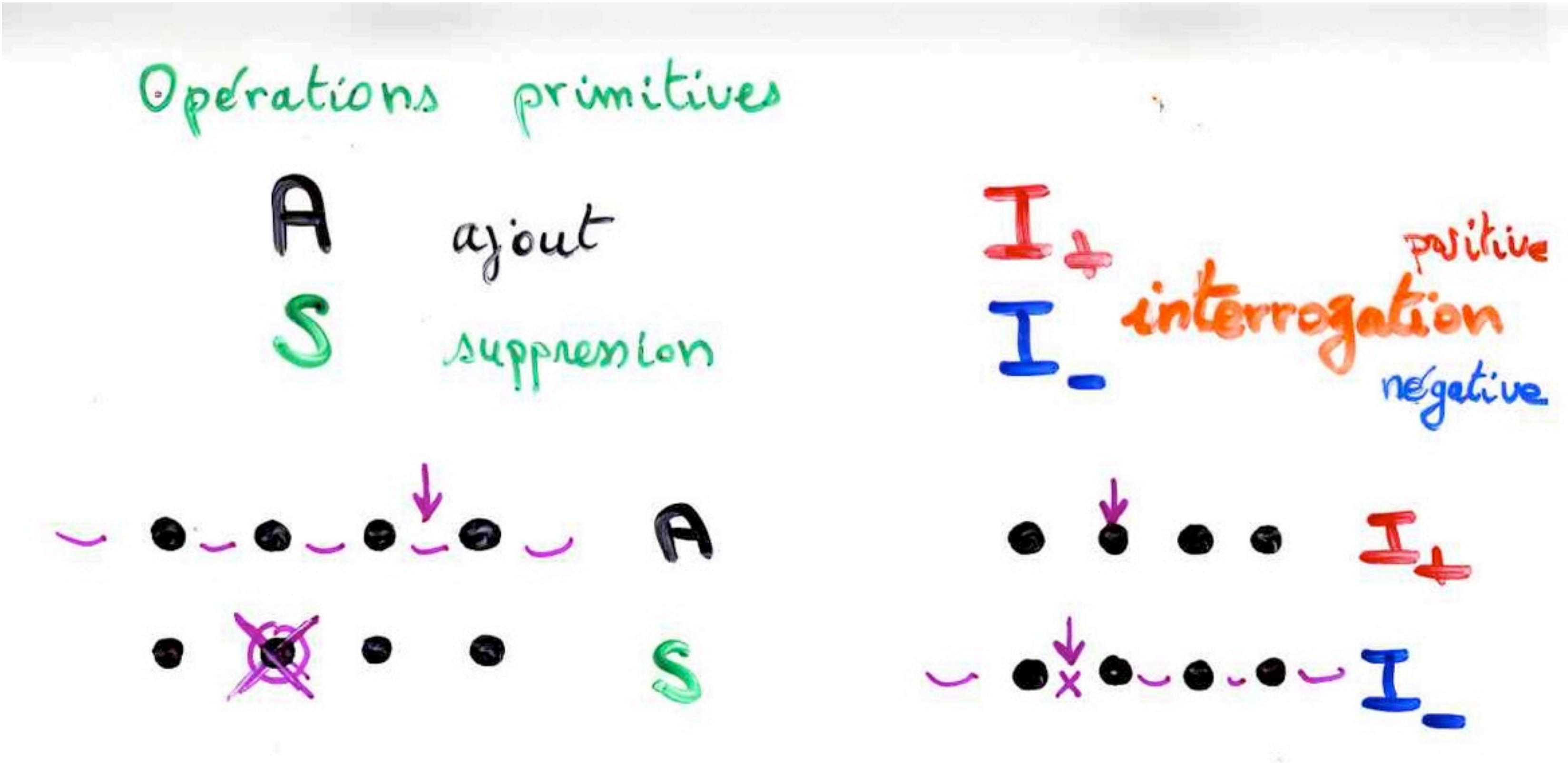
8


24
17
12
10
8

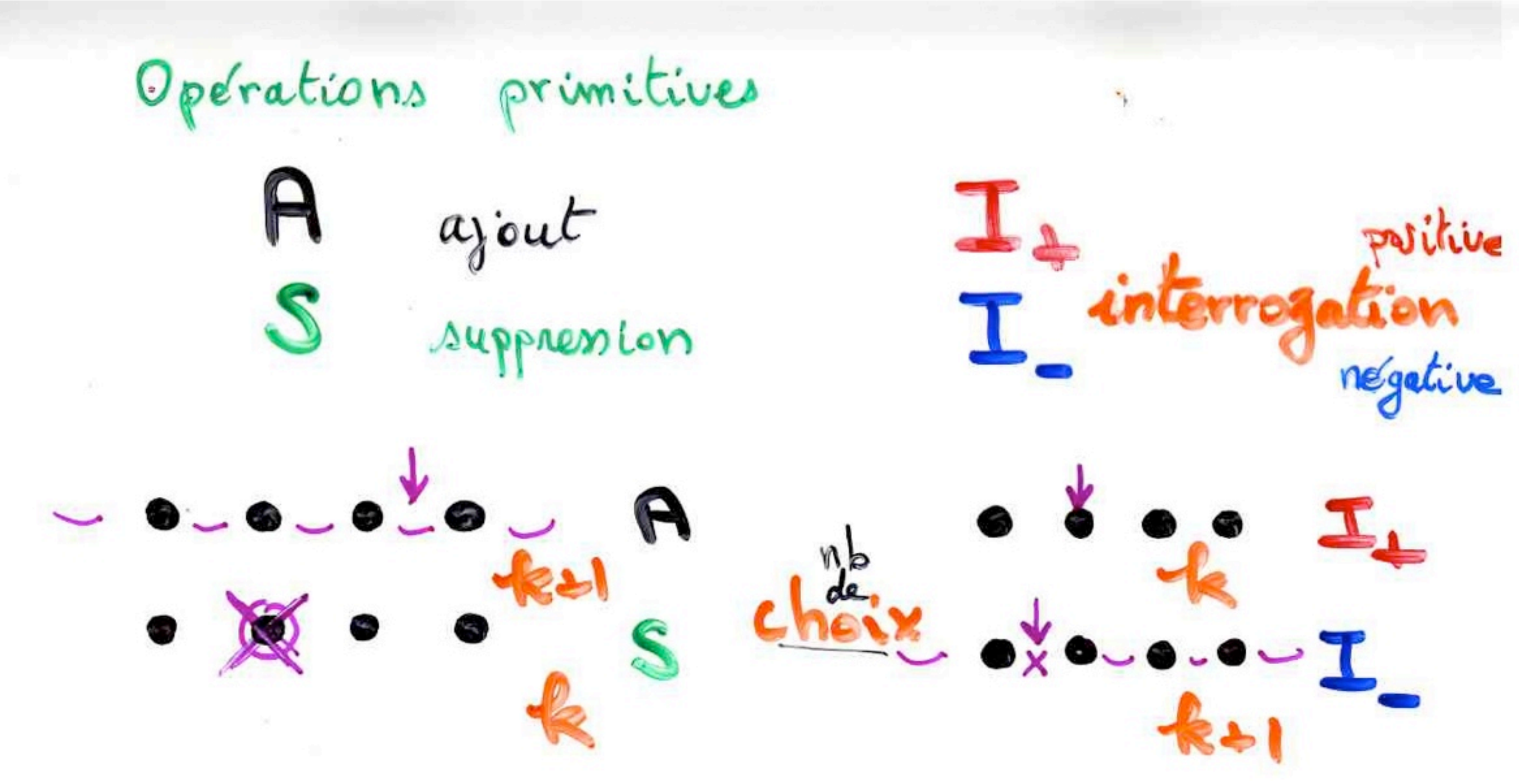


histoires de fichiers Françon, (1976)

Data structure histories

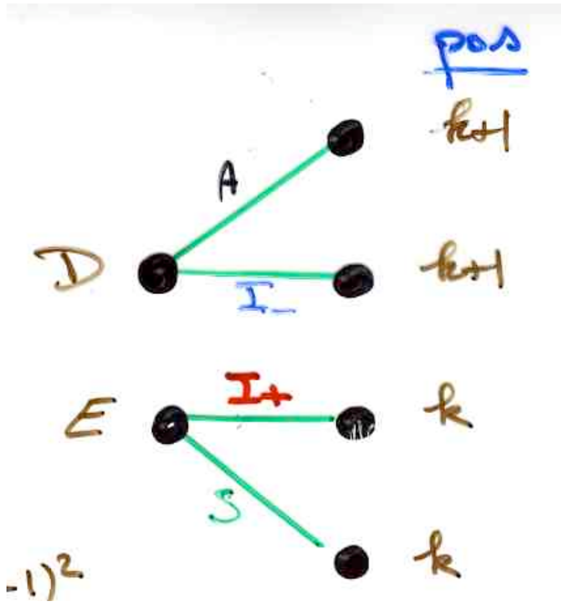


Primitive operations
for “dictionnaries” data structure:
add or delete any elements, asking questions
(with positive or negative answer)



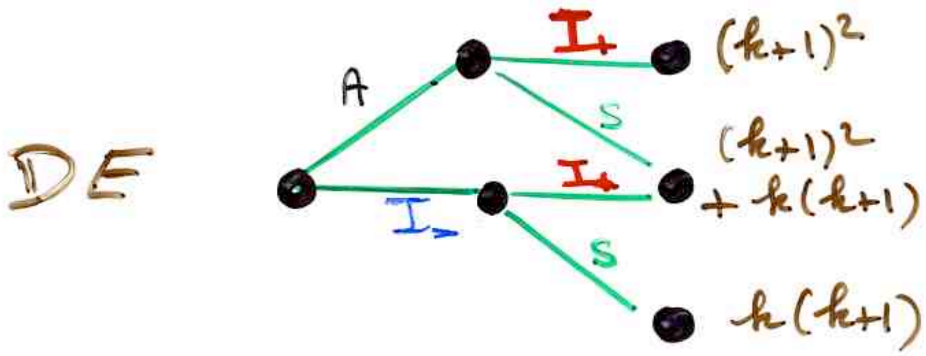
number of choices for each
primitive operations

$$\begin{cases} D = A + I_{-} \\ E = S + I_{+} \end{cases}$$

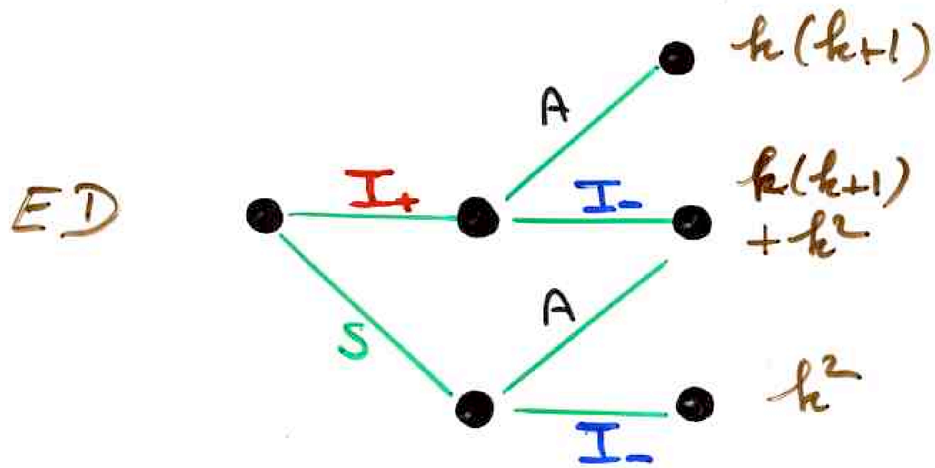


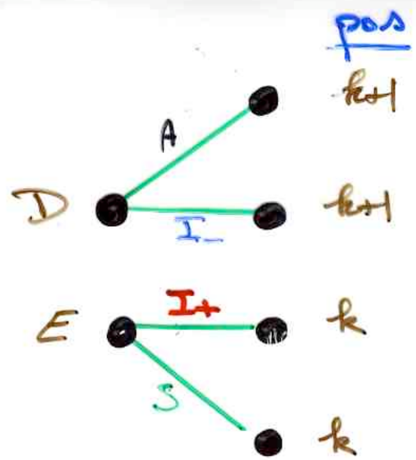
this corresponds to the n!
"restricted Laguerre histories"

for more details on the two kind of Laguerre histories,
see the slides of "petite école de combinatoire", LaBRI, 2006/07
on combinatorics of orthogonal polynomials, or the Lecture Notes,
LACIM, UQAM, Montréal, 1984 (on website xgv)


DE
A
I+
S
I-
(k+1)²
(k+1)² + k(k+1)
k(k+1)


DE − ED
k+1
2k+1
k


ED
I+
S
A
I-
k(k+1)
k(k+1) + k²
k²

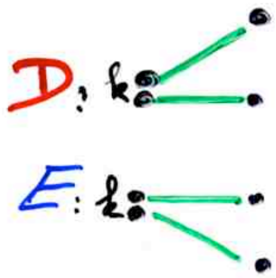

pos
D
A
I-
k+1
k+1
E
I+
S
k
k

$$DE - ED = E + D$$

$$\begin{cases} D = A + I_+ \\ E = S + I_- \end{cases}$$

$$DE = ED + E + D$$

aussi :



(k+1) possibilités partout

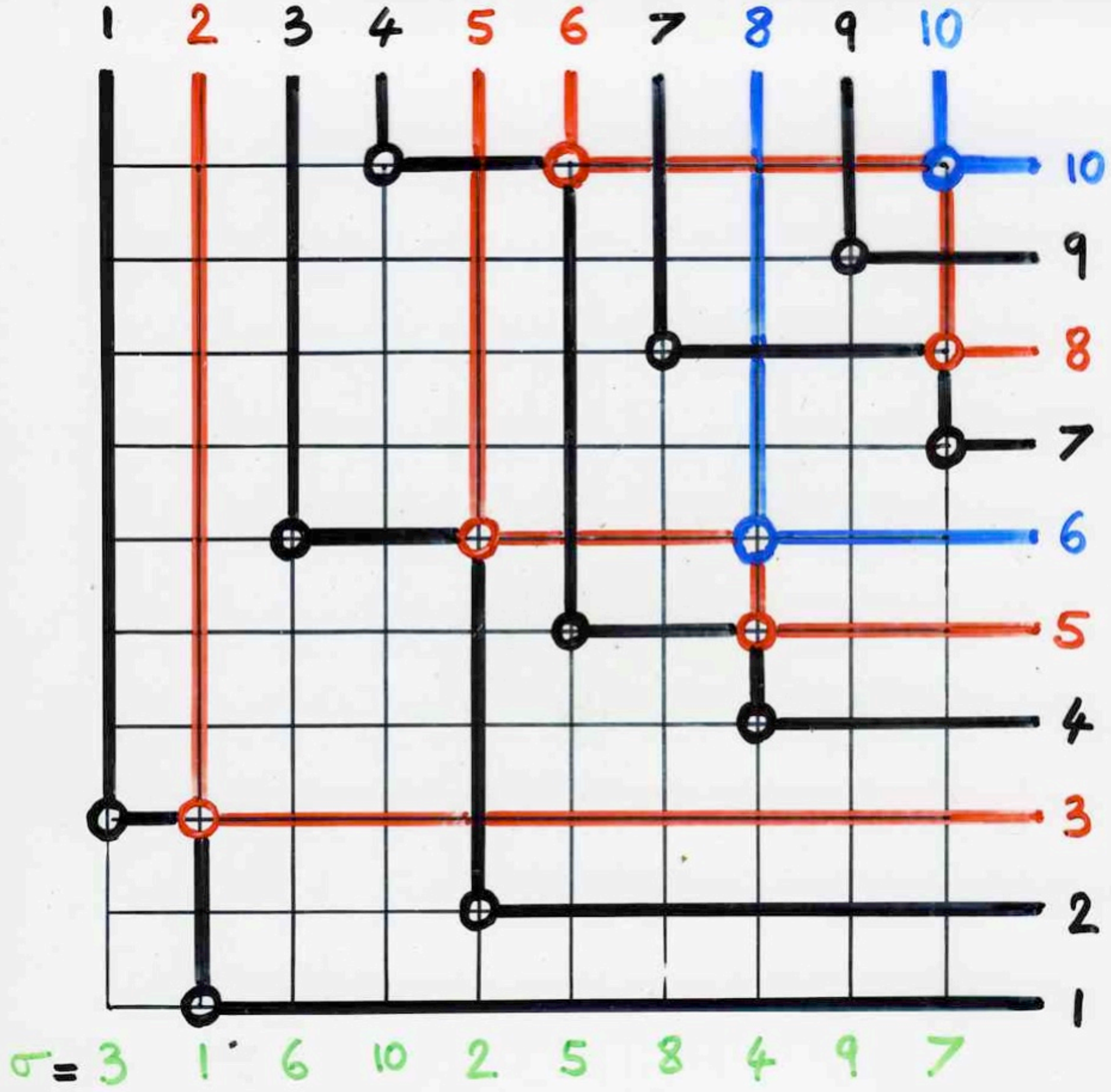
(histoires de Laguerre "larges")

this valuation corresponds to the (n+1)!
"enlarged Laguerre histories"

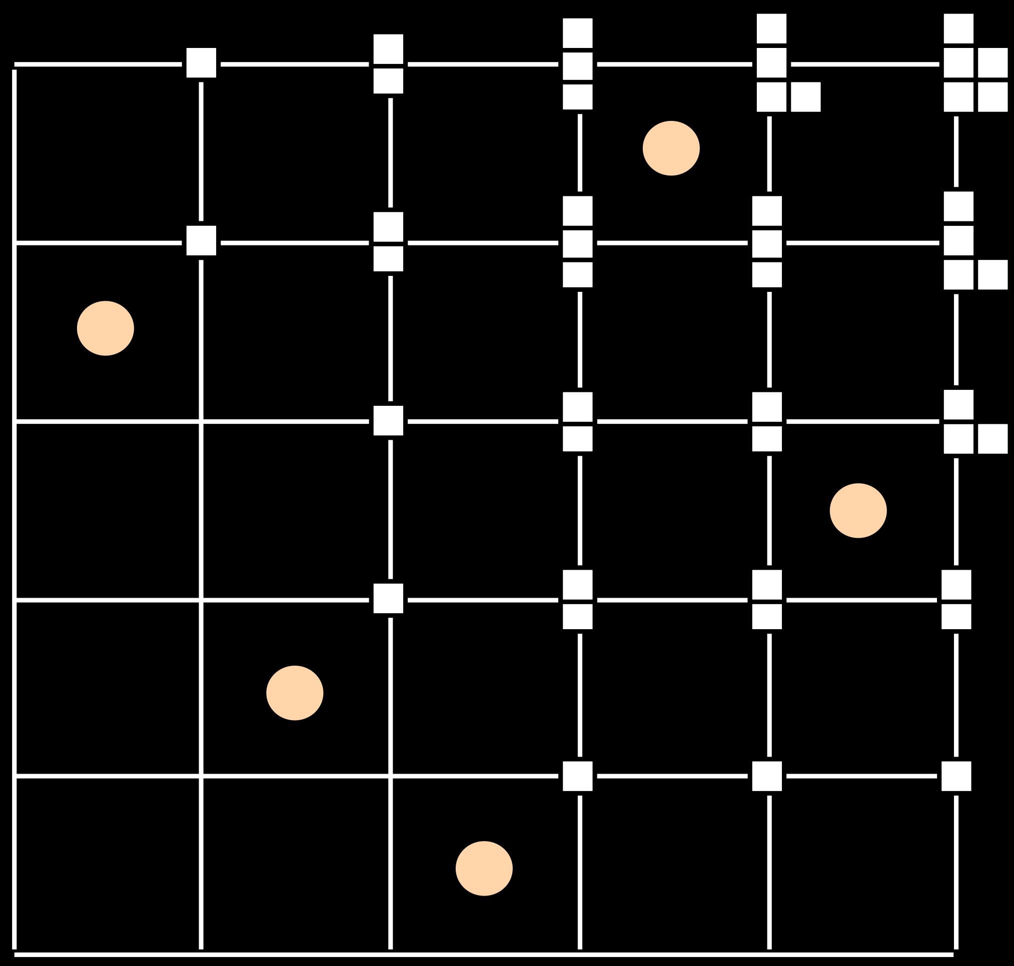
§7 local RSK
and geometric RSK

(the geometric construction with "light" and "shadow" for RSK leads to a simple proof of the fact that RSK and the "local rules" give the same bijection)

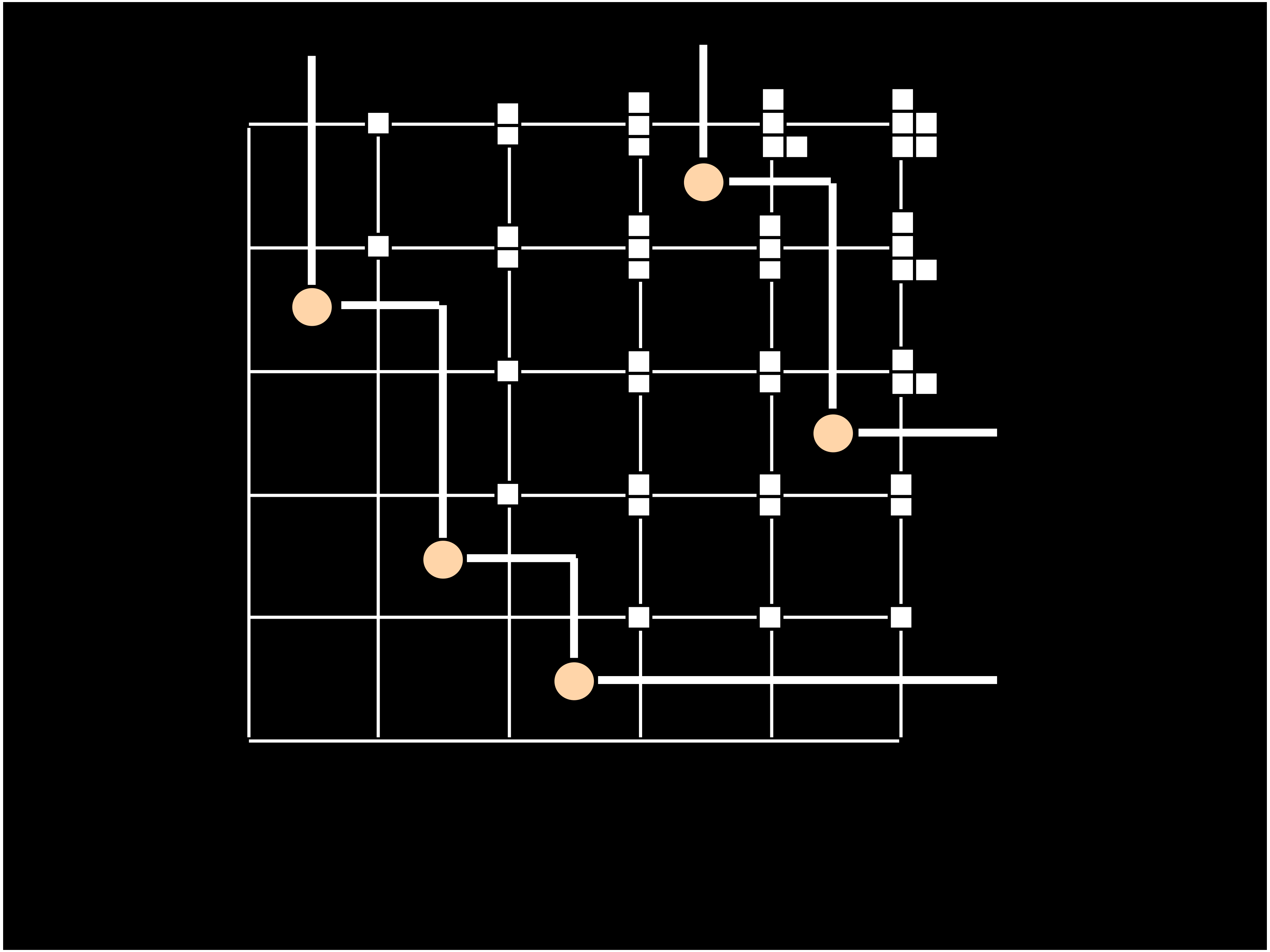
For a more complete introduction to RSK and Fomin's local rules,
see on my web site (page "exposés"):
Robinson-Schensted-Knuth: RSK1 (pdf, 9,1 Mo)
groupe de travail de combinatoire, Bordeaux, LaBRI, Février 2005
Robinson-Schensted-Knuth: RSK2 (pdf,10,8Mo)
groupe de travail de combinatoire, Bordeaux, LaBRI, Février 2005

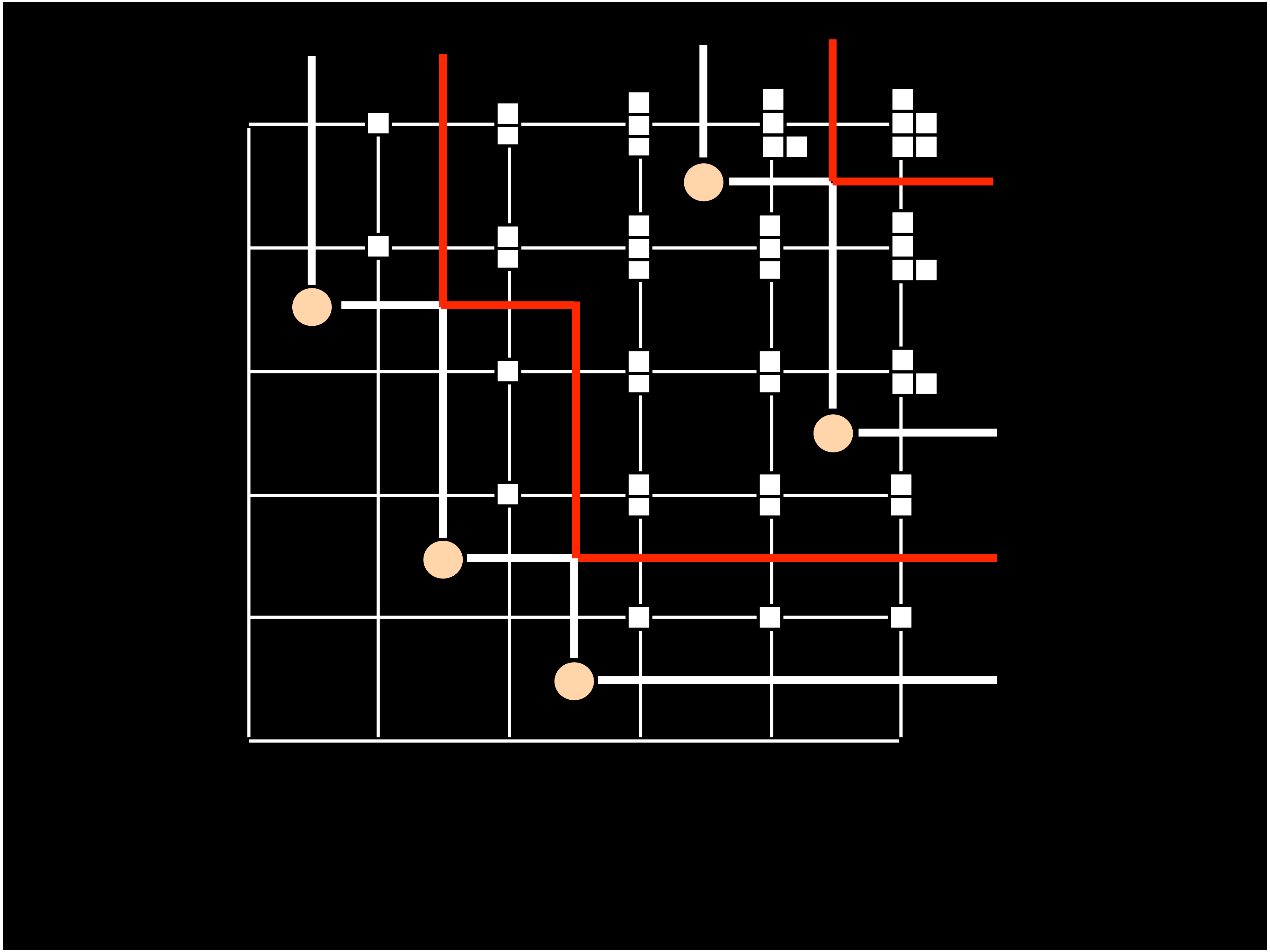


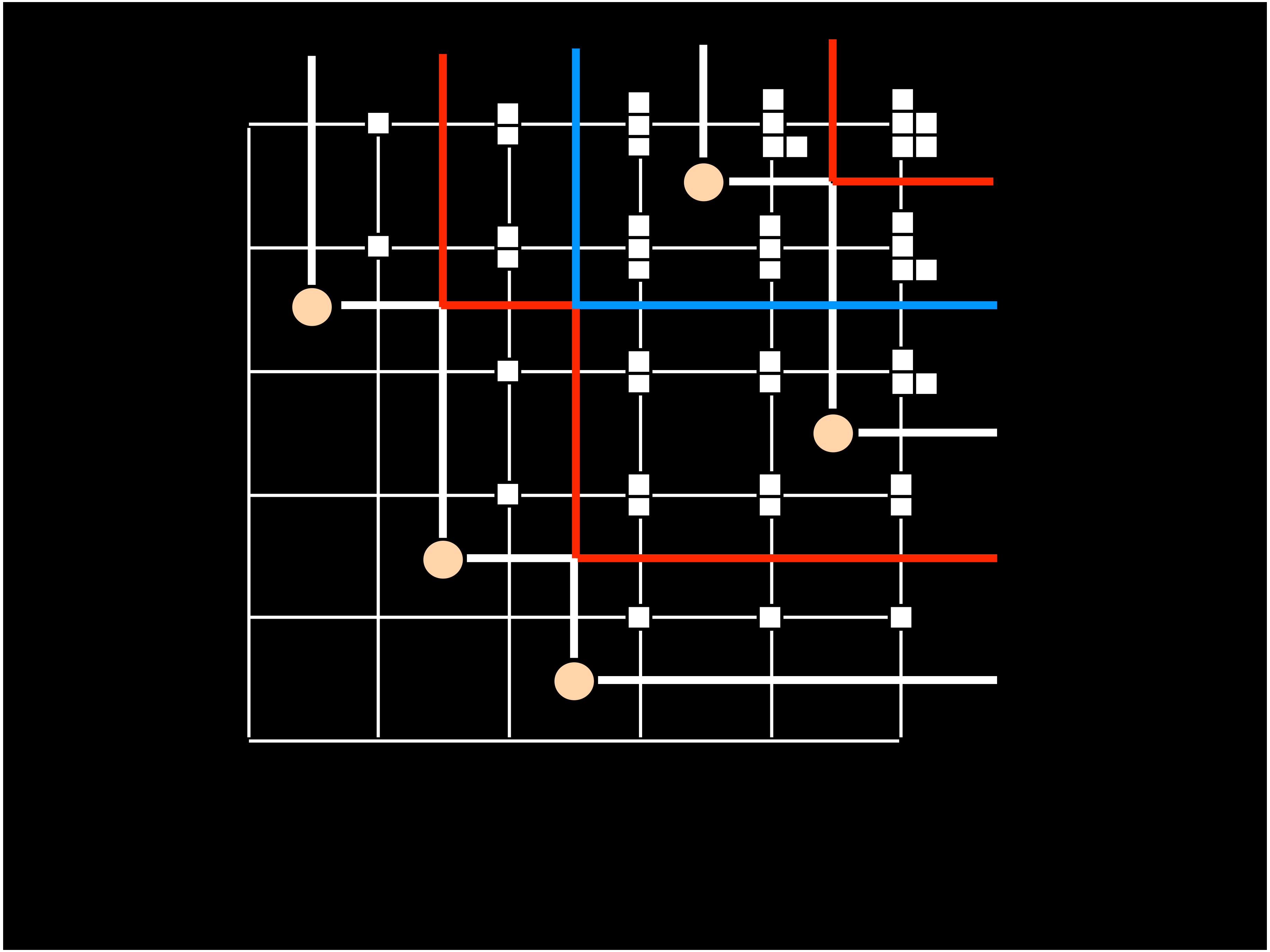
1 2 3 4 5 6 7 8 9 10
10
9
8
7
6
5
4
3
2
1
σ = 3 1 6 10 2 5 8 4 9 7

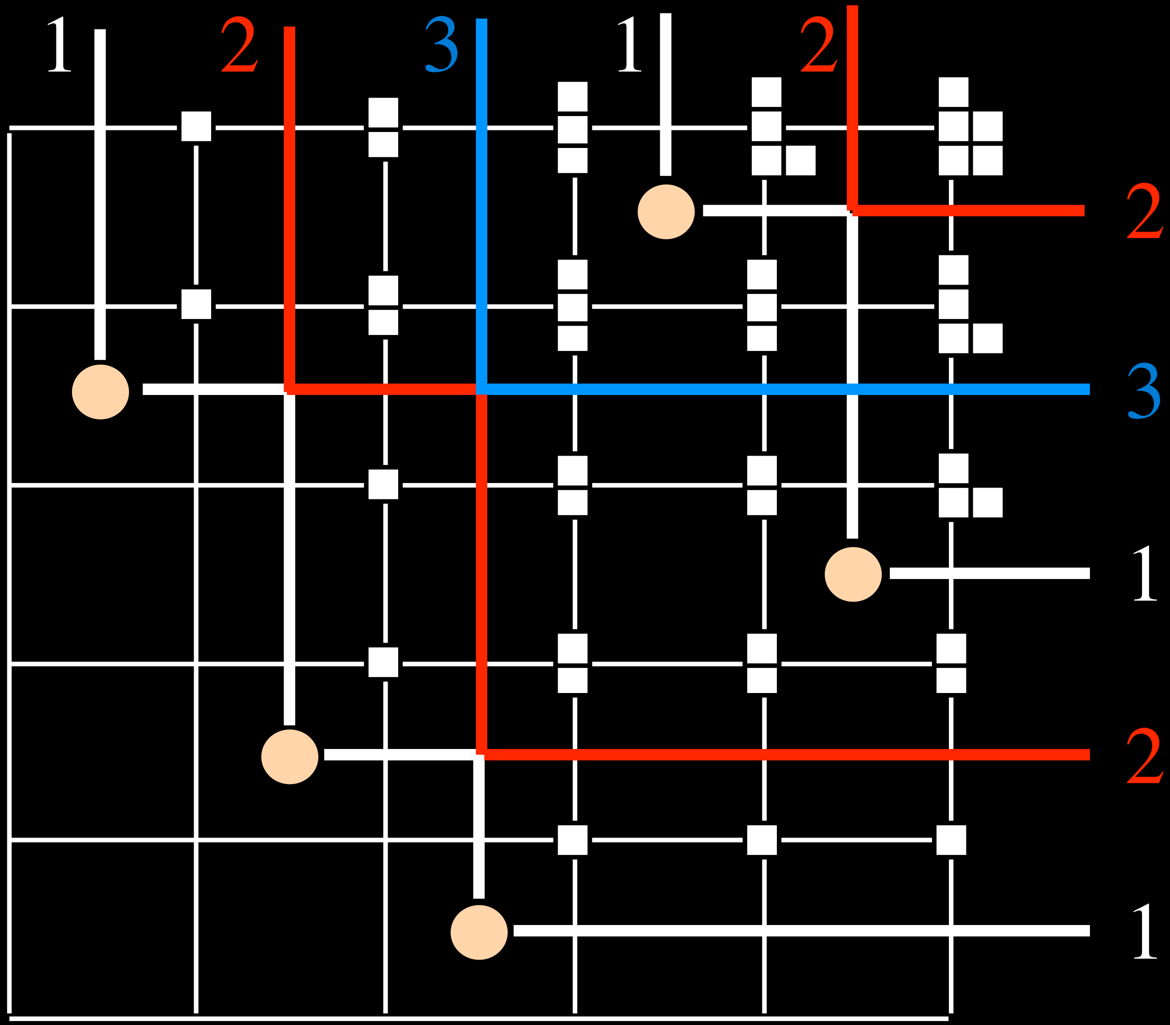


4 2 1 5 3

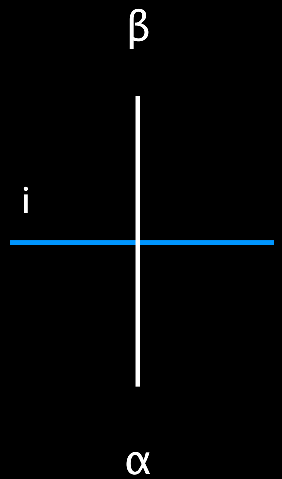




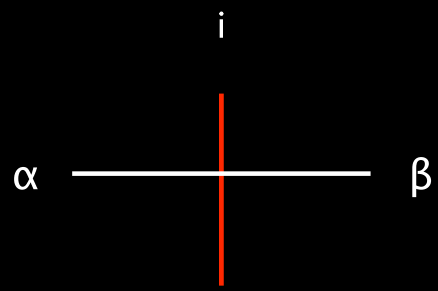


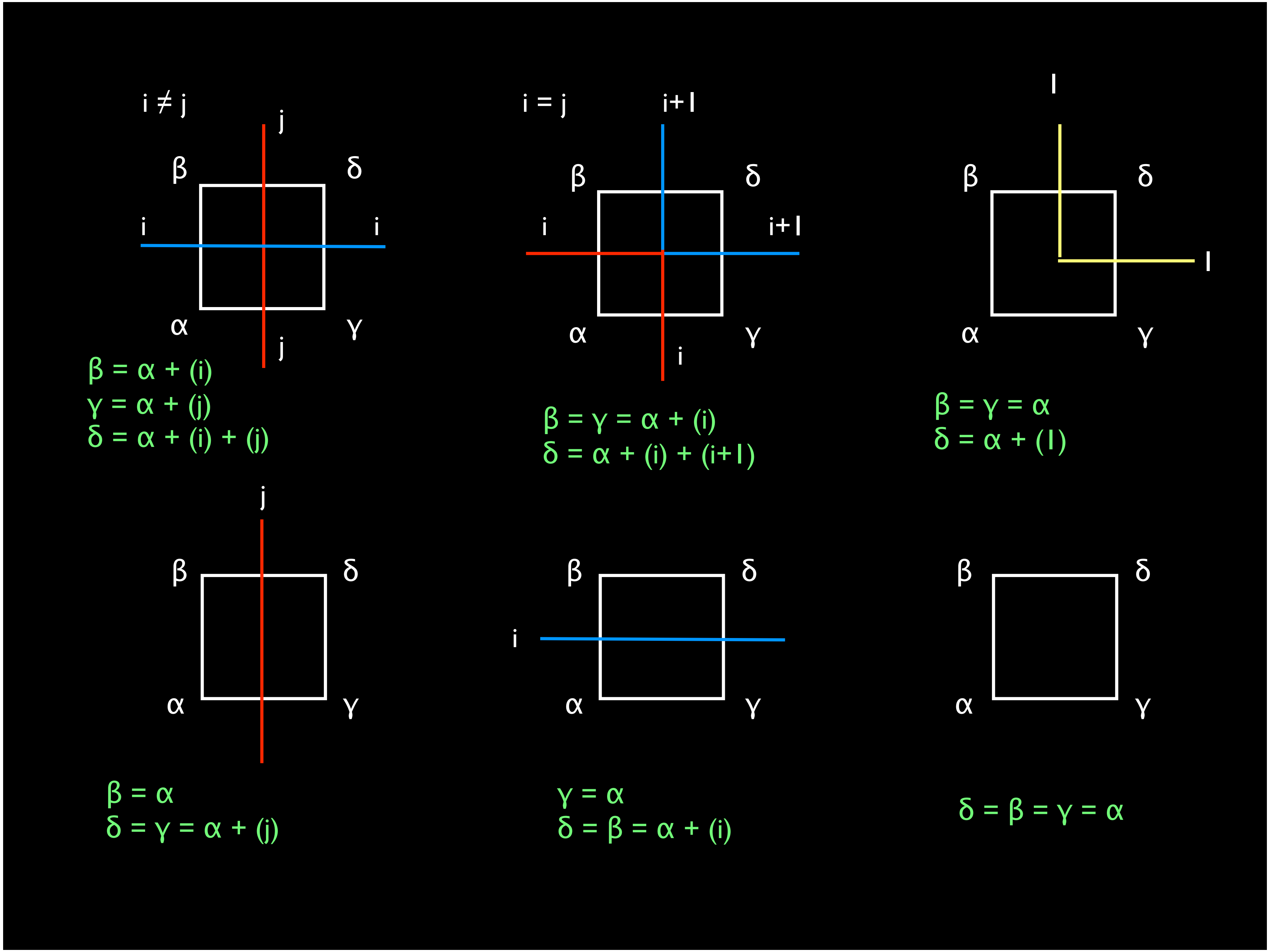


1
2
3
1
2
2
3
1
2
1

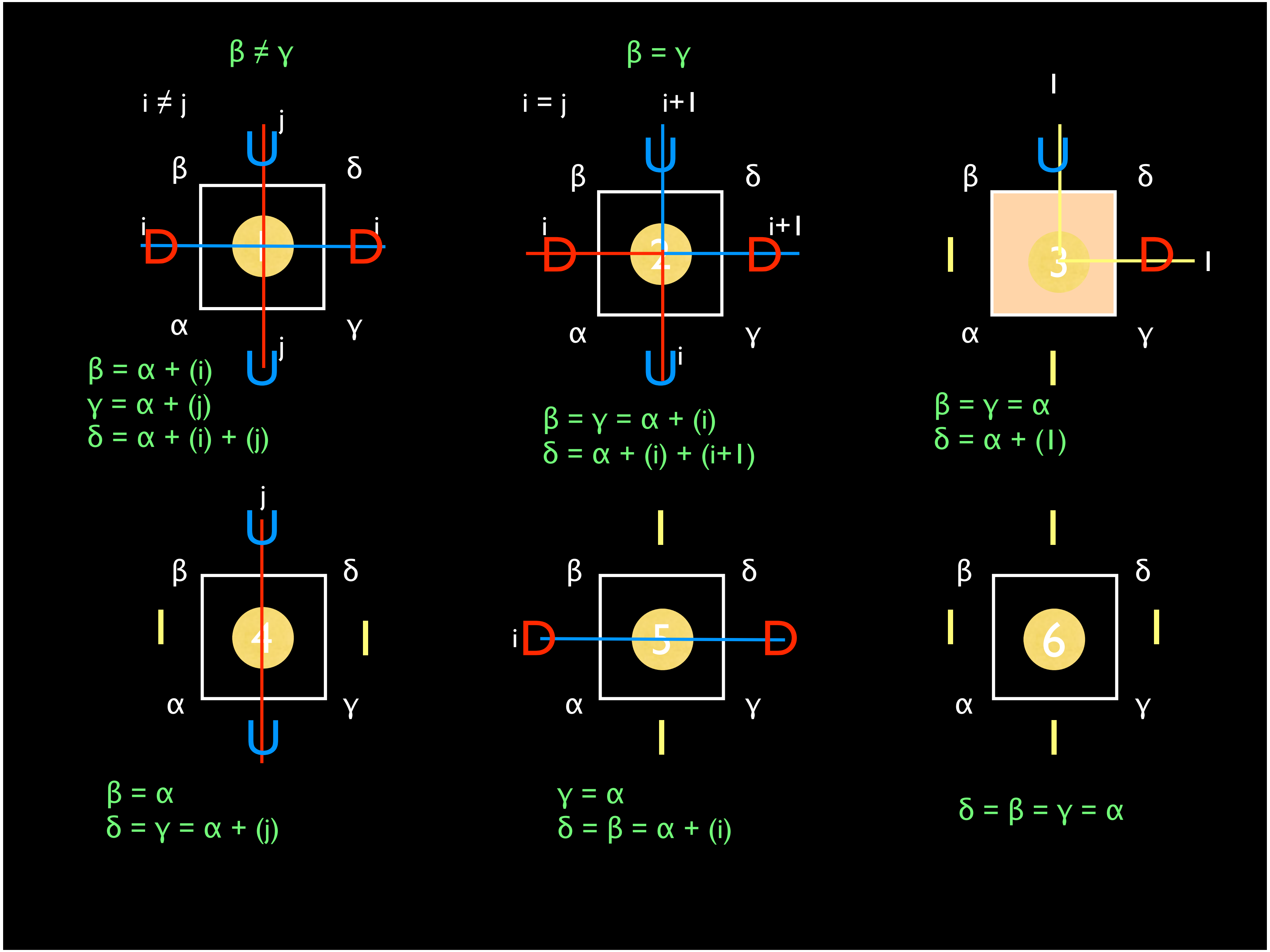


$\beta = \alpha + (i)$

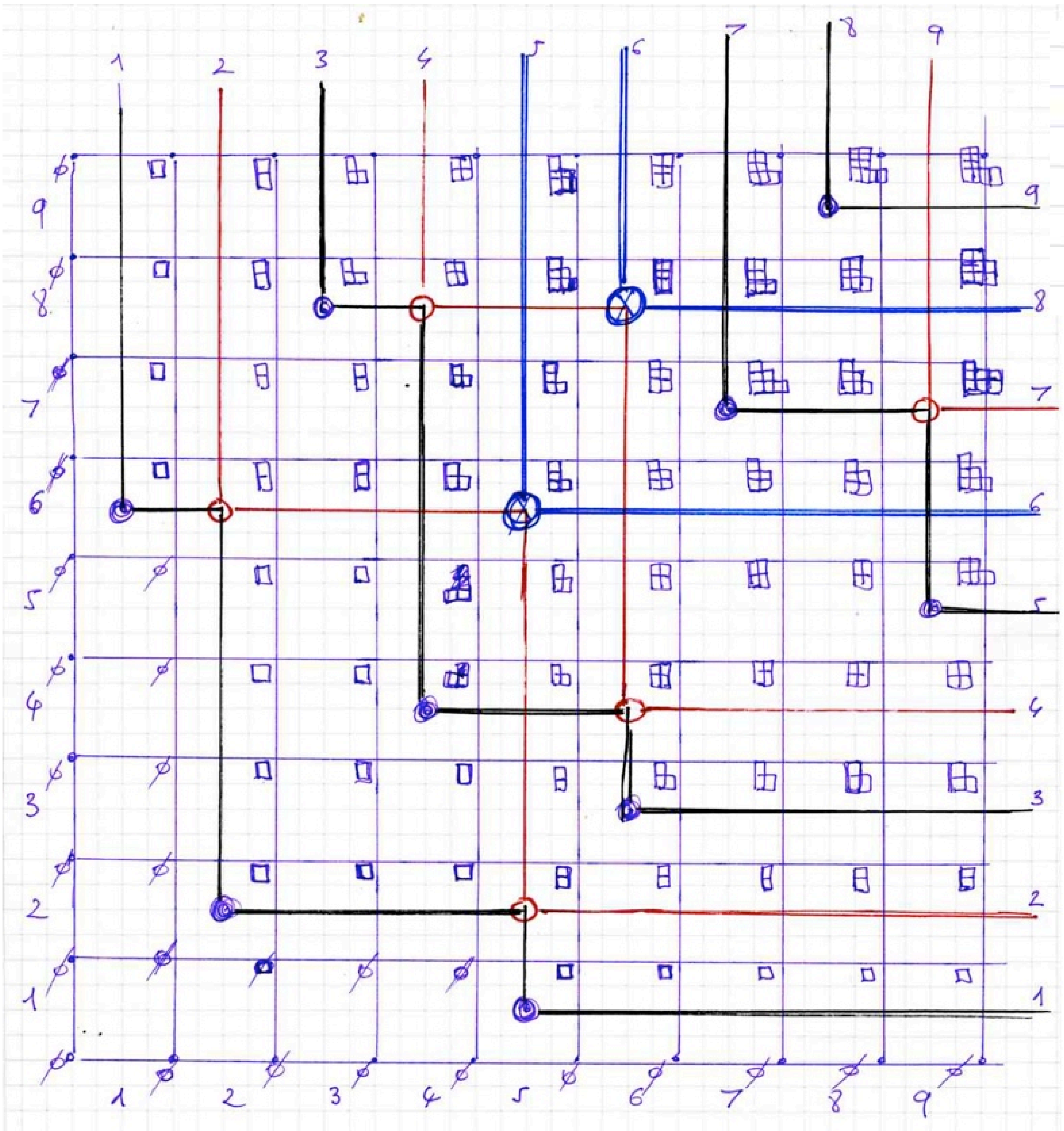




i ≠ j
j
β
δ
i
i
α
γ
j
β = α + (i)
γ = α + (j)
δ = α + (i) + (j)
i = j
i+l
β
δ
i
i+l
α
γ
i
β = γ = α + (i)
δ = α + (i) + (i+l)
l
β
δ
l
α
γ
β = γ = α
δ = α + (l)
j
β
δ
α
γ
β = α
δ = γ = α + (j)
β
δ
i
α
γ
γ = α
δ = β = α + (i)
β
δ
α
γ
δ = β = γ = α



β ≠ γ
i ≠ j
β = α + (i)
γ = α + (j)
δ = α + (i) + (j)
β = γ
i = j
i+1
β = γ = α + (i)
δ = α + (i) + (i+1)
β = γ = α
δ = α + (1)
β = α
δ = γ = α + (j)
γ = α
δ = β = α + (i)
δ = β = γ = α



5	6		
2	4	9	
1	3	7	8

Q

another example with

6 2 8 4 1 3 7 9 5

6	8		
2	4	7	
1	3	5	9

P

aurore boréale Nunavik xgv